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Research article

Chance-constrained set-membership filtering for complex networks over full-duplex relay networks with missing measurements

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Abstract: This paper explores the chance-constrained set-membership filtering problem for complex networks subject to missing measurements and long-distance transmissions. To enhance the transmission reliability, a full-duplex relay was placed between the sensor and the remote filter, supplemented by a self-interference suppression mechanism to mitigate relay-induced disturbances. Missing measurements were modeled using Bernoulli random variables, while transmission uncertainties arising from long-distance transmission were characterized by stochastic channel parameters. The primary objective was to construct a filter that confines the filtering error within a predefined ellipsoidal bound at a specified probability level. To this end, sufficient conditions in the form of recursive linear matrix inequalities were derived, from which the corresponding filter gains were obtained. Within this framework, two optimization schemes were further formulated to achieve locally optimal filtering performance. A numerical result validated the effectiveness of the proposed method.

Keywords: complex networks; chance-constrained filtering; set-membership filtering; full-duplex relay; missing measurements
Mathematics Subject Classification: 93E11, 93E03

1. Introduction

Complex networks (CNs) have attracted significant interest due to their broad applications in various domains, including vehicular and sensor communication networks [1], microgrid systems [2, 3], and networked systems [4, 5]. Typically, a CN is characterized by large-scale interconnections among its nodes and edges, which signify the relationships and interactions between different entities. Significant effort has been devoted to filtering and state estimation problems, since these play a fundamental role in analyzing and regulating networked dynamical behaviors [6, 7]. Nonetheless, the complex interconnections among nodes, coupled with uncertainties introduced by the network, present considerable obstacles to attaining reliable state estimation, thereby requiring additional research.

Due to the inherent difficulty in describing pervasive noise using probabilistic models, many engineering systems face challenges when applying traditional filtering methods. Approaches such as Kalman filtering necessitate prior knowledge of noise distributions, yet such information is often difficult to obtain in practical applications. Set-membership filtering has emerged as a powerful alternative to overcome this limitation by providing bounded guarantees for filtering errors under unknown-but-bounded disturbances [8–10]. The set-membership filtering method provides a deterministic estimation framework by computing recursive ellipsoid sets encompassing the actual system states. This method ensures robustness without relying on precise statistical assumptions, making it particularly suitable for control networks characterized by modeling uncertainties and randomness [11, 12]. Nevertheless, traditional set-membership filtering techniques may be excessively conservative, with stringent requirements that are challenging to meet in real-world scenarios.

To alleviate such conservatism, researchers have introduced chance-constrained set-membership filtering, commonly referred to as probability-guaranteed filtering [13, 14]. This approach relaxes the strict deterministic constraints by requiring the estimation error to remain within a prescribed ellipsoidal region only with a given probability, thereby offering enhanced flexibility. As a result, the chance-constrained formulation is more suitable for practical filtering problems, such as target tracking and weapon shooting tests, where excessively stringent deterministic guarantees may be unnecessary or difficult to satisfy. Numerous studies have demonstrated that this probabilistic framework can achieve satisfactory performance while mitigating unnecessary conservatism and offering more design freedom in networked estimation problems [15, 16]. Related probability-guaranteed schemes have also been reported for fusion estimation [17, 18] and consensus control [19].

In practical networked systems, due to sensor malfunctions or unstable communication environments, measurement information may be intermittently unavailable during the transmission process [20, 21]. Accordingly, control and filtering problems with missing measurements have received sustained attention [22–24], with further developments reported for networked systems subject to intermittent observations and data losses [25, 26]. One commonly adopted approach in the literature is to model random data unavailability as binary-valued packet dropout events, which are governed by a sequence of independent Bernoulli random variables [27–29]. In particular, a binary indicator is added at each time step, taking the value of 1 when the corresponding measurement is successfully received, and 0 otherwise. In addition to accurately representing the stochastic arrival behavior of measurement signals, this probabilistic modeling framework also accounts for a number of network-induced imperfections that arise in realistic communication environments.

Owing to the limited transmission capacity of sensors and significant attenuation in long-distance communication, direct delivery of measurement signals to remote filters is often impractical [30]. To enhance transmission reliability, deploying a relay has emerged as an effective solution. Various relaying strategies have been developed, including amplify-and-forward protocols [31, 32], decode-and-forward protocols [33], and filter-and-forward protocols [34]. Among these, amplify-and-forward relaying, which receives sensor signals, amplifies them, and forwards them to the destination, has attracted considerable interest due to its structural simplicity and convenient implementation [35–37]. This relay-aided mechanism has recently been incorporated into filtering, estimation, and fault diagnosis problems for networked systems [38, 39].

It is worth noting that, in addition to the relaying protocol, the duplex operation mode of the relay also plays a crucial role in determining the signal transmission performance. Mobile communication systems are generally categorized into simplex, half-duplex, and full-duplex modes [40–42]. Among them, full-duplex relays have attracted considerable attention owing to their capability of simultaneous transmission and reception, which leads to higher spectral efficiency compared with simplex or half-duplex schemes. Nevertheless, full-duplex relay networks inherently suffer from the self-interference problem, where the transmitted signal leaks into the receiving channel, thereby degrading the quality of the received signal [43]. Such self-interference constitutes one of the fundamental technical difficulties associated with the adoption of full-duplex relays. Therefore, effective suppression of relay-induced self-interference is essential for ensuring reliable signal transmission. For instance, self-interference cancellation techniques have been adopted in [44, 45] to mitigate the adverse effects caused by full-duplex relays.

With the above considerations in mind, the present study is confronted with the following three fundamental challenges: 1) how to handle the self-interference and the resulting dynamic coupling caused by full-duplex relays under missing measurements and stochastic channel effects, 2) how to formulate a chance-constrained set-membership filtering objective for the considered complex networks, and 3) how to deal with the coupled effects of network dynamics, relay transmission, and unknown-but-bounded noises in a recursive and tractable manner. In response to the above challenges, this paper investigates the chance-constrained set-membership filtering problem for complex networks with full-duplex relays, where the relay adopts the amplify-and-forward protocol. The main contributions of this article are summarized as follows.

1. The chance-constrained set-membership filtering problem for CNs is investigated. The proposed system model integrates unknown-but-bounded noise, missing measurements, and full-duplex relays, thereby capturing key features of practical networked systems.
2. To enhance signal strength and improve system reliability, an amplify-and-forward-based full-duplex relay scheme is adopted. In addition, a novel filter is constructed to effectively accommodate delays caused by self-interference and the occurrence of missing measurements.
3. A chance-constrained set-membership filtering algorithm is developed to guarantee that the true system state is contained within the estimation set with a prescribed probability. By employing recursive convex optimization techniques, two optimal solutions are developed, one aiming at minimizing the ellipsoidal bound, while the other focuses on maximizing the constrained probability.

The remainder of this article is structured as follows. Section 2 formulates the chance-constrained set-membership filtering problem for CNs under missing measurements and full-duplex relays. Primary findings are presented in Section 3, outlining solvability conditions for the filtering problem based on recursive linear matrix inequalities. Section 4 provides a numerical simulation example for illustration. Concluding remarks are given in Section 5.

Notations

$\mathbb{R}^p$	Set of p real vectors
$\mathbb{R}^{p\times q}$	Set of $p \times q$ real matrices
Z^T/Z^{-1}	Transpose/inverse of matrix Z
$\mathrm{tr}(Z)$	Trace of matrix Z

$\mathrm{diag}\{\cdots\}$	Block-diagonal matrix
$\mathrm{diag}_N\{Z_i\}$	Block diagonal matrix $\mathrm{diag}\{Z_1, \cdots, Z_N\}$
$\mathbb{P}\{\mathcal{G}\}$	Probability of event $\mathcal{G}$
$\mathbb{E}\{A\}$	Expectation of random variable A
$\otimes$	Kronecker product
$*$	Symmetric terms
$M > (\geq)\ 0$	Positive definite (non-negative definite)

2. Problem formulation

2.1. System model

Consider a CN composed of N nodes described by

$$\begin{cases} x_{i,s+1} = A_{i,s}x_{i,s} + \sum_{j=1}^{N} \delta_{ij}\Gamma x_{j,s} + B_{i,s}\omega_{i,s}, \\ \quad y_{i,s} = \lambda_{i,s}C_{i,s}x_{i,s} + \upsilon_{i,s}, \quad i = 1, 2, \cdots, N, \end{cases} \tag{2.1}$$

where $x_{i,s} \in \mathbb{R}^{n_x}$ and $y_{i,s} \in \mathbb{R}^{n_y}$ denote the system state and measurement output, respectively. $\Delta = [\delta_{ij}]_{N\times N}$ represents the external coupling configuration matrix associated with the network topology, and Γ denotes the internal coupling structure matrix. $\omega_{i,s} \in \mathbb{R}^{n_\omega}$ and $\upsilon_{i,s} \in \mathbb{R}^{n_y}$ are the unknown process noise and measurement noise, respectively. The coefficient matrices $A_{i,s} \in \mathbb{R}^{n_x\times n_x}$, $B_{i,s} \in \mathbb{R}^{n_x\times n_\omega}$, and $C_{i,s} \in \mathbb{R}^{n_y\times n_x}$ are assumed to be known. The random variable $\lambda_{i,s}$ follows a Bernoulli distribution to characterize the measurement of missing phenomena, and satisfies

$$\mathbb{P}\{\lambda_{i,s} = 1\} = \bar{\lambda}_i, \ \ \mathbb{P}\{\lambda_{i,s} = 0\} = 1 - \bar{\lambda}_i, \tag{2.2}$$

where $\bar{\lambda}_i$ is a known constant.

2.2. Full-duplex relay network

As shown in Figure 1, signal transmission is considered over full-duplex relay networks. The transmission between sensors and the remote filter consists of two stages, namely, the sensor-to-relay transmission and the relay-to-filter transmission, while the effect of full-duplex self-interference is also taken into account. This relay configuration is adopted to facilitate long-distance transmission and improve the reliability of the measurement information available to the remote filter, while the adverse effect of self-interference is mitigated by a cancellation mechanism.

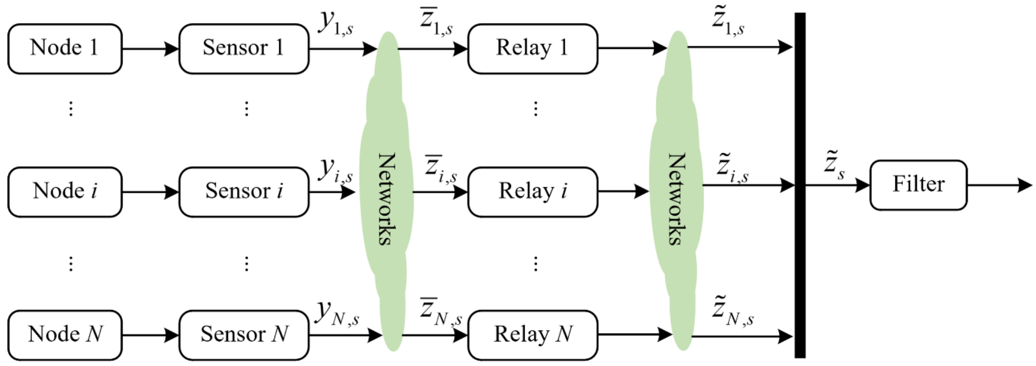

Figure 1. Filtering scheme for the CN over full-duplex relay networks.

