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Research article

A WAS neural network framework for computing and analyzing solutions of a generalized $(3+1)$-dimensional nonlinear Wave equation: Stability analysis

Waseem Razzaq[1], Asim Zafar[1], Ahmed Al Nuaim[2,*] and Abdullah Al Nuaim[2]

[1] Department of Mathematics, COMSATS University Islamabad, Vehari Campus, Pakistan
[2] Department of Management Information Systems, School of Business, King Faisal University, Al-Ahsa 31982, Saudi Arabia

* **Correspondence:** Email: aaalnuaim@kfu.edu.sa.

Abstract: This paper discusses the generalized non-linear (3 + 1)-dimensional wave equation by modeling and analyzing the dynamics of multi-dimensional nonlinear waves with the WAS-neural network technique. The suggested framework accurately models various wave forms such as bright, singular, and bright-dark solitons. Insofar as we know such neural network based solutions of this model are not reported before. To ensure the reliability and proficiency of the WAS neural network technique. The gain solutions are stable or not by executing the stability analysis on them. The graphical visualization in three-dimensional surface and two-dimensional plots are used. The findings validate that the WAS neural network technique is an efficient and strong alternative to classical techniques of higher-dimensional nonlinear wave equations, and has applications in fluid mechanics and engineering systems that have to deal with gas liquid interactions.

Keywords: the (3 + 1)-dimensional wave equation; WAS neural network method; analytical method; neural network; soliton solutions; stability analysis
Mathematics Subject Classification: 35Q55, 35C08, 37K10, 35A20

1. Introduction

The basic tools of modeling a broad spectrum of phenomena of waves in physics, engineering, and applied mathematics rely on nonlinear partial differential equations (NLPDEs). More specifically, the use of higher-dimensional formulations particularly in the (3 + 1)-dimensional has become more and more popular due to its successful ability to represent complex interactions between nonlinearity, dispersion, and external perturbations. These models occur naturally in a range of disciplines such as nonlinear optics, fluid mechanics, plasma physics, and gas-liquid interactions phenomena in the real world. Precise analytical solutions to NLPDEs have thus emerged as a popular research direction as they do not only provide new insights into the nonlinear dynamics of waves, but also act as a reference point to analytical approximations and numerical simulations [1, 2]. In the last few years, numerous methods for exact solutions have been suggested to obtain different special type solutions like periodic solitons, dark solitons, bright solitons, and singular soliton solutions to nonlinear evolution equations. Some of the most popular ones include the Kudryashov technique [3], the tanh-expansion technique [4], the F-expansion technique [5], the extended (G'/G)-expansion technique [6], the ϕ^6-expansion technique [7], the (G'/G^2)-expansion technique [8], the $\exp(\Omega(\zeta))$-expansion technique [9] and the sub-equation technique [10], among others. The analysis of NLPDEs has recently progressed further with the use of neural network techniques due to the development of new analytical approaches. These techniques have proven successful in yielding a range of classes of exact solutions to a range of nonlinear models, which provide very useful information on nonlinear waves. As an example, in [11] researchers used a neural network method for exact solutions. In [12], researchers applied a deep linear neural network framework to study the nonlinear dynamics. In [13], investigators analyzed the wave propagation pattern in a Korteweg–de Vries model in an symbolic computation approach framework which revealed intricate dynamics. Physics-informed neural networks (PINNs) were used in [14] for the prediction of uncertain quantities. PINNs wrer used by Panichi in [15] for multivariate partial differential equations. Altogether, these works shows the increasing significance of algebraic methods in the analytical analysis of non-linear evolution equations.

There are classical nonlinear equations like the Korteweg-de Vries equation [16], the Biswas-Arshed equation [17], the Schrodinger equation [18], the Drinfeld Wilson Sokolov equation [19], the Boussinesq equation [20], and the generalized Bogoyavlvensky–Konopelchenko equation [21]. They have been widely studied to explain complex wave phenomena. These equations have been instrumental in enriching the study of soliton dynamics, nonlinear dispersion, and the interaction of waves in many physical situations. Nonetheless, the rising complexity of modern physical and engineering challenges and multidimensional wave propagation necessitate the exploration of more formalized non-linear models that can offer further complex wave phenomena.

In the present work, we take a nonlinear wave equation of the kind of generalized (3+1)-dimensional equation with fixed coefficients, expressed as [22]

$$(v_t + a_1 v\, v_x + a_2\, v_{xxx} + a_3\, v_x)_x + a_4\, v_{yy} + a_5\, v_{zz} = 0, \tag{1.1}$$

where the smooth wave envelope is represented by the term of the wave, t is the time, x,y,z are spatial variables, and $a_i (i = 1, 2, 3, 4, 5)$ with real parameters used to deal with anisotropic and nonlinear dispersive effects. This model is commonly used to get the propagation of non-linear acoustic waves in gas bubbles and compressible liquids. With gas bubbles, dispersion and compressibility properties of the medium change give further intricate wave dynamics in pure liquids. Particularly, the temporal evolution is controlled by the term, denoted by v_t, the nonlinear convective term $a_1 v\, v_x$ is the steepening of the wave, $a_2 v_{xxx}$ is the dispersion of the bubble, and $a_3 v_x$ is the diffusion or dispersion of the spatial heterogeneities. The transverse components $a_4 v_{yy}$ and $a_5 v_{zz}$ are used to describe the effect of diffraction in the y- and z-planes that are necessary in the life-like three-dimensional propagation of waves.

However, recent studies of nonlinear wave equations in higher dimensions have shown solution structures that are rich and with significant applications in fluid dynamics and shallow water. The bilinear Baklund transformations and similarity reductions studied in [23] with a (3 + 1)-

dimensional shallow water wave model provided some insight of the integrability properties of aquatic environments. The same model was represented as resonant Y-type solitons and interaction waves built in [24] and interaction waves with emphasis given to nonlinear resonance mechanisms, as reported in [25]. Similarity-reduction methods have been used in [26] to obtain reduced forms of the extended shallow water wave equations. In [27], multi-soliton, breather, and hybrid wave solutions were obtained. Collectively, these works indicate the growing popularity of analytical solutions of the nonlinear wave equations of $(3+1)$-dimensions and their applicability to theory and practice. Following these advancements, in this article we examine a nonlinear generalized $(3+1)$-dimensional wave model with the WAS-neural network method [28]. This method has some advantages:

- Capability of solving highly nonlinear and complex partial differential equations (PDEs).
- No need to use traveling wave transformation to convert PDEs into ordinary differential equations.
- Utilizes adjustable hidden layers to effectively manage and reduce computational complexity.

Most specific solutions, such as bright, dark, singular, periodic, and singular periodic solitons are obtained and discussed. The resultant wave behaviors are demonstrated by the help of the contour plots, surface plots, and three-dimensional visualizations. Such results are related to the theoretical knowledge of non-linear waves and provide the possibility of application in engineering, fluid structure interactions, and gas liquid dynamics.

The rest of this paper is structured as follows. In Section 2, the major steps of the proposed analytical method are described. Section 3 represents the applications of the WAS-NN technique. Section 4 graphical illustration is given for the obtained solutions using 3-dimensional surface and 2-dimensional plot. In Section 5, we provide a stability analysis of the obtained solutions. Lastly, Section 6 concludes and summarizes the paper.

2. Description of the technique

This section presents the WAS neural network (WAS-NN) technique to construct exact solutions of nonlinear partial differential equations (nPDEs) written in the general form

$$P(v, v_x, v_t, v_{xx}, v_{xt}, v_{tt}, \ldots) = 0, \tag{2.1}$$

where P denotes a polynomial function of the dependent variable v and its partial derivatives with respect to the spatial and temporal variables. To derive exact solutions of Eq (2.1), a trial solution is formulated using a neural network model, as illustrated in Figure 1. The output of the neural network is regarded as an analytical representation of the solution. Accordingly, the NN-based trial solution is expressed as

$$v = \sum_{l_n \in L_n} \varpi_{l_{n,u}} F_{l_n}(\zeta_{l_n}), \tag{2.2}$$

where

- $\varpi_{l_{n,u}}$ is the weight connecting neuron l_n in the final hidden layer to the output neuron u.
- $F(\cdot)$ denotes the activation function.
- $L_n = \{I_{n-1+1}, I_{n-1+2}, \ldots, I_n\}$ represents the index set of neurons in the *nth* (final hidden) layer.

