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Research article

Solving intuitionistic fuzzy integro-differential equations using artificial neural networks and newton-cotes methods

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Abstract: In this paper, a modified method based on artificial neural network and Newton-Cotes methods with positive coefficients was developed and applied for the first time in the literature to solve the intuitionistic fuzzy integro-differential equations (IFFIDEs), where the parameters and variables are considered intuitionistic fuzzy numbers. The triangular intuitionistic fuzzy number (TIFN) was used for expressing intuitionistic fuzzy variables and parameters. The fuzzification of the deterministic r-cut and β-cut solutions leads to the artificial neural intuitionistic numerical solution. The main reason for using neural networks is their applicability in handling the intuitionistic fuzzy variables and parameters of IFFIDEs. A numerical example was given to demonstrate the proposed method. The results agree with the theoretical prediction and highlight how the combination of artificial neural networks and Newton-Cotes method enhances the accuracy and efficiency of solving IFFIDEs, making our proposed method valuable for application in fields such as engineering, medicine, and physics.

Keywords: intuitionistic fuzzy numbers; intuitionistic fuzzy integro-differential equations; intuitionistic fuzzy Fredholm integro-differential equations; artificial neural network; Newton-Cotes methods
Mathematics Subject Classification: 30E10, 34K05

1. Introduction

The Integro-differential equations (IDEs) combine the major features of differential and integral equations, which are considered an important tool for modeling various phenomena across several fields, including engineering, economics, and physics [1]. The IDEs are classified based on the limits of integration into two types, including Fredholm integro-differential equations (F-IDEs) and Volterra integro-differential equations (V-IDEs). In particular, the classification of IDEs is based on whether the limits of integration are constant or variable. The IDEs with constant limits are known as the F-IDEs, where the name of F-IDEs refers to the mathematician Erik Fredholm [2]. F-IDEs are used for modeling complex systems in several fields, including medicine, physics, engineering, and biology [3]. F-IDEs usually relate to boundary value problems. The unknown function relies on values under fixed integration, which often makes solving the F-IDEs more complicated. By merging the fuzzy logic into F-IDEs, the systems can model where uncertainty and imprecision exist. Fuzzy sets aim to represent and handle the vague or incomplete information that xommonly appear in real-world problems, such as in medicine, physics, and engineering [4–6]. The use of fuzzy numbers in F-IDEs improves their ability to handle uncertainty by providing more accurate and realistic solutions in the modeling of complex systems where data is often unavailable. Therefore, F-IDEs and uncertainty (fuzzy) play a fundamental tool in handling mathematical problems. These lead to the fuzzy Fredholm integro-differential equations (FFIDEs). The FFIDEs are useful for modeling real systems with uncertainty and memory effects. They appear in engineering problems such as vibration and damping, and in medicine, such as disease spread and drug concentration models [7,8]. Intuitionistic fuzzy numbers improve the model by representing uncertainty using membership, non-membership, and hesitation.

Artificial Neural Networks (ANNs) are mathematical models inspired by the human brain based on implementation through computer software. ANNs act as parallel distributed processors made of interconnected elements known as neurons. Information is processed by these neurons by passing signals from one neuron to another through connection links. Every link contains a weight that affects the passing signal. The nonlinear activation function of each neuron generates an output from its input. There are feedback connections in ANNs in which outputs are passed back to previous layers. Similarly, there are feed-forward connections in which signals flow in one direction. The backpropagation algorithm is generally involved in learning ANNs, and it identifies the change in weights that results in differences between actual and predicted outputs in various cycles of learning. The performance of ANN is then calculated using error metrics such as mean squared error (MSE) or root mean squared error (RMSE) [9,10]. On the other hand, Fuzzy Neural Networks (FNNs) have attracted considerable interest because they manage complex, nonlinear systems that include uncertain or imprecise data. By merging the flexibility of neural networks with fuzzy logic, FNNs model and approximate real-world problems where traditional methods may not succeed [11]. Therefore, FNNs can learn from data and respond to uncertainties, making them effective for solving problems in engineering, medicine, and science. Some researchers have examined the use of FNNs for solving fuzzy integral and integro-differential equations. Fadravi et.al (2014) [12] presented a new method for solving linear second-kind Fredholm fuzzy integral equations using ANNs. Modern research has explored and studied intuitionistic fuzzy generalizations within neural-network frameworks, containing intuitionistic fuzzy deep neural network formulas, in addition to fuzzy physics based on neural networks for vagueness-aware learning [13,14]. Furthermore, researchers have examined intuitionistic fuzzy integral equations in Fredholm type from a numerical viewpoint. However, numerical frameworks that merge neural-network-current approximation with IFFIDEs remain insufficiently examined, which motivates the suggested method in this paper.

The presented method transforms the fuzzy integral equation into two parametric linear Fredholm integral equations. Then, the Fredholm integral equations are solved numerically with neural networks based on using optimizing weights through an unconstrained optimization problem. Numerical examples are given to show the proposed approach's reliability and accuracy by highlighting its ability to handle complex fuzzy equations well in real-life applications. Moreover, Mosleh (2014) [15] introduced a modified method for solving second-kind Fredholm integro-differential equations with fuzzy initial values by combining FNNs with Newton-Cotes methods. The FNNs are used as universal approximators by adjusting fuzzy weights with a learning algorithm based on a cost function. Numerical examples are presented to indicate that the proposed method provides accurate solutions and is effective in solving complex fuzzy integro-differential equations in scientific fields. Additionally, Jafarian and Nia, (2013) [16] presented a feedback neural network approach for approximating solutions of linear Fredholm integral equations of the second kind. The presented method involves substituting the n-th truncation of the Taylor series into the integral equations and employing a two-layer feedback neural network to adjust the coefficients with a learning algorithm based on gradient descent. The results show that the proposed FNN method outperforms traditional numerical methods, such as the trapezoidal quadrature rule, and provides accurate approximations with high efficiency. Thereafter, a novel hybrid approach combining FNNs and Newton-Cotes methods was presented by Mosleh and Otadi, (2017) [17] for solving systems of fuzzy linear Fredholm integro-differential equations. The hybrid approach uses FNNs as universal approximators to calculate fuzzy parameters in the system, with a learning algorithm designed to adjust fuzzy weights. Numerical simulations validate the effectiveness of the presented approach and demonstrate their ability to provide accurate approximations of fuzzy solutions in complex integral equations.

After the introduction of fuzzy sets and fuzzy numbers by Zadeh in 1965 [18], several extensions of fuzzy set theory have emerged, one of which is the intuitionistic fuzzy set, which was first proposed by the mathematician Atanassov in 1983 [19]. The work of Atanassov marked a turning point in extending the concept of a classic fuzzy set by adding two functions, independent of each other, which are membership degree T and non-membership degree F for every element in the universe of discourse. IFS enables effective representation for the degree of belongingness and no belongingness, where the sum of these degrees is less than or equal to one. The remaining value, called the hesitation margin or indeterminacy, captures the amount of uncertainty or ignorance for the element's class membership. This framework has therefore provided a more flexible mathematical tool to model imprecision, vagueness, and incomplete information in various fields such as decision-making, pattern recognition, and artificial intelligence [20,21]. Thus, IFS is a crucial tool utilized in situations where available information is not sufficient to determine the element's membership status precisely.

