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Research article

p-adic Bochner–Riesz operators and their functional dynamics

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Abstract: In this article, we introduced and investigated two new families of p-adic pseudo-differential operators arising from Bochner–Riesz-type constructions in the non-Archimedean setting. The first family is given by p-adic Bochner–Riesz operators, while the second consists of operators whose symbols are naturally derived from and closely related to the Bochner–Riesz framework. These classes of operators provide new examples of nonlocal operators in p-adic analysis and offer a broader perspective on the role of radial symbols in the study of evolution processes over ultrametric spaces. Our main objective was to analyze the evolutionary dynamics generated by these operators through the construction and study of the associated convolution semigroups. In particular, we derived and investigated the corresponding evolution equations and examined the fundamental kernels that govern their solutions. Special attention was devoted to understanding how the different symbolic structures of the two operator families affect the analytical and qualitative properties of the resulting kernels, including their regularity, propagation behavior, and semigroup characteristics. The results revealed significant differences between the dynamics generated by the two classes of operators, highlighting the influence of the underlying symbols on the associated evolution processes. Furthermore, our approach establishes new connections between harmonic analysis on p-adic fields, pseudo-differential operator theory, and non-Archimedean evolution equations. These findings contribute to the development of p-adic analysis and provide a foundation for future applications in non-Archimedean mathematical physics, stochastic processes, and operator theory.

Keywords: p-adic analysis; p-adic pseudo-differential operators; Bochner–Riesz operators; evolution equations; convolution semigroups; m-dissipative operators
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1. Introduction

Bochner–Riesz operators occupy a central position in classical harmonic analysis on Euclidean spaces, where they arise naturally as regularization devices in problems related to Fourier inversion and summability. Since their introduction, these operators have played a fundamental role in the study of convergence of Fourier integrals, restriction phenomena, dispersive partial differential equations, and smoothing effects associated with evolution equations. The analysis of classical Bochner–Riesz operators is deeply intertwined with the geometry of the Euclidean unit sphere, oscillatory integral techniques, and subtle curvature effects, leading to a rich and highly nontrivial theory whose behavior depends delicately on both the dimension and the order of the operator. For further details, the reader is referred to [6, 10, 13, 15], and the references therein.

In contrast, the non-Archimedean setting presents a radically different analytical and geometric landscape. The field of p-adic numbers and its higher-dimensional analogues are totally disconnected ultrametric spaces that lack the smooth geometric structures underlying the classical theory. Consequently, many of the standard tools of Euclidean harmonic analysis do not transfer directly to the p-adic context.

Nevertheless, p-adic analysis has evolved into a rich and independent framework, motivated both by its intrinsic mathematical interest and by a wide range of applications. For instance, non-Archimedean methods have been successfully employed in mathematical models arising in epidemiology and porous media flows [1, 3]. They also play an important role in p-adic mathematical physics and the theory of ultrametric pseudo-differential equations [5, 7]. Connections with stochastic analysis and mathematical physics have been explored through non-Archimedean white noise theory, stochastic pseudo-differential equations, and Coulomb gas models [20, 21]. Applications to geophysical diffusion processes have been investigated in [9], while the development of non-Archimedean pseudo-differential operators and their applications has been studied in [16–19]. See also the references therein for further developments.

Despite the importance of Bochner–Riesz operators in the Archimedean setting, their p-adic counterparts have not been systematically investigated. In fact, p-adic Bochner–Riesz operators are not part of the classical theory of pseudo-differential operators on local fields, and their introduction requires new ideas adapted to the ultrametric structure of $\mathbb{Q}_p^n$. The purpose of this article is precisely to fill this gap by introducing and studying two new families of p-adic pseudo-differential operators, both inspired by Bochner–Riesz-type constructions but exhibiting markedly different analytical behaviors.

The first family introduced in this work consists of what we call p-adic Bochner–Riesz operators. These operators are defined via truncated radial symbols of the form

$$(1 - \|\xi\|_p)_+^\alpha,$$

which are compactly supported and piecewise constant with respect to the p-adic norm. Although this expression formally resembles the classical Bochner–Riesz multiplier, its p-adic realization leads to fundamentally new phenomena. In particular, the associated kernels are radial, explicitly computable, and generate generalized convolution semigroups on $\mathbb{Q}_p^n$. This semigroup structure, established in Theorem 4.2, has no direct analogue in the classical Bochner–Riesz theory and constitutes one of the main novelties of the present work.

A second family of pseudo-differential operators is introduced through symbols of exponential type naturally associated with the p-adic Bochner–Riesz multipliers. These operators, while closely related to the first family, give rise to kernels with substantially different qualitative properties. Most notably, although the corresponding symbols are strictly positive, the associated kernels fail to be positive and are not integrable. This unexpected behavior reflects the oscillatory nature of the p-adic Fourier transform and highlights a sharp contrast with both classical diffusion operators and standard p-adic heat kernels.

A central theme of this article is the study of the evolutionary dynamics generated by these two families of operators. For the p-adic Bochner–Riesz operators, we prove in Theorem 5.2 that the family $(\mathfrak{B}_\alpha)_{\alpha>0}$ converges strongly, as $\alpha \to 0^+$, to the Fourier projection onto the frequency ball B^n_{-1}. More precisely, if

$$Pf = \mathcal{F}^{-1}(\mathbf{1}_{B^n_{-1}}\widehat{f}),$$

then

$$\lim_{\alpha\to 0^+} \|\mathfrak{B}_\alpha f - Pf\|_{L^p(\mathbb{Q}_p^n)} = 0.$$

This phenomenon reflects the fact that the symbol $(1 - \|\xi\|_p)_+^\alpha$ converges pointwise to the characteristic function of the ball B^n_{-1} rather than to the constant function 1.

The corresponding fundamental solution, introduced in Section 5, is analyzed in detail. In Lemma 5, we derive an explicit representation formula for this kernel and show that, despite the positivity of the symbol, the kernel itself is non-positive away from the origin. This phenomenon has no classical analogue and underscores the distinctive nature of p-adic evolution equations driven by Bochner–Riesz-type operators. The well-posedness of the associated Cauchy problem is then established in Theorem 5.5, providing a complete description of the solution in terms of convolution with the fundamental kernel.

The second family of operators is investigated from an operator-theoretic perspective. We show that the corresponding pseudo-differential operator is self-adjoint and dissipative, and we prove in Theorem 6.2 that it is, in fact, m-dissipative on $L^2(\mathbb{Q}_p^n)$. This result allows us to apply the Hille–Yosida–Phillips theory and associate a contraction semigroup with the operator. The abstract inhomogeneous evolution problem generated by this semigroup is solved in Theorem 6.4, yielding a variation-of-constants formula that further illustrates the dynamical richness of the model.

Beyond the specific results obtained, this article opens several directions for future research. We conclude the introduction by formulating two open problems that naturally arise from our analysis.

Open Problem 1. Determine the precise L^p–L^q mapping properties of p-adic Bochner–Riesz operators for $p \neq q$, and investigate whether non-Archimedean analogues of the classical Bochner–Riesz summability thresholds exist.

Open Problem 2. Study stochastic processes and Markov dynamics associated with suitable modifications of the p-adic Bochner–Riesz kernels, with the goal of understanding whether probabilistic interpretations can be developed despite the lack of kernel positivity.

The structure of the paper is as follows. In Section 2, we recall the necessary background on Fourier analysis and distributions on $\mathbb{Q}_p^n$. In Section 3, we introduce truncated Bochner–Riesz-type symbols and establish their fundamental analytical properties. In Section 4, we construct the convolution semigroups associated with the corresponding kernels. In Section 5, we define and analyze p-adic Bochner–Riesz pseudo-differential operators and their fundamental solutions. In Section 6, we introduce the second family of pseudo-differential operators and study their dissipativity and semigroup generation properties. Finally, in Section 7, we provide a detailed comparison between the Archimedean and non-Archimedean Bochner–Riesz frameworks, highlighting both the analogies and the profound structural differences.

2. Fourier analysis on $\mathbb{Q}_p^n$

Let p be a fixed prime number. We denote by $\mathbb{Q}_p$ the field of p-adic numbers, which admits the representation

$$\mathbb{Q}_p = \left\{ \sum_{i=k}^{\infty} a_i p^i : k \in \mathbb{Z},\ a_i \in \{0, 1, \ldots, p-1\},\ a_k \neq 0 \right\}.$$

The field $\mathbb{Q}_p$ is obtained as the completion of the rational numbers $\mathbb{Q}$ with respect to the p-adic absolute value $|\cdot|_p$, defined by

$$|x|_p = \begin{cases} 0, & \text{if } x = 0, \\ p^{-\gamma}, & \text{if } x = p^{\gamma}\dfrac{a}{b}, \end{cases}$$

where $a, b \in \mathbb{Z}$ are integers not divisible by p. The integer $\gamma = \mathrm{ord}_p(x)$, with the convention $\mathrm{ord}_p(0) = +\infty$, is called the *p-adic order* of x.