The neural network contains two types of trainable parameters,

- weight coefficients $\varpi_{i,j}$ and
- bias parameters b_l,

which define the connections between neuron i in one layer and neuron j in the next layer. The internal state of the l_i-th neuron in the *ith* layer is defined as

$$\zeta_{l_i} = \sum_{l_{i-1} \in L_{i-1}} \varpi_{l_{i-1},l_i} F_{l_{i-1}}(\zeta_{l_{i-1}}) + b_{l_i}, \qquad i = 1, 2, \ldots, n.$$

The WAS-NN method exploits the analytical properties of the Riccati equation together with the flexibility of neural network architectures. In particular, the Riccati equation is employed to model nonlinear activation behavior consistent with soliton-type structures. Its intrinsic nonlinearity is well suited to capture the solution characteristics of the nonlinear models considered.

$$\phi'(\zeta) = \phi^2(\zeta) + \Omega. \tag{2.3}$$

As a result, it gives a family of solutions for different values of the parameter Ω. The corresponding results of the Riccati equation depends on the sign of parameter Ω and can be classified as follows.
Case I. $\Omega < 0$.

$$\phi(\zeta) = \begin{cases} -\sqrt{-\Omega}\, \tanh\left(\sqrt{-\Omega}\,\zeta\right), \\ -\sqrt{-\Omega}\, \coth\left(\sqrt{-\Omega}\,\zeta\right), \\ \sqrt{-\Omega}\left[-\tanh\left(2\sqrt{-\Omega}\,\zeta\right) \pm i\, \operatorname{sech}\left(2\sqrt{-\Omega}\,\zeta\right)\right], \\ \sqrt{-\Omega}\left[-\coth\left(2\sqrt{-\Omega}\,\zeta\right) \pm \operatorname{csch}\left(2\sqrt{-\Omega}\,\zeta\right)\right], \\ -\dfrac{\sqrt{-\Omega}}{2}\left[\tanh\left(\dfrac{\sqrt{-\Omega}}{2}\,\zeta\right) + \coth\left(\dfrac{\sqrt{-\Omega}}{2}\,\zeta\right)\right]. \end{cases} \tag{2.4}$$

Case II. $\Omega > 0$.

$$\phi(\zeta) = \begin{cases} \sqrt{\Omega}\, \tan\left(\sqrt{\Omega}\,\zeta\right), \\ -\sqrt{\Omega}\, \cot\left(\sqrt{\Omega}\,\zeta\right), \\ \sqrt{\Omega}\left[\tan\left(2\sqrt{\Omega}\,\zeta\right) \pm \sec\left(2\sqrt{\Omega}\,\zeta\right)\right], \\ \sqrt{\Omega}\left[-\cot\left(2\sqrt{\Omega}\,\zeta\right) \pm \csc\left(2\sqrt{\Omega}\,\zeta\right)\right], \\ \dfrac{\sqrt{\Omega}}{2}\left[\tan\left(\dfrac{\sqrt{\Omega}}{2}\,\zeta\right) - \cot\left(\dfrac{\sqrt{\Omega}}{2}\,\zeta\right)\right]. \end{cases} \tag{2.5}$$

Case III. $\Omega = 0$.

$$\phi(\zeta) = -\frac{1}{\zeta}. \tag{2.6}$$

The proposed approach consists of two interconnected stages defined as follows:

1. Incorporating explicit solutions of the Riccati equation into the neural network through the choice of activation functions;
2. employing the resulting neural network structure to construct trial solutions and reduce the nonlinear partial differential equation (nPDE) to an equivalent algebraic system.

The essential procedure of the WAS-NN technique can be outlined as follows.
Step 1. Design of the WAS-NN architecture:

- *Input layer:* Includes the independent variables, such as x, t, or variables in higher-dimensional settings;
- *hidden layers:* Composed of neurons activated by functions derived from the Riccati equation;
- *output layer:* Produces the trial solution $v(x, y, z, t)$.

Step 2. Forward evaluation: The WAS-NN model is evaluated through forward propagation to generate a trial function $v(x, y, z, t)$ representing a candidate solution of the nPDE.
Step 3. Substitution into the governing equation: The obtained trial function is inserted into the original nPDE, which transforms the differential equation into an algebraic expression.
Step 4. Matching of coefficients: Terms with the same functional structure are grouped together, and the coefficients of the resulting basis functions are equated to zero, leading to a system of algebraic equations.
Step 5. Determination of unknown parameters: The resulting algebraic system is solved to compute the unknown coefficients. Substituting these values into the trial function yields an exact or semi-analytical solution of the original nPDE.

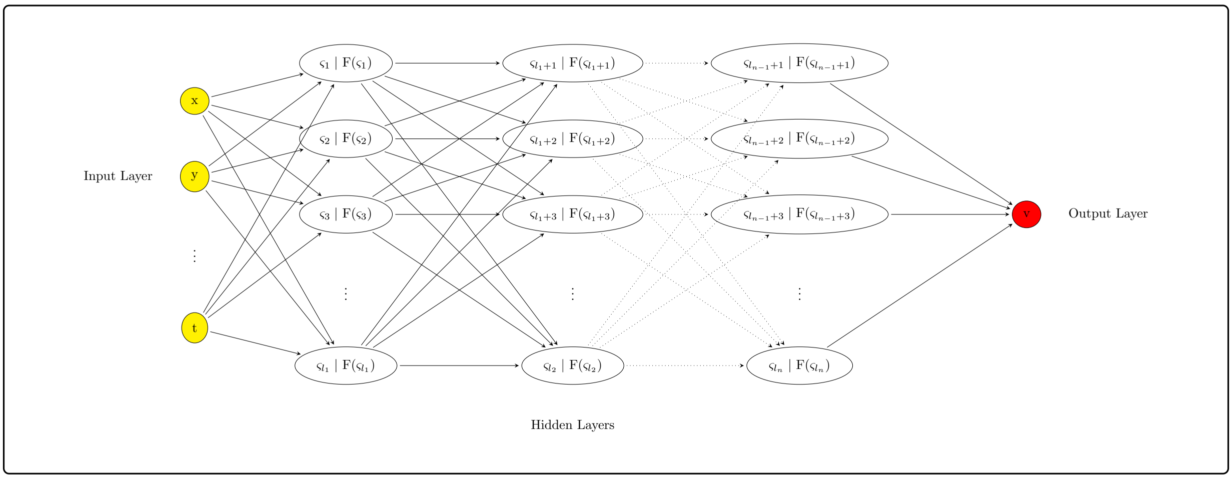


Figure 1. Neural networks model.

3. Applications of WAS-NN technique

3.1. 4-2-1 neural network structure

We choose the 4-2-1 neural network structure as illustrated in Figure 2.

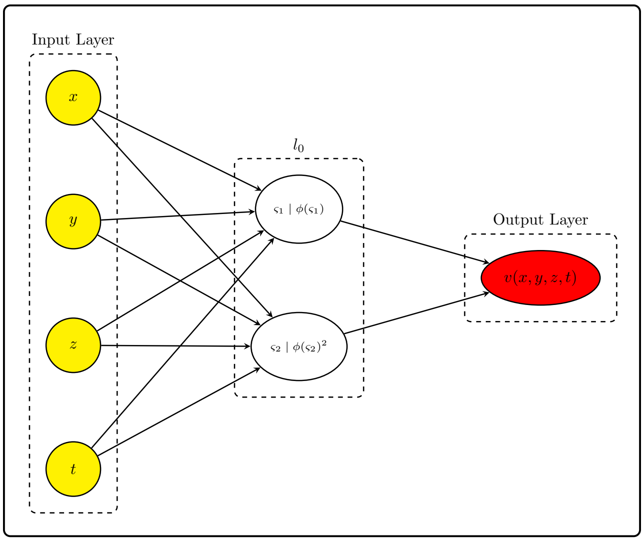


Figure 2. 4-2-1 neural network model.

$$\begin{cases} \varsigma_1 = x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + t\varpi_{\{4,1\}} + b_1, \\ \varsigma_2 = x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + t\varpi_{\{4,2\}} + b_2, \\ \varsigma = F(\varsigma_1)\,\varpi_{\{1,3\}} + F(\varsigma_2)\,\varpi_{\{2,3\}} + b_3. \end{cases} \tag{3.1}$$

Here, the first layer consists of x, y, z, t and a hidden layer consisting of the identity function $(\cdot)$ and the square function $(\cdot)^2$. The output layer consists of

$$\begin{cases} \varsigma_1 = x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + t\varpi_{\{4,1\}} + b_1, \\ \varsigma_2 = x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + t\varpi_{\{4,2\}} + b_2, \\ v = \phi(\varsigma_1)\,\varpi_{\{1,3\}} + (\phi(\varsigma_2))^2\varpi_{\{2,3\}} + b_3. \end{cases} \tag{3.2}$$

