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Research article

Lyapunov stability of nonlinear systems with impulsive disturbances and delayed impulses via event-triggered impulsive control

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Abstract: In this research, we used an event-triggered impulsive control (ETIC) approach to study the Lyapunov stability of nonlinear systems subject to impulsive disturbances and impulse delays. Based on impulsive control theory, a novel event-triggering mechanism (ETM) that incorporates impulsive disturbance information was developed. The proposed ETM adopts an intermittent monitoring scheme, under which the impulsive control instants are autonomously dictated, ensuring that the controller is activated exactly once between two consecutive impulsive perturbations. As a result, the divergent dynamic behaviors induced by disturbances can be rapidly and effectively restrained. Furthermore, sufficient conditions were derived to exclude the occurrence of Zeno behavior and to guarantee the global asymptotic stability (GAS) of the closed-loop system under the ETIC framework. In addition, by utilizing linear matrix inequality (LMI) techniques, the synchronization of chaotic systems was successfully achieved. Theoretical analysis revealed that the stability of the system depends not only on the designed ETM but also on the presence of impulse delays. Finally, two numerical simulation examples were provided to demonstrate the effectiveness and feasibility of the proposed approach.

Keywords: Lyapunov stability; delay impulsive systems; Zeno behavior; impulsive disturbance; event-triggered impulsive control
Mathematics Subject Classification: 93C10

1. Introduction

Impulsive systems, as an important class of hybrid systems, have received considerable interest in recent years owing to their broad applicability across numerous domains, including biological systems, satellite engineering and control engineering [1–3]. The stability analysis of such systems serves as the core topic of numerous studies, and relevant results have been thoroughly investigated in a large number of studies [4–6]. In general, from the viewpoint of impulsive influences, the existing literature on stability analysis of impulsive systems can be broadly classified into two categories, namely, impulsive perturbation and impulsive control. Specifically, impulsive perturbation mainly focuses on the robustness of systems under destabilizing impulsive effects, while impulsive control aims to stabilize systems by applying stabilizing impulses.

Impulsive control, regarded as a class of control strategies characterized by discontinuous state updates, has been extensively applied in practical scenarios such as bioengineering [1] and satellite orbit transfer [2] owing to its unique advantages including low control cost, strong confidentiality, and great robustness. The determination of controller updating times is a key factor in the design of impulsive control systems, which is essential for improving the expected performance of the system. A direct and intuitive solution is to design a suitable time-based triggering scheme [7, 8]. Time-triggered impulsive control, which is developed under the time-triggered control framework, has attracted widespread attention and has been studied extensively. Under this strategy, the control signals are updated according to a pre-set time interval. Time-triggered control has been widely applied due to its simple implementation and favorable stability. However, a fixed sampling period may lead to unnecessary resource waste in the communication process.

To overcome this limitation, event-triggered control (ETC) has been recognized as an effective control strategy in recent years [9–11]. Under the ETC framework, control actions are executed only when certain triggering conditions are satisfied, such as the system state reaching a specified threshold or the control error exceeding a predefined bound. Compared with conventional time-triggered schemes, this strategy can significantly reduce the frequency of control updates, thereby minimizing unnecessary resource consumption to the greatest extent. In practical applications, researchers have proposed various ETC methods to meet the specific requirements of control systems. For instance, in [12], a dynamic event-triggered adaptive output feedback tracking control scheme was proposed for multi-agent systems subject to time-varying input delays. In [13], a new ETC strategy was proposed for nonlinear feedback systems based on a dynamic event-triggering mechanism (ETM), which can significantly prolong the average trigger interval while ensuring system stability and L_p gain performance. Moreover, for four-wheel independent steering systems, Luan et al. [14] proposed an event-triggered stability control strategy based on dynamic lateral force redistribution to achieve adaptive and smooth switching between different steering modes while ensuring the system's maneuverability and stability.

Nevertheless, most of the above studies focus on specific types of systems, which limits their universality. In particular, when the system state undergoes abrupt changes at specific instants (i.e., impulsive behavior occurs), the above methods may no longer be applicable, posing challenges to both theoretical analysis and practical applications. The event-triggered impulsive control (ETIC) approach integrates the strengths of both impulsive control and ETC, thereby achieving complementary benefits from the combination of the two methods [15, 16]. Specifically, the impulsive control action is activated only when the system dynamic characteristics break through a pre-set threshold or satisfy a state-dependent trigger event. Between two adjacent impulsive control instants, the system is not subject to any control intervention. This design enables ETIC to perform control tasks according to the actual demands of system dynamics, thereby effectively alleviating the problem of communication transmission redundancy caused by time-triggered impulsive control. Lately, several traditional ETIC techniques have been developed and discussed in [17–19]. Among them, an event-based impulsive control strategy was developed to guarantee the Lyapunov stability of nonlinear dynamical systems in [17], but this study neglected the possible perturbations in the system.

In practical systems, disturbances are unavoidable and may significantly degrade system performance. To address these multifaceted challenges, researchers have proposed sophisticated control strategies for such systems. For instance, considering the nonlinearities and external disturbances in direct-drive electro-hydraulic actuators, Tan et al. [20] proposed a composite control method combining model predictive control and improved active disturbance rejection control, which effectively improved the dynamic response and disturbance rejection ability of the system. Similarly, in the trajectory tracking control of mobile robots, Chen et al. [21] designed a robust iterative learning control scheme based on a neural network compensator for unknown parameters and external disturbances, achieving high-precision tracking performance. Although these methods effectively deal with continuous nonlinearities and uncertainties, their control frameworks do not specifically analyze or mitigate the destructive intermittent impulsive disturbances in these scenarios.

To address the impact of impulsive perturbations, the academic community has recently proposed numerous control strategies aimed at stabilizing systems and analyzing their stability under such perturbations in [22–24]. For example, Liu et al. [23] achieved the asymptotic stability of reaction–diffusion systems under disturbances by designing an event-triggered impulsive control mechanism with Zeno-free conditions and deriving stability criteria based on impulsive control theory. Fang and Li [24] presented a novel ETM integrating impulsive disturbance information, which guaranteed that the impulsive controller was triggered exactly once during the interval between two successive impulsive disturbances, thereby effectively suppressing the divergent behavior induced by impulsive perturbations.

Although considerable advancements have been achieved in the study of ETIC, most existing studies largely ignore the delay issue. Impulsive delays play a crucial role in the design and analysis of impulsive systems because the sampling, transmission, and processing of impulsive signals require a certain amount of time [25–27]. Recently, the impacts of delays on the stability and control performance of impulsive systems have attracted increasing attention [28–30], and various effective control strategies have been proposed for this issue [31–33]. In [31], the synchronization problem of inertial Cohen-Grossberg neural networks was analyzed, considering impulses that depend on state-dependent delays. In [32], the effect of impulsive delays on achieving synchronization in uncertain chaotic neural networks was investigated. Lv et al. [33] proposed an event-triggered delayed impulsive control strategy based on a quadratic Lyapunov function for complex dynamical networks with a coupling delay. This strategy can tolerate network issues such as packet loss and ensures exponential synchronization with less conservatism while avoiding Zeno behavior.

Motivated by the previous analysis, this study focuses on the Lyapunov stability of nonlinear delayed impulsive systems under anti-impulsive disturbance ETIC, while taking impulsive perturbations into account. The following is a summary of this paper's primary contributions:

(*i*) Compared with previous studies [22–24], this work fully accounts for the effects of delayed impulses, provides a clear characterization of how trigger parameters, impulsive intensity, and impulse delays are interrelated, and investigates how variations in these parameters affect system stability, clearly demonstrating the stabilizing effect of impulsive delay on global asymptotic stability (GAS).

(*ii*) Different from conventional ETMs, this paper introduces a novel ETM that leverages impulsive disturbance information. The proposed method intermittently monitors system dynamics with minimal resource usage, enabling timely suppression of divergent behavior caused by impulsive perturbations and effectively preventing Zeno phenomena.

