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Research article

Numerical analysis of fractional nonlinear chemical wave models using the α-Laplace homotopy perturbation method

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Abstract: This paper investigates nonlinear fractional chemical-wave models governed by the Atangana–Baleanu–Caputo (ABC) fractional derivative. The motivation of this study arose from the need to model memory-dependent effects in nonlinear chemical-wave propagation, which cannot be adequately described using classical integer-order models. To solve the considered system, an α-Laplace homotopy perturbation method (α-LHPM) was developed by combining the α-Laplace transform with He's homotopy perturbation technique. The proposed approach provides recursive series solutions with rapid convergence and reduced computational complexity. Existence, uniqueness, and Hyers–Ulam stability results were also established under suitable assumptions. As an application, the fractional Belousov–Zhabotinsky dynamical system (BZDS) was analyzed for different fractional orders $0 < \sigma \leq 1$. Numerical simulations showed that decreasing the fractional-order parameter slows the wave propagation and produces smoother solution profiles due to stronger memory effects. In the classical case $\sigma = 1$, the obtained numerical solutions showed excellent agreement with the exact solutions, with absolute errors of order 10^{-5} for ζ and 10^{-4} for ω. These results demonstrate that the proposed α-LHPM is an accurate and efficient semi-analytical tool for solving nonlinear fractional chemical-wave models.

Keywords: fractional dynamical systems; fractional derivatives; α-Laplace transform; homotopy perturbation method
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1. Introduction

Fractional calculus extends the usual concepts of differentiation and integration to non-integer orders. It provides a useful way to describe processes where the current state depends not only on the present but also on past history. This idea goes back to Leibniz's discussion with L'Hôpital in 1695 about derivatives of order $1/2$. Later, mathematicians such as Riemann, Liouville, and Grünwald developed this idea into a well-known field [1]. Further developments can be found in [2]. Over time, different types of fractional operators have been introduced, including the Caputo–Fabrizio, Hilfer, Katugampola, and ABC derivatives. Fractional models are widely used because they can describe memory and hereditary effects better than classical models. In many real problems, the behavior of a system depends on its past states, which is difficult to represent using standard models [3]. Related work is also discussed in [4]. Depending on the operator, different types of memory effects can be modeled, such as power-law or exponential decay. For this reason, fractional calculus has become an important tool in modeling and analysis [5]. Alhazmi et al. [6] introduced a chaos control and stability framework for the glucose–insulin regulatory system using Atangana–Baleanu fractal-fractional derivatives. Almutairi and Saber [7] introduced a time-fractal fractional derivative approach with a power-law kernel for analyzing the Burke–Shaw system. These models are used in many fields such as viscoelasticity, diffusion, heat transfer, control theory, and biology [8–11]. In many cases, they give results that match observations better than classical models. Because of this, the field has attracted increasing attention in recent years. Partial differential equations are commonly used to describe physical processes such as diffusion and wave propagation. However, classical equations are local and may not be suitable for systems with memory effects. Fractional partial differential equations (FPDEs) extend these models by using fractional derivatives instead of integer-order derivatives [12]. These equations have been applied in many areas such as physics, biology, and finance [13]. Additional applications are discussed in [14]. Duan et al. [15] introduced a study on global dynamics and mixed-mode oscillations in nonlinear ship rolling models and, in [16] introduced the analysis of strange nonchaotic attractors in quasiperiodically forced piecewise smooth systems.

Many analytical and numerical methods have been developed to solve fractional models. These include transform methods, decomposition methods, homotopy techniques, and numerical schemes. For example, the Sinh–Gordon expansion is discussed in [17], while algebraic methods are presented in [18]. The fractional Newton method (FNM) is studied in [19], and Laplace transform methods are given in [1]. Modified Adams–Bashforth schemes are described in [20,21], the homotopy perturbation technique in [22], the Boussinesq-type wave equation transform in [23], and variational iteration methods are presented in [24]. Sumudu decomposition is discussed in [25], Adomian decomposition appears in [26], and the Laplace residual power series method in [27]. Kernel Hilbert space methods are studied in [28], and scaling methods are presented is discussed in [29]. The q-homotopy analysis transform method (q-HATM) is introduced in [30], residual power series method [31], monotone iterative scheme [32, 33], and numerical treatment method [34] have also been explored. Alsallami et al. [35] introduced a time-fractional analysis of the Belousov–Zhabotinsky system using singular and non-singular kernels. These methods aim to provide accurate and practical solutions. Ali et al. [36] introduced optical soliton solutions for the time-fractional $q-$deformed sinh–Gordon equation via a semi-analytic method. Fractional chemical-wave models have important applications in several real-world phenomena involving memory-dependent transport and diffusion processes. Such models arise in chemical reaction dynamics, biological pattern formation, neuronal signal propagation, cardiac tissue dynamics, and reaction–diffusion systems with hereditary effects. In particular, the fractional-order parameter in the ABC operator provides a realistic description of memory and nonlocal temporal behavior that cannot be captured by classical integer-order models.

In this work, we consider the following general fractional nonlinear system:

$$\begin{cases} {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\zeta(\xi_1,\xi_2) = \zeta_{\xi_1\xi_1}(\xi_1,\xi_2) \\ \quad + \mathcal{R}_1\left\{\xi_1,\xi_2,\zeta(\xi_1,\xi_2),\zeta(\xi_1,\xi_2-\rho),\varpi\left[\xi_1,\xi_2-\mathcal{G}(\zeta(\xi_1,\xi_2))\right]\right\}, \\ {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\varpi(\xi_1,\xi_2) = \varpi_{\xi_1\xi_1}(\xi_1,\xi_2) \\ \quad + \mathcal{R}_2\left\{\xi_1,\xi_2,\varpi(\xi_1,\xi_2),\varpi(\xi_1,\xi_2-\rho),\zeta\left[\xi_1,\xi_2-\mathcal{G}(\varpi(\xi_1,\xi_2))\right]\right\}, \\ \zeta(\xi_1,\xi_2) = \phi_1(\xi_1,\xi_2), \quad 0 \le \xi_2 \le \chi_0, \\ \varpi(\xi_1,\xi_2) = \phi_2(\xi_1,\xi_2), \quad 0 \le \xi_2 \le \chi_0, \\ \zeta(\xi_1,0) = k_0(\xi_1), \quad \varpi(\xi_1,0) = \hat{k}_0(\xi_1), \end{cases} \tag{1.1}$$

for all $(\xi_1,\xi_2) \in \Delta \times [0,\xi_0]$, where $\Delta \subset \mathbb{R}$ is a compact set, ${}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}$ is the ABC operator of order $0 < \sigma \le 1$, $\chi_0 = \max\{\rho, \sup_t \mathcal{G}(t)\}$ with $\chi_0 < \xi_0$, $\rho > 0$, $\mathcal{G}$ is a continuous map on $(0,\chi_0]$, $\phi_{1,2}$ are bounded continuous maps, and $\mathcal{R}_{1,2}$ are continuous.

The use of the ABC fractional derivative in the present Belousov–Zhabotinsky dynamical system (BZDS) model is motivated by the memory-dependent nature of chemical wave propagation. In contrast to classical reaction–diffusion models, the fractional formulation incorporates hereditary effects and delayed transport behavior. The ABC operator is particularly suitable because its non-singular Mittag–Leffler kernel provides a smooth memory decay without introducing singularities near the initial time. Moreover, the fractional-order parameter σ controls the strength of the memory effect: when $\sigma = 1$, the model reduces to the classical BZDS, while smaller values of σ correspond to stronger memory and slower chemical-wave propagation. Therefore, the ABC fractional formulation provides a flexible framework for describing nonlocal temporal effects in nonlinear chemical-wave dynamics.

If $\mathcal{R}_1 = \zeta[1-\zeta-\delta_1\varpi] + \delta'_1\varpi$ and $\mathcal{R}_2 = -\delta_2\zeta\varpi + \delta_3\varpi$, then system (1.1) represents the fractional BZDS [37], where ζ and ϖ denote the concentrations of the reacting species. Passing to a fractional form allows us to include memory effects, which can give a more realistic description of wave behavior. Polynomial schemes are discussed in [37]. The q-HATM is used in [38], and expansion techniques (ET) methods are presented in [39]. Fractional sub-equation (FS) procedures are studied in [40], and transform methods are discussed in [41].

The motivation of this work comes from the fact that chemical wave processes are affected by memory effects that cannot be described well by classical models. In this paper, we study fractional nonlinear systems related to chemical wave dynamics. We use the α-Laplace homotopy perturbation method (α-LHPM) to obtain approximate solutions. The proposed method combines the ABC fractional operator with the α-Laplace transform to build series solutions that can be used for numerical computation. As an application, the method is applied to the fractional BZDS to study how the fractional-order parameter affects wave propagation.

The novelty of this work lies in the development of the α-Laplace homotopy perturbation method (α-LHPM) for solving nonlinear fractional dynamical systems involving the Atangana–Baleanu–Caputo (ABC) fractional derivative. In contrast to the classical homotopy perturbation method, the proposed approach incorporates the α-Laplace transform within the perturbation framework, leading to a convenient recursive formulation for constructing approximate solutions of nonlinear fractional models. Moreover, unlike standard Laplace-based approaches, the proposed method is formulated directly within the α-Laplace setting, which provides additional flexibility in treating fractional-order operators and preserving the nonlocal structure of the considered system.

The motivation for combining the α-Laplace transform with the ABC fractional derivative is closely related to the memory-dependent nature of the investigated chemical-wave model. The ABC operator employs a non-singular Mittag–Leffler kernel, which is suitable for describing hereditary and memory effects appearing in complex diffusion and reaction processes. On the other hand, the α-Laplace transform allows the fractional system to be converted into an equivalent algebraic form that facilitates the recursive construction of the homotopy series solution. Consequently, the proposed α-LHPM provides a systematic framework for handling nonlinear fractional terms while preserving the memory characteristics induced by the ABC operator.

The proposed α-LHPM differs from the classical HPM in that the latter is constructed directly from the differential formulation without employing an integral transform. It also differs from the standard Laplace-HPM by replacing the classical Laplace transform with the α-Laplace transform in the solution procedure. This modification leads to a more suitable operational framework for the present fractional system and yields rapidly convergent series approximations for the nonlinear chemical-wave model.

Fractional-order models have recently demonstrated significant modeling capability in several complex applications involving memory-dependent dynamics, including animal learning processes, behavioral learning systems, and fractional blood-flow models. Motivated by these developments, the present work investigates the fractional Belousov–Zhabotinsky system within the ABC fractional framework and develops an efficient computational scheme for obtaining approximate analytical solutions.

The rest of the paper is organized as follows. Section 2 presents the preliminary definitions and analytical framework associated with the ABC fractional operator and the equivalent integral formulation of the system. Section 3 introduces the proposed α-LHPM together with its convergence and stability analysis. Section 4 presents the numerical simulations and discusses the obtained results. Finally, Section 5 concludes the paper.

2. Preliminaries

This section presents the main concepts and analytical tools used throughout the paper. We first recall the definitions of the ABC fractional integral and derivative, together with some of their fundamental properties. These operators are employed to describe the memory-dependent behavior of the considered nonlinear chemical-wave system. Next, the fractional system is rewritten in an equivalent integral form by applying the ABC fractional integral operator. This formulation allows the problem to be studied within a suitable Banach space equipped with the supremum norm. The corresponding nonlinear operator is then introduced, and several auxiliary results concerning continuity, Lipschitz conditions, and contraction properties are established. These results provide the analytical framework required for proving the existence and uniqueness of solutions and for studying the convergence behavior of the proposed numerical method in the subsequent sections.

