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Research article

Adaptive designated-time fuzzy event-triggered tracking of constrained nonlinear systems and its applications in robotic systems

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Abstract: This paper develops a designated-time adaptive tracking control strategy for nonlinear systems subject to multiple sources of uncertainty, including output constraints, unmodeled dynamics, and unknown disturbances. By incorporating an enhanced fuzzy logic system for parameter estimation and a bounded command filtering approach, this work proposes a systematic tracking control scheme that effectively avoids the issue of computational complexity explosion. To address the significant uncertainties induced by constraints, an asymmetric barrier Lyapunov function is used for analysis and design under the condition of known control coefficients. Furthermore, a controller constructed based on an event-triggered mechanism ensures uniform boundedness and designated-time convergence of all signals. The feasibility and effectiveness of the proposed control method are validated through a practical application case.

Keywords: designated-time stabilization; output constraint; fuzzy logic system; event-triggered mechanism
Mathematics Subject Classification: 93D05, 68T27

1. Introduction

Constraints as an inherent property of dynamical systems, universally restrict the evolution ranges of their states, outputs, or inputs. Whether in mechanical, aerospace, or robotics domains, neglecting these limitations may lead to system failure [1–3]. Therefore, developing control theories that can actively handle and strictly adhere to constraints is of fundamental importance. To prevent constraints from being violated, researchers have developed various strategies including reference governors [4,5] and prescribed performance control [6–8], among which the barrier Lyapunov function (BLF) approach has become a crucial tool for addressing constraint-related issues [9–11]. However, the aforementioned control methods based on the assumption of constant constraints are difficult to directly apply to dynamic scenarios where constraint boundaries vary over time. To address this theoretical gap, Tee et al. pioneered the extension of the classical BLF framework to the time-varying domain, proposing an asymmetric BLF, which provides a key theoretical tool for handling time-varying constraints [12–14]. Despite such significant progress, the control design problem for output-constrained nonlinear systems which requires simultaneously addressing system nonlinearity, time-varying characteristics, and strict output constraints still lacks systematic research. This underexplored area constitutes the core starting point of the work presented in this paper. As is well known, traditional backstepping often leads to the so-called "explosion of complexity" phenomenon, which complicates controller design. Dynamic surface control (DSC), introduced in [15–17], addressed the numerous constraints required for nonlinear systems but did not consider compensating for errors arising from DSC, making it more difficult to achieve better control quality. The command-filtered backstepping method effectively overcomes the accumulation of errors introduced by filter dynamics in traditional designs by systematically incorporating an error compensation mechanism into the control structure, thereby ensuring the stability of the closed-loop system and tracking accuracy [18–20].

Event-triggered mechanisms are an innovative control strategy whose core lies in updating control signals only when the system state meets specific trigger conditions, thereby replacing the traditional periodic time-triggered control approach [21–23]. Compared to time-triggered mechanisms that always operate at fixed intervals, the event-triggered mechanism significantly enhances resource efficiency by reducing unnecessary communication and computation. The event-triggered strategy is especially well-suited for nonlinear systems operating under resource constraints [21, 24, 25]. Its principal merit lies in the fact that control-signal updates are determined by real-time state variations, thereby substantially lowering the rate of control actuation and the overall energy usage without compromising system performance [26–28]. Hence, the event-triggered mechanism holds significant application value in resource-sensitive systems such as unmanned aerial vehicles, the internet of things, and smart microgrids, establishing itself as a key energy-saving technology in modern control engineering.

While finite-time and fixed-time control techniques [29–31] provide benefits such as a fast convergence rate and enhanced robustness, the convergence duration remains dependent on the initial state and predetermined parameters [32–34]. A key feature of the predefined-time stability framework is that the settling time can be explicitly prescribed by adjusting control parameters. However, a well-known challenge in its implementation is that controllers incorporating fractional power terms are prone to singularity issues when differentiated. To tackle this, Tong et al. proposed a fuzzy-based predefined-time control approach that incorporates hyperbolic tangent terms to address this issue [35–37]. Building on earlier work, Sun et al. introduced designated-time control utilizing a time-scale function, which offers a solution to the unbounded control magnitude problem in nonlinear system predefined-time control, further progressing the development of such methods [38].

Despite significant progress in designated-time stability, a research gap remains in achieving indicator constraints while addressing unmodeled dynamics. This has inspired our exploration in this study: How to ensure that the event-triggered mechanism can effectively respond to changes in system states while maintaining the stability of nonlinear systems, especially in the presence of output constraints and unmodeled dynamics. In comparison with prior research, the key contributions of this paper are summarized as follows:

(1) This study presents, for the first time, a designated-time control strategy for nonlinear systems with multiple uncertainties such as asymmetric time-varying output constraints and unmodeled

dynamics. In contrast to conventional finite-time and fixed-time control whose convergence time relies on initial conditions, as well as most predefined-time control schemes that ignore output constraints or unmodeled dynamics, the proposed framework innovatively combines the strengths of existing finite-time and predefined-time control methods. It not only achieves designated-time stability with a user-prescribed convergence time, but also significantly reduces the conservatism of the settling time bound through structural optimization. Unlike existing results that can handle only partial uncertainties at most, the proposed approach achieves a more balanced performance among fast convergence, strong robustness, and constraint compliance.

(2) To handle complex uncertainties including unknown parameters and unmodeled dynamics, this paper introduces an improved fuzzy logic system for online estimation. Different from traditional backstepping-based fuzzy control that suffers from the "explosion of complexity", a bounded command filtering approach with an error compensation mechanism is incorporated. Compared with conventional dynamic surface control and command-filtered backstepping that ignore constraint effects or lack designated-time performance, the proposed composite scheme ensures high approximation accuracy while systematically simplifying the controller structure. This advantage makes the proposed method more feasible and practical in engineering applications than existing adaptive fuzzy control methods.

(3) To address asymmetric time-varying output constraints, an asymmetric barrier Lyapunov function is adopted for system analysis and design. Unlike most existing BLF-based control methods that adopt static triggering or fail to ensure designated-time convergence, this work integrates the constraint-handling mechanism with an event-triggered mechanism. Compared with traditional periodic time-triggered control, the proposed approach reduces the communication and computational burden significantly while strictly guaranteeing constraint satisfaction. As a result, the designed controller achieves a unified co-design of constraint enforcement, designated-time convergence, and resource efficiency, which is rarely realized in closely related works.

2. Problem statement and preliminaries

2.1. Preliminaries

Definition 1 [38]. The following nonlinear system is considered:

$$\dot{x} = s(x(t), t), \tag{2.1}$$

where x represents the system's state variable. The function $s(x(t), t)$ possesses local Lipschitz continuity with $s(0, t) = 0$.

(1) If there exists a connected set $\mathscr{H}(t_0, x_0) \subset \mathscr{R}^n$ (containing the origin $0 \in \mathscr{H}(t_0, x_0)$) such that for any given designated time instant $T_a^* \in \mathscr{R}^+\backslash\{0\}$ the solution $x(t)$ of system (2.1) beginning from any initial condition $(t_0, x_0) \in \mathscr{R}^+ \times \mathscr{R}^n$ is defined on $[t_0, \infty)$, and remains within $x(t) \in \mathscr{H}(t_0, x_0)$, $\forall t \geq t_0 + T_a(x_0, t_0)$, where the entry time $T_a(x_0, t_0) = \sup_{x(t)} \inf\{T_a^s(x(t)) \in \mathscr{R}^+ \mid x(t) \in \mathscr{H}(t_0, x_0), \forall\, t \geq t_0 + T_a^s(x(t))\}$ satisfies $T_a(x_0, t_0) \leq T_a^*$ for all $(x_0, t_0) \in \mathscr{R}^n \times \mathscr{R}^+$, then $\mathscr{H}(t_0, x_0)$ is termed a designated time attractive region within the designated time T_a^*.

(2) If there exists a connected set $\mathscr{M}(t_0, x_0) \subset \mathscr{R}^n$ (containing $0 \in \mathscr{R}^n$) and a neighborhood $\mathscr{N}(0, \varepsilon)$ of the origin (where $\varepsilon > 0$ is a sufficiently small constant normal number) such that $\mathscr{M}(t_0, x_0)$ is designated-time attractive at the designated time $T_a^* \in \mathscr{R}^+\backslash\{0\}$, and every system solution $x(t)$

originating from the initial state $x_0 \in \mathscr{M}(t_0, x_0)$ satisfies $x(t) \in \mathscr{N}(0, \varepsilon)$, $\forall t \geq t_0 + T_a^* + T_s$ with $T_s \in \mathscr{R}^+$ being a finite time, then the system (2.1) is said to possess designated-time stability.

Define the function $\mu(t) = \mu_1(t) + a$, where $t \in [t_0, \infty)$ and the constant $a > 0$ ensures that the resulting controller is well-defined for all $t \in [T_p + t_0, \infty)$. In addition, $\mu_1(t)$ satisfies

$$\begin{cases} \mu_1(t) > 0, & t \in [t_0, T_p + t_0), \\ \mu_1(t) = 0, & t \in [T_p + t_0, \infty), \end{cases}$$

and its derivative $\dot{\mu}_1(t) < 0$ is bounded on $[t_0, T_p + t_0)$.

Remark 1. Compared with conventional finite-time and fixed-time control [29, 33, 35], achieving designated-time convergence under asymmetric time-varying output constraints poses more complicated mathematical challenges. On one hand, finite-/fixed-time convergence depends on initial conditions and cannot be preassigned explicitly, and it is difficult to be compatible with asymmetric time-varying barrier Lyapunov functions for constraint enforcement; on the other hand, standard predefined-time control usually involves fractional power terms, which easily cause singularity when differentiating virtual control laws, and also suffers from unbounded control magnitude and large conservatism of the settling-time bound. The time-scaling function $\mu(t)$ introduced in this paper can effectively overcome the above drawbacks, which not only allows the convergence time to be explicitly predefined by users and independent of initial states, but also avoids fractional power structures to eliminate differentiation singularity and unbounded control magnitude. Meanwhile, it can be naturally integrated with asymmetric time-varying barrier Lyapunov functions to achieve designated-time stability and tracking under strict output constraints, and significantly reduce the conservatism of the settling-time bound to realize a better balance between fast convergence and robustness.

