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Research article

Exploring the truncated M-fractional exact soliton solutions and modulation instability of the Heimburg model

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Abstract: This research investigated the different types of exact soliton solutions of the biomathematics model known as the truncated M-fractional Heimburg model. This concerned equation describes the transmission of electromechanical pulses in nerves as well as to describe the flow of blood through blood vessels. For our purpose, we used the modified extended tanh function scheme and the modified (G'/G^2)-expansion scheme. The obtained exact soliton solutions were the periodic, kink, singular, dark, bright, and other soliton solutions. The obtained solutions were dynamically explained by using 2D, 3D, and contour graphs. The obtained results were nonexistent due to the use of a novel definition of fractional derivatives. Both schemes were not used for the concerned model in the literature. The effect of the fractional derivative on the solutions was explained by using 2D graphs. Next we gained the steady-state solutions with the help of modulation instability analysis. The obtained solutions are useful for various purposes like blood flow simulation, vascular disease modeling, hemodynamic analysis, medical device design, physiological research, etc.

Keywords: Heimburg model; novel analytical techniques; dynamical analysis; modulation instability; exact solitons
Mathematics Subject Classification: 35Q51, 35Q92, 35R11, 35R60, 60H15

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1. Introduction

Most naturally occurring phenomena are represented in the form of nonlinear fractional partial differential equations (NLFPDEs). Different techniques have been developed to obtain the exact and numerical solutions for the NLFPDEs such as the extended sinh-Gordon equation expansion technique [1], the extended simple equation technique [2], the generalized first integral technique [3], the modified simplest equation technique [4], the (G'/G)-expansion technique [5], PID controllers [6], the improved $(G'/G)-$expansion method [7], the polynomial complete discriminant system method, the dynamical system method [8], the semi-inverse method [9, 10], and the Hirota bilinear method [11]. etc.

We used the modified extended tanh function and modified (G'/G^2)-expansion techniques in this research. Both techniques are used for different models. For example, the modified extended tanh function technique is used for the Broer-Kaup-Kupershmidt equation [12], the Bogoyavlenskii equation [13], the concatenation equation [14], the Lakshmanan-Porsezian-Daniel equation [15], etc. Similarly, the modified (G'/G^2)-expansion technique is used for the classical Boussinesq system of equations [16], the Kadomtsev Petviashvili-modified equal width equation [17], the Wazwaz Kaur Boussinesq equation [18], the generalized Bretherton equation [19], the coupled nonlinear Higgs equation [20], etc.

Soliton theory is the study of solitary waves that maintain their shape and speed over long distances, often arising in nonlinear systems. These "solitons" have particle-like properties and are found in various fields including optics, fluids, nonlinear PDEs, and signal processing, etc.

We study the biomembranes and nerves model, which was developed by Heimburg and Jackson. The concerned biomathematics model describes the mechanical procedure in biomembranes. Assumed by the equation, the nerve axon is represented as a cylinder-shaped biomembrane that, at the right termperature, lower than usual, transitions from a fluid to a gel structure. Consider the following Heimburg model:

$$v_{tt} - ((1 + av + bv^2)v_x)_x + v_{xxxx} - \lambda v_{xxt} = 0. \tag{1.1}$$

Here, $v = v(x, t)$ represents the wave profile and a, b, and λ are the non-zero constants.

Different types of schemes are applied to gain different results for Eq (1), including the kink- and periodic-shaped solitons obtained by utilizing the $\exp(-\phi(\zeta))$-expansion scheme [21]. By using the Hirota bilinear scheme, homoclinic breather wave, lump wave, and periodic cross kink wave solutions are obtained in [22]. By utilizing the extended sinh-Gordon equation expansion technique, the Jacobi elliptic doubly periodic function solution, and solitary wave solutions are obtained in [23].

The motivation of this research is to discover the novel exact wave solutions of the nonlinear Heimburg equation along the truncated M-fractional derivative (TMFD). The impact of the TMFD on the obtained solutions is shown by a 2D plot. The use of the truncated M-fractional derivative provides the solutions nearer to the numerical solutions of the governing model. The truncated M-fractional derivative fulfills the properties of both integer and fractional-order derivatives. The obtained solutions do not exist in the literature. Moreover, we performed modulation instability analysis (MI) to obtain the steady-state solutions of the concerning system. Both techniques do not need any intermediate transformation. The used methods are simple and useful for other nonlinear fractional models in applied sciences and engineering.

The truncated M-fractional Heimburg model explains the following:

(i) Models realistic nerve pulse propagation;
(ii) Finite memory and dissipation in biological tissues;
(iii) Thermal and mechanical effects ignored by electrical models;
(iv) Stability and soliton dynamics in complex media.

This paper contains the following sections: Section 2 explains the modified extended tanh function and modified (G'/G^2)-expansion schemes. Section 3 provides the mathematical analysis and exact wave solutions of the concerned model. Section 4 gives the graphical representation of the obtained solutions. Section 5 describes the modulation instability. Section 6 provides the results analysis. At the end, a conclusion is given in Section 7.

1.1. Truncated M-fractional derivative

Definition 1.1. Consider $w(y) : [0, \infty) \to \Re$, and hence the TMFD of w of degree α is given as [24]:

$$D_{M,y}^{\alpha,\beta} w(y) = \lim_{h\to 0} \frac{w\,(y\,E_\beta(hy^{1-\epsilon})) - w(y)}{h}, \quad \alpha \in (0,1], \;\; \beta > 0,$$

where $E_\beta(.)$ shows a truncated Mittag-Leffler function [25]

$$E_\beta(z) = \sum_{j=0}^{i} \frac{z^j}{\Gamma(\beta j + 1)}, \;\; \varrho > 0 \;\text{ and }\; z \in \mathbf{C}.$$

Theorem 1.1. Suppose $a, b \in \Re$, and g, f are $\epsilon-$ differentiable for $y > 0$, from [24]:

$$(a)\; D_{M,y}^{\alpha,\beta}(ag(y) + bf(y)) = aD_{M,y}^{\alpha,\beta} g(y) + bD_{M,y}^{\alpha,\beta} f(y).$$

$$(b)\; D_{M,y}^{\alpha,\beta}(g(y).f(y)) = g(y)D_{M,y}^{\alpha,\beta} f(y) + f(y)D_{M,y}^{\alpha,\beta} g(y).$$

$$(c)\; D_{M,y}^{\alpha,\beta}(f(g(y))) = D_{M,y}^{\alpha,\beta} g(y).D_{M,y}^{\alpha,\beta} f(g(y)).$$

$$(d)\; D_{M,y}^{\alpha,\beta}(\frac{g(y)}{f(y)}) = \frac{f(y)D_{M,y}^{\alpha,\beta} g(y) - g(y)D_{M,y}^{\alpha,\beta} f(y)}{(f(y))^2}.$$

$$(e)\; D_{M,y}^{\alpha,\beta}(C) = 0, \;\; \text{where } C \text{ is a constant.}$$

$$(f)\; D_{M,y}^{\alpha,\beta} g(y) = \frac{y^{1-\alpha}}{\Gamma(\beta+1)} \frac{dg(y)}{dy}.$$

2. Description of schemes

2.1. Modified extended tanh function scheme

Here, we provide the main description of the scheme.

Consider a nonlinear fractional PDE:

$$\chi(v, v^2, ..., \frac{\partial^\epsilon v}{\partial t^\alpha}, \frac{\partial^{2\alpha} v}{\partial t^{2\alpha}}, ..., \frac{\partial v}{\partial x}, \frac{\partial^2 v}{\partial x^2}, ...) = 0, \tag{2.1}$$

where $v = v(x, t)$ denotes a wave profile. Suppose the wave transformation is given as:

$$v(x, t) = V(\delta), \;\; \delta = x - \mu \frac{\Gamma(1+\beta)}{\alpha} t^\alpha. \tag{2.2}$$

Here, μ represents the wave speed. Substituting Eq (2.2) in Eq (2.1), we get a nonlinear ODE:

$$\chi(V, V^2, \mu V', \mu v^2 V'', ...) = 0. \tag{2.3}$$

Consider the solution of Eq (2.3) shown as:

$$V(\delta) = a_0 + \sum_{i=1}^{m} a_i \psi^i(\delta) + \sum_{i=1}^{m} b_i \psi^{-i}(\delta). \tag{2.4}$$

Here $a_0, a_i, b_i (i = 1, 2, 3, ..., m)$ are unknowns.