(*iii*) This work examines how chaotic systems synchronize under impulsive disturbances within the ETIC framework and linear matrix inequalities (LMIs), demonstrating the practicality and effectiveness of the suggested ETIC approach.

This paper is structured in the following manner: Section 2 presents the system model, introduces relevant definitions, and outlines some preliminary theoretical concepts. In Section 3, the main results are developed. Section 4 illustrates how these theoretical findings can be applied to the synchronization of chaotic systems in the presence of impulsive disturbances. Section 5 includes two numerical examples to demonstrate the validity and effectiveness of the proposed approach. Finally, Section 6 concludes the paper.

Notations: Let $\mathbb{N}_+$ denote the set of positive integers, $\mathbb{R}$ represent the set of real numbers, and $\mathbb{R}_+$ denote the set of positive real numbers. The n-dimensional real space is represented by $\mathbb{R}^n$, equipped with the Euclidean norm $|\cdot|$, while $\mathbb{R}^{n\times n}$ denotes the space of $n\times n$ real matrices. The symbol I is used to represent an identity matrix whose order is determined by the context, and $\star$ is employed to represent the symmetric block corresponding to a given matrix element. For a matrix X, X^{-1} and X^{T} denote its inverse and transpose, respectively. A symmetric matrix X is said to be positive definite (negative definite) if $X > 0$ ($X < 0$). For vectors $\epsilon_1, \epsilon_2 \in \mathbb{R}^i$ with $1 \le i \le n$, we define $C(\epsilon_1, \epsilon_2)$ as the set of all continuous functions ϕ mapping ϵ_1 to ϵ_2. Let $\mathcal{K} = \{\phi \in C(\mathbb{R}_+, \mathbb{R}_+) \mid \phi(\epsilon) > 0$ whenever $\epsilon > 0$ and ϕ increases strictly with $\epsilon\}$ and $\mathcal{K}_\infty = \{\phi \in \mathcal{K},\ \phi(\epsilon) \to +\infty$ as $\epsilon \to +\infty\}$, $\Omega(h) = [1, 2, 3, \ldots, h]$.

2. Model description and preliminaries

A nonlinear impulsive system subject to impulsive disturbances is given below:

$$\begin{cases} \dot{x}(t) = \Psi(x(t)), & t \ge t_0,\ t \ne \mu_q, \\ x(t) = \Upsilon_q(x(t^-)), & t = \mu_q,\ q \in \mathbb{N}_+, \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ represents the state vector, $\dot{x}_i(t)$ represents the upper right-hand derivative of $x_i(t)$, and the functions $\Psi,\ \Upsilon_q \in C(\mathbb{R}^n, \mathbb{R}^n)$ for $\forall q \in \mathbb{N}_+$ are supposed to satisfy certain essential conditions. These conditions ensure that the solution $x(t) = x(t; t_0, x_0)$ of system (2.1) is existent and unique within the corresponding intervals, with $\Psi(0) = 0$ and $\Upsilon_q(0) = 0$. The sequence $\{\mu_q, q \in \mathbb{N}_+\}$ characterizes a set of impulsive disturbance instants with the initial condition $\mu_0 = t_0$. It is strictly increasing on $\mathbb{R}_+$ and can be either an infinite sequence or a finite sequence. In this study, we make the assumption that system (2.1) exhibits right continuity at all impulsive jump instants.

It should be noted that system (2.1) is subject to impulsive disturbances. To ensure the Lyapunov stability for system (2.1), an impulsive control strategy will be incorporated subsequently. The following is a mathematical explanation of impulsive control:

$$x(t) = \Pi_r(x(t^- - \tau)), \quad t = \nu_r, r \in \mathbb{N}_+, \tag{2.2}$$

where the function $\Pi_r \in C(\mathbb{R}^n, \mathbb{R}^n)$, and it holds that $\Pi_r(0) = 0$. The parameter $\tau > 0$ represents a constant time delay and the sequence $\{\nu_r,\ r \in \mathbb{N}_+\}$ represents a set of impulsive control instants, which is dictated by the ETM to be designed in the subsequent sections.

Therefore, by integrating system (2.1) with impulsive control input (2.2), the corresponding impulsive system can be written as

$$\begin{cases} \dot{x}(t) = \Psi(x(t)), & t \geq t_0,\ t \neq \mu_q,\ t \neq \nu_r, \\ x(t) = \Upsilon_q(x(t^-)), & t = \mu_q,\ q \in \mathbb{N}_+, \\ x(t) = \Pi_r(x(t^- - \tau)), & t = \nu_r,\ r \in \mathbb{N}_+. \end{cases} \tag{2.3}$$

In this system, the impulsive perturbation sequence $\{\mu_q\}$ and the impulsive control sequence $\{\nu_r\}$ are incorporated. For the coincident points of $\{\mu_q\}$ and $\{\nu_r\}$, we make the assumption that impulsive disturbance arises first, followed by the activation of impulsive control, which indicates that $x(t) = \Upsilon_q(x(t^-))$ occurs prior to the activation of $x(t) = \Pi_r(x(t^- - \tau))$.

Definition 2.1. *[24] Consider the impulsive perturbation sequence $\{\mu_i\}$ and the impulsive control sequence $\{\nu_i\}$. Let $x(t) = x(t; t_0, x_0, \{\mu_i\}, \{\nu_i\})$ denote the solution of system (2.3) corresponding to the initial condition (t_0, x_0). Under these conditions, system (2.3) is referred to as*

(1) stable, if for any $t_0 \geq 0$ and $\varepsilon > 0$, there exists $\delta = \delta(t_0, \varepsilon)$ such that for every solution $x(t)$ of the system satisfying $|x_0| < \delta$, we have $|x(t)| < \varepsilon$, $\forall t \geq t_0$;
(2) globally asymptotically stable, if it is stable and satisfies $\lim_{t\to+\infty} |x(t)| = 0$, for $\forall t_0 \geq 0$, i.e., it is globally attractive.

Definition 2.2. *[34] Consider a function $V : \mathbb{R}^n \to \mathbb{R}_+$ that is locally Lipschitz. We introduce the upper right-hand Dini derivative of V with respect to the dynamics described in (2.3) as follows:*

$$D^+V(x(t)) = \lim_{\Upsilon\to 0^+} \sup \frac{1}{\Upsilon}[V(x(t) + \Upsilon\Psi(x(t))) - V(x(t))].$$

Lemma 2.1. *[16] With any given real matrices Φ_1, Φ_2, positive definite matrix X, and positive number σ, the following inequality holds:*

$$\Phi_1^T\Phi_2 + \Phi_2^T\Phi_1 \leq \sigma\Phi_1^T X\Phi_1 + \sigma^{-1}\Phi_2^T X^{-1}\Phi_2.$$

3. Main results

Based on the above problem formulation, the main challenge lies in ensuring system stability in the presence of impulsive disturbances and impulse delays, while avoiding excessive control actions. To address these issues, we propose an ETIC strategy that integrates a novel triggering mechanism with impulse delays. The key idea is to activate control actions only when necessary, thereby reducing resource consumption while maintaining stability. In this section, we present the detailed design of the proposed method and the corresponding theoretical analysis. Specifically, to achieve the GAS of system (2.3), a set of sufficient conditions is derived in this section, with the elimination of Zeno behavior guaranteed. Suppose that the impulsive disturbance sequence $\{\mu_q\}$ fulfills the following properties:

$$\eta_{q-1} < \mu_q - \mu_{q-1} \leq \lambda_{q-1}, \quad q \in \mathbb{N}_+. \tag{3.1}$$