Lemma 1 [39]. Defining the command filter

$$\begin{cases} \dot{\Delta}_{i1} = \Delta_{i2}, \\ \varrho^2 \dot{\Delta}_{i2} = -\operatorname{sat}_{\varepsilon_b}\{\operatorname{sig}(\varrho\Delta_{i2})^g\} - \operatorname{sat}_{\varepsilon_b}\{\operatorname{sig}(\phi_g(\Delta_{i1}, \alpha_i, \varrho\Delta_{i2}))^{\frac{g}{2-g}}\}, \end{cases}$$

where $\varrho > 0$, $0 < g < 1$.

$$\phi_g(\Delta_{i1}, \alpha_i, \Delta_{i2}) = \Delta_{i1} - \alpha_i + \frac{\operatorname{sig}(\varrho\Delta_{i2})^{2-g}}{2-g},$$

$$\operatorname{sat}_{\gamma_b}(\beta) = \begin{cases} \beta, & |\beta| < \gamma_b, \\ \gamma_b \operatorname{sign}(\beta), & |\beta| \geq \gamma_b, \end{cases}$$

with α_i being the continuous input signal, $\Delta_{i1} = \tilde{\alpha}_i$ being the output signal, and there exists $\jmath_i > 0$ such that $|\Delta_{i1} - \alpha_i| \leq \jmath_i$.

Remark 2. Compared to existing finite-time command filters such as those presented in [15, 18], the filter designed in this paper introduces a composite structure combining a saturation function $sat_{\gamma_b}(\cdot)$ and a specific power term $sig(\cdot)^g$, achieving two key improvements: First, when the initial error is large or the signal changes rapidly, the filter can leverage its finite-time convergence property to quickly track the input signal. Second, the saturation mechanism effectively limits the amplitude of the filter's internal states, thereby suppressing oscillations caused by excessive regulation. This design not only guarantees closed-loop stability within a finite time, but also enhances adaptability to practical engineering challenges such as actuator saturation and nonsmooth dynamics, making it particularly suitable for control scenarios that require high dynamic response while strictly avoiding overshoot and oscillation.

Lemma 2 [40]. For $o \in \mathscr{R}$ and $s > 0$, there exists $0 \le |o| - o\tanh(\frac{o}{s}) \le 0.2785s$.

Lemma 3 [12]. For $|w| < 1$, $\log\frac{1}{1-w^2} < \frac{w^2}{1-w^2}$ holds.

Lemma 4 [41]. For $\zeta_j \in \mathscr{R}$ and $0 < r \le 1$, we have $(\sum_{j=1}^n |\zeta_j|)^r \le \sum_{j=1}^n |\zeta_j|^r \le n^{1-m}(\sum_{j=1}^n |\zeta_j|)^r$.

Lemma 5 [42]. Given any continuous nonlinear function $p(\varsigma)$ defined on a compact set Ω, for an arbitrary accuracy $\varepsilon > 0$, there exists a fuzzy logic system of the form $\Gamma^{\mathrm{T}}S(\varsigma)$ that satisfies $\sup_{\varsigma\in\Omega}|p(\varsigma) - \Gamma^{\mathrm{T}}S(\varsigma)| \le \varepsilon$. Here, $J > 1$ is the number of fuzzy rules, $\Gamma = [\Gamma_1, \ldots, \Gamma_J]^{\mathrm{T}}$ is the weight vector, and $S(\varsigma) = [S_1(\varsigma), \ldots, S_J(\varsigma)]^{\mathrm{T}}$ represents the vector of fuzzy basis functions. Each basis function is defined as $S_l(\varsigma) = \frac{\prod_{i=1}^n \psi_{\bar{\Psi}_i^l}(\varsigma_i)}{\sum_{l=1}^J(\prod_{i=1}^n \psi_{\bar{\Psi}_i^l}(\varsigma_i))}$, where $\psi_{\bar{\Psi}_i^l}(\varsigma_i)$ is typically chosen as a Gaussian membership function.

2.2. *Problem statement*

Consider the single-link robotic manipulators with external disturbances and unmodeled dynamics [43] (as shown in Figure 1). The system dynamic equation may be expressed as

$$\mathcal{G}\ddot{q} + \mathcal{Z}\dot{q} + \Psi\sin(q) = u + \tau_d, \tag{2.2}$$

where $\dot{q}$ indicates the angular velocity of the link, q represents the angular position, and the external disturbance is provided by τ_{di}. The mechanical inertia is $\mathcal{G}$, the parameter $\Psi = mgL$ represents the product of the mass m, the gravitational acceleration $g = 10\mathrm{N/kg}$, the damping coefficient is $\mathcal{Z}$, and the link length is L.

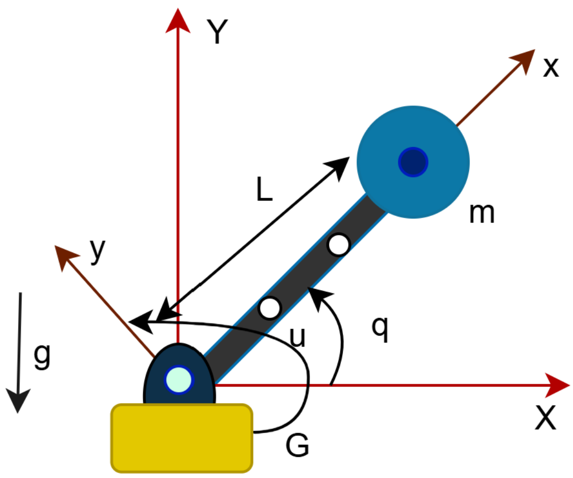

Figure 1. Single-link robotic manipulator.

Now, (2.2) is written in the following form:

$$\begin{cases} \dot{\Xi} = h(\Xi, \varsigma), \\ d\varsigma_i = (\varsigma_{i+1} + \Psi_i(\bar{\varsigma}_i) + \tau_{di}(\Xi, \varsigma))dt, \\ d\varsigma_n = (u + \Psi_n(\varsigma) + \tau_{dn}(\Xi, \varsigma))dt, \\ y = \varsigma_1, \end{cases} \tag{2.3}$$

where $\bar{\varsigma}_i = [\varsigma_1, \varsigma_2, \ldots, \varsigma_i]^{\mathrm{T}} \in \mathbb{R}^i$ represents the vector of the first i states, the full state vector is $\varsigma = [\varsigma_1, \varsigma_2, \ldots, \varsigma_n]^{\mathrm{T}} \in \mathbb{R}^n$, $u \in \mathbb{R}$ is the control input, and $y \in \mathbb{R}$ is the system output. The output $y(t)$ is subject to asymmetric time-varying constraints $k_{c1}(t) < y(t) < k_{c2}(t)$, with the constraint boundaries $k_{c1}(t)$ and $k_{c2}(t)$ being differentiable functions that satisfy $-k_{c1}(t) \neq k_{c2}(t)$. The unknown functions $\Psi_i(\cdot)$ are smooth and satisfy $\Psi_i(0) = 0$, the terms $\tau_{di}(\cdot)$ denote external disturbances satisfying the Lipschitz condition, $h(\cdot)$ is a smooth function, and Ξ represents the unmodeled dynamics of the system. This formulation characterizes a class of nonlinear controlled plants with complex dynamics, incorporating unmodeled dynamics, external disturbances, and asymmetric output constraints.

Assumption 1 [39]. For the reference signal $y_d(t)$ and its derivative $\dot{y}_d(t)$, it is assumed that there exist positive constants K_{c1}, K_{c2}, and $\bar{Y}_2$, along with time-dependent bounds $\underline{y}_1(t)$ and $\bar{y}_1(t)$ satisfying $\underline{y}_1(t) < \bar{y}_1(t)$, such that the following inequalities hold:

$$K_{c1} < k_{c1}(t) < \underline{y}_1(t) \leq y_d(t) \leq \bar{y}_1(t) < k_{c2}(t) < K_{c2},$$

and

$$|\dot{y}_d(t)| \leq \bar{Y}_2.$$

Assumption 2 [44]. The disturbances $\tau_{di}(\Xi, \varsigma)$ are assumed to satisfy a growth condition expressed by two nonnegative smooth functions $\bar{\tau}_{di1}(\|\varsigma\|)$ and $\bar{\tau}_{di2}(\|\Xi\|)$, such that the following inequality holds for each i:

$$|\tau_{di}(\Xi, \varsigma)| \leq \bar{\tau}_{di1}(\|\varsigma\|) + \bar{\tau}_{di2}(\|\Xi\|).$$

Definition 2 [44]. A system described by $\dot{\Xi} = h(\Xi, \varsigma)$ is said to be exponentially input-to-state practically stable (exp-ISPS) if there exists an exp-ISPS Lyapunov function $V(\Xi)$, κ_∞ functions ψ_3, ψ_4, ψ_5, and positive constants $\overline{a}_1$ and a_2 such that the following conditions are satisfied:

$$\psi_3(\|\Xi\|) \leq V(\Xi) \leq \psi_4(\|\Xi\|),$$

$$\frac{\partial V(\Xi)}{\partial \Xi} h(\Xi, \varsigma) \leq -\overline{a}_1 V(\Xi) + \psi_5(|\varsigma_1|) + a_2.$$

Remark 3. The aforementioned assumptions and definitions collectively form the foundation for the design and stability analysis of the control scheme proposed in this paper. Unlike existing studies [45–47] which typically require bounded higher-order derivatives of the reference signal, Assumption 1 adopted in this work only necessitates a bounded first-order derivative. This reduces the prior requirements on the desired trajectory and enhances the practical applicability of the scheme. The introduced time-varying bounds $\underline{y}_1(t)$ and $\overline{y}_1(t)$ explicitly ensure that the reference signal y_d remains consistently away from the time-varying constraint boundaries, providing a feasible tracking region for output-constrained control. Assumption 2 decomposes the composite disturbance into a sum of smooth functions depending on the measurable state ς and the unmodeled dynamics Ξ, respectively, offering a structural basis for subsequent approximation and compensation using tools such as fuzzy logic systems. Definition 2, which describes exponential input-to-state practical stability (exp-ISPS), serves as a key and relatively relaxed stability framework for handling nonlinear systems with unmodeled dynamics. Together, these conditions enable the method proposed herein to address the tracking problem for complex nonlinear systems with unmodeled dynamics, asymmetric time-varying output constraints, and composite disturbances under weaker assumptions.