The measurement signal is first transmitted to the relay, and the transmission process is modeled as

$$z_{i,s} = \sqrt{l_{i,s}^{(1)} t_{i,s}^{(1)}} y_{i,s} + \mu_{i,s}^{(1)}, \tag{2.3}$$

where $z_{i,s} \in \mathbb{R}^{n_y}$ denotes the transmitted measurement signal from the sensor to the relay, $l_{i,s}^{(1)}$ is the transmission power, and $t_{i,s}^{(1)}$ represents the random channel parameter from the sensor to the relay, satisfying $\mathbb{E}\{t_{i,s}^{(1)}\} = \bar{t}_{i,s}^{(1)}$ and $\mathbb{E}\{(t_{i,s}^{(1)} - \bar{t}_{i,s}^{(1)})^2\} = \sigma_{i,s}^{(1)}$, where $\bar{t}_{i,s}^{(1)}$ and $\sigma_{i,s}^{(1)}$ are known positive parameters. $\mu_{i,s}^{(1)}$ is an unknown-but-bounded transmission noise over the sensor-to-relay channel.

Due to the self-interference of the full-duplex relay, the received signal at the relay is given by

$$\bar{z}_{i,s} = z_{i,s} + \sqrt{l_{i,s-1}^{(2)} t_{i,s}^{(2)}} m_{i,s} + m_{i,s}^c, \tag{2.4}$$

where $m_{i,s}$ and $m_{i,s}^c$ denote the self-interference term generated by the full-duplex relay and its corresponding cancellation term, respectively. $l_{i,s}^{(2)}$ is the transmission power of the relay, and $t_{i,s}^{(2)}$ is the random channel parameter from the relay to itself, satisfying $\mathbb{E}\{t_{i,s}^{(2)}\} = \bar{t}_{i,s}^{(2)}$ and $\mathbb{E}\{(t_{i,s}^{(2)} - \bar{t}_{i,s}^{(2)})^2\} = \sigma_{i,s}^{(2)}$, where $\bar{t}_{i,s}^{(2)} > 0$ and $\sigma_{i,s}^{(2)} > 0$ are known parameters. The self-interference term $m_{i,s}$ is defined as

$$m_{i,s} = \begin{cases} 0, & s = 0, \\ \beta_{i,s-1}\bar{z}_{i,s-1}, & s > 0, \end{cases} \tag{2.5}$$

where $\beta_{i,s}$ is a known amplification coefficient. To mitigate the negative impact of self-interference on system performance, a cancellation term $m_{i,s}^c$ is introduced as

$$m_{i,s}^c = \begin{cases} 0, & s = 0, \\ -\sqrt{l_{i,s-1}^{(2)} \bar{t}_{i,s}^{(2)}} m_{i,s}, & s > 0. \end{cases} \tag{2.6}$$

Remark 1. *Due to self-interference, the signal actually received by the relay, $\bar{z}_{i,s}$, includes not only the transmitted signal $z_{i,s}$ from the sensor, but also the signal $m_{i,s}$ sent by the relay itself at time $s-1$ (i.e., the self-interference term $\beta_{i,s-1}\bar{z}_{i,s-1}$). Since the self-interference signal $m_{i,s}$ corresponds to a one-step delay, its transmission probability is associated with the transmission power at time $s-1$, namely, $l_{i,s-1}^{(2)}$.*

Next, the signal $\bar{z}_{i,s}$ is amplified and forwarded to the remote filter. The signal actually received by the remote filter is expressed as

$$\tilde{z}_{i,s} = \beta_{i,s}\sqrt{l_{i,s}^{(2)}}t_{i,s}^{(3)}\bar{z}_{i,s} + \mu_{i,s}^{(2)}, \tag{2.7}$$

where $t_{i,s}^{(3)}$ denotes the random channel parameter from the relay to the filter, satisfying $\mathbb{E}\{t_{i,s}^{(3)}\} = \bar{t}_{i,s}^{(3)}$ and $\mathbb{E}\{(t_{i,s}^{(3)} - \bar{t}_{i,s}^{(3)})^2\} = \sigma_{i,s}^{(3)}$, with $\bar{t}_{i,s}^{(3)} > 0$ and $\sigma_{i,s}^{(3)} > 0$ being known parameters. $\mu_{i,s}^{(2)}$ is the transmission noise over the relay-to-filter channel, which is unknown but bounded.

Remark 2. *The signal transmission process mainly includes the following steps. First, the measurement signal $y_{i,s}$ is transmitted by the sensor and converted into $z_{i,s}$ through the sensor-to-relay channel. Then, the full-duplex relay receives $z_{i,s}$ together with the self-interference signal. To eliminate the effect of self-interference, a compensation strategy is introduced in* (2.5)–(2.6). *Finally, the processed signal is amplified and forwarded by the relay, and the remote filter finally receives $\tilde{z}_{i,s}$, as shown in* (2.7).

Remark 3. *Compared with the half-duplex amplify-and-forward relay, the full-duplex amplify-and-forward relay can receive and forward the measurement signal simultaneously, rather than in two separate transmission phases. This feature improves the transmission efficiency and is therefore advantageous for remote filtering over long-distance communication channels. On the other hand, simultaneous transmission and reception inevitably give rise to self-interference at the relay side. Hence, the considered full-duplex relay model is equipped with a self-interference cancellation mechanism to mitigate this adverse effect.*

Assumption 1. *The noises $\omega_{i,s}$, $\upsilon_{i,s}$, $\mu_{i,s}^{(1)}$, and $\mu_{i,s}^{(2)}$ satisfy*

$$\begin{cases} \omega_{i,s} \in \mathfrak{W}_{i,s} \triangleq \{\omega_{i,s} : \omega_{i,s}^T \mathcal{W}_{i,s}^{-1}\omega_{i,s} \le 1\}, \\ \upsilon_{i,s} \in \mathfrak{V}_{i,s} \triangleq \{\upsilon_{i,s} : \upsilon_{i,s}^T \mathcal{V}_{i,s}^{-1}\upsilon_{i,s} \le 1\}, \\ \mu_{i,s}^{(1)} \in \mathfrak{U}_{i,s}^{(1)} \triangleq \{\mu_{i,s}^{(1)} : (\mu_{i,s}^{(1)})^T (\mathcal{U}_{i,s}^{(1)})^{-1}\mu_{i,s}^{(1)} \le 1\}, \\ \mu_{i,s}^{(2)} \in \mathfrak{U}_{i,s}^{(2)} \triangleq \{\mu_{i,s}^{(2)} : (\mu_{i,s}^{(2)})^T (\mathcal{U}_{i,s}^{(2)})^{-1}\mu_{i,s}^{(2)} \le 1\}, \end{cases} \tag{2.8}$$

where $\mathcal{W}_{i,s} > 0$, $\mathcal{V}_{i,s} > 0$, $\mathcal{U}_{i,s}^{(1)} > 0$, and $\mathcal{U}_{i,s}^{(2)} > 0$.

2.3. Filter design

Define

$$\begin{aligned} x_s &= \begin{bmatrix} x_{1,s}^T & x_{2,s}^T & \cdots & x_{N,s}^T \end{bmatrix}^T, \\ \bar{z}_s &= \begin{bmatrix} \bar{z}_{1,s}^T & \bar{z}_{2,s}^T & \cdots & \bar{z}_{N,s}^T \end{bmatrix}^T, \\ \tilde{z}_s &= \begin{bmatrix} \tilde{z}_{1,s}^T & \tilde{z}_{2,s}^T & \cdots & \tilde{z}_{N,s}^T \end{bmatrix}^T, \\ \omega_s &= \begin{bmatrix} \omega_{1,s}^T & \omega_{2,s}^T & \cdots & \omega_{N,s}^T \end{bmatrix}^T, \\ \upsilon_s &= \begin{bmatrix} \upsilon_{1,s}^T & \upsilon_{2,s}^T & \cdots & \upsilon_{N,s}^T \end{bmatrix}^T, \\ \mu_s^{(1)} &= \begin{bmatrix} (\mu_{1,s}^{(1)})^T & (\mu_{2,s}^{(1)})^T & \cdots & (\mu_{N,s}^{(1)})^T \end{bmatrix}^T, \\ \mu_s^{(2)} &= \begin{bmatrix} (\mu_{1,s}^{(2)})^T & (\mu_{2,s}^{(2)})^T & \cdots & (\mu_{N,s}^{(2)})^T \end{bmatrix}^T, \end{aligned}$$

$$
\begin{aligned}
A_s &= \mathrm{diag}_N\{A_{i,s}\}, \quad B_s = \mathrm{diag}_N\{B_{i,s}\}, \\
C_s &= \mathrm{diag}_N\{C_{i,s}\}, \quad \Lambda_s = \mathrm{diag}_N\{\lambda_{i,s} I\}, \\
\Omega_s &= \mathrm{diag}_N\{\beta_{i,s} I\}, \quad L_s^{(1)} = \mathrm{diag}_N\{\sqrt{l_{i,s}^{(1)}} I\}, \\
L_s^{(2)} &= \mathrm{diag}_N\{\sqrt{l_{i,s}^{(2)}} I\}, \quad T_s^{(1)} = \mathrm{diag}_N\{t_{i,s}^{(1)} I\}, \\
T_s^{(3)} &= \mathrm{diag}_N\{t_{i,s}^{(3)} I\}, \quad \tilde{T}_s^{(2)} = \mathrm{diag}_N\{(t_{i,s}^{(2)} - \bar{t}_{i,s}^{(2)}) I\}.
\end{aligned}
$$

Then, it follows that

$$
\begin{aligned}
x_{s+1} &= (A_s + \Delta \otimes \Gamma) x_s + B_s \omega_s, \\
\bar{z}_s &= T_s^{(1)} \Lambda_s L_s^{(1)} C_s x_s + \tilde{T}_s^{(2)} \Omega_{s-1} L_{s-1}^{(2)} \bar{z}_{s-1} + L_s^{(1)} T_s^{(1)} \upsilon_s + \mu_s^{(1)}, \\
\tilde{z}_s &= \Omega_s L_s^{(1)} L_s^{(2)} T_s^{(1)} T_s^{(3)} \Lambda_s C_s x_s + \Omega_{s-1} \Omega_s L_{s-1}^{(2)} L_s^{(2)} T_s^{(3)} \tilde{T}_s^{(2)} \bar{z}_{s-1} \\
&\quad + \Omega_s L_s^{(1)} L_s^{(2)} T_s^{(1)} T_s^{(3)} \upsilon_s + \Omega_s L_s^{(2)} T_s^{(3)} \mu_s^{(1)} + \mu_s^{(2)}.
\end{aligned}
$$