By using the homogenous balance principle in Eq (2.4), we get the value of m. Here, $\psi(\zeta)$ satisfies a given ODE:

$$\psi'(\delta) = \Omega + \psi^2(\delta), \tag{2.5}$$

where, Ω denotes the constant and the results of Eq (2.5) are given as [26]:

Family 1: If $\Omega < 0$:

$$\psi(\delta) = -\sqrt{-\Omega} \tanh(\sqrt{-\Omega}\, \delta), \tag{2.6}$$

or

$$\psi(\delta) = -\sqrt{-\Omega} \coth(\sqrt{-\Omega}\, \delta). \tag{2.7}$$

Family 2: If $\Omega > 0$:

$$\psi(\delta) = \sqrt{\Omega} \tan(\sqrt{\Omega}\, \delta), \tag{2.8}$$

or

$$\psi(\delta) = -\sqrt{\Omega} \cot(\sqrt{\Omega}\, \delta). \tag{2.9}$$

Family 3: If $\Omega = 0$:

$$\psi(\delta) = \frac{-1}{\delta}. \tag{2.10}$$

Putting Eq (2.4) in Eq (2.3) along with Eq (2.5), and by manipulating the system, we get the results of Eq (2.1).

The modified extended tanh function method has several advantages:

(i) It can be applied to various nonlinear evolution equations.
(ii) It provides exact traveling wave solutions, including solitons and kink solutions.
(iii) It involves straightforward calculations and simple algebraic manipulations.
(iv) It is effective for equations with complex nonlinearities.

2.2. *Modified (G'/G^2)−expansion scheme*

Here, we provide some main points given in [27].

Point 1:

Consider Eqs (2.1)–(2.3).

Point 2:

Consider the solution of Eq (2.3) given as

$$Q(\delta) = \sum_{i=0}^{m} \alpha_i (\frac{G'}{G^2})^i, \tag{2.11}$$

where, $\alpha_i (i = 0, 1, 2, 3, ..., m)$ are unknowns while $\alpha_i \neq 0$. The novel profile $G = G(\delta)$ satisfies the equation

$$(\frac{G'}{G^2})' = \lambda_0 + \lambda_1 (\frac{G'}{G^2})^2, \tag{2.12}$$

where λ_0 and λ_1 represent the parameters. The solutions of Eq (2.12) depending on a are given as:
Type 1: When $\lambda_0 \lambda_1 < 0$, we have

$$(\frac{G'}{G^2}) = -\frac{\sqrt{|\lambda_0 \lambda_1|}}{\lambda_1} + \frac{\sqrt{|\lambda_0 \lambda_1|}}{2} (\frac{C_1 \sinh(\sqrt{\lambda_0 \lambda_1}\ \delta) + C_2 \cosh(\sqrt{\lambda_0 \lambda_1}\ \delta)}{C_1 \cosh(\sqrt{\lambda_0 \lambda_1}\ \delta) + C_2 \sinh(\sqrt{\lambda_0 \lambda_1}\ \delta)}). \tag{2.13}$$

Type 2: When $\lambda_0 \lambda_1 > 0$, we have

$$(\frac{G'}{G^2}) = \sqrt{\frac{\lambda_0}{\lambda_1}} (\frac{C_1 \cos(\sqrt{\lambda_0 \lambda_1}\ \delta) + C_2 \sin(\sqrt{\lambda_0 \lambda_1}\ \delta)}{C_1 \sin(\sqrt{\lambda_0 \lambda_1}\ \delta) - C_2 \sin(\sqrt{\lambda_0 \lambda_1}\ \delta)}). \tag{2.14}$$

Type 3: When $\lambda_0 = 0$ and $\lambda_1 \neq 0$, we have

$$(\frac{G'}{G^2}) = -\frac{C_1}{\lambda_1 (C_1 \delta + C_2)}. \tag{2.15}$$

Here C_1 and C_2 are parameters.
Point 3:

Substituting Eq (2.11) in Eq (2.14), along with Eq (2.12), and summing up the coefficients of each power of $(\frac{G'}{G^2})^i$ equal to zero, after manipulating, the achieved set has $\alpha_i, \lambda_0, \lambda_1$, and v.
Point 4:

Using α_i, v are obtained in Step 3 in Eq (2.3), one can gain the results of Eq (2.1).

Here are some advantages of the modified (G'/G^2)-expansion scheme:
(i) It simplifies the process of finding exact solutions for complex nonlinear equations.
(ii) It can generate various types of solutions, including hyperbolic, trigonometric, and rational solutions.
(iii) It does not require specific initial or boundary conditions to find solutions.
(iv) It can be extended to solve fractional partial differential equations.

3. Mathematical analysis and exact solitons

Consider the concerned model in the sense of the TMFD given as

$$D_{M,t}^{2\alpha,\beta} v - D_{M,x}^{\alpha,\beta}((1 + av + bv^2) D_{M,x}^{\alpha,\beta} v) + D_{M,x}^{4\alpha,\beta} v - \lambda D_{M,x}^{2\alpha,\beta}(D_{M,t}^{\alpha,\beta} v) = 0. \tag{3.1}$$

Applying the given wave relation in Eq (3.1), we have

$$v(x, t) = V(\delta), \quad \delta = \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \mu \frac{t^\alpha}{\alpha}). \tag{3.2}$$

$D_{M,t}^{2\alpha,\beta} v = \mu^2 V''$, $D_{M,x}^{4\alpha,\beta} v = V^{(4)}$, $D_{M,x}^{2\alpha,\beta}(D_{M,t}^{\alpha,\beta} v) = -\mu V^{(3)}$, $D_{M,x}^{\alpha,\beta}((1+av+bv^2) D_{M,x}^{\alpha,\beta} v) = aVV'' + a(V')^2 + bV^2 V'' + 2bV(V')^2 + V''$.

We obtain the following:

$$-aVV'' - a(V')^2 - bV^2 V'' - 2bV(V')^2 + V^{(4)} + \lambda \mu V^{(3)} + (\mu^2 - 1)V'' = 0. \tag{3.3}$$

3.1. *Novel wave solutions by the modified extended* tanh *function scheme*

Equation (2.4) changes into the given relation if $m = 1$.

$$V(\delta) = \alpha_1\phi(\delta) + \alpha_0 + \frac{\beta_1}{\phi(\delta)}. \tag{3.4}$$

Inserting Eq (3.4) with Eq (2.5) in Eq (3.3), and manipulating the set of equations, we get the given results:
Set 1:

$$\{\alpha_0 = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = -\frac{\sqrt{6}}{\sqrt{b}}, \beta_1 = 0, \mu = \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.5}$$

Case 1:

$$\begin{aligned} v(x,t) = &-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\tanh(\sqrt{-\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))), \end{aligned} \tag{3.6}$$

or

$$\begin{aligned} v(x,t) = &-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\coth(\sqrt{-\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))). \end{aligned} \tag{3.7}$$

Case 2:

$$\begin{aligned} v(x,t) = &-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(\sqrt{\Omega}\tan(\sqrt{\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))), \end{aligned} \tag{3.8}$$

or

$$\begin{aligned} v(x,t) = &-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{\Omega}\cot(\sqrt{\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))). \end{aligned} \tag{3.9}$$

Case 3:

$$v(x,t) = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2-4b)} + a\left(\lambda^2-6\right)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})}. \tag{3.10}$$

Set 2:

$$\{\alpha_0 = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = -\frac{\sqrt{6}}{\sqrt{b}}, \beta_1 = 0, \mu = -\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.11}$$

Case 1:

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\tanh(\sqrt{-\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))), \end{aligned} \tag{3.12}$$

or

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\coth(\sqrt{-\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))). \end{aligned} \tag{3.13}$$

Case 2:

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(\sqrt{\Omega}\tan(\sqrt{\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))), \end{aligned} \tag{3.14}$$

or

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &- \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{\Omega}\cot(\sqrt{\Omega}\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))). \end{aligned} \tag{3.15}$$

Set 3:

$$\{\alpha_0 = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))} + a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = -\frac{\sqrt{6}}{\sqrt{b}}, \beta_1 = \frac{\sqrt{6}\Omega}{\sqrt{b}}, \mu = \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.16}$$

Case 1:

$$\begin{aligned} v(x,t) =& -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+ \frac{\sqrt{6}}{\sqrt{b}}(\sqrt{-\Omega}\tanh(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega}) \end{aligned}$$

$$
- \frac{\Omega}{\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})}), \tag{3.17}
$$

or

$$
\begin{aligned}
v(x,t) = &- \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\
&+ \frac{\sqrt{6}}{\sqrt{b}}(\,\sqrt{-\Omega}\coth(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega}) \\
&- \frac{\Omega}{\sqrt{-\Omega}\coth(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})}).
\end{aligned} \tag{3.18}
$$

