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Research article

Self-triggered delay impulsive control for nonlinear systems with exogenous disturbances

Biwen Li and Yuan Cai*

School of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, China

* **Correspondence:** Email: 2111493067@qq.com.

Abstract: This paper investigates input-to-state practical stability (ISpS) for a class of nonlinear dynamic systems subject to time-delayed impulsive effects and external inputs under self-triggered impulsive control (STIC). The considered framework features distinct external disturbances affecting both the continuous dynamics and the impulsive dynamics, with flexible delay characteristics associated with the impulses. To address the challenge of Zeno behavior, a self-triggering mechanism (STM) is proposed that dynamically determines impulse instants based on the combined information of the system state and external disturbances. By employing Lyapunov-based methods, sufficient conditions ensuring ISpS are derived. The theoretical results are further illustrated through an application example, where both the STM and the control gains are designed via matrix inequalities. Finally, two numerical examples are provided to demonstrate the validity and effectiveness of the proposed approach.

Keywords: input-to-state stability (ISS); nonlinear systems; self-triggered impulsive control; delayed impulsive; Zeno behavior
Mathematics Subject Classification: 93C30

1. Introduction

Input-to-state stability (ISS) holds a key position in control research. In 1989, Sontag first proposed the concept of ISS for continuous systems, laying the foundation for characterizing the relationship between the dynamic characteristics of the system and external disturbances [1]. Currently, a large number of studies have been conducted around this concept, generating many results that are both theoretically significant and practically valuable [2–4]. As a derivative concept of ISS, input-to-state practical stability (ISpS) has also been introduced into the discussion of control problems.

Impulsive systems are a type of hybrid Systems that combines the characteristics of continuous dynamics and discrete dynamics [5–7] and has wide applications in numerous research fields, including automatic control, image processing, and signal processing. From the perspective of impulsive effects, the stability research of impulsive systems can be divided into two major directions: impulsive perturbation and impulsive control. Among them, impulsive perturbation focuses on analyzing the robustness of unstable impulsive systems, whereas impulsive control takes the stability of stabilizing impulsive systems as the core objective. As a common discontinuous control method, impulsive control only needs to send control signals at specific moments, which not only has a simple structure and good robustness but also has cost advantages, thus attracting extensive attention from researchers [8, 9]. Currently, scholars have achieved many innovative results in the ISS problem of impulsive systems based on time-triggered impulsive control (TTIC) [10–12], but this control method has inherent drawbacks: It cannot fully take into account the real-time dynamics of the system and is prone to increasing the control cost. This may lead to a situation where unnecessary control operations are still performed when the system is already in a stable state, thereby causing resource waste. In contrast, the signal of event-triggered impulsive control (ETIC) is updated only when the preset event-triggering mechanism (ETM) is activated, and no control signals need to be transmitted between adjacent triggering moments. This feature gives it the advantage of saving communication resources, and a number of innovative studies on the ISS of nonlinear systems under ETIC have emerged in related fields [13–15]. In [16], researchers investigated input-to-state stability for nonlinear time-delay systems with delayed impulses under event-triggered impulsive control using a Lyapunov–Razumikhin approach and a forced impulse sequence to prevent Zeno behavior. Although significant progress has been made in theoretical research in this field, the practical application of event-triggered control strategies still faces challenges: its triggering conditions often require continuous or periodic monitoring of the system state, which poses high requirements for computing resources and communication capabilities. This problem is particularly prominent in resource-constrained systems.

In recent years, the self-triggered control (STC) technology has been successfully developed [17, 18]. The self-triggering mechanism only needs to rely on the system state at the most recent triggering moment to clearly predict the specific time of the next triggering. Moreover, STC shows extremely strong robustness against task delay. With these outstanding advantages, STC technology has become a focus of researchers [19–21]. In [19], researchers used the self-triggered communication strategy to explore the consensus problem of unknown linear multiagent systems; in [20], researchers proposed an STC scheme based on quadratic discounted cost for discrete-time linear systems in the presence of disturbances. For discrete-time nonlinear systems containing parameter uncertainties, Zhang et al. [21] developed a self-triggered adaptive model predictive control scheme. Inspired by the idea of integrating event-triggered control (ETC) and impulsive control, researchers have naturally thought of combining STC with impulsive control, resulting in self-triggered impulsive control (STIC). STIC can not only effectively reduce the requirement for continuous monitoring of system states but also further improve the utilization efficiency of resources. For example, Ding et al. [22] proposed a class of STIC schemes based on exponential decay functions for complex, networked physical systems. To alleviate the computational burden, Tan et al. [23] combined periodic sampling with an exponential event function to design a periodic STIC method for neural networks. Subsequently, Li and Wang [24] proposed a new STIC method suitable for nonlinear time-delay systems based on the comparison system method, which can clearly give the next triggering time.

