Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 11(6): 16144–16173.
DOI: 10.3934/math.2026664
Received: 09 April 2026
Revised: 22 May 2026
Accepted: 27 May 2026
Published: 08 June 2026

Research article

A robust SR-complex fuzzy MCDM framework with lie algebraic foundations for AI-assisted tuberculosis diagnosis

Manivannan Balamurugan[1,*], Ganesan Ellammal[1], Zaid Bassfar[2], Abdulaziz Mohammed Alanazi[3] and Kandhasamy Tamilvanan[4]

[1] Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai 600062, Tamil Nadu, India
[2] Faculty of Computing and Information Technology, University of Tabuk, PO. Box 741, Tabuk 71491, Saudi Arabia
[3] Department of Mathematics, University of Tabuk, PO. Box 741, Tabuk 71491, Saudi Arabia
[4] Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and technical Sciences, Saveetha University, Chennai 602105, Tamil Nadu, India

* **Correspondence:** Email: balamurugansvm@gmail.com.

Abstract: In this study, a square-root-complex fuzzy set (SR-CFS) frame is introduced to associate Lie algebraic structures for modeling interaction-aware hesitancy in multi-criteria decision making processes. Instead of using real-valued memberships as in classical fuzzy extensions, our method deals with squared and square-rooted complex membership values and phase elements simultaneously to show the strength of evidence, uncertainty, and disagreement among experts. In this structure, we define SR-complex fuzzy Lie subalgebras (SR-CFLSAs) and SR-complex fuzzy Lie ideals (SR-CFLIDs) and illustrate their basic properties. They consist of stability under Lie algebra homomorphisms, which ensure that the structure of uncertainty is preserved during algebraic operations. Based on these theories, we combine SR-CFS with the PROMETHEE II and VIKOR methods to develop an integrated decision-making system to deal with interdependent criteria effectively. The work includes an extensive case study on AI-assisted diagnosis of newborn tuberculosis, where we derive decision matrices from expert evaluations and clinical performance metrics.

Keywords: Lie algebra; SR-complex fuzzy set; SR-complex fuzzy subalgebra; SR-complex fuzzy Lie ideal; PROMETHEE II; VIKOR
Mathematics Subject Classification: 03E72, 08A72, 22E60, 62F07, 90C70

Abbreviations

MCDM	Multi-Criteria Decision Making
SR-CFS	SR-Complex Fuzzy Set
SR-CFLSA	SR-Complex Fuzzy Lie Subalgebra
SR-CFLID	SR-Complex Fuzzy Lie Ideal
PROMETHEE II	Preference Ranking Organization Method for Enrichment Evaluations II
VIKOR	VIekriterijumsko KOmpromisno Rangiranje
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
L	Lie Algebra
CFS	Complex Fuzzy Set
$[x, y]$	Lie bracket operation on L
$\gimel$	SR-complex fuzzy set defined on L
$\mu_\gimel(x)$	Primary complex membership degree of $x \in L$
$\nu_\gimel(x)$	Secondary complex membership degree of $x \in L$
$\lvert\mu_\gimel(x)\rvert^2$	Squared magnitude of $\mu_\gimel(x)$ representing evidence intensity
$\sqrt{\lvert\nu_\gimel(x)\rvert}$	Square-rooted magnitude of $\nu_\gimel(x)$ representing hesitation control
$\arg(\mu_\gimel(x))$	Phase of the complex membership capturing disagreement or ambiguity
f	Lie algebra homomorphism
$f^{-1}(\gimel)$	Preimage of SR-complex fuzzy set $\gimel$ under f

1. Introduction

Uncertainty, hesitation, and conflicting expert opinions are common challenges in scientific and engineering problems, particularly in multi-criteria decision-making (MCDM) environments where multiple interconnected criteria must be evaluated simultaneously. The effectiveness of MCDM methods largely depends on how uncertainty is represented and processed. To address this issue, Lotfi A. Zadeh introduced fuzzy set theory in 1965 [32], which allows partial membership representation and provides an efficient framework for handling vagueness and imprecision. Over time, several generalized fuzzy models were developed and successfully applied in artificial intelligence, pattern recognition, and decision-making systems.

However, many existing fuzzy MCDM approaches still rely on fixed membership structures and treat criteria independently, limiting their ability to capture interrelated and dynamic uncertainty. In practical decision-making processes, operations such as aggregation, ranking, and preference comparison may alter uncertainty information and produce inconsistent or ambiguous results. Moreover, many real-world interactions are non-commutative, meaning that the order of information processing significantly affects the final outcome. Therefore, conventional fuzzy MCDM methods lack a unified mathematical structure for preserving uncertainty consistently across multiple stages. To overcome these limitations, Lie algebraic structures provide a promising mathematical framework due to their closure properties, relational operators, and homomorphism stability. Originally developed by Sophus Lie for symmetry analysis and differential equations, Lie algebras later became important in geometry, representation theory, and physics [8, 12, 15]. Their ability to model interconnected and transformation-based systems makes them suitable for uncertainty-aware decision-making environments. Hence, integrating fuzzy set concepts with Lie algebraic structures offers a strong and systematic approach for handling dynamic and relational uncertainty in modern MCDM problems.

Rosenfeld [25] introduced fuzzy groups as an extension of classical group theory using fuzzy set theory. Akram [1, 2] further enriched this direction by developing interval-valued fuzzy Lie algebraic structures, allowing uncertainty to be represented within flexible membership intervals. Akram [4] proposed fuzzy Lie ideals over fuzzy fields. In addition, intuitionistic fuzzy Lie subalgebras, their Cartesian products, and homomorphic properties were investigated in [3], providing deeper insight into structure-preserving fuzzy algebraic mappings. Yehia [31] characterized disordered Lie subalgebras and ideals using closure conditions and Lie product behavior. Al-Masarwah et al. [5] extended multi-polar fuzzy ideal concepts in BCK/BCI-algebras, while Balamurugan et al. [9] introduced complex-valued numerical fuzzy ideals for richer uncertainty representation.

Kousar et al. [20] developed morphological fuzzy Lie algebraic structures, including subalgebras, ideals, homomorphisms, and series-based properties such as solvability and nilpotency. Al-Masarwah et al. [6] developed crossing cubic Lie algebra structures, including subalgebras, ideals, and homomorphisms. Shaqaqha [27, 28] extended fuzzy Lie algebra theory through intuitionistic, and Hom-Lie frameworks, emphasizing uncertainty representation and homomorphic preservation of algebraic properties. These studies expanded fuzzy Lie structures toward transformation-aware and nonlinear systems. Ramot et al. [24] introduced the concept of complex fuzzy sets, where the membership grades are represented by complex numbers. Furthermore, Jaber [16, 17] introduced complex intuitionistic fuzzy Lie sub-superalgebras and fuzzy bracket operations to model multidimensional uncertainty while preserving essential Lie algebraic characteristics within complex algebraic environments. Gao et al. [10] proposed a hybrid MCDM approach combining BWM and VIKOR under a Fermatean fuzzy environment for evaluating medical waste treatment technologies. Tamir et al. [29] generalized complex fuzzy sets by introducing complex-valued membership grades. Jaleel et al. [18] extended this further through interval-valued bipolar complex fuzzy sets and soft sets, unifying several fuzzy set frameworks.

Iqbal et al. [13] introduced an efficient convolutional neural network framework for accurate classification of human sperm head images. Their architecture incorporates multiple convolutional layers with varying filter dimensions while maintaining a reduced number of parameters, thereby enhancing computational efficiency without compromising classification performance. Iqbal et al. [14] developed an advanced deep convolutional neural network approach for multi-category skin lesion classification, achieving improved discriminative capability for complex dermatological datasets. Jiang et al. [19] presented a rough integrated asymmetric cloud model designed for large-scale group decision-making under a multi-granularity linguistic setting, enabling more flexible representation of decision-makers' preferences and uncertainties. Liu et al. [21] proposed the Cloud-TOPSIS-Sort methodology, which integrates the cloud model with the TOPSIS technique to effectively solve multi-criteria sorting problems by improving the representation and evaluation of uncertain decision information. Al-shami et al. [7] introduced square-root fuzzy sets as a novel extension of classical fuzzy sets, where membership grades are transformed using a square-root function to enhance flexibility in representing uncertainty and improve modeling capabilities in decision-making and uncertainty analysis.

Motivated by these limitations, the present study introduces an SR-CFS-based Lie algebraic framework that integrates squared complex-valued memberships, phase-dependent uncertainty representation, and secondary square-rooted memberships into a single mathematical structure. Unlike conventional fuzzy extensions, the proposed framework provides a more comprehensive representation of uncertainty while preserving algebraic stability under homomorphic transformations and operational mappings. This structure establishes a stronger theoretical foundation for modeling interconnected and transformation-aware uncertainty in advanced decision-making systems.

SR-CFS, PROMETHEE II, and VIKOR are integrated to give interaction-conscious MCDM approaches, demonstrating the theoretical framework's utility. These innovations, in an unstructured environment, formalize the old classification and compliance-based approaches. AI technology is used to provide a comprehensive analysis of newborn TB detection. Expert predictions and accurate skill assessments are strictly integrated into SR-complex fuzzy selection matrices. The investigations show better rank order and the embodiment of fuzzy estimates than uniform fuzzy MCDM techniques.

The main findings of the research are the following: Our improved SR-complex fuzzy model incorporates phase data, quadratic and square-root complex elements, and assesses ambiguity, evidence consistency, and different expert views.

1) We construct the algebraic structure of SR-complex fuzzy Lie using the theories of subalgebras and ideals, along with their invariance properties under homomorphic transformations.

2) Integrating the conceptual innovations with the VIKOR and PROMETHEE II algorithms produces structurally robust MCDM structures.

We will examine a real-world example of how our methodology can be used to make clinical decisions, evaluate how well it works, and what its reliability is. This will help us to demonstrate the practical application of our strategy. This study provides a theoretically sound and uncertainty-preserving basis for modern MCDM, by linking complex fuzzy moment theory with pseudo-algebraic structures.

The proposed strategy has the potential to promote research in areas such as learning-enhanced fuzzy models, algebraically standardized uncertainty computation, and interpretable outcome structures.

1.1. Research gap and aim of study

Several studies have enhanced MCDM techniques such as PROMETHEE II and VIKOR by integrating fuzzy environments to manage ambiguity and uncertain expert preferences. Gül et al. [11] incorporated fuzzy preference structures into PROMETHEE, while Sałabun et al. [26] demonstrated the effectiveness of fuzzy PROMETHEE II and VIKOR in complex decision analysis. Papathanasiou [23] highlighted the importance of consistent fuzzy transformations in VIKOR-based evaluations, and Naghizadeh et al. [22] improved decision reliability through a hybrid BWM–fuzzy VIKOR framework. In addition, Tufail et al. [30] introduced bimodal fuzzy representations capable of handling dual-sided uncertainty information.

Despite these advancements, existing fuzzy MCDM models mainly process criteria independently and lack mechanisms to represent nonlinear interdependencies among decision attributes. Furthermore, SR-complex fuzzy Lie algebraic structures have not yet been incorporated into PROMETHEE II and VIKOR methodologies. Therefore, there exists a significant research gap in developing an SR-complex fuzzy Lie algebra-based MCDM framework capable of modeling both complex uncertainty and relational interactions among criteria. The present work addresses this limitation by integrating SR-complex fuzzy sets with Lie algebraic concepts into PROMETHEE II and VIKOR decision-making techniques.