Case 2:

$$
\begin{aligned}
v(x,t) = &- \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\
&+ \frac{\sqrt{6}}{\sqrt{b}}(-\,\sqrt{\Omega}\tan(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega}) \\
&+ \frac{\Omega}{\sqrt{\Omega}\tan(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega})}),
\end{aligned} \tag{3.19}
$$

or

$$
\begin{aligned}
v(x,t) = &- \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\
&+ \frac{\sqrt{6}}{\sqrt{b}}(\,\sqrt{\Omega}\cot(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega}) \\
&- \frac{\Omega}{\sqrt{\Omega}\cot(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})}).
\end{aligned} \tag{3.20}
$$

Set 4:

$$
\{\alpha_0 = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))} - a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = -\frac{\sqrt{6}}{\sqrt{b}}, \beta_1 = \frac{\sqrt{6}\Omega}{\sqrt{b}}, \mu = -\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.21}
$$

Case 1:

$$
\begin{aligned}
v(x,t) = &\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))} - a(\lambda^2-6)}{2b(\lambda^2-6)} \\
&+ \frac{\sqrt{6}}{\sqrt{b}}(\,\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} + \frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})
\end{aligned}
$$

$$- \frac{\Omega}{\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})}), \tag{3.22}$$

or

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+ \frac{\sqrt{6}}{\sqrt{b}}(\,\sqrt{-\Omega}\coth(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega}) \\ &- \frac{\Omega}{\sqrt{-\Omega}\coth(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})}). \end{aligned} \tag{3.23}$$

Case 2:

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+ \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{\Omega}\tan(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega}) \\ &+ \frac{\Omega}{\sqrt{\Omega}\tan(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega})}), \end{aligned} \tag{3.24}$$

or

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+ \frac{\sqrt{6}}{\sqrt{b}}(\,\sqrt{\Omega}\cot(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega}) \\ &- \frac{\Omega}{\sqrt{\Omega}\cot(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})}). \end{aligned} \tag{3.25}$$

Set 5:

$$\{\alpha_0 = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = \frac{\sqrt{6}}{\sqrt{b}}, \beta_1 = 0, \mu = -\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.26}$$

Case 1:

$$\begin{aligned} v(x,t) =& -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+ \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega})), \end{aligned} \tag{3.27}$$

or

$$v(x,t) = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\coth(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})). \tag{3.28}$$

Case 2:

$$v(x,t) = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}}(\sqrt{\Omega}\tan(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})), \tag{3.29}$$

or

$$v(x,t) = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{\Omega}\cot(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})). \tag{3.30}$$

Set 6:

$$\{\alpha_0 = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = \frac{\sqrt{6}}{\sqrt{b}}, \beta_1 = 0, \mu = \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.31}$$

Case 1:

$$v(x,t) = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\tanh(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})), \tag{3.32}$$

or

$$v(x,t) = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\coth(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})). \tag{3.33}$$

Case 2:

$$v(x,t) = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}}(\sqrt{\Omega}\tan(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})), \tag{3.34}$$

or

$$\begin{aligned} v(x,t) =&\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+\frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{\Omega}\cot(\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})). \end{aligned} \tag{3.35}$$

Set 7:

$$\begin{aligned} &\{\alpha_0=-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}+a(\lambda^2-6)}{2b(\lambda^2-6)}, \\ &\alpha_1=\frac{\sqrt{6}}{\sqrt{b}},\beta_1=-\frac{\sqrt{6}\Omega}{\sqrt{b}},\mu=-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \end{aligned} \tag{3.36}$$

Case 1:

$$\begin{aligned} v(x,t) =&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}+a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+\frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega}) \\ &+\frac{\Omega}{\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})}), \end{aligned} \tag{3.37}$$

$$\begin{aligned} v(x,t) =&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}+a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+\frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\coth(\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega}) \\ &+\frac{\Omega}{\sqrt{-\Omega}\coth(\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})}). \end{aligned} \tag{3.38}$$

Case 2:

$$\begin{aligned} v(x,t) =&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}+a(\lambda^2-6)}{2b(\lambda^2-6)} \\ &+\frac{\sqrt{6}}{\sqrt{b}}(\,\sqrt{\Omega}\tan(\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega}) \\ &+\frac{\Omega}{\sqrt{\Omega}\tan(\,\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})}), \end{aligned} \tag{3.39}$$

$$\begin{aligned}v(x,t) =& -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}+a(\lambda^2-6)}{2b(\lambda^2-6)}\\ &+\frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{\Omega}\cot(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})\\ &+\frac{\Omega}{-\sqrt{\Omega}\cot(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})}).\end{aligned} \tag{3.40}$$

Set 8:

$$\{\alpha_0=\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)},\alpha_1=\frac{\sqrt{6}}{\sqrt{b}},\beta_1=-\frac{\sqrt{6}\Omega}{\sqrt{b}},\mu=\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.41}$$

Case 1:

$$\begin{aligned}v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)}\\ &+\frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\tanh(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})\\ &+\frac{\Omega}{\sqrt{-\Omega}\tanh(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})}),\end{aligned} \tag{3.42}$$

$$\begin{aligned}v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)}\\ &+\frac{\sqrt{6}}{\sqrt{b}}(-\sqrt{-\Omega}\coth(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})\\ &+\frac{\Omega}{\sqrt{-\Omega}\coth(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega})}).\end{aligned} \tag{3.43}$$

Case 2:

$$\begin{aligned}v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)}\\ &+\frac{\sqrt{6}}{\sqrt{b}}(\sqrt{\Omega}\tan(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})\\ &+\frac{\Omega}{\sqrt{\Omega}\tan(\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega})}),\end{aligned} \tag{3.44}$$

$$
\begin{aligned}
v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+4b(8\Omega-1))}-a(\lambda^2-6)}{2b(\lambda^2-6)} \\
&+\frac{\sqrt{6}}{\sqrt{b}}\Big(-\sqrt{\Omega}\cot\Big(\Gamma(\beta+1)\Big(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}\Big)\sqrt{\Omega}\Big) \\
&+\frac{\Omega}{-\sqrt{\Omega}\cot\Big(\Gamma(\beta+1)\big(\frac{x^\alpha}{\alpha}-\frac{\sqrt{3(a^2+32b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}\big)\sqrt{\Omega}\Big)}\Big).
\end{aligned}
\tag{3.45}
$$

Set 9:

$$
\begin{aligned}
\{\alpha_0 &= -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}, \\
\alpha_1 &= 0, \beta_1 = -\frac{\sqrt{6}\Omega}{\sqrt{b}}, \mu = -\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}.
\end{aligned}
\tag{3.46}
$$

Case 1:

$$
\begin{aligned}
v(x,t) =& -\frac{\sqrt{6}\Omega}{\sqrt{b}\Big(-\sqrt{-\Omega}\tanh\Big(\Gamma(\beta+1)\big(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}\big)\sqrt{-\Omega}\Big)\Big)} \\
&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)},
\end{aligned}
\tag{3.47}
$$

$$
\begin{aligned}
v(x,t) =& -\frac{\sqrt{6}\Omega}{\sqrt{b}\Big(-\sqrt{-\Omega}\coth\Big(\Gamma(\beta+1)\big(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}\big)\sqrt{-\Omega}\Big)\Big)} \\
&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}.
\end{aligned}
\tag{3.48}
$$

Case 2:

$$
\begin{aligned}
v(x,t) =& -\frac{\sqrt{6}\Omega}{\sqrt{b}\Big(\sqrt{\Omega}\tan\Big(\Gamma(\beta+1)\big(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}\big)\sqrt{\Omega}\Big)\Big)} \\
&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)},
\end{aligned}
\tag{3.49}
$$

$$
\begin{aligned}
v(x,t) =& -\frac{\sqrt{6}\Omega}{\sqrt{b}\Big(-\sqrt{\Omega}\cot\Big(\Gamma(\beta+1)\big(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}\big)\sqrt{\Omega}\Big)\Big)} \\
&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}.
\end{aligned}
\tag{3.50}
$$

Set 10:

$$\{\alpha_0 = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)},$$
$$\alpha_1 = 0, \beta_1 = -\frac{\sqrt{6}\Omega}{\sqrt{b}}, \mu = -\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.51}$$