Herein, the impulsive disturbance parameters satisfy $\eta_q, \lambda_q \in \mathbb{R}_+$. It should be noted that if the sequence $\{\mu_q\} = \{\mu_q, q \in \Omega(h)\}$ is finite in occurrence, then condition (3.1) holds for all $q \in \Omega(h)$. In this case, we set $\mu_q - \mu_{q-1} = \lambda_{q-1} \equiv 0$ for any $q \in \mathbb{N}_+ \setminus \Omega(h)$. For subsequent analysis, we define the set $\mathcal{F}^*$ as a class of impulsive disturbance sequences $\{\mu_q\}$ that satisfy condition (3.1). Following this, we define the ETM in the following manner:

$$\begin{aligned} \nu_r &= \min\{\nu_r^*, \tilde{\nu}_{r-1} + \eta_r\}, \\ \nu_r^* &= \inf\{t > \tilde{\nu}_{r-1} : V(x(t)) \geq e^{\theta_r} \min\{V(x(\nu_{r-1})), V(x(\mu_r))\}\}, \end{aligned} \tag{3.2}$$

where $r \in \mathbb{N}_+$, and $\theta_r \in \mathbb{R}_+$ denotes a triggered impulse parameter that satisfies $\theta_r > \zeta_r$, where $\zeta_r \in \mathbb{R}_+$ is a constant to be specified in subsequent sections. $V(x(t))$ represents the Lyapunov function associated with the state $x(t)$. If the sequence $\{\mu_q\} \in \mathcal{F}^*$ is infinite and unbounded, i.e., $\{\mu_q, q \in \mathbb{N}_+\}$, we set $\tilde{\nu}_{r-1} = \mu_r$. If the sequence $\{\mu_q\} \in \mathcal{F}^*$ is finite, i.e., $\{\mu_q, q \in \Omega(h)\}$, we define $\tilde{\nu}_{r-1} = \mu_r$ for all $r \in \Omega(h)$. For any $r \in \mathbb{N}_+ \setminus \Omega(h)$, we let $\Phi_r = \exp(\theta_r)V(x(\nu_{r-1}))$ and $\tilde{\nu}_{r-1} = \nu_{r-1}$. The term $\tilde{\nu}_{r-1} + \eta_r$ is defined as an auxiliary impulse instant, with η_r defined identically to that in condition (3.1). This mechanism ensures that at most one control action is triggered within each impulsive disturbance interval $[\mu_r, \mu_{r+1})$. It can suppress the divergence caused by the previous disturbance in a timely manner and avoid over-control simultaneously. The control is triggered when the system state $V(x(t))$ exceeds the threshold $e^{\theta_r} \min\{V(x(\nu_{r-1})), V(x(\mu_r))\}$; otherwise, the control will be triggered forcibly at the auxiliary time $\tilde{\nu}_{r-1} + \eta_r$. Furthermore, we assume that the parameters η_r and θ_r meet certain predefined conditions

$$\sum_{r=1}^{n} \min\left\{\eta_r, \frac{\theta_r - \zeta_r}{b}\right\} \to +\infty, \text{ as } n \to +\infty. \tag{3.3}$$

The main idea of ETM (3.2) is that a predefined event can be triggered at most once between two consecutive impulsive disturbance instants under the intermittent dynamics detection law. In other words, only a single control input is allowed to occur within each interval between two continuous disturbance actions, and no additional control input is required between adjacent impulsive control instants. This impulsive enhanced control strategy enables timely intervention in the system, effectively weakening the divergence caused by the previous impulsive disturbance. As a result, the restriction on impulsive control intensity can be relaxed. Moreover, since the impulsive control time is implicitly governed by the present event condition, the occurrence of Zeno behavior cannot be excluded. Therefore, before analyzing the system's Lyapunov stability, Zeno behavior must be ruled out.

Theorem 3.1. *Consider $\{\mu_q\} \in \mathcal{F}^*$. Suppose there is a locally Lipschitz function $V : \mathbb{R}^n \to \mathbb{R}_+$, $\omega_1, \omega_2 \in \mathcal{K}_\infty$, and constants $b, \eta_r, \lambda_r, \theta_r, \zeta_r, \xi_r \in \mathbb{R}_+$, with $\theta_r > \zeta_r$, such that for all $x \in \mathbb{R}^n$,*

(I_1) $D^+V(x(t)) \leq bV(x(t)), \quad t \geq t_0, t \neq \mu_q, t \neq \nu_r, q, r \in \mathbb{N}_+$;
(I_2) $V(\Upsilon_q(x)) \leq \exp(\zeta_q)V(x), \quad q \in \mathbb{N}_+$;
(I_3) $\omega_1(|x|) \leq V(x) \leq \omega_2(|x|)$;
(I_4) $V(\Pi_r(x)) \leq \exp(-\xi_r)V(x), \quad r \in \mathbb{N}_+$.

Consequently, the Zeno behavior in system (2.3) is eliminated by applying ETM (3.2). Furthermore, the impulsive control sequence $\{\nu_r\}$ meets

$$\nu_r - \nu_{r-1} \geq \min\left\{\eta_r, \frac{\theta_r - \zeta_r}{b}\right\}, \quad r \in \mathbb{N}_+.$$

Proof. Let $x(t) = x(t, t_0, x_0, \{\mu_q\}, \{\nu_r\})$ represent the solution of system (2.3) with (t_0, x_0), where $\{\mu_r\} \in \mathcal{F}^*$ and the impulsive control sequence $\{\nu_r\}$ is determined according to ETM (3.2). For simplicity, the

Lyapunov function is denoted by $V(x(t)) = V(t)$. Since ETM (3.2) guarantees that events are triggered infinitely, it is assumed that the impulsive control times meet the required conditions

$$t_0 < \nu_1 < \cdots < \nu_r < \cdots, \quad r \in \mathbb{N}_+.$$

According to the definition of disturbance sequence $\{\mu_q\}$, this sequence may be either finite or infinite and unbounded. Therefore, the exclusion of Zeno behavior is analyzed as below.

From one perspective, the impulsive disturbance sequence $\{\mu_q\} = \{\mu_q \mid q \in \mathbb{N}_+\}$ is infinite and unbounded, and it is assumed to satisfy

$$t_0 < \mu_1 < \cdots < \mu_r < \cdots, \quad q \in \mathbb{N}_+.$$

Before addressing the exclusion of Zeno behavior, we first show that the impulsive control instant $\nu_r \in [\mu_r, \mu_{r+1})$ for all $r \in \mathbb{N}_+$.

From the configuration of ETM (3.2), the impulsive control instant ν_r can be selected in two possible ways: the auxiliary impulse instant $\tilde{\nu}_{r-1} + \eta_r$ or the event-triggered impulse instant ν_r^*. If ν_r is chosen as the auxiliary impulse instant, denoted by $\nu_r = \tilde{\nu}_{r-1} + \eta_r$, it follows from (3.1) that $\nu_r \in [\mu_r, \mu_{r+1})$ for all $r \in \mathbb{N}_+$.

If ν_r corresponds to the triggered impulse instant, that is, $\nu_r = \nu_r^*$, then, (3.2) implies that $\mu_r \leq \nu_r^* \leq \tilde{\nu}_{r-1} + \eta_r < \mu_{r+1}$, indicating that for any $r \in \mathbb{N}_+$, $\nu_r = \nu_r^* \in [\mu_r, \mu_{r+1})$. Therefore, in both of the cases, the relation $\nu_r \in [\mu_r, \mu_{r+1})$ for all $r \in \mathbb{N}_+$ always holds. Next, for any $r \in \mathbb{N}_+$, the exclusion of Zeno behavior is established by considering two different cases.

