Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 11(6): 18845–18868.
DOI:10.3934/math.2026767
Received: 20 April 2026
Revised: 10 June 2026
Accepted: 15 June 2026
Published: 26 June 2026

Research article

Unified stability criteria for impulsive switched T-S fuzzy systems with time delays under the bounded admissible edge-dependent average dwell time

Suzhen Ran*

School of Foundation Studies, Zhejiang Pharmaceutical University, Ningbo 315500, China

* **Correspondence:** Email: ransz@zjpu.edu.cn.

Abstract: This paper is concerned with the input-to-state stability (ISS) problem for a class of impulsive switched nonlinear systems with time delays. By utilising the Takagi-Sugeno (T-S) fuzzy method, the subsystems and impulses are designed for different fuzzy rules and membership functions, which can increase the degree of freedom and flexibility of designing. Moreover, based on admissible edge-dependent average dwell time (AED-ADT) and mode-dependent bounded maximum average dwell time (MD-BMADT), we establish a unified bounded admissible edge-dependent average dwell time (BAED-ADT) criterion for nonlinear systems. Then, the ISS property is also derived for impulsive switched T-S fuzzy systems when the system is with all stable, some unstable, and all unstable subsystems. Meanwhile, the slow and fast switching methods are used for stable or unstable subsystems. By employing the new BAED-ADT method and admissible edge-dependent average impulsive interval (AED-AII) technique, some new sufficient criteria of a "unified" ISS for impulsive switched systems are derived by constructing the time-scheduled multiple Lyapunov functions. Finally, several illustrative examples are provided to illustrate our results.

Keywords: input-to-state stability (ISS); continuous-time impulsive switched system; Takagi-Sugeno fuzzy models; bounded admissible edge-dependent average dwell time (BAED-ADT); time-scheduled multiple Lyapunov functions
Mathematics Subject Classification: 93C30, 93C42

1. Introduction

As a type of hybrid systems, switched systems are composed of a set of continuous subsystems and switching rules that control switching between subsystems [1]. During the past few years, the switched models have been extensively investigated in physical or artificial systems, such as power converters, electrical engineering, flight control systems, computer science, network control systems, etc. [2–4]. Impulsive systems [5,6] are another special class of hybrid systems, in which the system are disturbed or failed, and the state of the system will suddenly jump at some certain moments. Overall consideration results in our present model: switched impulsive system. Up to now, there exist quite a few related works regarding this hybrid systems [7, 8].

In most existing results [3, 4], the authors concentrated on the linear switched systems, but they ignored the impulsive signals. Until now, because of the complexity of nonlinear systems, there exist few investigations on nonlinear impulsive switched systems. The Takagi-Sugeno (T-S) fuzzy model, which was proposed in [9], can approximate nonlinear function [10–12]. The T-S fuzzy model can analyze and synthesize the nonlinear systems and has been proved to be significant [9–11]. In recent years, many works on the T-S fuzzy model have been reported in the literature [12–14]. In [15, 16], the authors used the fuzzy systems to approximate the unknown functions. In addition, stability analysis is invariably one of the primary concerns in the domain of T-S fuzzy systems [17, 18]. However, a drawback in the above mentioned results is that they all designed the same fuzzy rules and membership functions for plants and impulses. Here, we raise a hypothesis: Is it possible to make the plants and impulses use different fuzzy rules and membership functions? If so, what difficulties will researchers encounter in the process? Can this approach further reduce the conservativeness of the article? As far as we know, little attention has been paid to the investigation on T-S fuzzy systems with the above assumption, which motivates our research.

Due to various reasons, the information usually can be slow in the transmission process, that is time delays [19, 20]. Meanwhile, time delays are the unavoidable phenomenon in many practical systems, which may trigger instability or bad system performance [21, 22]. For instance, in networked control systems, when modeling epidemic spreading processes, industrial processes systems, aircraft control systems, environmental systems and so on, there usually exist time delays, which can reduce the stability and performance. In [23], the time-varying delay was proposed to prove the maximum upper bound for linear systems. For time-delayed control systems [24, 25], some new stability analyses of systems with time-varying delays were obtained, which ensures the stability performance of the system under the given constraints. Therefore, the examination of nonlinear systems with time delays is necessary [26].

Besides, the impulsive signal will also affect the performance of the system. In [27], the authors proposed the average impulsive interval (AII) method to deal with the exponential stability of stochastic impulsive systems. Then, a new method called mode-dependent average impulsive interval (MD-AII) was adopted for switched neural networks in [28]. However, the AII and MD-AII methods are only affect current activated subsystem, but have no affect on the front activated subsystems. In order to reflect the relationship between current and front activated subsystems, a more general conception called the admissible edge-dependent average impulsive interval (AED-AII) was proposed in [29]. In the literature [30], the authors assumed that the switchings and impulses occur asynchronously. Here, we consider a more general case: The switchings and impulses occur asynchronously. The above mentioned synchronous phenomena can be considered as a special case of this paper.

It is worth noting that some existing research about switched systems [31, 32], required that there exists at least one stable subsystem. The main idea is that the activated stable subsystem in a sufficiently large time can compensate for the state dispersion, and the dwell time of each subsystem is $\tau(> 0)$, where τ is defined as average dwell time (ADT). In [33], by using the ADT method, the stability of the switched system was obtained. Then, a relaxed method named the bounded maximum average dwell time (BMADT) switching was proposed in [34], which widened the bound of the ADT in [35].

To improve the fast switching approach, the mode-dependent bounded maximum average dwell time (MD-BMADT) method was further proposed in [36]. However, the ADT and MD-BMADT switching methods have some conservativeness because the relationship between subsystems cannot be fully displayed. Therefore, based on the above definitions, a more precise concept called admissible edge-dependent average dwell time (AED-ADT) was proposed in [37]. Meanwhile, other advanced control strategies have also been developed to further improve transient performance and robustness. Sliding mode control with variable convergence rate for nonlinear impulsive stochastic systems was studied in [38]; asynchronously switched control with variable convergence rate under a persistent dwell-time scheme was investigated in [39]; and region stability/stabilization and H_∞ control for discrete-time impulsive T-S fuzzy systems was developed in [40]. Due to the admissible transition edge weights of the AED-ADT that rely on the corresponding admissible transition edge, so it is more flexible than other definitions. Then a question arises about whether the MD-BMADT and AED-ADT can be further improved so that the dwell time is related to the current and previous instants and the bound of dwell time can be further relaxed. This gives rise to the study of this paper.

Motivated by the above discussion, the main purpose of this work is to investigate the stability of the T-S fuzzy impulsive switched system with time delays. The main contributions of this work are summarised as follows:

1) It is the first attempt to introduce the scheme, in which the fuzzy rules and membership functions are different for the controlled object and impulse. Compared with the approach that used the same fuzzy rules and membership for controlled object and impulses in [13], the new scheme in this paper greatly increases the degree of freedom and the flexibility of the design and reduces the conservativeness.

2) This paper gives a novel definition of the bounded admissible edge-dependent average dwell time (BAED-ADT), which is an improvement based on the MD-BMADT approach. Compared with the MD-BMADT in [36], the admissible transition edge-dependent weights of the BAED-ADT depend on the corresponding admissible transition edge, while the MD-BMADT only requires that the average time of each mode is greater than τ. Besides, the lower bound of the dwell time can be set according to the actual switching signal through the new method, which is more flexible than the AED-ADT in [29].

3) By constructing a class of the time-scheduled Lyapunov functions, some unified stability conditions are obtained. Compared with the traditional function that is independent of time, the Lyapunov function in this paper can be designed to change over time, making the results more flexible and less conservative. Besides, different from the considered cases in [41], we also consider that all subsystems are unstable. Thus, we can obtain the unified stability condition for nonlinear impulsive switched systems.

1.1. Problem definition

As we all know, nonlinear systems have captured exceeding attention during the past few years [13–15]. Due to the complexity of nonlinear systems, so it is a challenge to examine nonlinear system. However, it is worth noting that the fuzzy method is a powerful tool to deal with the problems of nonlinear systems, and it has obtained speed developments [32]. When the model parameters of the controlled object are unknown or highly uncertain, and the classical control theory is not applicable to the them, the fuzzy method provides a feasible solution to the nonlinear problem.

The Takagi-Sugeno-Kang (TSK) method is one of the fuzzy methods [9], and the main idea of this method is to transform complex nonlinear problems into different small line segments. The nonlinear system is fitted by multiple linear systems, the input variables are deconstructed by the fuzzy algorithm, and the equations that represent the relationship between input and output are generated by fuzzy inference. Compared with other methods, the TSK approach can represent complex nonlinear systems only by using very little fuzzy information, and it also has a good nonlinear approximation ability. Thus, based on the many benefits of the TSK fuzzy approach, it is a good choice to study the nonlinear impulsive switched systems in this paper.

1.2. Article structure

The paper is organized as follows: In Section 2, we first introduce the T-S fuzzy model of the system and some basic definitions and notations. In Section 3, we get some unified criteria of input-to-state stability (ISS) for nonlinear impulsive switched systems with time delays. Several examples are also discussed to verify the efficiency and less conservativeness of the obtained results in Section 4. In Section 5, the paper is concluded.

2. Problem formulation and preliminaries

$\mathbb{Z}$ and $\mathbb{Z}^+$ denote the set of nonnegative integers and positive integers, respectively. $\mathbb{R}$ implies the set of real numbers, and $\mathbb{R}_{\geq t} := [t, +\infty)$; $\mathbb{R}^n$ means the n-dimensional real space equipped with the Euclidean norm denoted by $\|\cdot\|$; $\mathbb{R}^{n\times n}$ represents the real matrices with $n\times n$ dimensions. For $-\infty < a < b < \infty$, the notation $\mathcal{PC}([a,b];\mathbb{R}^n)$ expresses the functions from $[a,b]$ to $\mathbb{R}^n$. For $r = \max\{r_1, r_2\} > 0$, the norm of $\phi \in \mathcal{PC}([-r,0];\mathbb{R}^n)$ is denoted by $\|\phi\| = \sup_{-r\leq d\leq 0} |\phi(d)|$. A continuous function $\alpha : [0,a) \to [0,\infty)$ belongs to class $\mathcal{K}$ if $\alpha(0) = 0$ and is strictly increasing; $\alpha(\cdot)$ is class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. A function $\beta : [0,a) \times [0,\infty) \to [0,\infty)$ is class $\mathcal{KL}$ if $\beta(r,s)$ is of class $\mathcal{K}$ for each fixed s and decreases to zero as $s \to \infty$ for each fixed r.

The following continuous-time impulsive switched nonlinear system is researched:

$$\begin{cases} \dot{x}(t) = f_i(t, x_t, \omega(t)), t \in [t_k, t_{k+1}), \\ \quad i = \sigma(t),\ t \in \mathbb{R}_{\geq t_0} \setminus \mathcal{I}, k \in \mathbb{Z}, \\ \Delta x(t) = g_{ij}(t, x_{t^-}, \omega(t^-)),\ j = \delta(t),\ t \in \mathcal{I}, \\ x_{t_0} = \xi, \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the system state, $\dot{x}(t)$ denotes the right-hand derivative of $x(t)$, and $x_t = x(t+s), s \in [-r,0]$, $x_{t^-} = x(t+s), s \in [-r,0)$. $\omega(t) \in \mathcal{PC}(R^+; R^{n_x})$ denotes the input,and $\xi \in \mathcal{PC}([-r,0];\mathbb{R}^{n_x})$ is the initial data. The switching signal $\sigma(t)$ is a piecewise constant function of right continuous and takes values in the finite set $\aleph = \{1,2,...\}$. $\delta(t)$ is the impulsive signal, which is a piecewise constant function from the right of time. $\mathcal{I} = \{t_{0,1},...,t_{0,p},t_{1,1},...,t_{1,p},...\}$ and $\mathcal{S} = \{t_0,t_1,t_2,t_3,...\}$ are the impulsive time sequence and switching time sequence satisfying $t_0 < t_{0,1} < \cdots < t_{0,p} < t_1 < t_{1,1} < \cdots < t_{1,p} < \cdots$ to avoid accumulation points. The number of the impulses is finite, and the instants of impulses are random. $\mathcal{T} = \mathcal{I} \bigcup \mathcal{S}$. $f_i : \mathbb{R}^+ \times \mathcal{PC}([-r,0];\mathbb{R}^{n_x}) \times \mathbb{R}^{n_\omega} \to \mathbb{R}^{n_x}$, and $g_{ij} : \mathbb{R}^+ \times \mathcal{PC}([-r,0);\mathbb{R}^{n_x}) \times \mathbb{R}^{n_\omega} \to \mathbb{R}^{n_x}$. We suppose $f_i(t,0,0) \equiv 0, g_{ij}(t,0,0) \equiv 0$. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ indicates the state jump at t_k caused by impulses in which $x(t_k^-) = \lim_{t\to t_k^-} x(t)$ and $x(t_k^+) = \lim_{t\to t_k^+} x(t)$.