FNNs have gained significant attention due to their ability to handle complex nonlinear systems with uncertain or imprecise data. By combining the flexibility of neural networks with the power of fuzzy logic, these networks can model and approximate a wide range of real-world problems, where traditional deterministic approaches may fail. However, when FNNs are combined with intuitionistic fuzzy sets, which capture membership and non-membership degrees of uncertainty, the model becomes more powerful by providing a more comprehensive representation of uncertainty by incorporating a degree of hesitation or doubt, enhancing decision-making processes. The study of the IFFIDEs is comparatively limited in the literature. The essential concepts and computations of the intuitionistic fuzzy sets were presented by Atanassov as a generalization of classical fuzzy sets, providing a more flexible framework for handling vagueness. Later, the concept of intuitionistic fuzzy differential equations was formulated, and numerous theoretical contributions have considered existence and uniqueness results of the intuitionistic fuzzy integro-differential equations (IFFIDEs) under different conditions, including non-local conditions and semi-linear. On the other hand, these studies are mostly theoretical in nature and focus on analytical aspects. Particularly, numerical techniques for solving IFFIDEs, notably those including Fredholm-type kernels, have not been sufficiently investigated. This gap encourages the development of effective numerical frameworks for these equations. In conclusion, by combining FNNs with intuitionistic fuzzy sets, we can model and solve more complex systems where the data and relationships between variables are inherently uncertain, thus enhancing the decision-making processes across areas and advancing computational intelligence techniques. This motivates combining ANNs (to approximate the unknown solution) with the Newton–Cotes rule to compute the integral term efficiently on uniform nodes, overcoming limitations of traditional solvers for IFFIDEs. To the best of our knowledge, we are the first to combine artificial neural networks with the Newton–Cotes method for solving intuitionistic fuzzy Fredholm integro-differential equations, filling a gap in the literature where such a combined approach has not been studied. Therefore, our main aim of this paper is to extend and implement the FNNs to solve the IFFIDEs where the parameters and variables are considered intuitionistic fuzzy numbers. In particular, a novel modified method based on FNNs and Newton-Cotes methods is presented and applied to solve the IFFIDEs.

2. Preliminaries

Definition 1. Intuitionistic fuzzy sets (IFS) [22]: Let Y be a finite set of elements. An IFS $\widetilde{K}$ in Y is defined as:

$$\widetilde{K} = \{\langle x, \mu_{\widetilde{K}}(y), \nu_{\widetilde{K}}(y)\rangle \mid y \in Y\},$$

Such that the function $\mu_{\widetilde{K}} : Y \to [0,1]$ and $\nu_{\widetilde{K}} : Y \to [0,1]$, respectively, represent the degrees of non-membership and membership of the element $y \in Y$ to set $\widetilde{K}$ such that

$$\mu_{\widetilde{K}}(y) \in [0,1],\ \nu_{\widetilde{K}}(\mathrm{Y}) \in [0,1], \qquad 0 \leq \mu_{\widetilde{K}}(y) + \nu_{\widetilde{K}}(y) \leq 1.$$

Definition 2. (r, β)-cuts [22]: A subset $(r,\beta) -$ cut of Y, constructed by IFS $\widetilde{K}$, at $r \in [0,1]$, $\beta \in [0,1]$ are fixed numbers where $r + \beta \leq 1$ presented as follows:

$$\widetilde{K}_{r,\beta} = \{y \in Y : \mu_{\widetilde{K}}(y)) \geq r,\ \nu_{\widetilde{K}}(y) \leq \beta\},$$

$\widetilde{K}_{r,\beta}$ is a crisp (classic) set of elements that belong to $\widetilde{K}$ at least to the degree of r and that does not belong to $\widetilde{K}$ at most degrees of β.

Definition 3. Intuitionistic fuzzy number (IFN) [22]: An IFS $\widetilde{K}$ on real line is called IFN if it satisfies the following conditions:

1) There exists $y_0 \in \mathbb{R}$ such that $\mu_{\widetilde{K}}(y_0) = 1$ and $\nu_{\widetilde{K}}(y_0) = 0$.
2) The membership function $\mu_{\widetilde{K}}$ is convex,

   i. i.e., $\mu_{\widetilde{K}}(\lambda y_1 + (1-\lambda)y_2) \geq \min\{\mu_{\widetilde{K}}(y_1), \mu_{\widetilde{K}}(y_2);\ \forall y_1, y_2 \in \mathbb{R}, \lambda \in [0,1]\}$.
3) The non-membership function $\nu_{\widetilde{A}}$ is concave,

   i.e., $\nu_{\widetilde{k}}(\lambda y_1 + (1-\lambda)y_2) \leq \max\{\nu_{\widetilde{A}}(y_1), \nu_{\widetilde{A}}(y_2)\};\ \forall y_1, y_2 \in \mathbb{R}, \lambda \in [0,1]$.

Definition 4. Triangular intuitionistic fuzzy number (TIFN) [22]: A triangular intuitionistic fuzzy number $\widetilde{K}$ is denoted by $\widetilde{K} = (a_1, a_2, a_3), (a`_1, a_2, a`_3)$. An IFN $\widetilde{K}$ is called TIFN if its non-membership and membership function follow the rules:

$$\mu_{\widetilde{K}}(y) = \begin{cases} 0, & y \leq a_1 \\ \frac{y-a_1}{a_2-a_1}, & a_1 \leq y \leq a_2 \\ \frac{a_3-y}{a_3-a_2}, & a_2 \leq y \leq a_3 \\ 0, & y \geq a_3 \end{cases}$$

and

$$\nu_{\widetilde{K}}(x) = \begin{cases} 1, & y \leq a`_1 \\ \frac{a_2-y}{a_2-a`_1}, & a`_1 \leq y \leq a_2 \\ \frac{y-a_2}{a`_3-a_2}, & a_2 \leq y \leq a`_3 \\ 1, & y \geq a`_3 \end{cases}$$

where $a`_1 \leq a_1 \leq a_2 \leq a_3 \leq a`_3$.

$(r, \beta) -$ cut of $\widetilde{K} = (a_1, a_2, a_3), (a`_1, a_2, a`_3)$ may be represented as follows:

$$\widetilde{K}_{r,\beta} = \langle [\underline{K}(r), \overline{K}(r)], [\underline{K`}(\beta), \overline{k`}(\beta)] \rangle,$$

where

$$\underline{K}(r) = a_1 + r(a_2 - a_1),$$

$$\overline{K}(r) = a_3 - r(a_3 - a_2),$$

$$\underline{K`}(\beta) = a_2 - \beta(a_2 - a`_1).$$

and

$$\overline{K`}(\beta) = a_2 - \beta(a`_3 - a_2).$$

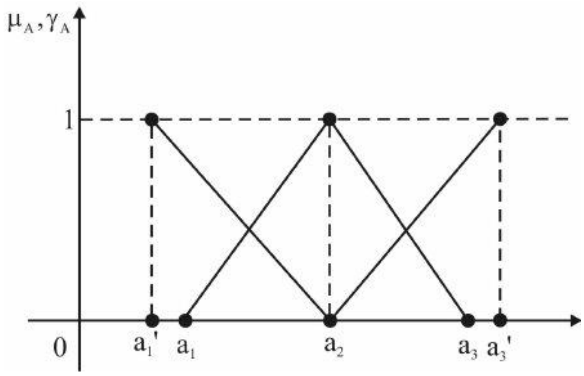


Figure 1. TIFN $\langle (a_1, a_2, a_3), (a'_1, a'_2, a'_3) \rangle$.