By using Eq (3.2) in Eq (1.1), we obtain following algebraic equations:

$$\begin{aligned}
2a_3b_3\Omega^2\varpi_{\{2,3\}}\varpi^2_{\{1,2\}} + 16a_2\Omega^3\varpi_{\{2,3\}}\varpi^4_{\{1,2\}} + 2a_1\Omega^2\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}} + a_3\Omega^2\varpi^2_{\{1,1\}}\varpi^2_{\{1,3\}} + 2a_4\Omega^2\varpi_{\{2,3\}}\varpi^2_{\{3,2\}} + \\
2a_5\Omega^2\varpi^2_{\{2,2\}}\varpi_{\{2,3\}} + 2\Omega^2\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}} &= 0, \\
2a_3b_3\Omega\varpi_{\{1,3\}}\varpi^2_{\{1,1\}} + 16a_2\Omega^2\varpi_{\{1,3\}}\varpi^4_{\{1,1\}} + 2a_3\Omega^2\varpi^2_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{2,3\}} + 2a_1\Omega\varpi_{\{1,3\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 2a_5\Omega\varpi_{\{1,3\}}\varpi^2_{\{2,1\}} \\
+2a_4\Omega\varpi_{\{1,3\}}\varpi^2_{\{3,1\}} + 2\Omega\varpi_{\{1,3\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} &= 0, \\
2a_3b_3\varpi_{\{1,3\}}\varpi^2_{\{1,1\}} + 40a_2\Omega\varpi_{\{1,3\}}\varpi^4_{\{1,1\}} + 2a_1\varpi_{\{1,3\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 2a_5\varpi_{\{1,3\}}\varpi^2_{\{2,1\}} + 2a_4\varpi_{\{1,3\}}\varpi^2_{\{3,1\}} + 2\varpi_{\{1,3\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} &= 0, \\
3a_3\varpi^2_{\{1,1\}}\varpi^2_{\{1,3\}} &= 0, \\
24a_2\varpi^4_{\{1,1\}}\varpi_{\{1,3\}} &= 0, \\
4a_3\Omega^2\varpi_{\{1,1\}}\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{2,3\}} &= 0, \\
8a_3b_3\Omega\varpi_{\{2,3\}}\varpi^2_{\{1,2\}} + 136a_2\Omega^2\varpi_{\{2,3\}}\varpi^4_{\{1,2\}} + 6a_3\Omega^2\varpi^2_{\{2,3\}}\varpi^2_{\{1,2\}} + 8a_1\Omega\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}} + 8a_4\Omega\varpi_{\{2,3\}}\varpi^2_{\{3,2\}} + 8a_5\Omega\varpi^2_{\{2,2\}}\varpi_{\{2,3\}} \\
+8\Omega\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}} &= 0, \\
2a_3\Omega\varpi_{\{1,3\}}\varpi_{\{2,3\}}\varpi^2_{\{1,1\}} + 8a_3\Omega\varpi^2_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{2,3\}} &= 0, \\
2a_3\varpi^2_{\{1,1\}}\varpi_{\{1,3\}}\varpi_{\{2,3\}} &= 0, \\
4a_3\Omega\varpi_{\{1,1\}}\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{2,3\}} &= 0, \\
4a_3\varpi_{\{1,1\}}\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{2,3\}} &= 0,
\end{aligned}$$

$$6a_3b_3\varpi_{\{2,3\}}\varpi_{\{1,2\}}^2+240a_2\Omega\varpi_{\{2,3\}}\varpi_{\{1,2\}}^4+16a_3\Omega\varpi_{\{2,3\}}^2\varpi_{\{1,2\}}^2+6a_1\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+6a_4\varpi_{\{2,3\}}\varpi_{\{3,2\}}^2+6a_5\varpi_{\{2,2\}}^2\varpi_{\{2,3\}}$$
$$+6\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}=0, 6a_3\varpi_{\{1,2\}}^2\varpi_{\{1,3\}}\varpi_{\{2,3\}}=0,$$
$$120a_2\varpi_{\{2,3\}}\varpi_{\{1,2\}}^4+10a_3\varpi_{\{2,3\}}^2\varpi_{\{1,2\}}^2=0,$$
$$4a_3\Omega\varpi_{\{1,1\}}^2\varpi_{\{1,3\}}^2+4a_3\Omega\varpi_{\{1,1\}}\varpi_{\{1,2\}}\varpi_{\{2,3\}}\varpi_{\{1,3\}}=0.$$

After using the computational Mathematica software, we get following set:

Set 1.

$$\left\{\varpi_{\{4,2\}}=\frac{-a_3b_3\varpi_{\{1,2\}}^2-8a_2\Omega\varpi_{\{1,2\}}^4-a_5\varpi_{\{2,2\}}^2-a_4\varpi_{\{3,2\}}^2}{(a_1+1)\,\varpi_{\{1,2\}}}, \varpi_{\{1,3\}}=0, \varpi_{\{2,3\}}=-\frac{12a_2\varpi_{\{1,2\}}^2}{a_3}\right\}. \tag{3.3}$$

Case I. $\Omega<0$.

$$v(x,y,z,t)=\frac{12a_2\Omega\varpi_{\{1,2\}}^2\tanh^2\left(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)}{a_3}+b_3, \tag{3.4}$$

$$v(x,y,z,t)=\frac{12a_2\Omega\varpi_{\{1,2\}}^2\coth^2\left(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)}{a_3}+b_3, \tag{3.5}$$

$$v(x,y,z,t)=\frac{12a_2\Omega\varpi_{\{1,2\}}^2}{a_3}(-\tanh\left(2\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)$$
$$\pm i\operatorname{sech}\left(2\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right))^2+b_3, \tag{3.6}$$

$$v(x,y,z,t)=\frac{12a_2\Omega\varpi_{\{1,2\}}^2}{a_3}(-\coth\left(2\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)$$
$$\pm i\operatorname{csch}\left(2\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right))^2+b_3, \tag{3.7}$$

$$v(x,y,z,t)=\frac{3a_2\Omega\varpi_{\{1,2\}}^2}{a_3}(\tanh\left(\frac{1}{2}\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)$$
$$+\coth\left(\frac{1}{2}\sqrt{-\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right))^2+b_3. \tag{3.8}$$

Case II. $\Omega>0$.

$$v(x,y,z,t)=\frac{12a_2\Omega\varpi_{\{1,2\}}^2\tan^2\left(\sqrt{\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)}{a_3}-b_3, \tag{3.9}$$

$$v(x,y,z,t)=\frac{12a_2\Omega\varpi_{\{1,2\}}^2\cot^2\left(\sqrt{\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)}{a_3}+b_3, \tag{3.10}$$

$$v(x,y,z,t)=\frac{12a_2\Omega\varpi_{\{1,2\}}^2}{a_3}(\tan\left(2\sqrt{\Omega}\,(t\varpi_{\{4,2\}}+x\varpi_{\{1,2\}}+y\varpi_{\{2,2\}}+z\varpi_{\{3,2\}}+b_2)\right)$$

$$\pm \sec\left(2\sqrt{\Omega}\left(t\varpi_{\{4,2\}} + x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2\right)\right))^2 - b_3, \quad (3.11)$$

$$v(x,y,z,t) = \frac{12a_2\Omega\varpi_{\{1,2\}}^2}{a_3}(-\cot\left(2\sqrt{\Omega}\left(t\varpi_{\{4,2\}} + x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2\right)\right)$$
$$\pm \csc\left(2\sqrt{\Omega}\left(t\varpi_{\{4,2\}} + x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2\right)\right))^2 - b_3, \quad (3.12)$$

$$v(x,y,z,t) = \frac{3a_2\Omega\varpi_{\{1,2\}}^2}{a_3}(\tan\left(\frac{1}{2}\sqrt{\Omega}\left(t\varpi_{\{4,2\}} + x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2\right)\right)$$
$$- \cot\left(\frac{1}{2}\sqrt{\Omega}\left(t\varpi_{\{4,2\}} + x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2\right)\right))^2 - b_3. \quad (3.13)$$

Case III. $\Omega = 0$.

$$v(x,y,z,t) = b_3 - \frac{12a_2\varpi_{\{1,2\}}^2}{a_3\left(t\varpi_{\{4,2\}} + x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2\right)^2}. \quad (3.14)$$

3.2. 4-2-2-1 neural network structure

We choose the 4-2-2-1 neural network structure as illustrated in Figure 3.

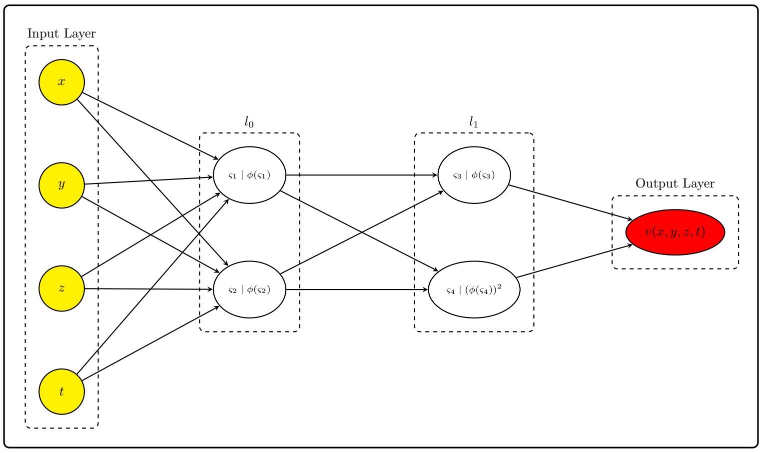


Figure 3. 4-2-2-1 neural network model.

$$\begin{cases} \varsigma_1 = x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + t\varpi_{\{4,1\}} + b_1, \\ \varsigma_2 = x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + t\varpi_{\{4,2\}} + b_2, \\ \varsigma_3 = \phi(\varsigma_1)\varpi_{\{1,3\}} + \phi(\varsigma_2)\varpi_{\{2,3\}} + b_3, \\ \varsigma_4 = \phi(\varsigma_1)\varpi_{\{1,4\}} + \phi(\varsigma_2)\varpi_{\{2,4\}} + b_4, \\ \varsigma = F(\varsigma_3)\varpi_{\{1,5\}} + F(\varsigma_4)\varpi_{\{2,5\}} + b_5. \end{cases} \quad (3.15)$$