It should be noted that most current studies have not paid sufficient attention to the impact of time delay [25–27]. However, in practical system applications, time delay often plays a significant role in the evolution of states. Particularly in the field of intelligent control, time delay is a key variable, as it not only may interfere with the real-time feedback of the system but also leads to a decline in control performance, and in severe cases, it can even destabilize the system. In related research, [26] analyzed the preset-time stability of impulsive systems under two triggering mechanisms, and Zhang [28] studied the cooperative output regulation problem of linear multiagent systems through distributed fixed-time error tracking control. However, none of these studies took the time-delay factor in the impulses into account. Although [29] investigated the local synchronization problem of time-delay systems under the framework of time-delayed excitation control, the triggering mechanism designed in this study was based on the comparison principle, which has certain application limitations. In [30], a periodic self-triggered intermittent impulsive control strategy for stabilizing complex networks with complex-valued random parameters was proposed, but its control mechanism was implicitly expressed, making the structure relatively complex and difficult to implement. In [31], researchers considered external disturbance factors, but the impulses and the external disturbances received by the system were completely consistent and still did not involve the delay issue in the impulses. Currently, research on the ISS problem based on self-triggered, time-delayed impulsive control under external disturbances is still relatively scarce, which undoubtedly is an open and valuable topic.

Based on the analysis and discussion in the previous sections, the research objective of this paper is clearly defined as follows: to explore the ISpS problem of nonlinear systems subject to external disturbances and with delayed impulses under STIC while avoiding the occurrence of Zeno behavior. This research process involves numerous challenges: impulse delay is liable to degrade the control system performance or even induce system instability, and the occurrence of external disturbances further increases the complexity of system stability analysis and control design. The key contributions of the present paper are outlined in the three aspects that follow:

(1) Unlike the conventional event-triggered mechanisms in [32, 33] that still necessitate continuous or periodic state monitoring to achieve ISpS, the self-triggered mechanism (STM) proposed in this paper represents a fundamental shift. By eliminating the need for inter-state sampling, our STM not only drastically reduces communication and computational overhead but also achieves resource efficiency.

(2) In contrast to [22–25], which overlook delays, our model explicitly incorporates time delays within the impulse, making the control strategy inherently more robust to constraints. In contrast to [29, 31], we discriminate between external disturbances affecting the continuous dynamics and those impacting the impulsive dynamics. This nuanced differentiation captures a wider range of practical engineering scenarios, thereby significantly broadening the applicability of our theoretical results.

(3) By leveraging linear matrix inequality (LMI) techniques, we establish a systematic codesign framework that synthesizes the two core components of the control strategy: the self-triggered mechanism (STM) and the optimal impulsive control gain. This matrix-inequality-based approach not only guarantees the desired ISpS property for nonlinear systems but also offers a computationally efficient design procedure.

The rest of the content within this paper is structured as outlined below. Relevant mathematical preliminaries and the system formulation are elaborated in Section 2. A novel STM featuring Zeno-freeness, along with the ISpS analysis, is introduced in Section 3. Illustrative examples with simulation results are offered in Section 4, and the conclusions are summarized in Section 5.

2. Model description and preliminaries

Notations: Z_+, R_+ and R are sets of positive integer numbers, non-negative real numbers, and real numbers, respectively. R^n and $R^{n\times m}$ represent the n-dimensional real space and the space of $n \times m$ real matrices, respectively. Let $|\cdot|$ denote the Euclidean norm. Assume $PC([-\tau, 0]; R^n) = \{\hbar : [-\tau, 0] \to R^n$ with norm $\|\hbar\|_\tau$, where $\|\hbar\|_\tau = \sup_{s\in[-\tau,0]} |\hbar(s)|\}$. A positive (negative) definite matrix $\Lambda \in R^{n\times n}$ is denoted by $\Lambda > 0$ ($\Lambda < 0$). The maximum (minimum) eigenvalue of the symmetric matrix Λ denotes $\lambda_{\max}(\Lambda)$ ($\lambda_{\min}(\Lambda)$). $\overline{\mathcal{A}} \vee \overline{\mathcal{B}} = \max(\overline{\mathcal{A}}, \overline{\mathcal{B}})$. We call a function $\alpha : R_+ \to R_+$ a class $\overline{\mathbf{K}}$ function when it has three key properties: continuity, strict monotonic increase, and $\alpha(0) = 0$. When such a function also has radial unboundedness, we further categorize it as $\overline{\mathbf{K}}_\infty$. For a function $\beta : R_+ \times R_+ \to R_+$, we define it as class $\overline{\mathbf{KL}}$ under two conditions: first, if we fix any non-negative time t, the function $\beta(\cdot, t)$ belongs to the class $\overline{\mathbf{K}}$; second, as t approaches positive infinity, $\beta(\cdot, t)$ converges to zero. Let F_0 stand for the collection of time sequences $\{t_k\}_{k\in Z_+}$ where $\inf_{k\in Z_+}\{t_k - t_{k-1}\} > 0$.