Triangular intuitionistic fuzzy numbers are used because they are easy and simple to compute. Thus, they can clarify the representation of the membership and non-membership functions while keeping the numerical calculations manageable. Their framework causes the r-cut and β-cut transformations to be straight, which oversimplify the implementation of the suggested method. The approach can be expanded to different types of intuitionistic fuzzy numbers in further research.

3. Fredholm integro differential equation in the intuitionistic fuzzy environment

In this section, the general form of the IFFIDE of the second kind is defined as follows:

$$\tilde{z}^{(n)}(s) = \tilde{f}(s) + \tilde{\lambda}\int_c^d k(s,t)\tilde{z}(t)dt\,, 0 \le s \le 1,$$
$$\tilde{z}(0) = \tilde{g}_{(1)}(t)\,, \tilde{z}'(0) = \tilde{g}_{(2)}(t)\,, \ldots, \tilde{z}^{(n-1)}(0) = \tilde{g}_{(n-1)}(t). \tag{1}$$

such that $\tilde{z}^{(n)}(s)$ is the n^{th} order derivative of the fuzzy intuitionistic function $\tilde{z}(s)$, and $\tilde{f}(s)$ is a known fuzzy intuitionistic function. $k(s,t)$ is a known function of the variables s and t, calling the kernel of IFFIDE and $k(s,t) \ge 0$. The fuzzy intuitionistic initial conditions are donated by

$$\tilde{z}(0) = \tilde{g}_{(1)}(t)\,, \tilde{z}'(0) = \tilde{g}_{(2)}(t)\,, \ldots, \tilde{z}^{(n-1)}(0) = \tilde{g}_{(n-1)}(t), \tag{2}$$

and c and d are the constants limits of IFFIDE.

The IFFIDE in equation (1) is then defuzzified based on utilizing the singular parametric formula under the r,β-cut framework explained in section 2 at $r,\beta \in [0,1]$, as follows:

$$\left[\tilde{z}^{(n)}(s)\right]_{r,\beta} = \left\{ \left[\underline{z}^{(n)}(s;r), \overline{z}^{(n)}(s;r)\right], \left[\underline{z}^{(n)}(s;\beta)\,, \overline{z}^{(n)}(s;\beta)\right] \right\}, \tag{3}$$

$$\left[\tilde{f}(s)\right]_{r,\beta} = \left\{ \left[\underline{f}(s;r), \overline{f}(s;r)\right], \left[\underline{f}(s;\beta), \overline{f}(s;\beta)\right] \right\}, \tag{4}$$

$$[\tilde{z}(t)]_{r,\beta} = \left\{ \left[\underline{z}(t;r), \overline{z}(t;r)\right], \left[\underline{z}(t;\beta), \overline{z}(t;\beta)\right] \right\}, \tag{5}$$

$$[\tilde{g}(0)]_{r,\beta} = \left\{ \left[\underline{g}(0;r), \overline{g}(0;r)\right], \left[\underline{g}(0;\beta), \overline{g}(0;\beta)\right] \right\}, \tag{6}$$

$$\left[\tilde{\lambda}\right]_{r,\beta} = \left\{ \left[\underline{\lambda}_r\, \overline{\lambda}_r\right], \left[\underline{\lambda}_\beta\,, \overline{\lambda}_\beta\right] \right\}. \tag{7}$$

for $, c \le s \le d\,, \tilde{\lambda} > 0$, and $r \in [0,1]$, and Eq (1) is rephrased to get the general formula of IFFIDE:

$$\begin{cases} \underline{z}^{(n)}(s;r) = \underline{f}(s;r) + \underline{\lambda}\int_c^d k(s,t)\underline{w}(t;r)dt \\ \overline{z}^{(n)}(s;r) = \overline{f}(s;r) + \overline{\lambda}\int_c^d k(s,t)\overline{w}(t;r)\,dt \end{cases}, \tag{8}$$

$$\begin{cases} \underline{z}^{(n)}(s;\beta) = \underline{f}(s;\beta) + \underline{\lambda}\int_c^d k(s,t)\underline{w}(t;\beta)dt \\ \overline{z}^{(n)}(s;\beta) = \overline{f}(s;\beta) + \overline{\lambda}\int_c^d k(s,t)\overline{w}(t;\beta)\,dt \end{cases}. \tag{9}$$

Equation (8) represents the lower intuitionistic bounds and upper intuitionistic bounds of the membership function for the general formula of IFFIDE. Moreover, equation (9) represents the lower intuitionistic bounds and upper intuitionistic bounds of the non-membership function of the general expression of IFFIDE.

4. Function approximation using intuitionistic fuzzy neural networks

FNNs provide mathematical solutions for certain real-life problems with outstanding generalizability. Additionally, a significant characteristic of multi-layer perceptron's is its ability to approximate the complex functions, making them useful for solving a wide range of problems resulting in applicability across a wide range of applications of real-life phenomena [23].

In this section, the ability for the feed-forward FNN to approximate functions is implemented to express a trial solution for the IFFIDE defined in Eq (8) and Eq (9) as the sum of two parts. The first part satisfies the initial conditions and has no adjustable parameters for the membership and non-membership functions. The second part uses two feed-forward FNNs: The first for the fuzzy membership function (T) and the second for the fuzzy non-membership function (F), both trained to meet the IFFIDEs). This type of fuzzy network is chosen for the architecture due to the ability of a multilayer perceptron with one hidden layer to approximate arbitrary functions closely.