Lemma 6. If the Lyapunov function $V(\Xi)$ for the system $\dot{\Xi} = h(\Xi, \varsigma)$ satisfies Definition 2, for a constant $\bar{h}$, any initial conditions $t_0 \geq 0$, $\Xi_0 = \Xi(t_0)$ and $r(t_0) = r_0 > 0$, any function $\bar{\Phi}$ such that $\bar{\Phi}(\varsigma_1) \geq \bar{w}(|\varsigma_1|)$, there exist a finite $T_1 = T_1(\bar{a}_1, r_0, \Xi_0) \geq 0$ and a function $B(t_0, t) \geq 0$ for $t \geq t_0$, and a signal can be portrayed as

$$\dot{r} = -\frac{\bar{h}}{\mu} r^p + \frac{\dot{\mu}}{\mu} r + \bar{\Phi}(\varsigma_1) + d_0, \quad r(t_0) = r_0, \tag{2.4}$$

where $p \in (0, 1)$, such that $B(t_0, t) = 0$ for $t \geq t_0 + T_1$ and $V(\Xi(t)) \leq r(t) + B(t_0, t)$.

Remark 4. Compared with standard predefined-time control methods [44, 46, 48], the proposed designated-time control scheme based on the time-scaling function $\mu(t)$ can effectively resolve the problem of unbounded control magnitude in conventional predefined-time approaches. Typical predefined-time control schemes rely on fractional power terms in their controller and virtual control design, which may lead to infinite control signals and singularity phenomena when performing repeated differentiation in the backstepping framework. In contrast, the proposed designated-time strategy removes fractional power terms and employs a smooth time-scaling structure $\mu(t)$ to achieve user-prescribed convergence time without causing singularity or unbounded control signals. As a result, the control signals remain uniformly bounded in the entire process, and the convergence time can be explicitly preassigned independent of initial conditions, which provides a more practical and reliable solution for constrained nonlinear systems with unmodeled dynamics and external disturbances.

Without losing generality, the equation $\bar{\Phi}(\varsigma_1) = \varsigma_1^2 \psi_0(\varsigma_1^2)$ is used in which $\psi_0 \geq 0$ is a smooth function.

This paper aims to design and implement an adaptive event-triggered tracking controller for system (2.3). The controller must fulfill the following three key objectives: (i) ensuring the system output tracks the reference signal y_d within a designated time frame; (ii) guaranteeing that all closed-loop signals remain uniformly bounded; and (iii) strictly maintaining the required output constraints at all times.

3. Design procedures

3.1. Controller design

The controller design begins with the definition of an error trajectory. Let the tracking error be $\delta_1 = \varsigma_1 - y_d$, and define the subsequent error variables as $\delta_i = \varsigma_i - \tilde{\alpha}_{i-1}$ for $i = 2, \ldots, n$, where $\tilde{\alpha}_{i-1}$ denotes the output of a command filter and α_{i-1} is its input.

To mitigate the effects of filtering errors, a set of compensation error signals is introduced:

$$\sigma_k = \delta_k - \xi_k, \quad k = 1, \ldots, n, \tag{3.1}$$

where ξ_k is a dynamically generated error compensation signal.

Since the functions $\bar{\Psi}_i(\omega_i)$ are unknown, they are approximated using fuzzy logic systems (FLSs) of the form:

$$\bar{\Psi}_i(\omega_i) = \theta_i^T \phi_i(\omega_i) + \varepsilon_i(\omega_i) \tag{3.2}$$

where θ_i is the ideal weight vector, $\phi_i(\omega_i)$ is the fuzzy basis function vector, and $\varepsilon_i(\omega_i)$ denotes the approximation error, bounded by $|\varepsilon_i(\omega_i)| \le \tilde{\varepsilon}_i$ with $\tilde{\varepsilon}_i > 0$. Let $\hat{\theta}_i$ be the estimate of θ_i, and define the estimation error as $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

Step 1: Based on Eq (3.1), the differential of the first compensation error signal is obtained as:

$$d\sigma_1 = (\varsigma_2 + \Psi_1 + \tau_{d1} - \dot{y}_d - \dot{\xi}_1)\,\mathrm{d}t. \tag{3.3}$$

An asymmetric time-varying barrier Lyapunov function (BLF) is then selected for the first step:

$$V_1 = \frac{\mu(\sigma_1)q}{2}\log\frac{k_{C1}^2}{k_{C1}^2 - \sigma_1^2} + \frac{(1-q(\sigma_1))\mu}{2}\log\frac{k_{C2}^2}{k_{C2}^2 - \sigma_1^2} + \frac{\mu}{2m_1}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1 + \frac{\mu}{2l_1}\tilde{\varphi}_1^2 + \frac{r}{l_0},$$

where the switching function is defined by

$$q(\sigma_1) = \begin{cases} 1, & \sigma_1 \le 0, \\ 0, & \sigma_1 > 0. \end{cases}$$

In this function, m_1, l_1, and l_0 are positive design constants. The term $\tilde{\varphi}_1 = \varphi_1 - \hat{\varphi}_1$ represents the estimation error for an unknown parameter φ_1, with $\hat{\varphi}_1$ being its estimate. The definition of φ_1 will be provided subsequently.

The design proceeds by introducing time-varying barrier boundaries defined as:

$$k_{C1} = y_d + \xi_1 - k_{c1}, \quad k_{C2} = k_{c2} - y_d - \xi_1.$$

These boundaries effectively translate the original asymmetric output constraints into a symmetric form relative to the compensated error σ_1.

To facilitate the analysis, a coordinate transformation is applied by defining the normalized error variables:

$$k_a = \frac{\sigma_1}{k_{C1}}, \quad k_b = \frac{\sigma_1}{k_{C2}}, \quad k = qk_a + (1-q)k_b,$$

where $q = q(\sigma_1)$ is the switching function defined previously. This transformation consolidates the two logarithmic terms in the original BLF. Consequently, the complex form of V_1 is simplified to the following compact expression:

$$V_1 = \frac{\mu}{2}\log\frac{1}{1-k^2} + \frac{\mu}{2m_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{\mu}{2l_1}\tilde{\varphi}_1^2 + \frac{r}{l_0}.$$

Using (3.3), $\dot{V}_1$ is given as

$$\begin{aligned}\dot{V}_1 =& \mu\gamma\sigma_1(\sigma_2 + \xi_2 + \alpha_1 + (\tilde{\alpha}_1 - \alpha_1) + \Psi_1 + \tau_{d1} - \dot{\xi}_1 - H - \dot{y}_d) - \frac{\mu}{m_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1 + \frac{\dot{\mu}}{2m_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{\dot{\mu}}{2l_1}\tilde{\varphi}_1^2 \\ &+ \frac{\dot{\mu}}{2}\log\frac{1}{1-k^2} - \frac{\mu}{l_1}\tilde{\varphi}_1\dot{\hat{\varphi}}_1 + \frac{\dot{r}}{l_0},\end{aligned} \tag{3.4}$$

where

$$\gamma = \frac{q}{k_{C1}^2 - \sigma_1^2} + \frac{1-q}{k_{C2}^2 - \sigma_1^2},$$

and

$$H = q\frac{\sigma_1}{k_{C1}}(\dot{y}_d + \dot{\xi}_1 - \dot{k}_{c1}) + (1-q)\frac{\sigma_1}{k_{C2}}(\dot{k}_{c2} - \dot{y}_d - \dot{\xi}_1).$$

Building upon Assumption 2 and Lemma 2, the impact of the unknown disturbance $\tau_{d1}(\Xi, \varsigma)$ on the dynamics can be bounded as follows:

$$\mu\gamma\sigma_1\tau_{d1}(\Xi, \varsigma) \le \mu\gamma|\sigma_1|\bar{\tau}_{d11}(\|\varsigma\|) + \mu\gamma|\sigma_1|\bar{\tau}_{d12}(\|\Xi\|), \tag{3.5}$$

$$\mu\gamma|\sigma_1|\bar{\tau}_{d11}(\|\varsigma\|) \le \mu\gamma\sigma_1\bar{\tau}_{d11}(\|\varsigma\|)\tanh\Big(\frac{\mu\gamma\sigma_1\bar{\tau}_{d11}(\|\varsigma\|)}{\epsilon_{11}}\Big) + 0.2785\epsilon_{11}, \tag{3.6}$$

where $\epsilon_{11} > 0$ is a constant.

Furthermore, by employing Definition 2, Lemma 4, and Young's inequality, the term involving the unmodeled dynamics can be processed. This yields a refined upper bound:

$$\begin{aligned}
\mu\gamma|\sigma_1|\bar{\tau}_{d12}(\|\Xi\|) &\le \mu\gamma|\sigma_1|\bar{\tau}_{d12}(\psi_3^{-1}(r(t) + B(t))) \\
&\le \mu\gamma|\sigma_1|\bar{\tau}_{d12}(\psi_3^{-1}(2r(t))) + \mu\gamma|\sigma_1|\bar{\tau}_{d12}(\psi_3^{-1}(2B(t))) \\
&\le \mu\gamma\sigma_1\bar{\tau}_{d12}(\psi_3^{-1}(2r(t)))\tanh\left(\frac{\mu\gamma\sigma_1\bar{\tau}_{d12}(\psi_3^{-1}(2r(t)))}{\epsilon_{12}}\right) \\
&\quad + 0.2785\epsilon_{12} + \mu^2\gamma^2\sigma_1^2 + \frac{1}{4}\bar{\tau}_{d12}^2(\psi_3^{-1}(2B(t))),
\end{aligned} \tag{3.7}$$

where $\epsilon_{12} > 0$ is a constant.