The following nonlinear system influenced by delayed impulses is considered:

$$\begin{aligned} &\dot{y}(t) = f(y(t), w_c(t)), t \neq t_k, t \geq t_0 \\ &y(t) = g\left(y(t^- - \tau), w_d(t^-)\right), t = t_k, k \in Z_+ \\ &y(t_0 + s) = \hbar(s), s \in [-\tau, 0]. \end{aligned} \tag{2.1}$$

Let $y(t) \in R^n$ serve as the state of the system. $\dot{y}(t)$ stands for the right-hand derivative of $y(t)$, and $\tau > 0$ is a fixed time delay. Additionally, $w_c(t), w_d(t) \in R^m$ are the locally bounded exogenous perturbation and impulsive perturbation input, respectively. The starting condition is defined by $\hbar(s) \in PC$; $f, g : R^n \times R^m \to R^n$, which satisfies $f(0, 0) = g(0, 0) = 0$, and f follows a local Lipschitz condition with respect to y and w_c. Let $\lim_{s\to t^+} y(s) = y(t^+)$ and $= \lim_{s\to t^-} y(s) = y(t^-)$. Suppose that the solution for System (2.1) and the input are both right-continuous while having left-hand limits at all time points.

Definition 2.1. *([34]) For a given constant $c > 0$, we say System (2.1) has the ISpS property relative to c if $\exists\beta \in \overline{\mathbf{KL}}$ and $r_1, r_2 \in \overline{\mathbf{K}}_\infty$ with the property that $\forall(\hbar, t_0)$ and $\forall(w_c, w_d)$. The solution of System (2.1) corresponding to these values meets the condition*

$$|y(t)| \leq \beta(\|\hbar\|_\tau, t - t_0) + r_1(\|w_c\|_{[t_0,t]}) + r_2\left(\max_{t_0\leq t_k\leq t}\{|w_d(t_k^- - \tau)|\}\right) + c, \quad t \geq t_0, \tag{2.2}$$

where $\|\cdot\|_I$ is the supremum norm on the interval I.

Remark 2.1. *In particular, if (2.2) holds with $c = 0$, then Definition 1 degenerates into the concept of classic ISS. Furthermore, when $w_c = w_d = 0$ and $c = 0$, it becomes Lyapunov asymptotically stable.*

Definition 2.2. *([9]) Take a function $V : R^n \to R_+$ that is locally Lipschitz continuous. We define the upper right-hand Dini derivative D^+V, which corresponds to System (2.1) as follows:*

$$D^+V(y) = \limsup_{h\to 0^+} \frac{1}{h}\Big(V(y + hf(y, w_c)) - V(y)\Big).$$

3. Main results

Within this section, we work within the STIC framework to account for the effects of delayed impulses: we outline criteria that enable nonlinear systems to achieve ISpS while preventing Zeno behavior. To simplify notation, we let $V(t) := V(t, y(t))$ and then proceed to examine the subsequent STM:

$$t_k = t_{k-1} + \int_{V(y(t_{k-1}))\vee\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)}^{\omega V(y(t_{k-1}))+\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)+\varepsilon} \frac{ds}{\psi(s)}, \quad k \in Z_+, \tag{3.1}$$

where V is the Lyapunov function for System (2.1), and $\psi \in K$, $\varphi \in K_\infty$ and $\omega > 1$, $\varepsilon > 0$ are triggering parameters. It is not difficult to observe that the key difference between STM (3.1) and the traditional ETM resides in the generation mode of the triggered time sequence. Specifically, the subsequent triggered time of STM (3.1) can be directly calculated by making use of the data associated with the current triggered time. In the following part, before exploring the stability characteristic of System (2.1), we will first confirm the feasibility of STM (3.1) by eliminating the occurrence of Zeno behavior.