Now, $\underline{z}_{T_r}\left(s;r;\underline{p}_r\right)$ is the lower bound of fuzzy trial approximate solution of Eq (8) and $\overline{z}_{T_r}\left(s;r;\overline{p}_r\right)$ is the upper bound of fuzzy trial approximate solution of Eq (8). Moreover, $\underline{z}_{T_\beta}\left(s;\beta;\underline{p}_\beta\right)$ is the lower bound of fuzzy trial approximate solution of Eq (9) and $\overline{z}_{T_\beta}\left(s;\beta;\overline{p}_\beta\right)$ is the upper bound of fuzzy trial solution of Eq (9), such that $\underline{p}_r,\ \overline{p}_r,\ \underline{p}_\beta, and\ \overline{p}_\beta$ are considered adjustable fuzzy parameters (in actuality $\underline{z}_{T_r}\left(s;r;\underline{p}_r\right)$, $\overline{z}_{T_r}\left(s;r;\overline{p}_r\right)$, $\underline{z}_{T_\beta}\left(s;\beta;\underline{p}_\beta\right)$, and $\overline{z}_{T_\beta}\left(s;\beta;\overline{p}_\beta\right)$ are approximations of $\underline{z}(s;r)$, $\overline{z}(s;r)$, $\underline{z}(s;\beta)$, and $\overline{z}(s;\beta)$, respectively). Thus, the problem of calculating the fuzzy approximate solutions in Eq (8) and Eq (9) at certain points in the range $[c,d]$ is equal to finding the fuzzy functionals $\underline{z}_{T_r}$, $\overline{z}_{T_r}$, $\underline{z}_{T_\beta}$, and $\overline{z}_{T_\beta}$ that meet the following constraints of the optimization problem outlined in [19]:

$$\begin{cases} \min_{\tilde{p}} \sum_{i=1}^{M} \left\{ \left(\underline{z}'_{T_r}\left(s;r;\underline{p}_r\right) - \underline{f}(s_i;r) - \underline{F}(s_i;r;\tilde{p}_r) \right)^2 + \left(\overline{z}'_{T_r}\left(s;r;\overline{p}_r\right) - \overline{f}(s_i;r) - \overline{F}(s_i;r;\tilde{p}_r) \right)^2 \right\} \\ \underline{z}_{T_r}\left(s_0;r;\underline{p}_r\right) = \underline{z}_0(r), \qquad \overline{z}_{T_r}\left(s_0;r;\overline{p}_r\right) = \overline{z}_0(r). \end{cases} \tag{10}$$

Such that $\tilde{p}_r = \left(\underline{p}_r\, ,\, \overline{p}_r\right)$ include all adjustable fuzzy parameters of neural network (weights and biases associated with the input and output layer) as the following:

$$\begin{cases} \underline{F}(s;r;\tilde{p}_r) = \underline{\lambda} \int_c^d k(s,t)\underline{z}_{T_r}(t;r;\tilde{p}_r)dt \\ \overline{F}(s;r;\tilde{p}_r) = \overline{\lambda} \int_c^d k(s,t)\overline{z}_{T_r}(t;r;\tilde{p}_r)dt \end{cases}. \tag{11}$$

and for the fuzzy non-membership function,

$$\begin{cases} \min_{\breve{p}} \sum_{i=1}^{M} \left\{ \left(\underline{z}'_{T_\beta}\left(s;\beta;\underline{p}_\beta\right) - \underline{f}`(s_i;\beta) - \underline{F}`(s_i;\beta;\breve{p}_\beta) \right)^2 + \left(\overline{z}'_{T}`\left(s;\beta;\overline{p}_\beta\right) - \overline{f}`(s_i;\beta) - \overline{F}`(s_i;\beta;\breve{p}_\beta) \right)^2 \right\} \\ \underline{z}_{T_\beta}\left(s_0;\beta;\underline{p}_\beta\right) = \underline{z}_0(\beta), \quad \overline{z}_{T_\beta}\left(s_0;\beta;\overline{p}_\beta\right) = \overline{z}_0(\beta). \end{cases} \tag{12}$$

Here, $\tilde{p}_\beta = \left(\underline{p}_\beta, \overline{p}_\beta\right)$ include all adjustable fuzzy parameters of the neural network (weights and biases associated with the input and output layer) as the following:

$$\begin{cases} \underline{F}(s;\beta;\tilde{p}_\beta) = \underline{\lambda} \int_c^d k(s,t) \underline{z}_{T_\beta}(t;\beta;\tilde{p}_\beta) dt \\ \overline{F}(s;\beta;\tilde{p}_\beta) = \overline{\lambda} \int_c^d k(s,t) \overline{z}_{T_\beta}(t;\beta;\tilde{p}_\beta) dt \end{cases}, \tag{13}$$

In this hybrid approach, the neural network approximates the unknown solution, while the Newton–Cotes method accurately evaluates the integral term, combining learning flexibility with numerical stability. Generally, the integration in Eq (13) cannot be obtained analytically. Therefore, in this situation, we naturally use numerical methods. Thus, to approximate the integral term, the Newton–Cotes quadrature rule is used through its compatibility and simplicity with uniformly spaced quantisation nodes. In contrast to Gaussian quadrature, it does not need special non-regular points, which is easy to directly implement among the intuitionistic fuzzy frame. Specifically, the uniform system enables concerted numerical dealing of the membership and non-membership functions in the suggested dual-network structure. A moderating-order scheme is implemented to confirm numerical stability. In this study, the numerical quadrature method R for the interval $[c;d]$ is used under fuzzy conditions with non-negative fuzzy weights g_j and N fuzzy nodes t_j, represented as follows:

$$Rf = \sum_{j=1}^{N} g_j f(t_j) = If - Ef = \int_c^d f(t_j) dt - Ef, \tag{14}$$

where Ef is the error. We ignore this rule-of-square error, and Eqs (10) and (12) are replaced by the following fuzzy approximate equations, respectively:

$$\begin{cases} \min_{\breve{p}} \sum_{i=1}^{M} \left\{ \begin{array}{l} \left(\underline{z}'_{T_r}\left(s;r;\underline{p}_r\right) - \underline{f}(s_i;r) - \underline{\lambda} \sum_{j=1}^{N} g_j k(s_j,t_j) \underline{z}_{T_r}(s_i;r;\breve{p}_r) \right)^2 + \\ \left(\overline{z}'_T(s_i;r;\overline{p}_r) - \overline{f}(s_i;r) - \overline{\lambda} \sum_{j=1}^{N} g_j k(s_j,t_j) \overline{z}_T(s_i;r;\breve{p}_r) \right)^2 \end{array} \right\} \\ \underline{z}_{T_r}\left(s_0;r;\underline{p}_r\right) = \underline{z}_0(r), \quad \overline{z}_{T_r}\left(s_0;r;\overline{p}_r\right) = \overline{z}_0(r) \end{cases} \tag{15}$$

and

$$\begin{cases} \min_{\breve{p}} \sum_{i=1}^{M} \begin{Bmatrix} \left(\underline{z}'_{T\beta}\left(s;\beta;\underline{p}\right) - \underline{f}(s_i;\beta) - \underline{\lambda} \sum_{j=1}^{N} g_j\, k(s_j,t_j) \underline{z}_{T\beta}(s_i;\beta;\breve{p}_\beta) \right)^2 \\ + \left(\overline{z}'_{T\beta}\left(s_i;\beta;\overline{p}_\beta\right) - \overline{f}(s_i;r) - \overline{\lambda} \sum_{j=1}^{N} g_j\, k(s_j,t_j) \overline{z}_{T\beta}(s_i;\beta;\breve{p}_\beta) \right)^2 \end{Bmatrix}. \\ \underline{z}_{T\beta}\left(s_0;\beta;\underline{p}_\beta\right) = \underline{z}_0(\beta),\ \overline{z}_{T\beta}\left(s_0;\beta;\overline{p}_\beta\right) = \overline{z}_0(\beta) \end{cases} \quad (16)$$

