Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 11(6): 15435–15447.
DOI: 10.3934/math.2026634
Received: 01 March 2026
Revised: 06 May 2026
Accepted: 12 May 2026
Published: 01 June 2026

Research article

Total domination concepts in interval-valued neutrosophic graphs

T. P. Sreelakshmi* **and K. Uma Samundesvari**

Department of Mathematics, Noorul Islam Centre for Higher Education, Kumaracoil, Tamil Nadu, India

* **Correspondence:** Email: sreelakshmitp.6.95@gmail.com.

Abstract: This paper studies total domination in interval-valued neutrosophic graphs (IVN_{graphs}), which extend classical graph structures by associating each node and edge with interval-valued degrees of truth, indeterminacy, and falsity. The main objective of this study is to generalize the concept of total domination from single-valued neutrosophic graphs to the interval-valued framework. We introduce formal definitions related to total domination and derive key conditions and properties of total dominating sets. An illustrative example is provided to support the theoretical developments and to demonstrate the applicability of the proposed framework. Extending the traditional domination theory to the interval-valued neutrosophic domain provides a more flexible and realistic model for handling uncertainties in complex network systems and offers a methodological foundation for further research in neutrosophic graph theory.

Keywords: interval-valued neutrosophic graph; domination; total domination; total domination number; neutrosophic-strong arcs
Mathematics Subject Classification: 03E72, 05C69, 05C72

1. Introduction

F. Smarandache [1] first introduced the idea of neutrosophic sets as the extended version of fuzzy and intuitionistic fuzzy sets by introducing three independent membership functions: truth, indeterminacy, and falsity. To offer a more realistic approach for handling uncertainty in the real world, Smarandache et al. [2] subsequently proposed single-valued neutrosophic sets, in which all membership functions are restricted to the unit interval [0,1]. Based on this, Wang et al. in the early 2000s suggested interval-valued neutrosophic sets where each degree of membership is represented as an interval instead of a single value. This sophistication allowed a more expressive representation of vagueness and incomplete information. Later, in 2016, Broumi et al. [3] defined and studied IVN_{graphs}, in which interval-based weights were put on both the nodes and the edges of the graph. These structures are especially applicable in modeling systems that can be characterized on positive and negative performance characteristics when the variability in reliability, efficiency, and risk are captured.

In parallel with these developments, the theory of domination in graph theory has been extensively studied over a number of decades. The concept of the domination number was initially conceived by Berge [4] as an extension of early combinatorial problems like coverage and control networks, and its systematic study was developed by Ore [5] and then generalized by the seminal survey of Cockayne and Hedetniemi [6]. Total domination, which was proposed by Cockayne [7], has become an important area of study due to its theoretical and practical significance. Subsequently, domination in fuzzy graph was studied in terms of strong arcs by Nagoorgani and Chandrasekharan [8]. Manjusha and Sunitha [9] expanded this line of research by examining domination and total domination in fuzzy graphs with strong arcs. More recently, Sreelakshmi and Uma Samundesvari [10] introduced total domination in uniform-single valued neutrosophic graphs using neutrosophic-strong arcs (NS_{arcs}). The main aim of this work is to extend the concept of total domination from single-valued neutrosophic graphs to IVN_{graphs} and to develop a corresponding theoretical framework for the analysis.

1.1. Research objectives

- To systematically extend the concept of total domination from classical, fuzzy, and single-valued neutrosophic graphs to the framework of IVN_{graphs}, thereby enabling richer uncertainty representation.
- To define total dominating sets using NS_{arcs}, where domination is determined through connectivity strength rather than conventional adjacency.
- To investigate the structural properties of total domination in IVN_{graphs} and establish theoretical results that characterize total dominating sets and total domination numbers.
- To demonstrate the applicability of the proposed theory by modeling and interpreting total domination in database graphs operating under uncertainty and variability.

1.2. Novelty and structure of the paper

In contrast to the existing domination studies, which are based on crisp adjacency, fuzzy effective edges, and single-valued neutrosophic relations, this paper formulates domination using NS_{arcs} in IVN_{graphs}.

Another explicit thing in this paper is the use of interval-valued truth, indeterminacy, and falsity values in the study of total domination. This interval-based approach simultaneously shows the best and worst case scenarios, providing an articulate and accurate model of uncertainty compared to single-valued models.

The structure of this paper is as follows: Section 2 provides a review of the fundamental definitions and concepts relevant to our study. In Section 3, we introduce a novel approach for identifying total dominating sets using NS_{arcs}, along with some definitions and theoretical results. Section 4 discusses total domination in some special uniform single-valued neutrosophic graphs. Section 5 explores a practical application of our findings. Finally, Section 6 concludes the paper by summarizing the key contributions and suggesting future research.

2. Preliminaries

This section presents the essential concepts and definitions in IVN_{graphs}, forming the basis for total domination. These preliminaries are essential for providing the groundwork for the theoretical framework in this paper. In this work, $G^* = (V, E)$ refers to a simple graph.