For an element $x = \sum\limits_{i=k}^{\infty} a_i p^i \in \mathbb{Q}_p$, $k = \mathrm{ord}_p(x)$, we define its *fractional part* by

$$\{x\}_p = \begin{cases} 0, & \text{if } x = 0 \text{ or } \mathrm{ord}_p(x) \geq 0, \\ \sum\limits_{i=k}^{-1} a_i p^i, & \text{if } \mathrm{ord}_p(x) < 0. \end{cases}$$

Let $\mathbb{Q}_p^n := \mathbb{Q}_p \times \cdots \times \mathbb{Q}_p$ denote the n-dimensional p-adic vector space, whose elements are written as $x = (x_1, \ldots, x_n)$ with $x_i \in \mathbb{Q}_p$. The norm on $\mathbb{Q}_p^n$ is defined by

$$\|x\|_p := \max_{1 \leq i \leq n} |x_i|_p,$$

which induces an ultrametric topology on $\mathbb{Q}_p^n$.

For $y \in \mathbb{Q}_p^n$ and $m \in \mathbb{Z}$, we define the ball and sphere of radius p^m centered at y as

$$B_m^n(y) = \{x \in \mathbb{Q}_p^n : \|x - y\|_p \leq p^m\}, \qquad S_m^n(y) = \{x \in \mathbb{Q}_p^n : \|x - y\|_p = p^m\},$$

respectively. For simplicity, we write $B_m^n = B_m^n(0)$ and $S_m^n = S_m^n(0)$. In particular, $B_0^n = \mathbb{Z}_p^n$ coincides with the ring of p-adic integers.

Balls and spheres are compact subsets of $\mathbb{Q}_p^n$. Moreover, the metric space $(\mathbb{Q}_p^n, \|\cdot\|_p)$ is totally disconnected and locally compact. The space $\mathbb{Q}_p^n$ carries a Haar measure $d^n x$, normalized by

$$\int_{\mathbb{Z}_p^n} d^n x = 1.$$

For $1 \leq \rho < \infty$, we denote by $L^\rho(\mathbb{Q}_p^n)$ the space of all complex-valued functions g on $\mathbb{Q}_p^n$ such that

$$\int_{\mathbb{Q}_p^n} |g(x)|^\rho \, d^n x < \infty.$$

Equipped with the norm

$$\|g\|_{L^\rho(\mathbb{Q}_p^n)} = \left(\int_{\mathbb{Q}_p^n} |g(x)|^\rho \, d^n x \right)^{1/\rho},$$

this space is a Banach space.

A function $\varphi : \mathbb{Q}_p^n \to \mathbb{C}$ is said to be *locally constant* if for every $x \in \mathbb{Q}_p^n$, there exists an integer $m = m(x)$ such that

$$\varphi(x') = \varphi(x) \quad \text{for all } x' \in B_m^n(x).$$

We denote by $\mathcal{E}(\mathbb{Q}_p^n)$ the vector space of all locally constant complex-valued functions on $\mathbb{Q}_p^n$.

A locally constant function with compact support is called a *Bruhat–Schwartz function* (or a test function). The space of such functions is denoted by $\mathcal{D}(\mathbb{Q}_p^n)$, and its dual $\mathcal{D}'(\mathbb{Q}_p^n)$ consists of all distributions on $\mathbb{Q}_p^n$. Any function $f \in L^1_{\mathrm{loc}}(\mathbb{Q}_p^n)$ induces a distribution by

$$\langle f, \varphi \rangle = \int_{\mathbb{Q}_p^n} f(x)\varphi(x) \, d^n x, \qquad \varphi \in \mathcal{D}(\mathbb{Q}_p^n).$$

For $f \in L^1(\mathbb{Q}_p^n)$, the Fourier transform of f is denoted and defined by

$$(\mathcal{F}_{x\to\xi} f)(\xi) = (\mathcal{F} f)(\xi) = \widehat{f}(\xi) = \int_{\mathbb{Q}_p^n} \chi_p(\xi \cdot x) \, f(x) \, d^n x, \qquad \xi \in \mathbb{Q}_p^n,$$

where $\xi \cdot x = \sum_{j=1}^n \xi_j x_j$ and $\chi_p(\xi \cdot x) = e^{2\pi i \{\xi \cdot x\}_p}$ is the standard additive character. The inverse Fourier transform of f is denoted and defined by

$$(\mathcal{F}^{-1}_{\xi\to x} f)(x) = (\mathcal{F}^{-1} f)(x) = \int_{\mathbb{Q}_p^n} \chi_p(-x \cdot \xi) \, f(\xi) \, d^n \xi.$$

The space $L^2(\mathbb{Q}_p^n)$ is a Hilbert space endowed with the inner product

$$(f, g) = \int_{\mathbb{Q}_p^n} f(x)\overline{g(x)} \, d^n x.$$

The Fourier transform extends uniquely to a unitary operator on $L^2(\mathbb{Q}_p^n)$ and satisfies the Parseval–Steklov identity

$$(f, g) = (\mathcal{F} f, \mathcal{F} g), \qquad \|f\|_{L^2(\mathbb{Q}_p^n)} = \|\mathcal{F} f\|_{L^2(\mathbb{Q}_p^n)}.$$

For further details and background material, we refer the reader to [2, 14, 19].

3. Bochner–Riesz-type kernels on $\mathbb{Q}_p^n$

In this section, we introduce a family of radial convolution kernels on $\mathbb{Q}_p^n$ associated with truncated Bochner–Riesz-type symbols. We analyze their basic analytic properties and study the limiting behavior of the symbols as the truncation parameter tends to zero, a fact that will be used in subsequent constructions.

We denote by $\mathbb{R}_+ := \{x \in \mathbb{R} : x \geq 0\}$ the set of non-negative real numbers and by $\mathbb{N} := \{1, 2, \ldots\}$ the set of natural numbers.

For a fixed $\alpha > 0$, we define the map

$$(1 - \|\cdot\|_p)_+^\alpha : \mathbb{Q}_p^n \to \mathbb{R}_+$$

by

$$(1 - \|\xi\|_p)_+^\alpha := (\max\{0, 1 - \|\xi\|_p\})^\alpha, \quad \xi \in \mathbb{Q}_p^n.$$

Remark 3.1. *In the classical Euclidean theory of Bochner–Riesz operators, one considers multipliers of the form*

$$(1 - |\xi|/R)_+^\alpha$$

and studies the limit $R \to \infty$, which corresponds to an expanding family of frequency cutoffs.

In the p-adic setting, the ultrametric structure implies a rigid scaling behavior: If $R = p^k$, then the change of variables $\xi \mapsto R\xi$ shows that the corresponding operator is unitarily equivalent (up to normalization) to the case $R = 1$. Therefore, varying R does not produce a genuine asymptotic regime but only a reparametrization of the same operator class.

For this reason, we fix $R = 1$ throughout and consider instead the limit $\alpha \to 0^+$, which recovers the sharp frequency cutoff

$$\mathbf{1}_{B_{-1}^n},$$

rather than the identity multiplier (see Lemma 2).

From now on, we shall refer to the function $(1-\|\cdot\|_p)_+^\alpha$ as the *truncated Bochner–Riesz-type symbol*.

Remark 3.2. *It is clear that $(1 - \|\cdot\|_p)_+^\alpha$ is a radial, non-negative, continuous function with*

$$supp((1 - \|\cdot\|_p)_+^\alpha) = B_{-1}^n.$$

The above implies that

$$\begin{aligned}
\int_{\mathbb{Q}_p^n} (1 - \|\xi\|_p)_+^\alpha \, d^n\xi &= \int_{B_{-1}^n} (1 - \|\xi\|_p)_+^\alpha \, d^n\xi \\
&= \sum_{j=1}^{\infty} (1 - p^{-j})^\alpha \int_{\|\xi\|_p = p^{-j}} d^n\xi \\
&= (1 - p^{-n}) \sum_{j=1}^{\infty} (1 - p^{-j})^\alpha p^{-nj}
\end{aligned}$$

$$\leq \sum_{j=1}^{\infty}(p^{-nj} - p^{-n(j+1)})$$
$$= p^{-n}.$$

Therefore,

$$(1 - \|\cdot\|_p)_+^{\alpha} \in L^1(\mathbb{Q}_p^n), \text{ for any } \alpha > 0.$$

In what follows, we formulate and establish a number of properties of the function $(1 - \|\cdot\|_p)_+^{\alpha}$, $\alpha > 0$, that will be required in the remainder of the paper.