The aggregation of pseudo-algebras serves a practical purpose; it provides:
(*i*) Completion under dependent operations.
(*ii*) Design stability under commutation.
(*iii*) Invariant uncertainty handling under aggregation.

In conventional fuzzy MCDM, aggregation algorithms can change the uncertain properties. Pseudo-algebraic aggregation ensures that SR-complex fuzzy options remain invariant under uniform transformations, preserving the uncertainty concept throughout multi-stage decision aggregation.

1.2. Novelty of the study

The primary innovations and contributions of this study can be outlined as follows:

(i) Introduction to square-root-CFS (SR-CFS): This paper introduces SR-CFS. Accordingly, the second-order CFS elements with square-root values are combined with the element states with square-root values to form a novel CFS. Unlike the currently popular intuitive, Pythagorean, hesitant, bipolar, or complex decision blocks, this dual modification manages hesitation while simultaneously helping to bring together dominant sources. It also creates a balanced representation of uncertainty.

(ii) SR-CFS handles confidence levels and uncertainty among decision makers with complex-valued elements. This is different from conventional regression extensions that rely only on real-valued elements. Furthermore, class and class-based operations are encouraged by the practical possibilities of improving consistency and clarity in uncertain decision-making environments.

(iii) New SR-complex fuzzy Lee algebra framework: This study integrates SR-CFS into the Lee algebra framework, paving the way for the advancement of SR-mixed fuzzy Lee sub-algebras and objectives. This framework ensures the resolution of uncertain representations, contact-sense integration, and system integrity through general algebraic operations in multivariate decision-making (MCDM) methods.

(iv) Safety properties: New findings based on the concept indicate that SR-CFLIDs preserve their properties under Lie algebraic transformations. This preservation property implies that uncertainties are invariant with respect to aggregation, transformation, and scaling. This is essential for multi-layered, scalable decision-making systems.

(v) Improved MCDM combination: The SR-CFS strategy, combined with PROMETHEE II and VIKOR algorithms, produces the most efficient SR-complex fuzzy VIKOR approaches. These modifications are complementary to the well-established ranking and consensus-based MCDM methods for handling dependence and two-dimensional uncertainty.

(vi) Real-life application: This research provides a robust model for accurate and reproducible detection of childhood tuberculosis using artificial intelligence technology. A systematic data collection approach is used to create decision-making teams. This activity demonstrates how the proposed framework can be understood and applied to important health problems.

(vii) Link to theory and application: This course raises the conceptual foundation and application relevance by relating abstract SR-complex fuzzy Lie algebra theorems to application possibilities.

1.3. Motivations

The objectives of this study are as follows:

(i) To construct and investigate SR-CFLSAs and SR-CFLIDs, focusing on their important algebraic operations such as addition, intersection, and complementation.

(ii) Investigating how SR-complex fuzzy Lie structures behave under Lie algebraic homomorphisms, especially epimorphisms, to ensure that structural properties remain unchanged.

(iii) To improve the fuzzy algebraic model by embedding SR-complex fuzzy moment theory in Lee algebras, thereby providing a rich mathematical tool for uncertainty and ambiguity.

(iv) To develop a novel MCDM framework using SR-complex fuzzy accounts to handle hesitant and uncertain expert judgments, in particular by integrating PROMETHEE II and VIKOR.

(v) Validate the capability of the proposed SR-complex fuzzy MCDM framework through real-world healthcare applications, specifically:

(a) AI-based tuberculosis (TB) diagnosis in children, and

(b) evaluation of wearable cardiac monitoring devices.

1.4. Highlights of the article

This paper presents a novel framework for modeling the associated uncertainty in multivariate decision making (MCDM). For this, SR-CFS constructed in a Lie algebraic framework are used.

Unlike traditional methods that rely on fixed-valued components, the proposed strategy enables a three-dimensional representation of uncertainty by combining square-complex components (to represent the intensity of the evidence), square-rooted secondary components (to control for hesitation), and cut components (to capture the divergent opinions of experts).

This work ensures that the indefiniteness is systematically preserved under transformations and integrations, and generates SR-CFLSAs and SR-CFLIDs; and also proves that they are invariant under Lie algebra homomorphisms. The framework is validated using a repeatable event test of an AI-assisted pediatric tuberculosis diagnosis, which is then combined with the VIKOR and PROMETHEE II methods. The comparative results indicate an increase in clarity, a strong commitment to expert bias, and a rank-ordered integrity, indicating a link between reliable decision-support structures and a brief, vague, false hypothesis.

2. Preliminaries

Definition 2.1. [12] A Lie algebra L over a field $\mathcal{F}$ is a vector space equipped with a bilinear operation, the Lie bracket, that satisfies the following properties:
(i) Bilinear: The Lie bracket $[,] : L \times L \to L$ is bilinear.
(ii) Skew-symmetry: For any $x, y \in L$,

$$[x, y] = -[y, x].$$

(iii) Jacobi identity: For any $x, y, z \in L$,

$$[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0.$$

A Lie subalgebra is a subspace K of L that is closed under the Lie bracket operation.

Definition 2.2. [7] Let L be a universe of discourse such that $\mu_{\gimel} : L \to [0,1]$ and $\lambda_{\gimel} : L \to [0,1]$ are mappings. Then, the SR-fuzzy set $\gimel$ in L is defined as a set of ordered triples:

$$\gimel = \{(x, \mu_{\gimel}(x),\ \lambda_{\gimel}(x)) \mid x \in L\},$$

where $\mu_{\gimel}(x)$ is the membership degree, and $\lambda_{\gimel}(x)$ is the non-membership degree such that $0 \leq (\mu_{\gimel}(x))^2 + \sqrt{\lambda_{\gimel}(x)} \leq 1$.

Definition 2.3. [24] A complex fuzzy set (shortly, CFS) $\gimel$ on L is defined as

$$\gimel = \{(x, \mu_{\gimel}(x) = a_{\gimel}(x) + i\, b_{\gimel}(x)) \mid x \in L\},$$

where $a_{\gimel}(x), b_{\gimel}(x) \in [0,1]$, $i = \sqrt{-1}$.

3. SR-complex fuzzy Lie subalgebra & SR-complex fuzzy Lie ideal

3.1. SR-complex fuzzy set

Definition 3.1. Let L be a universe of discourse. An SR-complex fuzzy set (Shortly, SR-CFS) $\gimel$ in L is defined as a set of ordered triples:

$$\gimel = \left\{\left(x, (\mu_{\gimel}(x))^2 = (r_A e^{iw_{\gimel}})^2,\ \sqrt{\lambda_{\gimel}(x)} = \sqrt{s_{\gimel} e^{iv_{\gimel}}}\right) \,\middle|\, x \in L\right\},$$

where:

- $\mu_{\gimel}(x) \in \mathbb{C}$ is the complex membership degree, and its square is given by:

$$(\mu_{\gimel}(x))^2 = (r_{\gimel} e^{iw_{\gimel}})^2 = r_{\gimel}^2 e^{i2w_{\gimel}},$$

  with $r_{\gimel} \in [0,1]$ and $w_{\gimel} \in [0, 2\pi)$.
- $\lambda_{\gimel}(x) \in \mathbb{C}$ is a secondary complex membership value, and its principal square-root is defined as:

$$\sqrt{\lambda_{\gimel}(x)} = \sqrt{s_{\gimel} e^{iv_{\gimel}}} = \sqrt{s_A} \cdot e^{iv_A/2},$$

  where $s_{\gimel} \in [0,1]$ and $v_{\gimel} \in [0, 2\pi)$.

Each element $x \in L$ is thus associated with a squared complex membership and the square-root of another complex membership, jointly representing richer uncertainty, and phase-dependent information than classical fuzzy sets.

A standard complex fuzzy set (CFS) [24] represents uncertainty using only one complex-valued membership function

$$\mu_{\gimel}(x) = r_{\gimel} e^{iw_{\gimel}},$$

where $r_{\gimel}$ denotes the membership amplitude and $w_{\gimel}$ represents phase information. Although CFS can model phase-dependent uncertainty, it lacks flexibility in strengthening or balancing uncertain information.

In contrast, the proposed SR-complex fuzzy set (SR-CFS) introduces both squared and square-root complex memberships:

$$(\mu_{\gimel}(x))^2 \quad \text{and} \quad \sqrt{\lambda_{\gimel}(x)}.$$

The squared membership

$$(\mu_{\gimel}(x))^2 = r_{\gimel}^2 e^{i2w_{\gimel}}$$

reduces smaller memberships more rapidly since

$$r_{\gimel}^2 \leq r_{\gimel}, \qquad 0 \leq r_{\gimel} \leq 1.$$

For example,

$$0.2^2 = 0.04, \qquad 0.8^2 = 0.64.$$

Thus, it increases discrimination among alternatives and strengthens reliable information.

Similarly, the square-root membership

$$\sqrt{\lambda_{\gimel}(x)} = \sqrt{s_{\gimel}}\, e^{i\nu_{\gimel}/2}$$

enhances weaker memberships because

$$\sqrt{s_{\gimel}} \geq s_{\gimel}.$$

For instance,

$$\sqrt{0.2} = 0.447, \qquad \sqrt{0.8} = 0.894.$$

Hence, it smooths uncertainty distribution and reduces dominance effects.

Therefore, unlike standard CFS, SR-CFS simultaneously provides stronger discrimination and balanced uncertainty handling, making it more effective for complex problems.

Definition 3.2. Let $\gimel = \left(\left(r_{\gimel} e^{i\omega_{\gimel}}\right)^2, \sqrt{s_{\gimel} e^{i\nu_{\gimel}}}\right)$ and $\daleth = \left(\left(r_{\daleth} e^{i\omega_{\daleth}}\right)^2, \sqrt{s_{\daleth} e^{i\nu_{\daleth}}}\right)$ be two SR-complex fuzzy numbers (SR-CFNs), where

$$r_{\gimel}, r_{\daleth}, s_{\gimel}, s_{\daleth} \in [0, 1]$$

and

$$\omega_{\gimel}, \omega_{\daleth}, \nu_{\gimel}, \nu_{\daleth} \in [0, 2\pi).$$

Then, the fundamental arithmetic operations on SR-complex fuzzy numbers are defined as follows:

1) **Addition operation**

$$\gimel \oplus \daleth = \left(\left(\sqrt{r_{\gimel}^2 + r_{\daleth}^2 - r_{\gimel}^2 r_{\daleth}^2}\; e^{i(\omega_{\gimel}+\omega_{\daleth})}\right)^2, \sqrt{(s_{\gimel} s_{\daleth}) e^{i(\nu_{\gimel}+\nu_{\daleth})}}\right).$$

2) **Multiplication operation**

$$\gimel \otimes \daleth = \left(\left(r_{\gimel} r_{\daleth}\; e^{i(\omega_{\gimel}+\omega_{\daleth})}\right)^2, \sqrt{(s_{\gimel} s_{\daleth}) e^{i(\nu_{\gimel}+\nu_{\daleth})}}\right).$$

3) **Scalar multiplication**

For any scalar $\lambda \geq 0$,

$$\lambda\gimel = \left(\left(\sqrt{1-(1-r_{\gimel}^2)^{\lambda}}\; e^{i\omega_{\gimel}}\right)^2, \sqrt{(s_{\gimel}^{\lambda}) e^{i\nu_{\gimel}}}\right).$$