Case 1:

$$v(x,t) = -\frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{-\Omega}\tanh(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega}))}$$
$$+ \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}, \tag{3.52}$$

$$v(x,t) = -\frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{-\Omega}\coth(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega}))}$$
$$+ \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}. \tag{3.53}$$

Case 2:

$$v(x,t) = -\frac{\sqrt{6}\Omega}{\sqrt{b}(\sqrt{\Omega}\tan(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega}))}$$
$$+ \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}, \tag{3.54}$$

$$v(x,t) = -\frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{\Omega}\cot(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega}))}$$
$$+ \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}. \tag{3.55}$$

Set 11:

$$\{\alpha_0 = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)},$$
$$\alpha_1 = 0, \beta_1 = \frac{\sqrt{6}\Omega}{\sqrt{b}}, \mu = \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.56}$$

Case 1:

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega}))} - \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}, \tag{3.57}$$

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{-\Omega}\coth(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega}))} - \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}. \tag{3.58}$$

Case 2:

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(\,\sqrt{\Omega}\tan(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega}))} - \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}, \tag{3.59}$$

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{\Omega}\cot(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} - \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{\Omega}))} - \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}. \tag{3.60}$$

Set 12:

$$\begin{aligned}\{\alpha_0 &= \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)},\\ \alpha_1 &= 0, \beta_1 = \frac{\sqrt{6}\Omega}{\sqrt{b}}, \mu = -\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\}.\end{aligned} \tag{3.61}$$

Case 1:

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{-\Omega}\tanh(\,\Gamma(\beta+1)(\frac{x^{\alpha}}{\alpha} + \frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^{\alpha}}{\alpha})\sqrt{-\Omega}))} + \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)}, \tag{3.62}$$

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{-\Omega}\coth(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{-\Omega}))} + \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}. \tag{3.63}$$

Case 2:

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(\sqrt{\Omega}\tan(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega}))} + \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}, \tag{3.64}$$

$$v(x,t) = \frac{\sqrt{6}\Omega}{\sqrt{b}(-\sqrt{\Omega}\cot(\Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\Omega-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})\sqrt{\Omega}))} + \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\Omega-4b)} - a(\lambda^2-6)}{2b(\lambda^2-6)}. \tag{3.65}$$

3.2. *Novel wave solutions by the modified (G'/G^2)−expansion technique*

When $m = 1$, Eq (2.11) changes into:

$$V(\delta) = \alpha_0 + \alpha_1\left(\frac{G'(\delta)}{G^2(\delta)}\right). \tag{3.66}$$

Here, α_0 and α_1 are unknowns. Putting Eq (3.66) along Eq (2.12) in Eq (3.3) provides the following results:
Set 1:

$$\{\alpha_0 = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = -\frac{\sqrt{6}\lambda_1}{\sqrt{b}}, \mu = \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.67}$$

Case 1:

$$\begin{aligned} v(x,t) = & -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)} + a(\lambda^2-6)}{2b(\lambda^2-6)} \\ & - \frac{\sqrt{6}\lambda_1}{\sqrt{b}}(-\frac{\sqrt{|\lambda_0\lambda_1|}}{\lambda_1} + \frac{\sqrt{|\lambda_0\lambda_1|}}{2}\ ((C_1\sinh(\sqrt{\lambda_0\lambda_1}\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})) \\ & + C_2\cosh(\sqrt{\lambda_0\lambda_1}\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} \\ & - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})))/(C_1\cosh(\sqrt{\lambda_0\lambda_1}\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})) \end{aligned}$$

$$+ C_2 \sinh(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))))). \tag{3.68}$$

Case 2:

$$\begin{aligned} v(x,t) =& -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)} \\ & - \frac{\sqrt{6}\lambda_1}{\sqrt{b}}(\sqrt{\frac{\lambda_0}{\lambda_1}} \ ((C_1 \cos(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})) \\ & + C_2 \sin(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} \\ & - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})))/(C_1 \sin(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})) \\ & - C_2 \sin(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))))). \end{aligned} \tag{3.69}$$

Case 3:

$$v(x,t) = -\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)} + \frac{\sqrt{6}}{\sqrt{b}}(\frac{C_1}{(C_1 \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}) + C_2)}). \tag{3.70}$$

Set 2:

$$\{\alpha_0 = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)}, \alpha_1 = -\frac{\sqrt{6}\lambda_1}{\sqrt{b}}, \mu = -\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\}. \tag{3.71}$$

Case 1:

$$\begin{aligned} v(x,t) =& \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)} \\ & - \frac{\sqrt{6}\lambda_1}{\sqrt{b}}(-\frac{\sqrt{|\lambda_0\lambda_1|}}{\lambda_1} + \frac{\sqrt{|\lambda_0\lambda_1|}}{2} \ ((C_1 \sinh(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})) \\ & + C_2 \cosh(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} \\ & + \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})))/(C_1 \cosh(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})) \\ & + C_2 \sinh(\sqrt{\lambda_0\lambda_1} \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))))). \end{aligned} \tag{3.72}$$

Case 2:

$$v(x,t) = \frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)}$$

$$\begin{aligned}
&-\frac{\sqrt{6}\lambda_1}{\sqrt{b}}(\sqrt{\frac{\lambda_0}{\lambda_1}}\ ((C_1\cos(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))\\
&+C_2\sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}\\
&+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})))/(C_1\sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))\\
&-C_2\sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))))).
\end{aligned}\tag{3.73}$$

Case 3:

$$v(x,t)=\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2-4b)}-a(\lambda^2-6)}{2b(\lambda^2-6)}+\frac{\sqrt{6}}{\sqrt{b}}(\frac{C_1}{(C_1\ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})+C_2)}).\tag{3.74}$$

Set 3:

$$\{\alpha_0=-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)},\alpha_1=\frac{\sqrt{6}\lambda_1}{\sqrt{b}},\mu=-\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\}.\tag{3.75}$$

Case 1:

$$\begin{aligned}
v(x,t)=&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}\\
&+\frac{\sqrt{6}\lambda_1}{\sqrt{b}}(-\frac{\sqrt{|\lambda_0\lambda_1|}}{\lambda_1}+\frac{\sqrt{|\lambda_0\lambda_1|}}{2}\ ((C_1\sinh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))\\
&+C_2\cosh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}\\
&+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})))/(C_1\cosh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))\\
&+C_2\sinh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))))).
\end{aligned}\tag{3.76}$$

Case 2:

$$\begin{aligned}
v(x,t)=&-\frac{\sqrt{\lambda^2(\lambda^2-6)(a^2+8b\lambda_0\lambda_1-4b)}+a(\lambda^2-6)}{2b(\lambda^2-6)}\\
&+\frac{\sqrt{6}\lambda_1}{\sqrt{b}}(\sqrt{\frac{\lambda_0}{\lambda_1}}\ ((C_1\cos(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))\\
&+C_2\sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}\\
&+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha})))/(C_1\sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta+1)(\frac{x^\alpha}{\alpha}+\frac{\sqrt{3(a^2+8b\lambda_0\lambda_1-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}))
\end{aligned}$$

$$- C_2 \sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}} \frac{t^\alpha}{\alpha}))))). \tag{3.77}$$

Case 3:

$$v(x,t) = -\frac{\sqrt{\lambda^2(\lambda^2 - 6)(a^2 - 4b)} + a(\lambda^2 - 6)}{2b(\lambda^2 - 6)} - \frac{\sqrt{6}}{\sqrt{b}}(\frac{C_1}{(C_1\ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} + \frac{\sqrt{3(a^2-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}) + C_2)}). \tag{3.78}$$

Set 4:

$$\{\alpha_0 = -\frac{\sqrt{\lambda^2(\lambda^2 - 6)(a^2 + 8b\lambda_0\lambda_1 - 4b)} + a(\lambda^2 - 6)}{2b(\lambda^2 - 6)}, \alpha_1 = \frac{\sqrt{6}\lambda_1}{\sqrt{b}}, \mu = \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\}. \tag{3.79}$$