Let ζ be any continuous function on $\Delta \times [0, \xi_0]$, where $\xi_0 > 0$. The ABC integral map, [42], of

$\zeta(\xi_1,\xi_2)$ for order $0<\sigma<1$ is

$$ {}^{ABC}\mathcal{I}^{\sigma}_{0,\xi_2}\zeta(\xi_1,\xi_2)=\frac{1-\sigma}{A^{\sigma}}\,\zeta(\xi_1,\xi_2)+\frac{\sigma}{\Gamma(\sigma)A^{\sigma}}\int_0^{\xi_2}(\xi_2-\varrho)^{\sigma-1}\zeta(\xi_1,\varrho)d\varrho,\quad 0<\sigma<1, \tag{2.1}$$

and the ABC derivative map of $\zeta(\xi_1,\xi_2)$ for order $n-1<\sigma\le n$ is

$$ {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\zeta(\xi_1,\xi_2)=\begin{cases}\dfrac{A^{\sigma}}{1-\sigma}\displaystyle\int_0^{\xi_2}E_{\sigma}\left(\frac{-\sigma(\xi_2-\varrho)^{\sigma}}{1-\sigma}\right)\zeta^{(n)}_{\varrho^n}(\xi_1,\varrho)d\varrho, & n-1<\sigma<n,\\ \zeta^{(n)}_{\xi_2^n}(\xi_1,\varrho), \quad \sigma:=n\in\mathbb{N}, & \end{cases} \tag{2.2}$$

where $\zeta^{(n)}_{\xi_2^n}:=\frac{\partial^n}{\partial\xi_2^n}\zeta$, $E_{\sigma}(\cdot)$ is the Mittag-Leffler function and $A^{\cdot}$ is a condition referred to as the normalization function with $A^0=A^1=1$. For $\xi_2\ge 0$ and for $n-1<\sigma<n$, we have ${}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\,{}^{ABC}\mathcal{I}^{\sigma}_{0,\xi_2}\zeta(\xi_1,\xi_2)=\zeta(\xi_1,\xi_2)$, ${}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\xi_2^{\delta}=\frac{\Gamma(\delta+1)}{\Gamma(\delta+1-\sigma)}\xi_2^{\delta-\sigma}$, and

$$ {}^{ABC}\mathcal{I}^{\sigma}_{0,\xi_2}\,{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\zeta(\xi_1,\xi_2)=\zeta(\xi_1,\xi_2)-\sum_{k=0}^{n-1}\frac{\xi_2^k}{k!}\zeta^{(k)}_{\xi_2}(\xi_1,\xi_2)\Big|_{\xi_2=0}, \tag{2.3}$$

where $\delta>-1$ and $n\in\mathbb{N}$.

Let $\mathcal{B}=C\big(\Delta\times[0,\xi_0],\mathbb{R}^2\big)$ be the Banach space of all continuous vector-valued functions on $\Delta\times[0,\xi_0]$, equipped with the supremum norm

$$\|(\zeta,\varpi)\|_{\mathcal{B}}=\max_{(\xi_1,\xi_2)\in\Delta\times[0,\xi_0]}\{|\zeta(\xi_1,\xi_2)|,|\varpi(\xi_1,\xi_2)|\}.$$

Since $\Delta\subset\mathbb{R}$ is compact, the product space $\Delta\times[0,\xi_0]$ is also compact. Therefore, the supremum norm is well defined on $\mathcal{B}$, and $\mathcal{B}$ becomes a Banach space under this norm.

In the sequel, all continuity and convergence arguments are considered with respect to the supremum norm.

Define the nonlinear operators

$$\mathbb{M}_{\zeta},\mathbb{M}_{\varpi}:\Delta\times[0,\xi_0]\times\mathcal{B}\to\mathbb{R}$$

by

$$\mathbb{M}_{\zeta}(\xi_1,\xi_2,\zeta,\varpi)=\zeta_{\xi_1\xi_1}(\xi_1,\xi_2)+\mathcal{R}_1(\xi_1,\xi_2,\zeta(\xi_1,\xi_2),\zeta(\xi_1,\xi_2-\rho),\varpi(\xi_1,\xi_2-\mathcal{G}(\zeta(\xi_1,\xi_2))))$$

and

$$\mathbb{M}_{\varpi}(\xi_1,\xi_2,\zeta,\varpi)=\varpi_{\xi_1\xi_1}(\xi_1,\xi_2)+\mathcal{R}_2(\xi_1,\xi_2,\varpi(\xi_1,\xi_2),\varpi(\xi_1,\xi_2-\rho),\zeta(\xi_1,\xi_2-\mathcal{G}(\varpi(\xi_1,\xi_2)))).$$

Applying the ABC fractional integral operator to system (1.1), together with property (2.3), yields the equivalent integral formulation

$$\zeta(\xi_1,\xi_2)=k_0(\xi_1)+\frac{1-\sigma}{A^{\sigma}}\mathbb{M}_{\zeta}(\xi_1,\xi_2,\zeta,\varpi)+\frac{\sigma}{\Gamma(\sigma)A^{\sigma}}\int_0^{\xi_2}(\xi_2-\varrho)^{\sigma-1}\mathbb{M}_{\zeta}(\xi_1,\varrho,\zeta,\varpi)\,d\varrho$$

and

$$\varpi(\xi_1,\xi_2) = \hat{k}_0(\xi_1) + \frac{1-\sigma}{A^\sigma}\mathbb{M}_\varpi(\xi_1,\xi_2,\zeta,\varpi) + \frac{\sigma}{\Gamma(\sigma)A^\sigma}\int_0^{\xi_2}(\xi_2-\varrho)^{\sigma-1}\mathbb{M}_\varpi(\xi_1,\varrho,\zeta,\varpi)\,d\varrho.$$

Define the operator

$$\mathcal{K}:\mathcal{B}\to\mathcal{B}$$

by

$$\mathcal{K}(\zeta,\varpi) = \left(\mathcal{K}_\zeta(\zeta,\varpi),\mathcal{K}_\varpi(\zeta,\varpi)\right),$$

where

$$\mathcal{K}_\zeta(\zeta,\varpi)(\xi_1,\xi_2) = k_0(\xi_1) + \frac{1-\sigma}{A^\sigma}\mathbb{M}_\zeta(\xi_1,\xi_2,\zeta,\varpi) + \frac{\sigma}{\Gamma(\sigma)A^\sigma}\int_0^{\xi_2}(\xi_2-\varrho)^{\sigma-1}\mathbb{M}_\zeta(\xi_1,\varrho,\zeta,\varpi)\,d\varrho$$

and

$$\mathcal{K}_\varpi(\zeta,\varpi)(\xi_1,\xi_2) = \hat{k}_0(\xi_1) + \frac{1-\sigma}{A^\sigma}\mathbb{M}_\varpi(\xi_1,\xi_2,\zeta,\varpi) + \frac{\sigma}{\Gamma(\sigma)A^\sigma}\int_0^{\xi_2}(\xi_2-\varrho)^{\sigma-1}\mathbb{M}_\varpi(\xi_1,\varrho,\zeta,\varpi)\,d\varrho.$$

Assume that there exists a constant $L > 0$ such that

$$\left|\mathbb{M}_\zeta(\xi_1,\xi_2,\zeta,\varpi) - \mathbb{M}_\zeta(\xi_1,\xi_2,\zeta_1,\varpi_1)\right| \le L\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}$$

and

$$|\mathbb{M}_\varpi(\xi_1,\xi_2,\zeta,\varpi) - \mathbb{M}_\varpi(\xi_1,\xi_2,\zeta_1,\varpi_1)| \le L\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}$$

for all $(\xi_1,\xi_2) \in \Delta\times[0,\xi_0]$. Then the nonlinear operators are said to satisfy the Lipschitz condition.

Lemma 2.1. Whenever $\mathbb{M}_\zeta$ and $\mathbb{M}_\varpi$ satisfy Lipschitz property, the corresponding operators $\mathcal{K}_\zeta$ and $\mathcal{K}_\varpi$ inherit the same Lipschitz property.

Proof. By the Lipschitz restriction of $\mathbb{M}_\zeta$ and $\mathbb{M}_\varpi$, we get $\gamma_\zeta,\gamma_\varpi > 0$, where

$$\left|\mathbb{M}_\zeta(\xi_1,\xi_2,\zeta,\varpi) - \mathbb{M}_\zeta(\xi_1,\xi_2,\zeta_1,\varpi_1)\right| \le \gamma_\zeta\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}$$

and

$$|\mathbb{M}_\varpi(\xi_1,\xi_2,\zeta,\varpi) - \mathbb{M}_\varpi(\xi_1,\xi_2,\zeta_1,\varpi_1)| \le \gamma_\varpi\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}.$$

Hence

$$\begin{aligned}
&\left|\mathcal{K}_\zeta(\zeta,\varpi)(\xi_1,\xi_2) - \mathcal{K}_\zeta(\zeta_1,\varpi_1)(\xi_1,\xi_2)\right| \le \frac{1-\sigma}{A^\sigma}\left|\mathbb{M}_\zeta(\xi_1,\xi_2,\zeta,\varpi) - \mathbb{M}_\zeta(\xi_1,\xi_2,\zeta_1,\varpi_1)\right| \\
&+ \frac{\sigma}{\Gamma(\sigma)A^\sigma}\left|\int_0^{\xi_2}(\xi_2-\varrho)^{\sigma-1}\,\mathbb{M}_\zeta(\xi_1,\varrho,\zeta,\varpi)d\varrho - \int_0^{\xi_2}(\xi_2-\varrho)^{\sigma-1}\,\mathbb{M}_\zeta(\xi_1,\varrho,\zeta_1,\varpi_1)d\varrho\right| \\
&\le \frac{(1-\sigma)\gamma_\zeta}{A^\sigma}\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}} \\
&+ \frac{\sigma}{\Gamma(\sigma)A^\sigma}\int_0^{\xi_2}|\xi_2-\varrho|^{\sigma-1}\,\left|\mathbb{M}_\zeta(\xi_1,\xi_2,\zeta,\varpi) - \mathbb{M}_\zeta(\xi_1,\xi_2,\zeta_1,\varpi_1)\right|d\varrho \\
&\le \frac{(1-\sigma)\gamma_\zeta}{A^\sigma}\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}} + \frac{\xi_{20}^\sigma\gamma_\zeta}{\Gamma(\sigma)A^\sigma}\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}.
\end{aligned}$$

Then we have

$$\left\|\mathcal{K}_\zeta(\zeta,\varpi)-\mathcal{K}_\zeta(\zeta_1,\varpi_1)\right\| \leq \left[\frac{(1-\sigma)\gamma_\zeta}{A^\sigma}+\frac{\xi_{20}^\sigma\gamma_\zeta}{\Gamma(\sigma)A^\sigma}\right]\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}. \tag{2.4}$$

Similarly, we have

$$\|\mathcal{K}_\varpi(\zeta,\varpi)-\mathcal{K}_\varpi(\zeta_1,\varpi_1)\| \leq \left[\frac{(1-\sigma)\gamma_\varpi}{A^\sigma}+\frac{\xi_{20}^\sigma\gamma_\varpi}{\Gamma(\sigma)A^\sigma}\right]\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}. \tag{2.5}$$

□

Consider the following conditions:

R1: The mappings $\mathbb{M}_\zeta$ and $\mathbb{M}_\varpi$ are continuous on $\Delta\times[0,\xi_0]\times\mathcal{B}$.