4) **Score function**

The score function of an SR-complex fuzzy number $\gimel$ is defined by

$$S(\gimel) = r_{\gimel}^2 - \sqrt{s_{\gimel}}.$$

5) **SR-complex fuzzy weighted averaging operator**
Let
$$\gimel_j = \left(\left(r_j e^{i\nu_j}\right)^2,\ \sqrt{s_j e^{i\nu_j}}\right),\ j = 1, 2, \ldots, n,$$
be a collection of SR-complex fuzzy numbers with weight vector
$$\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T,$$
where
$$\omega_j \in [0, 1] \text{ and } \sum_{j=1}^{n} \omega_j = 1.$$
Then, the SR-complex fuzzy weighted averaging operator (SR-CFWA) is defined as
$$\text{SR-CFWA}(\gimel_1, \gimel_2, \ldots, \gimel_n) = \left(\left(\sqrt{1 - \prod_{j=1}^{n}(1 - r_j^2)^{\omega_j}}\ e^{i\sum_{j=1}^{n}\omega_j\nu_j}\right)^2,\ \sqrt{\left(\prod_{j=1}^{n} s_j^{\omega_j}\right) e^{i\sum_{j=1}^{n}\omega_j\nu_j}}\right).$$

3.2. SR-complex fuzzy Lie subalgebra

Definition 3.3. Let L be a Lie algebra over F. An SR-CFS $\gimel$ on L is defined by
$$\gimel = \left\{\left(x,\ (\mu_\gimel(x))^2,\ \sqrt{\lambda_\gimel(x)}\right) \middle| x \in L\right\},$$
where
$$(\mu_\gimel(x))^2 = \left(r_\gimel(x)e^{iw_\gimel(x)}\right)^2 = r_\gimel^2(x)e^{i2w_\gimel(x)}$$
and
$$\sqrt{\lambda_\gimel(x)} = \sqrt{s_\gimel(x)e^{i\nu_\gimel(x)}} = \sqrt{s_\gimel(x)}\, e^{i\nu_\gimel(x)/2},$$
with
$$r_\gimel(x), s_\gimel(x) \in [0, 1], \qquad w_\gimel(x), \nu_\gimel(x) \in [0, 2\pi).$$

Then $\gimel$ is called an SR-CFLSA of L, if for all $x, y \in L$ and $\alpha \in F$, the following conditions hold:

1) $(\mu_\gimel(x + y))^2 \geq \min\left\{(\mu_\gimel(x))^2, (\mu_\gimel(y))^2\right\}$.
2) $(\mu_\gimel(\alpha x))^2 \geq (\mu_\gimel(x))^2$.
3) $(\mu_\gimel([x, y]))^2 \geq \min\left\{(\mu_\gimel(x))^2, (\mu_\gimel(y))^2\right\}$.
4) $\sqrt{\lambda_\gimel(x + y)} \leq \max\left\{\sqrt{\lambda_\gimel(x)},\ \sqrt{\lambda_\gimel(y)}\right\}$.
5) $\sqrt{\lambda_\gimel(\alpha x)} \leq \sqrt{\lambda_\gimel(x)}$.
6) $\sqrt{\lambda_\gimel([x, y])} \leq \max\left\{\sqrt{\lambda_\gimel(x)},\ \sqrt{\lambda_\gimel(y)}\right\}$.

Definition 3.4. Let $\gimel = \left\{\left((\mu_\gimel)^2 = (r_\gimel e^{iw_\gimel})^2,\ \sqrt{\lambda_\gimel(x)} = \sqrt{s_\gimel e^{i\nu_\gimel}}\right) \mid x \in \gimel\right\}$ and $\daleth = \left\{\left((\mu_\daleth)^2 = (r_\daleth e^{iw_\daleth})^2,\ \sqrt{\lambda_\daleth(x)} = \sqrt{s_\daleth e^{i\nu_\daleth}}\right) \mid x \in \daleth\right\}$ be two SR-CFS on L. Then, the Cartesian product $\gimel \times \daleth$ is defined as follows:
$$\begin{aligned}\gimel \times \daleth &= \left((\mu_\gimel)^2,\ \sqrt{\lambda_\gimel}\right) \times \left((\mu_\daleth)^2,\ \sqrt{\lambda_\daleth}\right)\\ &= \left((\mu_\gimel)^2 \times (\mu_\daleth)^2,\ \sqrt{\lambda_\gimel} \times \sqrt{\lambda_\daleth}\right),\end{aligned}$$

where,

$$((\mu_{\gimel} \times \mu_{\daleth})(x, y))^2 \quad = \quad \min\left\{(\mu_{\gimel}(x))^2,\ (\mu_{\daleth}(y))^2\right\},$$

and

$$\left(\sqrt{\lambda_{\gimel}} \times \sqrt{\lambda_{\daleth}}\right)(x, y) \quad = \quad \max\left\{\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\daleth}(x)}\right\}.$$

Definition 3.5. Consider two SR-CFSs, $\gimel = \left((\mu_{\gimel})^2, \sqrt{\lambda_{\gimel}}\right)$ and $\daleth = ((\mu_{\daleth})^2, \sqrt{\lambda_{\daleth}})$ of L. Then, $[\gimel\daleth] = \left\{< x,\ [(\mu_{\gimel})^2(\mu_{\daleth})^2](x),\ [\sqrt{\lambda_{\gimel}}\sqrt{\lambda_{\daleth}}](x) >: x \in L\right\}$ is the SR-CFS formed by their product with degrees defined as:

$$[(\mu_{\gimel})^2(\mu_{\daleth})^2](x) \quad = \quad \begin{cases} \sup\left\{\min\left\{(\mu_{\gimel}(y))^2, (\mu_{\daleth}(z))^2\right\}\right\}, & \text{if } x = yz, \\ 0, & \text{if } x \neq yz, \end{cases}$$

and

$$[\sqrt{\lambda_{\gimel}}\sqrt{\lambda_{\daleth}}](x) \quad = \quad \begin{cases} \inf\left\{\max\left\{\sqrt{\lambda_{\gimel}(y)},\ \sqrt{\lambda_{\daleth}z)}\right\}\right\}, & \text{if } x = yz, \\ 0, & \text{if } x \neq yz. \end{cases}$$

The SR-CFS

$$<< \gimel\daleth >>= \left\{< x,\ << (\mu_{\gimel})^2(\mu_{\daleth})^2 >> (x),\ << \sqrt{\lambda_{\gimel}}\sqrt{\lambda_{\daleth}} >> (x) > :\ x \in L\right\}$$

generated by $\gimel$ and $\daleth$ is formalized as follows:

$$<< (\mu_{\gimel})^2(\mu_{\daleth})^2 >> (x) = \begin{cases} \sup\left\{\min_{i\in N}\left\{\min\left\{(\mu_{\gimel}(y_i))^2,\ (\mu_{\daleth}(z_i))^2\right\}\right\}\right\}, & \text{if } x = \sum_{i=1}^{n} y_i z_i, \\ 0, & \text{if } x \neq \sum_{i=1}^{n} y_i z_i. \end{cases}$$

This condition ensures that when criteria interact (via the Lie bracket), the resulting uncertainty remains within the same fuzzy structure. In MCDM, this means that aggregated criteria interactions do not introduce artificial or inconsistent uncertainty.

3.3. SR-complex fuzzy Lie ideal

Definition 3.6. Let $\gimel = \left\{\left(x,\ (\mu_{\gimel}(x))^2,\ \sqrt{\lambda_{\gimel}(x)}\right) \middle| x \in L\right\}$ be an SR-CFS on a Lie algebra L. Then, $\gimel$ is called an SR-CFLID of L if, for all $\alpha \in F$ and $x, y \in L$, the following conditions hold:

1) $(\mu_{\gimel}(x + y))^2 \geq \min\left\{(\mu_{\gimel}(x))^2, (\mu_{\gimel}(y))^2\right\}$ and $\sqrt{\lambda_{\gimel}(x + y)} \leq \max\left\{\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\gimel}(y)}\right\}$.
2) $(\mu_{\gimel}(\alpha x))^2 \geq (\mu_{\gimel}(x))^2$ and $\sqrt{\lambda_{\gimel}(\alpha x)} \leq \sqrt{\lambda_{\gimel}(x)}$.
3) If $(\mu_{\gimel}([x, y]))^2 \geq (\mu_{\gimel}(x))^2$ and $\sqrt{\lambda_{\gimel}([x, y])} \leq \sqrt{\lambda_{\gimel}(x)}$, then $\gimel$ is called a left SR-CFLID.
4) If $(\mu_{\gimel}([x, y]))^2 \geq (\mu_{\gimel}(y))^2$ and $\sqrt{\lambda_{\gimel}([x, y])} \leq \sqrt{\lambda_{\gimel}(y)}$, then $\gimel$ is called a right SR-CFLID.
5) If $(\mu_{\gimel}([x, y]))^2 \geq \max\left\{(\mu_{\gimel}(x))^2, (\mu_{\gimel}(y))^2\right\}$ and $\sqrt{\lambda_{\gimel}([x, y])} \leq \min\left\{\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\gimel}(y)}\right\}$, then $\gimel$ is called a two-sided SR-CFLID.

Theorem 3.7. *If* $\gimel$ *and* $\daleth$ *are two SR-CFLIDs of L, then the function* $\gimel \cup \daleth : L \to [0,1]$ *defined by*

$$(\gimel \cup \daleth)(x) = \left(\max\left\{(\mu_{\gimel}(x))^2,\ (\mu_{\daleth}(x))^2\right\},\ \min\left\{\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\daleth}(x)}\right\}\right)$$

is an SR-CFLID of L.

Proof. Let $x, y \in L$ and $\alpha \in F$. Then,

$$\begin{aligned}
(\mu_{\gimel\cup\daleth}(x+y))^2 &= \max\left\{(\mu_{\gimel}(x+y))^2, (\mu_{\daleth}(x+y))^2\right\} \\
&\geq \max\left\{\min\left\{(\mu_{\gimel}(x))^2,\ (\mu_{\gimel}(y))^2\right\},\ \min\left\{(\mu_{\daleth}(x))^2,\ (\mu_{\daleth}(y))^2\right\}\right\} \\
&= \max\left\{\min\left\{(\mu_{\gimel}(x))^2,\ (\mu_{\daleth}(x))^2\right\},\ \min\left\{(\mu_{\gimel}(y))^2, (\mu_{\daleth}(y))^2\right\}\right\} \\
&= \min\left\{\max\left((\mu_{\gimel}(x))^2,\ (\mu_{\daleth}(x))^2\right),\ \max\left((\mu_{\gimel}(y))^2, (\mu_{\daleth}(y))^2\right)\right\} \\
(\mu_{\gimel\cup\daleth}(x+y))^2 &\geq \min\left\{(\mu_{\gimel\cup\daleth}(x))^2,\ (\mu_{A\cup B}(y))^2\right\},
\end{aligned}$$

$$\begin{aligned}
(\mu_{\gimel\cup\daleth}(\alpha x))^2 &= \max\left\{(\mu_{\gimel}(\alpha x))^2,\ (\mu_{\daleth}(\alpha x))^2\right\} \\
&\geq \max\{(\mu_{\gimel}(x))^2,\ \left(\mu_{\daleth}(x))^2\right\} \\
&= (\mu_{\gimel\cup\daleth}(x))^2,
\end{aligned}$$