Case 1:

$$\begin{aligned} v(x,t) = &- \frac{\sqrt{\lambda^2(\lambda^2 - 6)(a^2 + 8b\lambda_0\lambda_1 - 4b)} + a(\lambda^2 - 6)}{2b(\lambda^2 - 6)} \\ &+ \frac{\sqrt{6}\lambda_1}{\sqrt{b}}(-\frac{\sqrt{|\lambda_0\lambda_1|}}{\lambda_1} + \frac{\sqrt{|\lambda_0\lambda_1|}}{2}\ \ ((C_1 \sinh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha})) \\ &+ C_2 \cosh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} \\ &- \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha})))/(C_1 \cosh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha})) \\ &+ C_2 \sinh(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha}))))). \end{aligned} \tag{3.80}$$

Case 2:

$$\begin{aligned} v(x,t) = &- \frac{\sqrt{\lambda^2(\lambda^2 - 6)(a^2 + 8b\lambda_0\lambda_1 - 4b)} + a(\lambda^2 - 6)}{2b(\lambda^2 - 6)} \\ &+ \frac{\sqrt{6}\lambda_1}{\sqrt{b}}(\sqrt{\frac{\lambda_0}{\lambda_1}}\ \ ((C_1 \cos(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha})) \\ &+ C_2 \sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} \\ &- \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha})))/(C_1 \sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha})) \\ &- C_2 \sin(\sqrt{\lambda_0\lambda_1}\ \ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2 + 8b\lambda_0\lambda_1 - 4b)}}{\sqrt{2b(\lambda^2 - 6)}}\frac{t^\alpha}{\alpha}))))). \end{aligned} \tag{3.81}$$

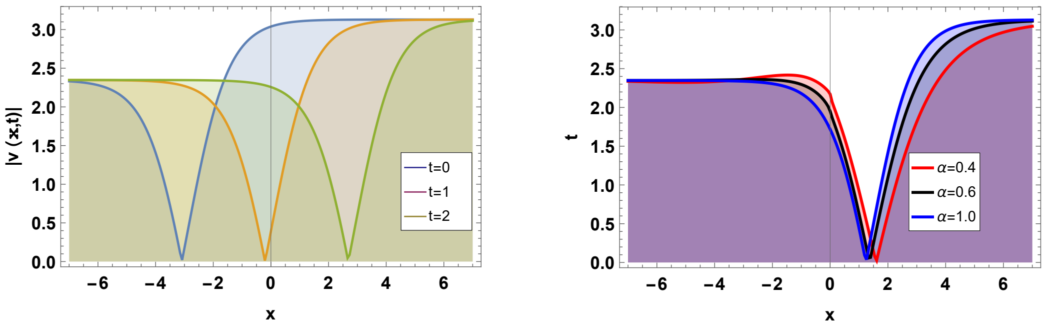
Case 3:

$$v(x,t) = -\frac{\sqrt{\lambda^2(\lambda^2 - 6)(a^2 - 4b)} + a(\lambda^2 - 6)}{2b(\lambda^2 - 6)} + \frac{\sqrt{6}\lambda_1}{\sqrt{b}}(-\frac{C_1}{\lambda_1(C_1\ \Gamma(\beta + 1)(\frac{x^\alpha}{\alpha} - \frac{\sqrt{3(a^2-4b)}}{\sqrt{2b(\lambda^2-6)}}\frac{t^\alpha}{\alpha}) + C_2)}). \tag{3.82}$$

4. Graphical representation

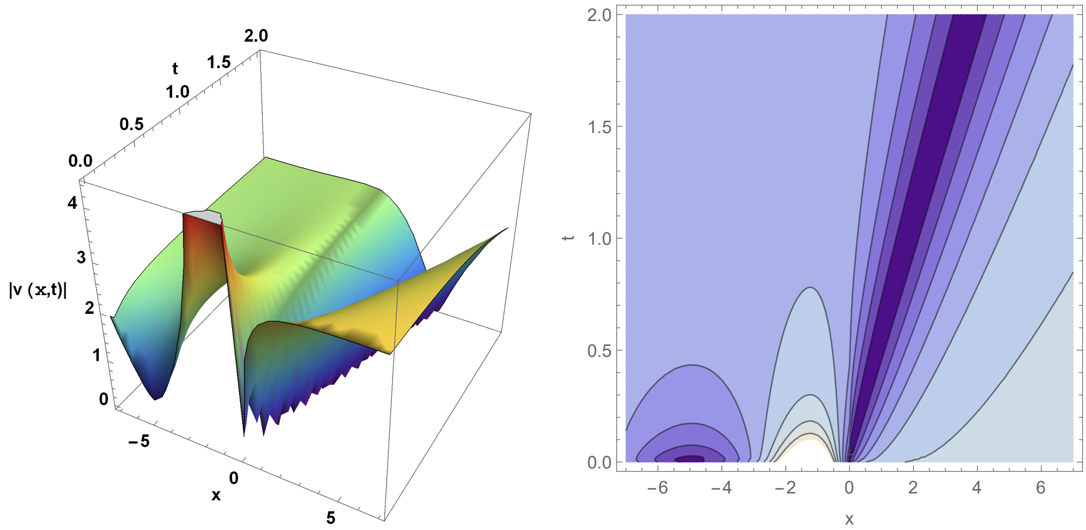
Graphical representation for achieved results of the truncated M-fractional nonlinear Heimburg equation will be explained here as shown in Figures 1–6.

We used contour, 3D, and 2D graphs to explain the dynamical behavior of the results. Plots when $\alpha = 0.4, 0.6, 1$ are also drawn to show the impact of the fractional derivative. To avoid the complex or singular solutions, $\lambda^2 > 6$ and $b \neq 0$ should hold for all the obtained solutions.



(a) 2D plot

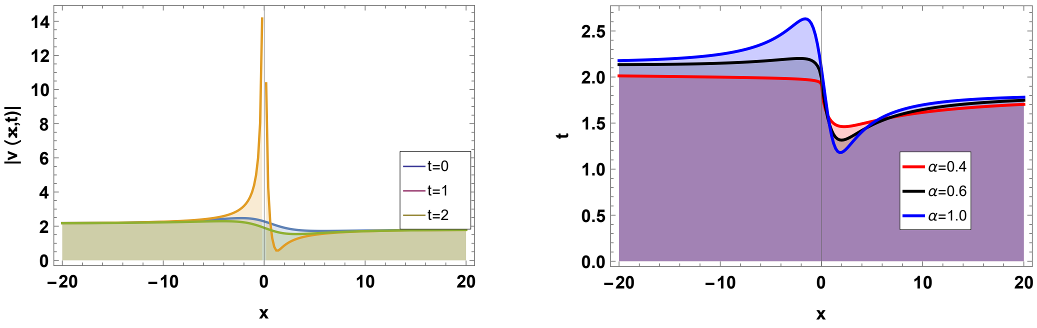
(b) 2D plot



(c) 3D plot for $\alpha = 0.5$

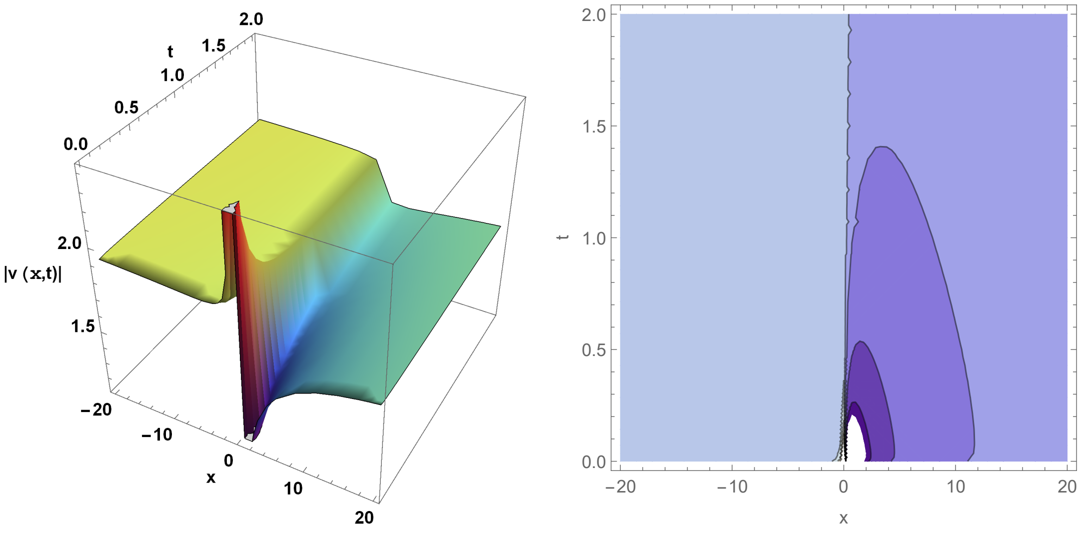
(d) Contour graph when $\alpha = 0.5$

Figure 1. The graph of $|v(x,t)|$ is denoted through Eq (3.6) for: $a = 0.06, b = 0.4, \Omega = -0.5, \lambda = 0.5$, and $\beta = 1$. (a) represents the 2D graph for $x \in (-7, 7)$ for $\alpha = 1$, with a blue shape for t=0, an orange shape for t=1, and a green shape for t=2. (b) is the two-dimensional plot if $-7 < x < 7$ and $0 < t < 2$, with a red shape if α is 0.4, a black shape if α is 0.6, and a blue shape if α is 1. We can observe that the phase of the wave changes with the change in the value of α. (c) is a 3D graph for $\alpha = 0.5$ with $0 < t < 2$. (d) shows a contour graph when α is 0.5 when $t \in (0, 2)$. Kink soliton solutions have various applications such as in condensed matter physics, particle physics, biological systems, and optical fibers.