Definition 2.1. *[11] Let Λ denote a universe of discourse. A neutrosophic set X on Λ can be expressed as*

$X = \{\langle v, T_\Lambda(v), I_\Lambda(v), and F_\Lambda(v)\rangle, v \in \Lambda\}$, where the mappings T_Λ, I_Λ, F_Λ assign to each element $v \in \Lambda$ its degree of truth, indeterminacy, and falsity membership, respectively. These functions take values in the interval $]0^-, 1^+[$, subject to the requirement that for all $v \in \Lambda$,

$$0^- \leq T_\Lambda(v) + I_\Lambda(v) + F_\Lambda(v) \leq 3^+. \tag{2.1}$$

Definition 2.2. *[12] A single-valued neutrosophic graph (SVN_{graph}) on G^* consists of the pair $G = (\Phi, \Psi)$, where*

- *T_Φ, I_Φ, and F_Φ are the functions from V to $[0, 1]$ defined as degree of truth-membership (T_{MF}), indeterminacy-membership (I_{MF}), and falsity-membership (F_{MF}) of $v_i \in V$ such that*

$$0 \leq T_\Phi(v_i) + I_\Phi(v_i) + F_\Phi(v_i) \leq 3. \tag{2.2}$$

- *Similarly, T_Ψ, I_Ψ and F_Ψ are functions from $E \rightarrow [0, 1]$, where*

$$T_\Psi(v_i, v_j) \leq \wedge \left\{T_\Phi(v_i), T_\Phi(v_j)\right\}, \tag{2.3}$$

$$I_\Psi(v_i, v_j) \geq \vee \left\{I_\Phi(v_i), I_\Phi(v_j)\right\}, \tag{2.4}$$

$$F_\Psi(v_i, v_j) \geq \vee \left\{F_\Phi(v_i), F_\Phi(v_j)\right\}, \tag{2.5}$$

$$0 \leq T_\Psi(v_i, v_j) + I_\Psi(v_i, v_j) + F_\Psi(v_i, v_j) \leq 3, \tag{2.6}$$

$\forall (v_i, v_j) \in E; (i, j = 1, 2, ..., n)$.

Definition 2.3. *[3] An IVN_{graph} on G^* is defined as $G = (\Phi, \Psi)$, where $\Phi = \left\langle [T_\phi{}^-, T_\phi{}^+], [I_\phi{}^-, I_\phi{}^+], [F_\phi{}^-, F_\phi{}^+] \right\rangle$ is an IVN-set on V and $\Psi = \left\langle [T_\psi{}^-, T_\psi{}^+], [I_\psi{}^-, I_\psi{}^+], [F_\psi{}^-, F_\psi{}^+] \right\rangle$ is an IVN-relation on E, which holds the following conditons:*

- *$V = \{v_1, v_2, v_3, ..., v_n\}$ such that the values of $T_\phi{}^-$, $T_\phi{}^+$, $I_\phi{}^-$, $I_\phi{}^+$, $F_\phi{}^-$, and $F_\phi{}^+$ run from V to [0,1] and $0 \leq T_\phi{}^+ + I_\phi{}^+ + F_\phi{}^+ \leq 3$; $v_i \in V$.*
- *The values of $T_\psi{}^-$, $T_\psi{}^+$, $I_\psi{}^-$, $I_\psi{}^+$, $F_\psi{}^-$, and $F_\psi{}^+$ are from $E \rightarrow [0, 1]$ such that*

$$T_\psi{}^-(v_i, v_j) \leq T_\phi{}^-(v_i) \wedge T_\phi{}^-(v_j), \tag{2.7}$$

$$T_\psi{}^+(v_i, v_j) \leq T_\phi{}^+(v_i) \wedge T_\phi{}^+(v_j), \tag{2.8}$$

$$I_\psi{}^-(v_i, v_j) \geq I_\phi{}^-(v_i) \vee I_\phi{}^-(v_j), \tag{2.9}$$

$$I_\psi{}^+(v_i, v_j) \geq I_\phi{}^+(v_i) \vee I_\phi{}^+(v_j), \tag{2.10}$$

$$F_\psi{}^-(v_i, v_j) \geq F_\phi{}^-(v_i) \vee F_\phi{}^-(v_j), \tag{2.11}$$

$$F_\psi{}^+(v_i, v_j) \geq F_\phi{}^+(v_i) \vee F_\phi{}^+(v_j), \tag{2.12}$$

$\forall (v_i, v_j) \in E$.

3. Total domination in IVN_{graph}

Definition 3.1. *Let $G = (\Phi, \Psi)$ be an IVN_{graph}. Then the vertex cardinality is*

$$|V| = \textstyle\sum_{v_i \in V} \frac{1+T_{\phi}{}^{+}(v_i)-T_{\phi}{}^{-}(v_i)+1+I_{\phi}{}^{+}(v_i)-I_{\phi}{}^{-}(v_i)+1+F_{\phi}{}^{-}(v_i)-F_{\phi}{}^{+}(v_i)}{2}, \tag{3.1}$$

$$\text{and } |E| = \textstyle\sum_{(v_i,v_j) \in E} \frac{1+T_{\phi}{}^{+}(v_i,v_j)-T_{\phi}{}^{-}(v_i,v_j)+1+I_{\phi}{}^{+}(v_i,v_j)-I_{\phi}{}^{-}(v_i,v_j)+1+F_{\phi}{}^{-}(v_i,v_j)-F_{\phi}{}^{+}(v_i,v_j)}{2} \tag{3.2}$$

is the edge cardinality.