$$\begin{aligned}
(\mu_{\gimel\cup\daleth}([x,y]))^2 &= \max\left\{(\mu_{\gimel}([x,y]))^2, (\mu_{\daleth}([x,y]))^2\right\} \\
&\geq \max\left\{\min\left\{(\mu_{\gimel}(x))^2, (\mu_{\gimel}(y))^2\right\},\ \min\left\{(\mu_{\daleth}(x))^2,\ (\mu_{\daleth}(y))^2\right\}\right\} \\
&= \min\left\{\max((\mu_{\gimel}(x))^2,\ (\mu_{\daleth}(x))^2), \max((\mu_{\gimel}(y))^2,\ (\mu_{\daleth}(y))^2)\right\} \\
&\geq \min\left\{(\mu_{A\cup B}(x))^2,\ (\mu_{A\cup B}(y))^2\right\},
\end{aligned}$$

$$\begin{aligned}
\sqrt{\lambda_{\gimel\cup\daleth}(x+y)} &= \min\left\{\sqrt{\lambda_{\gimel}(x+y)},\ \sqrt{\lambda_{\daleth}(x+y)}\right\} \\
&\leq \min\left\{\max(\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\gimel}(y)}),\ \max(\sqrt{\lambda_{\daleth}(x)},\ \sqrt{\lambda_{\daleth}(y)})\right\} \\
&= \max\left\{\min(\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\daleth}(x)}),\ \min(\sqrt{\lambda_{\gimel}(y)},\ \sqrt{\lambda_{\daleth}(y)})\right\} \\
&\leq \max(\sqrt{\lambda_{\gimel\cup\daleth}(x)},\ \sqrt{\lambda_{A\cup B}(y)}),
\end{aligned}$$

$$\begin{aligned}
\sqrt{\lambda_{\gimel\cup\daleth}(\alpha x)} &= \min\left\{\sqrt{\lambda_{\gimel}(\alpha x)},\ \sqrt{\lambda_{\daleth}(\alpha x)}\right\} \\
&\leq \min\left\{\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\daleth}(x)}\right\} \\
&\leq \sqrt{\lambda_{\gimel\cup\daleth}(x)},
\end{aligned}$$

and

$$\begin{aligned}
\sqrt{\lambda_{\gimel\cup\daleth}([x,y])} &= \min\left\{\sqrt{\lambda_{\gimel}([x,y])},\ \sqrt{\lambda_{\daleth}([x,y])}\right\} \\
&\leq \min\left\{\max(\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\gimel}(y)}),\ \max(\sqrt{\lambda_{\daleth}(x)},\ \sqrt{\lambda_{\daleth}(y)})\right\} \\
&= \max\left\{\min(\sqrt{\lambda_{\gimel}(x)},\ \sqrt{\lambda_{\daleth}(x)}),\ \min(\sqrt{\lambda_{\gimel}(y)},\ \sqrt{\lambda_{\daleth}(y)})\right\} \\
&\leq \max(\sqrt{\lambda_{A\cup B}(x)},\ \sqrt{\lambda_{A\cup B}(y)}).
\end{aligned}$$

Therefore, $\gimel \cup \daleth$ is a SR-CFLID of L. □

Corollary 3.8. *Let* $\gimel$ *and* $\daleth$ *be two SR-CFLIDs of L. Then, the function* $\gimel \cap \daleth : L \to [0,1]$ *is a SR-CFLID of L, defined by* $\gimel \cap \daleth = (\min\{(\mu_\gimel(x))^2,\ (\mu_\daleth(x))^2\},\ \max\{\sqrt{\lambda_\gimel(x)},\ \sqrt{\lambda_\daleth(x)}\})$.

Proof. Straightforward. □

Definition 3.9. Let $f : L_1 \to L_2$ be a homomorphism of L. For any SR-CFS $\gimel = ((\mu_\gimel)^2,\ \sqrt{\lambda_\gimel})$ in L_2, we define a SR-CFS $\gimel^f = ((\mu_\gimel^f)^2,\ \sqrt{\lambda_\gimel^f})$ in L_1 by

$$(\mu_\gimel^f)^2(x) = (\mu_\gimel(f(x)))^2,\ \sqrt{\lambda_\gimel^f(x)} = \sqrt{\lambda_\gimel(f(x))}$$

for all $x \in L_1$. Clearly, $\gimel^f(x_1) = \gimel^f(x_2) = \gimel(x)$ for all $x_1, x_2 \in f^{-1}(x)$.

Definition 3.10. Let f be a map from a set L_1 to a set L_2. If $\gimel = ((\mu_\gimel)^2,\ \sqrt{\lambda_\gimel})$ and $\daleth = ((\mu_\daleth)^2,\ \sqrt{\lambda_\daleth})$ are SR-CFSs in L_1 and L_2 respectively, then the pre-image of $\daleth$ under f, denoted by $f^{-1}(\daleth)$, is a SR-CFS defined by

$$f^{-1}(\daleth) = (f^{-1}(\mu_\daleth)^2,\ f^{-1}(\sqrt{\lambda_\daleth})).$$

Theorem 3.11. *Let* $f : L_1 \to L_2$ *be an epimorphism of L. If* $\gimel = ((\mu_\gimel)^2,\ \sqrt{\lambda_\gimel})$ *is a SR-CFLID of* L_2*, then* $f^{-1}(\gimel^c) = (f^{-1}(\gimel))^c$.

Proof. Let $\gimel = ((\mu_\gimel)^2,\ \sqrt{\lambda_\gimel})$ is a SR-CFS in L_2. Then, for $x \in L_1$,

$$\begin{aligned} f^{-1}((\mu_\gimel^c(x))^2) &= (\mu_\gimel^c(f(x)))^2 \\ &= 1 - (\mu_\gimel(f(x)))^2 \\ &= 1 - f^{-1}((\mu_\gimel(x))^2) \\ &= (f^{-1}((\mu_\gimel)^2))^c(x), \end{aligned}$$

$$\begin{aligned} f^{-1}(\sqrt{\lambda_\gimel^c(x)}) &= \sqrt{\lambda_\gimel^c(f(x))} \\ &= 1 - \sqrt{\lambda_\gimel(f(x))} \\ &= 1 - f^{-1}(\sqrt{\lambda_\gimel(x)}) \\ &= (f^{-1}(\sqrt{\lambda_\gimel}))^c(x). \end{aligned}$$

Hence,

$$\begin{aligned} f^{-1}(\gimel^c) &= f^{-1}((\mu_\gimel^c)^2,\ \sqrt{\lambda_\gimel^c}) \\ &= (f^{-1}((\mu_\gimel^c)^2),\ f^{-1}(\sqrt{\lambda_\gimel^c})) \\ &= (f^{-1}((\mu_\gimel)^2)^c),\ (f^{-1}(\sqrt{\lambda_\gimel}))^c) \\ &= (f^{-1}(\gimel))^c. \end{aligned}$$

□

3.4. Preservation under Lie algebra homomorphisms

Definition 3.12. Let $f : L_1 \to L_2$ be a Lie algebra epimorphism. If $\gimel$ is an SR-complex fuzzy Lie ideal of L_2, then its preimage $f^{-1}(\gimel)$ is an SR-CFLID of L_1.

Theorem 3.13. *Let $f : L_1 \to L_2$ be a Lie algebra homomorphism. If $\gimel$ is an SR-CFLID of L_2, then the inverse image $f^{-1}(\gimel)$ is an SR-CFLID of L_1, where*

$$f^{-1}(\gimel)(x) = \gimel(f(x)), \qquad x \in L_1.$$

Proof. Let $\gimel$ be an SR-CFLID of L_2. For each $x \in L_1$, define

$$f^{-1}(\gimel)(x) = \Big((\mu_\gimel(f(x)))^2, \sqrt{\lambda_\gimel(f(x))}\Big).$$

That is,

$$\mu_{f^{-1}(\gimel)}(x) = \mu_\gimel(f(x)), \ \lambda_{f^{-1}(\gimel)}(x) = \lambda_\gimel(f(x)).$$

We prove that $f^{-1}(\gimel)$ satisfies the conditions of an SR-CFLID of L_1.
For any $x, y \in L_1$, since f is a Lie algebra homomorphism,

$$f(x + y) = f(x) + f(y).$$

Hence,

$$\mu_{f^{-1}(\gimel)}(x + y) = \mu_\gimel(f(x + y)) = \mu_\gimel(f(x) + f(y)).$$

Since $\gimel$ is an SR-CFLID of L_2,

$$\mu_\gimel(f(x) + f(y)) \geq \min\{\mu_\gimel(f(x)), \mu_\gimel(f(y))\}.$$

Therefore,

$$\mu_{f^{-1}(\gimel)}(x + y) \geq \min\{\mu_{f^{-1}(\gimel)}(x), \mu_{f^{-1}(\gimel)}(y)\}.$$

Similarly,

$$\lambda_{f^{-1}(\gimel)}(x + y) \leq \max\{\lambda_{f^{-1}(\gimel)}(x), \lambda_{f^{-1}(\gimel)}(y)\}.$$

For any scalar a and $x \in L_1$, the linearity of f gives

$$f(ax) = af(x).$$

Thus,

$$\mu_{f^{-1}(\gimel)}(ax) = \mu_\gimel(f(ax)) = \mu_\gimel(af(x)) \geq \mu_\gimel(f(x)) = \mu_{f^{-1}(\gimel)}(x),$$

and, similarly,

$$\lambda_{f^{-1}(\gimel)}(ax) = \lambda_\gimel(af(x)) \leq \lambda_\gimel(f(x)) = \lambda_{f^{-1}(\gimel)}(x).$$

Now, for $x, y \in L_1$, since f preserves the Lie bracket,

$$f([x, y]) = [f(x), f(y)].$$

Therefore,

$$\mu_{f^{-1}(\gimel)}([x, y]) = \mu_\gimel(f([x, y])) = \mu_\gimel([f(x), f(y)]).$$

Since $\gimel$ is an SR-CFLID of L_2,

$$\mu_{\gimel}([f(x), f(y)]) \geq \max\{\mu_{\gimel}(f(x)), \mu_{\gimel}(f(y))\}.$$

Hence,

$$\mu_{f^{-1}(\gimel)}([x, y]) \geq \max\{\mu_{f^{-1}(\gimel)}(x), \mu_{f^{-1}(\gimel)}(y)\}.$$

Similarly,

$$\lambda_{f^{-1}(\gimel)}([x, y]) \leq \min\{\lambda_{f^{-1}(\gimel)}(x), \lambda_{f^{-1}(\gimel)}(y)\}.$$

Thus, $f^{-1}(\gimel)$ satisfies all defining conditions of an SR-complex fuzzy Lie ideal of L_1. Therefore, $f^{-1}(\gimel)$ is an SR-CFLID of L_1. □

SR-complex fuzzy Lie ideals represent uncertainty components that remain stable under interactions with the entire decision space. Grading can vary significantly, especially with overly weighted assessments.

This consistency ensures that representations of consistent quality are maintained when the decision-making domain is narrowed, varied, or minimized. This consistency is essential for multiple criteria decision-making (MCDM) methods to be reliable and easy to understand. Methods such as PROMETHEE II and VIKOR incorporate SR-complex fuzzy estimators built on a Lie algebraic system by replacing conventional truth-valued option approaches with SR-complex fuzzy option relations. Because of this connection, these two approaches can operate with a sense of connection and under two-sided uncertainty, which preserves their basic ranking mechanisms.