(a) 2D plot

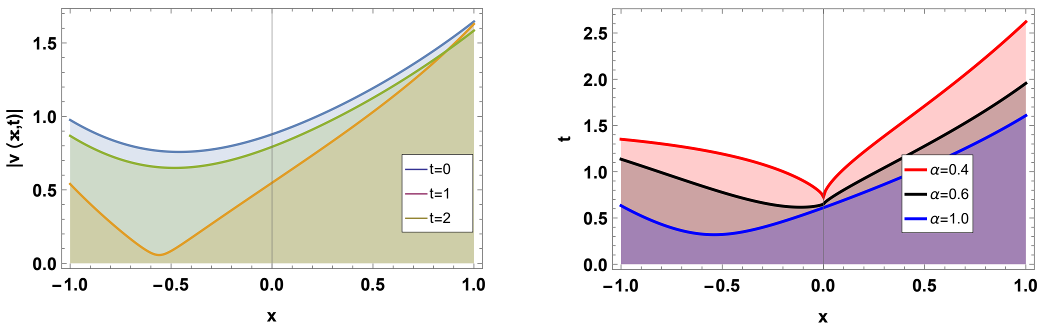
(b) 2D plot



(c) 3D plot for $\alpha = 0.5$

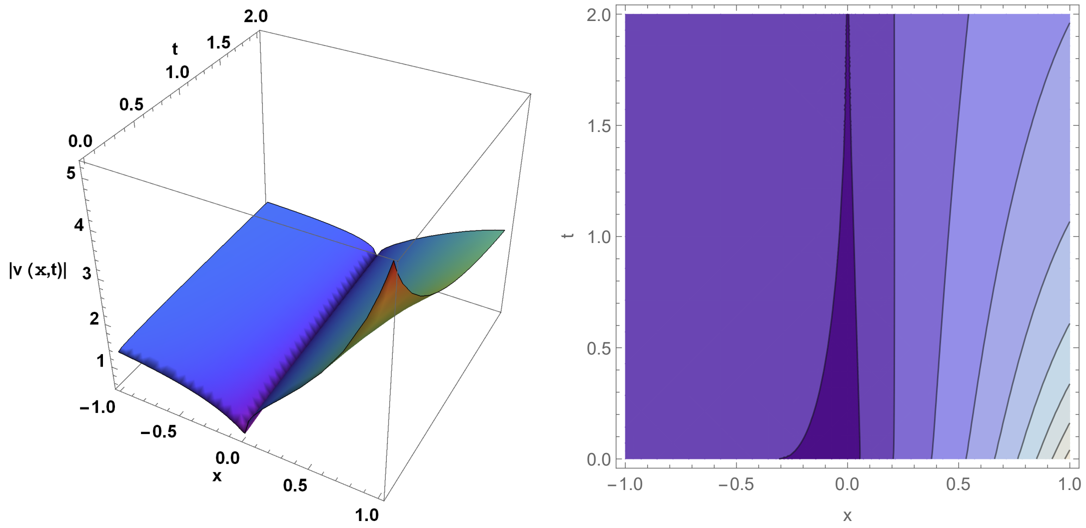
(d) Contour graph when $\alpha = 0.5$

Figure 2. The graph for $|v(x,t)|$ is denoted through Eq (3.7) for: $a = 3.8, b = 1, \Omega = -0.006, \lambda = 0.8$, and $\beta = 1$. (a) represents the 2D plot when $x \in (-20, 20)$ for $\alpha = 1$, with a blue shape for t=0, an orange shape for t=1, and a green shape for t=2. (b) is the two-dimensional plot if $-20 < x < 20$ and $0 < t < 2$, with a red shape if α is 0.4, a black shape if α is 0.6, and a blue shape if α is 1. It is seen that the phase of the wave solution is shifted along the variation of α. (c) is a 3D graph for $\alpha = 0.5$ with $0 < t < 2$. (d) shows a contour graph when α is 0.5 if $t \in (0, 2)$. Singular solitons have applications in different fields including nonlinear optics, fluid dynamics, plasma physics, and mathematical models.



(a) 2D plot

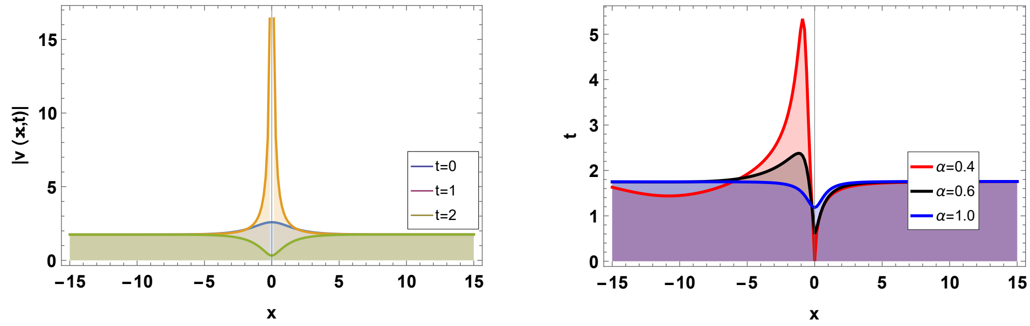
(b) 2D plot



(c) 3D plot for $\alpha = 0.5$

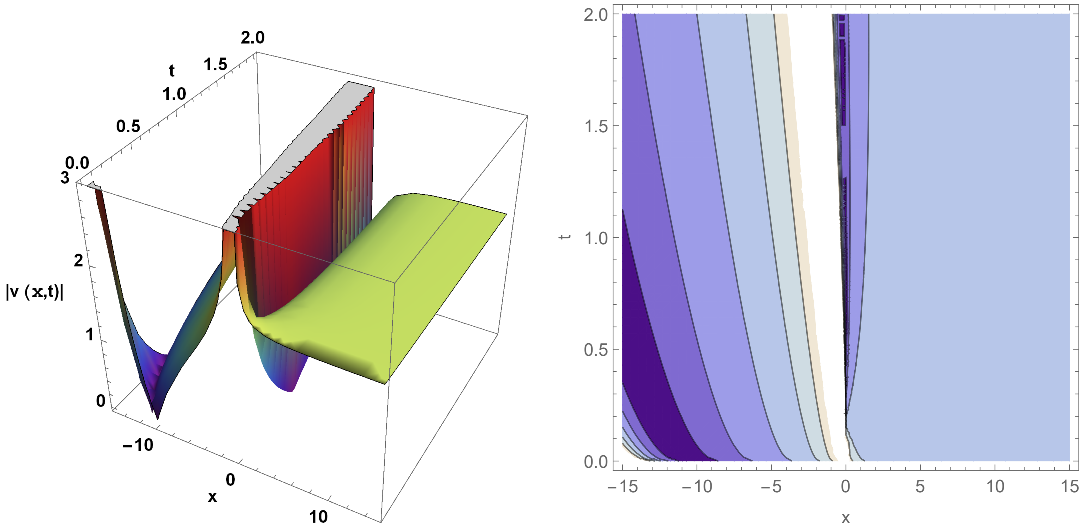
(d) Contour graph when $\alpha = 0.5$

Figure 3. The graph of $|v(x, t)|$ is represented by Eq (3.8) for: $a = 1.2, b = 1.1, \Omega = 0.4, \lambda = 0.4$, and $\beta = 1$. (a) shows the 2D plot for $x \in (-1, 1)$ for $\alpha = 1$, with a blue shape for $t = 0$, an orange shape for $t = 1$, and a green shape for $t = 2$. (b) is the two-dimensional plot if $-1 < x < 1$ and $0 < t < 2$, with a red shape if α is 0.4, a black shape if α is 0.6, and a blue shape if α is 1. We can observe that the phase of the wave solution changes with the change in α. (c) is a 3D graph for $\alpha = 0.5$ with $0 < t < 2$. (d) shows the contour graph when α is 0.5 when $t \in (0, 2)$. Periodic soliton solutions have applications in nonlinear optics, water waves, plasma physics, and Bose-Einstein condensates.