Here, the first layer consists of x, y, z, t and a hidden layer consisting of the identity function $(\cdot)$ and the square function $(\cdot)^2$. The output layer consists of

$$
\begin{cases}
\varsigma_1 = x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + t\varpi_{\{4,1\}} + b_1, \\
\varsigma_2 = x\varpi_{\{1,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + t\varpi_{\{4,2\}} + b_2, \\
\varsigma_3 = \phi(\varsigma_1)\,\varpi_{\{1,3\}} + \phi(\varsigma_2)\,\varpi_{\{2,3\}} + b_3, \\
\varsigma_4 = \phi(\varsigma_1)\,\varpi_{\{1,4\}} + \phi(\varsigma_2)\,\varpi_{\{2,4\}} + b_4, \\
\ v = (\varsigma_3)\,\varpi_{\{1,5\}} + (\varsigma_4)^2\,\varpi_{\{2,5\}} + b_5.
\end{cases}
\tag{3.16}
$$

By using Eq (3.16) in Eq (1.1), we obtain following algebraic system of equations:

$$
\begin{aligned}
&16\Omega^3 a_2\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4 + 16\Omega^3 a_2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^3 + \Omega^2 a_3\varpi_{\{1,3\}}^2\varpi_{\{1,5\}}^2\varpi_{\{1,1\}}^2 \\
&+6\Omega^2 a_3 b_4^2\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2 + 2\Omega^2 a_3 b_5\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 2\Omega^2 a_3 b_3\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 \\
&+4\Omega^2 a_3 b_4\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 12\Omega^2 a_3 b_4^2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}} + 2\Omega^2 a_3\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,1\}} \\
&+4\Omega^2 a_3 b_4\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 16\Omega^3 a_2\varpi_{\{1,2\}}^3\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 4\Omega^2 a_3 b_5\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} \\
&+4\Omega^2 a_3 b_4\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 4\Omega^2 a_3 b_3\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 2\Omega^2\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} \\
&+2\Omega^2 a_1\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 2\Omega^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,1\}} + 2\Omega^2 a_1\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,1\}} + \Omega^2 a_3\varpi_{\{1,2\}}^2\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}^2 \\
&+6\Omega^2 a_3 b_4^2\varpi_{\{1,2\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2 + 2\Omega^2 a_4\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{3,1\}}^2 + 2\Omega^2 a_4\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{3,2\}}^2 + 2\Omega^2 a_5\varpi_{\{1,4\}}^2\varpi_{\{2,1\}}^2\varpi_{\{2,5\}} + 16\Omega^3 a_2\varpi_{\{1,2\}}^4\varpi_{\{2,4\}}^2\varpi_{\{2,5\}} \\
&+2\Omega^2 a_3 b_5\varpi_{\{1,2\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}} + 2\Omega^2 a_5\varpi_{\{2,2\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}} + 2\Omega^2 a_3 b_3\varpi_{\{1,2\}}^2\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}} + 4\Omega^2 a_5\varpi_{\{1,4\}}\varpi_{\{2,1\}}\varpi_{\{2,2\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}} \\
&+4\Omega^2 a_3 b_4\varpi_{\{1,2\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}} + 4\Omega^2 a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}\varpi_{\{3,2\}} + 2\Omega^2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}} \\
&+2\Omega^2 a_1\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}} + 2\Omega^2\varpi_{\{1,2\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,2\}} + 2\Omega^2 a_1\varpi_{\{1,2\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,2\}} = 0, \\
&32a_2 b_4\Omega^2\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4 + 12a_3 b_4\Omega^2\varpi_{\{1,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2 + 24a_3 b_4\Omega^2\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}} \\
&+12a_3 b_4\Omega^2\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2 + 2a_3 b_3\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{1,1\}}^2 + 4a_3 b_4^3\Omega\varpi_{\{1,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2 \\
&+2a_3 b_5\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{1,1\}}^2 + 4a_3 b_4 b_5\Omega\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 2a_3 b_4^2\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 \\
&+4a_3 b_3 b_4\Omega\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 4a_1 b_4\Omega\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 4a_4 b_4\Omega\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}^2 \\
&+4a_5 b_4\Omega\varpi_{\{1,4\}}\varpi_{\{2,1\}}^2\varpi_{\{2,5\}} + 16a_2\Omega^2\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{1,1\}}^4 + 6a_3\Omega^2\varpi_{\{1,3\}}\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 \\
&+4a_3\Omega^2\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 8a_3\Omega^2\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 2a_3\Omega^2\varpi_{\{1,2\}}^2\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}} \\
&+4a_3\Omega^2\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}} + 2a_1\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 2a_5\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,1\}}^2 \\
&+2a_4\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{3,1\}}^2 + 4b_4\Omega\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 2\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} = 0, \\
&136\Omega^2 a_2\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4 + 64\Omega^2 a_2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^3 + 4\Omega a_3\varpi_{\{1,3\}}^2\varpi_{\{1,5\}}^2\varpi_{\{1,1\}}^2 \\
&+6\Omega^2 a_3\varpi_{\{1,4\}}^4\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2 + 24\Omega a_3 b_4^2\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2 + 8\Omega a_3 b_5\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 \\
&+8\Omega a_3 b_3\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 16\Omega a_3 b_4\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 12\Omega^2 a_3\varpi_{\{1,2\}}\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}} \\
&+12\Omega a_3 b_4^2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}} + 2\Omega a_3\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,1\}} + 4\Omega a_3 b_4\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} \\
&+16\Omega^2 a_2\varpi_{\{1,2\}}^3\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 4\Omega a_3 b_5\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 4\Omega a_3 b_4\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} \\
&+4\Omega a_3 b_3\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 8\Omega\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 8\Omega a_1\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} \\
&+2\Omega\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,1\}} + 2\Omega a_1\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,1\}} + 6\Omega^2 a_3\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2 \\
&+8\Omega a_4\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{3,1\}}^2 + 8\Omega a_5\varpi_{\{1,4\}}^2\varpi_{\{2,1\}}^2\varpi_{\{2,5\}} + 4\Omega a_5\varpi_{\{1,4\}}\varpi_{\{2,1\}}\varpi_{\{2,2\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}} \\
&+4\Omega a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}\varpi_{\{3,2\}} + 2\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}} + 2\Omega a_1\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}} = 0, \\
&80a_2 b_4\Omega\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4 + 36a_3 b_4\Omega\varpi_{\{1,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2 + 24a_3 b_4\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}} \\
&+2a_3 b_3\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{1,1\}}^2 + 4a_3 b_4^3\varpi_{\{1,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2 + 2a_3 b_5\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{1,1\}}^2 \\
&+4a_3 b_4 b_5\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 2a_3 b_4^2\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 4a_3 b_3 b_4\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 \\
&+4a_1 b_4\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 4a_4 b_4\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}^2 + 4a_5 b_4\varpi_{\{1,4\}}\varpi_{\{2,1\}}^2\varpi_{\{2,5\}} \\
&+40a_2\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{1,1\}}^4 + 18a_3\Omega\varpi_{\{1,3\}}\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2 + 4a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} \\
&+8a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}} + 2a_1\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}} + 2a_5\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,1\}}^2
\end{aligned}
$$

$$+2a_4\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{3,1\}}^2+4b_4\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}+2\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}=0,$$

$$18a_3b_4^2\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+6a_3b_5\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+6a_3b_3\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2$$
$$+12a_3b_4\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+240a_2\Omega\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4+48a_2\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^3$$
$$+16a_3\Omega\varpi_{\{1,4\}}^4\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+12a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}+3a_3\varpi_{\{1,3\}}^2\varpi_{\{1,5\}}^2\varpi_{\{1,1\}}^2$$
$$+6a_1\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}+6a_4\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{3,1\}}^2+6a_5\varpi_{\{1,4\}}^2\varpi_{\{2,1\}}^2\varpi_{\{2,5\}}+6\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}=0,$$

$$48a_2b_4\varpi_{\{1,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4+24a_3b_4\varpi_{\{1,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+24a_2\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{1,1\}}^4+12a_3\varpi_{\{1,3\}}\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2=0,$$

$$120a_2\varpi_{\{1,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4+10a_3\varpi_{\{1,4\}}^4\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2=0,$$

$$32a_2b_4\Omega^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4+12a_3b_4\Omega^2\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+24a_3b_4\Omega^2\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}$$
$$+12a_3b_4\Omega^2\varpi_{\{1,1\}}^2\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2+4a_3b_4^3\Omega\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2$$
$$+2a_3b_3\Omega\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,2\}}^2+2a_3b_5\Omega\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{1,2\}}^2+2a_3b_4^2\Omega\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2$$
$$+4a_3b_4b_5\Omega\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_3b_3b_4\Omega\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_1b_4\Omega\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}$$
$$+4a_4b_4\Omega\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,2\}}^2+4a_5b_4\Omega\varpi_{\{2,2\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}+16a_2\Omega^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{1,2\}}^4$$
$$+6a_3\Omega^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_3\Omega^2\varpi_{\{1,1\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}+8a_3\Omega^2\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}$$
$$+2a_3\Omega^2\varpi_{\{1,1\}}^2\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}+4a_3\Omega^2\varpi_{\{1,1\}}^2\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}$$
$$+2a_1\Omega\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+2a_4\Omega\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{3,2\}}^2+2a_5\Omega\varpi_{\{1,5\}}\varpi_{\{2,2\}}^2\varpi_{\{2,3\}}+$$
$$4b_4\Omega\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+2\Omega\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}=0,$$