4. Proposed SR-complex fuzzy Lie algebra-based MCDM framework

This immutability property ensures that volatile metaphors remain safe when the place where a decision is made is minimized, modified, or compressed. If a multi-component decision-making system (MCDM) is to be scalable and easy to interpret, consistency is of utmost importance. Both PROMETHEE II and VIKOR handle SR-complex fuzzy estimators derived from within the Lie algebra framework by replacing conventional truth-valued priority functions with SR-complex fuzzy priority relations. As a result of this integration, these two strategies are able to operate under conditions of contact-sensitivity and two-sided uncertainty, while keeping their important ranking methods intact.

4.1. Conceptual overview of the proposed framework

In real-world multi-factorial decision-making (MCDM) systems, such as artificial intelligence-assisted medical decision-making, expert predictions, and system results are subjected to various transformations, including standardization, aggregation, and comparison. When using conventional real-valued fuzzy sets, these approaches can avoid representing uncertainty. The proposed SR-complex fuzzy framework solves this problem by ensuring that the uncertainty remains systematically constant throughout the decision-making algorithm.

Here is how the framework works:

1) Experts' language ratings and AI performance metrics are collected at the input level.

2) Uncertainty modeling: Ratings are expressed using SR-FSs representing disagreement, hesitation, and intensity.

3) The integration and correlation of criteria within a Boolean algebraic framework is referred to as correlation algebraic consistency. It ensures closure and correlation awareness.

4) Conclusion: In order to provide robust rankings, SR-complex fuzzy data is input into VIKOR and PROMETHEE II.

5) This method ensures that the uncertainty representations remain consistent even after integration and transformation by embedding the fuzzy uncertainty within a Boolean algebra. This is very important for accurate multi-level resolution. The comparative analysis of fuzzy set extensions is presented in Table 1.

Table 1. Comparative analysis of fuzzy set extensions.

Model	Captures Hesitation	Captures Disagreement	Phase Information	Interaction Invariance
Intuitionistic FS	✓	×	×	×
Pythagorean FS	✓	×	×	×
Complex FS	Partial	✓	✓	×
SR-CFS	✓	✓	✓	✓

Existing fuzzy extensions generally treat hesitation and disagreement as scalar magnitudes. In contrast, the proposed SR-CFS separates uncertainty into three structurally distinct components:

- **Evidence intensity (squared magnitude)**
- **Controlled hesitation (square-root component)**
- **Expert disagreement (phase angle)**

This tri-component modeling enables dual-sided uncertainty representation that is not available in classical fuzzy, intuitionistic fuzzy, Pythagorean fuzzy, or standard complex fuzzy frameworks.

5. Experimental study

In real clinical settings, medical experts often provide different opinions due to lack of data, uncertainty of symptoms, and variations in patient conditions. The proposed SR-complex fuzzy framework increases the reliability of decision by simultaneously incorporating membership, non-membership, and hesitation and phase information, such that the model can represent the uncertainty and disagreement more effectively than conventional fuzzy approaches.

The phase terms are responsible for the variability of expert opinions. The hesitation component is responsible for the incomplete or ambiguous medical information frequently encountered in tuberculosis diagnosis. The ranking results are relatively stable with minor variations or noisy inputs in expert evaluations, which demonstrates the robustness of the proposed framework.

Therefore, the proposed approach may assist clinicians in reaching more dependable and consistent decisions in complicated healthcare settings where uncertainty and inconsistent assessments are unavoidable.

Tuberculosis (TB) is a leading cause of death in young children. Early detection is challenging, especially in low-income countries, where symptoms are vague and access to trained radiologists is limited. However, recent developments in artificial intelligence (AI) technology have made it possible to develop automated diagnostic tools that can analyze clinical and radiographic information and assist physicians. However, selecting an AI-assisted diagnostic method suitable for screening for tuberculosis in infants is a complex decision involving multiple criteria (MCDM). This is because a compromise decision has to be made considering many factors such as disease severity, accuracy, sensitivity, interpretability, and potential for use.

These criteria are often evaluated under conditions of ambiguity, hesitation, and expert disagreement, making standard unambiguous or true-valued fuzzy techniques ineffective. This justifies the use of the proposed SR-complex fuzzy decision framework, which can describe evidence strength, hesitation, and expert disagreement all at once.

5.1. Description of alternatives and evaluation criteria

Five AI-assisted TB diagnostic tools, denoted by G_1, G_2, G_3, G_4, and G_5 were considered based on recommendations from pediatric pulmonologists and recent clinical AI literature. The evaluation was conducted using six clinically relevant criteria:

- P_1: Diagnostic accuracy
- P_2: Sensitivity for early-stage TB
- P_3: Specificity
- P_4: Robustness across imaging conditions
- P_5: Interpretability of AI predictions and Deployment feasibility in low-resource settings

Criteria weights were determined through expert consensus and normalized to ensure

$$\sum_{j=1}^{6} w_j = 1. \tag{5.1}$$

5.2. Expert panel and data sources

An expert panel consisting of:

- one pediatric infectious disease specialist.
- one radiologist with TB screening experience.
- one AI healthcare systems engineer.

Quantitative indicators of performance (accuracy, sensitivity, specificity) were developed from validated clinical studies and pilot implementations. Qualitative criteria were evaluated through structured expert questionnaires on the basis of a predefined linguistic scale.

5.3. Algorithm for SR-complex fuzzy PROMETHEE II and SR-complex fuzzy VIKOR method

Algorithm 1: SR-complex fuzzy PROMETHEE II method.

Input: Alternatives $T = \{T_1, T_2, \ldots, T_m\}$, criteria $P = \{P_1, P_2, \ldots, P_n\}$, weights w_j

Output: Complete ranking of alternatives

Step 1: Construct the SR-complex fuzzy decision matrix

$$D = [f_j(T_i)]_{m \times n},$$

where each evaluation is represented by an SR-complex fuzzy number

$$f_j(T_i) = \left((r_{ij}e^{i\omega_{ij}})^2, \sqrt{s_{ij}e^{i\nu_{ij}}}\right).$$

Step 2: Normalize the SR-complex fuzzy evaluations.

For benefit and cost criteria,

$$\widetilde{f_{ij}} = \left(\left(\frac{r_{ij}}{\max_i r_{ij}} e^{i\omega_{ij}}\right)^2, \sqrt{\left(\frac{s_{ij}}{\max_i s_{ij}}\right) e^{i\nu_{ij}}}\right). \text{ and } \widetilde{f_{ij}} = \left(\left(\frac{\min_i r_{ij}}{r_{ij}} e^{i\omega_{ij}}\right)^2, \sqrt{\left(\frac{\min_i s_{ij}}{s_{ij}}\right) e^{i\nu_{ij}}}\right).$$

Step 3: Compare SR-complex fuzzy numbers using the score function

$$S(f_j(T_i)) = r_{ij}^2 - \sqrt{s_{ij}}.$$

Step 4: Compute the deviation between alternatives

$$d_j(T_i, T_k) = S(\widetilde{f_j}(T_i)) - S(\widetilde{f_j}(T_k)).$$

Step 5: Determine the preference degree using the preference function

$$P_j(T_i, T_k) = F_j(d_j(T_i, T_k)), \; 0 \leq P_j(T_i, T_k) \leq 1.$$

Step 6: Aggregate the complex preference values using the SR-complex fuzzy weighted averaging operator

$$\pi(T_i, T_k) = \sum_{j=1}^{n} w_j P_j(T_i, T_k).$$

Step 7: Compute the positive preference flow

$$\phi^+(T_i) = \frac{1}{m-1} \sum_{k=1}^{m} \pi(T_i, T_k).$$

Step 8: Compute the negative preference flow

$$\phi^-(T_i) = \frac{1}{m-1} \sum_{k=1}^{m} \pi(T_k, T_i).$$

Step 9: Compute the net preference flow

$$\phi(T_i) = \phi^+(T_i) - \phi^-(T_i).$$

Step 10: Rank the alternatives according to descending values of $\phi(T_i)$.

Algorithm 2: SR-complex fuzzy VIKOR method.

Input: Alternatives $T = \{T_1, T_2, \ldots, T_m\}$, criteria $P = \{P_1, P_2, \ldots, P_n\}$, weights w_j with $\sum_{j=1}^{n} w_j = 1$, and strategy coefficient $v \in [0, 1]$

Output: Compromise ranking of alternatives

Step 1: Construct the SR-complex fuzzy decision matrix

$$D = [f_j(T_i)]_{m\times n},$$

where

$$f_j(T_i) = \left((r_{ij}e^{i\omega_{ij}})^2, \sqrt{s_{ij}e^{i\nu_{ij}}}\right), \; r_{ij}, s_{ij} \in [0, 1].$$

Step 2: Normalize the SR-complex fuzzy values.

For benefit and cost criteria,

$$\widetilde{f_{ij}} = \left(\left(\frac{r_{ij}}{\max_i r_{ij}}e^{i\omega_{ij}}\right)^2, \sqrt{\left(\frac{s_{ij}}{\max_i s_{ij}}\right)e^{i\nu_{ij}}}\right)$$

and

$$\widetilde{f_{ij}} = \left(\left(\frac{\min_i r_{ij}}{r_{ij}}e^{i\omega_{ij}}\right)^2, \sqrt{\left(\frac{\min_i s_{ij}}{s_{ij}}\right)e^{i\nu_{ij}}}\right).$$

Step 3: Convert each normalized SR-complex fuzzy value into a comparable crisp score

$$S(\widetilde{f_{ij}}) = \widetilde{r}_{ij}^2 - \sqrt{\widetilde{s}_{ij}}.$$

Step 4: Determine the best and worst score values for each criterion

$$f_j^* = \max_i S(\widetilde{f_{ij}}), \; f_j^- = \min_i S(\widetilde{f_{ij}}).$$

Step 5: Compute the normalized distance measure

$$d_{ij} = \frac{f_j^* - S(\widetilde{f_{ij}})}{f_j^* - f_j^-}. \text{ If } f_j^* = f_j^-, \text{ set } d_{ij} = 0.$$

Step 6: Calculate the group utility measure

$$S_i = \sum_{j=1}^{n} w_j d_{ij}.$$

Step 7: Calculate the individual regret measure

$$R_i = \max_j\{w_j d_{ij}\}.$$

Step 8: Identify $S^* = \min_i S_i, S^- = \max_i S_i,$ and $R^* = \min_i R_i, R^- = \max_i R_i$.

Step 9: Compute the VIKOR compromise index

$$Q_i = v\frac{S_i - S^*}{S^- - S^*} + (1 - v)\frac{R_i - R^*}{R^- - R^*}.$$

If $S^- = S^*$ or $R^- = R^*$, the corresponding fraction is taken as zero.

Step 10: Rank alternatives in ascending order of Q_i. The smaller value of Q_i indicates the better compromise alternative.

Step 11: Select the compromise solution based on acceptable advantage and stability conditions.

5.4. Selection of AI-assisted diagnostic tool for TB detection in infants

This study addresses the challenges of making decisions in a complex environment with multiple criteria using a framework called SR-complex fuzzy set (SR-CFS). It combines clinical assessments such as interpretability and ranking ability with metrics such as diagnostic accuracy, sensitivity, and specificity when evaluating the performance of artificial intelligence (AI)-assisted TB detection tools. These assessments are conducted by a team that includes an AI health systems engineer, a pediatric infectious disease specialist, and a radiologist with experience in tuberculosis screening. The PROMETHEE II method integrates multivariate and uncertain assessments using best-of-breed decision-making logic. In particular, it is well suited to create a balance between pairwise comparisons, partial dominance, and competing criteria in a clinical decision-making context. Furthermore, PROMETHEE II provides robust rankings of diagnostic options when used in an SR-CFS approach, while retaining uncertainty, hesitation, and expert disagreement.