Define $\bar{x}_s = [x_s^T \ \bar{z}_{s-1}^T]^T$ and $\bar{w}_s = [\omega_s^T \ \upsilon_s^T \ (\mu_s^{(1)})^T \ (\mu_s^{(2)})^T]^T$. The augmented system can be represented as

$$
\bar{x}_{s+1} = \bar{A}_s \bar{x}_s + \bar{B}_s \Theta_s \bar{w}_s, \tag{2.9}
$$

where

$$
\bar{A}_s = \begin{bmatrix} A_s + \Delta \otimes \Gamma & 0 \\ T_s^{(1)} \Lambda_s L_s^{(1)} C_s & \tilde{T}_s^{(2)} \Omega_{s-1} L_{s-1}^{(2)} \end{bmatrix}, \quad \bar{B}_s = \begin{bmatrix} B_s & 0 & 0 & 0 \\ 0 & L_s^{(1)} & I & 0 \end{bmatrix}, \quad \Theta_s = \mathrm{diag}\{I, T_s^{(1)}, I, I\}.
$$

Based on the augmented state $\bar{x}_s$, the signal $\tilde{z}_s$ received by the filter can be rewritten as

$$
\tilde{z}_s = \bar{F}_s \Phi_s \bar{C}_s \bar{x}_s + \bar{D}_s \Psi_s \bar{w}_s, \tag{2.10}
$$

where

$$
\begin{aligned}
\bar{F}_s &= \begin{bmatrix} \Omega_s L_s^{(1)} L_s^{(2)} & \Omega_{s-1} \Omega_s L_{s-1}^{(2)} L_s^{(2)} \end{bmatrix}, \\
\Phi_s &= \mathrm{diag}\{T_s^{(1)} T_s^{(3)} \Lambda_s, T_s^{(3)} \tilde{T}_s^{(2)}\}, \\
\bar{C}_s &= \begin{bmatrix} C_s & 0 \\ 0 & I \end{bmatrix}, \quad \bar{D}_s = \begin{bmatrix} 0 & \Omega_s L_s^{(1)} L_s^{(2)} & \Omega_s L_s^{(2)} & I \end{bmatrix}, \\
\Psi_s &= \mathrm{diag}\{I, T_s^{(1)} T_s^{(3)}, T_s^{(3)}, I\}.
\end{aligned}
$$

The remote filter is constructed as

$$
\hat{x}_{s+1} = K_s \hat{x}_s + G_s \tilde{z}_s, \tag{2.11}
$$

where $\hat{x}_s$ denotes the estimate of the state $\bar{x}_s$, and K_s and G_s are the filter gains to be designed. Furthermore, define the filtering error as $e_s = \bar{x}_s - \hat{x}_s$.

2.4. Design objective

Assumption 2. *The initial state $\bar{x}_0$ and the initial estimate $\hat{x}_0$ are assumed to satisfy*

$$\mathbb{E}\{(\bar{x}_0 - \hat{x}_0)^T \mathfrak{Q}_0^{-1}(\bar{x}_0 - \hat{x}_0)\} \leq 1 - \mathbf{p}, \tag{2.12}$$

where $\mathfrak{Q}_0 > 0$ is a known matrix and $\mathbf{p}$ $(0 < \mathbf{p} < 1)$ is a prescribed probability level.

The design objective of this paper is to, for a given sequence of positive definite matrices $\{\mathfrak{Q}_s\}_{s\in[0,\mathcal{T}]}$, design the filter (2.11) such that

$$\mathbb{P}\{(\bar{x}_s - \hat{x}_s)^T \mathfrak{Q}_s^{-1}(\bar{x}_s - \hat{x}_s) \leq 1\} \geq \mathbf{p}. \tag{2.13}$$

3. Main results

Prior to presenting theorems, we outline the following lemma to facilitate our derivations.

Lemma 1. *[46] Given a matrix $\mathfrak{M} > 0$ and a vector ℓ, we define an ellipsoid $\aleph$ as:*

$$\aleph \triangleq \{\wp | (\wp - \ell)^T \mathfrak{M}(\wp - \ell) \leq 1\}.$$

Here, $\wp \in \mathbb{R}^{n_\wp}$ is a random variable. If the following inequality:

$$\mathbb{E}\{(\wp - \ell)^T \mathfrak{M}(\wp - \ell)\} \leq 1 - \mathbf{p}$$

holds, we have

$$\mathbb{P}\{\wp \in \aleph\} \geq \mathbf{p}.$$

Next, we will derive the sufficient conditions for inequality (2.13) for all $s \in [0, \mathcal{T}]$.

Theorem 1. *Given the filter parameters K_s and G_s, for the prescribed sequence of positive definite matrices $\{\mathfrak{P}_s\}_{s\in[0,\mathcal{T}]}$ and the sequence of positive scalars $\{\tau_{m,s}\}_{s\in[0,\mathcal{T}]}$ $(m = 1, 2, \cdots, 8)$, if there exists a sequence of nonnegative scalars $\{\varpi_{m,s}\}_{s\in[0,\mathcal{T}-1]}$ $(m = 1, 2, \cdots, 5)$ such that the following recursive inequality holds:*

$$\begin{bmatrix} \aleph_s^1 & * \\ \aleph_s^2 & \aleph_s^3 \end{bmatrix} \leq 0, \tag{3.1}$$

where

$$\begin{aligned}
\aleph_s^1 &= \mathrm{diag}\{-1 + \sum_{\iota=1}^{5} \varpi_{\iota,s}, -\varpi_{1,s} I_{Nn_r}, -(\varpi_{2,s}\bar{\mathcal{W}}_s + \varpi_{3,s}\bar{\mathcal{V}}_s + \varpi_{4,s}\bar{\mathcal{U}}_s^{(1)} + \varpi_{5,s}\bar{\mathcal{U}}_s^{(2)})\}, \\
\aleph_s^3 &= \mathrm{diag}\{-\mathfrak{P}_{s+1}, -(1 + \tau_{1,s} + \tau_{2,s} + \tau_{3,s} + \tau_{4,s})^{-1}\tilde{\mathfrak{P}}_{s+1}, -(1 + \tau_{1,s}^{-1} + \tau_{4,s} + \tau_{5,s})^{-1}\mathfrak{P}_{s+1}, \\
&\quad - (1 + \tau_{2,s}^{-1} + \tau_{4,s}^{-1} + \tau_{6,s})^{-1}\mathfrak{P}_{s+1}, -(1 + \tau_{3,s}^{-1} + \tau_{5,s}^{-1} + \tau_{6,s}^{-1})^{-1}\mathfrak{P}_{s+1}\}, \\
\Pi_s &= \begin{bmatrix} (\hat{A}_s - G_s\bar{F}_s\bar{\Phi}_s\bar{C}_s - K_s)\hat{x}_s & (\hat{A}_s - G_s\bar{F}_s\bar{\Phi}_s\bar{C}_s)\Xi_s & \bar{B}_s\bar{\Theta}_s - G_s\bar{D}_s\bar{\Psi}_s \end{bmatrix}, \\
\aleph_s^2 &= \begin{bmatrix} \Pi_s^T & \check{\Pi}_{1,s}^T & \check{\Pi}_{2,s}^T & \check{\Pi}_{3,s}^T & \check{\Pi}_{4,s}^T \end{bmatrix}^T, \quad \check{\Pi}_{1,s}^T = (I_2 \otimes \mathcal{I}) \begin{bmatrix} \check{A}_s\hat{x}_s & \check{A}_s\Xi_s & 0 \end{bmatrix}, \\
\check{\Pi}_{2,s}^T &= \begin{bmatrix} -G_s\bar{F}_s\check{\Phi}_s\bar{C}_s\hat{x}_s & -G_s\bar{F}_s\check{\Phi}_s\bar{C}_s\Xi_s & 0 \end{bmatrix},
\end{aligned}$$

$$
\begin{aligned}
&\check{\Pi}_{3,s}^T = \begin{bmatrix} 0 & 0 & \bar{B}_s\check{\Theta}_s \end{bmatrix}, \ \ \check{\Pi}_{4,s}^T = \begin{bmatrix} 0 & 0 & -G_s\bar{D}_s\check{\Psi}_s \end{bmatrix},\\
&\hat{A}_s = \begin{bmatrix} A_s + \Delta\otimes\Gamma & 0 \\ \bar{T}_s^{(1)}\bar{\Lambda}_s L_s^{(1)} C_s & 0 \end{bmatrix}, \ \ \check{\mathfrak{A}}_s = \begin{bmatrix} \sqrt{\bar{\Lambda}_s\Sigma_s^{(1)} + \bar{\Lambda}_s(I-\bar{\Lambda}_s)(\bar{T}_s^{(1)})^2} L_s^{(1)} C_s & 0 \\ 0 & \Sigma_s^{(2)}\Omega_s L_{s-1}^{(2)} \end{bmatrix},\\
&\bar{\Theta}_s = \mathrm{diag}\{I, \bar{T}_s^{(1)}, I, I\}, \ \ \bar{\Phi}_s = \mathrm{diag}\{\bar{T}_s^{(1)}\bar{T}_s^{(3)}\bar{\Lambda}_s, 0\}, \ \ \bar{\Psi}_s = \mathrm{diag}\{I, \bar{T}_s^{(1)}\bar{T}_s^{(3)}, \bar{T}_s^{(3)}, I\},\\
&\check{\Theta}_s = \mathrm{diag}\{0, \sqrt{\Sigma_s^{(1)}}, 0, 0\}, \ \ \tilde{\mathfrak{P}}_{s+1} = I_2\otimes\mathfrak{P}_{s+1}, \ \ \check{\Phi}_s^{11} = \Sigma_s^{(1)}\Sigma_s^{(3)} + \Sigma_s^{(1)}(\bar{T}_s^{(3)})^2 + \Sigma_s^{(3)}(\bar{T}_s^{(1)})^2,\\
&\check{\Phi}_s = \mathrm{diag}\{\sqrt{\check{\Phi}_s^{11}\bar{\Lambda}_s + (\bar{T}_s^{(1)})^2(\bar{T}_s^{(3)})^2\bar{\Lambda}_s(I-\bar{\Lambda}_s)}, \sqrt{\Sigma_s^{(3)} + (\bar{T}_s^{(3)})^2\Sigma_s^{(2)}}\}, \ \ \mathcal{I} = \begin{bmatrix} 0 \\ I \end{bmatrix},\\
&\check{\Psi}_s = \mathrm{diag}\{0, \sqrt{\Sigma_s^{(1)}\Sigma_s^{(3)} + \Sigma_s^{(1)}(\bar{T}_s^{(3)})^2 + \Sigma_s^{(3)}(\bar{T}_s^{(1)})^2}, \sqrt{\Sigma_s^{(3)}}, 0\},\\
&\bar{\mathcal{W}}_s = \mathrm{diag}\{\mathcal{W}_s^{-1}, 0, 0, 0\}, \ \ \bar{\mathcal{V}}_s = \mathrm{diag}\{0, \mathcal{V}_s^{-1}, 0, 0\}, \ \ \bar{\mathcal{U}}_s^{(1)} = \mathrm{diag}\{0, 0, (\mathcal{U}_s^{(1)})^{-1}, 0\},\\
&T_s^{(1)} = \mathrm{diag}\{\bar{t}_{i,s}^{(1)} I\}, \ \ \bar{\mathcal{U}}_s^{(2)} = \mathrm{diag}\{0, 0, 0, (\mathcal{U}_s^{(2)})^{-1}\}, \ \ T_s^{(2)} = \mathrm{diag}\{\bar{t}_{i,s}^{(2)} I\},\\
&T_s^{(3)} = \mathrm{diag}\{\bar{t}_{i,s}^{(3)} I\}, \ \ \Sigma_s^{(1)} = \mathrm{diag}\{\sigma_{i,s}^{(1)} I\}, \ \ \Sigma_s^{(2)} = \mathrm{diag}\{\sigma_{i,s}^{(2)} I\}, \ \ \Sigma_s^{(3)} = \mathrm{diag}\{\sigma_{i,s}^{(3)} I\},
\end{aligned}
$$

then the following inequality holds:

$$
\mathbb{E}\{(\bar{x}_{s+1} - \hat{x}_{s+1})^T\mathfrak{P}_{s+1}^{-1}(\bar{x}_{s+1} - \hat{x}_{s+1})\} \leq 1. \tag{3.2}
$$

Proof. The proof is carried out by mathematical induction. First, let $\mathfrak{P}_0 \triangleq (1-\mathbf{p})\mathfrak{Q}_0$.