Case I. $\nu_r = \tilde{\nu}_{r-1} + \eta_r$. Since $\nu_{r-1} < \mu_r$, it follows that

$$\nu_r - \nu_{r-1} > \nu_r - \mu_r = \tilde{\nu}_{r-1} + \eta_r - \mu_r = \eta_r.$$

Case II. $\nu_r = \nu_r^*$. From the definition of ETM (3.2), one has

$$V(\nu_r^-) = \exp(\theta_r) \min\{V(\nu_{r-1}), V(\mu_r)\}. \tag{3.4}$$

Using the relation $\mu_{r-1} \leq \nu_{r-1} < \mu_r \leq \nu_r < \mu_{r+1}$, we show that

$$\nu_r - \nu_{r-1} \geq \frac{\theta_r - \zeta_r}{b}.$$

First, suppose that $V(\nu_r^-) = \exp(\theta_r)V(\nu_{r-1})$ holds in (3.4). From condition (I_1), we can conclude that

$$V(\mu_r^-) \leq \exp(b(\mu_r - \nu_{r-1}))V(\nu_{r-1}),$$

which combined with condition (I_2) implies that

$$V(\mu_r) \leq \exp(\zeta_r)V(\mu_r^-) \leq \exp(b(\mu_r - \nu_{r-1}) + \zeta_r)V(\nu_{r-1}).$$

Since $\nu_{r-1} < \mu_r \leq \nu_r$, condition (I_1) further yields

$$V(\nu_r^-) \leq \exp(b(\nu_r - \mu_r))V(\mu_r) \leq \exp(b(\nu_r - \nu_{r-1}) + \zeta_r)V(\nu_{r-1}).$$

From the combination of the inequality above and (3.4), it follows that

$$\nu_r - \nu_{r-1} \geq \frac{\theta_r - \zeta_r}{b}.$$

Next, assume that $V(\nu_r^-) = \exp(\theta_r)V(\mu_r)$ holds in (3.4). From condition (I_1), one has

$$V(\nu_r^-) \leq \exp(b(\nu_r - \mu_r))V(\mu_r), \tag{3.5}$$

which together with (3.4) implies

$$\nu_r - \nu_{r-1} > \nu_r - \mu_r \geq \frac{\theta_r}{b}.$$

Therefore, in both cases, it holds that

$$\nu_r - \nu_{r-1} \geq \min\left\{\eta_r, \frac{\theta_r - \zeta_r}{b}\right\}, \quad r \in \mathbb{N}_+.$$

From another perspective, suppose that sequence $\{\mu_q\}$ is finite, i.e., $\{\mu_q\} = \{\mu_q, q \in \Omega(h)\}$ with

$$t_0 < \mu_1 < \cdots < \mu_h, \quad h \in \mathbb{N}_+.$$

Following arguments similar to the infinite case, we have $\nu_r \in [\mu_r, \mu_{r+1})$ for $r \in \Omega(h-1)$ and $\nu_h \geq \mu_h$. Moreover,

$$\nu_r - \nu_{r-1} \geq \min\left\{\eta_r, \frac{\theta_r - \zeta_r}{b}\right\}, \quad r \in \Omega(h).$$

Since no further impulsive disturbances occur on the interval $[\mu_h, +\infty)$, it follows from ETM (3.2) and condition (I_1) that

$$\nu_r - \nu_{r-1} \geq \min\left\{\eta_r, \frac{\theta_r}{b}\right\}, \quad r \in \mathbb{N}_+ \setminus \Omega(h).$$

Consequently, regardless of whether the impulsive disturbances are finite or infinite, one always has

$$\nu_r - \nu_{r-1} \geq \min\left\{\eta_r, \frac{\theta_r - \zeta_r}{b}\right\}, \quad r \in \mathbb{N}_+,$$

which further implies

$$\nu_r \geq t_0 + \sum_{i=1}^{r} \min\left\{\eta_i, \frac{\theta_i - \zeta_i}{b}\right\}.$$

Recalling (3.3), it follows that $\nu_r \to +\infty$ as $r \to +\infty$. Consequently, the occurrence of Zeno behavior in system (2.3) under ETM (3.2) is eliminated, which completes the proof.

Theorem 3.2. *Under the framework of Theorem 3.1, if there exist functions $V : \mathbb{R}^n \to \mathbb{R}_+$, two class-$\mathcal{K}_\infty$ functions ω_1 and ω_2, and constants $b, \lambda_r, \theta_r, \zeta_r, \xi_r \in \mathbb{R}_+$, such that all conditions (I_1)–(I_4) stated in Theorem 3.1 hold, together with the additional conditions listed below:*

(I_5)

$$m_r + \sum_{i=1}^{r-1}(m_i - \xi_i) \to -\infty, \text{ as } r \to +\infty, \forall r \in \mathbb{N}_+,$$

where $m_i = \max\{b\lambda_{i-1} + \zeta_i, \theta_i\}$, $i \in \mathbb{N}_+$, then system (2.3) achieves GAS under ETM (3.2).

Proof. In practice, the impulsive disturbance sequence $\{\mu_q\} \in \mathcal{F}^*$ may be an infinite and unbounded sequence or a finite sequence. Accordingly, the analysis is divided into two situations: impulsive disturbances occurring infinitely or only a finite number of times.

First, we analyze the scenario in which impulsive disturbances occur infinitely many times, that is, $\{\mu_q\} = \{\mu_q \mid q \in \mathbb{N}_+\}$. As shown in the proof of Theorem 3.1, it holds that $\nu_1 \geq \mu_1$, which implies that the interval $[t_0, \mu_1)$ is not subjected to any control input. Then, from condition (I_1) and (3.1), it can be concluded that for any $t \in [t_0, \mu_1)$,

$$V(t) \leq \exp(b(t - t_0))V(t_0) \leq \exp(b\lambda_0)V(t_0). \tag{3.6}$$

Based on condition (I_2) at the occurrence of the first impulsive disturbance μ_1,

$$V(\mu_1) \leq \exp(\zeta_1)V(\mu_1^-) \leq \exp(b(\mu_1 - t_0) + \zeta_1)V(t_0) \leq \exp(b\lambda_0 + \zeta_1)V(t_0). \tag{3.7}$$

From ETM (3.2), it follows that, for all $t \in [\mu_1, \nu_1)$,

$$V(t) \leq \exp(\theta_1) \min\{V(t_0), V(\mu_1)\} \leq \exp(\theta_1)V(t_0). \tag{3.8}$$

By combining (3.6), (3.7), and (3.8), one can infer that, for all $t \in [t_0, \nu_1)$,

$$V(t) \leq \exp\left(\max\{b\lambda_0 + \zeta_1, \theta_1\}\right) V(t_0) = \exp(m_1)V(t_0).$$

From condition (I_4) and the preceding analysis, at the first impulsive control instant ν_1, we have

$$V(\nu_1) \leq \exp(-\xi_1)V(\nu_1^-) \leq \exp\left(m_1 - \xi_1\right) V(t_0).$$

In a similar manner, we can conclude that for all $t \in [\nu_1, \nu_2)$,

$$V(t) \leq \exp(m_2)V(\nu_1) \leq \exp\left(m_1 + m_2 - \xi_1\right) V(t_0).$$

Moreover, considering the second impulsive control time ν_2,

$$V(\nu_2) \leq \exp(-\xi_2)V(\nu_2^-) \leq \exp\left(\sum_{i=1}^{2}(m_i - \xi_i)\right) V(t_0).$$

It can be inferred by repeating the aforementioned process that for all $t \in [\nu_{r-1}, \nu_r)$, $r \in \mathbb{N}_+$,

$$V(t) \leq \exp(m_r)V(\nu_{r-1}) \leq \exp\left(m_r + \sum_{i=1}^{r-1}(m_i - \xi_i)\right) V(t_0). \tag{3.9}$$

Second, consider the case where impulsive disturbances occur only finitely many times. For any $q \in \mathbb{N}_+ \setminus \Omega(h)$, suppose that the disturbance sequence is given by $\{\mu_q\} = \{\mu_q, q \in \Omega(h)\}$ with $\mu_q = \mu_h$. According to the analysis in Theorem 3.1, it follows that $\nu_r \in [\mu_r, \mu_{r+1})$ for $r \in \Omega(h-1)$, and $\nu_h \geq \mu_h$. It is straightforward that inequality (3.9) holds for all $r \in \Omega(h)$. Moreover, at the h-th impulsive control instant ν_h, one has

$$V(\nu_h) \leq \exp(-\xi_h)V(\nu_h^-) \leq \exp\left(\sum_{i=1}^{h}(m_i - \xi_i)\right) V(t_0).$$