Definition 3.2. *Let $G = (\Phi, \Psi)$ be an IVN_{graph}. If G has a path P of length k from vertex v_1 to v_2, given by $P = v_1, a_1, a_2, ..., (a_{k-1} = v_2)$, then the strength of a path P is defined as ($T_{\psi}{}^{-^{k}}(v_1, v_2)$, $T_{\psi}{}^{+^{k}}(v_1, v_2)$, $I_{\psi}{}^{-^{k}}(v_1, v_2)$, $I_{\psi}{}^{+^{k}}(v_1, v_2)$, $F_{\psi}{}^{-^{k}}(v_1, v_2)$, $F_{\psi}{}^{+^{k}}(v_1, v_2)$), where*

$$T_{\psi}{}^{-^{k}}(v_1, v_2) = \wedge \left\{T_{\psi}{}^{-}(v_1, a_1), T_{\psi}{}^{-}(a_1, a_2), ..., T_{\psi}{}^{-}(a_{k-2}, (a_{k-1} = v_2))\right\}, \tag{3.3}$$

$$T_{\psi}{}^{+^{k}}(v_1, v_2) = \wedge \left\{T_{\psi}{}^{+}(v_1, a_1), T_{\psi}{}^{+}(a_1, a_2), ..., T_{\psi}{}^{+}(a_{k-2}, (a_{k-1} = v_2))\right\}, \tag{3.4}$$

$$I_{\psi}{}^{-^{k}}(v_1, v_2) = \vee \left\{I_{\psi}{}^{-}(v_1, a_1), I_{\psi}{}^{-}(a_1, a_2), ..., I_{\psi}{}^{-}(a_{k-2}, (a_{k-1} = v_2))\right\}, \tag{3.5}$$

$$I_{\psi}{}^{+^{k}}(v_1, v_2) = \vee \left\{I_{\psi}{}^{+}(v_1, a_1), I_{\psi}{}^{+}(a_1, a_2), ..., I_{\psi}{}^{+}(a_{k-2}, (a_{k-1} = v_2))\right\}, \tag{3.6}$$

$$F_{\psi}{}^{-^{k}}(v_1, v_2) = \vee \left\{F_{\psi}{}^{-}(v_1, a_1), F_{\psi}{}^{-}(a_1, a_2), ..., F_{\psi}{}^{-}(a_{k-2}, (a_{k-1} = v_2))\right\}, \tag{3.7}$$

$$F_{\psi}{}^{+^{k}}(v_1, v_2) = \vee \left\{F_{\psi}{}^{+}(v_1, a_1), F_{\psi}{}^{+}(a_1, a_2), ..., F_{\psi}{}^{+}(a_{k-2}, (a_{k-1} = v_2))\right\}. \tag{3.8}$$

Definition 3.3. *Let $G = (\Phi, \Psi)$ be an IVN_{graph}. The strength of connectedness between two vertices v_1 and v_2 is defined as $T_{\psi}{}^{-^{\alpha}}(v_1, v_2)$, $T_{\psi}{}^{+^{\alpha}}(v_1, v_2)$, $I_{\psi}{}^{-^{\alpha}}(v_1, v_2)$, $I_{\psi}{}^{+^{\alpha}}(v_1, v_2)$, $F_{\psi}{}^{-^{\alpha}}(v_1, v_2)$, and $F_{\psi}{}^{+^{\alpha}}(v_1, v_2)$, where*

$$T_{\psi}{}^{-^{\alpha}}(v_1, v_2) = \vee \left\{T_{\psi}{}^{-^{k}}(v_1, v_2) : k = 1, 2..., n\right\}, \tag{3.9}$$

$$T_{\psi}{}^{+^{\alpha}}(v_1, v_2) = \vee \left\{T_{\psi}{}^{+^{k}}(v_1, v_2) : k = 1, 2..., n\right\}, \tag{3.10}$$

$$I_{\psi}{}^{-^{\alpha}}(v_1, v_2) = \wedge \left\{I_{\psi}{}^{-^{k}}(v_1, v_2) : k = 1, 2..., n\right\}, \tag{3.11}$$

$$I_{\psi}{}^{+^{\alpha}}(v_1, v_2) = \wedge \left\{I_{\psi}{}^{+^{k}}(v_1, v_2) : k = 1, 2..., n\right\}, \tag{3.12}$$

$$F_{\psi}{}^{-^{\alpha}}(v_1, v_2) = \wedge \left\{F_{\psi}{}^{-^{k}}(v_1, v_2) : k = 1, 2..., n\right\}, \tag{3.13}$$

$$F_{\psi}{}^{+^{\alpha}}(v_1, v_2) = \wedge \left\{F_{\psi}{}^{+^{k}}(v_1, v_2) : k = 1, 2..., n\right\}. \tag{3.14}$$

Definition 3.4. *An edge (v_i, v_j) of an IVN_{graph} is said to be an NS_{arc} if the following conditions hold:*

$$T_{\psi}{}^{-}(v_i, v_j) \geq T_{\psi}{}^{-^{\infty}}(v_i, v_j), \tag{3.15}$$

$$T_{\psi}{}^{+}(v_i, v_j) \geq T_{\psi}{}^{+^{\infty}}(v_i, v_j), \tag{3.16}$$

$$I_{\psi}{}^{-}(v_i, v_j) \leq I_{\psi}{}^{-^{\infty}}(v_i, v_j), \tag{3.17}$$

$$I_{\psi}{}^{+}(v_i, v_j) \leq I_{\psi}{}^{+^{\infty}}(v_i, v_j), \tag{3.18}$$

$$F_{\psi}{}^{-}(v_i, v_j) \leq F_{\psi}{}^{-^{\infty}}(v_i, v_j), \tag{3.19}$$

$$F_{\psi}{}^{+}(v_i, v_j) \leq F_{\psi}{}^{+^{\infty}}(v_i, v_j). \tag{3.20}$$

Definition 3.5. *An edge (v_i, v_j) of an IVN_{graph} is said to be a neutrosophic-weak arc (NW_{arc}) if the following conditions hold:*

$$T_{\psi}^{-}(v_i, v_j) < T_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.21}$$

$$T_{\psi}^{+}(v_i, v_j) < T_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.22}$$

$$I_{\psi}^{-}(v_i, v_j) > I_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.23}$$

$$I_{\psi}^{+}(v_i, v_j) > I_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.24}$$

$$F_{\psi}^{-}(v_i, v_j) > F_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.25}$$

$$F_{\psi}^{+}(v_i, v_j) > F_{\psi}^{+^{\infty}}(v_i, v_j). \tag{3.26}$$

Remark 3.1. *(1) Strength of connectivity between the vertices v_i and v_j is indicated by $CONN_{G(v_i,v_j)}$.*
(2) On the basis of connectedness, the NS_{arc} is generally distinguished into three main categories.