Theorem 3.1. *Suppose there exist locally Lipschitz function $V : R^n \to R_+, \psi \in \overline{\mathbf{K}}, \alpha_1, \alpha_2, \phi \in \overline{\mathbf{K}}_\infty$ and constants $\varepsilon > 0$ and $d \in (0, 1)$ satisfying $1 < \omega < 1/d$ with the property that for $\forall t \in R_+, k \in N_+$ satisfy the following:*

(A_1) $\alpha_1(|y|) \le V(t, y) \le \alpha_2(|y|)$ for $\forall y \in R^n$;
(A_2) $V(g(y(t_k - \tau), w_d(t_k - \tau))) \le dV(y(t_k - \tau)) + \phi(|w_d(t_k - \tau)|)$ for $\forall y \in R^n$ $\forall w_d(t) \in R^m$;
(A_3) $D^+V(t, y) \le \psi(V(y))$ for $\forall t \in [t_k, t_{k-1})$ whenever $V(y) \ge \varphi(\|w_c\|_{[t_0,t_{k-1})})$.

It is then demonstrated that System (2.1) incorporated with STM (3.1) is free from Zeno behavior.

Proof. To begin, we demonstrate that for $\forall t \in [t_k, t_{k-1})$,

$$y(t) \le \omega V(y(t_{k-1})) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \varepsilon. \tag{3.2}$$

We can easily see that (3.2) holds for $t = t_{k-1}$. Let us verify the validity of (3.2) for $\forall \bar{t} \in [t_k, t_{k-1})$. We need to take two distinct scenarios into account.

Case (1): Let $\Omega := \left\{t \in [t_{k-1}, \bar{t}], V(t) < \varphi(\|w_c\|_{[t_0,t_{k-1}]})\right\}$. $V(\bar{t}) > \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)$, which leads to two situations.

(i) If $\Omega = \emptyset$, then $V(t) > \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)$ for $\forall t \in [t_{k-1}, \bar{t}]$. It follows from (A_3) in Theorem 3.1 that

$$\int_{V(t_{k-1})}^{V(\bar{t})} \frac{ds}{\psi(s)} = \int_{t_{k-1}}^{\bar{t}} \frac{d(V(t))}{\psi(V(t))} \le \bar{t} - t_{k-1} \le t_k - t_{k-1}. \tag{3.3}$$

By the definition of STM (3.1), it holds that

$$t_k - t_{k-1} = \int_{V(y(t_{k-1}))\vee\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)}^{\omega V(y(t_{k-1}))+\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)+\varepsilon} \frac{ds}{\psi(s)}. \tag{3.4}$$

Then, combining (3.3) and (3.4), it is straightforward to recognize that

$$V(\bar{t}) \le \omega V(y(t_{k-1})) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \varepsilon. \tag{3.5}$$

(ii) We define $t^* = \sup\left\{t \in [t_{k-1}, \bar{t}] : V(t) < \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)\right\}$. If $\Omega \neq \emptyset$, because $V(\bar{t}) > \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)$, it implies that $V(t^*) = \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)$ with $t^* \in (t_{k-1}, \bar{t})$, and $V(t) > \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)$ for $\forall t \in [t^*, \bar{t}]$. Hence, based on (A_3) in Theorem 3.1, it follows that

$$\int_{\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)}^{V(\bar{t})} \frac{ds}{\psi(s)} = \int_{V(t^*)}^{V(\bar{t})} \frac{ds}{\psi(s)} \leq \bar{t} - t^*.$$

According to (3.4) and $\bar{t} - t^* \leq t_k - t_{k-1}$, one can further obtain

$$\int_{\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)}^{V(\bar{t})} \frac{ds}{\psi(s)} \leq \int_{V(y(t_{k-1}))\vee\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)}^{\omega V(y(t_{k-1}))+\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)+\varepsilon} \frac{ds}{\psi(s)},$$

which yields (3.5).

Case (2): $V(\bar{t}) \leq \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)$. It is evident that ineq (3.5) holds true in this case.

To summarize the above arguments, we can conclude that (3.2) holds for $\forall t \in [t_k, t_{k-1})$, and this specifically guarantees that

$$V(t_k^-) \leq \omega V(t_{k-1}) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \varepsilon, \quad \forall k \in Z_+.$$

Subsequently, we can infer that

$$V(t) \leq \omega V_0 + \varphi(|w_c(t_0)|) + \varepsilon, \quad t \in [t_0, t_1).$$