Since $\nu_h \geq \mu_h$ and μ_h is the last impulsive disturbance instant, no further impulsive disturbances occur on the interval $[\nu_h, +\infty)$. According to ETM (3.2) and condition (I_1), for any $t \in [\nu_{r-1}, \nu_r)$, $r \in \mathbb{N}_+ \backslash \Omega(h)$, one has

$$V(t) \leq \exp(\theta_r) V(\nu_{r-1}) \leq \exp\left(\theta_r + \sum_{i=h+1}^{r-1} (\theta_i - \xi_i) + \sum_{i=1}^{h} (m_i - \xi_i)\right) V(t_0).$$

Together with $\theta_i \leq m_i$ for all $t \in [\nu_{r-1}, \nu_r)$, one can infer that

$$V(t) \leq \exp(m_r) V(\nu_{r-1}) \leq \exp\left(m_r + \sum_{i=1}^{r-1} (m_i - \xi_i)\right) V(t_0).$$

Therefore, regardless of whether the sequence of impulsive disturbances $\{\mu_q\}$ is infinite or finite, it holds that for all $t \in [\nu_{r-1}, \nu_r)$, $r \in \mathbb{N}_+$,

$$V(t) \leq \exp(m_r) V(\nu_{r-1}) \leq \exp\left(m_r + \sum_{i=1}^{r-1} (m_i - \xi_i)\right) V(t_0).$$

By recalling condition (I_5), the above inequality implies that $V(t) \to 0$ as $t \to +\infty$. Consequently, system (2.3) is globally asymptotically stable under ETM (3.2). This completes the proof.

Remark 3.1. *In light of ETM (3.2), Theorems 3.1 and 3.2 provide several sufficient conditions for the GAS of system (2.3), where the delay effects in the impulses are thoroughly considered. To be specific, condition (I_1) dictates the continuous dynamic evolution of system (2.3) in the continuous interval; condition (I_2) dictates the destructive strength of impulsive disturbances at the impulsive disturbance instants μ_q; condition (I_3) ensures that V can effectively characterize the magnitude of the system state and is equivalent to the Euclidean norm; condition (I_4) regulates the abrupt changes or jumping behavior of the system state; and condition (I_5) depicts the link between impulsive disturbance and impulse control parameters. These conditions ensure the GAS of system (2.3), irrespective of whether impulsive disturbances occur finitely or infinitely many times.*

Remark 3.2. *According to Theorem 3.1, for any positive integer $r \in \mathbb{N}_+$, the following inequality holds:*

$$\nu_r - \nu_{r-1} \geq \min\left\{\eta_r, \ \frac{\theta_r - \zeta_r}{b}\right\}.$$

Within the framework of the proposed ETM, each impulsive control time is categorized as either an event-triggered impulsive time or an auxiliary impulsive time. When the r-th impulsive control time corresponds to an event-triggered instant, it follows that $\nu_r - \nu_{r-1} \geq \frac{\theta_r - \zeta_r}{b}$, which leads to an interesting finding: The occurrence of event-triggered instants is directly governed by both the magnitude of impulsive disturbances and the behavior of the system's continuous-time dynamics. Intuitively, systems subjected to strongly destabilizing impulsive disturbances require immediate control intervention to regain stability. Examining the inequality reveals that larger values of ζ_r (or b) result in a smaller control interval $\nu_r - \nu_{r-1}$, signaling a more pressing need for control action. Conversely, in the presence of weakly destabilizing impulsive disturbances, the control demand is diminished, allowing for a larger allowable control interval. Specifically, a smaller ζ_r leads to a larger $\nu_r - \nu_{r-1}$, which aligns with practical expectations. Thus, the ETM developed in this work effectively integrates information from both the continuous and discrete dynamics of the system. By adjusting the impulsive generator based *on the system's inherent characteristics, the framework guarantees the effective achievement of the intended control performance.*

Remark 3.3. *Under the framework of ETM (3.2), Theorem 3.2 presents sufficient conditions for the GAS of system (2.3). Recently, several representative studies have investigated the ETIC problem [17–19], yet none of these existing results consider systems affected by impulsive disturbances. In our ETM design, we effectively leverage information about impulsive disturbances and introduce auxiliary impulsive instants. Based on this ETM, impulsive disturbances and control actions occur alternately. It can be observed that, in most ETIC approaches, system measurements are monitored continuously [17–19]. In contrast, the proposed ETM implements an intermittent detecting scheme, which corresponds to a piecewise continuous monitoring approach. Specifically, the monitoring of the system state is activated at the moment an impulsive disturbance occurs and remains continuously active within the subsequent inter-disturbance interval* $[\mu_r, \mu_{r+1})$. *The* inf *operator in (3.2) is realized during this active window. Once an event is triggered (or the auxiliary instant is reached), intensive monitoring is effectively suspended until the next disturbance instant* μ_{r+1} *re-activates it. This design confines the continuous observation to critical periods following disturbances, thereby reducing the overall monitoring overhead compared to a fully continuous time-triggered approach. Notably, for systems with a finite number of impulsive disturbances, after the final disturbance, the system transitions to continuous monitoring without further interruptions. Thus, the proposed ETM effectively reduces the monitoring resource costs and the communication load.*

Remark 3.4. *Numerous studies have investigated the stability of nonlinear systems using ETIC strategies [22–24]. However, these studies have not explicitly considered the influence of impulsive delays in their analysis. This may lead to overly optimistic stability conditions and can suffer from performance degradation or even instability when implemented in real systems with communication or processing lags. Existing delay-inclusive models often treat delays conservatively, which may result in higher control frequency or resource consumption. While impulsive delays are considered in [27], this work is limited to linear systems and has certain restrictions. Additionally, the ETM designed in [33] is highly conservative. In contrast, this study focuses on the GAS of nonlinear systems with delayed impulsive disturbances within the ETIC framework, with impulsive disturbances and control actions occurring alternately. The proposed method explicitly incorporates delay information into the ETM, achieving a better balance between stability robustness and resource efficiency. Moreover, no explicit upper bound is required on the duration between two successive triggering instants, and Zeno behavior is naturally eliminated by the proposed ETM.*

4. Application

In this section, the theoretical results presented above are reformulated as LMIs, which can be effectively addressed by employing the LMI toolbox provided in MATLAB. These LMIs are then employed to address the drive-response synchronization problem for a certain class of chaotic systems. The following chaotic system is considered:

$$\dot{x}(t) = Mx(t) + N\Psi(x(t)), \quad t \geq t_0, \tag{4.1}$$

where $x(t) \in \mathbb{R}^n$ denotes the system state, $M, N \in \mathbb{R}^{n\times n}$, and $\Psi \in C(\mathbb{R}^n, \mathbb{R}^n)$ is a Lipschitz continuous nonlinear function with Lipschitz constant matrix L, satisfying $\Psi(0) = 0$. It is worth noting that many neural network models can be described by system (4.1). The drive-response synchronization problem has been widely investigated in the literature. However, most existing studies do not take into account the possible influence of impulsive disturbances on the response system. Taking impulsive disturbances into consideration, the response system can be expressed as follows:

$$\dot{y}(t) = My(t) + N\Psi(y(t)) + \omega(t) + u(t), \quad t \geq t_0, \tag{4.2}$$

with

$$\omega(t) = \sum_{q\in\mathbb{N}_+} Ge(\mu_q^-)\,\delta(t-\mu_q), \quad u(t) = \sum_{r\in\mathbb{N}_+} Ue(\nu_r^-)\,\delta(t-\nu_r).$$

Here, $\omega(t)$ represents the instantaneous disturbance affecting response system (4.2) during its interaction with drive system (4.1), and $G \in \mathbb{R}^{n\times n}$ is the associated disturbance intensity matrix. The Dirac control input $u(t)$ is applied to realize drive-response synchronization, where $U \in \mathbb{R}^{n\times n}$ represents the impulsive control gain matrix. The synchronization error is given by $e(t) = y(t) - x(t)$. The sequence $\{\mu_q\} \in \mathcal{F}^*$ indicates the impulsive disturbance instants, which are assumed to be infinite and unbounded, while $\{\nu_r\}$ represents the impulsive control instants generated by the ETM to be designed subsequently.