Lemma 1. *For fixed $\alpha > 0$, the function $(1-\|\cdot\|_p)_+^{\alpha}$, $\alpha > 0$, is non-increasing with respect to the p-adic norm. More precisely, if $\xi_1, \xi_2 \in \mathbb{Q}_p^n$ satisfy*

$$\|\xi_1\|_p \leq \|\xi_2\|_p,$$

then

$$(1 - \|\xi_1\|_p)_+^{\alpha} \geq (1 - \|\xi_2\|_p)_+^{\alpha}.$$

Proof. Let $\xi_1, \xi_2 \in \mathbb{Q}_p^n$ be such that $\|\xi_1\|_p \leq \|\xi_2\|_p$. We consider the following cases:

Case 1: $\xi_1 \in B_{-1}^n$ and $\xi_2 \in \mathbb{Q}_p^n \setminus B_{-1}^n$. Then $\|\xi_1\|_p < 1$ and $\|\xi_2\|_p \geq 1$, so that

$$(1 - \|\xi_1\|_p)_+^{\alpha} = (1 - \|\xi_1\|_p)^{\alpha} > 0, \qquad (1 - \|\xi_2\|_p)_+^{\alpha} = 0.$$

Hence,

$$(1 - \|\xi_1\|_p)_+^{\alpha} > (1 - \|\xi_2\|_p)_+^{\alpha}.$$

Case 2: $\xi_1, \xi_2 \in B_{-1}^n$. In this case, $0 \leq \|\xi_1\|_p \leq \|\xi_2\|_p < 1$, and therefore

$$1 - \|\xi_1\|_p \geq 1 - \|\xi_2\|_p.$$

Since the function $t \mapsto t^{\alpha}$ is increasing on $[0, 1]$ for $\alpha > 0$, it follows that

$$(1 - \|\xi_1\|_p)_+^{\alpha} = (1 - \|\xi_1\|_p)^{\alpha} \geq (1 - \|\xi_2\|_p)^{\alpha} = (1 - \|\xi_2\|_p)_+^{\alpha}.$$

Case 3: $\xi_1, \xi_2 \in \mathbb{Q}_p^n \setminus B_{-1}^n$. Then $\|\xi_1\|_p \geq 1$ and $\|\xi_2\|_p \geq 1$, and consequently

$$(1 - \|\xi_1\|_p)_+^{\alpha} = (1 - \|\xi_2\|_p)_+^{\alpha} = 0.$$

Combining the three cases, we conclude that

$$(1 - \|\xi_1\|_p)_+^{\alpha} \geq (1 - \|\xi_2\|_p)_+^{\alpha},$$

which proves the lemma. □

Lemma 2. *For every $\xi \in \mathbb{Q}_p^n$, one has*

$$\lim_{\alpha \to 0^+} (1 - \|\xi\|_p)_+^{\alpha} = \mathbf{1}_{B_{-1}^n}(\xi),$$

where $\mathbf{1}_{B_{-1}^n}(\xi)$ denotes the indicator function of the ball B_{-1}^n.

Proof. Fix $\xi \in \mathbb{Q}_p^n$. We analyze the limit

$$\lim_{\alpha\to 0^+}(1-\|\xi\|_p)_+^\alpha$$

by distinguishing cases according to the value of $\|\xi\|_p$.

Case 1: $0 < \|\xi\|_p \leq p^{-1}$. In this case, $0 < 1-\|\xi\|_p < 1$, and hence

$$(1-\|\xi\|_p)_+^\alpha = (1-\|\xi\|_p)^\alpha.$$

Using the exponential representation,

$$(1-\|\xi\|_p)^\alpha = \exp\left(\alpha\log(1-\|\xi\|_p)\right).$$

Since $\log(1-\|\xi\|_p) < 0$ is a finite real number, we obtain

$$\lim_{\alpha\to 0^+} \alpha\log(1-\|\xi\|_p) = 0,$$

and therefore

$$\lim_{\alpha\to 0^+}(1-\|\xi\|_p)_+^\alpha = 1.$$

Case 2: $\|\xi\|_p = 1$. In this case,

$$1-\|\xi\|_p = 0,$$

and consequently

$$(1-\|\xi\|_p)_+^\alpha = 0^\alpha = 0 \qquad \text{for all } \alpha > 0.$$

Hence,

$$\lim_{\alpha\to 0^+}(1-\|\xi\|_p)_+^\alpha = 0.$$

Note that no indeterminate expression of the form 0^0 appears, since the function is never evaluated at $\alpha = 0$.

Case 3: $\|\xi\|_p > 1$. In this case,

$$1-\|\xi\|_p < 0,$$

and therefore

$$(1-\|\xi\|_p)_+^\alpha = 0 \qquad \text{for all } \alpha > 0.$$

It follows immediately that

$$\lim_{\alpha\to 0^+}(1-\|\xi\|_p)_+^\alpha = 0.$$

Case 4: $\|\xi\|_p = 0$. In this case,

$$1-\|\xi\|_p = 1,$$

and therefore

$$(1-\|\xi\|_p)_+^\alpha = 1 \qquad \text{for all } \alpha > 0.$$

It follows immediately that

$$\lim_{\alpha\to 0^+}(1-\|\xi\|_p)_+^\alpha = 1.$$

Combining the four cases, we conclude that

$$\lim_{\alpha\to 0^+}(1-\|\xi\|_p)_+^\alpha = \begin{cases} 1, & \text{if } \|\xi\|_p < 1, \\ 0, & \text{if } \|\xi\|_p \geq 1, \end{cases}$$

which is precisely the indicator function of the ball B^n_{-1}. □

Remark 3.3. *The limit in Lemma 2 does not involve any ambiguity of the form* 0^0*, since the exponent* α *is always strictly positive and the value* $\alpha = 0$ *is never substituted into the function.*

We now introduce a family of functions defined on $\mathbb{Q}_p^n$, denoted by $K_\alpha(x)$ for $\alpha > 0$, which we refer to as *Bochner–Riesz kernels*.

For $\alpha > 0$, we denote and define

$$K_\alpha(x) := \int_{\mathbb{Q}_p^n} \chi_p(-x\cdot\xi)(1-\|\xi\|_p)_+^\alpha \, d^n\xi, \quad x \in \mathbb{Q}_p^n.$$

Lemma 3. *For any fixed* $\alpha > 0$*,* K_α *is a radial, non-negative function and*

$$\lim_{\|x\|_p\to\infty} K_\alpha(x) = 0.$$

Proof. Let $x \in \mathbb{Q}_p^n$ be arbitrary. We distinguish two cases. First, if $x = 0$, then

$$K_\alpha(0) = \sum_{j=1}^{\infty}(1-p^{-j})^\alpha \int_{\|\xi\|_p=p^{-j}} d^n\xi = (1-p^{-n})\sum_{j=1}^{\infty}(1-p^{-j})^\alpha p^{-nj} > 0.$$

Assume next that $x \neq 0$. Let $x = p^\gamma x_0$, where $\gamma \in \mathbb{Z}$ and $\|x_0\| = 1$. Then $\|x\|_p = p^{-\gamma}$ and

$$K_\alpha(x) = \int_{\mathbb{Q}_p^n} \chi_p(-p^\gamma\xi\cdot x_0)(1-\|\xi\|_p)_+^\alpha \, d^n\xi.$$

Making the change of variable $\mu = p^\gamma\xi$, we have that $\|\mu\|_p = p^{-\gamma}\|\xi\|_p$ and $d^n\mu = p^{-n\gamma}d^n\xi$. Then,

$$\begin{aligned} K_\alpha(x) &= p^{n\gamma}\int_{\mathbb{Q}_p^n} \chi_p(-\mu\cdot x_0)(1-p^\gamma\|\mu\|_p)_+^\alpha \, d^n\mu \\ &= \|x\|_p^{-n}\sum_{-\infty<j<\infty}(1-\|x\|_p^{-1}p^j)_+^\alpha \int_{\|\mu\|_p=p^j}\chi_p(-\mu\cdot x_0)d^n\mu. \end{aligned}$$

Using the formula

$$\int_{\|\mu\|_p=p^j} \chi_p(-\mu\cdot x_0)d^n\mu = \begin{cases} p^{nj}(1-p^{-n}), & \text{if } j \leq 0, \\ -p^{n(j-1)}, & \text{if } j = 1, \\ 0, & \text{if } j \geq 2, \end{cases}$$

and the identity

$$(1-p^{-n})\sum_{j=0}^{\infty} p^{-nj} = 1,$$

we obtain

$$\begin{aligned}
K_\alpha(x) &= \|x\|_p^{-n}\left((1-p^{-n})\sum_{j=-\infty}^{0} p^{nj}(1-\|x\|_p^{-1}p^{j})_+^\alpha - (1-\|x\|_p^{-1}p)_+^\alpha\right)\\
&= \|x\|_p^{-n}\left((1-p^{-n})\sum_{j=0}^{\infty} p^{-nj}(1-\|x\|_p^{-1}p^{-j})_+^\alpha - (1-\|x\|_p^{-1}p)_+^\alpha\right)\\
&= \|x\|_p^{-n}(1-p^{-n})\sum_{j=0}^{\infty} p^{-nj}\left((1-\|x\|_p^{-1}p^{-j})_+^\alpha - (1-\|x\|_p^{-1}p)_+^\alpha\right)\\
&\geq 0.
\end{aligned}$$

Additionally, the equality

$$K_\alpha(x) = \|x\|_p^{-n}(1-p^{-n})\sum_{j=0}^{\infty} p^{-nj}\left((1-\|x\|_p^{-1}p^{-j})_+^\alpha - (1-\|x\|_p^{-1}p)_+^\alpha\right)$$

also implies that K_α is a radial function and that

$$\lim_{\|x\|_p\to\infty} K_\alpha(x) = 0.$$

□

4. Generalized convolution semigroups on $\mathbb{Q}_p^n$

Definition 1. A family $(\mu_t)_{t>0}$ of positive bounded measures on $\mathbb{Q}_p^n$ with the properties

(i) $\mu_t(\mathbb{Q}_p^n) \leq 1$ for $t > 0$,
(ii) $\mu_t * \mu_s = \mu_{t+s}$ for $t, s > 0$

is called a *generalized convolution semigroup* on $\mathbb{Q}_p^n$.