Using (A_2) in Theorem 3.1, for triggering instant t_1, we have

$$\begin{aligned} V(t_1) &\leq dV(t_1^- - \tau) + \phi(|w_d(t_1^- - \tau)|) \\ &\leqslant \begin{cases} d[\omega V_0 + \varphi(|w_c(t_0)|) + \varepsilon] + \phi(|w_d(t_1^- - \tau)|), t_0 < t_1 - \tau < t_1 \\ dV_0 + \phi(|w_d(t_1^- - \tau)|), -\tau < t_1 - \tau < t_0 \end{cases} \\ &\leq \omega dV_0 + d\varphi(|w_c(t_0)|) + d\varepsilon + \phi(|w_d(t_1 - \tau)|) \end{aligned}$$

and

$$\begin{aligned} V(t) &\leq \omega V(t_1) + \varphi\left(\|w_c\|_{[t_0,t_1]}\right) + \varepsilon \\ &\leq \omega[\omega dV_0 + d\varphi(|w_c(t_0)|) + d\varepsilon + \phi(|w_d(t_1^- - \tau)|)] + \varphi\left(\|w_c\|_{[t_0,t_1]}\right) + \varepsilon \\ &\leq \omega^2 dV_0 + (1 + \omega d)\varphi(\|w_c\|_{[t_0,t_1]}) + \omega\phi(|w_d(t_1^- - \tau)|) + (1 + \omega d)\varepsilon, \quad t \in [t_1, t_2). \end{aligned}$$

Analogously, at triggering instant t_2,

$$\begin{aligned} V(t_2) &\leq dV(t_2^- - \tau) + \phi(|w_d(t_2^- - \tau)|) \\ &\leq \begin{cases} d[\omega^2 dV_0 + (1 + \omega d)\varphi(\|w_c\|_{[t_0,t_1]}) + \omega\varphi(|w_d(t_1^- - \tau)|) + (1 + \omega d)\varepsilon] \\ \quad + \phi(|w_d(t_2^- - \tau)|),\ t_1 < t_2 - \tau < t_2 \\ d[\omega V_0 + \varphi(|w_c(t_0)|) + \varepsilon] + \phi(|w_d(t_2^- - \tau)|),\ t_0 < t_2 - \tau < t_1 \\ dV_0 + \phi(|w_d(t_2^- - \tau)|),\ -\tau < t_2 - \tau < t_0 \end{cases} \\ &\leq \omega^2 d^2 V_0 + (1 + \omega d)d\,\varphi(\|w_c\|_{[t_0,t_1]}) + \omega d\,\phi(|w_d(t_1^- - \tau)|) + \phi(|w_d(t_2^- - \tau)|) + (1 + \omega d)d\varepsilon. \end{aligned}$$

Likewise, one can also acquire

$$
\begin{aligned}
V(t) &\le \omega V(t_2) + \varphi\left(\|w_c\|_{[t_0,t_2]}\right) + \varepsilon \\
&\le \omega[\omega^2 d^2 V_0 + (1+\omega d)d\varphi(\|w_c\|_{[t_0,t_2]}) + \omega d\phi(|w_d(t_1^- - \tau)|) \\
&\quad + \phi(|w_d(t_2^- - \tau)|) + (1+\omega d)d\varepsilon] + \varphi\left(\|w_c\|_{[t_0,t_2]}\right) + \varepsilon \\
&\le \omega^3 d^2 V_0 + \left[1 + \omega d + \omega^2 d^2\right]\varphi(\|w_c\|_{[t_0,t_2]}) + \omega^2 d\,\phi(|w_d(t_1^- - \tau)|) \\
&\quad + \omega\,\phi(|w_d(t_2^- - \tau)|) + \left[1 + \omega d + \omega^2 d^2\right]\varepsilon, \quad t \in [t_2, t_3).
\end{aligned}
$$

At triggering instant t_3,

$$
\begin{aligned}
V(t_3) &\le dV(t_3^- - \tau) + \phi(|w_d(t_3^- - \tau)|) \\
&\le \begin{cases}
\omega^3 d^3 V_0 + \left[1 + \omega d + \omega^2 d^2\right] d\,\varphi(\|w_c\|_{[t_0,t_2]}) + \omega^2 d^2\,\phi(|w_d(t_1^- - \tau)|) + \omega d\,\phi(|w_d(t_2^- - \tau)|) \\
+ \left[1 + \omega d + \omega^2 d^2\right]\varepsilon + \phi(|w_d(t_3^- - \tau)|), t_2 < t_3 - \tau < t_3 \\
\omega^2 d^2 V_0 + (1+\omega d)d\,\varphi(\|w_c\|_{[t_0,t_1]}) + \omega d\,\phi(|w_d(t_1^- - \tau)|) + (1+\omega d)d\varepsilon \\
+\phi(|w_d(t_3^- - \tau)|), t_1 < t_3 - \tau < t_2 \\
\omega d V_0 + d\,\varphi(|w_c(t_0)|) + d\varepsilon + \phi(|w_d(t_3^- - \tau)|), t_0 < t_3 - \tau < t_1 \\
dV_0 + \phi(|w_d(t_3^- - \tau)|), -\tau < t_3 - \tau < t_0
\end{cases} \\
&\le \omega^3 d^3 V_0 + \left[1 + \omega d + \omega^2 d^2\right] d\,\varphi(\|w_c\|_{[t_0,t_2]}) + \omega^2 d^2\,\phi(|w_d(t_1^- - \tau)|) + \omega d\,\phi(|w_d(t_2^- - \tau)|) \\
&\quad + \left[1 + \omega d + \omega^2 d^2\right]\varepsilon + \phi(|w_d(t_3^- - \tau)|).
\end{aligned}
$$