The error system corresponding to drive system (4.1) and response system (4.2) can be expressed as follows:

$$\begin{cases} \dot{e}(t) = Me(t) + NF(e(t)), & t \geq t_0,\ t \neq \mu_q,\ t \neq \nu_r, \\ e(t) = (I+G)e(t^-), & t = \mu_q,\ q \in \mathbb{N}_+, \\ e(t) = (I+U)e(t^-), & t = \nu_r,\ r \in \mathbb{N}_+. \end{cases} \tag{4.3}$$

Here, $F(e(t)) = \Psi(y(t)) - \Psi(x(t))$. The assumptions on the sequences $\{\nu_r\}$ and $\{\mu_q\}$ are the same as those adopted for system (2.3). The goal is to design $\{\nu_r\}$ and U so that error system (4.3) achieves GAS. This ensures drive-response synchronization between drive system (4.1) and response system (4.2). In accordance with Theorem 3.2, the following conditions are obtained.

Theorem 4.1. *Consider $\{\mu_q\} \in \mathcal{F}^*$ with $\eta_r = \eta$, $\lambda_r = \lambda$. Suppose there exist positive constants $b, \theta, \zeta, \xi \in \mathbb{R}_+$, with $\theta > \zeta$, E is an $n\times n$ positive definite matrix, R is an $n\times n$ positive definite diagonal matrix, and H is an $n \times n$ real matrix, for which $\xi > \max\{b\lambda + \zeta, \theta\}$ and*

$$\begin{bmatrix} EM + M^TE - bE + LRL & EN \\ \star & -R \end{bmatrix} \leq 0, \tag{4.4}$$

$$\begin{bmatrix} -e^{\zeta}E & (I+G)^TE \\ \star & -E \end{bmatrix} \leq 0, \tag{4.5}$$

$$\begin{bmatrix} -e^{-\xi}E & E+H \\ \star & -E \end{bmatrix} \leq 0. \tag{4.6}$$

With the impulsive control gain matrix $U = E^{-1}H^T$ and the following event-triggered mechanism, error system (4.3) is guaranteed to be globally asymptotically stable.

$$\begin{aligned} \nu_r &= \min\{\nu_r^*, \mu_r + \eta\}, \\ \nu_r^* &= \inf\{t \geq \mu_r : e^T(t)Ee(t) \geq \exp(\theta)\min\{e^T(\nu_{r-1})Ee(\nu_{r-1}), e^T(\mu_r)Ee(\mu_r)\}\}. \end{aligned} \tag{4.7}$$

Proof. For simplicity, let the Lyapunov function be defined as $V(e(t)) = V(t)$, and consider $V(t) = e^T(t)Ee(t)$ as a Lyapunov candidate. Using LMI (4.4), together with Young's inequality along with the Schur complement, it follows that, for any t in the continuous-time interval, the following inequality always holds:

$$\begin{aligned} D^+V(t) &= 2e^T(t)(E(Me(t) + NF(e(t)))) \\ &= e^T(t)(EM + M^TE)e(t) + 2e^T(t)ENF(e(t)) \\ &\le e^T(t)(EM + M^TE)e(t) + e^T(t)L^TRL\,e(t) + e^T(t)ENR^{-1}N^TE\,e(t) \\ &\le e^T(t)(EM + M^TE + L^TRL + ENR^{-1}N^TE)e(t) \\ &\le bV(t). \end{aligned}$$

Applying LMIs (4.5) and (4.6) along with the Schur complement, one can respectively obtain the following results at the impulsive disturbance instant μ_q and the impulsive control instant ν_r:

$$\begin{aligned} V(\mu_q) &= e^T(\mu_q)Ee(\mu_q) \\ &\le e^T(\mu_q^-)\,(I + G)^TE(I + G)\,e(\mu_q^-) \\ &\le \exp(\zeta)\,V(\mu_q^-) \end{aligned}$$

and

$$\begin{aligned} V(\nu_r) &= e^T(\nu_r)Ee(\nu_r) \\ &= e^T(\nu_r^-)\,(I + U)^TE(I + U)\,e(\nu_r^-) \\ &\le \exp(-\xi)\,V(\nu_r^-). \end{aligned}$$

It is evident that all the conditions specified in Theorems 3.1 and 3.2 hold, which ensures that error system (4.3) is globally asymptotically stable under ETM (4.7). Consequently, response system (4.2) achieves synchronization with drive system (4.1). This completes the proof.

5. Numerical examples

This section uses two numerical examples to show the viability and validity of the suggested theoretical results.

Example 5.1. Examine a nonlinear system that experiences impulsive disturbance

$$\begin{cases} \dot{x}(t) = \tan\Upsilon(x(t)), & t \ge 0,\ t \ne \mu_q, \\ x(t) = 1.25x(t^-), & t = \mu_q,\ q \in \mathbb{N}^+. \end{cases} \tag{5.1}$$

Consider a sequence of uncertain impulsive disturbances $\{\mu_q\} \in \mathcal{F}^*$ with $\mu_{2n-1} \in [0.5n - 0.2 - \nu, 0.5n - 0.2 + \nu]$, $\mu_{2n} \in [0.5n - \nu, 0.5n + \nu]$, where $\nu = 0.03$, $n \in \mathbb{N}^+$. This indicates that the exact q-th impulsive disturbance moment is stochastically selected within the predefined q-th time interval. In the absence of control input, system (5.1) fails to achieve GAS with $x(0) = 1$. In what follows, a dedicated ETIC strategy will be proposed to ensure the GAS of system (5.1). For this goal, an impulsive control law is first considered:

$$x(t) = \Pi(x(t^- - \tau)),\ t = \nu_r,\ r \in \mathbb{N}^+,$$

where $\tau = 0.1$ is a constant delay, ν_r denotes the impulsive control instant dictated by the ETM to be designed, and Π denotes the control gain. Based on the disturbance sequence $\{\mu_q\}$, the parameters are chosen as $\eta_r = \eta = 0.14$ and $\lambda_r = \lambda = 0.36$. Considering the Lyapunov function $V(x(t)) = |x(t)|$, and applying conditions (I_1) and (I_2), along with the inequality $\theta_r > \zeta_r$ in Theorem 3.1, the values $b = 1$, $\zeta_q = \zeta = 0.27$, and $\theta_r = \theta = 0.45$ are selected. Furthermore, using condition (I_5) in Theorem 3.2, we obtain $\xi_r = \xi = 0.73$, and applying condition (I_4) of Theorem 3.1, the control gain is determined as $\Pi = 0.48$. The ETM is then specified as follows:

$$\begin{aligned} \nu_r &= \min\{\nu_r^*\ ,\ \mu_r + 0.14\}, \\ \nu_r^* &= \inf\left\{t \geq \mu_r\ :\ |x(t)| \geq e^{0.45} \min\{|x(\nu_{r-1})|, |x(\mu_r)|\}\right\}. \end{aligned} \tag{5.2}$$

In the simulation, system (5.1) remains globally asymptotically stable under ETM (5.2), as illustrated by Figure 1. However, when all other parameters are kept unchanged, a slight increase in the impulsive disturbance strength to $\Upsilon = 1.85$ results in $\Upsilon \notin [-1.31, 1.31]$ (i.e., $\Upsilon \notin [-e^{\zeta}, e^{\zeta}]$), thereby violating condition (I_2). Consequently, the stability of system (5.1) is destroyed, as shown in Figure 2.