Remark 4.1. *Let $\mathcal{B}$ denote the σ-algebra of Borel subsets of $\mathbb{Q}_p^n$. In other words, $\mathcal{B}$ is the σ-algebra generated by the open subsets of $\mathbb{Q}_p^n$ with respect to the p-adic topology.*

Moreover, if $B \in \mathcal{B}$, we shall use the notation

$$\int_B K_\alpha(x)\, d^n x$$

to denote the measure of B induced by K_α.

The purpose of the next result is to show that the family $(K_\alpha)_{\alpha>0}$ gives rise to a generalized convolution semigroup on $\mathbb{Q}_p^n$. To this end, we will check that the associated measures $K_\alpha(x)\,d^nx$ are positive, bounded, and satisfy properties (i)–(ii) of Definition 1.

Theorem 4.2. *The family $(K_\alpha)_{\alpha>0}$ determines a generalized convolution semigroup on $\mathbb{Q}_p^n$.*

Proof. We verify the conditions of Definition 1 step by step.

(i) By the definition of K_α for $\alpha > 0$, together with Remark 3.2 and [14, Theorem (1.1), – p. 117], the function K_α is continuous. Hence, using the topology of $\mathbb{Q}_p^n$, for any family of n-dimensional balls $\{B_i\}_{i\in I}$, with I finite or countable, such that

$$B_i \cap B_j = \varnothing \quad \text{for all } i \neq j,$$

one has

$$\int_{\bigsqcup B_i} K_\alpha(x)\,d^nx = \sum_i \int_{B_i} K_\alpha(x)\,d^nx.$$

Since $K_\alpha(x) \geq 0$, it follows that the family $(K_\alpha)_{\alpha>0}$ defines positive measures on $\mathbb{Q}_p^n$.
On the other hand, for any $\alpha > 0$, we have that

$$\begin{aligned}
&\mathcal{F}(K_\alpha)(0) = 1,\\
&\mathcal{F}(K_\alpha)(\xi) = \int_{\mathbb{Q}_p^n} \chi_p(\xi\cdot x)K_\alpha(x)d^nx,\\
&\mathcal{F}(K_\alpha)(0) = \int_{\mathbb{Q}_p^n} K_\alpha(x)d^nx.
\end{aligned}$$

Consequently,

$$\int_{\mathbb{Q}_p^n} K_\alpha(x)d^nx = 1. \tag{4.1}$$

Therefore, $(K_\alpha)_{\alpha>0}$ determines a family of bounded, positive measures on $\mathbb{Q}_p^n$.

(ii) For any choice of $\alpha_1, \alpha_2 > 0$ and $x \in \mathbb{Q}_p^n$, we have that

$$\begin{aligned}
K_{\alpha_1+\alpha_2}(x) &= \int_{\mathbb{Q}_p^n} \chi_p(-x\cdot\xi)(1-\|\xi\|_p)_+^{\alpha_1+\alpha_2}\,d^n\xi\\
&= \int_{\mathbb{Q}_p^n} \chi_p(-x\cdot\xi)(1-\|\xi\|_p)_+^{\alpha_1}(1-\|\xi\|_p)_+^{\alpha_2}\,d^n\xi\\
&= \mathcal{F}^{-1}_{\xi\to x}(1-\|\xi\|_p)_+^{\alpha_1} * \mathcal{F}^{-1}_{\xi\to x}(1-\|\xi\|_p)_+^{\alpha_2}\\
&= (K_{\alpha_1} * K_{\alpha_2})(x).
\end{aligned}$$

□

Our aim in what follows is to establish the strong continuity of the convolution semigroup $(K_\alpha)_{\alpha>0}$ on the spaces $L^\rho\left(\mathbb{Q}_p^n\right)$.

Lemma 4. *Let $\alpha > 0$ and let $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$. Then the convolution*

$$(K_\alpha * \varphi)(x) = \int\limits_{\mathbb{Q}_p^n} K_\alpha(y)\,\varphi(x-y)\,d^n y$$

is a test function. More precisely,

$$K_\alpha * \varphi \in \mathcal{D}(\mathbb{Q}_p^n).$$

Proof. Let $\alpha > 0$ be fixed and let $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$. By definition,

$$(K_\alpha * \varphi)(x) = \int_{\mathbb{Q}_p^n} K_\alpha(y)\,\varphi(x-y)\,d^n y, \qquad x \in \mathbb{Q}_p^n.$$

By virtue of (4.1) and [2, (4.4.3), – p. 60], the kernel K_α induces a regular distribution on $\mathbb{Q}_p^n$. Therefore, by [2, Proposition 4.7.7], we have that $K_\alpha * \varphi$ is a locally constant function on $\mathbb{Q}_p^n$, i.e.,

$$K_\alpha * \varphi \in \mathcal{E}(\mathbb{Q}_p^n).$$

We next prove that $K_\alpha * \varphi$ has compact support. Without loss of generality, let us assume that $supp(\varphi) = B_R^n$ for some $R \in \mathbb{Z}$.

If $(K_\alpha * \varphi)(x) \neq 0$, then there exists $y \in \mathbb{Q}_p^n$ such that

$$K_\alpha(y) \neq 0 \quad \text{and} \quad \varphi(x-y) \neq 0,$$

i.e.,

$$y \in supp(K_\alpha) = B_{-1}^n \quad \text{and} \quad x - y \in B_R^n \ \ (\text{ equivalently, } y \in x - B_R^n).$$

Hence,

$$(K_\alpha * \varphi)(x) = \int\limits_{B_{-1}^n \cap \left(x - B_R^n\right)} K_\alpha(y)\,\varphi(x-y)\,d^n y,$$

and consequently, $supp(K_\alpha * \varphi)$ is contained in a compact subset of $\mathbb{Q}_p^n$, and thus $K_\alpha * \varphi$ has compact support.

Combining the above arguments, we conclude that $K_\alpha * \varphi$ is a locally constant function with compact support, that is,

$$K_\alpha * \varphi \in \mathcal{D}(\mathbb{Q}_p^n).$$

□

5. p-adic Bochner–Riesz-type pseudo-differential operators

In this section, we introduce a family of pseudo-differential operators on $\mathbb{Q}_p^n$ whose symbols are of Bochner–Riesz-type. We establish their basic mapping properties on $L^\rho(\mathbb{Q}_p^n)$, prove a strong convergence result, and study the fundamental kernel naturally associated with these operators.

Remark 5.1. *As a consequence of Theorem 4.2 together with the p-adic version of Young's inequality (see [2, Theorem 5.2.2]), it follows that for every function* $f \in L^\rho(\mathbb{Q}_p^n)$, $1 \le \rho < \infty$, *and for any* $\alpha > 0$, *the estimate*

$$\|K_\alpha * f\|_{L^\rho(\mathbb{Q}_p^n)} \le \|f\|_{L^\rho(\mathbb{Q}_p^n)}$$

holds.

For $f \in L^\rho(\mathbb{Q}_p^n)$, $1 \le \rho < \infty$, and $\alpha > 0$, we define the pseudo-differential operator

$$\begin{aligned}(\mathfrak{B}_\alpha f)(x) &:= \int_{\mathbb{Q}_p^n} \chi_p(-x \cdot \xi)\,(1 - \|\xi\|_p)_+^\alpha\, \widehat{f}(\xi)\, d^n\xi \\ &= (K_\alpha * f)(x), \qquad x \in \mathbb{Q}_p^n.\end{aligned}$$

Consequently, by Remark 5.1, the operator

$$\mathfrak{B}_\alpha : L^\rho(\mathbb{Q}_p^n) \to L^\rho(\mathbb{Q}_p^n)$$

is well-defined and acts as a contraction on $L^\rho(\mathbb{Q}_p^n)$. We refer to $\mathfrak{B}_\alpha$ as *the p-adic Bochner–Riesz operator of order* α.