R2: The mappings $\mathbb{M}_\zeta$ and $\mathbb{M}_\varpi$ have the Lipschitz property with corresponding Lipschitz constants $\gamma_\zeta,\gamma_\varpi>0$, that is,

$$\left|\mathbb{M}_\zeta(\xi_1,\xi_2,\zeta,\varpi)-\mathbb{M}_\zeta(\xi_1,\xi_2,\zeta_1,\varpi_1)\right| \leq \gamma_\zeta\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}$$

and

$$|\mathbb{M}_\varpi(\xi_1,\xi_2,\zeta,\varpi)-\mathbb{M}_\varpi(\xi_1,\xi_2,\zeta_1,\varpi_1)| \leq \gamma_\varpi\,\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}$$

for all $(\xi_1,\xi_2)\in\Delta\times[0,\xi_0]$.

Theorem 2.2. Assume that conditions R1-R2 hold and $F_\sigma(\gamma_\zeta+\gamma_\varpi)<1$, where

$$F_\sigma=\frac{1-\sigma}{A^\sigma}+\frac{\xi_0^\sigma}{\Gamma(\sigma)A^\sigma}.$$

Then the system (1.1) admits a unique solution on $\Delta\times[0,\xi_0]$.

Proof. For any $(\zeta,\varpi),(\zeta_1,\varpi_1)\in\mathcal{B}$, using the Lipschitz assumptions, we obtain

$$\begin{aligned}&\|\mathcal{K}(\zeta,\varpi)-\mathcal{K}(\zeta_1,\varpi_1)\|_{\mathcal{B}}\\ &\leq F_\sigma(\gamma_\zeta+\gamma_\varpi)\|(\zeta,\varpi)-(\zeta_1,\varpi_1)\|_{\mathcal{B}}.\end{aligned}$$

Since

$$F_\sigma(\gamma_\zeta+\gamma_\varpi)<1,$$

the operator $\mathcal{K}$ is a contraction mapping. Hence, by the Banach fixed point theorem, $\mathcal{K}$ admits a unique fixed point in $\mathcal{B}$, which is the unique solution of system (1.1). □

3. On the α-LHPM

The proposed α-LHPM is presented in a detailed step-by-step form in order to clarify the implementation procedure for solving the considered nonlinear fractional system. First, we develop the proposed α-LHPM for solving the nonlinear fractional system (1.1). The method combines the α-Laplace transform with He's homotopy perturbation framework in order to construct rapidly convergent series solutions for fractional nonlinear models involving the ABC operator. We begin by introducing the definition and some basic properties of the α-Laplace transform, together with its interaction with the ABC fractional derivative. The transformed formulation is then combined with the homotopy perturbation expansion to derive recursive relations for the successive approximation terms. After constructing the iterative scheme, we investigate the convergence behavior of the resulting series solution and establish its relation to the unique solution of the original fractional system. Finally, Hyers–Ulam stability and convergence properties of the truncated α-LHPM approximations are analyzed under suitable assumptions.

First, we introduce the concept of the α-Laplace transform and some facts about it with the ABC fractional operator. Let ζ be a map on $\Delta \times [0, \xi_0]$. The Laplace transform $\mathbb{L}_{\xi_2}$ [43] of $\zeta(\xi_1, \xi_2)$ with respect to ξ_2 is defined by

$$\mathbb{L}_{\xi_2}\left[\zeta(\xi_1, \xi_2)\right] = \int_0^{\infty} \zeta(\xi_1, \xi_2) e^{-\theta\xi_2} d\xi_2 \quad Re(\theta) > 0. \tag{3.1}$$

For every $\alpha \in [0, 1)$, define the α-Laplace transform $\mathbb{L}_{\alpha\xi_2}$ of $\zeta(\xi_1, \xi_2)$ with respect to ξ_2 is defined by

$$\zeta_{\mathbb{L}_{\alpha\xi_2}}(\xi_1, \theta) := \mathbb{L}_{\alpha\xi_2}\left[\zeta(\xi_1, \xi_2)\right] = \theta^{\alpha-1} \int_0^{\infty} \zeta(\xi_1, \xi_2) e^{-\theta^{\alpha}\xi_2} d\xi_2 \quad Re(\theta) > 0. \tag{3.2}$$

The factor $\theta^{\alpha-1}$ appearing in Eq (4.2) is a direct consequence of the fractional integral structure associated with the α-Laplace transform. Specifically, the kernel of the fractional operator introduces a power-law term of the form $\theta^{\alpha-1}$, which accounts for the nonlocal memory effects of the system. This behavior is consistent with standard formulations in fractional calculus. The inverse α-Laplace transform $\zeta^{-1}_{\mathbb{L}_{\alpha\xi_2}}$ of $\zeta_{\mathbb{L}_{\alpha\xi_2}}(\xi_1, \theta)$ is

$$\zeta(\xi_1, \xi_2) = \zeta^{-1}_{\mathbb{L}_{\alpha\xi_2}}\left[\zeta_{\mathbb{L}_{\alpha\xi_2}}(\xi_1, \theta)\right]. \tag{3.3}$$

It is easy to see that the α-Laplace transform and its inverse are linear maps and when $\alpha \to 1$. The operator $\mathbb{L}_{\alpha\xi_2}$ tends to the classical Laplace transform $\mathbb{L}_{\xi_2}$. Also, note that $\mathbb{L}_{\alpha\xi_2}\left[\xi_2^n\right] = \frac{n!}{\theta^{n\alpha+1}}$.

Theorem 3.1. The transform $\mathbb{L}_{\alpha\xi_2}$ of $\zeta^{(m)}_{\xi_2}(\xi_1, \xi_2)$ is given by

$$\mathbb{L}_{\alpha\xi_2}\left[\zeta^{(m)}_{\xi_2}(\xi_1, \xi_2)\right] = \theta^{m\alpha}\zeta_{\mathbb{L}_{\alpha\xi_2}}(\xi_1, \theta) - \sum_{j=0}^{m-1} \theta^{(m-j)\alpha-1}\ \zeta^{(j)}_{\xi_2}(\xi_1, \xi_2)\Big|_{\xi_2=0}. \tag{3.4}$$

Proof. The result follows by mathematical induction on m. For $m = 1$, the formula is verified directly using the definition of the α-Laplace transform. Assume that the result holds for some integer m. Then, by applying the linearity of the transform and the induction hypothesis to the $(m + 1)$-th derivative, the desired formula is obtained. This completes the proof. □

Theorem 3.2. The transform $\mathbb{L}_{\alpha\xi_2}$ of the ABC operator is

$$\begin{aligned} \mathbb{L}_{\alpha\xi_2}\left[{}^{ABC}\mathscr{D}^{\sigma}_{0,\xi_2}\zeta(\xi_1, \xi_2)\right] &= \frac{A^{\sigma}}{\sigma + (1-\sigma)\theta^{\sigma\alpha}} \\ &\times \left[\theta^{(\sigma+n)\alpha-1}\zeta_{\mathbb{L}_{\alpha\xi_2}}(\xi_1, \theta) - \sum_{j=0}^{n-1} \theta^{(\sigma+n-j-1)\alpha-1}\left[\zeta^{(j)}_{\xi_2}(\xi_1, \xi_2)\Big|_{\xi_2=0}\right]\right] \end{aligned} \tag{3.5}$$

where $n - 1 < \sigma < n$.

Proof. Let $\lambda = \dfrac{\sigma}{1-\sigma}$ and define the kernel $K_\sigma(\xi_1,\xi_2) = E_\sigma(-\lambda\xi_2^\sigma)$, so that the ABC derivative can be written as the convolution

$$\begin{aligned} {}^{ABC}\mathcal{D}^\sigma_{0,\xi_2}\zeta(\xi_1,\xi_2) &= \frac{A^\sigma}{1-\sigma}\int_0^{\xi_2} K_\sigma(\xi_1,\xi_2-\varrho)\,\zeta^{(n)}_{\varrho^n}(\xi_1,\varrho)\,d\varrho \\ &= \frac{A^\sigma}{1-\sigma}\left[K_\sigma(\xi_1,\xi_2) * \zeta^{(n)}_{\xi_2}(\xi_1,\xi_2)\right] \end{aligned} \tag{3.6}$$

where

$$\zeta(\xi_1,\xi_2) * \varpi(\xi_1,\xi_2) = \int_0^{\xi_2} \zeta(\xi_1,\xi_2-\tau) * \varpi(\xi_1,\tau)d\tau.$$

By definition of the α-Laplace transform, $\mathbb{L}_{\alpha\xi_2}\zeta(\xi_1,\xi_2) = \theta^{\alpha-1}\mathbb{L}_{\xi_2}[\zeta](\xi_1,\theta^\alpha)$ we have

$$\begin{aligned} \mathbb{L}_{\alpha\xi_2}\left[{}^{ABC}\mathcal{D}^\sigma_{0,\xi_2}\zeta(\xi_1,\xi_2)\right] &= \frac{A^\sigma}{1-\sigma}\,\mathbb{L}_{\alpha\xi_2}\left[K_\sigma(\xi_1,\xi_2) * \zeta^{(n)}_{\xi_2}(\xi_1,\xi_2)\right] \\ &= \frac{A^\sigma}{1-\sigma}\,\theta^{\alpha-1}\mathbb{L}_{\xi_2}\left[K_\sigma * \zeta^{(n)}_{\xi_2}\right](\xi_1,\theta^\alpha) \\ &= \frac{A^\sigma}{1-\sigma}\,\theta^{\alpha-1}\mathbb{L}_{\xi_2}\left[K_\sigma\right](\xi_1,\theta^\alpha)\,\mathbb{L}_{\xi_2}\left[\zeta^{(n)}_{\xi_2}\right](\xi_1,\theta^\alpha) \\ &= \frac{A^\sigma}{1-\sigma}\,\theta^{\alpha-1}\frac{(\theta^\alpha)^{\sigma-1}}{(\theta^\alpha)^\sigma+\lambda}\,\mathbb{L}_{\xi_2}\left[\zeta^{(n)}_{\xi_2}\right](\xi_1,\theta^\alpha) \\ &= \frac{A^\sigma}{\sigma+(1-\sigma)\theta^{\sigma\alpha}}\,\theta^{\sigma\alpha-1}\,\mathbb{L}_{\xi_2}\left[\zeta^{(n)}_{\xi_2}\right](\xi_1,\theta^\alpha). \end{aligned} \tag{3.7}$$

From Theorem 3.1, we have

$$\begin{aligned} \mathbb{L}_{\alpha\xi_2}\left[{}^{ABC}\mathcal{D}^\sigma_{0,\xi_2}\zeta(\xi_1,\xi_2)\right] &= \frac{A^\sigma}{\sigma+(1-\sigma)\theta^{\sigma\alpha}}\,\theta^{\sigma\alpha-1} \\ &\times\left[\theta^{n\alpha}\zeta_{\mathbb{L}_{\alpha\xi_2}}(\xi_1,\theta) - \sum_{j=0}^{n-1}\theta^{(n-j)\alpha-1}\,\zeta^{(j)}_{\xi_2}(\xi_1,\xi_2)\Big|_{\xi_2=0}\right] \\ &= \frac{A^\sigma}{\sigma+(1-\sigma)\theta^{\sigma\alpha}} \\ &\times\left[\theta^{(\sigma+n)\alpha-1}\zeta_{\mathbb{L}_{\alpha\xi_2}}(\xi_1,\theta) - \sum_{j=0}^{n-1}\theta^{(\sigma+n-j-1)\alpha-1}\left[\zeta^{(j)}_{\xi_2}(\xi_1,\xi_2)\Big|_{\xi_2=0}\right]\right]. \end{aligned} \tag{3.8}$$