Each trial solution of $\underline{z}_T(r)$ and $\overline{z}_T(r)$ for each membership function employs a single feed-forward FNN, where the corresponding fuzzy networks are defined by $\underline{N}(r)$ and $\overline{N}(r)$, with fuzzy membership adjustable parameters $\underline{p}(r)$ and $\overline{p}(r)$, respectively. Additionally, each membership function employs a single feed-forward FNN, where the corresponding fuzzy network are defined by $\underline{N}(\beta)$ and $\overline{N}(\beta)$, with fuzzy non-membership adjustable parameters $\underline{p}(\beta)$ and $\overline{p}(\beta)$, respectively. Thus, the related fuzzy trial functions are presented in the following formula [24]:

For membership components:

$$\begin{aligned} \underline{z}_{T_r}\left(s_i;r;\underline{p}_r\right) &= \underline{z}(s_0,r) + (s - s_0) + \underline{N}_r\left(s;r;\underline{p}_r\right), \\ \overline{z}_{T_r}\left(s_i;r;\overline{p}_r\right) &= \overline{z}(s_0,r) + (s - s_0) + \overline{N}_r\left(s;r;\overline{p}_r\right). \end{aligned} \quad (17)$$

For non-membership components:

$$\begin{aligned} \underline{z}_{T\beta}\left(s_i;\beta;\underline{p}_\beta\right) &= \underline{z}`(s_0,\beta) + (s - s_0) + \underline{N}_\beta\left(s;\beta;\underline{p}_\beta\right), \\ \overline{z}_{T\beta}\left(s_i;\beta;\overline{p}_\beta\right) &= \overline{z}`(s_0,\beta) + (s - s_0) + \overline{N}_\beta\left(s;\beta;\overline{p}_\beta\right), \end{aligned} \quad (18)$$

where $\underline{N}(\mathrm{r}), \overline{N}(r)$, $\underline{N}(\beta)$ and $\overline{N}(\beta)$ are the single-output membership and non- membership function of feed-forward FNNs with intuitionistic adjustable parameters $\underline{p}$ (r), $\overline{p}(r)$, $\underline{p}(\beta)$, and $\overline{p}(\beta)$, respectively, for the lower and upper bounds of membership and non-membership functions, respectively. We assume that this intuitionistic solution satisfies the intuitionistic fuzzy initial condition in Eqs (17) and (18). According to Eqs (17) and (18), it is straight forward to show that:

For membership components:

$$\begin{aligned} \underline{z}'_{T_r}\left(s;r;\underline{p}_r\right) &= \underline{N}_r\left(s;r;\underline{p}_r\right) + (s - s_0)\frac{\partial \underline{N}_r}{\partial s}, \\ \overline{z}'_{T_r}\left(s;r;\overline{p}_r\right) &= \overline{N}_r\left(s;r;\overline{p}_r\right) + (s - s_0)\frac{\partial \overline{N}_r}{\partial s}, \end{aligned} \quad (19)$$

and for non-membership components:

$$\underline{z}'_{T\beta}\left(s;\beta;\underline{p}_\beta\right) = \underline{N}_\beta\left(s;\beta;\underline{p}_\beta\right) + (s - s_0)\frac{\partial \underline{N}_\beta}{\partial s},$$
$$\overline{z}'_{T\beta}\left(s;\beta;\overline{p}_\beta\right) = \overline{N}_\beta\left(s;\beta;\overline{p}_\beta\right) + (s - s_0)\frac{\partial \overline{N}_\beta}{\partial s}. \tag{20}$$

Let us now assume a multilayer intuitionistic perceptron having a single hidden layer involving $H\ sigmoid$ units and an intuitionistic fuzzy linear output unit for membership and non-membership components, as shown in Figures 1–4.

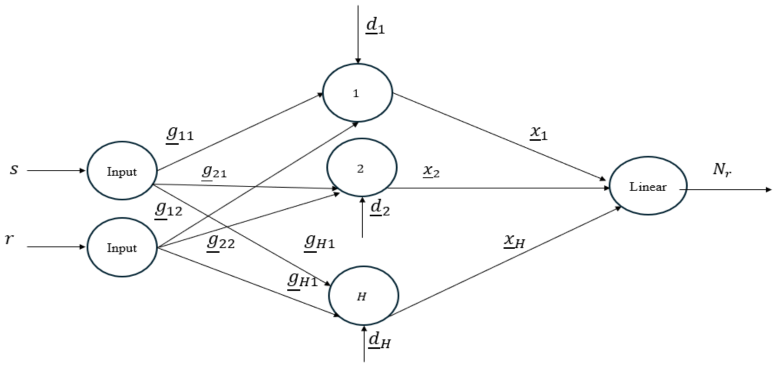


Figure 2. A three-layer sensor with two inputs (s, r) and a single hidden layer that involves H = 10 neurons with bias terms, and a one linear output neuron $\underline{N}_r$.

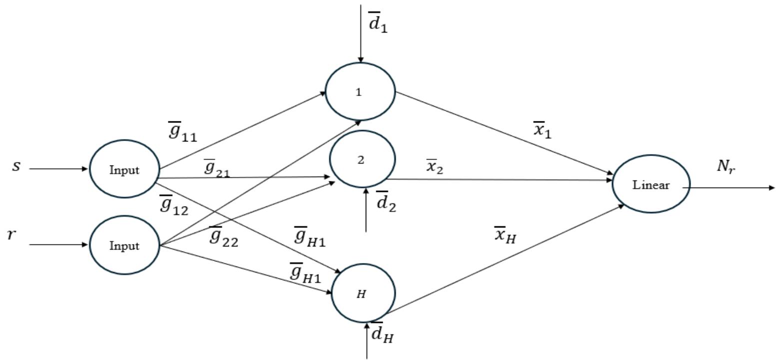


Figure 3. A three-layer sensor with two inputs (s, r) and a single hidden layer that involves H = 10 neurons with bias terms, and a one linear output neuron $\overline{N}_r$.

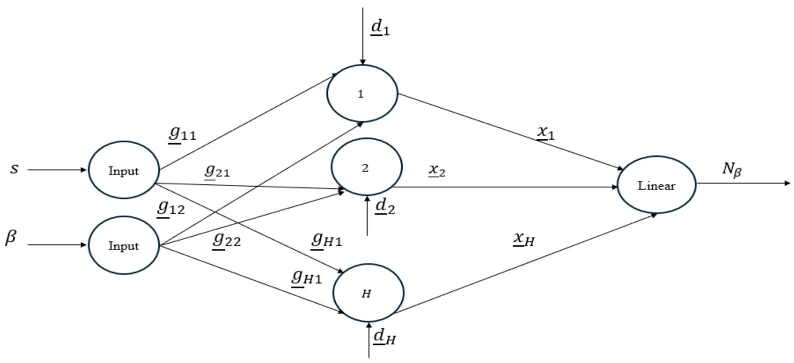


Figure 4. A three-layer sensor with two inputs (s, β) and a single hidden layer that involves H = 10 neurons with bias terms, and a one linear output neuron $\underline{N}_\beta$.