Theorem 5.2 (Strong convergence to the low-frequency projection)**.** *Let* $1 \le \rho < \infty$. *For* $f \in L^\rho(\mathbb{Q}_p^n)$, *we define the Fourier projection operator*

$$Pf := \mathcal{F}^{-1}(\mathbf{1}_{B_{-1}^n} \widehat{f}).$$

Then, for every $f \in L^\rho(\mathbb{Q}_p^n)$, *one has*

$$\lim_{\alpha \to 0^+} \|\mathfrak{B}_\alpha f - Pf\|_{L^\rho(\mathbb{Q}_p^n)} = 0.$$

Proof. Fix $1 \le \rho < \infty$ and let $f \in L^\rho(\mathbb{Q}_p^n)$. By the density of $\mathcal{D}(\mathbb{Q}_p^n)$ in $L^\rho(\mathbb{Q}_p^n)$ (see [14, (3.8), – p. 122]), given any $\varepsilon > 0$, there exists a function $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$ such that

$$\|f - \varphi\|_{L^\rho(\mathbb{Q}_p^n)} < \frac{\varepsilon}{3}. \tag{5.1}$$

We write

$$K_\alpha * f - Pf = K_\alpha * (f - \varphi) + (K_\alpha * \varphi - P\varphi) + P(\varphi - f).$$

Hence, by the triangle inequality,

$$\begin{aligned}\|K_\alpha * f - Pf\|_{L^\rho(\mathbb{Q}_p^n)} \le\ & \|K_\alpha * (f - \varphi)\|_{L^\rho(\mathbb{Q}_p^n)} \\ & + \|K_\alpha * \varphi - P\varphi\|_{L^\rho(\mathbb{Q}_p^n)} \\ & + \|P(\varphi - f)\|_{L^\rho(\mathbb{Q}_p^n)}.\end{aligned} \tag{5.2}$$

By Remark 5.1, we have

$$\|K_\alpha * (f - \varphi)\|_{L^\rho(\mathbb{Q}_p^n)} \le \|f - \varphi\|_{L^\rho(\mathbb{Q}_p^n)} < \frac{\varepsilon}{3}.$$

On the other hand, since

$$Pf = \mathcal{F}^{-1}(\mathbf{1}_{B^n_{-1}}\widehat{f}),$$

the operator P is a convolution operator associated with the kernel

$$\mathcal{F}^{-1}(\mathbf{1}_{B^n_{-1}}).$$

Therefore, by the p-adic Young inequality,

$$\|P(\varphi - f)\|_{L^p(\mathbb{Q}_p^n)} \leq \|\varphi - f\|_{L^p(\mathbb{Q}_p^n)} < \frac{\varepsilon}{3}.$$

It remains to analyze the middle term in (5.2). Since $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$, by Lemma 4, [2, Theorem 4.8.2], and [14, (3.8), – p. 122], we have

$$K_\alpha * \varphi - P\varphi, \quad \mathcal{F}(K_\alpha * \varphi - P\varphi) \in \mathcal{D}(\mathbb{Q}_p^n),$$

and

$$\widehat{\varphi} \in L^1(\mathbb{Q}_p^n).$$

Furthermore, for every $\xi \in \mathbb{Q}_p^n$,

$$\mathcal{F}(K_\alpha * \varphi - P\varphi)(\xi) = \Big((1 - \|\xi\|_p)_+^\alpha - \mathbf{1}_{B^n_{-1}}(\xi)\Big)\widehat{\varphi}(\xi).$$

Moreover,

$$\Big|(1 - \|\xi\|_p)_+^\alpha - \mathbf{1}_{B^n_{-1}}(\xi)\Big| \leq 1.$$

Hence, by the dominated convergence theorem together with Lemma 2, we obtain

$$\lim_{\alpha \to 0^+} \|K_\alpha * \varphi - P\varphi\|_{L^p(\mathbb{Q}_p^n)} = 0.$$

Combining the previous estimates in (5.2), we conclude that

$$\limsup_{\alpha \to 0^+} \|K_\alpha * f - Pf\|_{L^p(\mathbb{Q}_p^n)} \leq \frac{2\varepsilon}{3}.$$

Since $\varepsilon > 0$ is arbitrary, the result follows. □

We proceed to study key properties of the fundamental solution corresponding to the operator $\mathfrak{B}_\alpha$.

We define the *fundamental solution* or *Bochner–Riesz fundamental kernel* associated with the p-adic Bochner–Riesz operator $\mathfrak{B}_\alpha$ by

$$Z_t(x) = Z(x,t) := \int_{\mathbb{Q}_p^n} \chi_p(-x \cdot \xi)\, e^{-t(1-\|\xi\|_p)_+^\alpha}\, d^n\xi, \qquad x \in \mathbb{Q}_p^n,\ t > 0. \tag{5.3}$$

Lemma 5. *Let $t > 0$. The fundamental solution $Z_t(\cdot)$ associated with the p-adic Bochner–Riesz operator satisfies the following properties:*

(i) The function $Z_t(\cdot)$ admits the representation

$$Z_t(x) = \begin{cases} +\infty, & \text{if } x = 0, \\ (1-p^{-n})\|x\|_p^{-n}\left(\sum\limits_{j=0}^{\infty} p^{-nj}\Big(e^{-t(1-p^{-j}\|x\|_p^{-1})_+^{\alpha}} - e^{-t(1-p\|x\|_p^{-1})_+^{\alpha}}\Big)\right) \le 0, & \text{if } x \neq 0. \end{cases}$$

(ii) For all $s > 0$,

$$Z_t * Z_s = Z_{t+s}.$$

Proof. (i) Let $t > 0$ and $x \in \mathbb{Q}_p^n$ be arbitrary but fixed. If $x = 0$, then by Remark 3.2, we have that

$$\begin{aligned} Z_t(0) &= \int\limits_{\mathbb{Q}_p^n} e^{-t(1-\|\xi\|_p)_+^{\alpha}}\, d^n\xi \\ &= \int\limits_{B_{-1}^n} e^{-t(1-\|\xi\|_p)_+^{\alpha}}\, d^n\xi + \int\limits_{\mathbb{Q}_p^n \setminus B_{-1}^n} d^n\xi \\ &= +\infty. \end{aligned}$$

Assume that $x = p^{\gamma}x_0 \neq 0$, where $\|x_0\|_p = 1$ and $\gamma = ord_p(x)$. By performing the change of variables $u = p^{\gamma}\xi$, we obtain

$$\begin{aligned} Z_t(x) &= \int\limits_{\mathbb{Q}_p^n} \chi_p(-p^{\gamma}\xi \cdot x_0)\, e^{-t(1-\|\xi\|_p)_+^{\alpha}}\, d^n\xi \\ &= p^{n\gamma} \int\limits_{\mathbb{Q}_p^n} \chi_p(-u \cdot x_0)\, e^{-t(1-p^{\gamma}\|u\|_p)_+^{\alpha}}\, d^n u \\ &= \|x\|_p^{-n} \sum_{-\infty<j<\infty} e^{-t(1-p^{j}\|x\|_p^{-1})_+^{\alpha}} \int\limits_{\|u\|_p = p^j} \chi_p(-u \cdot x_0)\, d^n u. \end{aligned}$$

By using the formula

$$\int\limits_{\|u\|_p = p^j} \chi_p(-u \cdot x_0)\, d^n u = \begin{cases} p^{nj}(1-p^{-n}), & \text{if } j \le 0, \\ -p^{n(j-1)}, & \text{if } j = 1, \\ 0, & \text{if } j \ge 2, \end{cases}$$

together with Lemma 1 and the identity

$$(1-p^{-n})\sum_{j=0}^{\infty} p^{-nj} = 1,$$

we obtain

$$Z_t(x) = \|x\|_p^{-n}\left((1-p^{-n})\sum_{j=-\infty}^{0} p^{nj} e^{-t(1-p^{j}\|x\|_p^{-1})_+^{\alpha}} - e^{-t(1-p\|x\|_p^{-1})_+^{\alpha}}\right)$$

$$= \|x\|_p^{-n} \left((1-p^{-n}) \sum_{j=0}^{\infty} p^{-nj} e^{-t(1-p^{-j}\|x\|_p^{-1})_+^\alpha} - e^{-t(1-p\|x\|_p^{-1})_+^\alpha} \right)$$

$$= (1-p^{-n})\|x\|_p^{-n} \left(\sum_{j=0}^{\infty} p^{-nj} \left(e^{-t(1-p^{-j}\|x\|_p^{-1})_+^\alpha} - e^{-t(1-p\|x\|_p^{-1})_+^\alpha} \right) \right)$$

$$\leq 0.$$

(ii) Let $t > 0$ and $x \in \mathbb{Q}_p^n$ be arbitrary but fixed. For $s > 0$, we have that

$$\begin{aligned} Z_{t+s}(x) &= \int_{\mathbb{Q}_p^n} \chi_p(-x \cdot \xi)\, e^{-(t+s)(1-\|\xi\|_p)_+^\alpha}\, d^n\xi \\ &= \int_{\mathbb{Q}_p^n} \chi_p(-x \cdot \xi)\, e^{-t(1-\|\xi\|_p)_+^\alpha}\, e^{-s(1-\|\xi\|_p)_+^\alpha}\, d^n\xi \\ &= (Z_t * Z_s)(x). \end{aligned}$$

□

Remark 5.3. *The sign behavior of the kernel $Z_t(x)$ is markedly different from the corresponding kernel in the classical Archimedean setting.*

Indeed, in the Euclidean theory, Bochner–Riesz-type kernels are oscillatory and can be expressed in terms of Bessel functions; see, for instance, [11, 12]. Consequently, they exhibit alternating sign behavior and oscillatory decay at infinity.