In this paper, we assume $\aleph = \aleph_s \bigcup \aleph_u = \aleph_b \bigcup \aleph_c = \aleph_{ss} \bigcup \aleph_{uu}$, where $\aleph_s$, $\aleph_u$, $\aleph_b$, $\aleph_c$, $\aleph_{ss}$ and $\aleph_{uu}$ denote the sets of the stable subsystems, unstable subsystems, slow switching approach, fast switching approach, switching stable subsystems, and switching unstable subsystems, respectively. The impulsive switched nonlinear system (2.1) can be described by fuzzy rules [9], and the fuzzy rule p for ith subsystem is represented as follows.

Plant rule p: If $z_{i1}(t)$ is M_{ip1}, $z_{i2}(t)$ is M_{ip2},..., and $z_{il}(t)$ is M_{ipl}, then,

$$\dot{x}(t) = A_{ip}x(t) + B_{ip}x(t-r) + C_{ip}\omega(t),\ i \in \aleph,\ p \in \{1, 2, ..., m\}, \tag{2.2}$$

where M_{ipl} is the fuzzy sets, m is the number of fuzzy rules, $z_{i1}, z_{i2}, ..., z_{il}$ are the premise variable vectors, Marked for review in case this could be rearranged as "Aip, Bip, Cip are known constant matrices". $A_{ip}, B_{ip} \in \mathbb{R}^{n\times n}$ and $C_{ip} \in \mathbb{R}^{n\times m}$ are known constant matrices.

To increase the design flexibly and decrease the conservatism, the impulse of the ith subsystem may own a different membership function.

Impulse rule ν: If $\varsigma_{ij1}(t)$ is $N_{ij\nu1}$, $\varsigma_{ij2}(t)$ is $N_{ij\nu2}$,..., and $\varsigma_{ij\theta}(t)$ is $N_{ij\nu\theta}$, then

$$\Delta x(t) = D_{ij\nu}x(t-r) + E_{ij\nu}\omega(t),\ t \in \mathcal{I},\ \nu \in \{1, 2, ..., m\},$$

where $N_{ij\nu\theta}$ is the fuzzy sets, m is the number of fuzzy rules, $\varsigma_{ij1}, \varsigma_{ij2}, ..., \varsigma_{ij\theta}$ are the premise variable vectors, and $D_{ij\nu}, E_{ij\nu} \in \mathbb{R}^{n\times n}$ are known constant matrices.

Utilising fuzzy blending, one can obtain the ith subsystem as

$$\begin{cases} \dot{x}(t) = \sum\limits_{p=1}^{m} h_{ip}(z_i(t))[A_{ip}x(t) + B_{ip}x(t-r) + C_{ip}\omega(t)], \\ \Delta x(t) = \sum\limits_{\nu=1}^{m} h_{ij\nu}(\varsigma_{ij}(t))[D_{ij\nu}x(t-r) + E_{ij\nu}\omega(t)], \end{cases} \tag{2.3}$$

where $z_i(t) = [z_{i1}(t), z_{i2}(t), \cdots, z_{il}(t)]^T$, $h_{ip}(z_i(t)) = \prod\limits_{k=1}^{l} M_{ipk}(z_{ik}(t))/\sum\limits_{p=1}^{m}\prod\limits_{k=1}^{l} M_{ipk}(z_{ik}(t)), \varsigma_{ij}(t) = [\varsigma_{ij1}(t),\ \varsigma_{ij2}(t), \cdots, \varsigma_{ij\theta}(t)]^T$ and $h_{ij\nu}(\varsigma_{ij}(t)) = \prod\limits_{q=1}^{\theta} N_{ij\nu q}(\varsigma_{ijq}(t))/\ \sum\limits_{\nu=1}^{m}\prod\limits_{q=1}^{\theta} N_{ij\nu q}(\varsigma_{ijq}(t))$ are the normalised membership functions along with $h_{ip}(z_i(t)) \geq 0$, $\sum\limits_{p=1}^{m} h_{ip}(z_i(t)) = 1$, and $h_{ij\nu}(\varsigma_{ij}(t)) \geq 0$, $\sum\limits_{\nu=1}^{m} h_{ij\nu}(\varsigma_{ij}(t)) = 1$. For the sake of simplicity, let's replace $h_{ip}(z_i(t)), h_{ij\nu}(\varsigma_{ij}(t))$ with $h_{ip}(t), h_{ij\nu}(t)$.

Remark 1. *In this paper, we choose the new approach to design the fuzzy system. In the existing results [13], the authors used the method where the controlled object and the impulses have the same fuzzy rules. The rules of impulses must be consistent with the fuzzy rules of the controlled object, where the rules contain the same premise variable vectors, fuzzy sets, fuzzy rule numbers, membership functions, etc. It diminishes the flexibility of impulse design and exists as somewhat conservative. However, the new method in this paper allows that the controlled object has different elements from the impulse, which increases the flexibility of the design and reduces the conservativeness.*

Definition 1. *[42] System* (2.1) *is said to be ISS under a certain class of impulsive and switching signals $\mathcal{T}$ if there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$, independent of the choice of impulsive and switching signals in $\mathcal{T}$, such that for each initial condition ξ and for every input function $\omega(t)$, the solution of* (2.1) *exists for all $t \geq 0$ and satisfies*

$$|x(t)| \leq \beta(|\xi|_r, t-t_0) + \gamma(\|\omega[t_0, t]\|).$$

Definition 2. *[37] In an oriented graph, $\wp$ $\epsilon(j,i)$ is denoted as an admissible transition edge of $\wp$ for $(j,i) \in \aleph \times \aleph$ $(j \neq i)$, and an oriented edge from mode j to mode i is acceptable, i.e., $\varepsilon(j,i)$ exists. Λ denotes the set of admissible transition edges. $\mu_{j,i}$ and Γ express an admissible transition edge-dependent weight (ATEDW) and a set of ATEDWs, which depicts the switching character from the jth subsystem to the ith subsystem, respectively.*

Definition 3. *[29, 37]. For a switching signal $\sigma(t)$ and any $i, j \in \aleph$, $\epsilon(j,i) \in \Lambda$, $N_{\sigma(j,i)}(t_0, t)$ denotes the switching number from j to i over the range $[t_0, t)$, and $T_{(j,i)}(t_0, t)$ expresses the entire duration of subsystem i in the range $[t_0, t)$, where mode j is the previously active subsystem and $t \geq t_0 \geq 0$. We think $\tau_{s(j,i)}$ is a slow admissible edge-dependent average dwell time (SAED-ADT) of $\sigma(t)$ if there are positive numbers $N^0_{\sigma(j,i)}$ such that*

$$N_{\sigma(j,i)}(t_0, t) \leq N^0_{\sigma(j,i)} + \frac{T_{(j,i)}(t_0, t)}{\tau_{s(j,i)}}, \ \forall t \geq t_0 \geq 0,$$

where $N^0_{\sigma(j,i)}$ is called as the slow admissible edge-dependent chatter bound.

Based on the definition of [34], we propose a modified definition as follows.

Definition 4. *For a given switching signal $\sigma(t)$ at the switching times t_k, $i \in \aleph_c$, $j \in \aleph$, $\theta(j,i) \in \mathbb{Z}$. For any $t \in [t_k, t_{k+1})$, we have*

$$T_{\theta(j,i)}(t, t_0) = \sum_{i=1}^{k} T_{\theta(j,i)}(t_i, t_{i-1}) + T_{\theta(j,i)}(t, t_{k+1}),$$

where

$$T_{\theta(j,i)}(t, t_i) = \begin{cases} 0, \ t \leq t_i + \theta(j,i), \\ t - t_i - \theta(j,i), \ t > t_i + \theta(j,i), \end{cases}$$

$T_{\theta(j,i)}(t_i, t_{i-1})$ is the divergence time, and $\theta(j,i) \geq 0$ is the compensation bound.

Definition 5. *For a fast switching signal $\sigma(t) \in \aleph_c$, $t \geq t_0 \geq 0$, $i, j \in \aleph$ and $\epsilon(j,i) \in \Lambda$, $N_{\sigma(j,i)}(t_0, t)$ denotes the switching number from mode j to mode i over the range $[t_0, t)$, and $T_{\theta(j,i)}(t_0, t)$ expresses the entire duration of subsystem i in the range $[t_0, t)$, where mode j is the previously active subsystem. If there exists $\theta(j,i) \in \mathbb{Z}$, positive numbers $N^0_{\sigma(j,i)}$, $\tau_{f(j,i)}$, and $T^0_{(j,i)}$ such that*

$$N_{\sigma(j,i)}(t_0, t) \geq \frac{T_{(j,i)}(t_0, t)}{\tau_{f(j,i)} + \theta(j,i)} - N^0_{\sigma(j,i)}, \forall t \geq t_0 \geq 0,$$

$$T_{\theta(j,i)} \leq \frac{\tau_{f(j,i)} T_{(j,i)}(t_0, t)}{\tau_{f(j,i)} + \theta(j,i)} + T^0_{(j,i)},$$

where $N^0_{\sigma(j,i)}$ is called as the fast admissible edge-dependent chatter bound and $\tau_{f(j,i)}$ is named the fast switching BAED-ADT.

Remark 2. *The proposed BAED-ADT reduces to the MD-BMADT when $\theta(j,i) = 0$ and impulses are absent, reduces to the AED-ADT when only slow switching is used, and reduces to the standard ADT when edge-dependence and impulses are removed.*

Definition 6. *[29] Consider a hybrid system with impulsive and switched signals. For a switching signal $\sigma(t)$ and any $j, i \in \aleph$, $\epsilon(j,i) \in \Lambda$, $N_{\delta(j,i)}(t_0, t)$ denotes the impulse number from mode j to mode i over the range $[t_0, t)$, and $T_{(j,i)}(t_0, t)$ expresses the entire duration of subsystem i in the range $[t_0, t)$, where mode j is the previously active subsystem, $t \geq t_0 \geq 0$. We think the ith subsystem has an AED-AII $\tau_{\delta(j,i)}$, if there is positive number $N^0_{\delta(j,i)}$ and $\tau_{\delta(j,i)}$ such that*

$$\frac{T_{(j,i)}(t_0, t)}{\tau_{\delta(j,i)}} - N^0_{\delta(j,i)} \leq N_{\delta(j,i)}(t_0, t),$$

and

$$N_{\delta(j,i)}(t_0, t) \leq \frac{T_{(j,i)}(t_0, t)}{\tau_{\delta(j,i)}} + N^0_{\delta(j,i)}, \ \forall t \geq t_0 \geq 0,$$

where $N^0_{(j,i)}$ is called as the admissible edge-dependent elasticity number.

3. Main results

First, we will prove the stability conditions for nonlinear impulsive switched systems and that the subsystems contain stable and unstable subsystems.

Lemma 1. *For the impulsive switched non-linear system, constants $r_1,\ r_2 > 0$; $\lambda_i < 0,\ i \in \aleph_s$; $\lambda_i > 0,\ i \in \aleph_u$; $\mu^{11}_{j,i} > 1,\ \mu^{12}_{j,i} > 0,\ \eta^1_{j,i} > 1, \tau^*_{s(i,b)} > 0,\ \xi_b = (\lambda_b \tau^*_{s(i,b)} + \ln \widetilde{\mu}_{i,b})/\tau^*_{s(i,b)} + \ln \rho_{j,i}/\tau^*_{\delta(i,b)},\ \widetilde{\mu}_{j,i} = \max\{\mu^{11}_{j,i}, \eta^1_{j,i}\} + \mu^{12}_{j,i},\ 0 < \rho^{11}_{j,i} < 1,\ \rho^{12}_{j,i} > 0,\ k_{j,i} > 0,\ i, j \in \aleph,\ b \in \aleph_b$; $0 < \mu^{21}_{j,i} < 1,\ \mu^{22}_{j,i} > 0,\ \eta^2_{j,i} > 0,\ \tau^*_{f\theta(i,c)} > 0,\ \xi_c = (\lambda_c \tau^*_{f\theta(i,c)} + \ln \overline{\mu}_{i,c})/(\tau^*_{f\theta_c(i,c)} + \theta_c) + \ln \rho_{j,i}/\tau^*_{\delta(i,c)},\ \theta_c > 0,\ \overline{\mu}_{j,i} = \max\{\mu^{21}_{j,i}, \eta^2_{j,i}\} + \mu^{22}_{j,i},\ \rho^{21}_{j,i} > 1,\ \rho^{22}_{j,i} > 0,\ i, j \in \aleph,\ c \in \aleph_c$,;and if there exists functions $V_{\sigma(t)}(x(t)) : \mathbb{R}^n \to \mathbb{R},\ \alpha_1,\ \alpha_2,\ \alpha_3 \in \mathcal{K}_\infty$ such that*

$$\begin{aligned} &\alpha_1(\|\phi(0)\|) \leq V^1_i(t, \phi(0)) \leq \alpha_2(\|\phi(0)\|), \\ 0 \leq &V^2_i(t, \phi) \leq \alpha_3(\|\phi\|_r),\ i \in \aleph,\ r = \max\{r_1, r_2\}, \end{aligned} \tag{3.1}$$