From Assumption 2, it follows that

$$
\mathbb{E}\{(\bar{x}_0 - \hat{x}_0)^T\mathfrak{P}_0^{-1}(\bar{x}_0 - \hat{x}_0)\} \leq 1. \tag{3.3}
$$

Next, assume that $\mathbb{E}\{(\bar{x}_s - \hat{x}_s)^T\mathfrak{P}_s^{-1}(\bar{x}_s - \hat{x}_s)\} \leq 1$. If (3.1) is satisfied, then (3.2) follows. There exists a vector $r_s \in \mathbb{R}^{Nn_r}$ satisfying $\mathbb{E}\{r_s^T r_s\} \leq 1$ such that

$$
e_s = \Xi_s r_s, \tag{3.4}
$$

where Ξ_s satisfies $\mathfrak{P}_s = \Xi_s\Xi_s^T$.

From (2.9)–(2.11), one has

$$
\begin{aligned}
e_{s+1} = {} & (\hat{A}_s - G_s\bar{F}_s\bar{\Phi}_s\bar{C}_s - K_s)\hat{x}_s + (\hat{A}_s - G_s\bar{F}_s\bar{\Phi}_s\bar{C}_s)e_s + (\bar{B}_s\bar{\Theta}_s - G_s\bar{D}_s\bar{\Psi}_s)\bar{w}_s\\
& + \tilde{A}_s\hat{x}_s + \tilde{A}_s e_s - G_s\bar{F}_s\tilde{\Phi}_s\bar{C}_s\hat{x}_s - G_s\bar{F}_s\tilde{\Phi}_s\bar{C}_s e_s + \bar{B}_s\tilde{\Theta}_s\bar{w}_s - G_s\bar{D}_s\tilde{\Psi}_s\bar{w}_s,
\end{aligned} \tag{3.5}
$$

where

$$
\begin{aligned}
&\tilde{A}_s = \begin{bmatrix} 0 & 0 \\ (T_s^{(1)}\Lambda_s - \bar{T}_s^{(1)}\bar{\Lambda}_s)L_s^{(1)}C_s & \tilde{T}_s^{(2)}\Omega_{s-1}L_{s-1}^{(2)} \end{bmatrix},\\
&\tilde{\Theta}_s = \mathrm{diag}\{0, T_s^{(1)} - \bar{T}_s^{(1)}, 0, 0\},\\
&\tilde{\Phi}_s = \mathrm{diag}\{T_s^{(1)}T_s^{(3)}\Lambda_s - \bar{T}_s^{(1)}\bar{T}_s^{(3)}\bar{\Lambda}_s, T_s^{(3)}\tilde{T}_s^{(2)}\},\\
&\tilde{\Psi}_s = \mathrm{diag}\{0, T_s^{(1)}T_s^{(3)} - \bar{T}_s^{(1)}\bar{T}_s^{(3)}, T_s^{(3)} - \bar{T}_s^{(3)}, 0\}.
\end{aligned}
$$

Define $\phi_s = [1 \ r_s^T \ \bar{w}_s^T]^T$. Then, (3.5) can be rewritten as

$$
e_{s+1} = \Pi_s\phi_s + \tilde{\Pi}_{1,s}\phi_s + \tilde{\Pi}_{2,s}\phi_s + \tilde{\Pi}_{3,s}\phi_s + \tilde{\Pi}_{4,s}\phi_s, \tag{3.6}
$$

where

$$\tilde{\Pi}_{1,s} = \begin{bmatrix} \tilde{A}_s \hat{x}_s & \tilde{A}_s \Xi_s & 0 \end{bmatrix}, \ \tilde{\Pi}_{3,s} = \begin{bmatrix} 0 & 0 & \bar{B}_s \tilde{\Theta}_s \end{bmatrix},$$
$$\tilde{\Pi}_{2,s} = \begin{bmatrix} -G_s \bar{F}_s \tilde{\Phi}_s \bar{C}_s \hat{x}_s & -G_s \bar{F}_s \tilde{\Phi}_s \bar{C}_s \Xi_s & 0 \end{bmatrix},$$
$$\tilde{\Pi}_{4,s} = \begin{bmatrix} 0 & 0 & -G_s \bar{D}_s \tilde{\Psi}_s \end{bmatrix}.$$

According to (3.6), we have

$$\begin{aligned}
&\mathbb{E}\{e_{s+1}^T \mathfrak{P}_{s+1}^{-1} e_{s+1}\} \\
&= \mathbb{E}\{\phi_s^T (\Pi_s + \tilde{\Pi}_{1,s} + \tilde{\Pi}_{2,s} + \tilde{\Pi}_{3,s} + \tilde{\Pi}_{4,s})^T \mathfrak{P}_{s+1}^{-1} (\Pi_s + \tilde{\Pi}_{1,s} + \tilde{\Pi}_{2,s} + \tilde{\Pi}_{3,s} + \tilde{\Pi}_{4,s}) \phi_s\} \\
&= \mathbb{E}\{\phi_s^T \Pi_s^T \mathfrak{P}_{s+1}^{-1} \Pi_s \phi_s + \phi_s^T \tilde{\Pi}_{1,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{1,s} \phi_s + \phi_s^T \tilde{\Pi}_{2,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{2,s} \phi_s + \phi_s^T \tilde{\Pi}_{3,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{3,s} \phi_s \\
&\quad + \phi_s^T \tilde{\Pi}_{4,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{4,s} \phi_s + 2\phi_s^T \tilde{\Pi}_{1,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{2,s} \phi_s + 2\phi_s^T \tilde{\Pi}_{1,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{3,s} \phi_s + 2\phi_s^T \tilde{\Pi}_{1,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{4,s} \phi_s \\
&\quad + 2\phi_s^T \tilde{\Pi}_{2,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{3,s} \phi_s + 2\phi_s^T \tilde{\Pi}_{2,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{4,s} \phi_s + 2\phi_s^T \tilde{\Pi}_{3,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{4,s} \phi_s\} \\
&\leq \mathbb{E}\{\phi_s^T \Pi_s^T \mathfrak{P}_{s+1}^{-1} \Pi_s \phi_s\} + (1 + \tau_{1,s} + \tau_{2,s} + \tau_{3,s} + \tau_{4,s}) \mathbb{E}\{\phi_s^T \tilde{\Pi}_{1,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{1,s} \phi_s\} \\
&\quad + (1 + \tau_{1,s}^{-1} + \tau_{4,s} + \tau_{5,s}) \mathbb{E}\{\phi_s^T \tilde{\Pi}_{2,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{2,s} \phi_s\} + (1 + \tau_{2,s}^{-1} + \tau_{4,s}^{-1} + \tau_{6,s}) \mathbb{E}\{\phi_s^T \tilde{\Pi}_{3,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{3,s} \phi_s\} \\
&\quad + (1 + \tau_{3,s}^{-1} + \tau_{5,s}^{-1} + \tau_{6,s}^{-1}) \mathbb{E}\{\phi_s^T \tilde{\Pi}_{4,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{4,s} \phi_s\},
\end{aligned} \tag{3.7}$$

where $\tau_{m,s}$ $(m = 1, 2, \cdots, 8)$ are given positive scalars.

Based on the statistical properties of the random variable $\tilde{\mathfrak{A}}_s$, it can be derived that

$$\mathbb{E}\{\phi_s^T \tilde{\Pi}_{1,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{1,s} \phi_s\} \leq \phi_s^T \check{\Pi}_{1,s}^T \tilde{\mathfrak{P}}_{s+1}^{-1} \check{\Pi}_{1,s} \phi_s. \tag{3.8}$$

Considering measurement missing and random channel parameters, from the definitions of $\tilde{\Phi}_s$, $\tilde{\Theta}_s$, and $\tilde{\Psi}_s$, one obtains

$$\begin{aligned}
\mathbb{E}\{\phi_s^T \tilde{\Pi}_{2,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{2,s} \phi_s\} &= \phi_s^T \check{\Pi}_{2,s}^T \mathfrak{P}_{s+1}^{-1} \check{\Pi}_{2,s} \phi_s, \\
\mathbb{E}\{\phi_s^T \tilde{\Pi}_{3,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{3,s} \phi_s\} &= \phi_s^T \check{\Pi}_{3,s}^T \mathfrak{P}_{s+1}^{-1} \check{\Pi}_{3,s} \phi_s, \\
\mathbb{E}\{\phi_s^T \tilde{\Pi}_{4,s}^T \mathfrak{P}_{s+1}^{-1} \tilde{\Pi}_{4,s} \phi_s\} &= \phi_s^T \check{\Pi}_{4,s}^T \mathfrak{P}_{s+1}^{-1} \check{\Pi}_{4,s} \phi_s.
\end{aligned} \tag{3.9}$$

Combining (3.7) and (3.9), it follows that

$$\mathbb{E}\{e_{s+1}^T \mathfrak{P}_{s+1}^{-1} e_{s+1}\} \leq \phi_s^T \Upsilon_s \phi_s, \tag{3.10}$$

where

$$\begin{aligned}
\Upsilon_s = {} & \Pi_s^T \mathfrak{P}_{s+1}^{-1} \Pi_s + (1 + \tau_{1,s} + \tau_{2,s} + \tau_{3,s} + \tau_{4,s}) \check{\Pi}_{1,s}^T \tilde{\mathfrak{P}}_{s+1}^{-1} \check{\Pi}_{1,s} + (1 + \tau_{2,s}^{-1} + \tau_{4,s}^{-1} + \tau_{6,s}) \check{\Pi}_{2,s}^T \mathfrak{P}_{s+1}^{-1} \check{\Pi}_{2,s} \\
& + (1 + \tau_{2,s}^{-1} + \tau_{4,s}^{-1} + \tau_{6,s}) \check{\Pi}_{3,s}^T \mathfrak{P}_{s+1}^{-1} \check{\Pi}_{3,s} + (1 + \tau_{3,s}^{-1} + \tau_{5,s}^{-1} + \tau_{6,s}^{-1}) \check{\Pi}_{4,s}^T \mathfrak{P}_{s+1}^{-1} \check{\Pi}_{4,s}.
\end{aligned}$$