Based on (3.5)–(3.7), (3.4) can be rewritten as

$$\begin{aligned}
\dot{V}_1 \le{}& \mu\gamma\sigma_1(\sigma_2 + \xi_2 + \alpha_1 + (\tilde{\alpha}_1 - \alpha_1) + \Psi_1 + \bar{\tau}_{d1} - \dot{y}_d - \dot{\xi}_1 - H + \mu\gamma\sigma_1) \\
&+ \frac{\dot{\mu}}{\mu}V_1 - \frac{\mu}{m_1}\tilde{\theta}_1^{\mathrm{T}}\dot{\hat{\theta}}_1 - \frac{\mu}{l_1}\tilde{\varphi}_1\dot{\hat{\varphi}}_1 + \frac{1}{l_0}\Big(-\frac{\bar{h}}{\mu}r^p + \bar{\Phi}(\varsigma_1)\Big) + R_1,
\end{aligned} \tag{3.8}$$

where

$$\bar{\tau}_{d1}(\varsigma, \gamma, \sigma_1, r) = \bar{\tau}_{d11}(\|\varsigma\|)\tanh\left(\frac{\mu\gamma\sigma_1\bar{\tau}_{d11}(\|\varsigma\|)}{\epsilon_{11}}\right) + \bar{\tau}_{d12}(\psi_3^{-1}(2r(t)))\tanh\left(\frac{\mu\gamma\sigma_1\bar{\tau}_{d12}}{\epsilon_{12}}\right),$$

and

$$R_1 = 0.2785(\epsilon_{11} + \epsilon_{12}) + \frac{d_0}{l_0} + \frac{1}{4}\bar{\tau}_{d12}^2(\psi_3^{-1}(2B(t))).$$

Notice the function $\frac{\bar{\Phi}(\varsigma_1)}{\mu\gamma\sigma_1 l_0}$ is discontinuous at $\sigma_1 = 0$, and the function $\tanh^2(\frac{\mu\gamma\sigma_1}{\lambda_1})$ is introduced. Therefore, one obtains

$$\begin{aligned}
\dot{V}_1 \le{}& \gamma\sigma_1\Big(\sigma_2 + \xi_2 + \alpha_1 + (\tilde{\alpha}_1 - \alpha_1) + \Psi_1 + \bar{\tau}_{d1} - \dot{\xi}_1 - H - \dot{y}_d + \mu\gamma\sigma_1 \\
&+ \frac{16}{\mu\gamma\sigma_1}\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0}\Big) - \frac{\mu}{m_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1 - \frac{\mu}{l_1}\tilde{\varphi}_1\dot{\hat{\varphi}}_1 \\
&- \frac{\bar{h}}{l_0\mu}r^p + \frac{\dot{\mu}}{\mu}V_1 + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + R_1.
\end{aligned} \tag{3.9}$$

According to (3.2), $\bar{\Psi}_1(\omega_1) = \Psi_1(\varsigma_1, \sigma_1, k_{C1}, k_{C2}) + \bar{\tau}_{d1}(\varsigma, \gamma, \sigma_1) = \theta_1^T \phi_1(\omega_1) + \varepsilon_1(\omega_1)$. And, using $\varphi_1 = \bar{\varepsilon}_1 + \gamma_1 > 0$, $\dot{V}_1$ is given as

$$\begin{aligned}\dot{V}_1 \leq \mu\gamma\sigma_1\Big(&\sigma_2 + \xi_2 + \alpha_1 + (\tilde{\alpha}_1 - \alpha_1) + \theta_1^T\phi_1 - \dot{y}_d - \dot{\xi}_1 - \gamma_1 \operatorname{sign}(\xi_1) - H + \mu\gamma\sigma_1 \\ &+ \frac{16}{\mu\gamma\sigma_1}\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0}\Big) + \frac{\dot{\mu}}{\mu}V_1 - \frac{\mu}{m_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1 - \frac{\mu}{l_1}\tilde{\varphi}_1\dot{\hat{\varphi}}_1 \\ &- \frac{\bar{h}}{l_0\mu}r^p + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \mu\gamma|\sigma_1|\varphi_1 + R_1.\end{aligned} \tag{3.10}$$

Using Lemma 2, the following inequality is obtained:

$$\mu\gamma|\sigma_1|\varphi_1 - \varphi_1\mu\gamma\sigma_1 \tanh(\frac{\mu\gamma\sigma_1}{q_1}) \leq 0.2785 q_1\varphi_1, \tag{3.11}$$

subsequently, the virtual controller and adaptive laws are designed as follows:

$$\alpha_1 = -c_1\xi_1 - \frac{k_1}{\mu^{2-p}}\gamma^{p-1}\delta_1^{2p-1} - \hat{\theta}_1^T\phi_1 + \dot{y}_d - \tilde{Y}_1(t)\sigma_1 - \mu\gamma\sigma_1 - \hat{\varphi}_1\tanh\left(\frac{\mu\gamma\sigma_1}{q_1}\right) - \frac{16}{\mu\gamma\sigma_1}\tanh^2\left(\frac{\mu\gamma\sigma_1}{\lambda_1}\right)\frac{\bar{\Phi}(\varsigma_1)}{l_0}, \tag{3.12}$$

$$\dot{\hat{\theta}}_1 = m_1\gamma\sigma_1\phi_1 - \frac{\iota_1}{\mu}\hat{\theta}_1, \tag{3.13}$$

$$\dot{\hat{\varphi}}_1 = l_1\gamma\sigma_1\tanh(\frac{\mu\gamma\sigma_1}{q_1}) - \frac{\tau_1}{\mu}\hat{\varphi}_1, \tag{3.14}$$

$$\dot{\xi}_1 = -c_1\xi_1 - \frac{k_1}{\mu^{2-p}}\gamma^{p-1}\xi_1^{2p-1} + \xi_2 + (\tilde{\alpha}_1 - \alpha_1) - \gamma_1 \operatorname{sign}(\xi_1), \tag{3.15}$$

where ι_1, τ_1 are positive design parameters, and design parameters $k_1 > 0, q_1 > 0$. The exponent $p = \frac{2n}{2n+1}$, with $n \geq 2$ being a integer. Consider

$$\tilde{Y}_1(t) = \max\{|\frac{\dot{k}_{C1}}{k_{C1}}|, |\frac{\dot{k}_{C2}}{k_{C2}}|\},$$

which results in the inequality

$$\tilde{Y}_1(t) + q\frac{\dot{k}_{C1}}{k_{C1}} + (1-q)\frac{\dot{k}_{C2}}{k_{C2}} \geq 0.$$

According to the fact $\tilde{\theta}_1^T\theta_1 \leq \frac{1}{2}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{1}{2}\theta_1^T\theta_1$ and $\tilde{\varphi}_1\varphi_1 \leq \frac{1}{2}\tilde{\varphi}_1^2 + \frac{1}{2}\varphi_1^2$, substituting (3.11)–(3.15) into (3.10) yields

$$\dot{V}_1 \leq \frac{\dot{\mu}}{\mu}V_1 - k_1\Big(\log\frac{1}{1-k^2}\Big)^p + \mu\gamma\sigma_1\sigma_2 - \frac{\bar{h}}{l_0\mu}r^p - \frac{\iota_1}{2m_1}\tilde{\theta}_1^T\tilde{\theta}_1 - \frac{\tau_1}{2l_1}\tilde{\varphi}_1^2 + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \bar{R}_1, \tag{3.16}$$

where $\bar{R}_1 = R_1 + 0.2785q_1\varphi_1 + \frac{\iota_1}{2m_1}\theta_1^T\theta_1 + \frac{\tau_1}{2l_1}\varphi_1^2$.

Step i ($2 \leq i \leq n-1$)**:** By applying the error dynamics defined in (3.1), the differential of the ith compensation error signal is derived as:

$$d\sigma_i = (\varsigma_{i+1} + \Psi_i + \tau_{di} - \dot{\tilde{\alpha}}_{i-1} - \dot{\xi}_i)dt. \tag{3.17}$$

A Lyapunov function candidate V_i is then selected, constructed recursively as:

$$V_i = V_{i-1} + \frac{\mu}{2}\sigma_i^2 + \frac{\mu}{2m_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \frac{\mu}{2l_i}\tilde{\varphi}_i^2,$$

where $m_i > 0, l_i > 0$ are positive design constants. The term $\tilde{\varphi}_i = \varphi_i - \hat{\varphi}_i$ represents the parameter estimation error, with $\hat{\varphi}_i$ being the estimate of the unknown constant φ_i.

With the aid of (3.17), one obtains

$$\begin{aligned}\dot{V}_i \leq{}& \frac{\dot{\mu}}{\mu}V_i - k_1\Big(\log\frac{1}{1-k^2}\Big)^p + \mu\sigma_{i-1}\sigma_i - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{i}k_j\sigma_j^{2p} - \sum_{j=1}^{i-1}\frac{\iota_j}{2m_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j - \sum_{j=1}^{i-1}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2\\ &+ \mu\sigma_i\Big(k_i\sigma_i^{2p-1} + \xi_{i+1} + \sigma_{i+1} + \alpha_i + (\tilde{\alpha}_i - \alpha_i) + \Psi_i + \tau_{di} - \dot{\tilde{\alpha}}_{i-1} - \dot{\xi}_i\Big)\\ &- \frac{\mu}{m_i}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i - \frac{\mu}{l_i}\tilde{\varphi}_i\dot{\hat{\varphi}}_i + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \sum_{j=1}^{i-1}\bar{R}_j.\end{aligned} \tag{3.18}$$

Building on Assumption 2 and Lemma 2, the disturbance term can be bounded as follows:

$$\mu\sigma_i\tau_{di}(\Xi,\varsigma) \leq \mu|\sigma_i|\bar{\tau}_{di1}(\|\varsigma\|) + \mu|\sigma_i|\bar{\tau}_{di2}(\|\Xi\|), \tag{3.19}$$

$$\mu|\sigma_i|\bar{\tau}_{di1}(\|\varsigma\|) \leq \mu\sigma_i\bar{\tau}_{di1}(\|\varsigma\|)\tanh\left(\frac{\mu\sigma_i\bar{\tau}_{di1}(\|\varsigma\|)}{\epsilon_{i1}}\right) + 0.2785\epsilon_{i1} \tag{3.20}$$

with $\epsilon_{i1} > 0$.