Hence the proof is complete. □

We now apply the proposed α-LHPM to the fractional nonlinear system (1.1). For convenience, the system is rewritten in the operator form

$$\begin{aligned} {}^{ABC}\mathcal{D}^\sigma_{0,\xi_2}\zeta &= F_1(\zeta) + F_2(\zeta,\varpi), \\ {}^{ABC}\mathcal{D}^\sigma_{0,\xi_2}\varpi &= G_1(\varpi) + G_2(\zeta,\varpi), \end{aligned} \tag{3.9}$$

where the linear operators are defined as

$$F_1(\zeta) = \zeta_{\xi_1\xi_1},\ \ G_1(\varpi) = \varpi_{\xi_1\xi_1},$$

and the nonlinear terms are

$$F_2(\zeta,\varpi)=\mathcal{R}_1\{\xi_1,\xi_2,\zeta,\zeta(\xi_2-\rho),\varpi[\xi_2-\mathcal{G}(\zeta)]\},$$

$$G_2(\zeta,\varpi)=\mathcal{R}_2\{\xi_1,\xi_2,\varpi,\varpi(\xi_2-\rho),\zeta[\xi_2-\mathcal{G}(\varpi)]\}.$$

Applying the α-Laplace transform $\mathbb{L}_{\alpha\xi_2}$ to the system gives

$$\begin{aligned}\mathbb{L}_{\alpha\xi_2}\left[{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\zeta\right]&=\mathbb{L}_{\alpha\xi_2}\left[F_1(\zeta)+F_2(\zeta,\varpi)\right],\\ \mathbb{L}_{\alpha\xi_2}\left[{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\varpi\right]&=\mathbb{L}_{\alpha\xi_2}\left[G_1(\varpi)+G_2(\zeta,\varpi)\right].\end{aligned}\tag{3.10}$$

Using Theorems 3.1 and 3.2, we obtain

$$\begin{aligned}\zeta_{\mathbb{L}_{\alpha\xi_2}}&=\frac{1}{\theta^{\alpha}}k_0(\xi_1)+\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left[F_1(\zeta)+F_2(\zeta,\varpi)\right],\\ \varpi_{\mathbb{L}_{\alpha\xi_2}}&=\frac{1}{\theta^{\alpha}}\hat{k}_0(\xi_1)+\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left[G_1(\varpi)+G_2(\zeta,\varpi)\right].\end{aligned}\tag{3.11}$$

Taking the inverse transform $\mathbb{L}^{-1}_{\alpha\xi_2}$ gives

$$\begin{aligned}\zeta(\xi_1,\xi_2)&=k_0(\xi_1)+\mathbb{L}^{-1}_{\alpha\xi_2}\left[\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left(F_1(\zeta)+F_2(\zeta,\varpi)\right)\right],\\ \varpi(\xi_1,\xi_2)&=\hat{k}_0(\xi_1)+\mathbb{L}^{-1}_{\alpha\xi_2}\left[\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left(G_1(\varpi)+G_2(\zeta,\varpi)\right)\right].\end{aligned}\tag{3.12}$$

Next, the homotopy perturbation expansion is introduced as

$$\zeta(\xi_1,\xi_2)=\sum_{n=0}^{\infty}p^n\zeta_n(\xi_1,\xi_2),\ \ \varpi(\xi_1,\xi_2)=\sum_{n=0}^{\infty}p^n\varpi_n(\xi_1,\xi_2).\tag{3.13}$$

Similarly, the nonlinear terms are expanded as

$$F_2(\zeta,\varpi)=\sum_{n=0}^{\infty}p^nH_{\zeta n},\ \ G_2(\zeta,\varpi)=\sum_{n=0}^{\infty}p^nH_{\varpi n}.\tag{3.14}$$

The He’s polynomials are defined by

$$H_{\zeta m}=\frac{1}{m!}\frac{\partial^m}{\partial p^m}F_2\left(\sum_{k=0}^{\infty}p^k\zeta_k,\sum_{k=0}^{\infty}p^k\varpi_k\right)_{p=0}$$

and

$$H_{\varpi m}=\frac{1}{m!}\frac{\partial^m}{\partial p^m}G_2\left(\sum_{k=0}^{\infty}p^k\zeta_k,\sum_{k=0}^{\infty}p^k\varpi_k\right)\Bigg|_{p=0}.$$

Substituting the expansions into the previous equations and equating the coefficients of like powers of p yields the recursive relations

$$p^0:\quad\zeta_0(\xi_1,\xi_2)=k_0(\xi_1),\ \ \varpi_0(\xi_1,\xi_2)=\hat{k}_0(\xi_1).\tag{3.15}$$

$$\begin{aligned} p^1: \quad \zeta_1 &= \mathbb{L}^{-1}_{\alpha\xi_2}\left[\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left(\zeta_{0\xi_1\xi_1}+H_{\zeta0}\right)\right], \\ \varpi_1 &= \mathbb{L}^{-1}_{\alpha\xi_2}\left[\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left(\varpi_{0\xi_1\xi_1}+H_{\varpi0}\right)\right]. \end{aligned} \tag{3.16}$$

$$\begin{aligned} p^2: \quad \zeta_2 &= \mathbb{L}^{-1}_{\alpha\xi_2}\left[\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left(\zeta_{1\xi_1\xi_1}+H_{\zeta1}\right)\right], \\ \varpi_2 &= \mathbb{L}^{-1}_{\alpha\xi_2}\left[\frac{1}{\mathbb{F}_{\sigma\theta}\theta^{\alpha\sigma+\alpha-1}}\mathbb{L}_{\alpha\xi_2}\left(\varpi_{1\xi_1\xi_1}+H_{\varpi1}\right)\right]. \end{aligned} \tag{3.17}$$

Continuing this procedure, the approximate solution is obtained as

$$\zeta(\xi_1,\xi_2)=\sum_{n=0}^{\infty}\zeta_n(\xi_1,\xi_2),\ \ \varpi(\xi_1,\xi_2)=\sum_{n=0}^{\infty}\varpi_n(\xi_1,\xi_2).$$

The resulting series converges rapidly to the solution of the nonlinear fractional system, demonstrating the computational efficiency of the proposed α-LHPM.

Theorem 3.3. Assume that conditions R1-R2 hold and

$$F_\sigma(\gamma_\zeta+\gamma_\varpi)<1.$$

If the α-LHPM series

$$\zeta(\xi_1,\xi_2)=\sum_{n=0}^{\infty}\zeta_n(\xi_1,\xi_2), \qquad \varpi(\xi_1,\xi_2)=\sum_{n=0}^{\infty}\varpi_n(\xi_1,\xi_2)$$

converges in $\mathcal{B}$, then its limit coincides with the unique solution of system (1.1) on $\Delta\times[0,\xi_0]$.

Proof. Assume that the α-LHPM series converges in $\mathcal{B}$, and denote its limit by

$$(\zeta^*,\varpi^*)=\left(\sum_{n=0}^{\infty}\zeta_n,\sum_{n=0}^{\infty}\varpi_n\right).$$

Since the convergence is in the Banach space $\mathcal{B}$, we have

$$\left\|\left(\sum_{n=0}^{N}\zeta_n,\sum_{n=0}^{N}\varpi_n\right)-(\zeta^*,\varpi^*)\right\|_{\mathcal{B}}\to 0 \quad \text{as} \quad N\to\infty.$$

The recursive terms of the α-LHPM are obtained by applying the α-Laplace transform to the integral formulation of system (1.1), introducing the homotopy expansion, and equating the coefficients of like powers of the embedding parameter. Therefore, the partial sums

$$(\zeta_N,\varpi_N)=\left(\sum_{n=0}^{N}\zeta_n,\sum_{n=0}^{N}\varpi_n\right)$$

satisfy the corresponding truncated operator equation associated with $\mathcal{K}$.

Passing to the limit as $N \to \infty$ and using the convergence in $\mathcal{B}$ together with the continuity of $\mathcal{K}$, we obtain

$$(\zeta^*, \varpi^*) = \mathcal{K}(\zeta^*, \varpi^*).$$

Thus, (ζ^*, ϖ^*) is a fixed point of the operator $\mathcal{K}$.

By Theorem 2.2, the condition

$$F_\sigma(\gamma_\zeta + \gamma_\varpi) < 1$$

implies that $\mathcal{K}$ is a contraction on $\mathcal{B}$. Hence, by the Banach fixed point theorem, $\mathcal{K}$ admits a unique fixed point in $\mathcal{B}$.

Since the fixed points of $\mathcal{K}$ in Theorem 2.2 are precisely the solutions of the integral formulation of system (1.1), and this integral formulation is equivalent to the original fractional system, the limit (ζ^*, ϖ^*) must coincide with the unique solution (ζ, ϖ) of system (1.1) on $\Delta \times [0, \xi_0]$.

Therefore,

$$\zeta(\xi_1, \xi_2) = \sum_{n=0}^{\infty} \zeta_n(\xi_1, \xi_2), \qquad \varpi(\xi_1, \xi_2) = \sum_{n=0}^{\infty} \varpi_n(\xi_1, \xi_2),$$

and the convergent α-LHPM series coincides with the unique solution of system (1.1). □

Here we study the stability of the solution of the fractional nonlinear system (1.1). In particular, we focus on Hyers–Ulam stability, which ensures that small perturbations in the governing equations produce only small deviations in the corresponding solution under the supremum norm.

Let $(\hat{\zeta}, \hat{\varpi})$ denote an approximate solution of (1.1) satisfying

$$\left| {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2} \hat{\zeta} - \mathbb{M}_\zeta(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) \right| \leq \Omega_1$$

and

$$\left| {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2} \hat{\varpi} - \mathbb{M}_\varpi(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) \right| \leq \Omega_2,$$

for all $(\xi_1, \xi_2) \in \Delta \times [0, \xi_0]$, where $\Omega_1, \Omega_2 > 0$.

Following [44], system (1.1) is said to possess Hyers–Ulam stability if there exists a unique solution (ζ, ϖ) satisfying

$$\|\hat{\zeta} - \zeta\| \leq \Omega_1' \Omega_1, \qquad \|\hat{\varpi} - \varpi\| \leq \Omega_2' \Omega_2,$$

where $\Omega_1', \Omega_2' > 0$ are constants independent of Ω_1 and Ω_2.

Lemma 3.4. Assume that conditions R1-R2 hold and

$$\mathbb{F}^\sigma(\gamma_\zeta + \gamma_\varpi) < 1.$$

Then system (1.1) is Hyers–Ulam stable in the supremum norm.

Proof. Let $(\hat{\zeta}, \hat{\varpi})$ be an approximate solution of system (1.1) satisfying

$$\left| {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2} \hat{\zeta} - \mathbb{M}_\zeta(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) \right| \leq \Omega_1$$

and

$$\left| {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2} \hat{\varpi} - \mathbb{M}_\varpi(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) \right| \leq \Omega_2,$$

for all $(\xi_1, \xi_2) \in \Delta \times [0, \xi_0]$.