$$32\Omega^2a_2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4+12\Omega^2a_3\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+12\Omega a_3b_4^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2$$
$$+2\Omega a_3\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,1\}}^2+4\Omega a_3b_4\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+48\Omega^2a_2\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2$$
$$+4\Omega a_3b_5\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4\Omega a_3b_4\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4\Omega a_3b_3\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2$$
$$+24\Omega^2a_3\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}+4\Omega\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}+4\Omega a_1\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}$$
$$+12\Omega^2a_3\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2+12\Omega a_3b_4^2\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2+4\Omega a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}^2$$
$$+4\Omega a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,2\}}^2+2\Omega a_3\varpi_{\{1,2\}}^2\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}+4\Omega a_3b_4\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}$$
$$+4\Omega a_5\varpi_{\{1,4\}}\varpi_{\{2,1\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}$$
$$+4\Omega a_5\varpi_{\{1,4\}}\varpi_{\{2,2\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}+32\Omega^2a_2\varpi_{\{1,2\}}^4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}$$
$$+4\Omega a_3b_5\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}+4\Omega a_3b_4\varpi_{\{1,2\}}^2\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}+4\Omega a_3b_3\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}$$
$$+4\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}+4\Omega a_1\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}=0,$$

$$48a_3b_4\Omega\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+24a_3b_4\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}+12a_3b_4\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2$$
$$+8a_3\Omega\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+16a_3\Omega\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}$$
$$+8a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}+2a_3\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}+4a_3\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}=0,$$

$$12a_3b_4^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+4a_3b_4\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4a_3b_5\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2$$
$$+4a_3b_4\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4a_3b_3\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+80a_2\Omega\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4$$
$$+36a_3\Omega\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+48a_2\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+24a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}$$
$$+4a_3\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2+2a_3\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,1\}}^2+4a_1\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}$$
$$+4a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}^2+4a_5\varpi_{\{1,4\}}\varpi_{\{2,1\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}+4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,1\}}=0,$$

$$36a_3b_4\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+6a_3\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+12a_3\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2=0,$$

$$48a_2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^4+24a_3\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2=0,$$

$$136\Omega^2a_2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4+64\Omega^2a_2\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^3+4\Omega a_3\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}^2\varpi_{\{1,2\}}^2$$
$$+6\Omega^2a_3\varpi_{\{2,4\}}^4\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+24\Omega a_3b_4^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+8\Omega a_3b_5\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2$$
$$+8\Omega a_3b_3\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+16\Omega a_3b_4\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+12\Omega^2a_3\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}$$
$$+12\Omega a_3b_4^2\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}+2\Omega a_3\varpi_{\{1,1\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,2\}}+4\Omega a_3b_4\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}$$
$$+16\Omega^2a_2\varpi_{\{1,1\}}^3\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}+4\Omega a_3b_5\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}+4\Omega a_3b_4\varpi_{\{1,1\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}$$
$$+4\Omega a_3b_3\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}+2\Omega\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,2\}}+2\Omega a_1\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}\varpi_{\{1,2\}}$$
$$+8\Omega\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+8\Omega a_1\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+6\Omega^2a_3\varpi_{\{1,1\}}^2\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2$$

$$
\begin{aligned}
&+8\Omega a_4\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{3,2\}}^2+8\Omega a_5\varpi_{\{2,2\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}+4\Omega a_5\varpi_{\{1,4\}}\varpi_{\{2,1\}}\varpi_{\{2,2\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\\
&+4\Omega a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}\varpi_{\{3,2\}}+2\Omega\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}+2\Omega a_1\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}=0,\\
&12a_3b_4\Omega\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+24a_3b_4\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}+48a_3b_4\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\\
&+2a_3\Omega\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4a_3\Omega\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}\\
&+8a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}+8a_3\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}+16a_3\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}=0,\\
&12a_3b_4^2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}+4a_3b_4\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}+4a_3b_5\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}\\
&+4a_3b_4\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}+4a_3b_3\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}+64a_2\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^3\\
&+24a_3\Omega\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+12a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}+12a_3\Omega\varpi_{\{1,2\}}\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}\\
&+64a_2\Omega\varpi_{\{1,2\}}^3\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}+24a_3\Omega\varpi_{\{1,2\}}^2\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2+2a_3\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,1\}}\\
&+2a_1\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,1\}}+4a_5\varpi_{\{1,4\}}\varpi_{\{2,1\}}\varpi_{\{2,2\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}+4a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,1\}}\varpi_{\{3,2\}}\\
&+2a_1\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}+2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,1\}}+2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,1\}}=0,\\
&12a_3b_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+24a_3b_4\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}+2a_3\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2\\
&+4a_3\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^2+4a_3\varpi_{\{1,2\}}\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}+8a_3\varpi_{\{1,2\}}\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}=0,\\
&48a_2\varpi_{\{1,2\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,1\}}^3+18a_3\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}^2+12a_3\varpi_{\{1,2\}}\varpi_{\{1,4\}}^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,1\}}=0,\\
&80a_2b_4\Omega\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4+36a_3b_4\Omega\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+24a_3b_4\Omega\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}+4a_3b_4^3\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2\\
&+2a_3b_3\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,2\}}^2+2a_3b_5\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{1,2\}}^2+2a_3b_4^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2\\
&+4a_3b_4b_5\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_3b_3b_4\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_1b_4\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}\\
&+4a_4b_4\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,2\}}^2+4a_5b_4\varpi_{\{2,2\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}+40a_2\Omega\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{1,2\}}^4\\
&+18a_3\Omega\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_3\Omega\varpi_{\{1,1\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}+8a_3\Omega\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}\\
&+2a_1\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+2a_4\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{3,2\}}^2+2a_5\varpi_{\{1,5\}}\varpi_{\{2,2\}}^2\varpi_{\{2,3\}}+4b_4\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}=0,\\
&12a_3b_4^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+4a_3b_4\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_3b_5\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2\\
&+4a_3b_4\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_3b_3\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+80a_2\Omega\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4\\
&+36a_3\Omega\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+48a_2\Omega\varpi_{\{1,1\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+24a_3\Omega\varpi_{\{1,1\}}\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}\\
&+4a_3\Omega\varpi_{\{1,1\}}^2\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2+2a_3\varpi_{\{1,3\}}\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}\varpi_{\{1,2\}}^2+4a_1\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}\\
&+4a_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{3,2\}}^2+4a_5\varpi_{\{1,4\}}\varpi_{\{2,2\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}+4\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}=0,\\
&12a_3b_4\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+24a_3b_4\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}+2a_3\varpi_{\{1,4\}}^2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2\\
&+4a_3\varpi_{\{1,3\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+4a_3\varpi_{\{1,1\}}\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}+8a_3\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}=0,\\
&4a_3\varpi_{\{1,2\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}^2\varpi_{\{1,4\}}^3+24a_3\varpi_{\{1,1\}}\varpi_{\{1,2\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,4\}}^2+4a_3\varpi_{\{1,1\}}^2\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,4\}}+48a_2\varpi_{\{1,1\}}^2\varpi_{\{1,2\}}^2\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,4\}}=0,\\
&18a_3b_4^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+6a_3b_5\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+6a_3b_3\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2\\
&+12a_3b_4\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+240a_2\Omega\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4+48a_2\Omega\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^3\\
&+16a_3\Omega\varpi_{\{2,4\}}^4\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+12a_3\Omega\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}+3a_3\varpi_{\{1,5\}}^2\varpi_{\{2,3\}}^2\varpi_{\{1,2\}}^2\\
&+6a_1\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}+6a_4\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{3,2\}}^2+6a_5\varpi_{\{2,2\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}+6\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{4,2\}}\varpi_{\{1,2\}}=0,\\
&36a_3b_4\varpi_{\{1,4\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+6a_3\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2+12a_3\varpi_{\{1,4\}}\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2=0,\\
&48a_2\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^3+18a_3\varpi_{\{1,4\}}^2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+12a_3\varpi_{\{1,1\}}\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}=0,\\
&48a_2b_4\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4+24a_3b_4\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2+24a_2\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{1,2\}}^4+12a_3\varpi_{\{1,5\}}\varpi_{\{2,3\}}\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^2=0,\\
&48a_2\varpi_{\{1,4\}}\varpi_{\{2,4\}}\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4+24a_3\varpi_{\{1,4\}}\varpi_{\{2,4\}}^3\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2=0,\ 120a_2\varpi_{\{2,4\}}^2\varpi_{\{2,5\}}\varpi_{\{1,2\}}^4+10a_3\varpi_{\{2,4\}}^4\varpi_{\{2,5\}}^2\varpi_{\{1,2\}}^2=0.
\end{aligned}
$$

After using the computational Mathematica software we get following set:

Set 1.

$$
\varpi_{\{2,3\}}=0,\varpi_{\{2,4\}}=0,\varpi_{\{2,5\}}=-\frac{12a_2\varpi_{\{1,1\}}^2}{a_3\varpi_{\{1,4\}}^2},b_4=\frac{\varpi_{\{1,3\}}\varpi_{\{1,5\}}\varpi_{\{1,4\}}a_3}{24\varpi_{\{1,1\}}^2a_2},
$$

$$
b_5=-\frac{1}{48\varpi_{\{1,1\}}^2a_3a_2}(384\,\Omega\,\varpi_{\{1,1\}}^4{a_2}^2+48\,\varpi_{\{1,1\}}^2\varpi_{\{1,5\}}a_2a_3b_3-\varpi_{\{1,3\}}^2\varpi_{\{1,5\}}^2{a_3}^2
$$

$$+ 48\,\varpi_{\{1,1\}}\varpi_{\{1,4\}}a_1a_2 + 48\,\varpi_{\{1,2\}}^2a_2a_5 + 48\,\varpi_{\{1,3\}}^2a_2a_4 + 48\,\varpi_{\{1,1\}}\varpi_{\{1,4\}}a_2). \quad (3.17)$$

Case I. $\Omega < 0$.