By contrast, the kernel $Z_t(x)$ obtained in the present non-Archimedean framework satisfies

$$Z_t(x) \leq 0, \qquad x \neq 0.$$

Thus, away from the origin, the kernel is purely non-positive.

This phenomenon reflects a fundamental structural difference between the Archimedean and p-adic settings. In the Euclidean case, oscillations are closely connected with the geometry of the sphere and the asymptotic properties of Bessel functions, whereas in the p-adic framework the ultrametric geometry produces a completely different sign structure for the associated kernel.

Remark 5.4. *Even though the symbol $e^{-t(1-\|\xi\|_p)_+^\alpha}$ is strictly positive, the associated kernel Z_t is not a positive function, a fact that stems from the oscillatory character of the p-adic Fourier transform. More precisely, one has $Z_t(x) \leq 0$ for all $x \neq 0$, and furthermore $Z_t \notin L^1(\mathbb{Q}_p^n)$ for every $t > 0$.*

We now investigate an evolution equation naturally associated with the p-adic Bochner–Riesz operator. The purpose of this result is to justify the introduction of the kernel Z_t and to describe its action through an explicit Fourier representation.

Theorem 5.5. *Let $1 \leq \rho < \infty$ and $\alpha > 0$. Given $f \in L^\rho(\mathbb{Q}_p^n)$, consider the Cauchy problem*

$$\begin{cases} \partial_t u(x,t) + \mathfrak{B}_\alpha u(x,t) = 0, & x \in \mathbb{Q}_p^n,\ t > 0, \\ u(x,0) = f(x), & x \in \mathbb{Q}_p^n, \end{cases} \tag{5.4}$$

where $\mathfrak{B}_\alpha$ denotes the p-adic Bochner–Riesz operator of order α. Then the problem (5.4) *admits a unique solution $u(\cdot, t) \in L^\rho(\mathbb{Q}_p^n)$ for every $t > 0$, whose Fourier transform is given by*

$$\widehat{u}(\xi, t) = e^{-t(1-\|\xi\|_p)_+^\alpha} \widehat{f}(\xi), \qquad \xi \in \mathbb{Q}_p^n,\ t > 0.$$

Consequently,

$$u(x, t) = Z_t * f(x), \qquad t > 0,$$

where Z_t, $t > 0$, is the fundamental solution associated with $\mathfrak{B}_\alpha$.

Proof. Let $f \in L^\rho(\mathbb{Q}_p^n)$ be fixed. We apply the p-adic Fourier transform with respect to the spatial variable x to Eq (5.4). Using the definition of $\mathfrak{B}_\alpha$ and the Fourier multiplier representation, we obtain, for every $\xi \in \mathbb{Q}_p^n$,

$$\partial_t \widehat{u}(\xi, t) + (1 - \|\xi\|_p)_+^\alpha \widehat{u}(\xi, t) = 0, \qquad t > 0, \tag{5.5}$$

with the initial condition

$$\widehat{u}(\xi, 0) = \widehat{f}(\xi).$$

For each fixed $\xi \in \mathbb{Q}_p^n$, Eq (5.5) is an ordinary differential equation in the variable t. Since $(1 - \|\xi\|_p)_+^\alpha$ does not depend on t, the equation is separable. Indeed, assuming $\widehat{u}(\xi, t) \neq 0$, we can write

$$\frac{1}{\widehat{u}(\xi, t)} \frac{d}{dt} \widehat{u}(\xi, t) = -(1 - \|\xi\|_p)_+^\alpha.$$

Integrating with respect to t, we obtain

$$\ln |\widehat{u}(\xi, t)| = -\, t(1 - \|\xi\|_p)_+^\alpha + C(\xi),$$

where $C(\xi)$ is a constant depending on ξ. Exponentiating both sides yields

$$\widehat{u}(\xi, t) = C_1(\xi)\, e^{-t(1-\|\xi\|_p)_+^\alpha}.$$

The constant $C_1(\xi)$ is determined by the initial condition $\widehat{u}(\xi, 0) = \widehat{f}(\xi)$, which gives $C_1(\xi) = \widehat{f}(\xi)$. Therefore,

$$\widehat{u}(\xi, t) = e^{-t(1-\|\xi\|_p)_+^\alpha} \widehat{f}(\xi), \qquad \xi \in \mathbb{Q}_p^n,\ t > 0.$$

Finally, taking the inverse Fourier transform, we conclude that

$$u(x, t) = Z_t * f(x),$$

where Z_t is the inverse Fourier transform of $e^{-t(1-\|\xi\|_p)_+^\alpha}$. This completes the proof. □

6. Evolution operators associated with p-adic Bochner–Riesz symbols

In this section, we introduce and study a pseudo-differential operator naturally associated with the p-adic Bochner–Riesz symbol. Although this operator differs from the Bochner–Riesz operator itself, it plays a fundamental role in the analysis of dissipativity, in the generation of contraction semigroups, and in the study of related evolution equations. Since the fundamental solution Z_t satisfies $Z_t(x) \leq 0$ for all $x \neq 0$, we shall refer to the function $-Z_t$ as the positive Bochner–Riesz kernel associated with the operator $\mathfrak{B}_\alpha$, $\alpha > 0$.

Definition 2. [4, Definition 2.2.2] An operator A in a Banach space X (endowed with the norm $\|\cdot\|$) is m-dissipative if

(i) A is dissipative, that is,
$$\|u-\lambda Au\| \geq \|u\|$$
for all $u \in D(A)$ (D(A) denotes the domain of the operator A) and all $\lambda > 0$;

(ii) for all $\lambda > 0$ and all $f \in X$, there exists $u \in D(A)$ such that $u - \lambda Au = f$.

Remark 6.1. *We recall the following characterization of dissipative operators on Hilbert spaces.*

(i) Let A be a linear operator with domain $D(A)$ in a Hilbert space X, endowed with the inner product $(\cdot,\cdot)$. The operator A is said to be dissipative if
$$Re(Af, f) \leq 0, \qquad \text{for all } f \in D(A).$$

(ii) If A is a self-adjoint operator on X such that $A \leq 0$, that is, $(Au, u) \leq 0$ for all $u \in D(A)$, then A is m-dissipative.

For further details, we refer the reader to [4, Corollary 2.4.8] and [8].

Fix $t > 0$ and $\alpha > 0$. For any $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$, we define the pseudo-differential operator $\mathcal{E}_t^\alpha$ by
$$\begin{aligned}(\mathcal{E}_t^\alpha \varphi)(x) &:= -\int_{\mathbb{Q}_p^n} \chi_p(-x\cdot\xi)\, e^{-t(1-\|\xi\|_p)_+^\alpha}\, \widehat{\varphi}(\xi)\, d^n\xi \\ &= -\mathcal{F}^{-1}\left(e^{-t(1-\|\xi\|_p)_+^\alpha}\widehat{\varphi}(\xi)\right) \\ &= -(Z_t * \varphi)(x), \qquad x \in \mathbb{Q}_p^n.\end{aligned}$$

The next results are devoted to showing that the pseudo-differential operator $\mathcal{E}_t^\alpha$ is m-dissipative on $L^2(\mathbb{Q}_p^n)$.