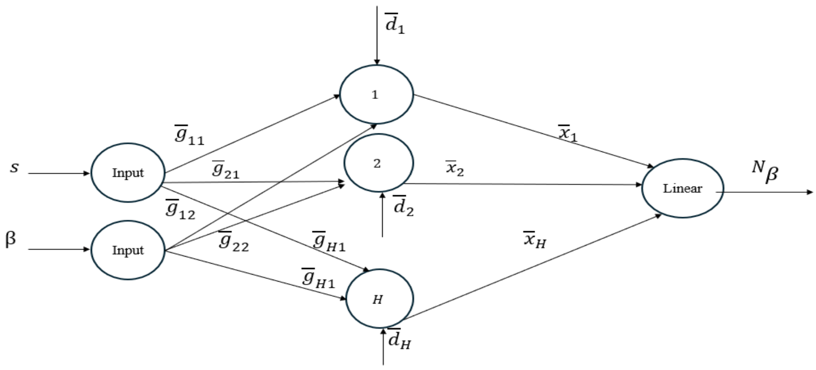


Figure 5. A three-layer sensor with two inputs (s, β) and a single hidden layer that involves H = 10 neurons with bias terms, and a one linear output neuron $\overline{N}_\beta$.

As seen in the above four figures, the figures illustrate the use of a three-layer FNN to approximate the lower and upper bounds of fuzzy membership and non-membership functions in a system described by an IFFIDE. Figure 1, Figure 2, and Figure 3 show the membership function, with $\underline{N}_r$ and $\overline{N}_r$ representing the lower and upper intuitionistic fuzzy solutions for membership, respectively. Figure 3 and Figure 4 show the non-membership function, where $\underline{N}_\beta$ and $\underline{N}_\beta$ represent the lower and upper fuzzy solutions for non-membership function. Each figure represents the FNNs processing two inputs to generate these intuitionistic fuzzy outputs, which are then used to solve the IFFIDE.

Now, we have:

For membership components:

$$\begin{aligned} \underline{N}_r &= \sum_{i=1}^{H} \underline{u}_{i_r} \sigma_r\left(\underline{z}_{i_r}\right), \qquad \underline{z}_{i_r} = \underline{g}_{i1} s + \underline{g}_{i2} r + \underline{d}_i \\ \overline{N}_r &= \sum_{i=1}^{H} \overline{u}_{i_r} \sigma_r\left(\overline{z}_{i_r}\right), \qquad \overline{z}_{i_r} = \overline{g}_{i1} s + \overline{g}_{i2} r + \overline{d}_i \end{aligned} \tag{21}$$

where $\sigma_r\left(\underline{z}_{i_r}\right)$ and $\sigma_r\left(\overline{z}_{i_r}\right)$ are the intuitionistic lower and intuitionistic upper bounds of the sigmoid transfer neural function, respectively. Then, the following are evaluated:

$$\begin{aligned} \frac{\partial \underline{N}_r}{\partial s} &= \sum_{i=1}^{H} \underline{u}_{i_r} \underline{g}_{i1} \sigma_r{}'\left(\underline{z}_{i_r}\right) \\ \frac{\partial \overline{N}_r}{\partial s} &= \sum_{i=1}^{H} \overline{u}_{i_r} \overline{g}_{i1} \sigma_r{}'\left(\overline{z}_{i_r}\right) \end{aligned} \tag{22}$$

where $\sigma_r{}'(\tilde{z})$ is the intuitionistic fuzzy first derivative of membership function of the sigmoid function. Furthermore, among the many possible sigmoid functions, we select the following fuzzy $\sigma_r(\tilde{z}) = \frac{1}{(1+e^{-z})}$ since all derivatives can be obtained from the fuzzy $\sigma_r(\tilde{z})$ in terms of the fuzzy sigmoid function. Thus, we have following:

$$\sigma_r'(\tilde{z}) = -\sigma_r^{\,2}(\tilde{z}) + \sigma_r\,(\tilde{z}). \tag{23}$$

For non-membership components:

$$\begin{aligned} \underline{N}_\beta &= \sum_{i=1}^{H} \underline{u}_{i\,\beta}\, \sigma_\beta\left(\underline{z}_{i_\beta}\right), \qquad \underline{z}_{i\,\beta} = \underline{g}_{i1}s + \underline{g}_{i2}\beta + \underline{d}_i \\ \overline{N}_\beta &= \sum_{i=1}^{H} \overline{u}_{i\,\beta}\, \sigma_\beta\left(\overline{z}_{i\beta}\right), \qquad \overline{z}_{i\beta} = \overline{g}_{i1}s + \overline{g}_{i2}\beta + \overline{d}_i \end{aligned} \tag{24}$$

where $\sigma_\beta(\tilde{z})$ is an intuitionistic fuzzy sigmoid transfer function. Then, the following are evaluated:

$$\begin{aligned} \frac{\partial \underline{N}_\beta}{\partial s} &= \textstyle\sum_{i=1}^{H} \underline{u}_{i\,\beta}\, \underline{g}_{i1} \sigma'_\beta\left(\underline{z}_{i_\beta}\right), \\ \frac{\partial \overline{N}_\beta}{\partial s} &= \textstyle\sum_{i=1}^{H} \overline{u}_{i\beta} \overline{g}_{i1}\, \sigma'_\beta\left(\overline{z}_{i\beta}\right), \end{aligned} \tag{25}$$

where $\sigma'_\beta\left(\overline{z}_{i\beta}\right)$ is the intuitionistic first derivative of the non-membership function of the sigmoid function. Furthermore, among the many possible sigmoid functions, we select the following fuzzy $\sigma_\beta(\tilde{z}) = \frac{1}{(1+e^{-z})}$ since all derivatives can be obtained from the fuzzy $\sigma_\beta`(z)$ in terms of the fuzzy sigmoid function itself. Thus, we have:

$$\sigma_\beta{}'\,(\tilde{z}) = -\sigma_\beta{}^2(\tilde{z}) + \sigma_\beta(\tilde{z}). \tag{26}$$

For reproducibility, the proposed ANN–Newton–Cotes method is summarized as follows: The integral term in the IFFIDE is approximated using a Newton–Cotes quadrature rule. The intuitionistic fuzzy solution bounds are represented by ANN-based trial functions. The ANN weights and biases are initialized with small random values and then updated iteratively using backpropagation by minimizing the mean squared error (MSE) of the equation residuals at selected collocation points until convergence. Training is stopped when the MSE reaches a prescribed tolerance or when the maximum number of epochs is achieved. In addition to MSE, RMSE may be used to evaluate approximation accuracy. In this study, a feed-forward neural network with one hidden layer is employed, where the hidden layer uses a sigmoid activation function, and the output layer is linear. The number of hidden neurons $H$and the number of Newton–Cotes nodes $N$are selected to balance accuracy and computational cost (e.g., $H = 10$ in the numerical example), and a small positive learning rate is used to ensure stable convergence.