Then there exist perturbation functions g_1 and g_2 defined on $\Delta \times [0, \xi_0]$ such that

$$ {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\zeta} = \mathbb{M}_{\zeta}(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) + g_1(\xi_1, \xi_2) $$

and

$$ {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\varpi} = \mathbb{M}_{\varpi}(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) + g_2(\xi_1, \xi_2), $$

where

$$ |g_1(\xi_1, \xi_2)| \le \Omega_1, \qquad |g_2(\xi_1, \xi_2)| \le \Omega_2. $$

Applying the ABC fractional integral operator yields

$$ \begin{aligned} \hat{\zeta}(\xi_1, \xi_2) = k_0(\xi_1) &+ \frac{1-\sigma}{A^{\sigma}}\left[\mathbb{M}_{\zeta}(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) + g_1(\xi_1, \xi_2)\right] \\ &+ \frac{\sigma}{\Gamma(\sigma)A^{\sigma}}\int_0^{\xi_2}(\xi_2 - \varrho)^{\sigma-1}\left[\mathbb{M}_{\zeta}(\xi_1, \varrho, \hat{\zeta}, \hat{\varpi}) + g_1(\xi_1, \varrho)\right] d\varrho \end{aligned} $$

and

$$ \begin{aligned} \hat{\varpi}(\xi_1, \xi_2) = \hat{k}_0(\xi_1) &+ \frac{1-\sigma}{A^{\sigma}}\left[\mathbb{M}_{\varpi}(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) + g_2(\xi_1, \xi_2)\right] \\ &+ \frac{\sigma}{\Gamma(\sigma)A^{\sigma}}\int_0^{\xi_2}(\xi_2 - \varrho)^{\sigma-1}\left[\mathbb{M}_{\varpi}(\xi_1, \varrho, \hat{\zeta}, \hat{\varpi}) + g_2(\xi_1, \varrho)\right] d\varrho. \end{aligned} $$

Let (ζ, ϖ) be the unique exact solution of system (1.1). Then

$$ \begin{aligned} |\hat{\zeta}(\xi_1, \xi_2) - \zeta(\xi_1, \xi_2)| \le{}& \frac{1-\sigma}{A^{\sigma}}\left[\left|\mathbb{M}_{\zeta}(\xi_1, \xi_2, \hat{\zeta}, \hat{\varpi}) - \mathbb{M}_{\zeta}(\xi_1, \xi_2, \zeta, \varpi)\right| + |g_1(\xi_1, \xi_2)|\right] \\ &+ \frac{\sigma}{\Gamma(\sigma)A^{\sigma}}\int_0^{\xi_2}(\xi_2 - \varrho)^{\sigma-1}\left[\left|\mathbb{M}_{\zeta}(\xi_1, \varrho, \hat{\zeta}, \hat{\varpi}) - \mathbb{M}_{\zeta}(\xi_1, \varrho, \zeta, \varpi)\right| + |g_1(\xi_1, \varrho)|\right] d\varrho. \end{aligned} $$

Using the Lipschitz condition together with the estimate on g_1, we obtain

$$ \begin{aligned} |\hat{\zeta}(\xi_1, \xi_2) - \zeta(\xi_1, \xi_2)| \le{}& \frac{1-\sigma}{A^{\sigma}}\left[\gamma_{\zeta}\|(\hat{\zeta}, \hat{\varpi}) - (\zeta, \varpi)\|_{\mathcal{B}} + \Omega_1\right] \\ &+ \frac{\sigma}{\Gamma(\sigma)A^{\sigma}}\int_0^{\xi_2}(\xi_2 - \varrho)^{\sigma-1}\left[\gamma_{\zeta}\|(\hat{\zeta}, \hat{\varpi}) - (\zeta, \varpi)\|_{\mathcal{B}} + \Omega_1\right] d\varrho. \end{aligned} $$

Since

$$ \int_0^{\xi_2}(\xi_2 - \varrho)^{\sigma-1}\, d\varrho = \frac{\xi_2^{\sigma}}{\sigma} \le \frac{\xi_0^{\sigma}}{\sigma}, $$

it follows that

$$ \|\hat{\zeta} - \zeta\|_{\infty} \le \mathbb{F}^{\sigma}\left[\gamma_{\zeta}\|(\hat{\zeta}, \hat{\varpi}) - (\zeta, \varpi)\|_{\mathcal{B}} + \Omega_1\right]. $$

Similarly,

$$ \|\hat{\varpi} - \varpi\|_{\infty} \le \mathbb{F}^{\sigma}\left[\gamma_{\varpi}\|(\hat{\zeta}, \hat{\varpi}) - (\zeta, \varpi)\|_{\mathcal{B}} + \Omega_2\right]. $$

Taking the maximum of the above inequalities yields

$$ \begin{aligned} &\|(\hat{\zeta}, \hat{\varpi}) - (\zeta, \varpi)\|_{\mathcal{B}} \\ &\le \mathbb{F}^{\sigma}(\gamma_{\zeta} + \gamma_{\varpi})\|(\hat{\zeta}, \hat{\varpi}) - (\zeta, \varpi)\|_{\mathcal{B}} + \mathbb{F}^{\sigma}\max\{\Omega_1, \Omega_2\}. \end{aligned} $$

Since

$$\mathbb{F}^{\sigma}(\gamma_{\zeta}+\gamma_{\varpi})<1,$$

we conclude that

$$\|(\hat{\zeta},\hat{\varpi})-(\zeta,\varpi)\|_{\mathcal{B}}\leq\frac{\mathbb{F}^{\sigma}}{1-\mathbb{F}^{\sigma}(\gamma_{\zeta}+\gamma_{\varpi})}\max\{\Omega_1,\Omega_2\}.$$

Hence,

$$\|\hat{\zeta}-\zeta\|_{\infty}\leq\Omega_1'\Omega_1$$

and

$$\|\hat{\varpi}-\varpi\|_{\infty}\leq\Omega_2'\Omega_2,$$

where

$$\Omega_1'=\Omega_2'=\frac{\mathbb{F}^{\sigma}}{1-\mathbb{F}^{\sigma}(\gamma_{\zeta}+\gamma_{\varpi})}.$$

Therefore, system (1.1) possesses Hyers–Ulam stability in the supremum norm. □

Theorem 3.5. Assume that the hypotheses of Lemma 3.4 hold. Let

$$(\hat{\zeta}_N,\hat{\varpi}_N)=\left(\sum_{n=0}^{N}\zeta_n,\sum_{n=0}^{N}\varpi_n\right)$$

be the truncated α-LHPM approximation of the solution (ζ,ϖ) of system (1.1). Suppose that there exist constants $\Omega_{1,N},\Omega_{2,N}>0$ such that

$$\left|{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\zeta}_N-\mathbb{M}_{\zeta}(\xi_1,\xi_2,\hat{\zeta}_N,\hat{\varpi}_N)\right|\leq\Omega_{1,N}\quad\text{and}\quad\left|{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\varpi}_N-\mathbb{M}_{\varpi}(\xi_1,\xi_2,\hat{\zeta}_N,\hat{\varpi}_N)\right|\leq\Omega_{2,N},$$

for all $(\xi_1,\xi_2)\in\Delta\times[0,\xi_0]$. Then there exist positive constants C_1 and C_2, independent of N, such that

$$\|\hat{\zeta}_N-\zeta\|_{\infty}\leq C_1\max\{\Omega_{1,N},\Omega_{2,N}\}\quad\text{and}\quad\|\hat{\varpi}_N-\varpi\|_{\infty}\leq C_2\max\{\Omega_{1,N},\Omega_{2,N}\}.$$

In particular, if $\Omega_{1,N}\to0,\ \Omega_{2,N}\to0$ when $N\to\infty$, then $(\hat{\zeta}_N,\hat{\varpi}_N)\to(\zeta,\varpi)$ uniformly on $\Delta\times[0,\xi_0]$.

Proof. Let

$$(\hat{\zeta}_N,\hat{\varpi}_N)=\left(\sum_{n=0}^{N}\zeta_n,\sum_{n=0}^{N}\varpi_n\right)$$

be the truncated α-LHPM approximation. By assumption, $(\hat{\zeta}_N,\hat{\varpi}_N)$ satisfies the residual inequalities

$$\left|{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\zeta}_N-\mathbb{M}_{\zeta}(\xi_1,\xi_2,\hat{\zeta}_N,\hat{\varpi}_N)\right|\leq\Omega_{1,N}$$

and

$$\left|{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\varpi}_N-\mathbb{M}_{\varpi}(\xi_1,\xi_2,\hat{\zeta}_N,\hat{\varpi}_N)\right|\leq\Omega_{2,N}.$$

Thus, $(\hat{\zeta}_N,\hat{\varpi}_N)$ is an approximate solution of system (1.1) with perturbation bounds $\Omega_{1,N}$ and $\Omega_{2,N}$.

By Lemma 3.4, system (1.1) is Hyers–Ulam stable. Hence, there exist positive constants C_1 and C_2, independent of N, such that

$$\|\hat{\zeta}_N-\zeta\|_{\infty}\leq C_1\max\{\Omega_{1,N},\Omega_{2,N}\}$$

and

$$\|\hat{\varpi}_N - \varpi\|_\infty \leq C_2 \max\{\Omega_{1,N}, \Omega_{2,N}\}.$$

In particular, if

$$\Omega_{1,N} \to 0, \qquad \Omega_{2,N} \to 0 \quad \text{as} \quad N \to \infty,$$

then

$$\|\hat{\zeta}_N - \zeta\|_\infty \to 0, \qquad \|\hat{\varpi}_N - \varpi\|_\infty \to 0.$$

Therefore,

$$(\hat{\zeta}_N, \hat{\varpi}_N) \to (\zeta, \varpi)$$

uniformly on $\Delta \times [0, \xi_0]$. □

4. Examples

This section demonstrates the practical use of the proposed α-LHPM method for solving nonlinear fractional partial differential equations. In the previous sections, the theoretical framework of the method was developed and some analytical results such as existence and stability of solutions are established. Here, we focus on applying these theoretical results to obtain practical numerical solutions. The α-LHPM combines the homotopy perturbation method with the α-Laplace transform, which provides a simple and effective technique for solving nonlinear fractional dynamical systems with memory effects. The solutions obtained by this method are expressed in the form of series that converge rapidly and provide accurate approximations. To explain the procedure, we first consider a general nonlinear fractional equation and derive the series solution step by step using He's polynomials. After that, the method is applied to the fractional Belousov-Zhabotinsky dynamical system (BZDS), which is a well-known model in chemical oscillation theory. This model is suitable for testing the method because it contains strong nonlinear interactions and memory-dependent behavior. The obtained approximations by the proposed method are compared with numerical results to verify their accuracy. The results show that the α-LHPM gives accurate and stable solutions with good computational efficiency. Therefore the method can be considered a useful tool for studying fractional models that appear in chemical kinetics, nonlinear science and wave propagation problems.