Definition 3.6. *An edge (v_i, v_j) in an IVN_{graph} is called α - NS_{arc} $(\alpha_{N-strong})$ if*

$$T_{\psi}^{-}(v_i, v_j) > T_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.27}$$

$$T_{\psi}^{+}(v_i, v_j) > T_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.28}$$

$$I_{\psi}^{-}(v_i, v_j) < I_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.29}$$

$$I_{\psi}^{+}(v_i, v_j) < I_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.30}$$

$$F_{\psi}^{-}(v_i, v_j) < F_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.31}$$

$$F_{\psi}^{+}(v_i, v_j) < F_{\psi}^{+^{\infty}}(v_i, v_j). \tag{3.32}$$

Definition 3.7. *An edge (v_i, v_j) in an IVN_{graph} is called β - NS_{arc} $(\beta_{N-strong})$ if*

$$T_{\psi}^{-}(v_i, v_j) = T_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.33}$$

$$T_{\psi}^{+}(v_i, v_j) = T_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.34}$$

$$I_{\psi}^{-}(v_i, v_j) = I_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.35}$$

$$I_{\psi}^{+}(v_i, v_j) = I_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.36}$$

$$F_{\psi}^{-}(v_i, v_j) = F_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.37}$$

$$F_{\psi}^{+}(v_i, v_j) = F_{\psi}^{+^{\infty}}(v_i, v_j). \tag{3.38}$$

Definition 3.8. *An edge (v_i, v_j) in an IVN_{graph} is called δ - NW_{arc} (δ_{N-weak}) if*

$$T_{\psi}^{-}(v_i, v_j) < T_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.39}$$

$$T_{\psi}^{+}(v_i, v_j) < T_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.40}$$

$$I_{\psi}^{-}(v_i, v_j) > I_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.41}$$

$$I_{\psi}^{+}(v_i, v_j) > I_{\psi}^{+^{\infty}}(v_i, v_j), \tag{3.42}$$

$$F_{\psi}^{-}(v_i, v_j) > F_{\psi}^{-^{\infty}}(v_i, v_j), \tag{3.43}$$

$$F_{\psi}^{+}(v_i, v_j) > F_{\psi}^{+^{\infty}}(v_i, v_j). \tag{3.44}$$

Remark 3.2. *(1) By Definitions 3.6 and 3.7, an* NS_{arc} *is one that is either* $\alpha_{N-strong}$ *or* $\beta_{N-strong}$.

(2) A path P *in* G *is an neutrosophic-strong path if it contains only* NS_{arcs}.

(3) An IVN_{graph} *is called a strong-*IVN_{graph} *if all edges in* G *are neutrosophic-strong.*

Definition 3.9. *The neighborhood vertices of an* NS_{arc} *in an* IVN_{graph} *are denoted as* $N_{s-arc}(u)$ *and defined as*

$$N_{s-arc}(v_i) = \left\{v_j \in V : (v_i, v_j) \text{ is a } NS_{arc}\right\}. \tag{3.45}$$

Definition 3.10. *Let* G *be an* IVN_{graph} *and* v_i, $v_j \in V$. *Then* v_i *dominates* v_j *if there exists an* NS_{arc} *between them.*

A dominating set $S \subseteq V$ *is a group of vertices such that every vertex outside the group is dominated by at least one vertex inside the group.*

The lower-domination number (minimum cardinality of a dominating set) and upper-domination number (maximum cardinality of a dominating set) are denoted by γ'_d *and* γ'_D.

Definition 3.11. *Let* G *be an* IVN_{graph}. $U \subseteq V$ *is a total dominating set* (TD_{set}) *of* G *if all vertices in* V *are dominated by a vertex in* U*, distinct from itself.*

A total dominating set S *is a minimal* TD_{set} *if none of its proper subsets form a* TD_{set}. *The minimum cardinality of this set is called total domination number and is denoted as* γ'_{td}.

Remark 3.3. *In crisp graphs, the total domination number is defined as the number of vertices in a minimal* TD_{set}. *However, in fuzzy graphs, intuitionistic fuzzy graphs, and neutrosophic graphs, it is defined as the sum of vertex cardinalities of the vertices in a minimal* TD_{set}.

Example 3.1. *In Figure 1,* c_2c_3, c_3, c_4, *and* c_4c_1 *are the* NS_{arcs}. *The set* $A = \{c_3, c_4\}$ *is the minimal* TD_{set} *of the given graph.*

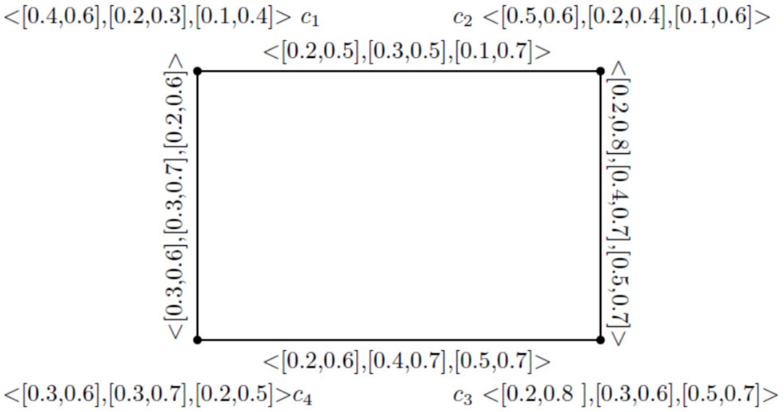

Figure 1. IVN_{graph}.