5. Numerical example

Example 1. Consider the following linear IFFIDE of the second kind:

$$\tilde{z}'(s) = \tilde{f}(s) + \int_0^s \ \widetilde{w}(t,\alpha)dt, \tag{27}$$

with

$$\tilde{z}(0) = [0.75 + 0.25r, 1.25 - 0.25r\,;\ 1 - 0.25\beta, 1 + 0.25\beta]$$

where $0 \le t, s \le 1,\ 0 \le r, \beta \le 1,\ k(s,t) = 1$ and $\tilde{f}(s) = \left[\underline{f}(s,r), \overline{f}(s,r); \underline{f}(s,\beta), \overline{f}(s,\beta)\right]$ i.e.,

$$\tilde{f}(s) = \{(e^s - s)[0.75 + 0.25r, 1.25 - 0.25r]\,, (e^s - s)[1 - 0.25\beta, 1 + 0.25\beta]\}$$

The exact solution is provided as:

$$\tilde{z}(s) = \{\, e^s\,[0.75 + 0.25r, 1.25 - 0.25r]\,, e^s[1 - 0.25\beta, 1 + 0.25\beta]\,\}$$

The functions of the trial for this problem are:

$$\begin{aligned} \underline{z}_{T_r}(s;r) &= (0.75 + 0.25r) + s\sum_{i=1}^{H} \frac{\underline{u}_{i_r}}{1 + e^{\left(-\underline{w}_{i1_r}\right)s - \left(\underline{w}_{i2_r}\right)r - \underline{b}_i}}, \\ \overline{z}_{T_r}(s;r) &= (1.25 - 0.25r) + s\sum_{i=1}^{H} \frac{\overline{u}_{i_r}}{1 + e^{\left(-\overline{w}_{i1_r}\right)s - \left(\overline{w}_{i2_r}\right)r - \overline{b}_i}}. \end{aligned} \tag{28}$$

Moreover,

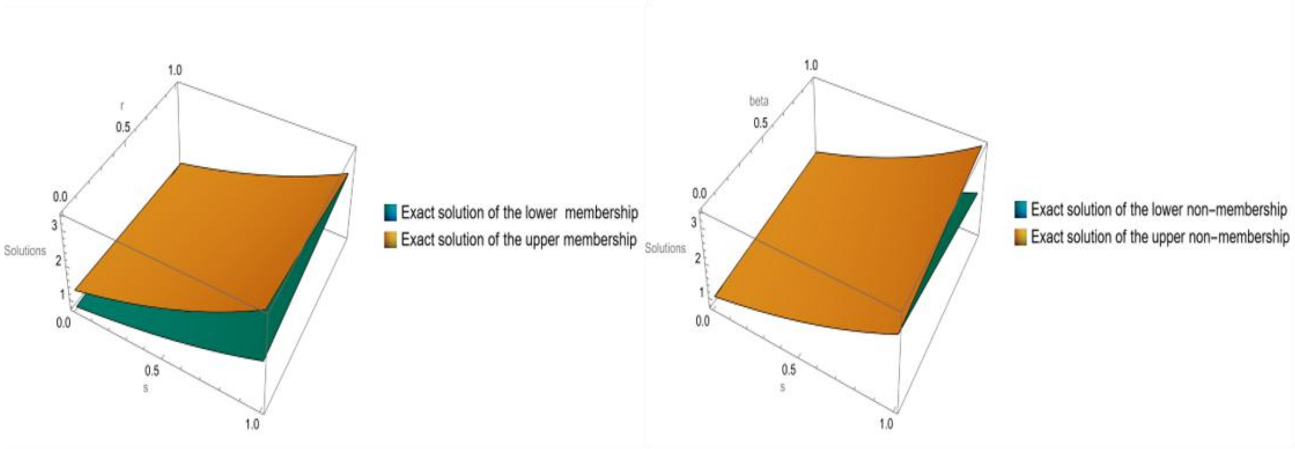
$$\begin{aligned} \underline{z}_{T_\beta}(s;\beta) &= (1 - 0.25\beta) + s\sum_{i=1}^{H} \frac{\underline{u}_{i_\beta}}{1 + e^{\left(-\underline{w}_{i1_\beta}\right)s - \left(\underline{w}_{i2_\beta}\right)\beta - \underline{b}_i}}, \\ \overline{z}_{T_\beta}(s;\beta) &= (1 + 0.25\beta) + s\sum_{i=1}^{H} \frac{\overline{u}_{i_\beta}}{1 + e^{\left(-\overline{w}_{i1_\beta}\right)s - \left(\overline{w}_{i2_\beta}\right)\beta - \overline{b}_i}}. \end{aligned} \tag{29}$$

Table 1. Intuitionistic exact and intuitionistic approximation and lower and upper solutions of membership for Eq (27) by artificial neural networks at $s = 0.2$ for all $r \in [0,1]$.

	Lower fuzzy solution of membership			Upper fuzzy solution of membership		
(r)	Exact Solution	Approximation Solution	Absolute Error	Exact Solution	Approximation Solution	Absolute Error
0	0.916052	0.916052	6.862×10^{-8}	1.526753	1.526753	1.143×10^{-7}
0.2	0.977122	0.977122	7.319×10^{-8}	1.465683	1.465683	1.097×10^{-7}
0.4	1.038192	1.038192	7.776×10^{-8}	1.404613	1.404613	1.052×10^{-7}
0.6	1.099262	1.099262	8.234×10^{-8}	1.343543	1.343542	1.006×10^{-7}
0.8	1.160332	1.160332	8.691×10^{-8}	1.282472	1.282472	9.606×10^{-8}
1	1.221402	1.221402	9.149×10^{-8}	1.221402	1.221402	9.149×10^{-8}

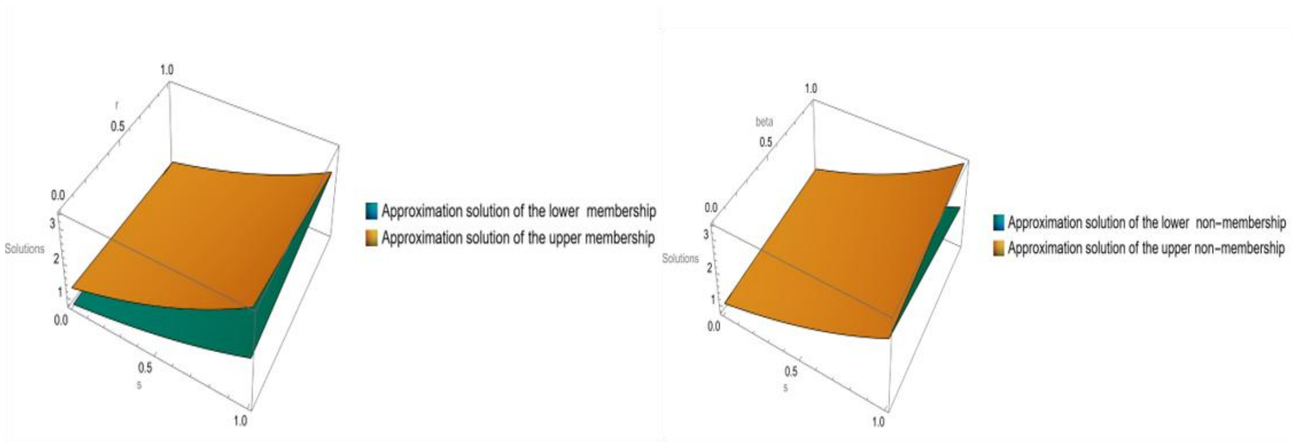
Table 2. Intuitionistic exact and intuitionistic approximation and lower and upper solutions of the non-membership for Eq (27) by artificial neural networks at $s = 0.2$ for all $\beta \in [0,1]$.