Note that

$$E\{r_s^T r_s\} \leq 1, \tag{3.11}$$
$$\omega_{i,s}^T \mathcal{W}_{i,s}^{-1} \omega_{i,s} \leq 1, \tag{3.12}$$
$$\upsilon_{i,s}^T \mathcal{V}_{i,s}^{-1} \upsilon_{i,s} \leq 1, \tag{3.13}$$

$$(\mu_{i,s}^{(1)})^T(\mathcal{U}_{i,s}^{(1)})^{-1}\mu_{i,s}^{(1)} \le 1, \tag{3.14}$$
$$(\mu_{i,s}^{(2)})^T(\mathcal{U}_{i,s}^{(2)})^{-1}\mu_{i,s}^{(2)} \le 1. \tag{3.15}$$

Furthermore, (3.12)–(3.15) can be written as

$$\begin{aligned}
\mathbb{E}\{\phi_s^T \mathrm{diag}\{-1, I_{Nn_r}, 0\}\phi_s\} &\le 0,\\
\phi_s^T \mathrm{diag}\{-1, 0, \bar{\mathcal{W}}_s\}\phi_s &\le 0,\\
\phi_s^T \mathrm{diag}\{-1, 0, \bar{\mathcal{V}}_s\}\phi_s &\le 0,\\
\phi_s^T \mathrm{diag}\{-1, 0, \bar{\mathcal{U}}_s^{(1)}\}\phi_s &\le 0,\\
\phi_s^T \mathrm{diag}\{-1, 0, \bar{\mathcal{U}}_s^{(2)}\}\phi_s &\le 0.
\end{aligned} \tag{3.16}$$

If there exist constants $\varpi_{m,s}$ ($m = 1, 2, 3, 4, 5$) such that

$$\begin{aligned}
&\Upsilon_s - \mathrm{diag}\{1, 0, 0\} - \varpi_{1,s}\mathrm{diag}\{-1, I_{Nn_r}, 0\} - \varpi_{2,s}\mathrm{diag}\{-1, 0, \bar{\mathcal{W}}_s\}\\
&- \varpi_{3,s}\mathrm{diag}\{-1, 0, \bar{\mathcal{V}}_s\} - \varpi_{4,s}\mathrm{diag}\{-1, 0, \bar{\mathcal{U}}_s^{(1)}\} - \varpi_{5,s}\mathrm{diag}\{-1, 0, \bar{\mathcal{U}}_s^{(2)}\} \le 0,
\end{aligned} \tag{3.17}$$

then (3.2) holds. Moreover, (3.17) holds if and only if (3.1) holds. The proof is now complete. □

Remark 4. *Although Theorem 1 is formulated in a set-membership form, it plays a key role in the analysis of the chance-constrained objective. In particular, according to Lemma 1, the probabilistic ellipsoidal constraint can be analyzed once condition* (3.2) *is ensured. Therefore, Theorem 1 is introduced to derive sufficient conditions for establishing* (3.2)*, thereby providing the bridge between the set-membership characterization and the subsequent chance-constrained performance analysis.*

Now, we are ready to present the design method of the desired filter.

Theorem 2. *For the prescribed scalar* $\mathbf{p}$*, the sequence of positive definite matrices* $\{\mathfrak{Q}_s\}_{s\in[0,\mathcal{T}]}$ *and the sequence of positive scalars* $\{\tau_{m,s}\}_{s\in[0,\mathcal{T}]}$ ($m = 1, 2, \cdots, 8$)*, if there exist a sequence of nonnegative scalars* $\{\varpi_{m,s}\}_{s\in[0,\mathcal{T}-1]}$ ($m = 1, 2, \cdots, 5$) *and matrices* $\{K_s, G_s\}_{s\in[0,\mathcal{T}-1]}$ *such that the following recursive inequality holds:*

$$\begin{bmatrix} \aleph_s^1 & * \\ \aleph_s^2 & \tilde{\aleph}_s^3 \end{bmatrix} \le 0, \tag{3.18}$$

where

$$\begin{aligned}
\tilde{\aleph}_s^3 = \mathrm{diag}\{&-(1-\mathbf{p})\mathfrak{Q}_{s+1}, -(1+\tau_{1,s}+\tau_{2,s}+\tau_{3,s}+\tau_{4,s})^{-1}(1-\mathbf{p})\tilde{\mathfrak{Q}}_{s+1},\\
&-(1+\tau_{1,s}^{-1}+\tau_{4,s}+\tau_{5,s})^{-1}(1-\mathbf{p})\mathfrak{Q}_{s+1},\\
&-(1+\tau_{2,s}^{-1}+\tau_{4,s}^{-1}+\tau_{6,s})^{-1}(1-\mathbf{p})\mathfrak{Q}_{s+1},\\
&-(1+\tau_{3,s}^{-1}+\tau_{5,s}^{-1}+\tau_{6,s}^{-1})^{-1}(1-\mathbf{p})\mathfrak{Q}_{s+1}\},
\end{aligned}$$

then the design objective (2.13) *is satisfied.*

Proof. Letting $\mathfrak{P}_s \triangleq (1-\mathbf{p})\mathfrak{Q}_s$, by means of Lemma 1, (3.18) follows directly from (3.1). The proof is complete. □

Algorithm 1 Computational Algorithm for K_s and G_s.

1: Set $s = 0$ and the maximum computation step $\mathcal{T}$. Set parameters $\{\mathfrak{Q}_s, \mathbf{p}\}_{s\in[0,\mathcal{T}]}$. Then, utilize $\mathfrak{P}_s \triangleq (1-\mathbf{p})\mathfrak{Q}_s$ and appropriately factorize $\{\mathfrak{P}_s\}$ to derive the sequence of matrices $\{\Xi_s\}$. Choose the initial values of $\bar{x}_0$ and $\hat{x}_0$ such that they satisfy (2.12).
2: Solve (3.19) for K_s and G_s.
3: Determine e_{s+1} using (3.5).
4: Assign $s = s+1$. If $s > \mathcal{T}$, exit. Otherwise, go to **2**.

It should be emphasized that the main objective of this paper is to establish recursive sufficient conditions under which the chance-constrained ellipsoidal filtering requirement in (2.13) can be guaranteed. Specifically, Theorems 1 and 2 provide the analytical foundation of the filter design by deriving the corresponding recursive matrix inequalities, while Algorithm 1 is used as a computational procedure for recursively solving these conditions and obtaining the filtering gains.

Two optimization problems (OPs) will be presented. The first seeks to get locally optimal filtering performance by minimizing $\mathfrak{Q}_s$ (in the sense of matrix trace). The second aims to maximize $\mathbf{p}$ at each time step, ensuring a local threshold probability that represents the minimum probability for confining errors within the desired ellipsoid.

Define

$$\mathfrak{F}_s \triangleq \{K_s, G_s, \varpi_{m,s}\}.$$

OP1: For a given probability $\mathbf{p}$, minimize the cost function $\mathfrak{S}_s = \mathrm{tr}(\mathfrak{Q}_s)$.

Corollary 1. *Given the probability* $\mathbf{p}$*, based on Theorem 2, if the following optimization problem is feasible:*

$$\min_{\mathfrak{F}_s, \mathfrak{Q}_{s+1}} \mathfrak{S}_{s+1} \quad s.t.\ (3.18), \tag{3.19}$$

then a sequence of minimizing matrices $\mathfrak{S}_s$ *(in the sense of matrix trace) can be guaranteed.*

Next, consider a time-varying probability, denoted by $\mathbf{p}_s$ at time s. The following optimization problem is proposed.

OP2: For a given $\{\mathfrak{Q}_s\}_{s\in[0,\mathcal{T}]}$, maximize $\mathbf{p}_s$.

Corollary 2. *Based on Theorem 2, if there exists* $\{\mathfrak{Q}_s\}_{s\in[0,\mathcal{T}]}$ *such that the following optimization problem is feasible:*

$$\min_{\mathfrak{F}_s} -\mathbf{p}_s \tag{3.20}$$

$$s.t. \begin{cases} 0 < \mathbf{p}_s < 1, \\ \begin{bmatrix} \aleph_s^1 & * \\ \aleph_s^2 & \hat{\aleph}_s^3 \end{bmatrix} \le 0, \end{cases} \tag{3.21}$$

where

$$\hat{\aleph}_s^3 = \mathrm{diag}\{-(1-\mathbf{p}_s)\mathfrak{Q}_{s+1}, -(1+\tau_{1,s}+\tau_{2,s}+\tau_{3,s}+\tau_{4,s})^{-1}(1-\mathbf{p}_s)\tilde{\mathfrak{Q}}_{s+1},$$

$$
\begin{aligned}
&-(1+\tau_{1,s}^{-1}+\tau_{4,s}+\tau_{5,s})^{-1}(1-\mathbf{p}_s)\mathfrak{Q}_{s+1},\\
&-(1+\tau_{2,s}^{-1}+\tau_{4,s}^{-1}+\tau_{6,s})^{-1}(1-\mathbf{p}_s)\mathfrak{Q}_{s+1},\\
&-(1+\tau_{3,s}^{-1}+\tau_{5,s}^{-1}+\tau_{6,s}^{-1})^{-1}(1-\mathbf{p}_s)\mathfrak{Q}_{s+1}\},
\end{aligned}
$$

then the probability level $\mathbf{p}_s$ *can be maximized.*

Remark 5. *Compared with existing studies on set-membership filtering for networked systems, the main contributions of this paper can be summarized from the following aspects.*

(1) A unified modeling framework is established to simultaneously incorporate missing measurements, unknown-but-bounded noises, and full-duplex relay-induced self-interference, which has rarely been addressed in an integrated manner in the existing literature.

(2) A novel augmented system representation is constructed to capture the dynamic coupling between relay transmission and network evolution, enabling the derivation of recursive filtering conditions in a tractable form.

(3) A chance-constrained set-membership filtering scheme is developed, which relaxes the traditional deterministic boundedness requirement by introducing probabilistic guarantees, thereby reducing conservatism while preserving robustness.

(4) Recursive LMI-based conditions are derived to ensure the probabilistic boundedness of the filtering error, and two optimization-based design schemes are further proposed to improve filtering performance.