By Definition 3.1, the vertex cardinality of each vertex is computed accordingly.

Therefore, $|A| = |c_3| + |c_4| = \frac{1+0.8-0.2+1+0.6-0.3+1+0.5-0.7}{2} + \frac{1+0.6-0.3+1+0.7-0.3+1+0.2-0.5}{2}$.

Hence, the total domination number is $\gamma'_{td} = 3.55$.

Definition 3.12. *An* IVN_{graph} *is called a uniform-*IVN_{graph} ($U - IVN_{graph}$) *of level* $[k_1{}^-, k_2{}^+]$, $[k_3{}^-, k_4{}^+]$, $[k_5{}^-, k_6{}^+]$ *if*

$$T_{\psi}{}^-(v_i, v_j) = k_1{}^- = T_{\phi}{}^-(v_i), \tag{3.46}$$

$$T_\psi{}^{+}(v_i, v_j) = k_2{}^{+} = T_\phi{}^{+}(v_i), \tag{3.47}$$

$$I_\psi{}^{-}(v_i, v_j) = k_3{}^{-} = I_\phi{}^{-}(v_i), \tag{3.48}$$

$$I_\psi{}^{+}(v_i, v_j) = k_4{}^{+} = I_\phi{}^{+}(v_i), \tag{3.49}$$

$$F_\psi{}^{-}(v_i, v_j) = k_5{}^{-} = F_\phi{}^{-}(v_i), \tag{3.50}$$

$$F_\psi{}^{+}(v_i, v_j) = k_6{}^{+} = F_\phi{}^{+}(v_i), \tag{3.51}$$

$\forall v_i \in V$, $(v_i, v_j) \in E$, and $k_1{}^{-}, k_2{}^{+}, k_3{}^{-}, k_4{}^{+}, k_5{}^{-}, k_6{}^{+} \in [0, 1]$.

Remark 3.4. *In a $U - IVN_{graph}$, all edges and vertices have the same values. Hence, the connectivity between any two vertices will coincide with the corresponding edge values. Therefore, the conditions in Definition 3.7 are satisfied for every edge, and hence all edges are $\beta_{N-strong}$.*

Definition 3.13. *A $U - IVN_{graph}$ $G = (\phi, \psi)$ is a complete $U - IVN_{graph}$ if the following conditions hold:*

$$T_\psi{}^{-}(v_i, v_j) = T_\phi{}^{-}(v_i) \wedge T_\phi{}^{-}(v_j), \tag{3.52}$$

$$T_\psi{}^{+}(v_i, v_j) = T_\phi{}^{+}(v_i) \wedge T_\phi{}^{+}(v_j), \tag{3.53}$$

$$I_\psi{}^{-}(v_i, v_j) = I_\phi{}^{-}(v_i) \vee I_\phi{}^{-}(v_j), \tag{3.54}$$

$$I_\psi{}^{+}(v_i, v_j) = I_\phi{}^{+}(v_i) \vee I_\phi{}^{+}(v_j), \tag{3.55}$$

$$F_\psi{}^{-}(v_i, v_j) = F_\phi{}^{-}(v_i) \vee F_\phi{}^{-}(v_j), \tag{3.56}$$

$$F_\psi{}^{+}(v_i, v_j) = F_\phi{}^{+}(v_i) \vee F_\phi{}^{+}(v_j), \tag{3.57}$$

$\forall (v_i, v_j) \in E$.

Theorem 3.1. *Suppose $G = (\phi, \psi)$ is a complete $U - IVN_{graph}$ with no isolated vertices. Then, $\gamma'_{td} = 2|v_i|$; $v_i \in V$, where all vertices have equal vertex cardinality.*

Proof. Let $G = (\phi, \psi)$ be a complete $U - IVN_{graph}$ without isolated vertices. By Definitions 3.12 and 3.13, all vertices and edges have identical values, and every pair of vertices is connected by $\beta_{N-strong}$ arcs.

In a complete $U - IVN_{graph}$, at least two vertices are required to totally dominate all vertices. Hence, a minimum TD_{set} contains exactly two vertices. Let $S = \left\{v_i, v_j\right\}$ be a TD_{set}. Since the graph is uniform, all vertices have the same vertex cardinality (Definition 3.1), i.e., $|v_i| = |v_j|$ $\forall v_i, v_j \in V$.

Therefore, the total domination number is given by $\gamma'_{td}(S) = |v_i| + |v_j| = |v_i| + |v_i|$.

Hence, $\gamma'_{td} = 2|v_i|$, $v_i \in V$. □

4. Total domination in some special $U - IVN_{graph}$

Theorem 4.1. *Let $G = (\phi, \psi)$ be a $U - IVN$ graph. If the total domination number of the corresponding crisp graph is k, then $\gamma'_{td}(G) = k|v_i|$, $v_i \in V$.*

Proof. Since G is a $U - IVN$ graph, all the vertices have the same values, and all the vertices have the same vertex cardinality (by Definition 3.1). Let $S = \{v_1, v_2, ..., v_k\}$ be a minimum TD_{set} of the corresponding crisp graph. Then, the corresponding TD_{set} of G also contains k vertices.

Therefore, the total domination number of G is given by $\gamma'_{td}(G) = |v_1| + |v_2| + ... + |v_k|$.