Furthermore, by applying Definition 2, Lemma 4, and Young's inequality, the term related to the unmodeled dynamics is bounded as:

$$\begin{aligned}\mu|\sigma_i|\bar{\tau}_{di2}(\|\Xi\|) &\leq \mu|\sigma_i|\bar{\tau}_{di2}(\psi_3^{-1}(r(t) + B(t)))\\ &\leq \mu\sigma_i\bar{\tau}_{di2}(\psi_3^{-1}(2r(t)))\tanh\left(\frac{\mu\sigma_i\bar{\tau}_{di2}}{\epsilon_{i2}}\right) + 0.2785\epsilon_{i2} + \mu^2\sigma_i^2 + \frac{1}{4}\bar{\tau}_{di2}^2(\psi_3^{-1}(2B(t))),\end{aligned} \tag{3.21}$$

with $\epsilon_{i2} > 0$.

Substituting the bounds from (3.19)–(3.21) into the expression for (3.18) yields

$$\begin{aligned}\dot{V}_i \leq{}& \frac{\dot{\mu}}{\mu}V_i - k_1\Big(\log\frac{1}{1-k^2}\Big)^p + \mu\sigma_{i-1}\sigma_i - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{i}k_j\sigma_j^{2p} - \sum_{j=1}^{i-1}\frac{\iota_j}{2m_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j - \sum_{j=1}^{i-1}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2\\ &+ \mu\sigma_i\Big(k_i\sigma_i^{2p-1} + \sigma_{i+1} + \xi_{i+1} + \alpha_i + (\tilde{\alpha}_i - \alpha_i) + \Psi_i + \bar{\tau}_{di} + \mu\sigma_i - \dot{\tilde{\alpha}}_{i-1} - \dot{\xi}_i\Big)\\ &- \frac{\mu}{m_i}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i - \frac{\mu}{l_i}\tilde{\varphi}_i\dot{\hat{\varphi}}_i + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \sum_{j=1}^{i-1}\bar{R}_j + R_i,\end{aligned} \tag{3.22}$$

where

$$\bar{\tau}_{di}(\varsigma,\sigma_i) = \bar{\tau}_{di1}(\|\varsigma\|)\tanh(\frac{\mu\sigma_i\bar{\tau}_{di1}(\|\varsigma\|)}{\epsilon_{i1}}) + \bar{\tau}_{di2}(\psi_3^{-1}(2r(t)))\tanh(\frac{\mu\sigma_i\bar{\tau}_{di2}}{\epsilon_{i2}}),$$

and

$$R_i = 0.2785(\epsilon_{i1}+\epsilon_{i2}) + \frac{1}{4}\bar{\tau}_{di2}^2(\psi_3^{-1}(2B(t))).$$

Similar to the approach outlined in (3.2), the combined term $F(\omega_i) = \Psi_i(\bar{\varsigma}_i,\sigma_i) + \bar{\tau}_{di}(\varsigma,\sigma_i,r)$ is represented via a fuzzy logic system as $F(\omega_i) = \theta_i^T\phi_i(\omega_i) + \varepsilon_i(\omega_i)$, where $\omega_i = (\varsigma,\sigma_i,r)$. Following this representation, the virtual controller and adaptive laws are designed as follows:

$$\alpha_i = -c_i\xi_i - \frac{k_i}{\mu^{2-p}}\gamma^{p-1}\delta_i^{2p-1} - \hat{\theta}_i^T\phi_i + \dot{\tilde{\alpha}}_{i-1} - \mu\sigma_i - \sigma_{i-1} - \hat{\varphi}_i\tanh\left(\frac{\mu\sigma_i}{q_i}\right), \tag{3.23}$$

$$\dot{\hat{\theta}}_i = m_i\sigma_i\phi_i - \frac{\iota_i}{\mu}\hat{\theta}_i, \tag{3.24}$$

$$\dot{\hat{\varphi}}_i = l_i\sigma_i\tanh\left(\frac{\mu\sigma_i}{q_i}\right) - \frac{\tau_i}{\mu}\hat{\varphi}_i, \tag{3.25}$$

$$\dot{\xi}_i = -c_i\xi_i - \frac{k_i}{\mu^{2-p}}\gamma^{p-1}\xi_i^{2p-1} + \xi_{i+1} + (\tilde{\alpha}_i - \alpha_i) - \gamma_i\,\mathrm{sign}(\xi_i), \tag{3.26}$$

where $\varphi_i = \bar{\varepsilon}_i + \gamma_i > 0$, and $q_i > 0$, $\iota_i > 0$, $\tau_i > 0$.

Building upon Lemma 2, the following inequality holds:

$$\mu|\sigma_i|\varphi_i - \varphi_i\mu\sigma_i\tanh(\frac{\mu\sigma_i}{q_1}) \le 0.2785q_i\varphi_i,$$

combining this result with the designed control laws and adaptive mechanisms given in (3.23)–(3.26), the time derivative of the Lyapunov function can be bounded as follows:

$$\begin{aligned}\dot{V}_i \le{}& \frac{\dot{\mu}}{\mu}\sigma_i - k_1\Big(\log\frac{1}{1-k^2}\Big)^p + \mu\sigma_i\sigma_{i+1} - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{i}k_j\sigma_j^{2p}\\ &- \sum_{j=1}^{i}\frac{\iota_j}{2m_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j - \sum_{j=1}^{i}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2 + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \sum_{j=1}^{i}\bar{R}_j,\end{aligned} \tag{3.27}$$

where $\bar{R}_i = R_i + 0.2785q_i\varphi_i + \frac{\iota_i}{2m_i}\theta_i^T\theta_i + \frac{\tau_i}{2l_i}\varphi_i^2$.

Step n: From (3.1), the differential of the final compensation error variable is derived as:

$$d\sigma_n = (u + \Psi_n + \tau_{dn} - \dot{\tilde{\alpha}}_{n-1} - \dot{\xi}_n)dt.$$

The overall Lyapunov function candidate for the nth step is chosen as:

$$\begin{aligned}V_n &= V_{n-1} + \frac{\mu}{2}\sigma_n^2 + \frac{\mu}{2m_n}\tilde{\theta}_n^{\mathrm{T}}\tilde{\theta}_n + \frac{\mu}{2l_n}\tilde{\varphi}_n^2\\ &= \frac{\mu}{2}\log\frac{1}{1-k^2} + \sum_{i=2}^{n}\frac{\mu}{2}\sigma_i^2 + \sum_{i=1}^{n}\frac{\mu}{2m_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \sum_{i=1}^{n}\frac{\mu}{2l_i}\tilde{\varphi}_i^2,\end{aligned} \tag{3.28}$$

where $m_n > 0, l_n > 0$, $\tilde{\varphi}_n = \varphi_n - \hat{\varphi}_n$, and $\hat{\varphi}_n$ is the estimate of the unknown constant φ_n.

Following the recursive design procedure from the previous steps, the time derivative of V_n can be bounded as:

$$\begin{aligned}\dot{V}_n \leq & \frac{\dot{\mu}}{\mu}V_n - k_1\Big(\log\frac{1}{1-k^2}\Big)^p - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{n-1}k_j\sigma_j^{2p} - \sum_{j=1}^{n-1}\frac{\iota_j}{2m_j}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i - \sum_{j=1}^{n-1}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2 \\ & + \mu\sigma_n\Big(\sigma_{n-1} + k_n\sigma_n^{2p-1} + u + \alpha_n - \alpha_n + \hat{\theta}_n^{\mathrm{T}}\phi_n - \dot{\tilde{\alpha}}_{n-1} + \mu\sigma_n - \dot{\xi}_n - \gamma_n\,\mathrm{sign}(\xi_n)\Big) \\ & - \frac{\mu}{m_n}\tilde{\theta}_n^{\mathrm{T}}(\dot{\hat{\theta}}_n - m_n\sigma_n^3\phi_n) - \frac{\mu}{l_n}\tilde{\varphi}_n\dot{\hat{\varphi}}_n + \mu|\sigma_n|\varphi_n + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \sum_{j=1}^{n-1}\bar{R}_j + R_n,\end{aligned} \tag{3.29}$$

where

$$F(\omega_n) = \Psi_n(\varsigma,\sigma_n) + \bar{\tau}_{dn}(\varsigma,\gamma,\sigma_n) = \theta_n^T\phi_n(\omega_n) + \varepsilon_n(\omega_n),$$

$$\bar{\tau}_{dn}(\varsigma,\sigma_n,r) = \bar{\tau}_{dn1}(\|\varsigma\|)\tanh\Big(\frac{\mu\sigma_n\bar{\tau}_{dn1}(\|\varsigma\|)}{\epsilon_{n1}}\Big) + \bar{\tau}_{dn2}(\psi_3^{-1}(2r(t)))\tanh\Big(\frac{\mu\sigma_n\bar{\tau}_{dn2}}{\epsilon_{n2}}\Big),$$

$$R_n = 0.2785(\epsilon_{n1} + \epsilon_{n2}) + \frac{1}{4}\bar{\tau}_{dn2}^2(\psi_3^{-1}(2B(t))),$$

and $\varphi_n = \bar{\varepsilon}_n + \gamma_n$ with $\omega_n = (\varsigma,\sigma_n)$.

Following the recursive design framework, the virtual controller and adaptive laws are designed as:

$$\alpha_n = -c_n\xi_n - \frac{k_n}{\mu^{2-p}}\gamma^{p-1}\delta_n^{2p-1} - \hat{\theta}_n^T\phi_n + \dot{\tilde{\alpha}}_{n-1} - \mu\sigma_n - \sigma_{n-1} - \hat{\varphi}_n\tanh\left(\frac{\mu\sigma_n}{q_n}\right), \tag{3.30}$$

$$\dot{\hat{\theta}}_n = m_n\sigma_n\phi_n - \frac{\iota_n}{\mu}\hat{\theta}_n, \tag{3.31}$$

$$\dot{\hat{\varphi}}_n = l_n\sigma_n\tanh\left(\frac{\mu\sigma_n}{q_n}\right) - \frac{\tau_n}{\mu}\hat{\varphi}_n, \tag{3.32}$$

$$\dot{\xi}_n = -c_n\xi_n - \frac{k_n}{\mu^{2-p}}\gamma^{p-1}\xi_n^{2p-1} + (\tilde{\alpha}_n - \alpha_n) - \gamma_n\,\mathrm{sign}(\xi_n), \tag{3.33}$$

where $c_n, k_n, q_n, \iota_n > 0$, and $\tau_n > 0$ are design parameters.