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(b_4 - \sqrt{-\Omega}\varpi_{\{1,4\}}\tanh\left(\sqrt{-\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right)\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(b_3 - \sqrt{-\Omega}\varpi_{\{1,3\}}\tanh\left(\sqrt{-\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right)\right) + b_5, \quad (3.18)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(b_4 - \sqrt{-\Omega}\varpi_{\{1,4\}}\coth\left(\sqrt{-\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right)\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(b_3 - \sqrt{-\Omega}\varpi_{\{1,3\}}\coth\left(\sqrt{-\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right)\right) + b_5, \quad (3.19)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\sqrt{-\Omega}\varpi_{\{1,4\}}\left(-\tanh\left(2\sqrt{-\Omega}\varsigma_1\right) \pm i\,\mathrm{sech}\left(2\sqrt{-\Omega}\varsigma_1\right)\right) + b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\sqrt{-\Omega}\varpi_{\{1,3\}}\left(-\tanh\left(2\sqrt{-\Omega}\varsigma_1\right) \pm i\,\mathrm{sech}\left(2\sqrt{-\Omega}\varsigma_1\right)\right) + b_3\right) + b_5, \quad (3.20)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\sqrt{-\Omega}\varpi_{\{1,4\}}\left(-\coth\left(2\sqrt{-\Omega}\varsigma_1\right) \pm i\,\mathrm{csch}\left(2\sqrt{-\Omega}\varsigma_1\right)\right) + b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\sqrt{-\Omega}\varpi_{\{1,3\}}\left(-\coth\left(2\sqrt{-\Omega}\varsigma_1\right) \pm i\,\mathrm{csch}\left(2\sqrt{-\Omega}\varsigma_1\right)\right) + b_3\right) + b_5, \quad (3.21)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(b_4 - \frac{1}{2}\sqrt{-\Omega}\varpi_{\{1,4\}}\left(\tanh\left(\frac{1}{2}\sqrt{-\Omega}\varsigma_1\right) + \coth\left(\frac{1}{2}\sqrt{-\Omega}\varsigma_1\right)\right)\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(b_3 - \frac{1}{2}\sqrt{-\Omega}\varpi_{\{1,3\}}\left(\tanh\left(\frac{1}{2}\sqrt{-\Omega}\varsigma_1\right) + \coth\left(\frac{1}{2}\sqrt{-\Omega}\varsigma_1\right)\right)\right) + b_5. \quad (3.22)$$

Case II. $\Omega > 0$.

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\sqrt{\Omega}\varpi_{\{1,4\}}\tan\left(\sqrt{\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right) + b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\sqrt{\Omega}\varpi_{\{1,3\}}\tan\left(\sqrt{\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right) + b_3\right) + b_5, \quad (3.23)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\sqrt{\Omega}\varpi_{\{1,4\}}\cot\left(\sqrt{\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right) + b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\sqrt{\Omega}\varpi_{\{1,3\}}\cot\left(\sqrt{\Omega}\left(t\varpi_{\{4,1\}} + x\varpi_{\{1,1\}} + y\varpi_{\{2,1\}} + z\varpi_{\{3,1\}} + b_1\right)\right) + b_3\right) + b_5, \quad (3.24)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\sqrt{\Omega}\varpi_{\{1,4\}}\left(\tan\left(2\sqrt{\Omega}\varsigma_1\right)\pm\sec\left(2\sqrt{\Omega}\varsigma_1\right)\right)+b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\sqrt{\Omega}\varpi_{\{1,3\}}\left(\tan\left(2\sqrt{\Omega}\varsigma_1\right)\pm\sec\left(2\sqrt{\Omega}\varsigma_1\right)\right)+b_3\right)+b_5, \quad (3.25)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\sqrt{\Omega}\varpi_{\{1,4\}}\left(-\coth\left(2\sqrt{\Omega}\varsigma_1\right)\pm\operatorname{csch}\left(2\sqrt{\Omega}\varsigma_1\right)\right)+b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\sqrt{\Omega}\varpi_{\{1,3\}}\left(-\coth\left(2\sqrt{\Omega}\varsigma_1\right)\pm\operatorname{csch}\left(2\sqrt{\Omega}\varsigma_1\right)\right)+b_3\right)+b_5, \quad (3.26)$$

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\frac{1}{2}\sqrt{\Omega}\varpi_{\{1,4\}}\left(\tan\left(\frac{\sqrt{\Omega}\varsigma_1}{2}\right)-\cot\left(\frac{\sqrt{\Omega}\varsigma_1}{2}\right)\right)+b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\frac{1}{2}\sqrt{\Omega}\varpi_{\{1,3\}}\left(\tan\left(\frac{\sqrt{\Omega}\varsigma_1}{2}\right)-\cot\left(\frac{\sqrt{\Omega}\varsigma_1}{2}\right)\right)+b_3\right)+b_5. \quad (3.27)$$

Case III. $\Omega = 0$.

$$v(x,y,z,t) = -\frac{12a_2\varpi_{\{1,1\}}^2\left(\frac{\varpi_{\{1,4\}}}{t\varpi_{\{4,1\}}+x\varpi_{\{1,1\}}+y\varpi_{\{2,1\}}+z\varpi_{\{3,1\}}+b_1}+b_4\right)^2}{a_3\varpi_{\{1,4\}}^2} + \varpi_{\{1,5\}}\left(\frac{\varpi_{\{1,3\}}}{t\varpi_{\{4,1\}}+x\varpi_{\{1,1\}}+y\varpi_{\{2,1\}}+z\varpi_{\{3,1\}}+b_1}+b_3\right)+b_5. \quad (3.28)$$

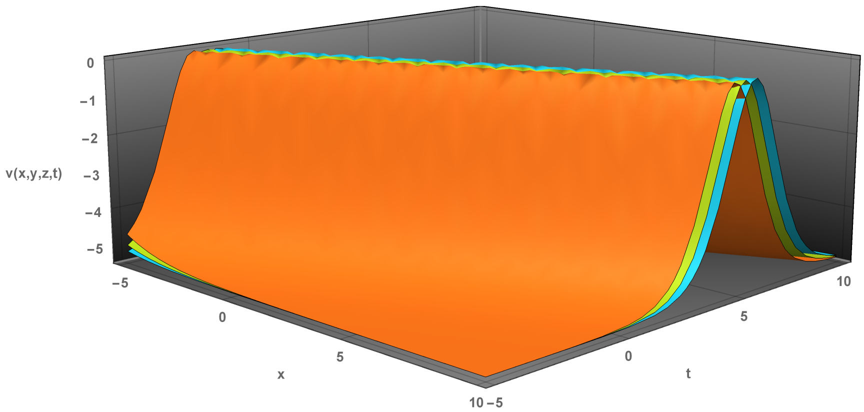
Verification: To ensure the validity and correctness of the obtained solutions, each solution is substituted back into the governing equation and verified symbolically. For instance, for the solution $v(x,y,z,t)$ given in (1.1), along with its corresponding derivatives, we obtain

$$\begin{aligned}\text{LHS} &= (v_t + a_1 v\, v_x + a_2 v_{xxx} + a_3 v_x)_x + a_4 v_{yy} + a_5 v_{zz}\\ &= 0,\end{aligned}$$

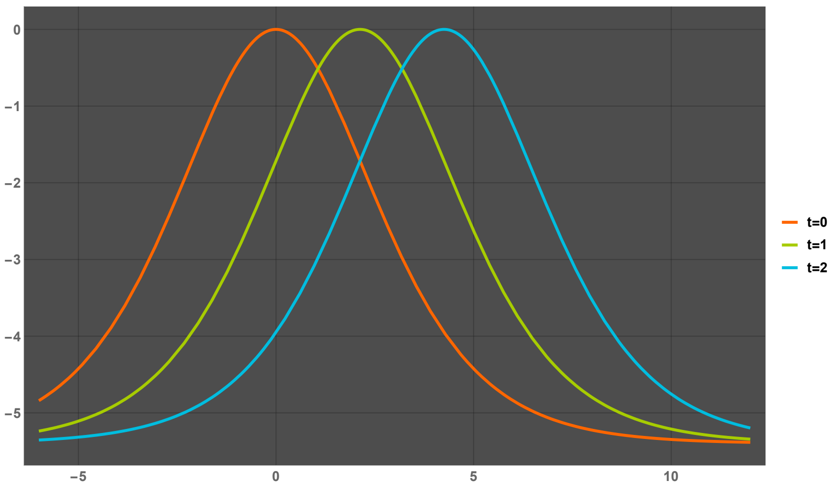
which confirms that the solution satisfies the governing equation identically. Similar symbolic substitutions and verifications have been performed for all other solutions presented in this work to ensure their accuracy and consistency.