4. Simulation

Consider the CN (2.1) with the following system parameters:

$$
\begin{aligned}
&N=5,\ \ n_x=2,\ \ \Gamma=\begin{bmatrix}1.35 & 0\\ 0 & 1.35\end{bmatrix},\\
&\Delta=\begin{bmatrix}-0.8 & 0.2 & 0.3 & 0.1 & 0.2\\ 0.2 & -0.8 & 0.3 & 0.1 & 0.2\\ 0.1 & 0.2 & -0.8 & 0.2 & 0.3\\ 0.3 & 0.3 & 0.1 & -0.8 & 0.1\\ 0.2 & 0.2 & 0.2 & 0.2 & -0.8\end{bmatrix},\\
&A_{1,s}=\begin{bmatrix}0.3 & 0\\ 0 & 0.2+0.01\cos(s)\end{bmatrix},\ \ A_{2,s}=\begin{bmatrix}0.3+0.02\cos(s) & 0\\ 0 & 0.6\end{bmatrix},\\
&A_{3,s}=\begin{bmatrix}0.6 & 0\\ 0 & 0.4+0.01\cos(0.5s)\end{bmatrix},\ \ A_{4,s}=\begin{bmatrix}0.6+0.01\cos(s) & 0\\ 0 & 0.3\end{bmatrix},\\
&A_{5,s}=\begin{bmatrix}0.3 & 0\\ 0 & 0.5+0.01\cos(0.5s)\end{bmatrix},\ \ B_{1,s}=\begin{bmatrix}0.3\\ 0.3+0.01\cos(s)\end{bmatrix},\\
&B_{2,s}=\begin{bmatrix}0.3+0.02\cos(s)\\ 0.4\end{bmatrix},\ \ B_{3,s}=\begin{bmatrix}0.4\\ 0.4+0.01\cos(0.5s)\end{bmatrix},\\
&B_{4,s}=\begin{bmatrix}0.3+0.01\cos(s)\\ 0.3\end{bmatrix},\ \ B_{5,s}=\begin{bmatrix}0.4\\ 0.4+0.01\cos(0.5s)\end{bmatrix},\\
&C_{1,s}=\begin{bmatrix}0.2+0.02\sin(s) & 0.2+0.02\sin(s)\end{bmatrix},
\end{aligned}
$$

$$C_{2,s} = \begin{bmatrix} 0.25 + 0.01\cos(s) & 0.2 + 0.02\cos(0.5s) \end{bmatrix},$$
$$C_{3,s} = \begin{bmatrix} 0.25 + 0.01\cos(s) & 0.25 + 0.02\sin(s) \end{bmatrix},$$
$$C_{4,s} = \begin{bmatrix} 0.3 + 0.01\sin(s) & 0.2 + 0.02\cos(0.5s) \end{bmatrix},$$
$$C_{5,s} = \begin{bmatrix} 0.3 + 0.01\cos(s) & 0.2 + 0.02\sin(s) \end{bmatrix}.$$

The process noise $\omega_{i,s}$, measurement noise $\upsilon_{i,s}$, sensor-to-relay transmission noise $\mu_{i,s}^{(1)}$, and relay-to-filter transmission noise $\mu_{i,s}^{(2)}$ are set as

$$\begin{aligned}
&\omega_{1,s} = 0.15\sin(2s), \quad \omega_{2,s} = 0.15\sin(3s),\\
&\omega_{3,s} = 0.2\cos(3s), \quad \omega_{4,s} = 0.15\cos(3s),\\
&\omega_{5,s} = 0.15\cos(s), \quad \upsilon_{1,s} = 0.2\sin(s),\\
&\upsilon_{2,s} = 0.25\cos(s), \quad \upsilon_{3,s} = 0.2\cos(s),\\
&\upsilon_{4,s} = 0.2\sin(3s), \quad \upsilon_{5,s} = 0.2\cos(3s),\\
&\mu_{1,s}^{(1)} = 0.1\sin(s), \quad \mu_{2,s}^{(1)} = 0.25\cos(s),\\
&\mu_{3,s}^{(1)} = 0.1\sin(s), \quad \mu_{4,s}^{(1)} = 0.1\cos(2s),\\
&\mu_{5,s}^{(1)} = 0.15\cos(s), \quad \mu_{1,s}^{(2)} = 0.15\sin(2s),\\
&\mu_{2,s}^{(2)} = 0.25\cos(s), \quad \mu_{3,s}^{(2)} = 0.1\sin(s),\\
&\mu_{4,s}^{(2)} = 0.1\cos(2s), \quad \mu_{5,s}^{(2)} = 0.25\sin(s).
\end{aligned}$$

Other parameters are chosen as $\mathcal{W}_{i,s} = \mathcal{V}_{i,s} = \mathcal{U}_{i,s}^{(1)} = \mathcal{U}_{i,s}^{(2)} = 0.5I_5$, $\mathbf{p} = 0.75$, and $\mathfrak{Q}_0 = 100I_{15}$. The probability of missing measurements is set as $\bar{\lambda}_i = 0.9$. The amplification factor, transmission power, and channel coefficients in different communication links are set to $\beta_{i,s} = 1$, $l_{i,s}^{(1)} = l_{i,s}^{(2)} = l_{i,s}^{(3)} = 1$, $\bar{t}_{i,s}^{(1)} = \bar{t}_{i,s}^{(2)} = \bar{t}_{i,s}^{(3)} = 0.95$, $\sigma_{i,s}^{(1)} = \sigma_{i,s}^{(2)} = \sigma_{i,s}^{(3)} = 0.01$. Moreover, the initial values $x_{i,0}$ and $\hat{x}_{i,0}$ are selected as

$$x_{1,0} = \begin{bmatrix} 0.5030 \\ 0.7090 \end{bmatrix}, \quad \hat{x}_{1,0} = \begin{bmatrix} 1.1049 \\ -0.5249 \end{bmatrix},$$
$$x_{2,0} = \begin{bmatrix} 0.5880 \\ 0.7810 \end{bmatrix}, \quad \hat{x}_{2,0} = \begin{bmatrix} 1.0809 \\ -0.5009 \end{bmatrix},$$
$$x_{3,0} = \begin{bmatrix} 0.5855 \\ 0.7615 \end{bmatrix}, \quad \hat{x}_{3,0} = \begin{bmatrix} 1.1089 \\ -0.4939 \end{bmatrix},$$
$$x_{4,0} = \begin{bmatrix} 0.6470 \\ 0.7855 \end{bmatrix}, \quad \hat{x}_{4,0} = \begin{bmatrix} 1.1009 \\ -0.5034 \end{bmatrix},$$
$$x_{5,0} = \begin{bmatrix} 0.5750 \\ 0.7720 \end{bmatrix}, \quad \hat{x}_{5,0} = \begin{bmatrix} 1.1546 \\ -0.5122 \end{bmatrix}.$$

The simulation results are shown in Figures 2–7. Specifically, Figures 2–6 depict the states $x_{i,s}$ ($i = 1, 2, 3, 4, 5$) and their corresponding estimates, and Figure 7 presents the measurement-missing situation of each node. It can be observed that the designed filter is capable of tracking the system states, which demonstrates the effectiveness of the proposed filtering method.

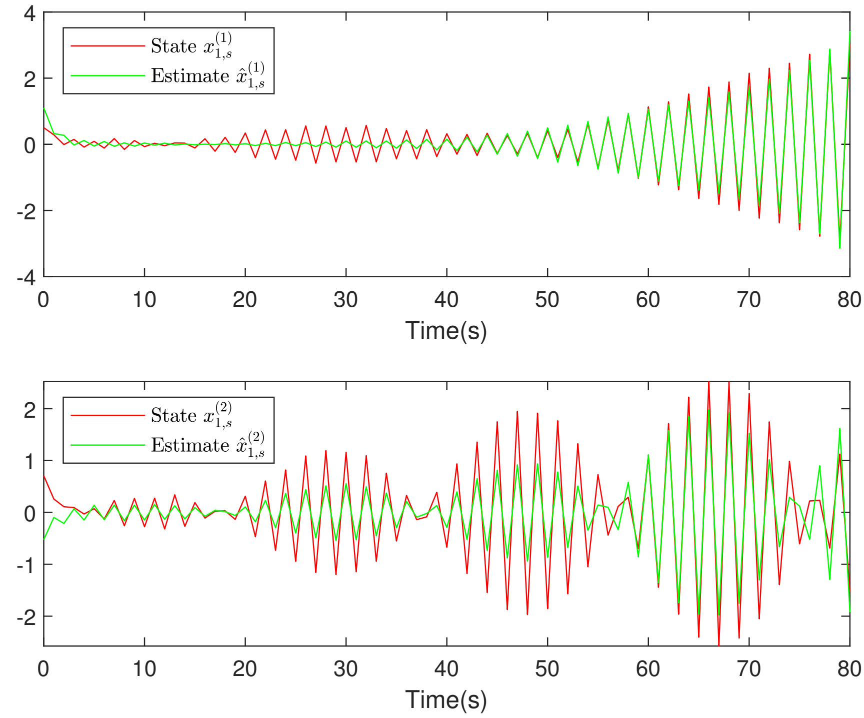

Figure 2. State of node 1 and its estimation.

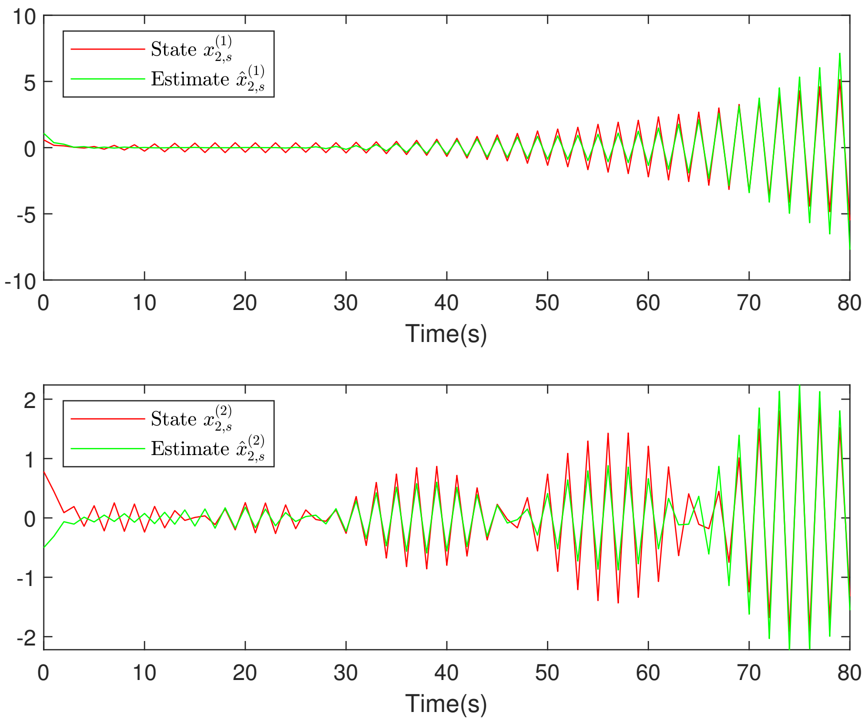

Figure 3. State of node 2 and its estimation.

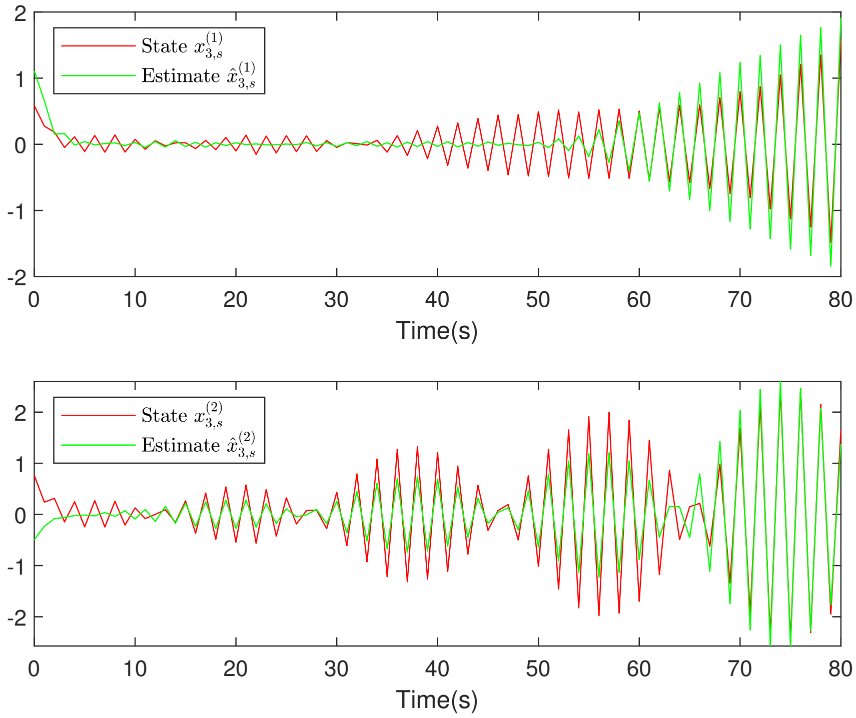

Figure 4. State of node 3 and its estimation.