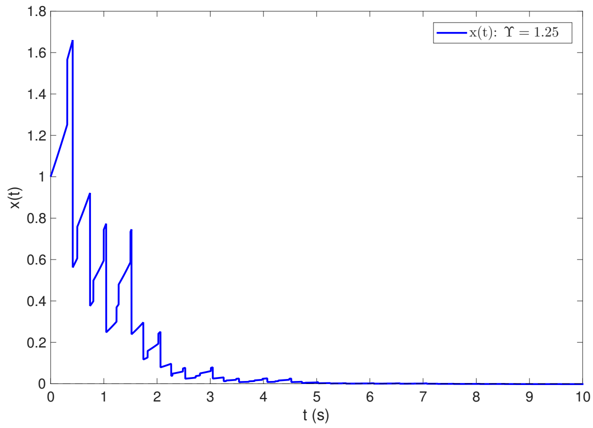

Figure 1. Dynamics of system (5.1) under ETM (5.2) with $x(t) : \Upsilon = 1.25$.

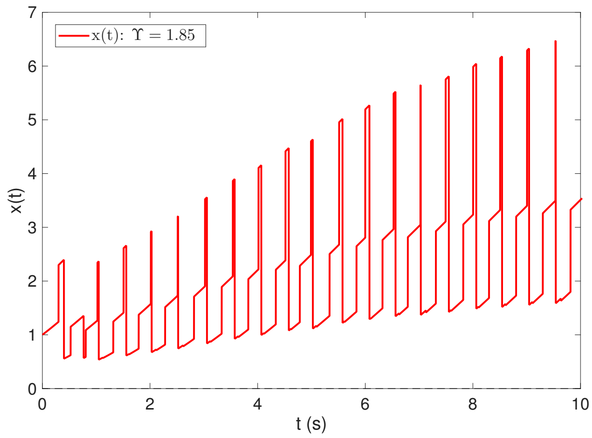

Figure 2. Dynamics of system (5.1) under ETM (5.2) with $x(t) : \Upsilon = 1.85$.

Figure 3 presents a comparative experiment on impulsive disturbance parameters. The comparison results show that the parameter constraints in the theorem are necessary conditions for system stability, and the impulsive disturbance intensity must remain within the upper bound of the Lyapunov function jump; otherwise, the control strategy becomes ineffective.

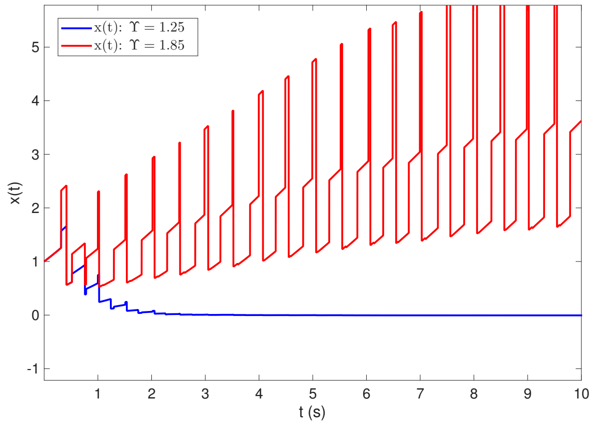

Figure 3. Dynamics of system (5.1) under ETM (5.2) with different parameter Υ.

The above simulation results confirm that the impulsive control sequence is a key factor in ensuring the GAS of system (5.1). From the simulation results presented in Figure 4, it is evident that the convergence speed of system (5.1) under ETM (5.2) decreases with the increase of delay τ, and more significant oscillations emerge for larger τ values. This indicates that although asymptotic stability remains preserved, an excessive delay can lead to a notable deterioration in the system's dynamic performance. It is therefore essential to select an appropriate τ value in practical applications to simultaneously guarantee the system's stability and satisfactory control performance.

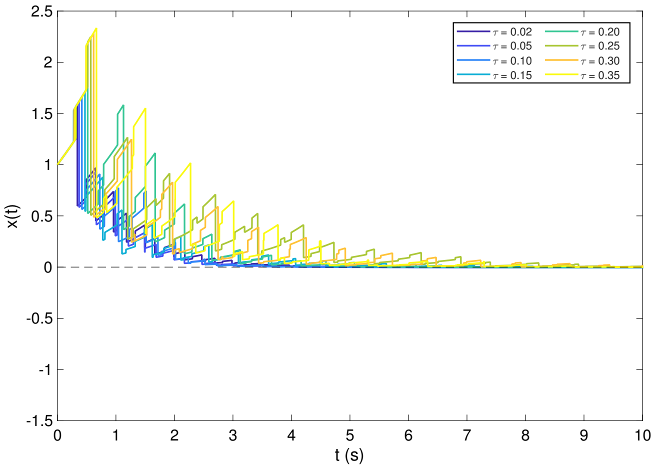

Figure 4. Effect of different delay τ on system stability.

Furthermore, system (5.1) attains GAS under ETM (5.2) for varying values of the triggered impulsive parameter θ. Figure 5 presents the distribution characteristics of impulsive control instants corresponding to different θ values. It can be observed that, as the triggered impulsive parameter θ

increases, the activation times of the impulsive control are delayed, and the probability that these times coincide with the auxiliary impulsive instants rises. This illustrates how sensitive the suggested ETIC approach is to the triggered impulsive parameter θ.

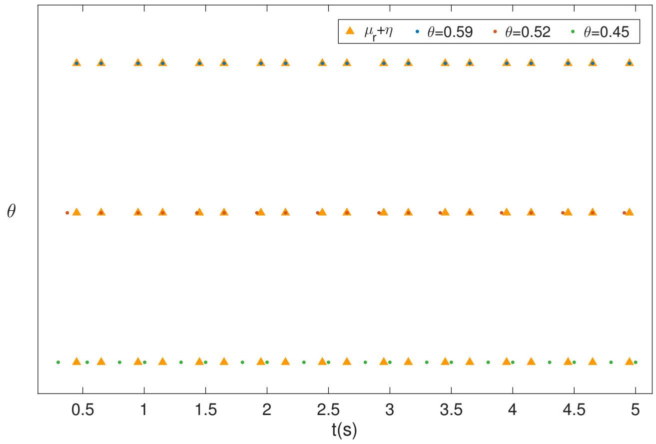

Figure 5. Activation instants of the impulsive control under ETM (5.2) for various values of the parameter θ.

Example 5.2. Consider the three-dimensional model of a chaotic cellular neural network (CCNN) as follows:

$$\dot{x}(t) = Mx(t) + N\Psi(x(t)) + C, \quad t \geq 0, \tag{5.3}$$

where $x \in \mathbb{R}^3$, the nonlinear functions are defined as $\Psi_1(a) = \Psi_2(a) = \Psi_3(a) = 0.5\,(|a+1| - |a-1|)$, and $C = [0, 0, 0]^T$. The system matrices M and N are given as follows:

$$M = -I, \quad N = \begin{bmatrix} 1.3 & -3.1 & -3.1 \\ -3.1 & 1.2 & -4.3 \\ -3.1 & 4.3 & 1.0 \end{bmatrix}.$$

CCNN system (5.3) exhibits a double-scroll chaotic attractor with $x_0 = [0.1, 0.1, 0.1]^T$, as shown in Figure 6. To study the synchronization of CCNN (5.3) under ETIC, a corresponding slave system is considered, which is taken as the three-dimensional version of system (4.2). The disturbance input is defined by $\omega(t) = G\,e(\mu_q^-)$, where the impulsive disturbance gain matrix is $G = 0.1I$, and the sequence of impulsive disturbance is $\{\mu_q\} = \{0.1q,\ q \in \mathbb{N}_+\} \in \mathcal{F}^*$. The impulsive control input is defined as $u(t) = Ue(\nu_r^-)$, associated with the control sequence $\{\nu_r\}$ to be designed later.