Diagnosing tuberculosis (TB) in children is a complex medical problem, as the disease is difficult to diagnose due to vague symptoms, inaccurate radiological images, and differences in interpretation between specialists. New tools developed with the help of artificial intelligence (AI) technology can help; however, choosing the best AI tool requires balancing a variety of criteria. Especially in hospitals with limited resources, the ease of use of the tool, its interpretability, accuracy, long-term use, and disease detection ability must be weighed.

This study addresses the inherent uncertainty and multidimensional choice problem by combining the SR-hard fuzzy multi-criteria decision-making (MCDM) framework with the PROMETHEE II hyperbola method. The proposed technique enables the simultaneous simulation of evidence strength, hesitation, and expert disagreement, which is not possible with conventional fuzzy models.

Step 1: Building an SR-hard fuzzy decision matrix with expert consensus and AI-driven metrics.

To illustrate the certainty and uncertainty in treatment evaluation, the expert-selected clinical decisions are combined with normalized AI discovery ability measures to create an SR-complex fuzzy decision matrix. The condition-specific items in this matrix assess the degree of disagreement between experts, reflecting the hesitation in choosing treatment options. At the same time, membership measures provide integrated skill estimates derived from AI measurements. Non-uniform grid angles are used to clearly show the differing opinions of experts.

The SR-complex fuzzy decision matrix that is produced is provided by:

$$\begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \quad P_1 \quad & \quad P_2 \quad & \quad P_3 \quad & \quad P_4 \quad & \quad P_5 \quad \end{array} \\ \begin{pmatrix} (0.00e^{i2\pi}, 0.40e^{i2\pi}) & (0.00e^{i2\pi}, 0.28e^{i2\pi}) & (0.15e^{i2\pi}, 0.41e^{i2\pi}) & (0.25e^{i2\pi}, 0.23e^{i2\pi}) & (0.14e^{i2\pi}, 0.27e^{i2\pi}) \\ (0.15e^{i2\pi}, 0.30e^{i2\pi}) & (0.44e^{i2\pi}, 0.43e^{i2\pi}) & (0.37e^{i2\pi}, 0.41e^{i2\pi}) & (0.33e^{i2\pi}, 0.31e^{i2\pi}) & (0.37e^{i2\pi}, 0.41e^{i2\pi}) \\ (0.42e^{i2\pi}, 0.62e^{i2\pi}) & (0.62e^{i2\pi}, 0.31e^{i2\pi}) & (0.54e^{i2\pi}, 0.46e^{i2\pi}) & (0.43e^{i2\pi}, 0.67e^{i2\pi}) & (0.53e^{i2\pi}, 0.49e^{i2\pi}) \\ (0.61e^{i2\pi}, 0.38e^{i2\pi}) & (0.44e^{i2\pi}, 0.52e^{i2\pi}) & (0.41e^{i2\pi}, 0.52e^{i2\pi}) & (0.33e^{i2\pi}, 0.64e^{i2\pi}) & (0.59e^{i2\pi}, 0.41e^{i2\pi}) \\ (0.57e^{i2\pi}, 0.43e^{i2\pi}) & (0.59e^{i2\pi}, 0.41e^{i2\pi}) & (0.60e^{i2\pi}, 0.38e^{i2\pi}) & (0.61e^{i2\pi}, 0.37e^{i2\pi}) & (0.62e^{i2\pi}, 0.55e^{i2\pi}) \end{pmatrix} \end{array}$$

Non-uniform phase angles increase the expressive power of the SR-complex fuzzy framework by explicitly reflecting variability in expert evaluations, making it more durable and interpretable than fixed-phase representations.

Step 2: Normalization of the SR-complex fuzzy decision matrix:

$$\tilde{d}_{ij} = \frac{|z_{ij}| - \min |z_j|}{\max |z_j| - \min |z_j|}. \tag{5.2}$$

The normalized matrix is

$$\begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_3 & P_4 & P_5 \end{array} \\ \begin{pmatrix} (0, 0.94) & (0, 0) & (0, 0.86) & (0, 0) & (0, 1) \\ (0, 0.28) & (1, 1) & (1, 0) & (0.39, 0.54) & (0.4, 0.75) \\ (0.6, 1) & (0.76, 0.85) & (0.6, 0.28) & (0.29, 0.54) & (0.6, 0.38) \\ (1, 0) & (0.62, 0.07) & (0.05, 1) & (0, 1) & (0.8, 0.25) \\ (0.56, 0.22) & (0.76, 0.85) & (0.55, 0.5) & (0.92, 0.12) & (1, 0) \end{pmatrix} \end{array}$$

Normalization preserves the structural and functional specificity of uncertainty by integrating all SR-related confounding estimates within a uniform range. The differences in each pair of estimates reflect the differing opinions of experts and the ambiguity caused by AI. Higher normalized scores indicate that a particular treatment regimen is more effective or diagnostically acceptable. This standardized method is used in the subsequent PROMETHEE II preference model to enable objective pairwise comparisons.

Step 3: The pairwise differences of the i^{th} alternative with respect to other alternatives are given by employing the formula

$$\Delta_{rs}^{(j)} = |\tilde{d}_{rj} - \tilde{d}_{sj}|. \tag{5.3}$$

Step 4: The preference function $\pi(T_r, T_s)$ for each pair:

$$\pi(T_r, T_s) = \frac{\sum_{j=1}^{n} w_j P_j(T_r, T_s)}{\sum_{j=1}^{n} w_j} \tag{5.4}$$

$$\begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_3 & P_4 & P_5 \end{array} \\ \begin{pmatrix} 0.0000 & 0.2000 & 0.1000 & 0.1200 & 0.1300 \\ 0.4500 & 0.0000 & 0.1400 & 0.3300 & 0.1300 \\ 0.4100 & 0.1900 & 0.0000 & 0.3200 & 0.1400 \\ 0.3300 & 0.2900 & 0.2200 & 0.0000 & 0.1700 \\ 0.4900 & 0.2500 & 0.2000 & 0.3300 & 0.0000 \end{pmatrix} \end{array}$$

The pairwise comparison values are presented in Table 2.

Table 2. Pairwise comparison values.

$T_r - P_r$	P_1	P_2	P_3	P_4	P_5 (Cost)
$T_1 - T_2$	0.34	2.00	0.14	1.39	0.75
$T_1 - T_3$	1.06	1.76	0.42	1.29	0.38
$T_1 - T_4$	0.06	1.07	1.05	1.00	0.25
$T_1 - T_5$	0.28	1.76	0.64	1.12	0.00
$T_2 - T_1$	1.00	0.00	0.00	0.46	0.60
$T_2 - T_3$	1.60	0.76	0.60	0.90	0.63
$T_2 - T_4$	0.72	0.07	0.05	0.61	0.50
$T_2 - T_5$	0.94	0.76	0.55	0.58	0.25
$T_3 - T_1$	0.40	0.15	0.40	0.46	0.40
$T_3 - T_2$	0.28	1.15	0.72	1.00	0.80
$T_3 - T_4$	0.00	0.22	0.45	0.71	0.87
$T_3 - T_5$	0.22	1.00	0.95	0.58	0.62
$T_4 - T_1$	0.00	0.38	0.86	0.00	0.20
$T_4 - T_2$	0.00	1.38	0.00	0.54	0.60
$T_4 - T_3$	0.60	1.14	0.28	0.54	0.80
$T_4 - T_5$	0.56	1.14	0.50	0.12	0.75
$T_5 - T_1$	0.44	0.15	0.45	0.08	0.00
$T_5 - T_2$	0.44	1.15	0.50	0.47	0.40
$T_5 - T_3$	1.04	1.00	0.78	0.37	0.60
$T_5 - T_4$	0.78	0.22	0.50	0.08	0.80

Step 5: The positive and negative outranking flows:

$$\phi^{+}(T_r) = \frac{1}{m-1}\sum_{\substack{s=1\\ s\neq r}}^{m} \pi(T_r, T_s), \tag{5.5}$$

and

$$\phi^{-}(T_r) = \frac{1}{m-1}\sum_{\substack{s=1\\ s\neq r}}^{m} \pi(T_s, T_r). \tag{5.6}$$

The positive and negative outranking flows are presented in Table 3.

Table 3. The positive and negative outranking flows.

$\phi^{+}(\alpha)$	$\phi^{+}(\alpha)$
0.1400	0.4200
0.2600	0.2300
0.2700	0.1700
0.2500	0.2800
0.3200	0.1400

Step 6: Identification of net flow and rank alternatives

$$\phi(T_r) = \phi^{+}(T_r) - \phi^{-}(T_r). \tag{5.7}$$

The net flow and ranking of alternatives are presented in Table 4.

Table 4. Net flow and rank alternatives.

$\phi^{+}(\alpha) - \phi^{-}(\alpha)$	Rank
−0.2800	5
0.0300	3
0.1000	2
−0.0300	4
0.1800	1

The findings show that alternative T_5 attains the largest net outranking flow, with T_3, T_2, T_4, and T_1 following closely behind. The improved quality of T_5 is generally due to its excellent reproducibility and its ability to maintain consistency across experts' differing opinions, even when other instruments exhibit similar levels of accuracy.

The results provide insights into how the proposed self-learning complexity framework distinguishes AI diagnostic tools that are visually indistinguishable with conventional measurement techniques and thus leads to more robust decision-making in critical clinical situations.

6. Sensitivity analysis

The sensitivity analysis was performed to examine the robustness of the proposed decision making model by varying the weights of individual criteria $P_1 - P_5$ and observing the changes in the net flow values of the alternatives. The results are shown in the form of grouped charts, where the horizontal axis reflects various weight levels of each criterion and the vertical axis reflects the net flow values of the alternatives. The different bars in each weight category correspond to the different alternatives $T_1 - T_5$, and allow a direct comparison of their performance under different weight conditions.

Tables 5–9 shows the effect of varying the weight of criterion P_1 to P_5 on the ranking values of the alternatives.

Table 5. Sensitivity analysis for P_1 weight values.

Weights	T_1	T_2	T_3	T_4	T_5
0.1	−0.32	−0.06	0.15	−0.04	0.16
0.2	−0.29	0.06	0.08	−0.02	0.21
0.3	−0.28	0.06	0.01	−0.01	0.25
0.4	−0.25	0.06	−0.05	0	0.31
0.5	−0.22	0.06	−0.13	0.01	0.36

Table 6. Sensitivity analysis for P_2 weight values.

Weights	T_1	T_2	T_3	T_4	T_5
0.1	−0.16	0.05	0.09	0.08	0.05
0.2	−0.24	0.07	0.09	−0.01	0.16
0.3	−0.31	0.10	0.11	−0.05	0.17
0.4	−0.37	0.13	0.14	−0.08	0.18
0.5	−0.42	0.16	0.16	−0.12	0.19

Table 7. Sensitivity analysis for P_3 weight values.

Weights	T_1	T_2	T_3	T_4	T_5
0.1	−0.32	0.09	0.14	−0.04	0.19
0.2	−0.29	0.07	0.11	−0.04	0.18
0.3	−0.26	0.06	0.10	−0.02	0.15
0.4	−0.23	0.05	0.07	−0.02	0.14
0.5	−0.21	0.04	0.05	0	0.12

Table 8. Sensitivity analysis for P_4 weight values.