Substituting the expressions from (3.30) and (3.33) into Inequality (3.29) yields the following bound for the time derivative of the final Lyapunov function:

$$\begin{aligned}\dot{V}_n \leq & \frac{\dot{\mu}}{\mu}V_n - k_1\left(\log\frac{1}{1-k^2}\right)^p - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{n}k_j\sigma_j^{2p} - \sum_{j=1}^{n}\frac{\iota_j}{2m_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j - \sum_{j=1}^{n}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2 \\ & + \mu\sigma_n(u - \alpha_n) + \left(1 - 16\tanh^2\left(\frac{\mu\gamma\sigma_1}{\lambda_1}\right)\right)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \sum_{j=1}^{n}\bar{R}_j,\end{aligned} \tag{3.34}$$

where the aggregated residual term is defined as $\bar{R}_n = R_n + 0.2785q_n\varphi_n + \frac{\iota_n}{2m_n}\theta_n^T\theta_n + \frac{\tau_n}{2l_n}\varphi_n^2$.

3.2. Event-triggered mechanism

To significantly improve the overall efficiency, a event-triggered mechanism is designed as follows:

$$\nu(t) = -(1 + \Theta(t))(\bar{o} \tanh(\frac{\sigma_n \bar{o}}{q_0}) + \alpha_n \tanh(\frac{\sigma_n \alpha_n}{q_0})), \tag{3.35}$$

$$u(t) = \nu(t_k), \forall t \in [t_k, t_{k+1}), \tag{3.36}$$

$$t_{k+1} = \inf\{t > t_k \mid |e(t)| > \Theta|u(t)| + o(t)\}, \tag{3.37}$$

$$\dot{\Theta} = -\rho_1 \alpha_n^2 \Theta, 0 < \Theta(0) < 1, \tag{3.38}$$

$$\dot{o} = -\rho_2 \alpha_n^2 o, o(0) > 0, \tag{3.39}$$

where $q_0 > 0$, $\rho_1 > 0$, and $\rho_2 > 0$, $\bar{o} = \frac{o(t)}{1-\Theta(t)}$ is an auxiliary error variable, and $t_k (k \in Z^+)$ denotes the update instant.

From the triggering condition, one can show that there exist bounded time-varying functions $z_a(t)$ and $z_b(t)$ satisfying $|z_a(t)| \leq 1$ and $|z_b(t)| \leq 1$, such that the actual control input can be equivalently expressed as

$$u(t) = \frac{\nu(t) - z_b(t)\, o(t)}{1 + z_a(t)\, \Theta(t)}. \tag{3.40}$$

Remark 5. Compared with the static event-triggered threshold that is fixed and cannot be adjusted online, the proposed dynamic event-triggered mechanism with adaptive thresholds $\Theta(t)$ and $o(t)$ has obvious superiorities. A static threshold is fixed and cannot adapt to real-time changes of system states, tracking errors, and virtual control signals, which may result in excessively dense triggering and unnecessary waste of communication and computational resources, or excessively sparse triggering and performance degradation. In contrast, the adaptive dynamic thresholds $\Theta(t)$ and $o(t)$ are constructed to be updated online according to the virtual control law α_n, which can flexibly adjust the triggering condition and realize a better trade-off between tracking performance and resource utilization. Moreover, such a dynamic structure can effectively exclude Zeno behavior and ensure a strictly positive lower bound for the inter-event intervals, which is more suitable for uncertain nonlinear systems with asymmetric time-varying output constraints and unmodeled dynamics.

To conclude, the control block diagram is summarized as depicted in Figure 2.

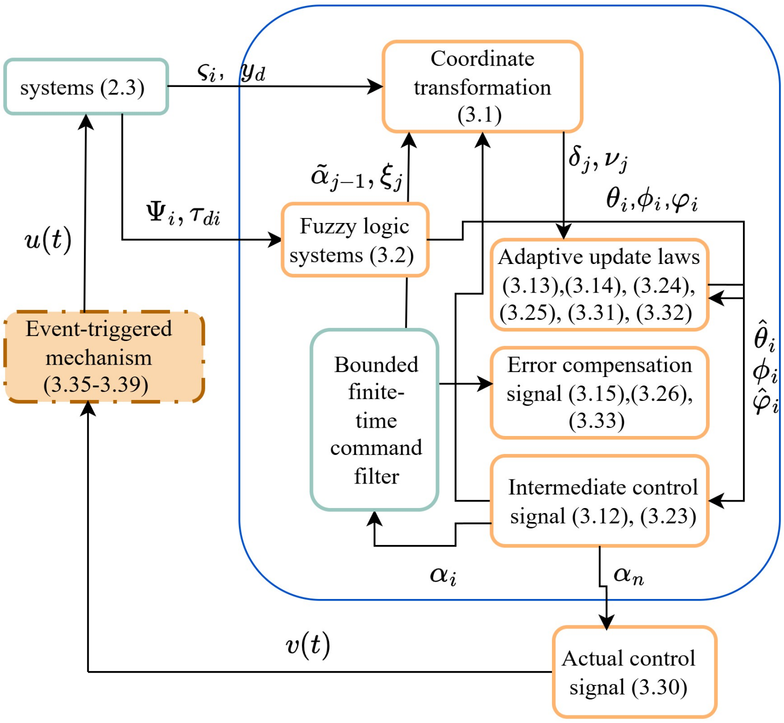

Figure 2. A schematic diagram of the overall control structure.

4. Main results

Theorem 1. Consider the nonlinear system (2.3) under Assumptions 1 and 2. By applying the event-triggered control mechanism defined in (3.35)–(3.39), the following closed-loop properties are guaranteed:

(I) All signals in the closed-loop system are uniformly bounded.

(II) The prescribed asymmetric time-varying output constraints are strictly satisfied at all times.

(III) The system output y tracks the reference signal y_d within a designated time interval.

Proof. (I) Substituting the equivalent representation of the control input from Eq (3.40) into Inequality (3.34) yields

$$\begin{aligned}\dot{V}_n \leq{} & \frac{\dot{\mu}}{\mu}V_n - k_1\Big(\log\frac{1}{1-k^2}\Big)^p - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{n} k_j\sigma_j^{2p} - \sum_{j=1}^{n}\frac{\iota_j}{2m_j}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i - \sum_{j=1}^{n}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2 \\ & + \mu\sigma_n\left(\frac{v(t)-z_b(t)o(t)}{1+z_a(t)\Theta(t)} - \alpha_n\right) + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \sum_{j=1}^{n}\bar{R}_j.\end{aligned} \tag{4.1}$$

Applying Lemma 2, the term involving the control input difference can be bounded as: $\mu\sigma_n(\frac{v(t)-z_b(t)o(t)}{1+z_a(t)\Theta(t)} - \alpha_n) \leq 0.557q_0$. Consequently, the time derivative of the Lyapunov function is further bounded by

$$\begin{aligned}\dot{V}_n \leq{}& \frac{\dot{\mu}}{\mu}V_n - k_1\Big(\log\frac{1}{1-k^2}\Big)^p - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{n}k_j\sigma_j^{2p}\\ &- \sum_{j=1}^{n}\frac{\iota_j}{2m_j}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i - \sum_{j=1}^{n}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2 + \Big(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Big)\frac{\bar{\Phi}(\varsigma_1)}{l_0} + \Gamma,\end{aligned} \tag{4.2}$$

with $\Gamma = \sum_{j=1}^{n}\bar{R}_j + 0.557q_0$.

To establish the boundedness of the error compensation signals ξ_i for $i = 1,\cdots,n$, a composite Lyapunov function candidate is defined as $V_\xi = \frac{\mu}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\xi_{i,j}^2$, whose derivative is

$$\begin{aligned}\dot{V}_\xi \leq{}& \frac{\dot{\mu}}{\mu}V_\xi + \mu\sum_{i=1}^{N}\Bigg(\sum_{j=1}^{n-1}\Big(\xi_{i,j}\Big(-c_{i,j}\xi_{i,j} - \frac{k_{i,j}}{\mu^{2-p}}\xi_{i,j}^{2p-1} + \xi_{i,j+1} + (\tilde{\alpha}_{i,j} - \alpha_{i,j})\Big) - l_{i,j}\,\mathrm{sign}(\xi_{i,j})\Big)\\ &+ \xi_{i,n}\Big(-c_{i,n}\xi_{i,n} - \frac{k_{i,n}}{\mu^{2-p}}\xi_{i,n}^{2p-1} - l_{i,n}\,\mathrm{sign}(\xi_{i,n})\Big)\Bigg).\end{aligned}$$

Under the given conditions, specifically, the bounded mismatch between the filter output and the virtual control law, $|\tilde{\alpha}_{i,j} - \alpha_{i,j}| \leq \Xi_{i.j}$, and the application of Young's inequality to handle cross terms, $\xi_{i,j}\xi_{i,j+1} \leq \frac{1}{2}\xi_{i,j}^2 + \frac{1}{2}\xi_{i,j+1}^2$, the derivative can be further bounded as:

$$\dot{V}_\xi \leq \frac{\dot{\mu}}{\mu}V_\xi - \frac{a_{\xi_0}}{\mu}V_\xi^p - \frac{a_{\xi_1}}{\mu}V_\xi - a_{\xi_2}V_\xi^{\frac{1}{2}} \leq \frac{\dot{\mu}}{\mu}V_\xi - \frac{a_{\xi_0}}{\mu}V_\xi^p, \tag{4.3}$$

where the positive constants are defined as $a_{\xi_0} = \min\{2^p c_{i,1},\ldots,2^p c_{i,n}\}$, $a_{\xi_1} = \min\{2(w_{i,1} - \frac{1}{2}),\ldots,2(w_{i,n} - \frac{1}{2})\}$, and $a_{\xi_2} = \min\{2|l_{i,j} - \Xi_{i,j}|, 2l_{i,n}\}$. Applying (3.1), the ξ_i are bounded.