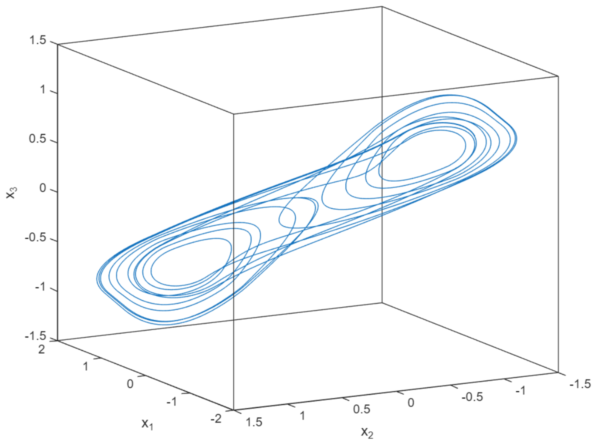

Figure 6. The chaotic attractor of CCNN (5.3).

Subsequently, consider the Lyapunov function $V(t) = x^T Ex$. Using MATLAB's LMI toolbox, the parameters $b = 12.3$, $\zeta = 0.27$, and $\xi = 1.55$ are chosen to satisfy LMIs (4.4)–(4.6) in Theorem 4.1. Consequently, the matrix E and the impulsive control gain matrix U can be computed as follows:

$$E = \begin{bmatrix} 0.2072 & 0.0383 & -0.0429 \\ 0.0383 & 1.1973 & -0.0016 \\ -0.0429 & -0.0016 & 0.2138 \end{bmatrix},$$

$$U = \begin{bmatrix} -1.0000 & -0.0000 & -0.0000 \\ -0.0000 & -1.0000 & -0.0000 \\ 0.0000 & -0.0000 & -1.0000 \end{bmatrix}.$$

Based on the impulsive disturbance sequence $\{\mu_q\}$, the parameters $\eta = 0.075$ and $\lambda = 0.1$ are selected. Then, according to the inequalities $\theta > \zeta$ and $\xi > \max\{b\lambda + \zeta, \theta\}$ in Theorem 4.1, the value $\theta = 0.9$ is chosen. Under these settings, the following is the definition of the ETM:

$$\begin{aligned} \nu_r &= \min\{\nu_r^*,\ 0.1r + 0.075\}, \\ \nu_r^* &= \inf\left\{t \geq \mu_r :\ e^T(t)Ee(t) \geq e^{0.9}\min\{e^T(\nu_{r-1})Ee(\nu_{r-1}), e^T(\mu_r)Ee(\mu_r)\}\right\}. \end{aligned} \tag{5.4}$$

The results of the simulation shown in Figure 7 reveal that the presented ETIC approach successfully stabilizes error system (4.3) under ETM (5.4), with the initial state of response system (4.2) set as $y_0 = [0.5, 0.1, 0.5]^T$. These results confirm that synchronization between systems (4.1) and (4.2) is effectively achieved. In particular, suppose that the control gain matrix U is replaced by

$$U^* = \begin{bmatrix} -1.0000 & -0.0000 & -0.0000 \\ -0.0000 & -1.9500 & -0.0000 \\ 0.0000 & -0.0000 & -1.0000 \end{bmatrix}.$$

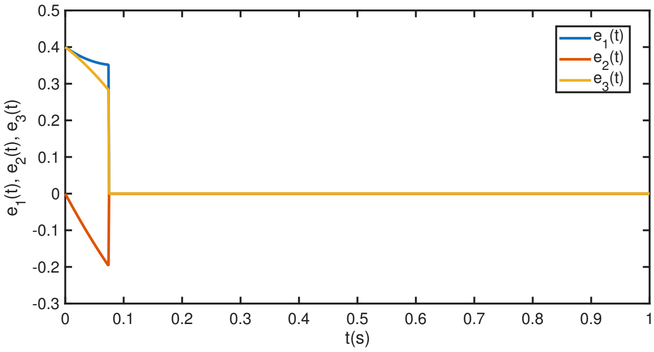

Figure 7. Dynamics of error system (4.3) with U under ETIC (5.4).

Under this circumstance, the LMIs in Theorem 4.1 are no longer satisfied. Simulation results shown in Figure 8 indicate that synchronization of CCNN (5.3) cannot be achieved, which demonstrates the sensitivity of the system performance to the control parameters and further confirms the effectiveness of theoretical analysis.

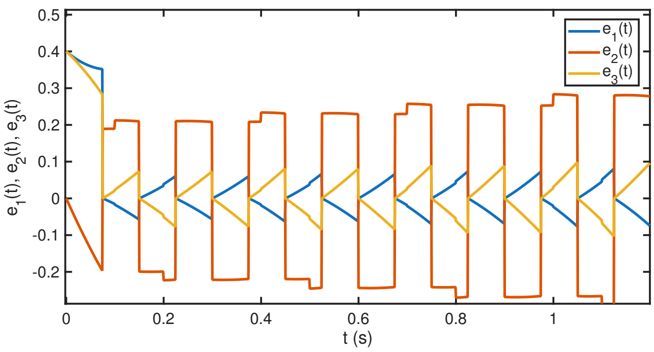

Figure 8. Dynamics of error system (4.3) with U^*.

Remark 5.1. *The proposed ETIC strategy can be effectively implemented in practical engineering systems with digital control architectures. In such systems, the controller is typically realized on embedded processors or industrial control units, where system states are sampled at discrete time instants. The ETM can be embedded into the controller as a logical decision module, which continuously (or intermittently) evaluates the triggering condition based on the latest available state measurements. Once the triggering condition is satisfied, an impulsive control action is executed through actuators, such as motors, valves, or power electronic devices. To reduce computational burden and communication overhead, the intermittent monitoring scheme introduced in this paper can be implemented by activating the detection module only within predefined time windows, while keeping it inactive otherwise. This is particularly suitable for networked control systems, where bandwidth and energy resources are limited. The impulsive nature of the control input also makes the proposed method attractive for systems where continuous actuation is either costly or infeasible. In addition, the proposed ETIC framework is well-suited for applications involving communication delays and sudden disturbances, such as robotic systems with network-induced delays, power systems subject to switching disturbances, and cyber-physical systems operating over shared communication* *networks. In these scenarios, the explicit incorporation of delay information into the triggering mechanism enhances robustness against latency and improves overall system performance. From an implementation perspective, practical issues such as sensor noise, sampling rate selection, actuator saturation, and computation delays should also be considered. These factors can be incorporated into the design by appropriately tuning the triggering parameters and ensuring feasibility of the derived LMI conditions. Therefore, the proposed control strategy provides a viable and flexible solution for real-world systems requiring efficient and reliable stabilization under impulsive effects and delays.*

6. Conclusions

In this paper, an ETIC framework is proposed that fully accounts for delay effects at impulsive instants and excludes Zeno behavior. The designed ETM integrates information regarding impulsive disturbances and is constrained by an intermittent monitoring rule. Moreover, sufficient conditions for the Lyapunov stability of nonlinear impulsive systems are derived. The derived findings are further used to study nonlinear impulsive control systems, based on which the corresponding ETM and impulsive controller are designed via LMI conditions. Finally, two numerical examples are provided to validate the proposed theoretical results. It should be noted that the proposed ETIC strategy cannot be directly applied if the information of impulsive disturbances is unavailable. Future research might concentrate on extending the ETIC design to eliminate the reliance on explicit disturbance information. In addition, the current analysis of implulse delays is relatively limited. More general delay forms, such as time-varying delays or distributed delays, could be considered in future work. Finally, this paper focuses on deterministic nonlinear systems. The proposed approach could be further extended to more complex systems, such as impulsive neural networks with stochastic disturbances, thereby enhancing its practical applicability and engineering adaptability.

Author contributions

X. Y. Wu: Writing–original draft; T. F. Xiao: Writing–review and editing; Jin-E Zhang: Supervision, Writing–review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this paper.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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