Weights	T_1	T_2	T_3	T_4	T_5
0.1	−0.30	0.05	0.12	−0.04	0.18
0.2	−0.32	0.07	0.12	−0.03	0.18
0.3	−0.34	0.08	0.11	−0.01	0.18
0.4	−0.36	0.10	0.09	0.03	0.18
0.5	−0.38	0.11	0.09	0.05	0.18

Table 9. Sensitivity analysis for P_5 weight values.

Weights	T_1	T_2	T_3	T_4	T_5
0.1	−0.32	−0.06	0.15	−0.04	0.16
0.2	−0.29	0.06	0.08	−0.02	0.21
0.3	−0.28	0.06	0.01	−0.01	0.25
0.4	−0.25	0.06	−0.05	0	0.31
0.5	−0.22	0.06	−0.13	0.01	0.36

Figure 1 shows the bar charts of criteria $T_1 - T_5$. We observe that the performance patterns of the alternatives change gradually with weight variation. The alternatives T_3 and T_5 tend to achieve better values for net flow in most weight cases, which indicates a good stability and a low sensitivity to weight changes. On the other hand, alternative T_1 has negative net flow values in all scenarios, indicating a worse performance regardless of the criteria importance. The bars for the alternatives T_2 and T_4 are of moderate height variation, indicating that their rankings are moderately affected by weight adjustments.

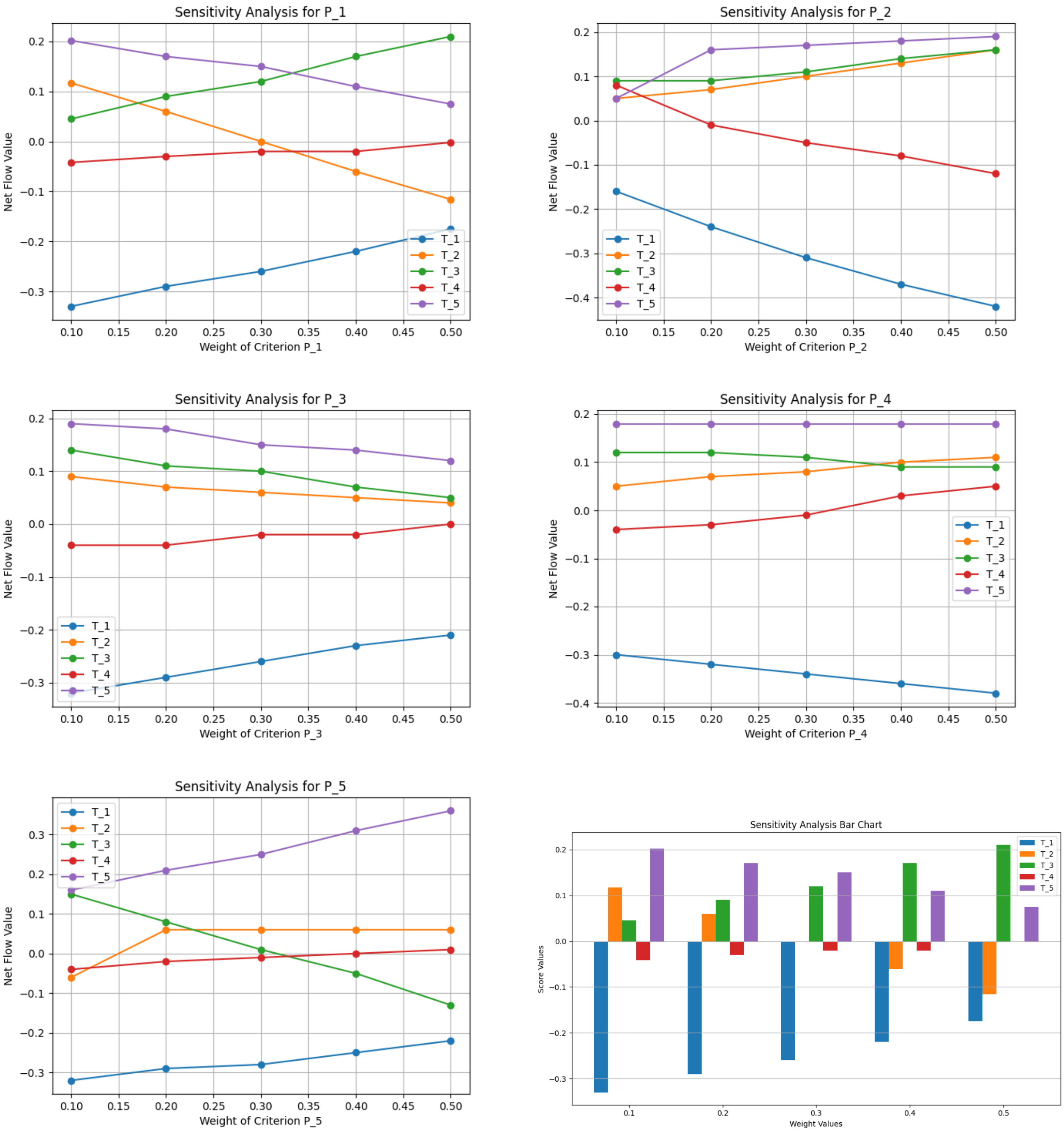

Figure 1. Sensitivity Analysis for $P_1 - P_5$.

Overall, the grouped bar charts indicate that there is no significant reversal of ranking between the best alternatives when changing the weights of criteria $P_1 - P_5$ in the range considered. This shows that the proposed model can obtain stable and consistent decision results and is insensitive to small changes in criterion weights.

Figure 2 shows the sensitivity analysis of the net flow values (ϕ_i) obtained from the SR-complex fuzzy PROMETHEE II method for different scenarios of criterion weights.

- The weight of the selected criterion is increased gradually from 0.15 to 0.50 with a step of 0.05 and the other weights are normalized proportionally.
- Figure 2 shows the evolution of net flow scores for five alternatives $T_1 - T_5$ against the weight range.
- In T_1, a sharp decreasing trend with high sensitivity and ranking decline from positive to negative values is observed.
- T_2 has slight fluctuations around zero, which shows moderate stability with weight variation.
- T_3 shows a regular increasing pattern with positive net flow values, which confirms strong ranking stability.
- T_4 displays a gradual downward trend and crosses the zero threshold, indicating moderate sensitivity.
- T_5 shows a steep upward trajectory and maintains the highest net flow values across all scenarios, indicating persistent dominance.
- The absence of major ranking reversals among top alternatives confirms the robustness and reliability of the proposed decision-making model.

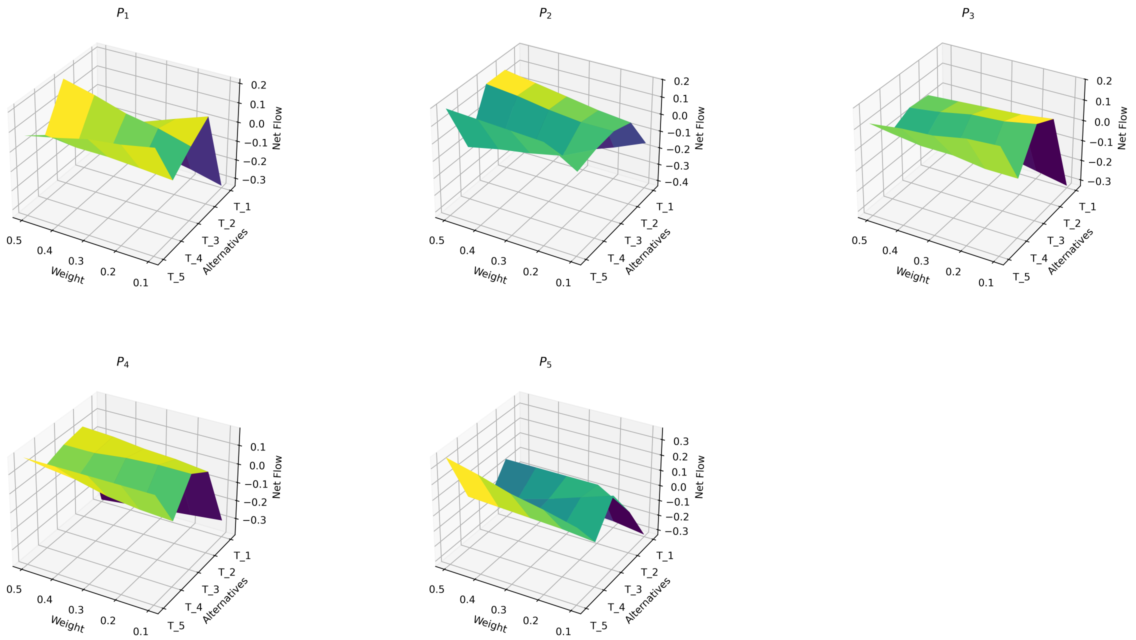

Figure 2. Sensitivity analysis of criteria $P_1 - P_5$ weights using 3D surface plots.

7. Comparative analysis

A comparative analysis was made to analyze the effectiveness of the proposed SR-CFS PROMETHEE II method as compared to TOPSIS and VIKOR methods. The ranking results from the three approaches are presented in Table 10.

Table 10. Ranking order of alternatives by different MCDM methods.

Methods	Ranking order
PROMETHEE II	$T_5 > T_3 > T_2 > T_4 > T_1$
TOPSIS	$T_2 > T_3 > T_5 > T_4 > T_1$
VIKOR	$T_2 > T_4 > T_3 > T_5 > T_1$

The ranking differences in Table 10 arise from the distinct evaluation mechanisms of the MCDM methods. PROMETHEE II uses pairwise preference flows, which favors T_5 due to its stronger outranking performance. In contrast, TOPSIS selects T_2 because it is closest to the ideal solution and farthest from the negative ideal solution. Similarly, VIKOR ranks T_2 first since it focuses on achieving the best compromise between group utility and individual regret. Thus, the variations in ranking reflect methodological differences rather than inconsistency in the results.

In Figure 3, the alternative T_1 is always ranked last in all methods. This means that the alternative T_1 performs badly and the methods are in strong agreement on the ranking of this alternative.

Moreover, T_2 is the best alternative in both TOPSIS and VIKOR, which indicates its closeness to the ideal solution and its ability to provide a balanced trade-off between group utility and individual regret. However, the proposed PROMETHEE II method selects T_5 as the best alternative, which shows a difference in the evaluation due to its outranking-based mechanism.

The different rankings of T_2, T_3, T_4, and T_5 are due to the fact that they use fundamentally different methods. TOPSIS ranks alternatives by their relative distances to ideal and negative-ideal solutions, while VIKOR is based on compromise ranking according to group utility and regret measures. On the contrary, the proposed PROMETHEE II method uses pairwise preference relations and net outranking flows, which allow a finer discrimination among the alternatives in the SR-CFS environment.

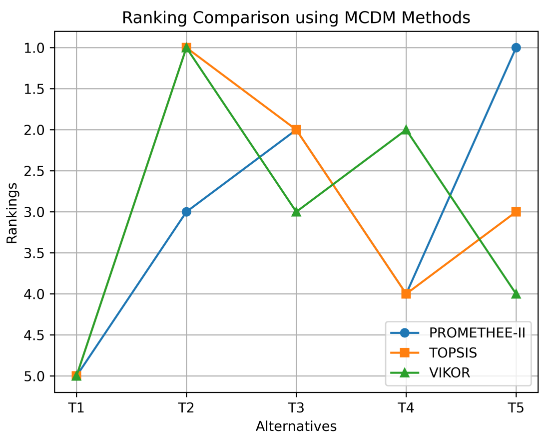

Figure 3. Ranking of alternatives using different methods.