$$D^+ V_i(t, \phi) \leq \lambda_i V_i(t, \phi) + \varphi(\|\omega(t)\|), \ \begin{cases} t \in [t_k, t_{k+1}),\ i \in \aleph_b, \\ t \in (t_k + \theta_i, t_{k+1}),\ i \in \aleph_c, \end{cases} \tag{3.2}$$

$$\begin{aligned} V^1_i(t_k, \phi(0)) &\leq \mu^{11}_{j,i} V^1_j(t_k, \phi(0)) + \varphi(\|\omega(t_k)\|) + \mu^{12}_{j,i} \sup_{d \in [-r_1, 0]} \{V^1_j(t_k + d, \phi(d)\}, \\ V^2_i(t_k, \phi) &\leq \eta^1_{j,i} V^2_j(t_k, \phi),\ i \in \aleph_b,\ j \in \aleph, \end{aligned} \tag{3.3}$$

$$\begin{aligned} V^1_i(t_k, \phi(0)) &\leq \mu^{21}_{j,i} V^1_j(t_k, \phi(0)) + \varphi(\|\omega(t_k)\|) + \mu^{22}_{j,i} \sup_{d \in [-r_1, 0]} \{V^1_j(t_k + d, \phi(d)\}, \\ V^2_i(t_k, \phi) &\leq \eta^2_{j,i} V^2_j(t_k, \phi),\ i \in \aleph_c,\ j \in \aleph, \end{aligned} \tag{3.4}$$

$$V^1_i(t, \phi(0) + g_{\sigma(t)}(t, \phi, \omega(t^-)) \leq \rho^{11}_{j,i} V^1_j(t^-, \phi(0)) + \rho^{12}_{j,i} \sup_{d \in [-r_2, 0]} \{V^1_j(t^- + d), \phi(d)\} + \varphi(\|\omega(t^-)\|), \tag{3.5}$$

$$V_i^2(t,\phi) \le k_{j,i} \sup_{d\in[-r_2,0]} \{V_j^1(t+d,\phi(d))\},\ i,j\in\aleph,$$

$$V_i^1(t,\phi(0)+g_{\sigma(t)}(t,\phi,\omega(t^-)) \le \rho_{j,i}^{21} V_j^1(t^-,\phi(0)) + \rho_{j,i}^{22} \sup_{d\in[-r_2,0]} \{V_j^1(t^-+d),\phi(d)\} + \varphi(\|\omega(t^-)\|), \tag{3.6}$$
$$V_i^2(t,\phi) \le \rho_{j,i}^{21} V_j^2(t,\phi),\ i,j\in\aleph,$$

then the system (2.1) is ISS if the switching and impulsive signals satisfy

$$\begin{cases} T^+/T^- < (\xi^* - \xi^-)/(\xi^+ - \xi^*),\ \xi^- \le \xi^* < 0, \\ \tau_{s(i,b)} \ge \tau^*_{s(i,b)},\ b\in\aleph_b, \\ \tau_{f\theta(i,c)} \le \tau^*_{f\theta(i,c)},\ c\in\aleph_c, \\ \tau_{\delta(j,m)} \ge \tau^*_{\delta(j,m)} = \ln\rho_{j,i}/[\xi_b - (\lambda_b\tau^*_{s(i,b)} + \ln\widetilde{\mu}_{i,b})/\tau^*_{s(i,b)}],\ j\in\mathcal{I},\ m\in\mathcal{I}_s,\ i\in\aleph,\ b\in\aleph_b, \\ \tau_{\delta(j,m)} \le \tau^*_{\delta(j,m)} = \ln\rho_{j,i}/[\xi_c - (\lambda_c\tau^*_{f\theta(i,c)} + \ln\overline{\mu}_{i,c})(\tau^*_{f\theta(i,c)} + \theta_i)],\ j\in\mathcal{I},\ m\in\mathcal{I}_u,\ i\in\aleph,\ c\in\aleph_c, \end{cases} \tag{3.7}$$

where $\xi^- = \max\limits_{i\in\aleph_{ss}}\{\xi_i\}$, $\xi^+ = \max\limits_{i\in\aleph_{uu}}\{\xi_i\}$ *and* T^- *and* T^+ *are respectively called the corresponding total running time of switching stable subsystems and switching unstable subsystems, in which* $T^- = \sum\limits_{i\in\aleph_{ss}} T_i(t,t_0)$, $\aleph_{ss} = \{i|\xi_i < 0, i\in\aleph\}$ *and* $T^+ = \sum\limits_{i\in\aleph_{uu}} T_i(t,t_0)$, $\aleph_{uu} = \{i|\xi_i > 0, i\in\aleph\}$.

Proof. First, for any $i,j\in\aleph,\ t\in[t_k,t_{k+1}),\ k\in\mathbb{Z}$, in view of (3.2), we can obtain

$$V_i(t,\phi) \le e^{\lambda_i(t-t_k)} V_i(x(t_k)) + \int_{t_k}^{t} e^{\lambda_i(t-s)}\varphi(\|\omega(s)\|)ds. \tag{3.8}$$

For any $i\in\aleph_b,\ j\in\aleph$, according to (3.3), we have

$$V_i(t_k,\phi) \le \mu_{j,i}^{11} V_j^1(t_k,\phi(0)) + \eta_{j,i}^1 V_j^2(t_k,\phi) + \mu_{j,i}^{12} \sup_{d\in[-r_1,0]}\{V_j^1(t_k+d,\phi(d))\} + \varphi(\|\omega(t_k)\|) \tag{3.9}$$
$$\le \widetilde{\mu}_{j,i} \sup_{d\in[-r_1,0]}\{V_j(t_k+d,\phi)\},$$

where $\widetilde{\mu}_{j,i} = \max\{\mu_{j,i}^{11}, \eta_{j,i}^1\} + \mu_{j,i}^{12}$.

In the same way, for any $i\in\aleph_c,\ j\in\aleph$, based on (3.4) we have

$$V_i(t,\phi) \le \overline{\mu}_{j,i} \sup_{d\in[-r_1,0]}\{V_j(t_k+d,\phi)\}, \tag{3.10}$$

where $\overline{\mu}_{j,i} = \max\{\mu_{j,i}^{21}, \eta_{j,i}^2\} + \mu_{j,i}^{22}$.

When stable impulses occur between any switching points, in view of (3.5), we can get

$$V_i(t_{k,m+1}) \le \rho_{j,i}^{11} V_i^1(t_{k,m+1}^-,\phi(0)) + \rho_{j,i}^{12} \sup_{d\in[-r_2,0]}\{V_i^1(t_{k,m+1}^- + d,\phi(d))\} + V_i^2(t_{k,m+1},\phi) + \varphi(\|\omega(t_{k,m+1})\|) \tag{3.11}$$
$$\le \widetilde{\rho}_{j,i} \sup_{d\in[-r_2,0]}\{V_j(t_{k,m+1}^- + d,\phi)\} + \varphi(\|\omega(t_{k,m+1})\|),$$

where $\widetilde{\rho}_{j,i} = \rho_{j,i}^{11} + \max\{\rho_{j,i}^{12}, 1-\rho_{j,i}^{11}\}$.

If $\lambda_i < 0$, i.e., subsystems are stable, combining (3.8) with (3.11), one can conclude

$$V_i(t_{k,m+1}) \leq \widetilde{\rho}_{j,i}[e^{-\lambda_i r_2} e^{\lambda_i(t_{k,m+1}-t_{k,m})} V_i(t_{k,m}) + \int_{t_{k,m}}^{t_{k,m+1}-r_2} e^{\lambda_i(t_{k,m+1}-r_2-s)} \varphi(\|\omega(s)\|)ds] + \varphi(\|\omega(t)\|). \tag{3.12}$$

If $\lambda_i > 0$, i.e., subsystems are unstable, combining (3.8) with (3.11), one can get

$$V_i(t_{k,m+1}) \leq \varphi(\|\omega(t)\|) + \widetilde{\rho}_{j,i}[e^{\lambda_i(t_{k,m+1}-t_{k,m})} V_i(t_{k,m}) + \int_{t_{k,m}}^{t_{k,m+1}} e^{\lambda_i(t_{k,m+1}-s)} \varphi(\|\omega(s)\|)ds]. \tag{3.13}$$

When unstable impulses occur between any switching points, by using (3.6), one can obtain

$$V_i(t_{k,m+1}) \leq \varphi(\|\omega(t_{k,m+1})\|) + \overline{\rho}_{j,i} \sup_{d\in[-r_2,0]} \{V_j(t_{k,m+1}^- + d, \phi)\}, \tag{3.14}$$

where $\overline{\rho}_{j,i} = \rho_{j,i}^{21} + \rho_{j,i}^{22}$.

Similarly, combining with (3.14) and (3.8), when $\lambda_i < 0$, we have

$$V_i(t_{k,m+1}) \leq \overline{\rho}_{j,i}[e^{-\lambda_i r_2} e^{\lambda_i(t_{k,m+1}-t_{k,m})} V_i(t_{k,m}) + \varphi(\|\omega(t)\|) + \int_{t_{k,m}}^{t_{k,m+1}-r_2} e^{\lambda_i(t_{k,m+1}-r_2-s)} \varphi(\|\omega(s)\|)ds]. \tag{3.15}$$

And when $\lambda_i > 0$, we get

$$V_i(t_{k,m+1}) \leq \varphi(\|\omega(t)\|) + \overline{\rho}_{j,i}[e^{\lambda_i(t_{k,m+1}-t_{k,m})} V_i(t_{k,m}) + \int_{t_{k,m}}^{t_{k,m+1}} e^{\lambda_i(t_{k,m+1}-s)} \varphi(\|\omega(s)\|)ds]. \tag{3.16}$$

For any $t \in [t_k, t_{k+1}),\ i \in \aleph_b,\ \lambda_i < 0$, in view of (3.8), (3.12), and (3.15), one can get

$$\begin{aligned}
V_i(x(t_{k+1})) &\leq e^{\lambda_i(t_{k+1}-t_{k,p})} V_i(x(t_{k,p})) + \int_{t_{k,p}}^{t_{k+1}} e^{\lambda_i(t_{k+1}-s)} \varphi(\|\omega(s)\|)ds \\
&\leq e^{-\lambda_i r_2} e^{\lambda_i(t_{k+1}-t_{k,p})} \rho_{i,p-1} e^{\lambda_i(t_{k,p}-t_{k,p-1})} V_i(x(t_{k,p-1})) + \int_{t_{k,p}}^{t_{k+1}} e^{\lambda_i(t_{k+1}-s)} \varphi(\|\omega(s)\|)ds \\
&+ e^{\lambda_i(t_{k+1}-t_{k,p})} \varphi(\|\omega(t)\|) + e^{\lambda_i(t_{k+1}-t_{k,p})} \rho_{i,p-1} \int_{t_{k,p-1}}^{t_{k,p}-r_2} e^{\lambda_i(t_{k,p}-r_2-s)} \varphi(\|\omega(s)\|)ds.
\end{aligned} \tag{3.17}$$

Through the same recursion process, we can get

$$\begin{aligned}
V_i(x(t_{k+1})) &\leq e^{-p\lambda_i r_2} e^{\lambda_i(t_{k+1}-t_k)} \rho_i^p * \widetilde{\mu}_{j,i} \sup_{d\in[-r_1,0]} \{V_j(t_k + d, \phi)\} + \gamma_0 \overline{\varphi}(\|\omega(t)\|) \\
&+ \sum_{n=1}^{p} e^{\lambda_i(t_{k+1}-t_{k,p-m+1})} \rho_i^n \overline{\varphi}(\|\omega(t)\|) e^{-\frac{m(m-1)}{2}\lambda_i r_2} (1 + \gamma_0),
\end{aligned} \tag{3.18}$$

where $\rho_{i,p} = 1,\ \rho_i = \max\{\rho_{i,p-m}, 1\},\ m = 1, ..., p;\ \gamma_0 = \max\{-\frac{1}{\lambda_i}\},\ \lambda_i < 0,\ i = 0, 1, ...;$ $\overline{\gamma}_0 = \max\{\frac{1}{\lambda_i}\},\ \lambda_i > 0,\ i = 0, 1, ...;$ and $\overline{\varphi}(\|\omega(t)\|) = \sup_{t_0 \leq s \leq t} \varphi(\|\omega(s)\|)$.