Repeating the above steps, one can infer that

$$
\begin{aligned}
V(t_k) &\le (\omega d)^k V_0 + \sum_{i=0}^{k-1} d(\omega d)^i \varphi(\|w_c\|_{[t_0,t_k]}) + \sum_{i=0}^{k-1} (\omega d)^i \phi\left(\max_{t_0 \le t_k \le t} |w_d(t_k^- - \tau)|\right) + \sum_{i=0}^{k-1} d(\omega d)^i \varepsilon \\
&\le (\omega d)^k V_0 + \frac{d}{1-\omega d}\varphi(\|w_c\|_{[t_0,t_k]}) + \frac{1}{1-\omega d}\phi\left(\max_{t_0 \le t_k \le t} |w_d(t_k^- - \tau)|\right) + \frac{d}{1-\omega d}\varepsilon.
\end{aligned} \tag{3.6}
$$

Note that the final inequality here stems from the fact that $\omega \in (1, 1/d)$. When we substitute (3.6) into (3.4), we can verify that $k \in Z_+$, in particular, makes sure that

$$
\begin{aligned}
t_k - t_{k-1} &= \int_{V(t_{k-1}) \vee \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)}^{\omega V(t_{k-1}) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \varepsilon} \frac{ds}{\psi(s)} \ge \int_{V(t_{k-1}) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right)}^{\omega V(t_{k-1}) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \varepsilon} \frac{ds}{\psi(s)} \\
&\ge \frac{\varepsilon}{\psi[\omega V(t_{k-1}) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \varepsilon]} \ge \frac{\varepsilon}{J}
\end{aligned}
$$

$$
J = \psi\left[\omega V_0 + \frac{1}{1-\omega d}\varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \frac{\omega}{1-\omega d}\phi\left(\max_{t_0 \le t_k \le t} |w_d(t_k^- - \tau)|\right) + \frac{\omega}{1-\omega d}\right].
$$

As a result, STM (3.1) will not bring about Zeno behavior, which concludes the proof.

Next, we can put forward the Lyapunov-driven conclusions for the ISpS property of System (2.1) when paired with STM (3.1). □

Theorem 3.2. *Under conditions in Theorem 3.1, for given constant $c > 0$, assume that the triggering parameters $\omega > 1$ and $\varepsilon > 0$ in SIM (3.1) satisfy*

$$0 < \frac{4\varepsilon}{1-\omega d} \le \alpha_1(c). \tag{3.7}$$

Then, System (2.1) with STM (3.1) is ISpS.

Proof. Note that (3.7) implies $\omega \in (1, 1/d)$ and $\varepsilon \in (0, \infty)$. Therefore, by leaning on Theorem 3.1, we can confirm that System (2.1) does not exhibit Zeno behavior. Given this setup, we can easily work out from (3.2) and (3.6) that $t \in [t_{k-1}, t_k)$, in turn, makes sure in particular that

$$\begin{aligned} V(t) &\le \omega V(t_{k-1}) + \varphi\left(\|w_c\|_{[t_0,t_{k-1}]}\right) + \varepsilon \\ &\le \omega(\omega d)^{k-1} V_0 + \frac{d}{1-\omega d}\varphi(\|w_c\|_{[t_0,t_k]}) + \frac{1}{1-\omega d}\phi\left(\max_{t_0\le t_k\le t} |w_d(t_k^- - \tau)|\right) + \frac{d}{1-\omega d}\varepsilon. \end{aligned}$$

Then, by using (A_1) in Theorem 3.1 and (3.7), one attains

$$|y(t)| \le \alpha_1^{-1}(4\omega(\omega d)^{k-1}\alpha_2(|y_0|)) + \alpha_1^{-1}(\frac{4\omega}{1-\omega d}\varphi(\|w_c\|_{[t_0,t]})) + \alpha_1^{-1}(\frac{4}{1-\omega d}\phi(\max_{t_0\le t_k\le t} |w_d(t_{k-1} - \tau)|) + c$$

for $\forall t \in [t_{k-1}, t_k)$. This indicates that System (2.1) with STM (3.1) is ISpS. □