4. Graphs

This section represents the wave dynamics of the obtained solutions in different behavior like bright, singular, and bright-dark. Figure 4 shows the bright behavior solution putting suitable numeric values of the parameters $a_1 = 1.7, a_3 = 0.1, b_2 = 0, b_3 = 0, \varpi_{\{1,2\}} = 0.3, \Omega = -1, a_2 = 0.5, a_5 = 0.7, \varpi_{\{2,2\}} = 0.3, a_4 = 0.6, \varpi_{\{3,2\}} = 0.9, and\ z = 0$. The solution is depicted through both 3D surface and 2D plots over the domain $x \in [-5, 10]$, where the orange, green, and blue curves correspond to $t = 0$, $t = 1$, and $t = 2$, respectively. Figure 5 shows the singular behavior solution putting suitable numeric values of the parameters $a_1 = 1.7, a_3 = -1.2, b_2 = -0.5, b_3 = -0.5, \varpi_{\{1,2\}} = 0.3, \Omega = -1, a_2 = 1.4, a_5 = -0.3, \varpi_{\{2,2\}} = 0.2, a_4 = 0.6, \varpi_{\{3,2\}} = 0.9$, and $z = 0$. The solution is depicted through both 3D surface and 2D plots over the domain $x \in [0, 10]$, where the orange, green, and blue curves correspond to $t = 0$, $t = 1$, and $t = 2$, respectively. Figure 6 shows the bright-dark behavior solution taking suitable numeric values of the parameters $a_1 = 1.7, a_3 = -1.2, b_2 = -0.5, b_3 = -0.5, \varpi_{\{1,2\}} = 0.3, \Omega = -1, a_2 = 1.4, a_5 = -0.3, \varpi_{\{2,2\}} = 0.2, a_4 = 0.6, \varpi_{\{3,2\}} = 0.9$, and $z = 0$. The solution is depicted through both 3D surface and 2D plots over the domain $x \in [-10, 15]$, where the orange, green, and blue curves correspond to $t = 0$, $t = 1$, and $t = 2$, respectively. Figure 7 shows the bright behavior solution putting suitable numeric values of the parameters $a_1 = 1.7, a_3 = 0.1, b_2 = 0, b_3 = 0, \varpi_{\{1,2\}} = 0.3, \Omega = -1, a_2 = 0.5, a_5 = 0.7, \varpi_{\{2,2\}} = 0.3, a_4 = 0.6, \varpi_{\{3,2\}} = 0.9$, and $z = 0$. The solution is depicted through both 3D surface and 2D plots over the domain $x \in [-5, 10]$, where the orange, green, and blue curves correspond to $t = 0$, $t = 1$, and $t = 2$, respectively.

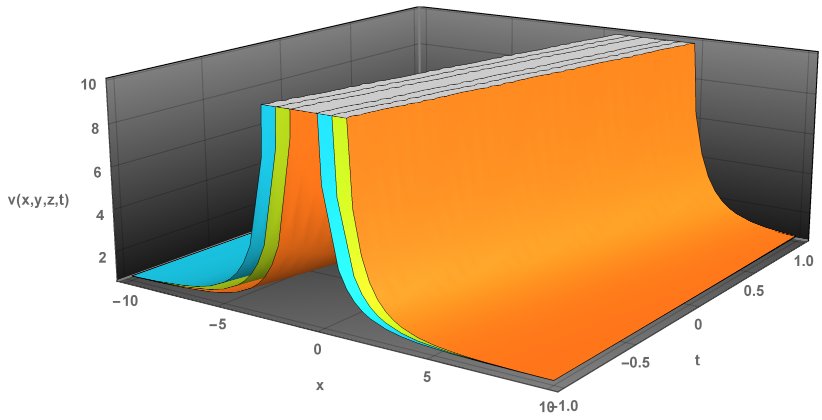


(a)

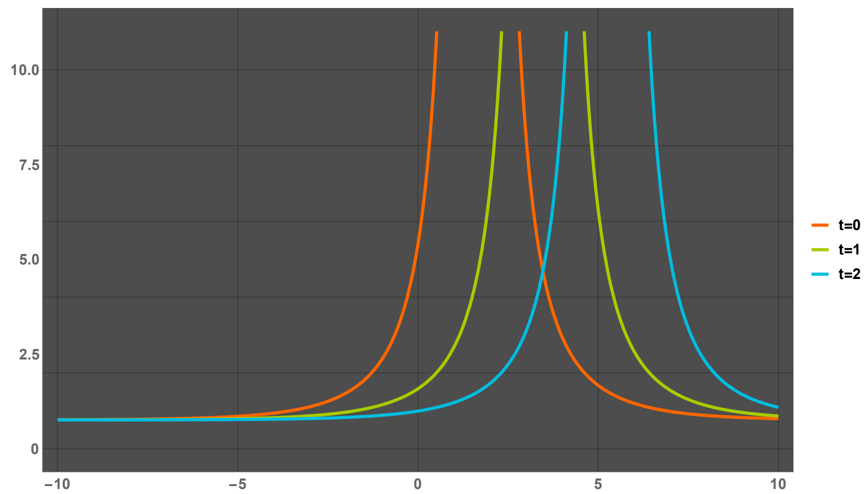


(b)

Figure 4. The plot shows the bright-type solution in 3D and 2D surfaces for the v(x,0,0,t) in orange, v(x,1,0,t) in green, and v(x,2,0,t) in blue, as appears in Eq (3.4).

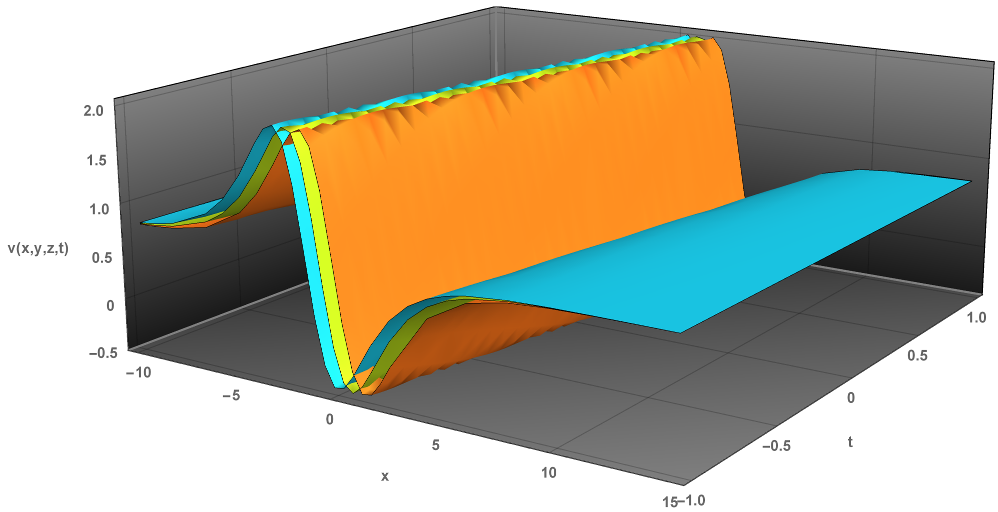


(a)

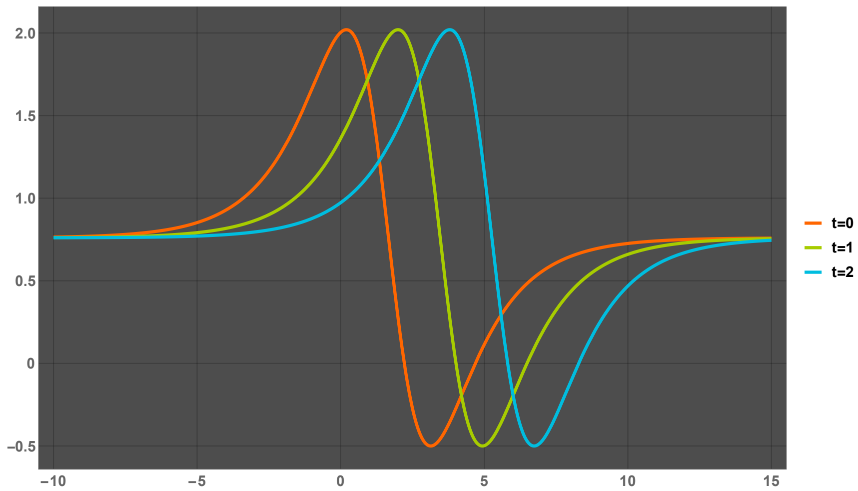


(b)

Figure 5. The plot shows the singular-type solution in 3D and 2D surfaces for the v(x,0,0,t) in orange, v(x,1,0,t) in green, and v(x,2,0,t) in blue, as appears in Eq (3.5).



(a)



(b)

Figure 6. The plot shows the bright-dark-type solution in 3D and 2D surfaces for the v(x,0,0,t) in orange, v(x,1,0,t) in green, and v(x,2,0,t) in blue, as appears in Eq (3.6).