Since all vertices have the same cardinality, say $|v_i|$, we get $\gamma'_{td}(G) = k|v_i|$.

Hence, the result follows. □

Remark 4.1. *In a $U - IVN_{graph}$, the total domination number is directly proportional to the number of vertices in a minimum TD_{set}.*

Definition 4.1. *The $U-IVN$ bull-graph is a simple planar, undirected $U-IVN$ graph with five vertices and five edges, formed by a triangle with two non-adjacent pendant edges attached to distinct vertices.*

Corollary 4.2. *If $G = (\phi, \psi)$ is a $U - IVN$ bull-graph, then $\gamma'_{td} = 2|v_i|$; $v_i \in V$.*

Proof. Let G be a $U - IVN$ bull-graph, where the vertex set is $V = \{v_1, v_2, v_3, v_4, v_5\}$ and the edge set is $E = \{v_1v_3, v_1v_2, v_2v_3, v_3v_5, v_2v_4\}$. Here, $\{v_1, v_2, v_3\}$ forms a triangle, and $\{v_3v_5, v_2v_4\}$ are the pendant edges of G. Since G is a $U - IVN_{graph}$, all the edges are considered to be NS_{arcs}. In G, the two vertices v_2 and v_3, which are connected to the triangle, form a minimum total dominating set of G. $\gamma'_{td} = |v_2| + |v_3|$.

Since G is uniform, all vertices have the same vertex cardinality (by Definition 3.1), i.e., $|v_2| = |v_3| = |v_i|$.

Hence, $\gamma'_{td} = 2|v_i|$. □

Definition 4.2. *The $U - IVN$ n-sunlet graph (S_n) is a graph with $2n$ vertices, constructed by joining n pendant edges to the vertices of a $U - IVN$ cycle graph C_n.*

Corollary 4.3. *For a $U - IVN$ n-sunlet graph, $\gamma'_{td}(S_n) = n|v_i|$; $v_i \in V$.*

Proof. Let S_n be a $U - IVN$ n-sunlet graph with $2n$ vertices, of which n are pendant vertices. Let $\{v_1, v_2, ..., v_n\}$ be the vertices of the cycle C_n and $\{o_1, o_2, ..., o_n\}$ be the corresponding pendant vertices, where each o_i is attached to v_i. Since S_n is a $U-IVN_{graph}$, all the edges are NS_{arcs}. To totally dominate all pendant vertices, all the cycle vertices v_i are needed.

Hence, $S = \{v_1, v_2, ..., v_n\}$ forms the minimum TD_{set} of S_n.

Therefore, $\gamma'_{td}(S_n) = |S| = |v_1| + |v_2| + ... + |v_n|$ becomes $\gamma'_{td}(S_n) = n|v_i|$ (Since G is uniform). □

Definition 4.3. *The $U - IVN$ wheel-graph (W_n) is formed by joining a single isolated vertex K_1 to every vertex of a $U - IVN$ cycle containing $(n - 1)$ vertices.*

Corollary 4.4. *If G is a $U - IVN$ wheel-graph (W_n), then $\gamma'_{td}(W_n) = 2|v_i|$; $v_i \in V$.*

Proof. Let G be a $U-IVN$ wheel-graph with the vertex set $V = \{u, v_1, v_2, ..., v_{n-1}\}$, where u is the central vertex of W_n. Since G is a $U - IVN_{graph}$, all of its edges are considered NS_{arcs}. As the central vertex u is connected to every other vertex, it must belong to the TD_{set}. By the definition of total domination, a TD_{set} requires a minimum of two vertices in general to totally dominate the whole graph. Hence, we can form a minimum TD_{set} by including u together with one vertex v_i.

Therefore, $\gamma'_{td}(W_n) = |u| + |v_i| = 2|v_i|$ (Since G is uniform, all vertices have the same vertex cardinalities).

□

Definition 4.4. *The $U - IVN$ Helm-graph (H_n) is obtained from a W_n by adjoining a pendant edge at each node of the rim C_n.*

Corollary 4.5. *Let G be a $U - IVN$ helm-graph, then $\gamma'_{td}(H_n) = n|v_i|$; $v_i \in V$.*

Proof. Let G be a $U - IVN$ helm-graph with $2n + 1$ vertices. Let u denote the central vertex, let $\{v_1, v_2, ..., v_n\}$ represent the rim vertices forming the cycle of H_n, and let $\{w_1, w_2, ..., w_n\}$ denote the pendant vertices.

Since G is a $U - IVN_{graph}$, in which each edge is an NS_{arc} it implies that all the vertices dominate each other. Now, the pendant vertex w_i is dominated by its corresponding rim vertices v_i. Thus, the set $\{v_1, v_2, ..., v_n\}$ totally dominates the pendant vertices.

So, $\{v_1, v_2, ..., v_n\} \subseteq TD_{set}(H_n)$.

Therefore, $n \leq |TD_{set}(H_n)|$.

The central vertex u is automatically dominated as there exist NS_{arcs} between u and each v_i. Furthermore, every rim vertex v_i is dominated by either v_{i-1} or v_{i+1}.

Thus, $|TD_{set}(H_n)| \leq n$, which leads to $\gamma'_{td}(H_n) = n|v_i|$. □

5. Application of IVN_{graph} in traffic signal control

The rapid growth of modern cities are so high that sometimes it is difficult to match the growth of our traffic system. Even though our governments are developing infrastructure such as wider roads and flyovers, the lack of an adaptive traffic signal system continues to hold us back. Most traffic lights operate on predefined cycles without considering the real-time traffic density. This can lead to situations where a road with only a few vehicles experiences a long green signal, while the other lane with heavy traffic is forced to wait. This type of signal control creates unnecessary delays and makes heavy traffic jams in junctions, especially in growing cities, which loses countless productive hours each year. This clearly shows that modern cities can't depend on outdated traffic control logic.