Remark 3.1. *As can be seen from the proofs of Theorems 3.1 and 3.2, introducing the triggering parameter ε is critical to guaranteeing the feasibility of the STM in (3.1). On one hand, it yields a uniformly strictly positive minimum interevent time, thus eliminating Zeno behavior and ensuring that STM (3.1) is feasible. On the other hand, it establishes an intrinsic relationship among impulse frequency, impulse intensity, and constant c, which is essential for achieving ISpS for a preassigned constant. It can be directly observed from (3.7) that for fixed ω and ε, a smaller c will narrow the value range of d.*

Remark 3.2. *Although the self-triggered schemes introduced in prior research works do not demand real-time collection of system state data, the procedure for calculating their trigger time points features notable implicitness and intricacy. Furthermore, the existence of external disturbances has not been adequately considered in any of the above-mentioned documents [29, 31]. On the contrary, the conclusions obtained in this paper fully account for the effects brought by both external and impulsive inputs, and the self-triggered impulsive control approach here is capable of ensuring that the investigated system possesses the ISpS characteristic corresponding to the preset constant c.*

Remark 3.3. *In Theorem 3.1, the condition A_3 associated with the designated function ψ defines the rate of state divergence over the time interval between any two consecutive impulse moments. Meanwhile, the requirement A_2 paired with the fixed constant d characterizes the magnitude of state jump generated by the impulsive control operation at discrete time points.*

4. Applications

In this section, we apply the self-triggered impulsive control strategies we've put forward to nonlinear systems, aiming to attain ISpS; meanwhile, the impulsive control gain is expressed via LMIs. Let's now look at the following nonlinear systems:

$$\dot{y}(t) = \overline{\mathcal{A}}y(t) + \overline{\mathcal{B}}g\left(y(t)\right) + \overline{\mathcal{H}}w(t) + n(t),\ t \ne t_k,\ t \ge t_0, \tag{4.1}$$

where $\overline{\mathcal{A}}$, $\overline{\mathcal{B}}$, and $\overline{\mathcal{H}}$ are predefined real matrices; $w(t)$ denotes the exogenous disturbance; g is a continuous function; and $n(t)$ serves as the Dirac control input. When affected by $n(t)$, the state $y(t)$ experiences a discrete jump at each impulse time t_k. We can then express System (4.1) in the form below:

$$\begin{cases} \dot{y}(t) = \overline{\mathcal{A}}y(t) + \overline{\mathcal{B}}g(y(t)) + \overline{\mathcal{H}}w(t), & t \geq t_0, \quad t \neq t_k \\ y(t) = (\bar{I} + K)y(t^- - \tau), & t = t_k, \quad k \in Z_+, \end{cases} \tag{4.2}$$

where $\{t_k\}_{k\in Z_+}$ refers to the impulse time instant, and K stands for the impulsive control gain matrix that we will specify in subsequent parts.

Assumption 1. [24] For nonlinear function g, we can find positive constants l and σ that satisfy $|g(y)| \leq l|y|^\sigma$ for every $y \in R^n$.

Theorem 4.1. *Given constant $c > 0$, suppose that there exist positive-definite matrices P, $Q \in R^{n\times n}$, and $M \in R^{m\times m}$; real matrix $W \in R^{n\times n}$; and positive constants ρ, d, and ω satisfying $1 < \omega < 1/d$ such that*

$$\begin{bmatrix} P\overline{\mathcal{A}} + \overline{\mathcal{A}}^T P - \rho P & P\overline{\mathcal{B}} & P\overline{\mathcal{H}} \\ * & -Q & 0 \\ * & * & -M \end{bmatrix} < 0 \tag{4.3}$$

$$\begin{bmatrix} -dP & W + P \\ * & -P \end{bmatrix} < 0. \tag{4.4}$$

Then, the ISpS is guaranteed for System (4.1) with STM

$$t_k = t_{k-1} + \int_{y^T(t_{k-1})Py(t_{k-1}) \vee \|w\|^2_{[t_0,\infty)}}^{\omega y^T(t_{k-1})Py(t_{k-1}) + \|w\|^2_{[t_0,\infty)} + \varepsilon} \frac{ds}{\psi(s)}, \tag{4.5}$$

where

$$\psi(s) := (\rho + \lambda_{\max}(M))\, s + \frac{\lambda_{\max}(Q)l^2}{[\lambda_{\min}(P)]^\sigma} s^\sigma$$

and

$$0 < \varepsilon \leq \frac{1}{4}c^2(1 - \omega d)\lambda_{\min}(P).$$

In addition, we set the impulse gain K to take the form $P^{-1}W^T$ in our design.