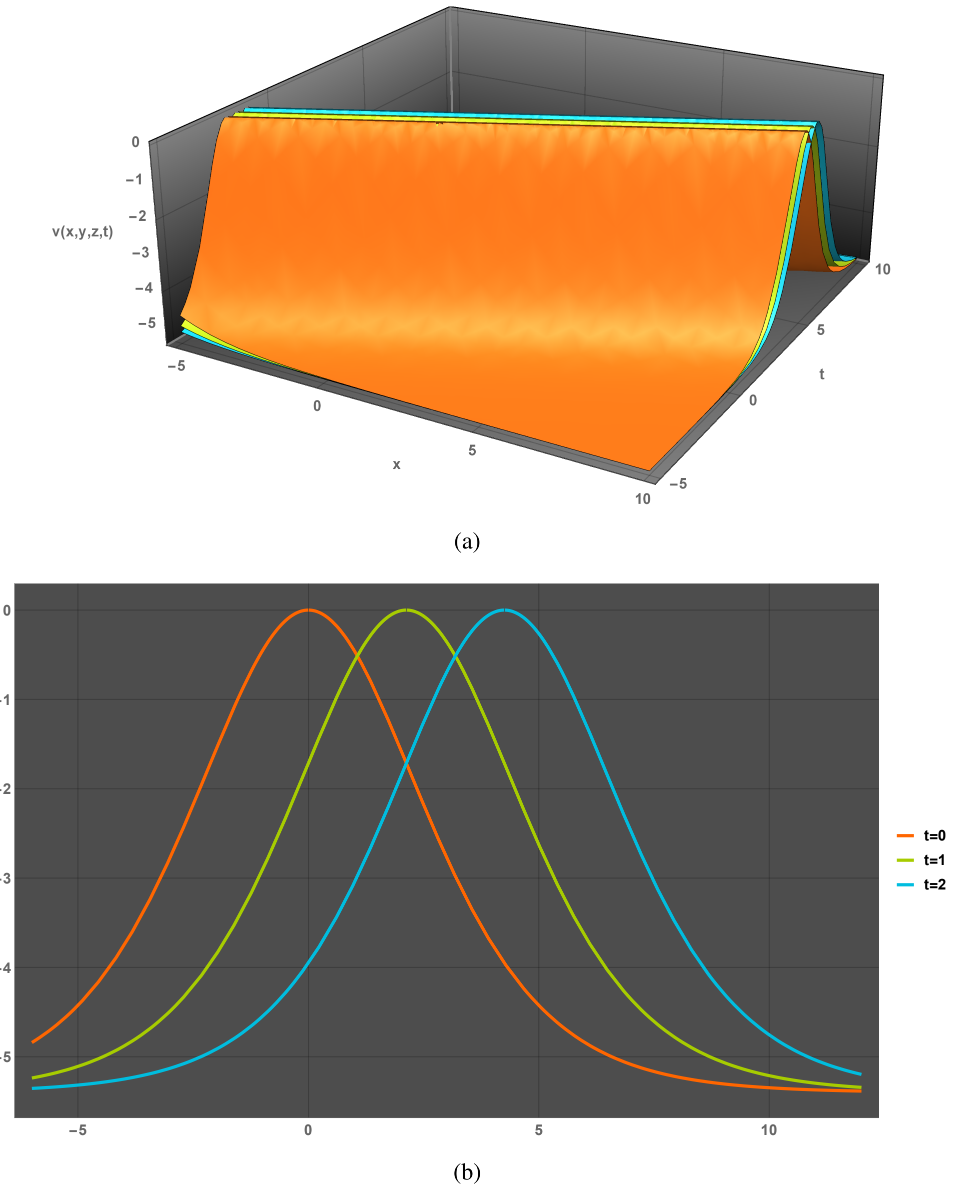


Figure 7. The plot shows the bright-type solution in 3D and 2D surfaces for the v(x,0,0,t) in orange, v(x,1,0,t) in green, and v(x,2,0,t) in blue, as appears in Eq (3.7).

4.1. Physical interpretation and practical relevance

Although the present study is primarily focused on the analytical construction of solutions, the obtained results possess significant practical relevance in various applied fields. In fluid dynamics, the derived solutions can be used to describe nonlinear wave propagation in compressible fluids containing gas bubbles, where phenomena such as wave steepening, dispersion, and soliton formation are commonly observed. In the context of nonlinear optics, the solutions provide insight into the behavior of optical pulses in nonlinear media, where similar mathematical models arise in the study of pulse propagation, modulation instability, and energy localization. Moreover, in engineering applications, the obtained wave structures can contribute to the analysis of stability and control of nonlinear wave phenomena in systems such as transmission lines, plasma waves, and mechanical vibrations. The availability of exact and analytical solutions also offers reliable benchmarks for testing numerical algorithms and validating computational models. Therefore, the proposed WAS-NN framework not only advances the theoretical understanding of nonlinear partial differential equations, but also supports practical modeling and analysis in a wide range of scientific and engineering applications.

5. Stability analysis

The stability analysis verifies whether the obtained solutions are stable or unstable. In this section, we investigate the stability of the derived solutions using the newly proposed neural network technique in conjunction with the Hamiltonian approach [28].

The momentum functional associated with the Hamiltonian structure is defined as

$$\mathfrak{M}(\varpi_{\{4,2\}}) = \frac{1}{2}\int_{-\infty}^{\infty} v(x,y,z,t)^2\,dx. \tag{5.1}$$

The stability criterion is given by

$$\frac{\partial \mathfrak{M}}{\partial \varpi_{\{4,2\}}} > 0, \tag{5.2}$$

where $\mathfrak{M}$ denotes the momentum and $\varpi_{\{4,2\}}$ represents the speed of waveform. If this condition holds, the obtained solution is considered stable.

Substituting the solution given in Eq (3.5) into the above Eq (5.1), and evaluating the integral over the finite interval $x \in [-1, 10]$, we obtain

$$\begin{aligned}\mathfrak{M}(\varpi_{\{4,2\}}) = \frac{12a_2\Omega\varpi_{\{1,2\}}^2}{a_3}\Big(&\frac{\sqrt{-\Omega}}{\Omega\varpi_{\{1,2\}}}\Big(\coth\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2)\Big)\\ &- \coth\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} - \varpi_{\{1,2\}} + b_2)\Big)\Big) + \Omega\varpi_{\{1,2\}}\\ &+ \frac{\sqrt{-\Omega}}{\Omega\varpi_{\{1,2\}}}\coth\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + 10\varpi_{\{1,2\}} + b_2)\Big)\\ &+ \Big(-\sqrt{-\Omega}\Big)\coth\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2)\Big) + 10\Omega\varpi_{\{1,2\}}\Big) + 11b_3.\end{aligned} \tag{5.3}$$

To examine the stability condition, we differentiate Eq (5.3) with respect to $\varpi_{\{4,2\}}$. This yields

$$\begin{aligned}\frac{\partial \mathfrak{M}}{\partial \varpi_{\{4,2\}}} = \frac{6a_2\Omega\varpi_{\{1,2\}}^2}{a_3}\Big(&\frac{t\Omega}{\Omega\varpi_{\{1,2\}}}\mathrm{csch}^2\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + 10\varpi_{\{1,2\}} + b_2)\Big)\\ &- t\Omega\,\mathrm{csch}^2\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2)\Big)\\ &+ \frac{\sqrt{-\Omega}}{\Omega\varpi_{\{1,2\}}}\Big(t\sqrt{-\Omega}\,\mathrm{csch}^2\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} - \varpi_{\{1,2\}} + b_2)\Big)\\ &- t\sqrt{-\Omega}\,\mathrm{csch}^2\Big(\sqrt{-\Omega}\,(t\varpi_{\{4,2\}} + y\varpi_{\{2,2\}} + z\varpi_{\{3,2\}} + b_2)\Big)\Big)\Big).\end{aligned} \tag{5.4}$$

For the selected parameter values, the numerical evaluation gives

$$\frac{\partial \mathfrak{M}}{\partial \varpi_{\{4,2\}}} = 0.696112 > 0.$$

Since the derivative of the momentum functional with respect to the velocity parameter is positive, the stability condition in Eq (5.2) is satisfied. Therefore, the obtained solution is stable.

6. Conclusions

In summary, the present research has managed to employ the WAS-NN technique to explore the generalized nonlinear wave model of the (3 + 1)-dimensional equation. The suggested computation system was capable of fully describing a wide variety of nonlinear waves such as bright, singular, and bright-dark soliton solutions. These neural-network-based study of the model under consideration have not been reported before to the best of our knowledge. The effect of parameters significant to the wave dynamics were vividly depicted by three-dimensional surface and two-dimensional plots, which verified the quality and strength of the WAS-NN methodology. In addition stability analysis applied on the gain solutions to check they are stable, this shows suggested method gives an effective alternative to classical methods of analysis to higher-dimensional nonlinear systems. The findings lead to the computational insights of nonlinear waves and show that neural network-based solvers are applicable in fluid mechanics and engineering processes including gas-liquid interactions. Several promising directions for future research can be identified based on the present study. First, the proposed WAS-NN framework can be extended to investigate higher-dimensional and more complex nonlinear partial differential equations arising in physics and engineering. Second, the incorporation of stochastic effects into the WAS-NN formulation would allow the analysis of systems influenced by randomness, such as those involving Brownian motion and noise-driven wave propagation, as well as also fractional derivative effects.

Author contributions

All authors contributed equally to this work. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflicts of interest

The authors declare that they have no conflict of interest.

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