(i) When $\sigma_1 \notin S_{\lambda_1}$, Lemma 5 gives the inequality

$$\Bigg(1 - 16\tanh^2\Big(\frac{\mu\gamma\sigma_1}{\lambda_1}\Big)\Bigg) \leq 0,$$

which, together with Inequality (4.2), leads to

$$\begin{aligned}\dot{V}_n \leq{}& \frac{\dot{\mu}}{\mu}V_n - k_1\Big(\log\frac{1}{1-k^2}\Big)^p - \frac{\bar{h}}{l_0\mu}r^p - \sum_{j=2}^{n}k_j\sigma_j^{2p}\\ &- \sum_{j=1}^{n}\frac{\iota_j}{2m_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j - \sum_{j=1}^{n}\frac{\tau_j}{2l_j}\tilde{\varphi}_j^2 + \Gamma.\end{aligned}$$

Using the algebraic inequality

$$|y|^d|z|^c \leq \frac{d}{d+c}\,\vartheta\,|y|^{d+c} + \frac{c}{d+c}\,\vartheta^{-\frac{d}{c}}|z|^{d+c},$$

and applying Lemma 4, we further obtain the bound

$$\begin{aligned}\dot{V}_n \leq{}& \frac{\dot{\mu}}{\mu}V_n - \tau_{d1}\Big(\frac{1}{2}\log\frac{1}{1-k^2} + \sum_{i=2}^{n}\frac{1}{2}\sigma_i^2 + \sum_{i=1}^{n}\frac{1}{2m_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \sum_{i=1}^{n}\frac{1}{2l_i}\tilde{\varphi}_i^2\Big)^p + Q_2\\ ={}& \frac{\dot{\mu}}{\mu}V_n - \frac{Q_1}{\mu}V_n^p + Q_2,\end{aligned} \tag{4.4}$$

where

$$Q_1 = \min\{1,\ 2^p k_j,\ \tau_j^p,\ \iota_j^p\}\ (j = 1, \ldots, n),$$

and

$$Q_2 = \Gamma + 2(1-p)p^{\frac{p}{1-p}} + (1-p)\Big(\frac{p}{a_1}\Big)^{\frac{p}{1-p}}.$$

From Inequality (4.4), the boundedness of $\sigma_i,\ \tilde{\theta}_i,\ \tilde{\varphi}_i\ (i = 1, \ldots, n)$ is established. Because $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\varphi}_i = \varphi_i - \hat{\varphi}_i$, the boundedness of the estimates $\hat{\theta}_i$ and $\hat{\varphi}_i$ follows directly. Finally, recalling the definitions of the tracking error δ_i in (3.1), the compensation error σ_i, and the boundedness of the compensation signals ξ_i, we conclude that δ_i and ς_i remain bounded. Hence, all closed-loop signals are uniformly bounded.

(*ii*) Case where $\sigma_1 \in S_{\lambda_1}$. This case indicates $|\sigma_1| \leq 0.2554\lambda_1$, i.e., σ_1 is bounded. Using the boundedness of ξ_1 and (3.1), it follows that δ_1 is also bounded. From (3.1), ς_1 is bounded. Thus, a constant $S > 0$ can be chosen such that $(1 - 16\tanh^2(\frac{\mu\gamma\sigma_1}{\lambda_1}))\frac{\bar{\Phi}(\varsigma_1)}{l_0} \leq S$. Following a reasoning analogous to case (*i*), we obtain $\dot{V}_n \leq -Q_1 V_n^p + \bar{Q}_2$ where $\bar{Q}_2 = Q_2 + S$. Then, (I) is proved.

(II) From (I), it has been shown that $\dot{V}_n \leq \frac{\dot{\mu}}{\mu}V_n - \frac{Q_1}{\mu}V_n^p + Q_2$ when $\sigma_1 \notin S_{\lambda_1}$, or $\dot{V}_n \leq \frac{\dot{\mu}}{\mu}V_n - \frac{Q_1}{\mu}V_n^p + \bar{Q}_2$ when $\sigma_1 \in S_{\lambda_1}$. For the subsequent analysis, the more general inequality $\dot{V}_n \leq \frac{\dot{\mu}}{\mu}V_n - \frac{Q_1}{\mu}V_n^p + \bar{Q}_2$ is considered when $\sigma_1 \in S_{\lambda_1}$.

Next, we verify the validity of statement (III) by examining two distinct cases.

Case I: For $t \in [t_0, t_0 + T_g)$, according to Lemma 2, the Lyapunov function satisfies $V_n(t) \leq \bar{D}\mu(t)$, where $\bar{D}$ is a positive constant. Consequently, at the instant $t_0 + T_g$, we have $V_n(t_0 + T_g) \leq \bar{D}a = \bar{M}$, with $\bar{M}$ being arbitrarily small. Similarly, the derivative of the compensation-error Lyapunov function obeys $\dot{V}_\xi \leq \frac{\dot{\mu}}{\mu}V_\xi$ for $t \in [t_0, t_0 + T_g)$. Applying Lemma 2 again guarantees the boundedness of all compensation signals $\xi_i\ (i = 1, \ldots, n)$, i.e., $|\xi_i| \leq M_i$ for some constants M_i.

Since $V_n \leq \bar{D}\mu(t)$, the inequality

$$\frac{\mu(t)}{2}\log\frac{1}{1-k^2} \leq \bar{D}\mu(t),$$

holds, which yields

$$|k| \leq \sqrt{1 - e^{-2\bar{D}}} < 1. \tag{4.5}$$

This directly implies $-k_{c1}(t) < \sigma_1(t) < k_{c2}(t)$.

By the definitions of $k_{c1}(t)$ and $k_{c2}(t)$, the above inequality passively ensures that the output always satisfies the prescribed constraints $k_{c1}(t) < y(t) < k_{c2}(t)$.

Furthermore, from Inequality (4.5) and the error definition (3.1), the tracking error $\delta_1(t) = y(t) - y_d(t)$ enters a small compact set

$$\mathbb{P} = \{\, \delta_1(t) \mid -(K_{c1} + M_1) < \delta_1(t) < K_{c2} + M_1 \,\}$$

within the designated time interval $[t_0, t_0 + T_g)$. In summary, by appropriately choosing the design parameter v, the quantities $|\xi_i(t)|$, $|\sigma_i(t)|$, $V_\xi(t)$, and $V_n(t)$ all converge to a small, globally attractive neighborhood within the pre-assigned time T_p. Hence, the boundedness of all closed-loop signals is guaranteed.

Case II: When $t \in [t_0 + T_g, \infty)$, once $\delta_1(t)$ enters the compact set $\mathbb{P}$ within the designated time T_p, Inequality (4.4) guarantees that it will remain in $\mathbb{P}$ for all subsequent time $t \geq T_g$. From (4.4), we obtain the differential inequality

$$\dot{V}_n(t) \leq -\frac{Q_1}{a} V_n^p(t) + \bar{Q}_2.$$

For any constant $b \in (0, 1)$, Lemma 2 yields a finite settling time

$$T^* = \frac{a}{(1-p)bQ_1}\left(V_n(t_0 + T_g)^{1-p} - \Big(\frac{a\bar{Q}_2}{(1-b)Q_1}\Big)^{\frac{1-p}{p}}\right) > 0.$$

Hence, the total convergence time is $T_s = t_0 + T_g + T^*$. Following the reasoning in [26], it can be shown that all signals in the closed-loop system remain recursively bounded. The same conclusion holds when the alternative bound

$$\dot{V}_n \leq \frac{\dot{\mu}}{\mu} V_n - \frac{Q_1}{\mu} V_n^p + Q_2$$

is satisfied (i.e., when $\sigma_1 \notin S_{\lambda_1}$).

It is essential to highlight that the designed control scheme effectively avoids Zeno behavior, as demonstrated in the following proof.

Proof. The objective is to show the existence of a constant $t_0 > 0$ such that the inter-execution interval is bounded below by t_0, i.e., $t_{k+1} - t_k \geq t_0$ for every positive integer k.

During the interval $t \in [t_k, t_{k+1})$, the control input is held constant, $u(t) = v(t_k)$, and consequently $\dot{u}(t) = 0$. Defining the measurement error as $\ell(t) = v(t) - u(t)$, its time derivative satisfies

$$\frac{d|\ell(t)|}{dt} = \operatorname{sign}(\ell(t))\, \dot{\ell}(t) \leq |\dot{v}(t)|. \tag{4.6}$$

The derivative $\dot{v}(t)$ exists and is bounded because $v(t)$ is constructed from bounded system signals. Let $\bar{v}$ be an upper bound for $|\dot{v}(t)|$.

At the triggering instant t_{k+1}, the error reaches the adaptive threshold, i.e.,

$$\lim_{t \to t_{k+1}} |\ell(t)| = \Theta(t)|u(t)| + o(t),$$

while at the previous update time t_k, we have $\ell(t_k) = 0$. Integrating inequality (4.6) over $[t_k, t_{k+1})$ yields

$$|\ell(t_{k+1})| \leq \bar{v}\,(t_{k+1} - t_k).$$

Combining the two expressions for $|\ell(t_{k+1})|$, we obtain

$$\Theta(t)|u(t)| + o(t) \leq \bar{v}\,(t_{k+1} - t_k),$$

which directly implies

$$t_{k+1} - t_k \geq \frac{\Theta(t)|u(t)| + o(t)}{\bar{v}} \equiv t_0 > 0.$$

Since the threshold parameters $\Theta(t)$ and $o(t)$ are positive and the control signal $u(t)$ is bounded, the lower bound t_0 is strictly positive. Hence, Zeno behavior is excluded.

5. Simulation example

Example 1. As an illustrative application, consider a robot manipulator system modeled by (2.2). Incorporating unmodeled dynamics and external disturbances, the system can be expressed in the following form:

$$\begin{cases} \dot{\Xi} = -\Xi + 0.6\varsigma_1^2 + \varsigma_2, \\ d\varsigma_1 = \varsigma_2 \, \mathrm{d}t, \\ d\varsigma_2 = (u - 10\sin(\varsigma_1) - 2\varsigma_2 + 0.15\Xi\cos(\varsigma_1\varsigma_2)) \, \mathrm{d}t, \\ y = \varsigma_1, \end{cases} \tag{5.1}$$

where $\mathcal{G} = 1$, $\mathcal{Z} = 2$, and $mgl = 10$.