The fractional BZDS model provides a useful representation for describing wave propagation phenomena observed in biological systems such as neuronal and cardiac tissues. Here, we illustrate the proposed approach by taking

$$\mathcal{R}_1 = \zeta[1 - \zeta - \delta_1 \varpi] + \delta_1' \varpi, \;\; \mathcal{R}_2 = -\delta_2 \zeta \varpi + \delta_3 \varpi$$

in the non-linear system (1.1), with the parameters $\delta_1 = 1$, $\delta_2 = 6$, and $\delta_3 = \delta_1' = 0$. Consider the BZDS system:

$$\begin{cases} {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2} \zeta(\xi_1, \xi_2) = \zeta_{\xi_1 \xi_1}(\xi_1, \xi_2) + \zeta(\xi_1, \xi_2) - \zeta^2(\xi_1, \xi_2) - \zeta(\xi_1, \xi_2)\varpi(\xi_1, \xi_2), \\ {}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2} \varpi(\xi_1, \xi_2) = \varpi_{\xi_1 \xi_1}(\xi_1, \xi_2) - 6\,\zeta(\xi_1, \xi_2)\varpi(\xi_1, \xi_2) + 6\,\varpi(\xi_1, \xi_2), \\ \zeta(\xi_1, 0) = \left(e^{\xi_1} + 1\right)^{-2}, \\ \varpi(\xi_1, 0) = -5\, e^{\xi_1} \left(e^{\xi_1} + 2\right)\left(e^{\xi_1} + 1\right)^{-2}. \end{cases} \tag{4.1}$$

Note that the exact solution of (4.1) at $\sigma = 1$ is

$$\zeta(\xi_1, \xi_2) = e^{10\xi_2}\left(e^{\xi_1} + e^{10\xi_2}\right)^{-2} \text{ and } \varpi(\xi_1, \xi_2) = -5\, e^{\xi_1}\left(e^{\xi_1} + 2e^{10\xi_2}\right)\left(e^{\xi_1} + e^{10\xi_2}\right)^{-2}.$$

Before applying the proposed α-LHPM to system (4.1), we note that this system satisfies the analytical framework developed in Sections 2 and 3. In particular, the nonlinear terms

$$\mathbb{M}_\zeta(\zeta, \varpi) = \zeta_{\xi_1\xi_1} + \zeta - \zeta^2 - \zeta\varpi \quad \text{and} \quad \mathbb{M}_\varpi(\zeta, \varpi) = \varpi_{\xi_1\xi_1} - 6\zeta\varpi + 6\varpi.$$

Clearly, the nonlinear functions $\zeta - \zeta^2 - \zeta\varpi$ and $-6\zeta\varpi + 6\varpi$ are polynomial functions in ζ and ϖ. Hence, they are continuous on every bounded subset of $\mathbb{R}^2$. Therefore, the operators $\mathbb{M}_\zeta$ and $\mathbb{M}_\varpi$ are continuous on the considered bounded domain. Now let $(\zeta, \varpi), (\zeta_1, \varpi_1) \in \mathscr{B}$. Assume that $\|(\zeta, \varpi)\|_{\mathscr{B}} \le R$ and $\|(\zeta_1, \varpi_1)\|_{\mathscr{B}} \le R$ for some $R > 0$. Then

$$|\zeta|, |\varpi|, |\zeta_1|, |\varpi_1| \le R.$$

For the first nonlinear term, we have

$$\begin{aligned}&\left|(\zeta - \zeta^2 - \zeta\varpi) - (\zeta_1 - \zeta_1^2 - \zeta_1\varpi_1)\right| \\ &\le |\zeta - \zeta_1| + |\zeta^2 - \zeta_1^2| + |\zeta\varpi - \zeta_1\varpi_1|.\end{aligned}$$

Since

$$|\zeta^2 - \zeta_1^2| = |\zeta - \zeta_1||\zeta + \zeta_1| \le 2R|\zeta - \zeta_1|$$

and

$$\begin{aligned}|\zeta\varpi - \zeta_1\varpi_1| &= |\zeta\varpi - \zeta_1\varpi + \zeta_1\varpi - \zeta_1\varpi_1| \\ &\le |\varpi||\zeta - \zeta_1| + |\zeta_1||\varpi - \varpi_1| \\ &\le R|\zeta - \zeta_1| + R|\varpi - \varpi_1|,\end{aligned}$$

we obtain

$$\left|(\zeta - \zeta^2 - \zeta\varpi) - (\zeta_1 - \zeta_1^2 - \zeta_1\varpi_1)\right| \le (1 + 3R)\|(\zeta, \varpi) - (\zeta_1, \varpi_1)\|_{\mathscr{B}}.$$

Thus, one may take $\gamma_\zeta = 1 + 3R$. Similarly, for the second nonlinear term,

$$\begin{aligned}&|(-6\zeta\varpi + 6\varpi) - (-6\zeta_1\varpi_1 + 6\varpi_1)| \\ &\le 6|\zeta\varpi - \zeta_1\varpi_1| + 6|\varpi - \varpi_1| \\ &\le 6R|\zeta - \zeta_1| + 6R|\varpi - \varpi_1| + 6|\varpi - \varpi_1| \\ &\le (12R + 6)\|(\zeta, \varpi) - (\zeta_1, \varpi_1)\|_{\mathscr{B}}.\end{aligned}$$

Therefore, take $\gamma_\varpi = 12R + 6$. Consequently, the contraction condition in Theorem 3.3 becomes

$$F_\sigma\,[(1 + 3R) + (12R + 6)] < 1,$$

or equivalently, $F_\sigma(15R + 7) < 1$. If this condition is satisfied on the considered bounded domain, then Theorem 3.3 implies that any convergent α-LHPM series generated for system (4.1) coincides with the unique solution of the fractional BZDS model. Moreover, if

$$(\hat{\zeta}_N, \hat{\varpi}_N) = \left(\sum_{n=0}^{N} \zeta_n, \sum_{n=0}^{N} \varpi_n\right)$$

denotes the truncated α-LHPM approximation and its residuals satisfy

$$\left|{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\zeta}_N - \mathbb{M}_{\zeta}(\hat{\zeta}_N, \hat{\varpi}_N)\right| \leq \Omega_{1,N}$$

and

$$\left|{}^{ABC}\mathcal{D}^{\sigma}_{0,\xi_2}\hat{\varpi}_N - \mathbb{M}_{\varpi}(\hat{\zeta}_N, \hat{\varpi}_N)\right| \leq \Omega_{2,N},$$

then Theorem 3.5 gives

$$\|\hat{\zeta}_N - \zeta\|_\infty \leq C_1 \max\{\Omega_{1,N}, \Omega_{2,N}\}$$

and

$$\|\hat{\varpi}_N - \varpi\|_\infty \leq C_2 \max\{\Omega_{1,N}, \Omega_{2,N}\}.$$

Hence, if $\Omega_{1,N} \to 0, \Omega_{2,N} \to 0$ when $N \to \infty$, then $(\hat{\zeta}_N, \hat{\varpi}_N) \to (\zeta, \varpi)$ uniformly on $\Delta \times [0, \xi_0]$. This confirms that the truncated α-LHPM approximation for system (4.1) is consistent with the convergence and stability framework established in Theorems 3.3 and 3.5.

Now apply the α-LHPM to get

$$\begin{aligned}
\mathbb{H}_{\zeta 0} &= 2\zeta_0\zeta_1 + \zeta_0\varpi_1 + \varpi_0\zeta_1 \\
\mathbb{H}_{\zeta 1} &= 2\zeta_0\zeta_2 + \zeta_1^2 + \zeta_0\varpi_2 + \varpi_1\zeta_1 + \zeta_2\varpi_0 \\
\mathbb{H}_{\zeta 2} &= 2\zeta_0\zeta_3 + 2\zeta_1\zeta_2 + \zeta_0\varpi_3 + \zeta_1\varpi_2 + \zeta_2\zeta_1 + \zeta_3\zeta_0 \\
&\vdots
\end{aligned} \tag{4.2}$$

and

$$\begin{aligned}
\mathbb{H}_{\varpi 0} &= \zeta_0\varpi_1 + \varpi_0\zeta_1 \\
\mathbb{H}_{\varpi 1} &= \zeta_0\zeta_2 + \zeta_1\varpi_1 + \varpi_2\zeta_0 \\
\mathbb{H}_{\varpi 2} &= \zeta_0\varpi_3 + \zeta_1\varpi_2 + \zeta_2\zeta_1 + \zeta_3\zeta_0 \\
&\vdots
\end{aligned} \tag{4.3}$$

Hence

$$\begin{aligned}
p^0 &: \zeta_0(\xi_1, \xi_2) = \frac{1}{(e^{\xi_1} + 1)^2} \\
p^1 &: \zeta_1(\xi_1, \xi_2) = \frac{10\xi_2^{\sigma}}{\Gamma(\sigma)} \frac{e^{\xi_1}}{(e^{\xi_1} + 1)^3} \\
p^2 &: \zeta_2(\xi_1, \xi_2) = \frac{50\,\xi_2^{2\sigma}}{\Gamma(2\sigma + 1)} \frac{e^{\xi_1}(2e^{\xi_1} - 1)}{(e^{\xi_1} + 1)^4} \\
p^3 &: \zeta_3(\xi_1, \xi_2) = \frac{50\,\xi_2^{3\sigma}}{\Gamma(3\sigma + 1)} \left[\frac{100}{\Gamma(\sigma)^2} \frac{e^{2\xi_1}}{(e^{\xi_1} + 1)^6} - \frac{e^{\xi_1}(15e^{2\xi_1} - 20e^{2\xi_1} + 6e^{\xi_1} - 5)}{(e^{\xi_1} + 1)^6}\right] \\
&\vdots
\end{aligned} \tag{4.4}$$

and

$$
\begin{aligned}
p^0 &: \varpi_0(\xi_1,\xi_2) = -5\, e^{\xi_1}\left(e^{\xi_1}+2\right)\left(e^{\xi_1}+1\right)^{-2} \\
p^1 &: \varpi_1(\xi_1,\xi_2) = \frac{-50\xi_2^{\sigma}}{\Gamma(\sigma)}\,\frac{e^{\xi_1}}{(e^{\xi_1}+1)^3} \\
p^2 &: \varpi_2(\xi_1,\xi_2) = \frac{-250\,\xi_2^{2\sigma}}{\Gamma(2\sigma+1)}\,\frac{e^{\xi_1}(2e^{\xi_1}-1)}{(e^{\xi_1}+1)^4} \\
p^3 &: \varpi_3(\xi_1,\xi_2) = \frac{-250\,\xi_2^{3\sigma}}{\Gamma(3\sigma+1)}\left[\frac{100}{\Gamma(\sigma)^2}\,\frac{e^{2\xi_1}}{(e^{\xi_1}+1)^6} - \frac{e^{\xi_1}(15e^{2\xi_1}-20e^{2\xi_1}+6e^{\xi_1}-5)}{(e^{\xi_1}+1)^6}\right] \\
&\vdots
\end{aligned}
\tag{4.5}
$$