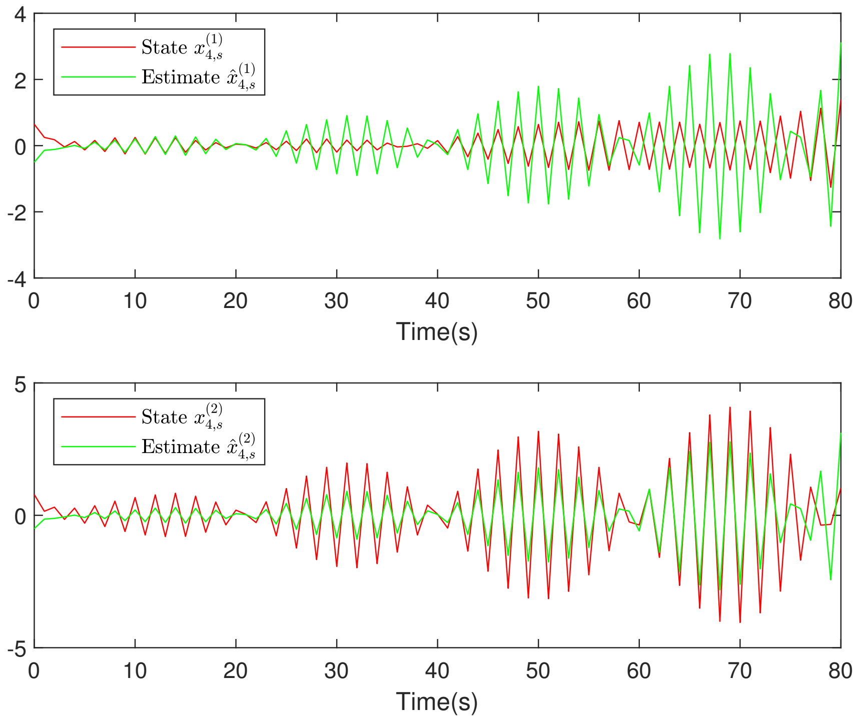

Figure 5. State of node 4 and its estimation.

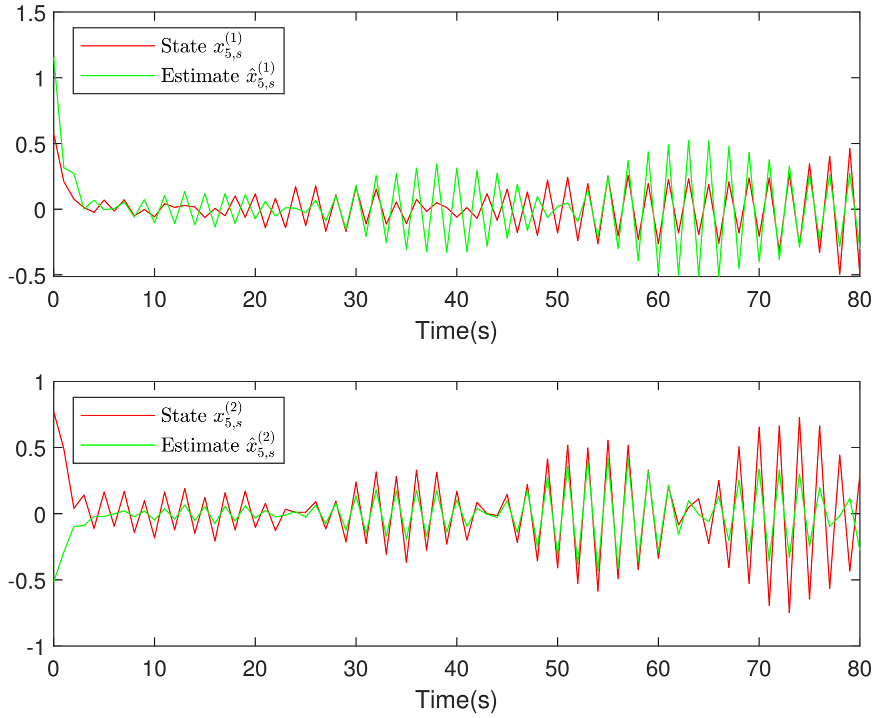

Figure 6. State of node 5 and its estimation.

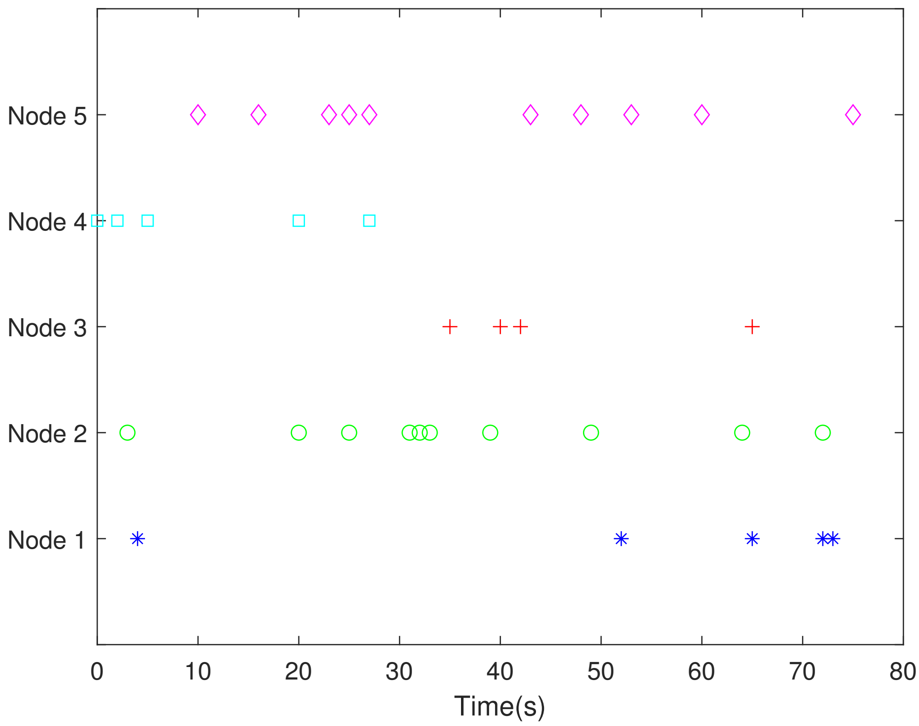

Figure 7. Measurement-missing situation of each node.

5. Conclusions

This paper has investigated the problem of chance-constrained set-membership filtering for CNs operating over full-duplex relay networks and subject to missing measurements. A unified probabilistic filtering framework has been developed by modeling missing measurements via Bernoulli random variables and capturing long-distance transmission uncertainties through stochastic channel parameters, while also accounting for relay-induced interference using a compensation mechanism. Recursive linear matrix inequalities have been derived to ensure that the filtering error remains probabilistically bounded within a specified ellipsoidal set. The corresponding filter gains can be recursively computed from these conditions. Furthermore, two optimization-based design schemes have been formulated to improve filtering performance under probabilistic constraints. Finally, a numerical simulation has been provided to verify the applicability of the proposed filtering scheme. Our future work will focus on extending the main results obtained in this article to nonlinear systems with privacy-preserving protocols [47, 48] and to nonlinear systems under half-duplex relay protocols [49, 50].

Author contributions

C. Hu: Conceptualization, methodology, software, writing–original draft; M. Shi: Conceptualization, methodology, writing–original draft, writing–review and editing; L. Ma: Conceptualization, investigation, supervision; J. Guo: Funding acquisition, resources, project administration. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there are no conflicts of interest.

References

1. A. A. Amin, A. Mubarak, H. U. Manzoor, Design of intelligent vehicular and sensor communication network: A comprehensive survey, *Syst. Sci. Control Eng.*, **13** (2025), 2529187. https://doi.org/10.1080/21642583.2025.2529187

2. Y. Liang, Y. Chen, D. Pan, H. Song, Privacy-preserving distributed entropy filtering for microgrids with innovation decomposition, *Int. J. Netw. Dyn. Intell.*, **4** (2025), 100004. https://doi.org/10.53941/ijndi.2025.100004

3. L. Sun, W. An, Y. Chen, P. Zhao, D. Ding, An overview of distributed economic dispatch of microgrids: Advances and challenges, *Syst. Sci. Control Eng.*, **13** (2025), 2467077. https://doi.org/10.1080/21642583.2025.2467077

4. L. Ma, Z. Wang, H. Liu, F. E. Alsaadi, F. E. Alsaadi, Neural-network-based filtering for a general class of nonlinear systems under dynamically bounded innovations over sensor networks, *IEEE Trans. Netw. Sci. Eng.*, **9** (2022), 1395–1408. https://doi.org/10.1109/TNSE.2022.3144484

5. Q. Yang, Y. Zhong, C. Liu, Y. Xu, T. Huang, L. Rutkowski, Partial-nodes-based estimation for complex networks with random inner coupling, *IEEE T. Syst. Man, Cy.-S.*, **56** (2026), 481–491. https://doi.org/10.1109/TSMC.2025.3626136

6. W. Xing, P. Shi, R. K. Agarwal, Y. Zhao, A survey on global pinning synchronization of complex networks, *J. Franklin I.*, **356** (2019), 3590–3611. https://doi.org/10.1016/j.jfranklin.2019.02.021

7. M. Shi, C. Gao, L. Ma, X. Yi, Distributed state estimation for complex networks under decode-and-forward relays: Handling transmission power constraints, *IEEE T. Ind. Inform.*, 2026, in press. https://doi.org/10.1109/TII.2026.3652805

8. F. Han, Z. Wang, H. Liu, H. Dong, G. Lu, Local design of distributed state estimators for linear discrete time-varying systems over binary sensor networks: A set-membership approach, *IEEE T. Syst. Man Cy.-S.*, **54** (2024), 5641–5654. https://doi.org/10.1109/TSMC.2024.3409611

9. G. Li, Z. Wang, X. Bai, Z. Zhao, H. Dong, Event-triggered set-membership filtering for active power distribution systems under fading channels: A zonotope-based approach, *IEEE T. Autom. Sci. Eng.*, **22** (2024), 1139–1151. https://doi.org/10.1109/TASE.2024.3360600

10. K. Zhu, Z. Wang, D. Ding, H. Dong, G. Wei, Encryption-decryption-based set-membership filtering for two-dimensional systems: On security and boundedness, *Automatica*, **173** (2025), 112091. https://doi.org/10.1016/j.automatica.2024.112091

11. C. Hu, S. Ding, Y. Jing, X. Xie, Event-based distributed set-membership estimation for complex networks: A coding-decoding method, *IEEE T. Netw. Sci. Eng.*, **11** (2024), 1619–1630. https://doi.org/10.1109/TNSE.2023.3326611