In the same way, if $i \in \aleph_b,\ \lambda_i > 0$, we can get

$$V_i(x(t_{k+1}) \leq e^{\lambda_i(t_{k+1}-t_k)} \rho_i^p \widetilde{\mu}_{j,i} \sup_{d\in[-r_1,0]} \{V_j(t_k + d, \phi)\} + \gamma_0 e^{-\lambda_i(t_{i,p}-t_{k+1})} \overline{\varphi}(\|\omega(t)\|)$$

$$+\sum_{n=1}^{p} e^{\lambda_i(t_{k+1}-t_{k,p-m+1})}\rho_i^n\overline{\varphi}(\|\omega(t)\|)(1+\overline{\gamma}_0 e^{-\lambda_i(t_{n,p-m}-t_{n,p-n+1})}). \tag{3.19}$$

For any $t \in [t_k, t_{k+1}),\ i \in \aleph_c,\ \lambda_i > 0$, in view of (3.8), (3.13), and (3.16), one can get

$$\begin{aligned} V_i(x(t_{k+1})) \leq{}& e^{\lambda_i(t_{k+1}-t_k)}\rho_i^p\overline{\mu}_{j,i}\sup_{d\in[-r_1,0]}\{V_j(t_k+d,\phi)\} + \overline{\gamma}_0 e^{-\lambda_i(t_{k,p}-t_{k+1})}\overline{\varphi}(\|\omega(t)\|) \\ &+\sum_{n=1}^{p} e^{\lambda_i(t_{k+1}-t_{k,p-m+1})} * \rho_i^n\overline{\varphi}(\|\omega(t)\|)(1+\overline{\gamma}_0 e^{-\lambda_i(t_{n,p-m}-t_{n,p-n+1})}). \end{aligned} \tag{3.20}$$

Thus, without loss of generality, for any $t \in [t_0, t)$, combining with (3.18)–(3.20), it is derived that

$$\begin{aligned} V(x(t)) \leq{}& e^{-p\lambda_k r_2} e^{\lambda_k(t_{k+1}-t_k)}\rho_k^p\widetilde{\mu}_{k-1,k} * \sup_{d\in[-r_1,0]}\{V_{k-1}(t_k+d,\phi)\} + \gamma_0\overline{\varphi}(\|\omega(t)\|) \\ &+\sum_{n=1}^{p} e^{\lambda_k(t_{k+1}-t_{k,p-n+1})}\rho_k^p\overline{\varphi}(\|\omega(t)\|)e^{-\frac{n(n-1)}{2}\lambda_k r_2}(1+\gamma_0) \\ \leq{}& e^{-p\lambda_k r_2} e^{\lambda_k(t_{k+1}-t_k)}\rho_k^p\widetilde{\mu}_{k-1,k}\sup_{d\in[-r_1,0]}\{e^{\lambda_{k-1}(t_k+d-t_{k-1})}\rho_{k-1}^p \\ &*\overline{\mu}_{k-2,k-1}\sup_{d\in[-r_1,0]}\{V_{k-2}(t_{k-1}+d,\phi)\}\sum_{n=1}^{p} e^{\lambda_{k-1}(t_k+d-t_{k-1,p-n+1})} \\ &*\rho_{k-1}^n\overline{\varphi}(\|\omega(t)\|)(1+\overline{\gamma}_0 e^{-\lambda_{k-1}(t_{n,p-n}-t_{n,p-n+1})}) \\ &+\overline{\gamma}_0 e^{-\lambda_{k-1}(t_{k-1,p}-t_k-d)}\overline{\varphi}(\|\omega(s)\|)\} \\ &+\sum_{n=1}^{p} e^{\lambda_k(t_{k+1}-t_{k,p-n+1})}\rho_k^p\overline{\varphi}(\|\omega(t)\|)e^{-\frac{n(n-1)}{2}\lambda_k r_2}(1+\gamma_0) + \gamma_0\overline{\varphi}(\|\omega(t)\|) \\ \leq{}& e^{-p\lambda_k r_2} e^{\lambda_k(t_{k+1}-t_k)}\rho_k^p\widetilde{\mu}_{k-1,k}e^{\lambda_{k-1}(t_k+d-t_{k-1})}\rho_{k-1}^p * \overline{\mu}_{k-2,k-1}\sup_{d\in[-r_1,0]}\{e^{\lambda_{k-2}(t_{k-1}-t_{k-2})}\rho_{k-2}^p\widetilde{\mu}_{k-3,k-2} \\ &*\sup_{d\in[-r_1,0]}\{V_{k-3}(t_{k-2}+d,\phi)\} + \sum_{n=1}^{p} e^{\lambda_{k-2}(t_{k-1}-t_{k-2,p-n+1})} * \rho_{k-2}^n\overline{\varphi}(\|\omega(t)\|)(1+\overline{\gamma}_0 e^{-\lambda_{k-2}(t_{n,p-n}-t_{n,p-n+1})}) \\ &+\gamma_0 e^{-\lambda_{k-2}(t_{k-2,p}-t_{k-1})}\overline{\varphi}(\|\omega(t)\|)\} + e^{-p\lambda_k r_2}e^{\lambda_k(t_{k+1}-t_k)} * \rho_k^p\widetilde{\mu}_{k-1,k}\sum_{n=1}^{p} e^{\lambda_{k-1}(t_k-t_{k-1,p-n+1})}\rho_{k-1}^n\overline{\varphi}(\|\omega(s)\|) \\ &*(1+\overline{\gamma}_0 e^{-\lambda_{k-1}(t_{n,p-n}-t_{n,p-n+1})}) + e^{-p\lambda_k r_2}e^{\lambda_k(t_{k+1}-t_k)} * \rho_k^p\widetilde{\mu}_{k-1,k}\overline{\gamma}_0 e^{-\lambda_{k-1}(t_{k-1,p}-t_k)}\overline{\varphi}(\|\omega(t)\|) + \gamma_0\overline{\varphi}(\|\omega(t)\|) \\ &+\sum_{n=1}^{p} e^{\lambda_k(t_{k+1}-t_{k,p-n+1})}\rho_k^p\overline{\varphi}(\|\omega(t)\|)e^{-\frac{-n(n-1)}{2}\lambda_k r_2}(1+\gamma_0). \end{aligned}$$

After the same recursion process as above, we can obtain

$$\begin{aligned} V(x(t)) \leq{}& (\prod_{b\in\aleph_b}\widetilde{\mu}_{a,b}^{N_b(t,t_0)}e^{-p\lambda_b r_2}\rho_b^{N_\delta(t,t_0)})e^{\sum_{b\in\aleph_b}\lambda_b T_b(t,t_0)} * (\prod_{c\in\aleph_c}\overline{\mu}_{a,c}^{N_c(t,t_0)}\rho_c^{N_\delta(t,t_0)})e^{\sum_{c\in\aleph_c}\lambda_c T_c(t,t_0)}V(x(t_0)) \\ &+[\sum_{b,c=1}^{k}(\prod_{b\in\aleph_b}\widetilde{\mu}_{a,b}^{N_b(t,t_0)}e^{-p\lambda_b r_2}\rho_b^{N_\delta(t,t_0)})e^{\sum_{b\in\aleph_b}\lambda_b T_b(t,t_0)} * (\prod_{c\in\aleph_c}\overline{\mu}_{a,c}^{N_c(t,t_0)}\rho_c^{N_\delta(t,t_0)})e^{\sum_{c\in\aleph_c}\lambda_c T_c(t,t_0)} \end{aligned}$$

$$* (A_1 + A_2)\overline{\varphi}(\|\omega(t)\|) + (A_3 + \gamma_0)]\overline{\varphi}(\|\omega(t)\|),$$

where $\hat{\gamma}_0 = \max\{\gamma_0, \overline{\gamma}_0\}$; $A_1 = \max\{\sum_{n=1}^{p} e^{\lambda_i(t_{i+1}-t_{i,p-n+1})}\ \rho_k^n(1 + \hat{\gamma}e^{-\lambda_i(t_{n,p-n}-t_{n,p-n+1})}), i = 1, ..., k-1\}$; $A_2 = \max\{\hat{\gamma}e^{-\lambda_i(t_{i,p}-t_{(i+1)})}, i = 1, ..., k-2\}$; and $A_3 = \max\{\sum_{n=1}^{p} e^{\lambda_k(t_{k+1}-t_{k,p-n+1})}\rho_k^p e^{-\frac{n(n-1)}{2}\lambda_k r_2}(1 + \gamma_0)\}$.

Then,

$$\begin{aligned}
V(x(t)) \leq\ & (\prod_{b\in\aleph_b} \overline{\mu}_{a,b}^{N_b^0 + \frac{T_b(t,t_0)}{\tau_{s(a,b)}}} e^{-p\lambda_b r_2}\rho_b^{\frac{T_b(t,t_0)}{\tau_{\delta(a,b)}} + N^0_{\delta(a,b)}}) \\
& * e^{\sum_{b\in\aleph_b} \lambda_b T_b(t,t_0)} (\prod_{c\in\aleph_c} \overline{\mu}_{a,c}^{\frac{T_c(t,t_0)}{\tau_{f(a,c)}+\theta_c} - N_c^0} \rho^{\frac{T_c(t,t_0)}{\tau_{\delta(a,c)}} - N^0_{\delta(a,c)}}) \\
& * e^{\prod_{c\in\aleph_c} \lambda_c(\frac{\tau_{f(a,c)}T_c(t,t_0)}{\tau_{f(a,c)}+\theta_c} + T_c^0)} V(x(t_0)) \\
& + [\sum_{b,c=1}^{k} (\prod_{b\in\aleph_b} \overline{\mu}_{a,b}^{N_b^0 + \frac{T_b(t,t_0)}{\tau_{s(a,b)}}} e^{-p\lambda_b r_2}\rho_b^{\frac{T_b(t,t_0)}{\tau_{\delta(a,b)}} + N^0_{\delta(a,b)}}) \\
& * e^{\sum_{b\in\aleph_b} \lambda_b T_b(t,t_0)} (\prod_{c\in\aleph_c} \overline{\mu}_{a,c}^{\frac{T_c(t,t_0)}{\tau_{f(a,c)}+\theta_c} - N_c^0} \rho^{\frac{T_c(t,t_0)}{\tau_{\delta(a,c)}} - N^0_{\delta(a,c)}}) \\
& * e^{\prod_{c\in\aleph_c} \lambda_c(\frac{\tau_{f(a,c)}T_c(t,t_0)}{\tau_{f(a,c)}+\theta_c} + T_c^0)} (A_1 + A_2) \\
& + (A_3 + \gamma_0)]\overline{\varphi}(\|\omega(t)\|) \\
& * e^{\sum_{c\in\aleph_c} ((\lambda_c\tau^*_{f(a,c)} + \ln\overline{\mu}_{a,c})/(\tau^*_{f(a,c)} + \theta_c) + \ln\rho_c/\tau^*_{\delta(a,c)})T_c(t,t_0)} \\
& * V(x(t_0)) + M\overline{\varphi}(\|\omega(t)\|) \\
& Be^{\sum_{b\in\aleph_b} \xi_b T_b(t,t_0) + \sum_{c\in\aleph_c} \xi_c T_c(t,t_0)} V(x(t_0)) + M\overline{\varphi}(\|\omega(t)\|) \\
& = Be^{\sum_{\overline{b}\in\aleph_s} \xi_{\overline{b}} T_{\overline{b}}(t,t_0) + \sum_{\overline{c}\in\aleph_u} \xi_{\overline{c}} T_{\overline{c}}(t,t_0)} V(x(t_0)) + M\overline{\varphi}(\|\omega(t)\|) \\
& \leq Be^{\xi^- T^- + \xi^+ T^+} V(x(t_0)) + M\overline{\varphi}(\|\omega(t)\|) \\
& \leq Be^{\xi^*(T^- + T^+)} V(x(t_0)) + M\overline{\varphi}(\|\omega(t)\|) \\
& = Be^{\xi^*(t-t_0)} V(x(t_0)) + M\overline{\varphi}(\|\omega(t)\|),
\end{aligned} \tag{3.21}$$

where $\xi_b = \sum_{b\in\aleph_b}((\lambda_b\tau^*_{s(a,b)} + \ln\widetilde{\mu}_{a,b})/\tau^*_{s(a,b)} + \ln\rho_b/\tau^*_{\delta(a,b)})$; $\xi_c = \sum_{c\in\aleph_c}((\lambda_c\tau^*_{f(a,c)} + \ln\overline{\mu}_{a,c})/(\tau^*_{f(a,c)} + \theta_c) + \ln\rho_c/\tau^*_{\delta(a,c)})$; and $M = \max\{\sum_{b,c=1}^{k}(A_1 + A_2) + (A_3 + \gamma_0)Be^{\xi_b T_b(t,t_0)}e^{\xi_c T_c(t,t_0)}\}$.