To overcome this, many cities across the world introduced the smart traffic control systems. Even though this reduced waiting time and improved traffic flow, their limitations still remain notable. These smart systems need reliable and continuous data to work properly. Bad weather, camera issues, poor visibility, network lag, or mismatched sensor data can confuse the system and lead to inaccurate traffic density assessment. As a result, the system again goes to fixed signal timings, losing its smart feature. This shows that the current system still struggles to handle real-life uncertainty and unpredictable conditions.

The classical graph models can't handle the changing behavior because the connections in these models are treated as either present or not. Even though fuzzy graph models improve on this idea, they use single membership value, which limits their ability to simultaneously represent reliability and uncertainty. This gap leads to the motivation of our present research. IVN_{graphs} solve this problem by assigning interval-based values to both vertex and edge: one for reliability, one for uncertainty, and one for possible failure. These interval-based values easily help to model complex and uncertain systems. Now, applying total domination on IVN_{graphs} ensures that we can select a set of key nodes such as major junctions or priority signal controllers. All other intersections remain connected to at least one of these nodes through a reliable traffic flow path. This ensures full network coverage despite sensor failure, communication issues, or uncertain signal data. Consequently, the system becomes more robust and continues to operate effectively. As a result, traffic control becomes more reliable. Practically, this supports better green-signal coordination and reduces congestion caused by local disruption.

5.1. Example

Consider an intelligent traffic control system modeled using a simple IVN_{graph} with four nodes and four edges, where the node v_1 represents the primary control unit at the junction in the local traffic network. It collects real-time information, makes local decisions, and updates signal timings accordingly. It is wirelessly connected to v_2, which is a peripheral junction. This handles moderate traffic from feeder roads joining the main corridor. v_2 acts as a supporting unit located around the junction. It collects data related to vehicle movement and congestion levels and passes it to the primary control unit. v_2 connectes to another nearby intersection, v_3. v_3 is a roadside unit connected to a CCTV node, which is supported by a sensor or IOT unit. Its role is to obtain video-based traffic details with the coordination of v_4, where v_4 acts as a master signal intersection directly integrated with the city traffic control center.

Each node and edge is assigned interval-based values for truth, indeterminacy, and falsity to represent reliability and uncertainty. The interval based values of nodes are given below.

$(T_{(v_1)}, I_{(v_1)}, F_{(v_1)}) = ([0.8, 0.9], [0.05, 0.1], [0.06, 0.12])$,
$(T_{(v_2)}, I_{(v_2)}, F_{(v_2)}) = ([0.7, 0.8], [0.08, 0.15], [0.09, 0.17])$,
$(T_{(v_3)}, I_{(v_3)}, F_{(v_3)}) = ([0.75, 0.85], [0.07, 0.1], [0.09, 0.17])$,
$(T_{(v_4)}, I_{(v_4)}, F_{(v_4)}) = ([0.88, 0.96], [0.02, 0.06], [0.01, 0.05])$.

The interval-based values of edges are given below.

$(T_{(v_1,v_2)}, I_{(v_1,v_2)}, F_{(v_1,v_2)}) = ([0.7, 0.8], [0.08, 0.15], [0.09, 0.17])$,
$(T_{(v_2,v_3)}, I_{(v_2,v_3)}, F_{(v_2,v_3)}) = ([0.69, 0.75], [0.08, 0.16], [0.09, 0.18])$,
$(T_{(v_3,v_4)}, I_{(v_3,v_4)}, F_{(v_3,v_4)}) = ([0.75, 0.85], [0.07, 0.1], [0.09, 0.17])$,
$(T_{(v_1,v_4)}, I_{(v_1,v_4)}, F_{(v_1,v_4)}) = ([0.8, 0.9], [0.05, 0.1], [0.06, 0.012])$.

The proposed intelligent traffic control system modeled using an IVN_{graphs} is illustrated in Figure 2.

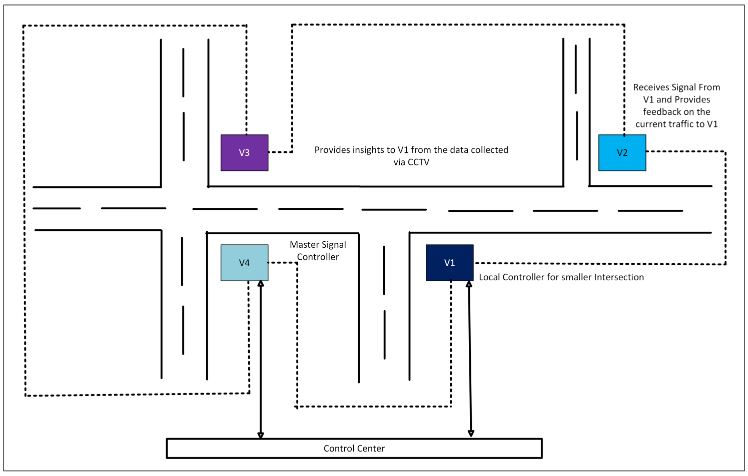

Figure 2. Proposed traffic control system.

The corresponding graph structure of the proposed traffic control system is shown in Figure 3.