Despite these differences a reasonable degree of consensus is observed among the methods, especially in terms of identifying the least preferred alternative and in maintaining close rankings among intermediate alternatives. This consistency adds to the credibility of the decision-making framework.

Overall, the proposed SR-CFS PROMETHEE II method provides a more flexible and discriminative ranking structure, making it suitable for complex decision-making problems involving uncertainty and fuzzy information.

Therefore, the proposed method not only ensures consistent decision outcomes but also enhances the sensitivity of ranking when compared to traditional MCDM approaches.

7.1. Advantages of the proposed framework

(i) When different performance ratings were found, especially when there were disagreements among experts, the PROMETHEE II method provided better results in decision-making than other alternatives.

(ii) In situations where significant agreements between criteria were required, the VIKOR method was helpful in reaching group consensus by providing a balanced compromise ranking.

(iii) By improving the handling of experts hesitation, ambivalence, and complex-valued opinions, both SR-complex fuzzy extensions performed better than conventional fuzzy and exact MCDM models in predicting experts' uncertainty.

According to these results, it is clear that SR-complex fuzzy MCDM methods provide improved and adaptive decision-making tools compared to their conventional counterparts. Therefore, while VIKOR is more suitable for consensus-based decision-making in complex healthcare environments, PROMETHEE II is better suited for extensive comparative studies in Table 11.

Table 11. Comparison of SR-CFS with existing fuzzy extensions.

Model	Hesitation	Dual uncertainty	Phase info	Structural invariance	Suitable for AI-MCDM
Intuitionistic FS	✓	×	×	×	Moderate
Pythagorean FS	✓	Partial	×	×	Moderate
Bipolar FS	Partial	✓	×	×	Limited
Complex FS	✓	Partial	✓	×	Moderate
SR-CFS (Proposed)	✓	✓	✓	✓	High

8. Conclusions

This study introduced an SR-complex fuzzy set (SR-CFS) framework within a Lie algebraic environment for handling uncertainty in multi-criteria decision-making problems. By combining squared and square-root complex membership structures with phase information, the proposed framework provides a flexible mechanism for representing hesitation, disagreement, and evidence intensity in uncertain decision environments. In addition, the concepts of SR-complex fuzzy Lie ideals and SR-complex fuzzy Lie subalgebras were formulated, and several of their basic algebraic properties were established. The homomorphic preservation results further indicate that the proposed structures maintain uncertainty information consistently under algebraic transformations.

To demonstrate applicability, the SR-CFS framework was integrated with PROMETHEE II and VIKOR methods for decision analysis. The obtained results show that the proposed approach can effectively model complex and hesitant evaluation information while providing meaningful ranking outcomes. The comparative study also indicates that the framework offers an alternative mathematical perspective for integrating complex fuzzy information with algebraic decision-making structures.

However, the proposed model has certain limitations. The inclusion of squared and square-root complex operations increases computational complexity when compared with conventional fuzzy approaches. Moreover, the performance of the model depends on the accurate selection of membership parameters and phase components, which may vary according to expert knowledge and application context. In large-scale decision problems, the computational burden and sensitivity to parameter variations may also influence the efficiency of the framework. Future work may focus on extending the proposed framework to hybrid fuzzy environments, large-scale decision-making problems, and real-world applications in medical diagnosis, risk assessment, and intelligent decision support systems.

Author contributions

Manivannan Balamurugan: Conceptualization, methodology, formal analysis, writing original draft preparation; Manivannan Balamurugan and Zaid Bassfar: Methodology, software, writing review and editing, supervision; Abdulaziz Mohammed Alanazi: Software, validation, investigation, writing review and editing; Ganesan Ellammal: Conceptualization, validation, formal analysis, writing original draft preparation; Kandhasamy Tamilvanan: Software, validation, investigation, writing review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that there is no conflict of interest in this paper.

References

1. M. Akram, Fuzzy Lie ideals of Lie algebras with interval-valued membership functions, *Quasigr. Relat. Syst.*, **16** (2008), 1–12.
2. M. Akram, Generalized fuzzy Lie subalgebras, *J. Gen. Lie Theory Appl.*, **4** (2008), 261–268. https://doi.org/10.4172/1736-4337.1000147
3. M. Akram, K. P. Shum, Intuitionistic fuzzy Lie algebras, *Southeast Asian Bull. Math.*, **31** (2007), 843–855.
4. M. Akram, K. P. Shum, Fuzzy Lie ideals over a fuzzy field, *Ital. J. Pure Appl. Math.*, **27** (2010), 281–292.
5. A. Al-Masarwah, A. G. Ahmad, m-polar fuzzy ideals of BCK/BCI-algebras, *J. King Saud Univ. Sci.*, **31** (2019), 1220–1226. https://doi.org/10.1016/j.jksus.2018.10.002
6. A. Al-Masarwah, N. Kdaisat, M. Abuqamar, K. Alsager, Crossing cubic Lie algebras, *AIMS Mathematics*, **9** (2024), 22112–22129. https://doi.org/10.3934/math.20241075

7. T. M. Al-shami, H. Z. Ibrahim, A. A. Azzam, A. I. El-Maghrabi, SR-fuzzy sets and their weighted aggregated operators in application to decision-making, *J. Funct. Spaces*, **2022** (2022), 3653225, 1–14. https://doi.org/10.1155/2022/3653225

8. Y. Bahturin, *Identical relations in Lie algebras*, Berlin, Boston: De Gruyter, 2021. https://doi.org/10.1515/9783110566659

9. M. Balamurugan, T. Ramesh, A. Al-Masarwah, K. Alsager, A new approach of complex fuzzy ideals in BCK/BCI-algebras, *Mathematics*, **12** (2024), 1583. https://doi.org/10.3390/math12101583

10. F. Gao, M. Han, S. Wang, J. Gao, A novel Fermatean fuzzy BWM-VIKOR based multi-criteria decision-making approach for selecting health care waste treatment technology, *Eng. Appl. Artif. Intel.*, **127** (2024), 107451. https://doi.org/10.1016/j.engappai.2023.107451

11. M. Gül, E. Çelik, A. T. Cumus, A. F. Güneri, A fuzzy logic-based PROMETHEE method for material selection problems, *Beni-Suef Univ. J. Basic Appl. Sci.*, **7** (2018), 68–79. https://doi.org/10.1016/j.bjbas.2017.07.002

12. J. E. Humphreys, *Introduction to Lie algebras and representation theory*, New York: Springer, 1972. https://doi.org/10.1007/978-1-4612-6398-2

13. I. Iqbal, G. Mustafa, J. Ma, Deep learning-based morphological classification of human sperm heads, *Diagnostics*, **10** (2020), 325. https://doi.org/10.3390/diagnostics10050325

14. I. Iqbal, M. Younus, K. Walayat, M. U. Kakar, J. Ma, Automated multi-class classification of skin lesions through deep convolutional neural network with dermoscopic images, *Comput. Med. Imag. Grap.*, **88** (2021), 101843. https://doi.org/10.1016/j.compmedimag.2020.101843

15. N. Jacobson, *Lie algebras*, New York: John Wiley and Son, Inc., 1962.

16. A. Jaber, On complex intuitionistic fuzzy Lie sub-superalgebras, arXiv: 2208.05765, 2022. https://doi.org/10.48550/arXiv.2208.05765

17. A. Jaber, R. Shaqbou'a, Complex intuitionistic fuzzy bracket product, *J. Appl. Math. Inform.*, **43** (2025), 1–21. https://doi.org/10.14317/jami.2025.001

18. A. Jaleel, T. Mahmood, W. Emam, S. Yin, Interval-valued bipolar complex fuzzy soft sets and their applications in decision making, *Sci. Rep.*, **14** (2024), 11589. https://doi.org/10.1038/s41598-024-58792-3

19. J. Jiang, X. Liu, Z. Wang, W. Ding, S. Zhang, H. Xu, Large group decision-making with a rough integrated asymmetric cloud model under multi-granularity linguistic environment, *Inform. Sci.*, **678** (2024), 120994. https://doi.org/10.1016/j.ins.2024.120994

20. S. Kousar, S. Arshad, N. Kausar, T. P. Hong, Construction of nilpotent and solvable Lie algebra in picture fuzzy environment, *Int. J. Comput. Intell. Syst.*, **16** (2023), 37. https://doi.org/10.1007/s44196-023-00213-w

21. X. Liu, C. Zhu, W. Ding, M. Deveci, S. Zhang, Cloud-f-divergence based probabilistic hesitant fuzzy multi-criteria sorting method: An application to medical insurance fraud, *Appl. Soft Comput.*, **195** (2026), 114976. https://doi.org/10.1016/j.asoc.2026.114976

22. A. Naghizadeh Vardin, R. Ansari, M. Khalilzadeh, J. Antuchevičienė, R. Bausys, An integrated decision support model based on BWM and fuzzy-VIKOR techniques for contractor selection in construction projects, *Sustainability*, **13** (2021), 6933. https://doi.org/10.3390/su13126933

23. J. Papathanasiou, An example on the use and limitations of MCDA: The case of fuzzy VIKOR, *Examples Counterexamples*, **1** (2021), 100001. https://doi.org/10.1016/j.exco.2020.100001

24. D. Ramot, R. Milo, M. Friedman, A. Kandel, Complex fuzzy sets, *IEEE T. Fuzzy Syst.*, **10** (2002), 171–186. https://doi.org/10.1109/91.995119

25. A. Rosenfeld, Fuzzy groups, *J. Math. Anal. Appl.*, **35** (1971), 512–517. https://doi.org/10.1016/0022-247X(71)90199-5

26. W. Sałabun, J. Wątróbski, A. Shekhovtsov, Are MCDA methods benchmarkable? A comparative study of TOPSIS, VIKOR, COPRAS and PROMETHEE II, *Symmetry*, **12** (2020), 1549. https://doi.org/10.3390/sym12091549

27. S. Shaqaqha, Fuzzy Hom–Lie ideals of Hom–Lie algebras, *Axioms*, **12** (2023), 630. https://doi.org/10.3390/axioms12070630

28. S. Shaqaqha, M. Y. Al-Deiakeh, On Lie homomorphisms of complex intuitionistic fuzzy Lie algebras, *Eur. J. Pure Appl. Math.*, **17** (2024), 3291–3303. https://doi.org/10.29020/nybg.ejpam.v17i4.5385

29. D. Tamir, L. Jin, A. Kandel, A new interpretation of complex membership grade, *Int. J. Intell. Syst.*, **26** (2011), 285–312. https://doi.org/10.1002/int.20454

30. F. Tufail, R. Gul, M. Shabir, S. K. Alharbi, H. A. E. W. Khalifa, An enhanced machine learning covering-based bipolar L-fuzzy rough set PROMETHEE model for battery storage systems in renewable energy, *Expert Syst. Appl.*, **287** (2025), 127951. https://doi.org/10.1016/j.eswa.2025.127951

31. S. E. Yehia, Fuzzy ideals and fuzzy subalgebras of Lie algebras, *Fuzzy Set. Syst.*, **80** (1996), 237–244. https://doi.org/10.1016/0165-0114(95)00109-3

32. L. A. Zadeh, Fuzzy sets, *Inform. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

AIMS Press