For any constants $c \geq 0$, $r \geq 0$ and any functions $q \in \mathcal{K}$, $\epsilon \in \mathcal{K}_\infty$, it always holds that

$$q(c + r) \leq q \cdot (Id + \epsilon)(c) + q \cdot (Id + \epsilon^{-1})(r). \tag{3.22}$$

From condition (3.1) and $\alpha = \alpha_2 + \alpha_3$, we have

$$|x(t)| \leq \alpha_1^{-1}(Id + \epsilon)(Be^{\xi^*(t-t_0)}\alpha(|\xi|_r))\alpha_1^{-1}(Id + \epsilon^{-1})M\overline{\varphi}(\|\omega(t)\|). \tag{3.23}$$

Then, system (2.1) is ISS. The proof is completed. $\square$

Remark 3. *In (3.2), the estimate of the upper bounds of the derivatives for Lyapunov function $\lambda_i < 0$ means the subsystem is stable, and $\lambda_i > 0$ means the subsystem is unstable. When the subsystem is stable, the slow switching is used in (3.3); when the subsystem is unstable, the methods of the slow switching and the fast switching can be used in (3.3) or (3.4). In [29], condition (2) of Theorem 1 just considers one case. Thus, the situation in [29] is a special case of this paper. In addition, in (3.5) and (3.6), impulses may occur between the switching points, which may be stable impulses to compensate the divergence state, or unstable impulses to affect the system as disturbances.*

Remark 4. *From the above discussion, we can get $\tau^*_{\sigma(j,i)} = -\frac{\ln\mu_{j,i}+\theta\frac{\ln\rho_{j,i}}{\tau^*_\delta}}{\lambda_i+\frac{\ln\rho_{j,i}}{\tau^*_\delta}}$. When the impulse disappears, the definition of BAED-ADT will be reduce to MD-BMADT in [34]. This means the boundary of the AED-ADT approach is more general and will be less conservative than the mode-dependent average dwell time (MDADT). Meanwhile, we can conclude that when $\xi_i < 0$, the subsystem is switching stable, and when $\xi_i > 0$, the subsystem is switching unstable. The switching stability of the subsystem is not directly related to the switching method and the stability of the subsystem itself. Thus, we get the unified stability result for the switched impulsive system.*

Theorem 1. *Assume that there exist $(h_{i\nu}(t) - \varrho_{i\nu}\mu_{i\nu}(t)) > 0, (0 < \varrho_{i\nu} < 1)$, constants $h_i,\ r_1,\ r_2,\ \lambda_i < 0$, $i \in \aleph_s$; $\lambda_i > 0,\ i \in \aleph_u$; $\mu^{11}_{j,i} > 1,\ \mu^{12}_{j,i} > 0,\ \eta^1_{j,i} > 1, \tau^*_{s(i,b)} > 0,\ \xi_b = (\lambda_b\tau^*_{s(i,b)} + \ln\widetilde{\mu}_{i,b})/\tau^*_{s(i,b)} + \ln\rho_{j,i}/\tau^*_{\delta(i,b)},\ \widetilde{\mu}_{j,i} = \max\{\mu^{11}_{j,i}, \eta^1_{j,i}\} + \mu^{12}_{j,i},\ 0 < \rho^{11}_{j,i} < 1,\ \rho^{12}_{j,i} > 0,\ k_{j,i} > 0,\ i, j \in \aleph,\ b \in \aleph_b$; $0 < \mu^{21}_{j,i} < 1,\ \mu^{22}_{j,i} > 0,\ \eta^2_{j,i} > 0, \tau^*_{f\theta(i,c)} > 0,\ \xi_c = (\lambda_c\tau^*_{f\theta(i,c)} + \ln\overline{\mu}_{i,c})/(\tau^*_{f\theta_c(i,c)} + \theta_c) + \ln\rho_{j,i}/\tau^*_{\delta(i,c)},\ \overline{\mu}_{j,i} = \max\{\mu^{21}_{j,i}, \eta^2_{j,i}\} + \mu^{22}_{j,i},\ 0 < \tau_{mc} \le \theta_c,\ \rho^{21}_{j,i} > 1,\ \rho^{22}_{j,i} > 0,\ i, j \in \aleph,\ c \in \aleph_c$; symmetric positive definite matrices $P_{i,q_i} > 0, q_i \in \{1, 2, ..., L_i\},\ L_i \in \mathbb{Z}^+,\ i \in \aleph,\ Q_i > 0$; and symmetric matrixes G_0 such that for $\forall d_i \in \{0, 1, ..., J_i - 1\},\ J_i \in \mathbb{Z}^+$*

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & 0 \\ * & -Q_i & 0 & 0 \\ * & * & -I & * \\ * & * & * & -\lambda_i r Q_i \end{bmatrix} \le 0, i \in \aleph_b, \tag{3.24}$$

where $L_{11} = (1-\alpha_i)R_i + \alpha_i R_{i+1}, L_{12} = ((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})B_{ip}, and L_{13} = ((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})C_{ip}$;

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ * & -Q_i & 0 \\ * & * & 0 \end{bmatrix} \le 0, i \in \aleph_c, \tag{3.25}$$

where $M_{11} = (1-\alpha_i)F_i + \alpha_i F_{i+1}, M_{12} = ((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})B_{ip}, and M_{13} = ((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})C_{ip}$;

$$\begin{bmatrix} N_{11} & N_{12} & N_{13} \\ * & -Q_i & 0 \\ * & * & 0 \end{bmatrix} \le 0, i \in \aleph_c, \tag{3.26}$$

where $N_{11} = (1-\alpha_i)H_i + \alpha_i H_{i+1}, N_{12} = ((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})B_{ip}, and N_{13} = ((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})C_{ip}$;

$$\begin{bmatrix} O_{11} & P_{i,J_i}B_{ip} & P_{i,J_i}C_{ip} & 0 \\ * & -Q_i & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\lambda_i r Q_i \end{bmatrix} \le 0, i \in \aleph, \tag{3.27}$$

where $O_1 = A_{ip}^T P_{i,J_i} + P_{i,J_i} A_{ip} + Q_i - \lambda_i P_{i,J_i}$;

$$\begin{bmatrix} P_{i,0} - \mu_{j,i}^{11} P_{j,q_j} & 0 & 0 \\ * & -\mu_{j,i}^{12} P_{j,q_j} & 0 \\ * & * & -I \end{bmatrix} \le 0,\ Q_i - \eta_{j,i}^1 Q_j \le 0,\ i \in \aleph, j \in \aleph_b, i \neq j, \tag{3.28}$$

$$\begin{bmatrix} P_{i,0} - \mu_{j,i}^{21} P_{j,q_j} & 0 & 0 \\ * & -\mu_{j,i}^{22} P_{j,J_j} & 0 \\ * & * & -I \end{bmatrix} \le 0,\ Q_i - \eta_{j,i}^2 Q_j \le 0,\ i \in \aleph, j \in \aleph_c, i \neq j, \tag{3.29}$$

$$\begin{bmatrix} P_i - \rho_{j,i}^{11} P_j & P_i D_{\nu p} & P_i E_{\nu p} \\ * & O_2 & D_{\nu p}^T P_i E_{\nu p} \\ * & * & E_{\nu p}^T P_i E_{\nu p} - I \end{bmatrix} \le 0,\ r - k_{j,i} \le 0,\ i, j, \nu \in \aleph, \tag{3.30}$$

where $O_2 = D_{\nu p}^T P_i D_{\nu p} - \rho_{j,i}^{12} P_j$;

$$\begin{bmatrix} P_i - \rho_{j,i}^{21} P_j & P_i D_{\nu p} & P_i E_{\nu p} \\ * & D_{\nu p}^T P_i D_{\nu p} - \rho_{j,i}^{22} P_j & D_{\nu p}^T P_i E_{\nu p} \\ * & * & E_{\nu p}^T P_i E_{\nu p} - I \end{bmatrix} \le 0,\ \rho_{j,i}^{21} > 1,\ i, j, \nu \in \aleph, \tag{3.31}$$

where $\Delta_i = J_i(P_{i,d_i+1} - P_{i,d_i})$, $R_i = A_{ip}^T P_{i,d_i} + P_{i,d_i} A_{ip} + \Delta_i/\tau_{si}^* + Q_i - \lambda_i P_{i,d_i}$, $R_{i+1} = A_{ip}^T P_{i,d_i+1} + P_{i,d_i+1} A_{ip} + \Delta_i/\tau_{si}^* + Q_i - \lambda_i P_{i,d_i+1}$; $F_i = A_{ip}^T P_{i,d_i} + P_{i,d_i} A_{ip} + \Delta_i/\tau_{mi} + Q_i$, $F_{i+1} = A_{ip}^T P_{i,d_i+1} + P_{i,d_i+1} A_{ip} + \Delta_i/\tau_{mc} + Q_i$; *and* $H_i = A_{ip}^T P_{i,d_i} + P_{i,d_i} A_{ip} + \Delta_i/\theta_c + Q_i$, $H_{i+1} = A_{ip}^T P_{i,d_i+1} + P_{i,d_i+1} A_{ip} + \Delta_i/\theta_c + Q_i$.

$$\Psi_{ipv}^j - G_j < 0, j = \{0, 1, 2, 3\}, \varrho_{iv} \Psi_{ipv}^j - \varrho_{iv} G_j + G_j < 0,$$

where

$$\Psi_{ipv}^0 = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix}, \Psi_{ipv}^1 = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Pi_{33} \end{bmatrix}, \Psi_{ipv}^2 = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ * & \Lambda_{22} & \Lambda_{23} \\ * & * & \Lambda_{33} \end{bmatrix}, \Psi_{ipv}^3 = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix},$$

$$\Xi_{11} = \Pi_{11} + \frac{\Delta_i}{\tau_{si}^*} I, \Xi_{12} = \Pi_{12} + \frac{\Delta_i}{\tau_{si}^*} D_{iv}, \Xi_{13} = \Pi_{13} + \frac{\Delta_i}{\tau_{si}^*} E_{iv}, \Xi_{22} = \Pi_{22} + D_{iv}^T \frac{\Delta_i}{\tau_{si}^*} D_{iv},$$

$$\Xi_{23} = \Pi_{23} + D_{iv}^T \frac{\Delta_i}{\tau_{si}^*} E_{iv}, \Xi_{33} = \Pi_{33} + E_{iv}^T \frac{\Delta_i}{\tau_{si}^*} E_{iv}, \Pi_{11} = A_{ip}^T P_i(t) + P_i(t) A_{ip} - \lambda_i P_i(t) + Q_i,$$

$$\Pi_{12} = A_{ip}^T P_i(t) D_{iv} + P_i(t) \overline{B}_i - \lambda_i P_i(t) D_{iv}, \Pi_{13} = A_{ip}^T P_i(t) E_{iv} + P_i(t) \overline{C}_i - \lambda_i P_i(t) E_{iv},$$

$$\overline{B}_i = A_{ip} D_{iv} + B_{ip}, \overline{C}_i = A_{ip} E_{iv} + C_{ip},$$

$$\Pi_{22} = \overline{B}_i^T P_i(t) D_{iv} + D_{iv}^T P_i(t) \overline{B}_i - \lambda_i D_{iv}^T P_i(t) D_{iv} - \lambda_i r Q_i - Q_i,$$

$$\Pi_{23} = \overline{B}_i^T P_i(t) E_{iv} + D_{iv}^T P_i(t) \overline{C}_i - \lambda_i D_{iv}^T P_i(t) E_{iv},$$

$$\Pi_{33} = \overline{C}_i P_i(t) E_{iv} + E_{iv}^T P_i(t) \overline{C}_i - \lambda_i E_{iv}^T P_i(t) E_{iv} - I,$$

$$\Lambda_{11} = \Gamma_{11} + \frac{\Delta_i}{\tau_{mc}} I, \Lambda_{12} = \Gamma_{12} + \frac{\Delta_i}{\tau_{mc}} D_{iv}, \Lambda_{13} = \Gamma_{13} + \frac{\Delta_i}{\tau_{mc}} E_{iv}, \Lambda_{22} = \Gamma_{22} + D_{iv}^T \frac{\Delta_i}{\tau_{mc}} D_{iv},$$

$$\Lambda_{23} = \Gamma_{23} + D_{iv}^T \frac{\Delta_i}{\tau_{mc}} E_{iv}, \Lambda_{33} = \Gamma_{33} + E_{iv}^T \frac{\Delta_i}{\tau_{mc}} E_{iv}, \Theta_{11} = \Gamma_{11} + \frac{\Delta_i}{\tau_c} I,\ \Gamma_{11} = A_{ip}^T P_i(t) + P_i(t) A_{ip} + Q_i,$$

$$\Theta_{12} = \Gamma_{12} + \frac{\Delta_i}{\tau_c} D_{iv},\ \Gamma_{12} = A_{ip}^T P_i(t) D_{iv} + P_i(t)\overline{B}_i, \Theta_{13} = \Gamma_{13} + \frac{\Delta_i}{\tau_c} E_{iv},\ \Gamma_{13} = A_{ip}^T P_i(t) E_{iv} + P_i(t)\overline{C}_i,$$

$$\Theta_{22} = \Gamma_{22} + D_{iv}^T \frac{\Delta_i}{\tau_c} D_{iv}, \Gamma_{22} = \overline{B}_i^T P_i(t) D_{iv} + D_{iv}^T P_i(t)\overline{B}_i - Q_i,$$

$$\Theta_{22} = \Gamma_{22} + D_{iv}^T \frac{\Delta_i}{\tau_c} E_{iv},\ \Gamma_{23} = \overline{B}_i P_i(t) E_{iv} + D_{iv}^T P_i(t)\overline{C}_i,$$

$$\Theta_{33} = \Gamma_{33} + E_{iv}^T \frac{\Delta_i}{\tau_c} E_{iv},\ \Gamma_{33} = \overline{C}_i P_i(t) E_{iv} + E_{iv}^T P_i(t)\overline{C}_i.$$

Then, the system (2.3) is ISS if the switching and impulsive signals satisfy (3.7).

Proof. First, we construct the multiple Lyapunov function $V_i(x(t)) = V_i^1(t, x(t)) + V_i^2(t, x_t)$ where $V_i^1(t, x(t)) = x(t)^T P_i(t) x(t)$ and $V_i^2(t, x_t) = \int_{t-r}^t x(s)^T Q_i x(s) ds,\ r = \max\{r_1, r_2\}$.