	Lower fuzzy solution of non-membership			Upper fuzzy solution of non-membership		
(β)	Exact Solution	Approximation Solution	Absolute Error	Exact Solution	Approximation Solution	Absolute Error
0	1.221402	1.221402	9.149×10^{-8}	1.221402	1.221402	9.149×10^{-8}
0.2	1.160332	1.160332	8.691×10^{-8}	1.282472	1.282472	9.606×10^{-8}
0.4	1.099262	1.099262	8.234×10^{-8}	1.343543	1.343542	1.006×10^{-7}
0.6	1.038192	1.038192	7.776×10^{-8}	1.404613	1.404613	1.052×10^{-7}
0.8	0.977122	0.977122	7.319×10^{-8}	1.465683	1.465683	1.097×10^{-7}
1	0.916052	0.916052	6.862×10^{-8}	1.526753	1.526753	1.143×10^{-7}



(a) (b)

Figure 6. Intuitionistic fuzzy lower and upper exact solutions for Eq (27) (a) of membership and (b) non-membership for all $s \in [0,1]$ and $r, \beta \in [0,1]$.



(a) (b)

Figure 7. Intuitionistic fuzzy lower and upper approximation solutions for Eq (27) (a) of membership and (b) non-membership for all $s \in [0,1]$ and $r, \beta \in [0,1]$.

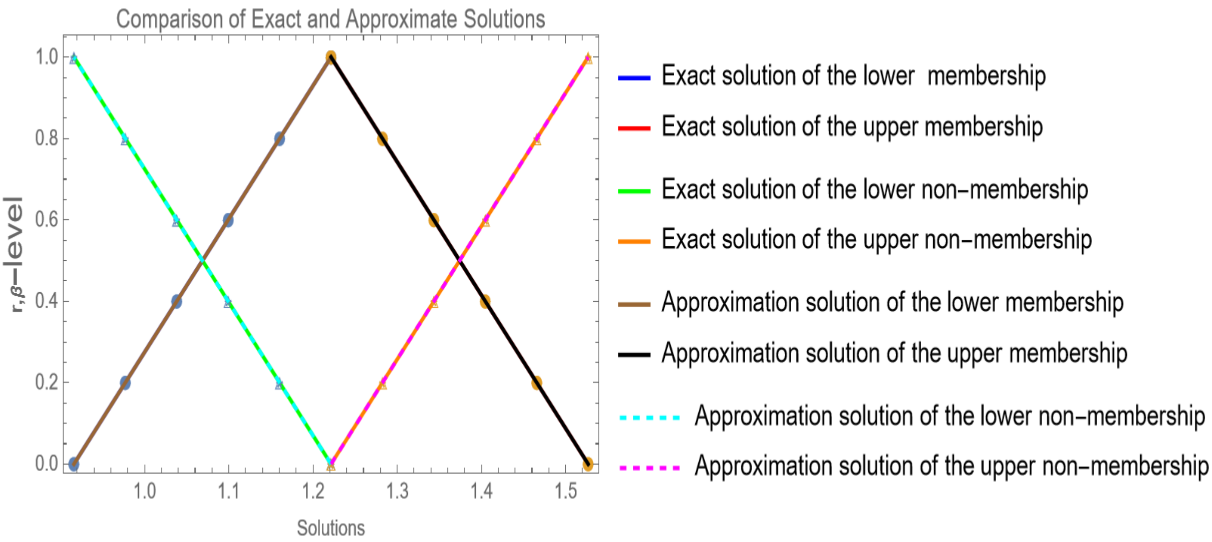


Figure 8. 2D plot comparing the intuitionistic exact and intuitionistic approximation solutions of Eq (29) using artificial neural networks for $r, \beta \in [0,1]$ at $s = 0.2$.

Upon examining the results presented in Figure 8, we can find many important remarks and conclusions regarding the performance and reliability of our proposed method for solving Eq (29) at $s = 0.2$. In general, it can be noted from Table 1, Table 2, and Figures 6–8 that the results obtained by the artificial neural networks are in good agreement with the exact solution for Eq (29) at $s = 0.2$ and all $r, \beta \in [0,1]$, attaining the intuitionistic triangular fuzzy number shape described in Section 2 for the two membership functions r-cut and non-membership β-cut functions, thereby satisfying the properties of fuzzy intuitionistic numbers. Furthermore, the results demonstrate that the FNN method accurately approximates the intuitionistic fuzzy membership and non-membership solutions, with the outputs closely aligning with the exact solutions. The FNN's ability to compute these fuzzy bounds effectively handles the imprecision inherent in intuitionistic fuzzy systems. This method's results are crucial for solving the IFFIDE, as the FNNs provide reliable approximations for membership and non-membership functions, as seen in the low absolute errors across the computed outputs.

Regarding error convergence, the accuracy of the proposed ANN–Newton–Cotes method improves when the number of hidden neurons H is increased, since a larger network provides a richer approximation space. Likewise, increasing the number of Newton–Cotes integration nodes N reduces the quadrature error in approximating the integral term. Therefore, the overall error decreases as H and N increase until it reaches a saturation level due to the training tolerance and machine precision. In our experiments, we select moderate values (e.g., $H = 10$) to balance accuracy, stability, and computational cost. The proposed method achieves errors of order 10^{-8} with CPU time in the order of a few seconds, using $H = 10$ neurons and a moderate number of Newton–Cotes nodes. Therefore, the overall error decreases as H and N increase, until it reaches a saturation level due to the training tolerance and machine precision.

6. Conclusions

In this paper, we solve the IFFIDEs based on develop, and apply a modified method based on artificial neural network and Newton-Cotes methods with a positive coefficient. The parameters and variables of IFFIDEs are considered intuitionistic fuzzy numbers. TIFN is used to represent the intuitionistic fuzzy variables and parameters. The fuzzification of the deterministic r-cut and β-cut solutions leads to the artificial neural intuitionistic numerical solution. The main reason for using neural networks is their applicability in handling the intuitionistic fuzzy variables and parameters of IFFIDEs. A numerical example is provided to illustrate the proposed method. The study shows that the results align with theoretical predictions, emphasizing how the integration of artificial neural networks and Newton-Cotes methods improves accuracy and efficiency in solving IFFIDEs. Although the proposed method shows high accuracy and efficiency, its implementation is limited to one-dimensional problems with triangular intuitionistic fuzzy numbers. Thus, researchers may extend it to more general fuzzy systems and higher-dimensional models.

Author Contributions

Hamzeh Zureigat: Conceptualization, methodology, software, validation, formal analysis, investigation, writing—original draft preparation, writing-review and editing; Murad Algazo: Formal analysis, resources, writing—original draft preparation, funding acquisition. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflicts of interest

The author declares no conflicts of interest.

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