(a) 2D plot

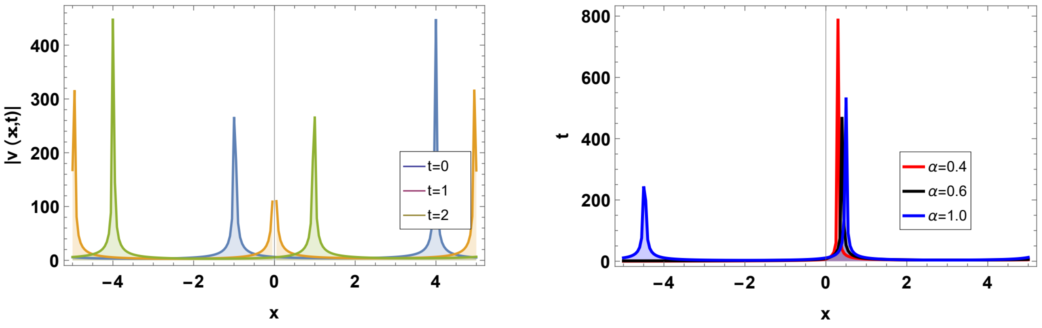
(b) 2D plot



(c) 3D plot for $\alpha = 0.5$

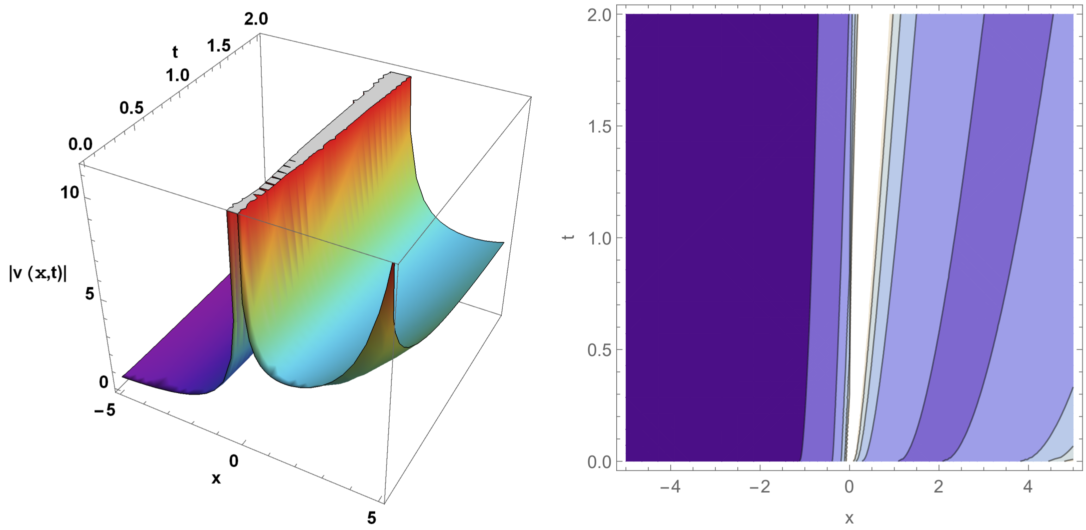
(d) Contour graph when $\alpha = 0.5$

Figure 4. The graph of $|v(x,t)|$ is represented by Eq (3.17) for: $a = -0.01, b = 1, \Omega = -0.04, \lambda = 4$, and $\beta = 1$. (a) shows the 2D plot when $x \in (-15, 15)$ for $\alpha = 1$, with a blue shape for t is zero, an orange shape for $t = 1$, and a green shape for $t = 2$. (b) is the two-dimensional plot if $-15 < x < 15$ and $0 < t < 2$, with a red shape if α is 0.4, a black shape if α is 0.6, and a blue shape if α is 1. It is observed that the phase of the waves shifts with the change in the value of α. (c) is a 3D graph for $\alpha = 0.5$ with $0 < t < 2$. (d) shows the contour graph when α is 0.5 when $t \in (0, 2)$.



(a) 2D plot

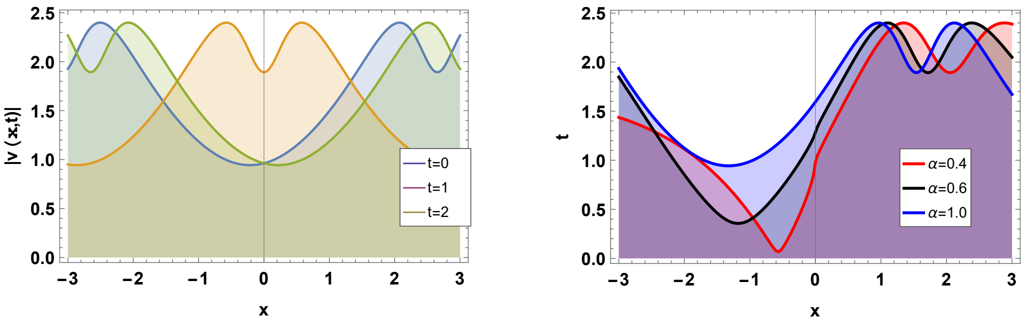
(b) 2D plot



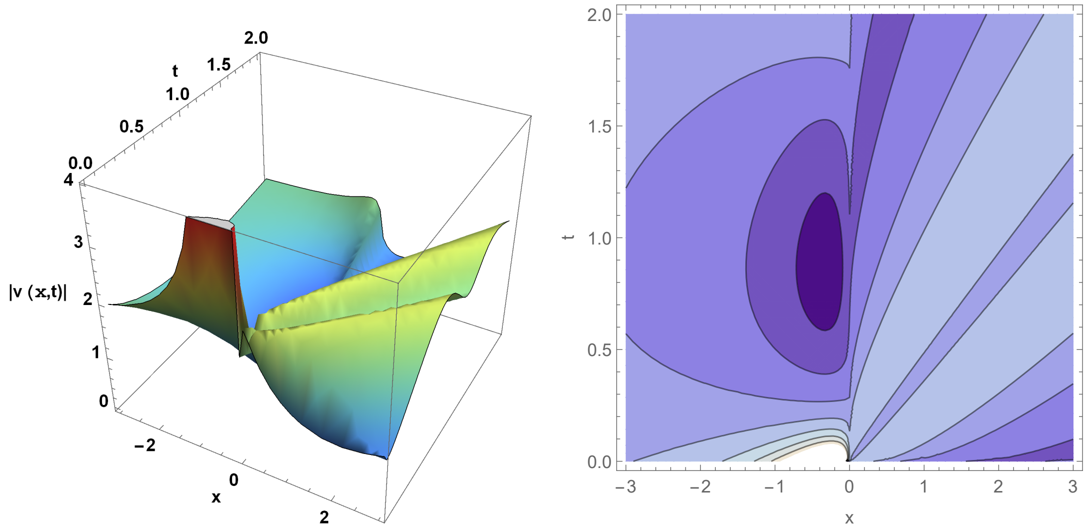
(c) 3D plot for $\alpha = 0.5$

(d) Contour graph when $\alpha = 0.5$

Figure 5. The graph for $|v(x,t)|$ is denoted through Eq (3.19) for: $a = 0.01, b = 0.2, \Omega = 0.1, \lambda = 1$, and $\beta = 1$. (a) represents the 2D plot for $x \in (-5, 5)$ for α is 1, with a blue shape for $t = 0$, an orange shape for t=1, and a green shape for $t = 2$. (b) is the two-dimensional plot if $-5 < x < 5$ and $0 < t < 2$, with a red shape if α is 0.4, a black shape if α is 0.6, and a blue shape if α is 1. We can see the α effects on the phase of the wave solution. (c) is a 3D graph for $\alpha = 0.5$ with $0 < t < 2$. (d) denotes the contour graph when α is 0.5 when $t \in (0, 2)$.