When $i \in \aleph_b$, the interval $[t_k, t_k + \tau_{si}^*)$ is divided into J_i subintervals $[t_k + h_i d_i, t_k + h_i(d_i + 1)),\ h_i = \tau_{si}^*/J_i,\ d_i = \{0, 1, ..., J_i - 1\}$. Then, the Lyapunov function matrix is defined as

$$P_i(t) = \begin{cases} (1 - \alpha_i) P_{i,d_i} + \alpha_i P_{i,d_i+1},\ t \in [t_k, t_k + \tau_{si}^*), \\ P_{i,J_i},\ t \in [t_k + \tau_{si}^*, t_{k+1}), i \in \aleph_b, \end{cases} \tag{3.32}$$

where $\alpha_i = (t - t_k - h_i d_i)/h_i$.

When $i \in \aleph_c$, a switching signal lower bound τ_{mc} satisfies $0 < \tau_{mc} \le \theta_c$. If $t_{k+1} - t_k \le \theta_c$, we can get by dividing the interval $[t_k, t_{k+1})$ into J_i subintervals, we can get

$$P_i(t) = (1 - \alpha_i) P_{i,d_i} + \alpha_i P_{i,d_i+1}, \tag{3.33}$$

where $d_i = (t_{k+1} - t_k)/J_i$. If $t_{k+1} - t_k > \theta_c$, the interval $[t_k, t_k + \theta_c)$ is divided into J_i subintervals. Then

$$P_i(t) = \begin{cases} (1 - \alpha_i) P_{i,d_i} + \alpha_i P_{i,d_i+1},\ t \in [t_k, t_k + \theta_c), \\ P_{i,J_i},\ t \in [t_k + \theta_c, t_{k+1}),\ i \in \aleph_c, \end{cases} \tag{3.34}$$

where $d_i = \theta_c/J_i$.

Case 1. Let $\varphi(\|\omega(t)\|) = \omega(t)^T \omega(t), \overline{x}(t) = \sup_{d \in [-r, 0]} \{x(t + d)\}$. For $\sigma(t_k) = i \in \aleph_b,\ t \in [t_k, t_k + \tau_{si}^*)$, based on the definition of $P_i(t)$, we can get

$$\dot{P}_i(t) = -\frac{1}{h_i} P_{i,d_i} + \frac{1}{h_i} P_{i,d_i+1} = \frac{J_i}{\tau_{si}^*}(P_{i,d_i+1} - P_{i,d_i}) = \frac{\Delta_i}{\tau_{si}^*},$$

and according to (3.24), we can obtain

$$\dot{V}_i(x(t)) - \lambda_i V_i(x(t)) - \varphi(\|\omega(t)\|) \le \sum_{p=1}^m h_{ip}(z_i(t)) \{x^T(t)[A_{ip}^T((1 - \alpha_i) P_{i,d_i}$$

$$
\begin{aligned}
&+ \alpha_i P_{i,d_i+1}) + ((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})A_{ip} + \frac{\Delta_i}{\tau^*_{si}} + Q_i - \lambda_i((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})]x(t) \\
&+ x^T(t-r)[B^T_{ip}((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})]x(t) + \omega^T(t)[C^T_{ip}((1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1})]x(t) \\
&- \lambda_i r\overline{x}^T(t)Q_i\overline{x}(t) - \omega^T(t)\omega(t)\} \leq 0.
\end{aligned} \tag{3.35}
$$

To get less conservative results, slack matrices are introduced with the following expression:

$$
\sum_{p=1}^{m}\sum_{v=1}^{m} h_{ip}(t)[(h_{iv}(t) - \mu_{iv}(t))G_j] = 0.
$$

If $t_{k,m} \in [t_k, t_k + \tau^*_{si})$, we have

$$
\begin{aligned}
&\dot{V}_i(x(t_{k,m})) - \lambda_i V_i(x(t_{k,m})) - \varphi(\|\omega(t_{k,m})\|) = \sum_{p=1}^{m}\sum_{v=1}^{m} h_{ip}(t_{k,m})h_{iv}(t_{k,m})\eta^T(t_{k,m})\Psi^0_{ipv}\eta(t_{k,m}) \\
&= \sum_{p=1}^{m}\sum_{v=1}^{m} h_{ip}(t_{k,m})\eta^T(t_{k,m})\{\mu_{iv}(t_{k,m})(\varrho_{iv}\Psi^0_{ipv} - \varrho_{iv}G_0 + G_0) + (h_{iv}(t_{k,m}) - \varrho_{iv}\mu_{iv}(t_{k,m}))(\Psi^0_{ipv} - G_0)\} \\
&< 0.
\end{aligned}
$$

When $t \in [t_k + \tau^*_{si}, t_{k+1})$, we can get $\dot{P}_i(t) = 0$. From (3.27), it concludes that

$$
\begin{aligned}
&\dot{V}_i(x(t)) - \lambda_i V_i(x(t)) - \varphi(\|\omega(t)\|) \leq \sum_{p=1}^{m} h_{ip}(z_i(t))\{x^T(t)[A^T_{ip}P_{i,J_i} + P_{i,J_i}A_{ip} \\
&+ Q_i - \lambda_i P_{i,J_i}]x(t) + x^T(t-r)(B^T_{ip}P_{i,J_i})x(t) + \omega^T(t)C_{ip}x(t) + x^T(t)P_{i,J_i}B_{ip}x(t-r) \\
&+ x^T(t)P_{i,J_i}C_{ip}\omega(t) - \lambda_i r\overline{x}(t) - \omega^T(t)\omega(t) - x^T(t-r)Q_i x(t-r)\} \\
&\leq 0.
\end{aligned} \tag{3.36}
$$

In the same way, if $t_{k,m} \in [t_k + \tau^*_{si}, t_{k+1})$, we have

$$
\dot{V}_i(x(t_{k,m})) - \lambda_i V_i(x(t_{k,m})) - \varphi(\|\omega(t_{k,m})\|) < 0, \tag{3.37}
$$

where $\eta(t) = [x(t), \overline{x}(t), \omega(t)]^T$. For $t \in [t_k, t_{k+1}), i \in \aleph_b$, $\dot{V}_i(x(t)) \leq \lambda_i V_i(x(t)) + \varphi(\|\omega(t)\|)$ in Lemma 1 holds if (3.24) and (3.27) are true.

Case 2. For $\sigma(t_k) = i \in \aleph_c$, when $t \in [t_k + \theta_c, t_{k+1})$, we can get $\dot{P}_i(t) = 0$. It is the same as Case 1, where for $t \in [t_k + \theta_c, t_{k+1}), i \in \aleph_c$, it is $\dot{V}_i(x(t)) \leq \lambda_i V_i(x(t)) + \varphi(\|\omega(t)\|)$. When $t \in [t_k, t_k + \theta_c)$, we can obtain $\dot{P}_i(t) = \frac{(P_{i,d_i+1} - P_{i,d_i})}{h_i} \leq \max\{\frac{\Delta_i}{\tau_{mc}}, \frac{\Delta_i}{\theta_c}\}$. Next, we will show $\dot{V}_i(x(t)) \leq 0$. From (3.25) and (3.26), we can get

$$
\begin{aligned}
\dot{V}_i(x(t)) \leq &\sum_{p=1}^{m} h_{ip}(z_i(t))\{x^T(t)[(1-\alpha_i) * (A^T_{ip}P_{i,d_i} + P_{i,d_i}A_{ip} + \frac{\Delta_i}{\tau_{mc}} + Q_i) \\
&+ \alpha_i(A^T_{ip}P_{i,d_i+1} + P_{i,d_i+1}A_{ip} + \frac{\Delta_i}{\tau_{mc}} + Q_i)] + x^T(t-r)B^T_{ip}[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]x(t) \\
&+ \omega^T(t)C^T_{ip}[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]x(t) \\
&+ x^T(t)[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]B_{ip}x(t-r) + x^T(t)[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]C_{ip}\omega(t)
\end{aligned} \tag{3.38}
$$

$$- x^T(t-r)Q_i x(t-r)\} \le 0,$$

and

$$\begin{aligned}\dot{V}_i(x(t)) \le \sum_{p=1}^{m} h_{ip}(z_i(t))\{x^T(t)[(1-\alpha_i) * (A_{ip}^T P_{i,d_i} + P_{i,d_i}A_{ip} + \frac{\Delta_i}{\theta_c} + Q_i)\\ + \alpha_i(A_{ip}^T P_{i,d_i+1} + P_{i,d_i+1}A_{ip} + \frac{\Delta_i}{\theta_c} + Q_i)] + x^T(t-r)B_{ip}^T[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]x(t)\\ + \omega^T(t)C_{ip}^T[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]x(t) + x^T(t)[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]B_{ip}x(t-r)\\ + x^T(t)[(1-\alpha_i)P_{i,d_i} + \alpha_i P_{i,d_i+1}]C_{ip}\omega(t) - x^T(t-r)Q_i x(t-r)\} \le 0. \end{aligned} \tag{3.39}$$

If $t_{k,m} \in [t_k, t_k + \theta_c)$, we show $\dot{V}_i(x(t_{k,m})) \le 0$ as follows:

$$\begin{aligned}\dot{V}_i(x(t_{k,m})) = \sum_{p=1}^{m}\sum_{v=1}^{m} h_{ip}(t_{k,m})\eta^T(t_{k,m})\{\mu_{iv}(t_{k,m})(\varrho_{iv}\Psi_{ipv}^2\\ - \varrho_{iv}G_2 + G_2) + (h_{iv}(t_{k,m}) - \varrho_{iv}\mu_{iv}(t_{k,m})) * (\Psi_{ipv}^2 - G_2)\} \le 0,\end{aligned}$$

and

$$\begin{aligned}\dot{V}_i(x(t_{k,m})) = \sum_{p=1}^{m}\sum_{v=1}^{m} h_{ip}(t_{k,m})\eta^T(t_{k,m})\{\mu_{iv}(t_{k,m})(\varrho_{iv}\Psi_{ipv}^3\\ - \varrho_{iv}G_3 + G_3) + (h_{iv}(t_{k,m}) - \varrho_{iv}\mu_{iv}(t_{k,m}))(\Psi_{ipv}^3 - G_3)\} \le 0.\end{aligned}$$

Case 3. If $\sigma(t_k) = i \in \aleph,\ \sigma(t_{k-1}) = j \in \aleph_b$, with (3.28), we can obtain

$$\begin{aligned}& V_i^1 - \mu_{j,i}^{12} \sup_{d\in[-r_1,0]}\{V_j^1(t_k + d, \phi(d))\} - \mu_{j,i}^{11}V_j^1(t_k) - \omega^T(t)\omega(t)\\ \le & x^T(t)P_{i,0}x(t) - \mu_{j,i}^{11}x^T(t)P_{j,q_j}x(t) - \mu_{j,i}^{12}\overline{x}^T(t)P_{j,q_j}\overline{x}^T(t) - \omega^T(t)\omega(t)\\ \le & 0, \end{aligned} \tag{3.40}$$

$$V_i^2 - \eta_{j,i}^1 V_j^2 = \int_{t-r}^{t}[x^T(s)Q_i x(s) - \eta_{j,i}^1 x^T(s)Q_j x(s)]ds \le 0. \tag{3.41}$$

Case 4. Same as Case 3, if $\sigma(t_k) = i \in \aleph,\ \sigma(t_{k-1}) = j \in \aleph_c$, with (3.29), we can obtain

$$V_i^1 - \mu_{j,i}^{21}V_j^1(t_k) - \omega^T(t)\omega(t) - \mu_{j,i}^{22}\sup_{d\in[-r_1,0]}\{V_j^1(t_k + d, \phi(d))\} \le 0, \tag{3.42}$$

$$V_i^2 - \eta_{j,i}^1 V_j^2 \le 0. \tag{3.43}$$

To sum up, (3.2)–(3.4) of Lemma 1 hold from Cases 1–4.

Now, we will show that (3.5) holds under (3.30).

$$\begin{aligned}& V_i^1(t, \phi(0) + g_{\sigma(t)}(t,\phi,\omega(t^-)) - \rho_{j,i}^{11}V_j^1(t^-,\phi(0)) - \rho_{j,i}^{12}\sup_{d\in[-r_2,0]}\{V_j^1(t^- + d), \phi(d)\} - \varphi(\|\omega(t^-)\|)\\ =& \sum_{v=1}^{m} h_{iv}(t)[(x(t) + D_{iv}x(t-r) + E_{iv}\omega(t))^T P_i(x(t) + D_{iv}x(t-r) + E_{iv}\omega(t)) - \rho_{j,i}^{11}x^T(t)P_j x(t)\end{aligned}$$

$$
\begin{aligned}
&-\rho_{j,i}^{12}\overline{x}^T(t)P_j\overline{x}(t)-\omega^T(t)\omega(t)] \\
\leq& 0 \qquad (3.44) \\
&V_i^2(t,\phi)-k_{j,i}\sup_{d\in[-r_2,0]}\{V_j^1(t+d,\phi(d))\}=\int_{t-r}^{t}x^T(s)Q_ix(s)ds-k_{j,i}\overline{x}^T(t)Q_i\overline{x}(t) \\
&\leq (r-k_{j,i})\overline{x}^T(t)Q_i\overline{x}(t)\leq 0. \qquad (3.45)
\end{aligned}
$$

Similarly, we can show (3.6) by using condition (3.31) and the same process.