Lemma 6. *The pseudo-differential operator $\mathcal{E}_t^\alpha$ is a dissipative operator.*

Proof. Let $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$ be arbitrary. Using the density of $\mathcal{D}(\mathbb{Q}_p^n)$ in $L^2(\mathbb{Q}_p^n)$ together with the Parseval–Steklov identity, we obtain
$$\begin{aligned}(\mathcal{E}_t^\alpha \varphi, \varphi) &= (-\mathcal{F}^{-1}(e^{-t(1-\|\xi\|_p)_+^\alpha}\widehat{\varphi}), \varphi) \\ &= -\int_{\mathbb{Q}_p^n} e^{-t(1-\|\xi\|_p)_+^\alpha}\, \widehat{\varphi}(\xi)\, \overline{\widehat{\varphi}(\xi)}\, d^n\xi \\ &= -\int_{\mathbb{Q}_p^n} e^{-t(1-\|\xi\|_p)_+^\alpha} \left|\widehat{\varphi}(\xi)\right|^2 d^n\xi \\ &\leq 0.\end{aligned}$$

The desired conclusion follows from Remark 6.1(i). □

Lemma 7. *The pseudo-differential operator $\mathcal{E}_t^\alpha$ is self-adjoint. More precisely,*
$$(\mathcal{E}_t^\alpha \varphi, \psi) = (\varphi, \mathcal{E}_t^\alpha \psi), \qquad \text{for all } \varphi, \psi \in \mathcal{D}(\mathbb{Q}_p^n).$$

Proof. Let $\varphi, \psi \in \mathcal{D}(\mathbb{Q}_p^n)$. By the Parseval–Steklov identity, we have

$$\begin{aligned}(\mathcal{E}_t^\alpha \varphi, \psi) &= \left(-\mathcal{F}^{-1}(e^{-t(1-\|\xi\|_p)_+^\alpha}\widehat{\varphi}), \psi\right) \\ &= -\int_{\mathbb{Q}_p^n} e^{-t(1-\|\xi\|_p)_+^\alpha}\, \widehat{\varphi}(\xi)\, \overline{\widehat{\psi}(\xi)}\, d^n\xi \\ &= -\int_{\mathbb{Q}_p^n} \widehat{\varphi}(\xi)\, \overline{e^{-t(1-\|\xi\|_p)_+^\alpha}\widehat{\psi}(\xi)}\, d^n\xi \\ &= (\varphi, -\mathcal{F}^{-1}(e^{-t(1-\|\xi\|_p)_+^\alpha}\widehat{\psi})) \\ &= (\varphi, \mathcal{E}_t^\alpha \psi),\end{aligned}$$

which proves the claim. □

Theorem 6.2. *The pseudo-differential operator $\mathcal{E}_t^\alpha$ is m-dissipative on $L^2(\mathbb{Q}_p^n)$.*

Proof. The conclusion follows from Remark 6.1 together with Lemmas 6 and 7. □

We now briefly recall some notions from the theory of contraction semigroups that will be used to study the abstract evolution problem associated with the pseudo-differential operator $\mathcal{E}_t^\alpha$. After introducing the notions of a contraction semigroup and its generator, we formulate and solve an inhomogeneous Cauchy problem in $L^2(\mathbb{Q}_p^n)$. The solution will be expressed through the semigroup generated by $\mathcal{E}_t^\alpha$, yielding a representation formula of variation-of-constants type.

Definition 3. [4, Definition 3.4.1] Let $(X, \|\cdot\|)$ be a Banach space. A one-parameter family $(T(t))_{t\geq 0} \subset \mathcal{L}(X)$ is a contraction semigroup in X provided that

(i) $\|T(t)\| \leq 1$ for all $t \geq 0$;

(ii) $T(0) = I$;

(iii) $T(s+t) = T(s)T(t)$ for all $s, t \geq 0$;

(iv) for all $x \in X$, the function $t \mapsto T(t)x$ belongs to $C([0,\infty), X)$.

Definition 4. [4, Definition 3.4.2] The generator of $(T(t))_{t\geq 0}$ is the linear operator L defined by

$$D(L) = \left\{ x \in X : \frac{T(t)x - x}{h} \text{ has a limit in } X \text{ as } h \to 0 \right\}$$

and

$$Lx = \lim_{h\to 0} \frac{T(t)x - x}{h}$$

for all $x \in D(L)$.

Remark 6.3. *By the Hille–Yosida–Phillips theorem (see [4, Theorem 3.4.4]), the pseudo-differential operator* $\mathcal{E}_t^\alpha$ *is the generator of a contraction semigroup on* $L^2(\mathbb{Q}_p^n)$*. We denote by* $(\mathcal{T}(t))_{t\geq 0}$ *the contraction semigroup generated by* $\mathcal{E}_t^\alpha$*.*

Theorem 6.4. *Let* $T > 0$*. Let* $u_0 \in L^2(\mathbb{Q}_p^n)$ *and* $f : [0, T] \to L^2(\mathbb{Q}_p^n)$*. We consider the abstract Cauchy problem*

$$\begin{cases} u \in C([0, T], D(\mathcal{E}_t^\alpha)) \cap C^1([0, T], L^2(\mathbb{Q}_p^n)), \\ \dfrac{d}{dt}u(t) = \mathcal{E}_t^\alpha u(t) + f(t), \quad t \in [0, T], \\ u(0) = u_0. \end{cases} \tag{6.1}$$

Then the unique solution of (6.1) *is given by*

$$u(t) = \mathcal{T}(t)u_0 + \int_0^t \mathcal{T}(t-s)f(s)\, ds, \quad t \in [0, T],$$

where $(\mathcal{T}(t))_{t\geq 0}$ *is the contraction semigroup on* $L^2(\mathbb{Q}_p^n)$ *generated by the m-dissipative pseudo-differential operator* $\mathcal{E}_t^\alpha$*.*

Proof. The existence, uniqueness, and regularity of the solution of the inhomogeneous Cauchy problem (6.1), together with the representation formula above, follow from standard results on inhomogeneous evolution equations associated with contraction semigroups; see, for instance, [4, Lemma 4.1.1], [4, Corollary 4.1.2], and Remark 6.3. □

7. Comparison between Archimedean and p-adic Bochner–Riesz operators

In the classical (Archimedean) framework, Bochner–Riesz operators naturally arise in harmonic analysis on $\mathbb{R}^n$ as regularization tools for the inversion of the Fourier transform and the summability of Fourier integrals. Given $\alpha > 0$, the classical Bochner–Riesz multiplier is defined by

$$(1 - |\xi|^2)_+^\alpha, \quad \xi \in \mathbb{R}^n,$$

and the associated operator plays a central role in problems related to the convergence of Fourier series, restriction phenomena, dispersive equations, and smoothing effects for evolution equations. The analysis of these operators is deeply influenced by the Euclidean geometry of $\mathbb{R}^n$, the smooth structure of spheres, and oscillatory integral techniques. In particular, the behavior of the Bochner–Riesz kernel is closely tied to the curvature properties of the unit sphere and delicate cancellation phenomena, leading to a highly non-trivial dependence on the dimension n and the order α.

In contrast, the p-adic framework developed in this work is governed by a fundamentally different geometric and analytic structure. The space $\mathbb{Q}_p^n$ is totally disconnected, ultrametric, and lacks any notion of curvature in the classical sense. As a consequence, the p-adic Bochner–Riesz symbol introduced here,

$$(1 - \|\xi\|_p)_+^\alpha,$$

exhibits a piecewise-constant radial behavior with respect to the p-adic norm and is supported on a compact ball rather than a smooth hypersurface. This discrete radial structure leads to sums over p-adic spheres instead of oscillatory integrals over Euclidean spheres, which drastically simplifies certain aspects of the analysis, while introducing new phenomena intrinsic to the non-Archimedean setting.

One of the most notable differences concerns the associated kernels. In the real case, Bochner–Riesz kernels typically exhibit oscillatory decay and may fail to be integrable depending on α and n. In the p-adic case, the Bochner–Riesz kernel K_α is radial, explicitly computable via p-adic spherical decompositions, and generates a convolution semigroup of probability measures. However, when considering the evolution kernel Z_t, despite the positivity of the symbol

$$e^{-t(1-\|\xi\|_p)_+^\alpha},$$

the corresponding kernel is not positive and satisfies $Z_t(x) \leq 0$ for all $x \neq 0$. This phenomenon has no direct analogue in the Archimedean theory and reflects the oscillatory nature of the p-adic Fourier transform combined with the ultrametric geometry.

From an operator-theoretic point of view, classical Bochner–Riesz operators are typically studied as bounded multipliers on $L^p(\mathbb{R}^n)$ and, in general, do not generate contraction semigroups. In contrast, the p-adic Bochner–Riesz framework naturally leads to evolution operators whose generators can be rigorously analyzed within the theory of dissipative operators on Hilbert spaces. In particular, the pseudo-differential operator E_t^α introduced in this work is shown to be self-adjoint, dissipative, and m-dissipative on $L^2(\mathbb{Q}_p^n)$, guaranteeing the existence of a contraction semigroup and allowing for a complete well-posed theory of the associated inhomogeneous evolution equations. This strong semigroup structure highlights a fundamental difference between the two contexts: Whereas the Archimedean Bochner–Riesz operator is primarily a summability operator, its p-adic counterpart admits a natural interpretation as a generator of non-Archimedean diffusion-type dynamics.