For simulation purposes, the initial conditions, reference signal, and output constraints are selected as follows: $\varsigma(0) = [0.2, -1.1]^T$, $(\xi_1(0), \xi_2(0)) = (0.2, 0.3)$, $(\Delta_{11}(0), \Delta_{12}(0)) = (-1, -0.2)$, $\hat{\theta}_1(0) = (0.2, 0.2, 0.5, 0.6, 0.1)$, $\hat{\theta}_2(0) = (0.3, 0.7, 0.8, 0.5, 0.8)$, $\hat{\varphi}_1(0) = 0.2$, $\hat{\varphi}_2(0) = 0.4$, $\Theta(0) = 0.3$, $\beta(0) = 0.3$, $y_r = 0.6\sin(t) + \sin(0.5t)$, $k_{c1} = 0.6\sin(t) + \sin(0.5t) - 0.3 - 0.8e^{-t}$ and $k_{c2} = 0.6\sin(t) + \sin(0.5t) + 0.18 + 1.2e^{-2t}$).

The simulation incorporates the following design parameters: $c_1 = 2, c_2 = 3, k_1 = 0.08, k_2 = 0.08, l_0 = 2, p = 0.8, q_1 = 1, q_2 = 1, \gamma_1 = 0.01, \gamma_2 = 0.01, q_0 = 0.6, \rho_1 = 0.01, \rho_2 = 0.01, \epsilon = 0.12$, $m_1 = 1, m_2 = 1, \iota_1 = 1, \iota_2 = 1, \tau_1 = 1, \tau_2 = 1, l_1 = 6, l_2 = 13$.

The outcomes are described in Figures 3–10. Figure 3 illustrates the trajectories of the output signal y and the desired signal y_d under the constraints k_{c1} and k_{c2} using the proposed scheme, along with the corresponding tracking error. Figure 4 shows the curve of the control input u under the dynamic event-triggered mechanism described in this paper, while Figure 5 illustrates the sequence of trigger time intervals under this mechanism, confirming that the system effectively avoids the Zeno phenomenon. Figure 6 shows the curve of the control input u under the traditional static event-triggered mechanism, while Figure 7 displays the sequence of trigger time intervals under the traditional static event-triggered mechanism, confirming that the dynamic event-triggered mechanism is more effective at conserving communication resources. As observed in Figure 3, the output y successfully tracks the desired signal y_d within the designated time, and the state variable ς_1 remains within the prescribed constraint region. Furthermore, Figure 8 presents the convergence behavior of the tracking error, and Figures 9 and 10 depict the trajectories of the compensation signal and the adaptive laws, respectively, both of which are ultimately bounded.

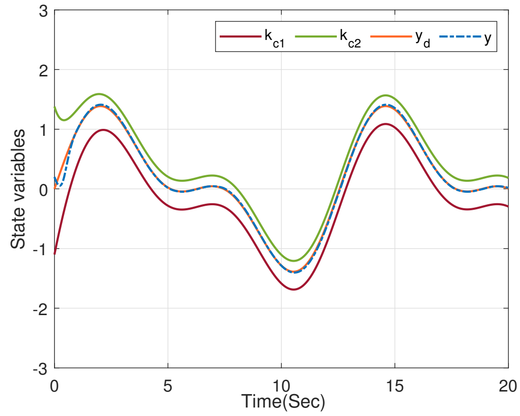

Figure 3. Trajectory of output signal under the proposed strategy.

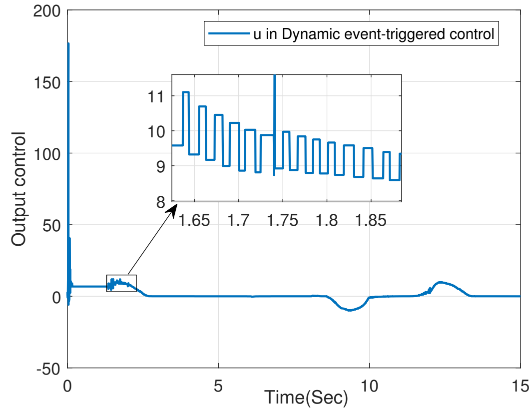

Figure 4. Control input in dynamic event-triggered control.

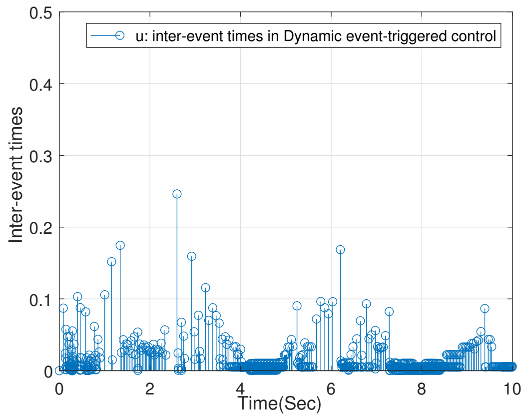

Figure 5. Interexecution time intervals in dynamic event-triggered control.

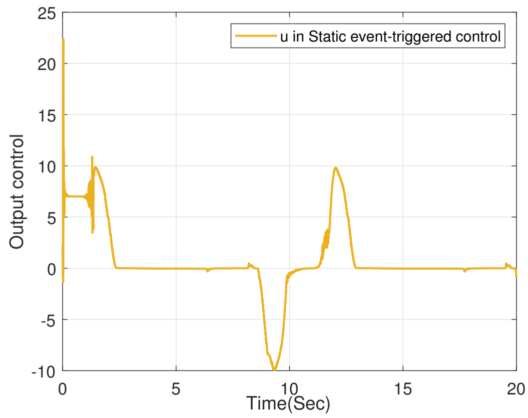

Figure 6. Control input in static event-triggered control.

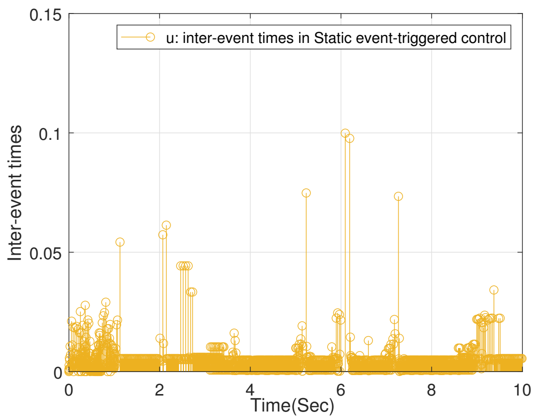

Figure 7. Interexecution time intervals in static event-triggered control.

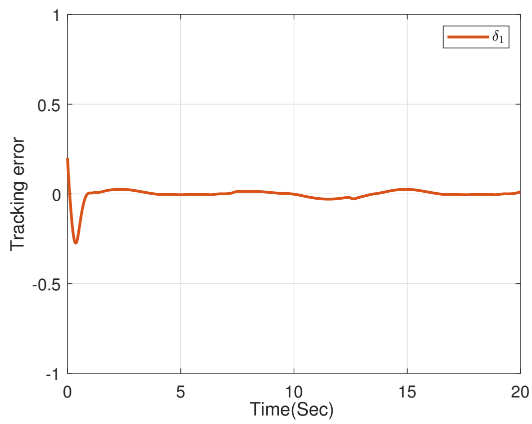

Figure 8. Trajectory of error.

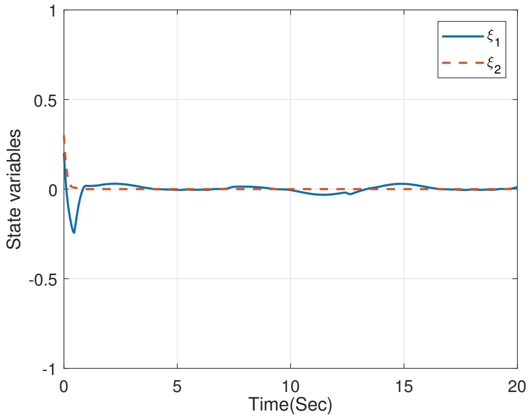

Figure 9. Compensation signals.

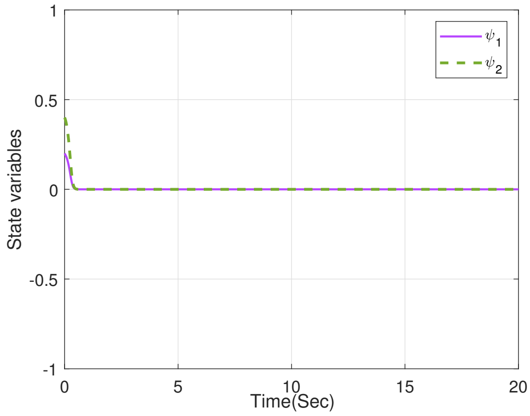

Figure 10. Adaptive parameters.

6. Conclusions

This paper develops a designated-time adaptive tracking control scheme for strict-feedback nonlinear systems with asymmetric time-varying output constraints, unmodeled dynamics, and external disturbances. Three main contributions are summarized as follows. First, different from conventional finite-time and predefined-time control, the proposed time-scale function based designated-time framework achieves arbitrarily preset settling time independent of initial conditions, reduces the conservatism of convergence time bounds, and eliminates the singularity and unbounded control magnitude drawbacks caused by fractional power terms. Second, fuzzy logic systems are utilized to approximate system unknown dynamics, and combined with bounded command filtering to avoid the complexity explosion problem in backstepping design. Third, by integrating an asymmetric time-varying barrier Lyapunov function into dynamic event-triggered control, the proposed method ensures output constraint satisfaction and greatly saves communication and computational resources. Theoretical analysis and simulation results verify the effectiveness and superiority of the scheme in tracking accuracy and transient performance. Furthermore, the applicability and inherent limitations of the method are discussed. Future work will extend this framework to nonlinear systems with full-state constraints, actuator faults, and non-strict-feedback structures, and improve robustness against complex noise and disturbances for practical robotic and industrial applications.

Author contributions

Lifang Qiu: Writing-original draft, methodology; Junsheng Zhao: Writing- reviewing and editing, resources. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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