According to Lemma 1, the system is ISS with the switching and impulsive signals satisfying (3.7). Thus, the theorem is proved. □

Remark 5. *Compared with [13, 36], we design different fuzzy rules and the membership functions for the controlled object and the impulse. By introducing the relaxation matrices, we get the membership functions relationship between the controlled object and the impulse, which makes the results have less conservativeness.*

4. Numerical examples

In this section, we present several examples to show the validity and effectiveness of the obtained conditions of stability. For convenience, the normalised membership functions of the impulsive switched T-S fuzzy systems are all given as $h_{i1}(t) = sin^2(x_1)$, $h_{i2}(t) = 1 - h_{i1}(t)$ *and* $h_{v1}(t) = h_{i2}(t)$, $h_{v2}(t) = h_{i1}(t)$.

Example 1. Consider the continuous-time impulsive switched T-S fuzzy system composing of the following three subsystems:

$$
A_{11}=\begin{bmatrix}-1.71 & 0.62\\ -0.93 & 0.46\end{bmatrix},\ A_{12}=\begin{bmatrix}-2.41 & 0.82\\ -1.22 & 0.45\end{bmatrix},\ B_{i1}=B_{i2}=\begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix}, C_{i1}=C_{i2}=\begin{bmatrix}0\\ 0.3\end{bmatrix},
$$

$$
D_{i1}=D_{i2}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ E_{i1}=E_{i2}=\begin{bmatrix}0.2\\ 0.2\end{bmatrix},\ i=\{1,2\}, A_{21}=\begin{bmatrix}0.52 & -0.84\\ 1.26 & 2.42\end{bmatrix},\ A_{22}=\begin{bmatrix}0.32 & -0.61\\ 0.91 & -1.8\end{bmatrix},
$$

$$
A_{31}=\begin{bmatrix}-3 & 0\\ 0 & -2\end{bmatrix},\ A_{32}=\begin{bmatrix}-2 & 0\\ 1 & -3\end{bmatrix},\ B_{31}=\begin{bmatrix}1/2 & 1/2\\ 0 & 1/2\end{bmatrix}, B_{32}=\begin{bmatrix}1/2 & 0\\ 1/2 & 0\end{bmatrix}, C_{31}=\begin{bmatrix}0.3\\ 0.2\end{bmatrix}, C_{32}=\begin{bmatrix}0.3\\ 0.3\end{bmatrix},
$$

$$
D_{31}=D_{32}=\begin{bmatrix}e^{-2}-1 & 0\\ 0 & e^{-2}-1\end{bmatrix}, E_{31}=E_{32}=\begin{bmatrix}0.2\\ 0.5\end{bmatrix}.
$$

Subsystems 1 and 2 are unstable, and Subsystem 3 is stable. Subsystem 1 is represented by the fuzzy-rule matrices A_{11}, A_{12} and the common coefficient matrices B_1, C_1, D_1, E_1, Subsystem 2 by A_{21}, A_{22} and the same B_1, C_1, D_1, E_1, Subsystem 3 by A_{31}, A_{32} and its own matrices $B_{31}, B_{32}, C_{31}, C_{32}, D_{31}, D_{32}, E_{31}, E_{32}$.So, we use the fast switching way for Subsystem 1 and slow switching way for Subsystems 2 and 3. The three subsystems all occur at stable impulses to compensate for the switching unstable subsystems to get the expected result.

Let $\lambda_1 = 0.42$, $\lambda_2 = 0.2, \lambda_3 = -0.3$; $\mu_{3,1}^{21} = 0.26$, $\mu_{3,1}^{12} = 0.1$; $\mu_{1,2}^{11} = 2.1$, $\mu_{1,2}^{12} = 0.2$; $\mu_{2,3}^{11} = 1.1$, $\mu_{2,3}^{12} = 0.3$; $\theta_1 = 2$; $\tau_{m1} = 0.1$, $\tau_{s(1,2)}^* = 0.1$, $\tau_{s(2,3)}^* = 0.6$, $\tau_{f(3,1)}^* = 2.5$; $J_i = 1$; $rho_{1,2}^{11} = 0.5$, $\rho_{2,3}^{11} = 0.6$, $\rho_{3,1}^{11} = 0.7$; *and* $\tau_{\delta(1,2)} = 0.7$, $\tau_{\delta(2,3)} = 0.9$, $\tau_{\delta(3,1)} = 3$. Through calculation, it has $\xi_1 =$

$(0.42*2.5+\ln 0.26)/2.5+\ln 0.7/3 = -0.2377 < 0$, $\xi_2 = (0.2*0.1+\ln 2.1)/0.1+\ln 0.5/0.7 = 6.6292 > 0$ and $\xi_3 = (-0.3*0.6+1.1)/0.6+\ln 0.6/0.9 = -0.7087 < 0$. Because Subsystems 1 and 3 are switching stable and subsystem 2 is switching unstable, it is $\xi^- = -0.2377$, $\xi^+ = 6.6292$. When $\xi^* = 1.1$, it obtains $\frac{T^+}{T^-} = 0.03225 \le \frac{\xi^*-\xi^-}{\xi^+-\xi^*} = 0.2419$. Given the initial state $x(t_0) = [-5, 5]^T$, the state response is shown in Figure 1 under the switching and impulsive signals satisfying $\tau_{f(3,1)} = 2 < \tau^*_{f(3,1)}$, $\tau_{s(1,2)} = 0.7 > \tau^*_{s(1,2)}$; $\tau_{s(2,3)} = 1.5 > \tau^*_{s(2,3)}$; and $\frac{T^+}{T^-} = 0.2 < 0.2419$. We can know the impulsive switched system will eventually converge to zero from Figures 1–3, i.e., the system is ISS according to Theorem 1.

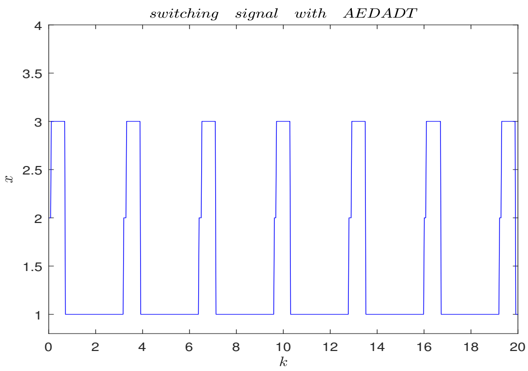

Figure 1. Switching signal.

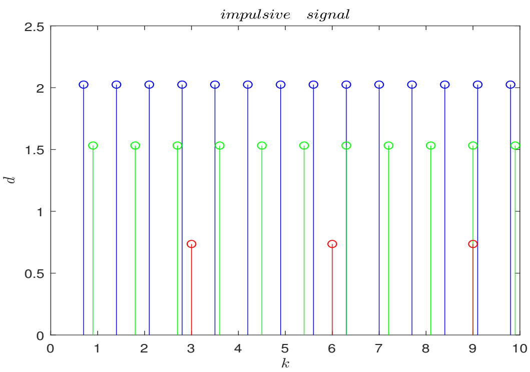

Figure 2. Impulsive signal.

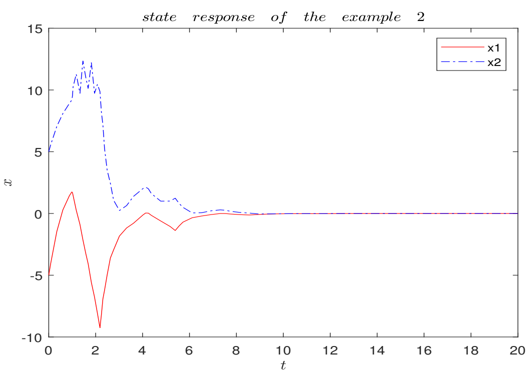

Figure 3. State response of the system.

Example 2. In [34], authors use the T-S fuzzy method to address the stability of the switched systems. Since there is only switching in [34], the authors only demonstrated fuzzy rules of each nonlinear subsystem. In this paper, not only are the fuzzy rules of each subsystem demonstrated, but the fuzzy rules of impulse are also given. So, in order to compare the two fuzzy methods and different dwell time approaches, we use the same data as in Example 4 of literature [34] as follows:

$$A_{11}\begin{bmatrix}-7.4721 & 5.5724\\ -7.5622 & 5.6623\end{bmatrix}, A_{12} = \begin{bmatrix}-9.5563 & 7.0562\\ -9.5760 & 7.0761\end{bmatrix},$$

$$A_{21} = \begin{bmatrix}-2.0326 & 1.8743\\ -0.7029 & 0.6826\end{bmatrix}, A_{22} = \begin{bmatrix}-3.3165 & 3.0118\\ -1.1294 & 1.0465\end{bmatrix}.$$

First, we represent the difference of the fuzzy rules in Table 1. Then, we analyze the difference from the simulation results. To ensure a fair and reproducible comparison, the initial condition is set as $x(t_0) = [1,\ -1]^{\mathrm{T}}$, and the external input is set to $\omega(t) \equiv 0$. All other parameters remain identical to those in [34]. Table 1 quantitatively compares the conservatism between the MD-BMADT method in [34] and the proposed BAED-ADT approach. The maximum allowable average dwell time (ADT) for fast switching τ^* is used as the primary conservatism indicator: A larger τ^* implies a less restrictive switching condition and thus lower conservatism.

Table 1. Quantitative comparison of conservatism between MD-BMADT and BAED-ADT.

Metric	MD-BMADT [34]	BAED-ADT (Proposed)	Improvement
Initial condition $x(t_0)$	$[1,\ -1]^{\mathrm{T}}$	$[1,\ -1]^{\mathrm{T}}$	–
Input $\omega(t)$	0	0	–
Max allowed ADT for subsystem 1 (τ_1^*)	1.4202	3.3577	136%
Max allowed ADT for subsystem 2 (τ_2^*)	1.2274	1.3756	12%
Feasible region (qualitative)	smaller	larger	less conservative
Peak state amplitude (approx.)	[−60, −68]	[−20, −26]	smaller
Convergence time (approx.)	30 s	21 s	faster

In [34], the largest upper bounds of the MD-BMADT are $\tau_1^* = 1.4202$, $\tau_1^* = 1.2274$. Here, both of the subsystems adopt fast switching and switching stable. From [34], let $\lambda_1 = 0.32$, $\lambda_2 = 0.12$, $\mu_{2,1}^{21} = 0.82$, $\mu_{1,2}^{21} = 0.95$, $\theta_1 = \theta_2 = 0.8$, $\tau_{m1} = \tau_{m2} = 0.6$, $J_i = 1$, $\rho_{2,1}^{11} = 0.9$, $\rho_{1,2}^{11} = 0.8$, $\tau_{\delta(2,1)}^* = 0.5$, $\tau_{\delta(1,2)}^* = 2.5$. According to Remark 4, the upper bounds of the BAED-ADT are obtained where $\tau_{f(2,1)}^* = -\frac{\ln\mu_{2,1}^{21}+\theta_1 * \frac{\ln\rho_{2,1}^{11}}{\tau_{\delta(2,1)}^*}}{\lambda_1+\frac{\ln\rho_{2,1}^{11}}{\tau_{\delta(2,1)}^*}} = 3.3577 > 1.4202$, $\tau_{f(1,2)}^* = -\frac{\ln\mu_{1,2}^{21}+\theta_2 * \frac{\ln\rho_{1,2}^{11}}{\tau_{\delta(1,2)}^*}}{\lambda_2+\frac{\ln\rho_{1,2}^{11}}{\tau_{\delta(1,2)}^*}} = 1.3756 > 1.2274$. It is obvious that the MD-BMADT method in [34] is a special case for the BAED-ADT method in this paper. Finally, we get the simulation result in Figure 4. From Figure 4, we can see that the state converges to zero when $t = 21$ and the peak of the state is about $[-20, -26]$. The state oscillations are relatively flat. Thus, we compare the state response graphs obtained by the two methods, as shown in Table 2. From Table 2, we can obtain that the bound of the BAED-ADT is large the MD-BMADT, i.e., the case of [34] is a special example of this paper. Meanwhile, the peak of [34] is larger than this paper, the state oscillations are relatively large, and the converge time is relatively long. Thus, we can get that the results in this paper have less conservativeness than [34]. Finally, according to Theorem 1, we can obtain that the system is ISS under the BAED-ADT.