Therefore the solution of (4.1) is

$$
\begin{aligned}
\zeta(\xi_1,\xi_2) = {} & \frac{1}{(e^{\xi_1}+1)^2} + \frac{10\xi_2^{\sigma}}{\Gamma(\sigma)}\,\frac{e^{\xi_1}}{(e^{\xi_1}+1)^3} + \frac{50\,\xi_2^{2\sigma}}{\Gamma(2\sigma+1)}\,\frac{e^{\xi_1}(2e^{\xi_1}-1)}{(e^{\xi_1}+1)^4} \\
& + \frac{50\,\xi_2^{3\sigma}}{\Gamma(3\sigma+1)}\left[\frac{100}{\Gamma(\sigma)^2}\,\frac{e^{2\xi_1}}{(e^{\xi_1}+1)^6} - \frac{e^{\xi_1}(15e^{2\xi_1}-20e^{2\xi_1}+6e^{\xi_1}-5)}{(e^{\xi_1}+1)^6}\right] + \cdots
\end{aligned}
\tag{4.6}
$$

and

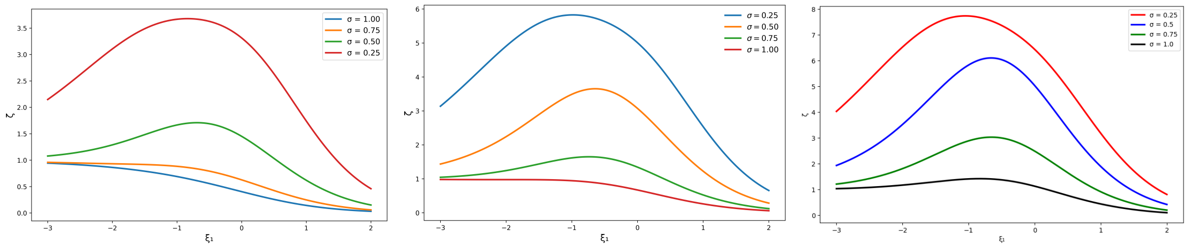
$$
\begin{aligned}
\varpi(\xi_1,\xi_2) = {} & -5\, e^{\xi_1}\left(e^{\xi_1}+2\right)\left(e^{\xi_1}+1\right)^{-2} - \frac{50\xi_2^{\sigma}}{\Gamma(\sigma)}\,\frac{e^{\xi_1}}{(e^{\xi_1}+1)^3} - \frac{250\,\xi_2^{2\sigma}}{\Gamma(2\sigma+1)}\,\frac{e^{\xi_1}(2e^{\xi_1}-1)}{(e^{\xi_1}+1)^4} \\
& - \frac{250\,\xi_2^{3\sigma}}{\Gamma(3\sigma+1)}\left[\frac{100}{\Gamma(\sigma)^2}\,\frac{e^{2\xi_1}}{(e^{\xi_1}+1)^6} - \frac{e^{\xi_1}(15e^{2\xi_1}-20e^{2\xi_1}+6e^{\xi_1}-5)}{(e^{\xi_1}+1)^6}\right] + \cdots
\end{aligned}
\tag{4.7}
$$

5. Numerical discussion

In this section, the numerical performance of the proposed α-LHPM is examined. The approximate solutions obtained by the method are compared with the exact analytical solution in the classical case $\sigma = 1$. The numerical values listed in Table 1, together with the graphical results shown in Figures 1–5, help illustrate the accuracy and convergence of the proposed method for the ABC fractional BZDS model.

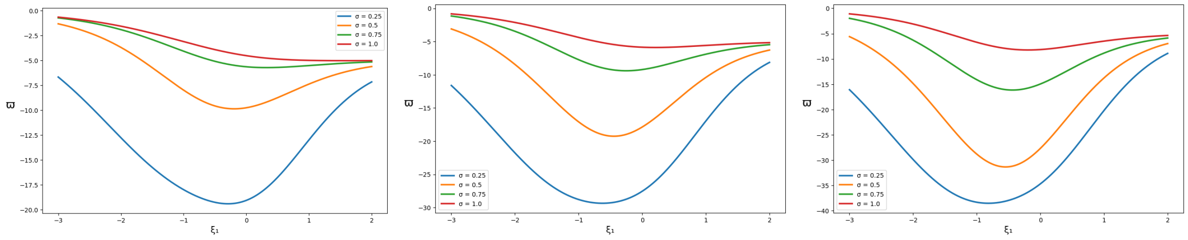
Table 1. Absolute error (AE) in solving α-LHPM of (4.1) at $\sigma = 1$.

ξ_1	ξ_2	$\zeta(\xi_1,\xi_2)_{\text{Exact}}$	$\zeta(\xi_1,\xi_2)_{\alpha\text{-LHPM}}$	AE_ζ	$\varpi(\xi_1,\xi_2)_{\text{Exact}}$	$\varpi(\xi_1,\xi_2)_{\alpha\text{-LHPM}}$	AE_ϖ
0.20	0.01	0.2041718	0.2041514	0.0000204	-3.8717761	-3.8713890	0.0003872
	0.02	0.2046827	0.2046622	0.0000205	-3.7500000	-3.7496250	0.0003750
	0.03	0.2041718	0.2041514	0.0000204	-3.6219843	-3.6216221	0.0003622
	0.04	0.2026494	0.2026292	0.0000203	-3.4884129	-3.4880640	0.0003488
	0.05	0.2001455	0.2001255	0.0000200	-3.3500790	-3.3497440	0.0003350
0.40	0.01	0.1665538	0.1665371	0.0000167	-4.0436517	-4.0432473	0.0004044
	0.02	0.1628843	0.1628680	0.0000163	-3.7454821	-3.7451075	0.0003745
	0.03	0.1582359	0.1582201	0.0000158	-3.4112804	-3.4109393	0.0003411
	0.04	0.1527745	0.1527593	0.0000153	-3.0564823	-3.0561767	0.0003065
	0.05	0.1466877	0.1466730	0.0000147	-2.6910445	-2.6907754	0.0002689
0.60	0.01	0.1348024	0.1347889	0.0000135	-4.3457480	-4.3453134	0.0004346
	0.02	0.1314961	0.1314830	0.0000131	-4.0319705	-4.0315673	0.0004032
	0.03	0.1272897	0.1272770	0.0000127	-3.6780249	-3.6776571	0.0003678
	0.04	0.1223629	0.1223507	0.0000122	-3.2993832	-3.2990532	0.0003299
	0.05	0.1169093	0.1168976	0.0000117	-2.9097560	-2.9094650	0.0002910
0.80	0.01	0.1086821	0.1086712	0.0000109	-4.6575273	-4.6570615	0.0004660
	0.02	0.1057272	0.1057166	0.0000106	-4.3273984	-4.3269656	0.0004327
	0.03	0.1020397	0.1020295	0.0000102	-3.9569812	-3.9565855	0.0003964
	0.04	0.0977564	0.0977466	0.0000098	-3.5612461	-3.5608900	0.0003561
	0.05	0.0930238	0.0930145	0.0000093	-3.1531704	-3.1528551	0.0003153



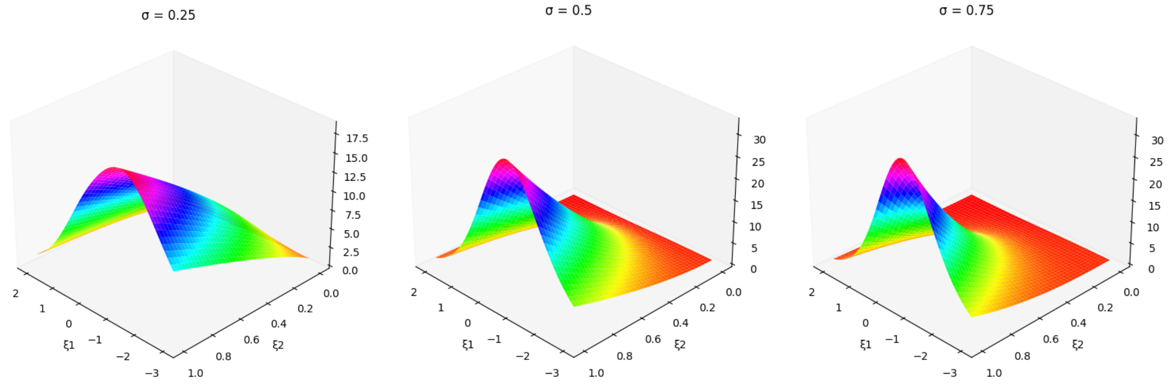
(a) Curves at $\xi_2 = 0.10$ (b) Curves at $\xi_2 = 0.20$ (c) Curves at $\xi_2 = 0.30$

Figure 1. The α-LHPM curves for the solutions $\zeta(\xi_1, \xi_2)$ of (4.1).



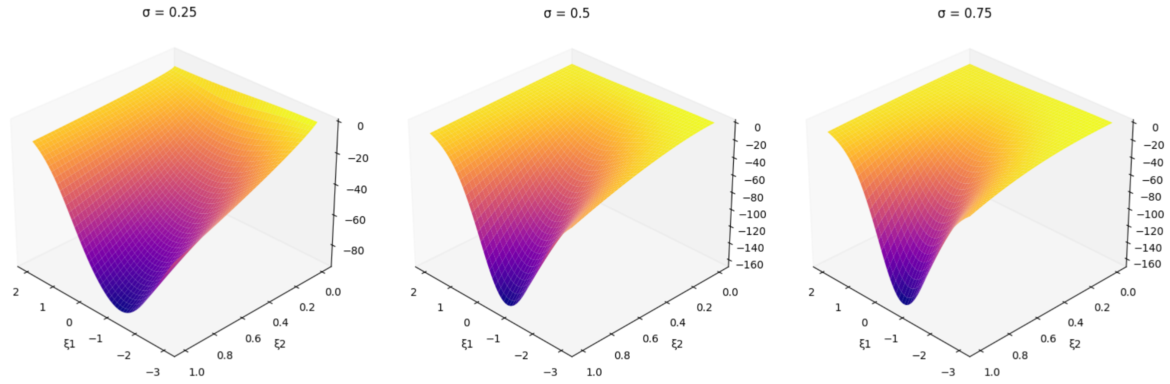
(a) Curves at $\xi_2 = 0.10$ (b) Curves at $\xi_2 = 0.20$ (c) Curves at $\xi_2 = 0.30$

Figure 2. The α-LHPM curves for the solutions $\varpi(\xi_1, \xi_2)$ of (4.1).



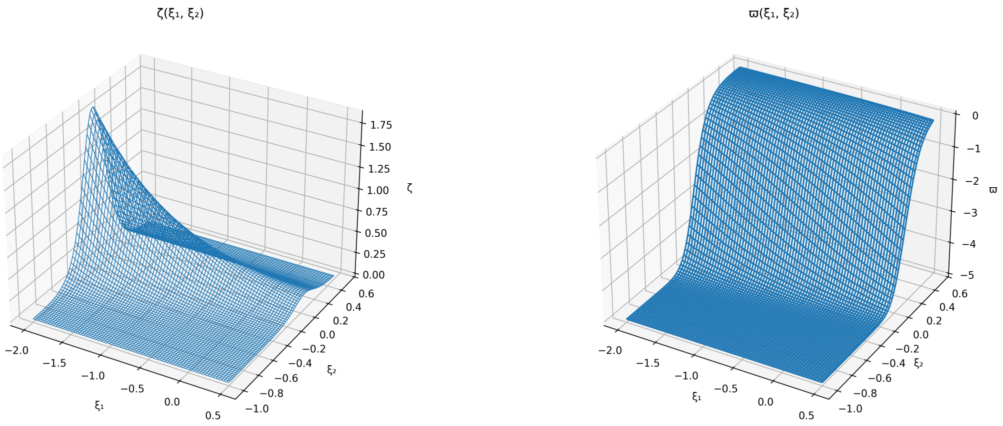
(a) $\sigma = 0.25$ (b) $\sigma = 0.5$ (c) $\sigma = 0.75$

Figure 3. The α-LHPM surface for the solutions $\zeta(\xi_1, \xi_2)$ of (4.1).



(a) $\sigma = 0.25$ (b) $\sigma = 0.5$ (c) $\sigma = 0.75$

Figure 4. The α-LHPM surface for the solutions $\varpi(\xi_1, \xi_2)$ of (4.1).