12. M. Luo, Y. Zhang, H. Yan, F. Yang, Ellipsoid-density-based set-membership global estimation for complex networked systems with absolute and relative measurements, *J. Franklin I.*, **362** (2025), 108007. https://doi.org/10.1016/j.jfranklin.2025.108007

13. E. Tian, Z. Wang, L. Zou, D. Yue, Probabilistic-constrained filtering for a class of nonlinear systems with improved static event-triggered communication, *Int. J. Robust Nonlin.*, **29** (2019), 1484–1498. https://doi.org/10.1002/rnc.4447

14. G. Wei, S. Liu, Y. Song, Y. Liu, Probability-guaranteed set-membership filtering for systems with incomplete measurements, *Automatica*, **60** (2015), 12–16. https://doi.org/10.1016/j.automatica.2015.06.037

15. M. Li, J. Liang, Probability-guaranteed distributed estimation for two-dimensional systems under stochastic access protocol, *IEEE T. Signal Inf. Pr.*, **10** (2024), 216–226. https://doi.org/10.1109/TSIPN.2024.3375596

16. M. Shi, C. Hu, J. Guo, L. Ma, X. Yi, Set-membership state estimation for complex networks with chance constraints under multi-modal deception attacks, *Asian J. Control*, **27** (2025), 1854–1865. https://doi.org/10.1002/asjc.3545
17. K. Chen, H. Song, P. Shi, W. A. Zhang, L. Yu, Probability-guaranteed distributed set-membership secure fusion estimation against nonlinear hybrid attacks, *IEEE T. Autom. Sci. Eng.*, **22** (2024), 5539–5550. https://doi.org/10.1109/TASE.2024.3424190
18. H. Song, M. Lai, Z. Hong, B. Chen, W. A. Zhang, L. Yu, Event-based probability-guaranteed set-membership secure fusion estimation for energy-constrained multi-sensor systems with asynchronous samplings, *IEEE T. Autom. Sci. Eng.*, **22** (2025), 13983–13994. https://doi.org/10.1109/TASE.2025.3558663
19. S. Yuan, L. Ma, C. Gao, Probability-guaranteed consensus control for nonlinear multi-agent systems under bit flips, *Int. J. Netw. Dyn. Intell.*, **4** (2025), 100020. https://doi.org/10.53941/ijndi.2025.100020
20. W. Chen, J. Hu, Z. Wu, S. Ma, A survey on fault detection for networked systems under communication constraints, *Syst. Sci. Control Eng.*, **13** (2025), 2460434. https://doi.org/10.1080/21642583.2025.2460434
21. Y. Liu, W. Yang, J. Zhou, Y. Luo, H_∞ filter for discrete-time periodic piecewise systems with missing measurements, *IEEE T. Syst. Man Cy.-S.*, **53** (2023), 5648–5657. https://doi.org/10.1109/TSMC.2023.3274392
22. Z. Cheng, L. Yang, Q. Yuan, Y. Long, H. Ren, Distributed consensus estimation for networked multi-sensor systems under hybrid attacks and missing measurements, *Sensors*, **24** (2024), 4071. https://doi.org/10.3390/s24134071
23. J. Hu, Z. Wang, G. P. Liu, H. Zhang, R. Navaratne, A prediction-based approach to distributed filtering with missing measurements and communication delays through sensor networks, *IEEE T. Syst. Man Cy.-S.*, **51** (2021), 7063–7074. https://doi.org/10.1109/TSMC.2020.2966977
24. W. Shi, J. Liu, H. K. Lam, J. Yu, Recursive estimator-based fuzzy adaptive control for discrete-time uncertain systems with state saturations and missing measurements, *IEEE T. Fuzzy Syst.*, **33** (2025), 908–918. https://doi.org/10.1109/TFUZZ.2024.3496781
25. Z. Sun, C. Han, Linear state estimation for multi-rate NCSs with multi-channel observation delays and unknown Markov packet losses, *Int. J. Netw. Dyn. Intell.*, **4** (2025), 100005. https://doi.org/10.53941/ijndi.2025.100005
26. Z. Yang, X. Zhang, W. Xiang, X. Lin, A novel particle filter based on one-step smoothing for nonlinear systems with random one-step delay and missing measurements, *Sensors*, **25** (2025), 318. https://doi.org/10.3390/s25020318
27. J. Liang, Z. Wang, X. Liu, Distributed state estimation for discrete-time sensor networks with randomly varying nonlinearities and missing measurements, *IEEE T. Neural Netw.*, **22** (2011), 486–496. https://doi.org/10.1109/TNN.2011.2105501
28. J. Fan, R. Jia, K. Zhang, J. Guo, Parameter identification of FIR systems with binary-valued observations: When the event-driven communication mechanism encounters DoS attacks, *Int. J. Syst. Sci.*, **56** (2025), 3156–3176. https://doi.org/10.1080/00207721.2025.2469814
29. R. Sakthivel, B. Kaviarasan, L. Rutkowski, V. T. Huynh, Resilient filtering of fuzzy multi-weighted complex dynamical networks against cyber attacks and missing measurements, *Commun. Nonlinear Sci.*, **157** (2026), 109731. https://doi.org/10.1016/j.cnsns.2026.109731

30. S. Liu, L. Wang, Y. Zhang, Y. A. Wang, H. Dong, Recursive filtering of networked systems with communication protocol scheduling: A survey, *Int. J. Syst. Sci.*, **56** (2025), 2499–2516. https://doi.org/10.1080/00207721.2024.2448775
31. Z. Liu, H. Yuan, H. Li, X. Guan, H. Yang, Robust power control for amplify-and-forward relaying scheme, *IEEE Commun. Lett.*, **19** (2015), 263–266. https://doi.org/10.1109/LCOMM.2014.2379718
32. W. Zhang, Y. Zhao, H. Wang, Z. Yang, Robust model predictive control for polyhedral uncertain systems under relay and redundancy protocol, *Int. J. Syst. Sci.*, **56** (2025), 1617–1632. https://doi.org/10.1080/00207721.2024.2428851
33. X. Meng, Z. Wang, F. Wang, Y. Chen, State estimation for nonlinear complex dynamical networks with random coupling strengths: A decode-and-forward relay-based strategy, *IEEE T. Syst. Man Cy.-S.*, **54** (2024), 4749–4760. https://doi.org/10.1109/TSMC.2024.3389971
34. G. Li, Z. Wang, X. Bai, Z. Zhao, G. Chen, Sequential fusion estimation for renewable energy microgrids under hybrid attacks: Handling filter-and-forward relays, *IEEE T. Ind. Inform.*, **21** (2025), 8224–8235. https://doi.org/10.1109/TII.2025.3582368
35. S. Cai, J. Liang, Recursive filtering for nonlinear systems with relay communication, energy harvesting and correlated noises, *Int. J. Netw. Dyn. Intell.*, **4** (2025), 100021. https://doi.org/10.53941/ijndi.2025.100021
36. D. Dai, J. Li, J. Zhang, B. Jiang, H. Dong, Energy-harvesting recursive filtering for delayed systems with amplify-and-forward relays: Cooperative communication manner, *J. Franklin I.*, **361** (2024), 107265. https://doi.org/10.1016/j.jfranklin.2024.107265
37. Y. Wang, Z. Wang, L. Zou, H. Dong, On H_∞ fuzzy proportional-integral observer design under amplify-and-forward relays and multirate measurements, *IEEE T. Fuzzy Syst.*, **32** (2024), 1873–1885. https://doi.org/10.1109/TFUZZ.2023.3337194
38. C. Xu, J. Sun, Y. Shen, W. Li, H. Dong, Fault estimation for complex networks with uncertain couplings and randomly occurring faults: A relay-aided strategy, *J. Franklin I.*, **362** (2025), 108157. https://doi.org/10.1016/j.jfranklin.2025.108157
39. Y. Zhong, T. Zhang, Z. Li, S. Bao, Y. Zhou, Distributed set-membership filtering for complex networks with amplify-and-forward relays: A two-step ellipsoid method, *Neurocomputing*, **622** (2025), 129317. https://doi.org/10.1016/j.neucom.2024.129317
40. D. Ciuonzo, A. Aubry, V. Carotenuto, Rician MIMO channel- and jamming-aware decision fusion, *IEEE T. Signal Process.*, **65** (2017), 3866–3880. https://doi.org/10.1109/TSP.2017.2686375
41. N. Nomikos, M. S. Talebi, T. Charalambous, R. Wichman, Bandit-based power control in full-duplex cooperative relay networks with strict-sense stationary and non-stationary wireless communication channels, *IEEE Open J. Commun. Soc.*, **3** (2022), 366–378. https://doi.org/10.1109/OJCOMS.2022.3154292
42. X. Pei, H. Yu, M. Wen, S. Mumtaz, S. AI Otaibi, M. Guizani, NOMA-based coordinated direct and relay transmission with a half-duplex/full-duplex relay, *IEEE T. Commun.*, **68** (2020), 6750–6760. https://doi.org/10.1109/TCOMM.2020.3017002
43. S. Zhang, X. Chai, K. P. Long, A. V. Vasilakos, L. Hanzo, Full duplex techniques for 5G networks: Self-interference cancellation, protocol design, and relay selection, *IEEE Commun. Mag.*, **53** (2015), 128–137. https://doi.org/10.1109/MCOM.2015.7105651

44. H. Tan, B. Shen, Q. Li, W. Qian, Recursive filtering for nonlinear systems with self-interferences over full-duplex relay networks, *IEEE/CAA J. Autom. Sin.*, **9** (2022), 2037–2040. https://doi.org/10.1109/JAS.2022.105965
45. L. Wang, Z. Wang, S. Liu, D. Peng, Unscented Kalman filtering over full-duplex relay networks under binary encoding schemes, *IEEE T. Autom. Control*, **70** (2025), 3441–3448. https://doi.org/10.1109/TAC.2024.3521281
46. L. Ma, Z. Wang, J. Hu, Q. L. Han, Probability-guaranteed envelope-constrained filtering for nonlinear systems subject to measurement outliers, *IEEE T. Autom. Control*, **66** (2021), 3274–3281. https://doi.org/10.1109/TAC.2020.3016767
47. Y. Hu, C. Zhang, S. Liu, Privacy-preserving distributed recursive filtering for state-saturated systems with quantization effects, *Int. J. Netw. Dyn. Intell.*, **4** (2025), 100012. https://doi.org/10.53941/ijndi.2025.100012
48. Y. Cai, X. Yang, Y. Yang, Q. Liu, Leader-following privacy-preserving consensus control of nonlinear multi-agent systems: A state decomposition approach, *Int. J. Syst. Sci.*, **56** (2025), 2284–2295. https://doi.org/10.1080/00207721.2024.2445726
49. W. Le, B. Shen, L. Zou, J. Sun, Stubborn state estimation for time-delay systems with relay communication network: The half-duplex case, *J. Franklin I.*, **362** (2025), 107465. https://doi.org/10.1016/j.jfranklin.2024.107465
50. S. Jain, M. Cardone, S. Mohajer, Optimality of energy-efficient scheduling and relaying for half-duplex relay networks, *IEEE J. Sel. Areas Inf.*, **3** (2022), 37–53. https://doi.org/10.1109/JSAIT.2022.3157829

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