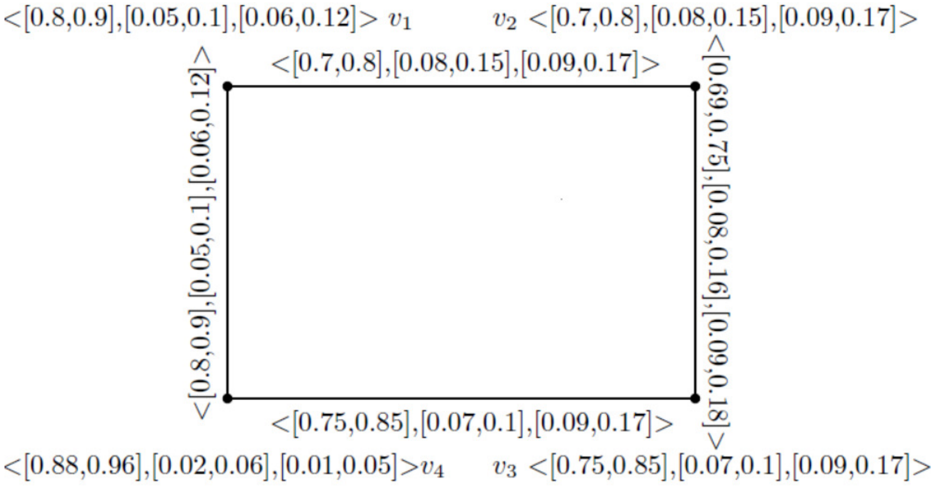

Figure 3. Graph structure of the proposed traffic control system.

In the constructed graph model of the traffic signal network, the edges (v_1, v_2), (v_3, v_4), and (v_1, v_4) are NS_{arc}, as determined by Definitions 3.3 and 3.4. Among all possible subsets of vertices, only the set $\{v_1, v_4\}$ can totally dominate the graph. Therefore, $\{v_1, v_4\}$ forms the TD_{set}, which ensures network-wide coordination. v_1 monitors v_2 through a stable short-range link. Similarly, v_4 monitors v_3 through a high-quality control connection. Both v_1 and v_4 maintain direct communication with each other by ensuring seamless co-ordination between local and central control layers. In real-world traffic control systems, the number of crossings, sensors, and communication links might grow across the city. As the size of the network increases, the concept of total domination becomes important for maintaining reliability and reducing resource usage. In large-scale systems, each node represents a traffic signal controller or a roadside sensing unit, and the edges represent communication links that transfer signal status, queue lengths, and environmental data. Over time, the performance of these connections may weaken due to weather conditions or equipment aging, and single-valued models may not adequately capture such dynamic behavior.

6. Conclusions

In this work, the concept of total domination has been extended to IVN_{graphs} to better handle uncertainty in complex networks. We introduce important concepts such as vertex cardinality, path strength, and connectivity and use them to develop results for total domination in $U - IVN_{graphs}$. The proposed work effectively captures uncertainty through interval parameters. The application of a traffic control system demonstrates the practical relevance of the proposed model.

The ideas presented can be further extended to other types of domination parameters and also can be applied to real-world problems involving complex and uncertain systems.

Author contributions

T. P. Sreelakshmi: Conceptualization, methodology, formal analysis, writing-original draft preparation; K. Uma Samundesvari: Supervision, validation, review and editing. All authors have read and approved the final version of the manuscript.

Use of Generative-AI tools declaration

The authors used AI-assisted language tool only for improving grammar and language clarity. All mathematical results, definitions, proofs and scientific content were developed and verified by authors.

Acknowledgments

The author would like to thank the reviewers and the editor for their valuable comments and suggestions, which helped to improve the quality of the manuscript.

Conflict of interest

There are no conflicts of interest to disclose.

References

1. F. Smarandache, Neutrosophic set - A generalization of the intuitionistic fuzzy set, *Int. J. Pure Appl. Math.*, **24** (2005), 287–297.
2. H. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman, Single valued neutrosophic sets, *viXra*, 2010.
3. S. Broumi, A. Bakali, M. Talea, F. Smarandache, P. Singh, Properties of interval-valued neutrosophic graphs, In: *Fuzzy multi-criteria decision-making using neutrosophic sets*, Cham: Springer, 2018, 173–202. https://doi.org/10.1007/978-3-030-00045-5_8
4. C. Berge, *The theory of graphs and its applications*, See Description, 1962.
5. O. Ore, *Theory of graphs*, Providence: American Mathematical Society, 1962.
6. E. J. Cockayne, S. T. Hedetniemi, Towards a theory of domination in graphs, *Networks*, **7** (1977), 247–261. https://doi.org/10.1002/net.3230070305
7. E. J. Cockayne, R. M. Dawes, S. T. Hedetniemi, Total domination in graphs, *Networks*, **10** (1980), 211–219. https://doi.org/10.1002/net.3230100304
8. A. Nagoorgani, V. T. Chandrasekaran, Domination in fuzzy graph, *Adv. Fuzzy Sets Syst.*, **1** (2006), 17–26.
9. O. T. Manjusha, M. S. Sunitha, Total domination in fuzzy graphs using strong arcs, *Ann. Pure Appl. Math.*, **9** (2015), 23–33.

10. T. P. Sreelakshmi, K. Uma Samundesvari, Total domination in uniform single valued neutrosophic graph, *Neutrosophic Sets Syst.*, **70** (2024), 305–313. https://doi.org/10.5281/zenodo.13175758

11. S. Broumi, M. Talea, A. Bakali, F. Smarandache, Single valued neutrosophic graphs, *J. New Theory*, **10** (2016), 86–101.

12. S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L. H. Son, et al., Uniform single valued neutrosophic graphs, *Neutrosophic Sets Syst.*, **17** (2017), 42–49.

AIMS Press