From an applications perspective, classical Bochner–Riesz operators are closely linked to problems in partial differential equations, dispersive estimates, and restriction theory. On the other hand, p-adic Bochner–Riesz operators fit naturally within the broader framework of non-Archimedean analysis and mathematical physics on ultrametric spaces. In particular, they provide explicit models for evolution equations on $\mathbb{Q}_p^n$, contribute to the construction of contraction semigroups, and offer new tools for the study of p-adic heat-type equations with nonlocal generators.

In summary, although real and p-adic Bochner–Riesz operators share a common conceptual origin as truncated Fourier multipliers, their analytic behavior, kernel structure, and operator-theoretic properties differ substantially. The p-adic framework replaces geometric complexity with arithmetic discreteness, leading to explicit formulas, strong semigroup properties, and new classes of evolution equations. These differences not only highlight the richness of non-Archimedean theory but also suggest that p-adic Bochner–Riesz operators constitute a natural and fertile extension of the classical theory into the realm of ultrametric analysis.

8. Discussion

The results obtained in this work provide a new perspective on the role of Bochner–Riesz-type constructions within the framework of p-adic harmonic analysis and pseudo-differential operator theory. Although the symbols considered here are inspired by the classical Bochner–Riesz multipliers, the ultrametric geometry of $\mathbb{Q}_p^n$ leads to phenomena that differ substantially from their Archimedean counterparts. Consequently, the operators introduced in this article should not be viewed merely as non-Archimedean analogues of classical Bochner–Riesz operators, but rather as a new class of p-adic evolution generators whose properties are intrinsically determined by the discrete and totally disconnected structure of the underlying space.

One of the principal findings of the paper is that the truncated radial symbol

$$(1 - \|\xi\|_p)_+^\alpha$$

generates a family of operators possessing a convolution semigroup structure. This behavior contrasts sharply with the classical Euclidean theory, where Bochner–Riesz operators are primarily studied as summability operators and are not naturally associated with semigroup dynamics. The existence of convolution semigroups reveals that, in the non-Archimedean setting, Bochner–Riesz-type symbols can be interpreted from an evolutionary viewpoint, providing a bridge between harmonic analysis and the theory of time-dependent equations over ultrametric spaces.

Another significant aspect of our analysis is the limiting behavior of the family $(\mathfrak{B}_\alpha)_{\alpha>0}$ as $\alpha \to 0^+$. Instead of converging to the identity operator, the family converges strongly to a Fourier projection onto a compact frequency ball. This phenomenon is a direct consequence of the discrete nature of the p-adic norm and has no exact analogue in the classical setting. The result illustrates how frequency localization in ultrametric spaces differs fundamentally from localization in Euclidean harmonic analysis. In particular, the limiting operator retains a nontrivial spectral truncation, reflecting the hierarchical organization of p-adic frequencies.

The analysis of the associated kernels reveals perhaps the most striking feature of the theory. Although the symbols defining the operators are nonnegative, the corresponding kernels are not positive. Such behavior contrasts with standard diffusion models, where positivity of the symbol is often associated with positivity-preserving semigroups and probabilistic interpretations. The appearance of sign-changing kernels demonstrates that positivity of the Fourier multiplier alone is insufficient to guarantee positivity of the inverse Fourier transform in the p-adic context. This observation highlights the delicate interaction between radial multipliers and the oscillatory structure of the non-Archimedean Fourier transform.

The second family of operators investigated in this work exhibits an even stronger departure from classical diffusion theory. While the associated symbols are strictly positive and generate self-adjoint operators, the resulting kernels are not integrable and fail to define probability densities. Nevertheless, the operator remains m-dissipative and therefore generates a contraction semigroup through the Hille–Yosida–Phillips theorem. This combination of dissipativity and lack of positivity illustrates that semigroup generation in the p-adic setting may occur independently of the probabilistic framework that frequently accompanies evolution equations in Euclidean analysis.

From a broader perspective, the results obtained here emphasize the importance of radial symbols as a source of nonlocal dynamics over ultrametric spaces. The two operator families studied in this paper demonstrate that relatively simple modifications of the symbolic structure can produce substantially different qualitative behaviors. The comparison between the corresponding kernels shows that spectral properties alone do not fully determine the analytical nature of the generated dynamics. Instead, subtle features of the symbol influence regularity, integrability, sign properties, and long-range interactions in ways that are specific to the non-Archimedean environment.

The present work also suggests several directions for further investigation. First, a complete characterization of the L^p–L^q boundedness properties of the introduced operators remains open. Such results would provide a non-Archimedean counterpart to the classical Bochner–Riesz problem and could reveal new threshold phenomena arising from ultrametric geometry. Second, it would be desirable to determine whether suitable modifications of the symbols can produce positive kernels while preserving the essential Bochner–Riesz structure. This question is closely related to the possibility of constructing associated Markov processes and obtaining probabilistic interpretations of the generated semigroups. Third, the spectral theory of these operators deserves further study, particularly regarding eigenfunction expansions, resolvent estimates, and connections with p-adic quantum models.

An additional aspect that emerges from our analysis is the role played by the field structure itself. The symbols considered here depend only on the p-adic norm and therefore reflect the hierarchical geometry of $\mathbb{Q}_p^n$. This suggests that analogous constructions could be developed over more general non-Archimedean local fields or ultrametric spaces. Understanding which properties are genuinely arithmetic and which are purely ultrametric in nature may lead to a broader theory of Bochner–Riesz-type operators beyond the setting of p-adic numbers.

In conclusion, the operators introduced in this article enrich the class of known p-adic pseudo-differential operators and reveal new mechanisms governing evolution equations on ultrametric spaces. The coexistence of convolution semigroups, non-positive kernels, strong convergence to spectral projections, and m-dissipative dynamics illustrates the diversity of behaviors that emerge from Bochner–Riesz-type constructions in the non-Archimedean setting. These findings not only contribute to the development of p-adic harmonic analysis but also open new avenues for research at the intersection of pseudo-differential operators, evolution equations, and non-Archimedean mathematical physics.

9. Conclusions

In this article, we introduced and analyzed two new families of p-adic pseudo-differential operators inspired by Bochner–Riesz-type constructions. Although both classes originate from radial symbols closely related to the classical Bochner–Riesz multiplier, our results demonstrate that they generate fundamentally different dynamics in the non-Archimedean setting. This highlights the richness of the interplay between symbolic structures, Fourier analysis, and evolution equations over ultrametric spaces.

For the first family, namely the p-adic Bochner–Riesz operators, we established the existence of an associated convolution semigroup and obtained explicit formulas for the corresponding kernels. We proved that these operators converge strongly, as the parameter $\alpha \to 0^+$, to a Fourier projection onto a compact frequency ball. Furthermore, we derived and studied the associated evolution equation, obtaining explicit representations of its solutions through convolution with the fundamental kernel. The analysis revealed a remarkable phenomenon: despite the non-negativity of the underlying symbol, the corresponding kernel is not positive. This behavior reflects the distinctive nature of the p-adic Fourier transform and has no direct analogue in the classical Bochner–Riesz theory.

For the second family of operators, defined through symbols naturally associated with the Bochner–Riesz framework, we established a complete operator-theoretic description. In particular, we proved that the corresponding pseudo-differential operator is self-adjoint, dissipative, and m-dissipative on $L^2(\mathbb{Q}_p^n)$. As a consequence, the Hille–Yosida–Phillips theorem yields a strongly continuous contraction semigroup, allowing us to solve the associated inhomogeneous evolution problem via a variation-of-constants formula. Unlike the first family, the kernels generated by these operators fail to be positive and are not integrable, illustrating once again that positivity of the symbol does not imply probabilistic behavior in the non-Archimedean setting.

Taken together, these results demonstrate that Bochner–Riesz-type symbols provide a fruitful source of new nonlocal operators on $\mathbb{Q}_p^n$. The semigroup structures, spectral properties, and qualitative behavior of the associated kernels reveal features that are specific to ultrametric analysis and that differ substantially from those encountered in Euclidean harmonic analysis. Consequently, the theory developed here expands the current landscape of p-adic pseudo-differential operators and contributes to a deeper understanding of evolution processes over non-Archimedean spaces.

Finally, the work opens several promising directions for future research. Among them are the study of sharp L^p–L^q estimates for the introduced operators, the investigation of spectral and asymptotic properties of the generated semigroups, the construction of related stochastic models, and the extension of the theory to more general local fields and ultrametric structures. We hope that the ideas developed in this paper will stimulate further interactions between p-adic harmonic analysis, pseudo-differential operator theory, and non-Archimedean mathematical physics.

Author contributions

All authors made equal and collaborative contributions to this research. The conception of the study, development of the theoretical framework, mathematical analysis, proof construction, validation of results, manuscript writing, revision, and final editing were performed jointly by all authors. No distinction in the level or nature of contributions is intended, as each author participated substantially and equally throughout all stages of the work. All authors have read and approved the final version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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