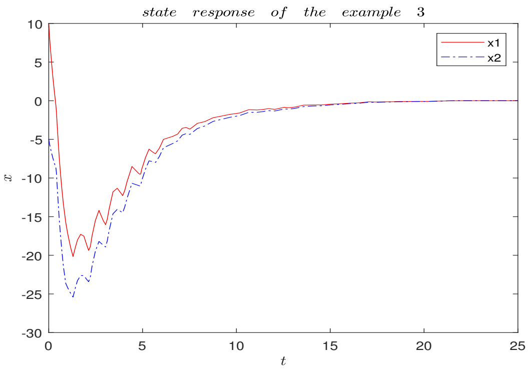

Figure 4. Simulation results for Example 3.

Table 2. Comparison between MD-BMADT and BAED-ADT.

Switching schemes	MD-BMADT	BAED-ADT
switching signal	$\tau_1^* = 1.4202$ $\tau_2^* = 1.2274$	$\tau_{f,(2,1)}^* = 3.3577$ $\tau_{f,(1,2)}^* = 1.3756$
time of convergence	$t = 30$	$t = 21$
amplitude of oscillation	*large*	*flat*
peak	$[-60, -68]$	$[-20, -26]$

5. Conclusions

In this paper, we investigate the unified ISS problem for continuous-time impulsive switched systems, which can be modeled by the fuzzy theorem. First, a novel method named the BAED-ADT approach is proposed. For stable subsystems, we adopt the slow switching method. For unstable subsystems, we can adopt either the slow switching method or fast switching method. Then, the switching stable subsystems and the stabilizing impulses can together compensate the state divergence caused by switching unstable subsystems or destabilizing impulses. By designing the time-scheduled Lyapunov function, the unified ISS conditions are concluded. Finally, several examples are provided to illustrate the flexibility and validity of the conclusions. Since the fuzzy method has great advantages in state estimation and model prediction for uncertain systems, we will plan to study this problem in future work.

Use of Generative-AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgements

The author thanks the editors and reviewers sincerely for their insightful suggestions which improved this work significantly. This work was supported by Zhejiang Pharmaceutical University 2024 Annual University-level Online Quality Courses for Vocational Undergraduate

Program.

Conflict of interest

The author declares no conflict of interest.

References

1. D. Liberzon, *Switching in systems and control*, Boston: Birkhauser, 2003. http://dx.doi.org/10.1007/978-1-4612-0017-8
2. J. Lian, C. Li, B. Xia, Sampled-data control of switched linear systems with application to an f-18 aircraft, *IEEE Trans. Ind. Electron.*, **64** (2017), 1332–1340. http://dx.doi.org/10.1109/TIE.2016.2618872
3. P. Wang, J. Zhao, Feedback dissipativity and stabilization for switched positive systems with a combined switch ing law, *IEEE Trans. Circuits II*, **67** (2019), 2572–2576. http://dx.doi.org/10.1109/TCSII.2019.2962283
4. P. Wang, D. Yang, Stability and L1-gain analysis for switched positive fuzzy systems with time-delay: a state dependent switching policy, *Fuzzy Set. Syst.*, **464** (2023), 108440. http://dx.doi.org/10.1016/j.fss.2022.11.011
5. J. M. Hu, Y. W. Wang, J. W. Xiao, W. Yang, L1-gain analysis and control of impulsive positive systems with interval uncer tainty and time delay, *J. Franklin I.*, **356** (2019), 9180–9205. http://dx.doi.org/10.1016/j.jfranklin.2019.08.010
6. T. Liu, B. Wu, L. Liu, Y. Wang Asynchronously finite-time control of discrete impulsive switched positive time-delay systems, *J. Franklin I.*, **352** (2015), 4503–4514. http://dx.doi.org/10.1016/j.jfranklin.2015.06.015
7. Y. H. Ju, F. W. Meng, Y. G. Sun, Exponential stability of switched linear impulsive time-varying system and its application, *J. Franklin I.*, **359** (2022), 5619–5633. http://dx.doi.org/10.1016/j.jfranklin.2022.05.024
8. L. J. Gao, Z. B. Cao, M. Zhang, Q. X. Zhu, Input-to-state stability for hybrid delayed systems with admissible edge-dependent switching signals, *J. Franklin I.*, **357** (2020), 8823–8850. http://dx.doi.org/10.1016/j.jfranklin.2020.06.008
9. T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. Syst. Man Cy.*, **SMC-15** (1985), 116–132. http://dx.doi.org/10.1109/TSMC.1985.6313399
10. H. J. Gao, X. M. Liu, J. Lam, Stability analysis and stabilization for discrete-time fuzzy systems with time-varying delay, *IEEE Trans. Syst. Man Cy. B*, **39** (2009), 306–317. http://dx.doi.org/10.1109/TSMCB.2008.2003449
11. M. Li, P. Shi, M. Liu, Y. Zhang, S. Wang, Event-triggered-based adaptive sliding mode control for T-S fuzzy systems with actuator failures and signal quantization, *IEEE Trans. Fuzzy Syst.*, **29** (2021), 1363–1374. http://dx.doi.org/10.1109/TFUZZ.2020.2974175

12. G. Yang, F. Hao, L. Zhang, B. Li, Actuator saturation control of continuous-time positive switched T-S fuzzy systems, *J. Franklin I.*, **358** (2021), 8862–8885. http://dx.doi.org/10.1016/j.jfranklin.2021.09.001
13. J. Zhang, W. H. Chen, X. Lu, Robust fuzzy stabilization of nonlinear time-delay systems subject to impulsive perturbations, *Commun. Nonlinear Sci.*, **80** (2020), 104953. http://dx.doi.org/10.1016/j.cnsns.2019.104953
14. J. H. Li, H. M. Wang, Fuzzy switching function-based sliding mode controller design for T-S fuzzy descriptor systems, *Inform. Sciences*, **624** (2023), 344–360. http://dx.doi.org/10.1016/j.ins.2022.12.073
15. W. Sun, S. Su, Y. Wu, J. Xia, V. Nguyen, Adaptive fuzzy control with high-order barrier Lyapunov functions for high-order uncertain nonlinear systems with full-state constraints, *IEEE Trans. Cybernetics*, **50** (2019), 3424–3432. http://dx.doi.org/10.1109/TCYB.2018.2890256
16. W. Sun, S. F. Su, Y. Wu, J. Xia, Novel adaptive fuzzy control for output constrained stochastic nonstrict feedback nonlinear systems, *IEEE Trans. Fuzzy Syst.*, **29** (2020), 1188–1197. http://dx.doi.org/10.1109/TFUZZ.2020.2969909
17. J. Joh, Y. H. Chen, R. Langari, On the stability issues of linear Takagi-Sugeno fuzzy models, *IEEE Trans. Fuzzy Syst.*, **6** (1998), 402–410. http://dx.doi.org/10.1109/91.705508
18. G. Feng, C. L. Chen, D. Sun, Y. Zhu, H_∞ controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions and bilinear matrix inequalities, *IEEE Trans. Fuzzy Syst.*, **13** (2005), 94–103. http://dx.doi.org/10.1109/TFUZZ.2004.839662
19. J. Lam, H. J. Gao, C. H. Wang, Stability analysis for continuous systems with two additive time-varying delay components, *Syst. Control Lett.*, **56** (2007), 16–24. http://dx.doi.org/10.1016/j.sysconle.2006.07.005
20. J. L. Zhang, H. G. Zhang, F. S. Yang, S. Wang, Robust fault detection filter design for a class of time-delay systems via equivalent transformation, *J. Control Theory Appl.*, **11** (2013), 54–60. http://dx.doi.org/10.1007/s11768-013-1097-z
21. J. H. Park, T. H. Li, Y. Liu, J. Chen, *Dynamic systems with time delays: stability and control*, Singapore: Springer-Nature, 2019. http://dx.doi.org/10.1007/978-981-13-9254-2
22. Y. Tian, Y. Sun, Exponential stability of switched nonlinear time-varying systems with mixed delays: comparison principle, *J. Franklin I.*, **357** (2020), 6918–6931. http://dx.doi.org/10.1016/j.jfranklin.2020.04.047
23. O. Kwon, S. Lee, M. Park, S. Lee, Augmented zero equality approach to stability for linear systems with time-varying delay, *Appl. Math. Comput.*, **381** (2020), 125329. http://dx.doi.org/10.1016/j.amc.2020.125329
24. L. Zhang, L. He, Y. Song, New results on stability analysis of delayed systems derived from extended Wirtinger's integral inequality, *Neurocomputing*, **283** (2018), 98–106. http://dx.doi.org/10.1016/j.neucom.2017.12.044
25. W. Zheng, H. Wang, F. Sun, S. Wen, Z. Zhang, H. Wang, New stability criteria for asymptotic stability of time-delay systems via integral inequalities and Jensen inequalities, *J. Inequal. Appl.*, **2019** (2019), 30. http://dx.doi.org/10.1186/s13660-019-1984-z

26. F. Long, L. Jiang, Y. He, M. Wu, Stability analysis of systems with time-varying delay via novel augmented Lyapunov Krasovskii functionals and an improved integral inequality, *Appl. Math. Comput.*, **357** (2019), 325–337. http://dx.doi.org/10.1016/j.amc.2019.04.004

27. D. Li, P. Cheng, L. Shang, Exponential stability analysis for stochastic functional differential systems with delayed impulsive effects: average impulsive interval approach, *Proceedings of 35th Chinese Control Conference (CCC)*, 2016, 1707–1712. http://dx.doi.org/10.1109/ChiCC.2016.7553338

28. X. Xie, X. Liu, H. Xu, Synchronization of delayed coupled switched neural networks: mode-dependent average impulsive interval, *Neurocomputing*, **365** (2019), 261–272. http://dx.doi.org/10.1016/j.neucom.2019.07.045

29. L. Gao, H. Liu, J. Park, Z. Cao, Input-to-state stability of discrete-time switched delayed systems with delay dependent impulses: admissible edge-dependent average impulsive interval, *Int. J. Robust Nonlin.*, **32** (2022), 6236–6266. http://dx.doi.org/10.1002/rnc.6132

30. X. Li, P. Li, Q. Wang, Input/output-to-state stability of impulsive switched systems, *Syst. Control Lett.*, **116** (2018), 1–7. http://dx.doi.org/10.1016/j.sysconle.2018.04.001

31. A. Y. Lu, G. H. Yang, Stabilization of switched systems with all modes unstable via periodical switching laws, *Automatica*, **122** (2020), 109150. http://dx.doi.org/10.1016/j.automatica.2020.109150

32. Y. Liu, Q. Zhu, N. Zhao, Event-triggered adaptive fuzzy control for switched nonlinear systems with state constraints, *Inform. Sciences*, **562** (2021), 28–43. http://dx.doi.org/10.1016/j.ins.2021.01.030

33. L. Zhang, H. Gao, Asynchronously switched control of switched linear systems with average dwell time, *Automatica*, **46** (2010), 953–958. http://dx.doi.org/10.1016/j.automatica.2010.02.021

34. X. Mao, H. Zhu, W. Chen, H. Zhang, New results on stability of switched continuous-time systems with all subsystems unstable, *ISA Trans.*, **87** (2019), 28–33. http://dx.doi.org/10.1016/j.isatra.2018.11.042

35. C. Liu, X. Mao, X. Xu, H. Zhang, Stability analysis of discrete-time switched T-S fuzzy systems with all subsystems unstable, *IEEE Access*, **7** (2019), 50412–50418. http://dx.doi.org/10.1109/ACCESS.2019.2911689

36. C. Liu, X. Mao, Q. Zheng, H. Zhang, Unified stability criteria for continuous-time switched T-S fuzzy systems, *IET Control Theory Appl.*, **14** (2020), 2455–2461. http://dx.doi.org/10.1049/iet-cta.2019.1421

37. L. Hou, X. Zhao, H. Sun, G. Zong, l_2-l_∞ filtering of discrete-time switched systems via admissible edge-dependent switching signals, *Syst. Control Lett.*, **113** (2018), 17–26. http://dx.doi.org/10.1016/j.sysconle.2017.10.005

38. P. He, H. Zhang, S. F. Su, A sliding mode control method with variable convergence rate for nonlinear impulsive stochastic systems, *IEEE Trans. Cybernetics*, **55** (2025), 2213–2222. http://dx.doi.org/10.1109/TCYB.2025.3551668

39. H. Geng, H. Zhang, S. Su, Asynchronously switched control with variable convergence rate for switched nonlinear systems: a persistent dwell-time scheme, *IEEE Trans. Fuzzy Syst.*, **32** (2024), 6695–6707. http://dx.doi.org/10.1109/TFUZZ.2024.3459860

40. T. Zhang, H. Zhang, X. Xie, Region stability/stabilization and H_∞ control for discrete-time impulsive Takagi-Sugeno fuzzy systems, *IEEE Trans. Fuzzy Syst.*, **32** (2024), 3410–3419. http://dx.doi.org/10.1109/TFUZZ.2024.3372936

41. T. Sun, T. Liu, X. M. Sun, Stability analysis of cyclic switched linear systems: an average cycle dwell time approach, *Inform. Sciences*, **544** (2021), 227–237. http://dx.doi.org/10.1016/j.ins.2020.07.053

42. E. D. Sontag, Smooth stabilization implies coprime factorization, *IEEE Trans. Automat. Contr.*, **34** (1989), 435–443. http://dx.doi.org/10.1109/9.28018

AIMS Press