(a) 2D plot (b) 2D plot



(c) 3D plot for $\alpha = 0.5$ (d) Contour graph when $\alpha = 0.5$

Figure 6. The graph for $|v(x,t)|$ is denoted through Eq (3.68) for: $a = -0.01, b = 0.2, \lambda_0 = -0.3, \lambda_1 = 1, \lambda = 1.4, C_1 = 0.2, C_2 = 0.5$, and $\beta = 1$. (a) represents the 2D plot if $x \in (-3,3)$ for $\alpha = 1$, with a blue shape for $t = 0$, an orange shape for $t = 1$, and a green shape for $t = 2$. (b) is the two-dimensional plot if $-3 < x < 3$ and $0 < t < 2$, with a red shape if α is 0.4, a black shape if α is 0.6, and a blue shape if α is 1. The phase of the wave solutions shifts with the change in the value of α. (c) is a 3D graph for $\alpha = 0.5$ with $0 < t < 2$. (d) denotes the contour graph when α is 0.5 when $t \in (0,2)$. Bright soliton solutions have applications in optical communications, nonlinear optics, Bose-Einstein condensed, and fluid dynamics.

5. MI analysis

The modulation instability study is carried out by perturbing a continuous wave (CW) steady-state solution of the governing equation, which is taken in the form

$$v(x,t) = V(x,t)\, e^{\iota\tau t}. \tag{5.1}$$

This state represents a uniform background of constant intensity over which small disturbances are introduced.

To examine the stability of a steady–state solution of the Heimburg equation, we consider a perturbed wave of the form [28, 29]

$$v(x,t) = \left(V(x,t) + \sqrt{\tau}\right) e^{\iota\tau t}, \tag{5.2}$$

where τ is a real constant associated with the carrier phase (or intrinsic temporal frequency) of the background wave, and $V(x,t)$ denotes a small complex-valued perturbation (envelope) superimposed on the steady state $\sqrt{\tau}$. In the context of the Heimburg model, τ is not an optical power but a model parameter governing the phase/frequency scale of the membrane or mechanical excitations.

Substituting Eq (5.2) into the governing equation (1) and retaining linear terms in V, we obtain

$$\iota\lambda\tau V_{xx} + 2\iota\tau V_t + \lambda V_{xxt} + V_{tt} - V_{xx} + V_{xxxx} - \tau^2 V - \tau^{5/2} = 0, \tag{5.3}$$

where λ is the dispersion parameter already defined in the main model. Equation (5.3) represents linearized perturbation dynamics around the uniform background state. We seek harmonic perturbations of the form

$$V(x,t) = A_1 e^{\iota(px-rt)} + A_2 e^{-\iota(px-rt)}, \tag{5.4}$$

where p is the perturbation wave number, r is the corresponding temporal frequency, and A_1, A_2 are small amplitudes. Substituting Eq (5.4) into Eq (5.3), collecting the coefficients of $e^{\iota(px-rt)}$ and $e^{-\iota(px-rt)}$, and setting the determinant of the resulting coefficient matrix to zero yields the dispersion relation

$$p^8 + 2p^6 - \lambda^2 p^4\tau^2 + \lambda^2 p^4 r^2 - 2p^4 r^2 - 2p^4\tau^2 + p^4 - 2p^2 r^2 - 2p^2\tau^2 + r^4 - 2r^2\tau^2 + \tau^4 = 0. \tag{5.5}$$

Solving Eq (5.5) for r gives the dispersion branches

$$r = \pm\sqrt{-\frac{1}{2}(\lambda^2-2)p^4 + p^2 + \tau^2 \pm \frac{1}{2}\sqrt{\lambda^2(\lambda^2-4)p^8 - 4\lambda^2 p^6 + 16p^4\tau^2 + 16p^2\tau^2}}. \tag{5.6}$$

The steady state becomes modulationally unstable when the radicand produces a negative real part inside the outer square root, i.e.,

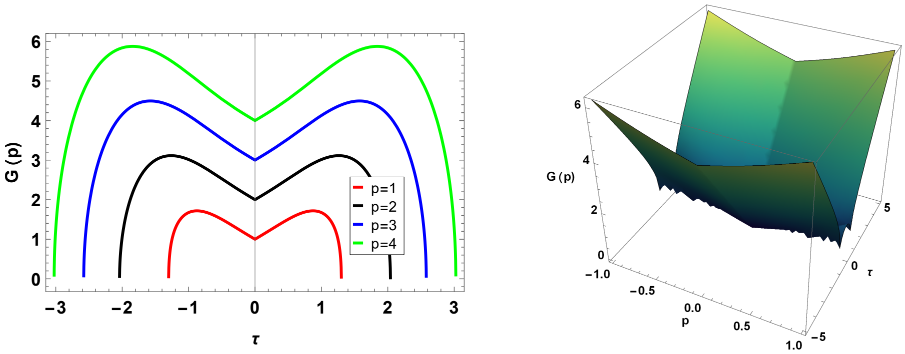
$$-\frac{1}{2}(\lambda^2-2)p^4 + p^2 + \tau^2 \pm \frac{1}{2}\sqrt{\lambda^2(\lambda^2-4)p^8 - 4\lambda^2 p^6 + 16p^4\tau^2 + 16p^2\tau^2} < 0. \tag{5.7}$$

Under this condition, r acquires a nonzero imaginary component and the perturbation grows exponentially in time.

Accordingly, the modulation–instability gain spectrum is defined as

$$G(p) = 2\,|Im(r(p))| = \sqrt{-\frac{1}{2}(\lambda^2-2)p^4 + p^2 + \tau^2 \pm \frac{1}{2}\sqrt{\lambda^2(\lambda^2-4)p^8 - 4\lambda^2 p^6 + 16p^4\tau^2 + 16p^2\tau^2}}. \tag{5.8}$$

All variables and parameters used here are consistent with the notation of the main governing equation, ensuring a unified physical interpretation within the Heimburg framework. The graphical interpretation of MI is shown in Figure 7.



(a) 2D plot (b) 3D plot

Figure 7. MI gain spectrum.

6. Results analysis

Here, we will compare the obtained results and the existing results of the concerned model. In [21], periodic- and kink-shaped soliton solutions are obtained by using the $\exp(-\phi(\xi))$-expansion function method. In [22], lump waves, multi-waves, homoclinic breather waves, and periodic cross kink solutions are gained by applying the Hirota bilinear method. In [23], solitary wave solutions, soliton solutions, and Jacobi elliptic doubly periodic function solutions are obtained through an extended sinh-Gordon equation expansion scheme. While we discovered the periodic wave solution, kink wave solution, singular wave solution, and many other soliton solutions with the help of the modified (G'/G^2)-expansion method and the modified extended tanh function method, these two methods were not used for the model under consideration. We performed modulation instability for the concerned equation and used the governing model with the truncated M-fractional derivative, which was not used earlier. We also analyzed the effect of a fractional-order derivative on the obtained solutions. To analyze the effects of the fractional order α on physical quantities, we can observe:

- Propagation velocity: Changes in α can alter dispersion relations, affecting velocity.
- Peak amplitude: Fractional order α can influence nonlinearity and dispersion balance, impacting amplitude.
- Pulse width: α can affect the balance of terms, altering pulse broadening or compression.

Table 1 shows the effects of α on the wave profile.

Table 1. Effect of α on the wave profile.

Value of α	Wave height (Amplitude)	Wave width
1.0	Highest	Narrow
0.6	Moderate	Medium
0.4	Lowest	Wide

7. Conclusions

Finally, we have successfully obtained the various types of wave solutions for the nonlinear biomembranes and nerves equation with a TMFD. By utilizing the modified (G'/G^2)-expansion method and the modified extended tanh function method, we obtained the kink, periodic, singular, dark, and bright soliton solutions involving trigonometric, hyperbolic trigonometric, and rational functions. The achieved solutions are represented by using 2D, 3D, and contour plots with the help of Mathematica software. All the obtained analytical solutions require the parameter conditions $\lambda^2 > 6$, $a^2 > 4b$, and $b > 0$. The obtained results are useful in different related fields. By using modulation instability analysis, we get the steady-state solution and stability for the concerned model. The used techniques are useful for other nonlinear fractional partial differential equations. In the future, we can compare the obtained exact solutions with the numerical solutions of the concerned model. We can perform bifurcation analysis, sensitivity analysis, and study chaotic behavior for the concerned model.

Authors contributions

Haitham Qawaqneh: Writing – original draft, conceptualization, methodology, formal analysis, supervision; Abdulaziz Khalid Alsharidi: Writing – review and editing, funding acquisition, project administration, methodology. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no competing interests in this paper.

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