(a) Surface plot of $\zeta(\xi_1, \xi_2)$ (b) Surface plot of $\varpi(\xi_1, \xi_2)$

Figure 5. The surface α-LHPM solutions of (4.1) at $\sigma = 1$.

5.1. Analysis of Table 1

Table 1 presents the exact and approximate values of $\zeta(\xi_1, \xi_2)$, and $\varpi(\xi_1, \xi_2)$, along with the absolute errors at selected spatial points $\xi_1 \in \{0.2, 0.4, 0.6, 0.8\}$ and time levels $\xi_2 \in [0.01, 0.05]$ when $\sigma = 1$. The results show that the approximate solutions produced by the α-LHPM are very close to the exact values. The absolute error of ζ is very small across the whole domain and is usually around 10^{-5}, while the error of ϖ is around 10^{-4}. This indicates a strong agreement between the approximate and exact solutions. It can also be observed that for a fixed value of ξ_2, the error of ζ slightly decreases as ξ_1 increases. For instance, when $\xi_2 = 0.01$, the error decreases from 2.04×10^{-5} at $\xi_1 = 0.20$ to 1.09×10^{-5} at $\xi_1 = 0.80$. This suggests that the approximation becomes slightly more accurate for larger spatial values. For the inhibitory variable ϖ, the error is a bit larger due to the nonlinear interaction term $-6\zeta\varpi$. However, the error remains small and well-controlled throughout the domain, which shows that the α-LHPM can effectively handle the nonlinear coupling in the system. Another observation is that the error changes very little as ξ_2 increases from 0.01 to 0.05. This means that the approximation remains stable over time and does not accumulate noticeable numerical error.

5.2. Analysis of figures

Figure 1 presents the spatial profiles of $\zeta(\xi_1, \xi_2)$ at three time levels $\xi_2 = 0.10$, $\xi_2 = 0.20$ and $\xi_2 = 0.30$. These curves describe how the activator concentration evolves in space over time. The profiles keep a smooth wave-like shape while their amplitude and position change gradually. As time increase the peak becomes more pronounced and moves slightly along the spatial axis, indicating the propagation of the wave. This type of behavior is typical for reaction–diffusion systems and reflects the dynamics of the BZDS model. The smooth curves also indicate that the α-LHPM produces stable solutions without artificial oscillations. Figure 2 shows the spatial profiles of the inhibitory variable $\varpi(\xi_1, \xi_2)$ at the same time levels. Unlike ζ, the solution ϖ forms negative trough-like shapes over the spatial domain. This reflects the inhibitory role of ϖ in the activator–inhibitor interaction of the BZDS system. As time increases, the depth and position of the trough change smoothly, showing how the inhibitory component responds to the propagation of the activator wave. The curves also show a phase difference compared to those of ζ which is common in nonlinear excitable systems. The smooth evolution again confirms the stability of the proposed method. Figures 3 and 4 display three-dimensional surface plots of $\zeta(\xi_1, \xi_2)$ and $\varpi(\xi_1, \xi_2)$ for different fractional orders $\sigma = 0.25$, $\sigma = 0.5$, and $\sigma = 0.75$.

These figures show that the fractional parameter has a clear effect on the system behavior. For smaller values of σ such as $\sigma = 0.25$, the surfaces appear smoother and more diffused, which indicates stronger memory effects and slower wave propagation. As σ increases toward the classical case $\sigma = 1$, the surfaces become sharper and more localized. This means that the memory effects become weaker and the dynamics approach the classical diffusion behavior. Figure 5 presents the complete surfaces of $\zeta(\xi_1, \xi_2)$ and $\varpi(\xi_1, \xi_2)$ when $\sigma = 1$. The surface of ζ forms a ridge-like structure corresponding to the propagation of the activator wave while the surface of ϖ forms a valley-like structure due to the inhibitory interaction.

These two complementary patterns clearly illustrate the coupled activator–inhibitor dynamics of the BZDS system. The smooth shapes of the surfaces also support the reliability of the α-LHPM approximation. From a modeling viewpoint, the fractional-order parameter σ measures the strength of temporal memory in the chemical-wave response. When σ is reduced, the system retains a longer influence from its previous states, which damps abrupt changes and produces smoother, slower profiles. As σ approaches one, this memory weakens and the behavior moves toward the classical reaction–diffusion picture. This interpretation is consistent with the standard physical role of fractional orders in hereditary transport and diffusion models.

5.3. Comparison with some methods

For fractional-order cases $\sigma \neq 1$, exact solutions are generally not available. Therefore, to assess the accuracy of the proposed α-LHPM, a comparison with well-known numerical methods, namely, the Adomian decomposition method (ADM) and the variational iteration method (VIM), is presented. The comparison is carried out at the same grid points used in the previous computations to ensure consistency and fairness, as shown in Table 2.

Table 2. Comparison of the α-LHPM with the ADM and VIM at $\sigma = 0.9$.

ξ_1	ξ_2	$\zeta_{\alpha-\text{LHPM}}$	ζ_{ADM}	ζ_{VIM}	$\varpi_{\alpha-\text{LHPM}}$	ϖ_{ADM}	ϖ_{VIM}
0.20	0.01	0.2031473821	0.2035129473	0.2032985614	-3.8602148392	-3.8609734125	-3.8606021947
	0.02	0.2036289154	0.2040215837	0.2038012496	-3.7395482715	-3.7403179284	-3.7400281163
	0.03	0.2031195736	0.2036021948	0.2034027781	-3.6128349207	-3.6136248712	-3.6132875019
	0.04	0.2017429816	0.2021345728	0.2019013652	-3.4806132751	-3.4815120483	-3.4810938572
	0.05	0.1993187649	0.1998421975	0.1996043287	-3.3438124972	-3.3447352081	-3.3442675398
0.40	0.01	0.1652849372	0.1656724831	0.1654382197	-4.0301457382	-4.0309821736	-4.0305129481
	0.02	0.1617482915	0.1621039476	0.1618921043	-3.7332948123	-3.7340182941	-3.7336128475
	0.03	0.1573291847	0.1577812945	0.1575329186	-3.4005128374	-3.4014382917	-3.4009271843
	0.04	0.1521184736	0.1525839472	0.1523371825	-3.0473482716	-3.0481975324	-3.0477812946
	0.05	0.1463819475	0.1467948213	0.1465928374	-2.6838194736	-2.6847129384	-2.6842981745
0.60	0.01	0.1336482917	0.1340738294	0.1338219475	-4.3302193847	-4.3310928374	-4.3306174839
	0.02	0.1304283746	0.1308729184	0.1306291847	-4.0185374928	-4.0193184726	-4.0189182746
	0.03	0.1263847291	0.1267928374	0.1265739184	-3.6667482915	-3.6676128473	-3.6671847295
	0.04	0.1215482917	0.1219384726	0.1217291846	-3.2903482719	-3.2911738492	-3.2907183947
	0.05	0.1162483917	0.1166847291	0.1164827391	-2.9032847291	-2.9041827395	-2.9037193847
0.80	0.01	0.1076482917	0.1080839472	0.1078291746	-4.6405482917	-4.6413827491	-4.6409183746
	0.02	0.1048291746	0.1052739184	0.1050283746	-4.3124374918	-4.3132847291	-4.3128371947
	0.03	0.1013847291	0.1017839475	0.1015827391	-3.9445482917	-3.9453829174	-3.9449182746
	0.04	0.0972483917	0.0976837491	0.0974928374	-3.5515482917	-3.5523847291	-3.5519274839
	0.05	0.0927482917	0.0931827394	0.0929283746	-3.1463482917	-3.1472847391	-3.1468291746

As observed from Table 2, the results obtained by the proposed α-LHPM are in very close agreement with those of ADM and VIM. This confirms the accuracy, stability, and reliability of the proposed method for solving nonlinear fractional systems. The consistency among the different methods further supports the validity of the obtained numerical solutions.

5.4. Convergence and accuracy of the α-LHPM

The numerical results indicate that the α-LHPM produces a rapidly convergent series solution. Only a few terms of the expansion are needed to obtain accurate approximations. The small errors observed in Table 1 confirm the fast convergence of the method. In addition, the graphical results do not show oscillations or numerical instability, which demonstrates the robustness of the approach. A full computational-cost comparison with alternative fractional solvers would require a common implementation platform and identical stopping criteria; such a benchmarking study is beyond the present manuscript and is therefore left for future work.

5.5. *Summary of observations*

From the numerical and graphical analysis, several observations can be made:

(1) The α-LHPM provides accurate approximations with errors of order 10^{-5}–10^{-4}.
(2) The method remains stable for nonlinear coupled systems.
(3) The fractional order σ strongly affects the propagation of the chemical waves.
(4) Smaller values of σ correspond to stronger memory effects and slower wave propagation.
(5) The method successfully captures the main dynamics of the fractional BZDS model.

Overall, these results show that the α-LHPM is an effective method for solving nonlinear fractional partial differential equations, particularly those related to chemical wave propagation and excitable media.

The obtained numerical results show that decreasing the fractional-order parameter produces slower wave propagation and smoother solution profiles. This behavior reflects stronger memory effects and delayed diffusion mechanisms, which are commonly observed in complex biological and chemical media such as neuronal tissues, cardiac excitation systems, and oscillatory chemical reactions.

From a physical viewpoint, the obtained results demonstrate that the fractional-order parameter plays an important role in controlling the propagation behavior of the chemical waves. Smaller fractional orders correspond to stronger memory effects, which slow down the diffusion process and produce smoother wave fronts. This behavior is physically consistent with nonlocal transport phenomena observed in complex media. Moreover, the obtained dynamics are closely related to several real applications such as oscillatory chemical reactions, neuronal signal transmission, cardiac excitation systems, and biological reaction–diffusion processes where memory-dependent effects cannot be neglected.

6. Conclusions

In this paper, an efficient α-LHPM has been developed for solving nonlinear fractional chemical-wave models involving the ABC fractional derivative. The proposed method combines the α-Laplace transform with He's homotopy perturbation framework to construct rapidly convergent semi-analytical series solutions for nonlinear fractional systems with memory effects. Theoretical results concerning existence, uniqueness, convergence, and Hyers–Ulam stability of the obtained solutions were established under suitable assumptions. The developed approach was successfully applied to the fractional BZDS, where the numerical results demonstrated high accuracy and strong agreement with the exact solution in the classical case $\sigma = 1$. The obtained absolute errors remained very small, confirming the efficiency and reliability of the proposed method. Furthermore, the numerical simulations showed that decreasing the fractional-order parameter leads to smoother wave profiles and slower chemical-wave propagation, reflecting the influence of memory effects in the fractional model.

The improved graphical representations also provided a clearer visualization of the dynamical behavior of the considered system.

Overall, the obtained results confirm that the proposed α-LHPM is a simple, accurate, and computationally effective tool for studying nonlinear fractional dynamical systems arising in chemical kinetics and wave propagation problems. Future work may focus on extending the method to more complex nonlinear fractional models and higher-dimensional systems.

Author contributions

Conceptualization: F.H.D., O.O.; Methodology: F.H.D., A.S., A.T.; Validation: F.H.D., A.S., O.O.; Investigation: F.H.D., K.A., A.T.; Resources: F.H.D., A.S., K.A.; Writing–original draft: A.S., K.A.; Writing–review and editing: F.H.D., O.O., A.T. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare that no AI tools were used in the preparation of this manuscript.

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Conflict of interest

The authors declare no conflicts of interest.

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