Proof. Let $V(t) = y^T(t)Py(t)$ and $\varphi(s) = s^2$. Subsequently, on the basis of the formulas (4.3) and (4.4) as well as the Schur complement, we can readily deduce that

$$\begin{aligned} V(t_k) &= y^T(t_k^- - \tau)(I + K)^T P(I + K)y(t_k^- - \tau) \\ &\leq dV^T(t_k^- - \tau), \end{aligned}$$

and $V(t) \geq \varphi(|w|)$ implies

$$D^+V(t) \leq y^T(t)\Big(P\overline{\mathcal{A}} + \overline{\mathcal{A}}^T P + P\overline{\mathcal{B}}Q^{-1}\overline{\mathcal{B}}^T P + P\overline{\mathcal{H}}M^{-1}\overline{\mathcal{H}}^T P\Big)y(t)$$

$$+ \lambda_{\max}(M)\, w^T(t)w(t) + g^T(y(t))Qg(y(t))$$
$$\leq \rho V(t) + \lambda_{\max}(M)\,|w|^2 + \frac{\lambda_{\max}(Q)\, l^2}{[\lambda_{\min}(P)]^{\sigma}}\left[y^T(t)Py(t)\right]^{\sigma}$$
$$\leq (\rho + \lambda_{\max}(M))V(t) + \frac{\lambda_{\max}(Q)\, l^2}{[\lambda_{\min}(P)]^{\sigma}}[V(t)]^{\sigma}$$

for $\forall t \in [t_{k-1}, t_k)$. Therefore, on the grounds of Theorem 3.2, the ISpS property is verified to be satisfied by System (4.1) associated with STM (4.5). The proof hereby comes to an end. □

Remark 4.1. *Leveraging matrix inequality methods, Theorem 4.1 outlines sufficient criteria for the ISpS of System (4.1) when paired with a sampled-time impulsive control (STIC) scheme. Note that the STIC strategy as designed relies on certain preset initialization parameters. To address this, a workable algorithm that integrates these parameter configurations is presented below. First, it is critical to preselect appropriate values for ρ and d, as these are needed to construct the sampled-time matrix (STM) in (4.5). In fact, the linear matrix inequality (LMI)-based conditions (4.3) and (4.4)—which involve ρ and d, respectively—govern the continuous and discrete dynamics of System (4.1). For this reason, d is chosen as the smallest value that closely matches the jump magnitude of the Lyapunov function V at impulse instants, ensuring that the condition (4.4) holds. Moreover, given the range $1 < \omega < 1/d$, increasing ω can help minimize d, which in turn lowers the impulse frequency. With these steps, the impulse gain K and the valid range for triggering parameters ω, ε can be determined, completing the design of the STIC strategy.*

5. Examples

In what follows, we put forward a pair of examples to show how the conclusions we've arrived at can be applied.

Example 1. *Let us look at this nonlinear impulsive system example that includes time-delayed impulsive actions as well as disturbance-driven inputs:*

$$\begin{aligned} \dot{y}(t) &= 0.9y^2(t) + w_c(t), && t \neq t_k; \\ y(t) &= 0.8y(t^- - 0.01) + w_d(t^-), && t = t_k. \end{aligned} \tag{5.1}$$

Figure 1 shows that the continuous-time behavior of System (5.1) does not satisfy the ISpS property. To make System (5.1) meet ISpS within the STIC framework, we pick $V(x) = |x|$ and $\rho(s) = |s|$, $\psi(s) = s+s^2$, and $d = 0.8$, specifically, $\omega \in (0,\ 1/d)$ and $\varepsilon \in (0, (1-\omega d)c/4)$. From this, we can conclude that System (5.1) achieves ISpS when controlled by STM, with the triggering time t_k defined as

$$t_k = t_{k-1} + \int_{|y(t_{k-1})| \vee \|w_c\|_{[t_0,\infty)}}^{\omega|y(t_{k-1})| + \|w_c\|_{[t_0,\infty)} + \varepsilon} \frac{ds}{s+s^2}. \tag{5.2}$$

For the parameters, take $\omega \in (1, 1.25)$ and $\varepsilon \in (0, (1-0.8\omega)c/4)$. In simulations, set $c = 2.2$, and use disturbance inputs $w_c(t) = 0.3\sin t, w_d(t) = 0.2\cos t$ and initial value $y(0) = 0.5$. When we choose $\omega = 1.1$ and $\varepsilon = 0.1$, the result in Figure 3 aligns with Theorem 3.2. On the other hand, if $\omega = 1.8 \notin (1, 1.25)$, System (5.1) may fail to reach ISpS. This matches the outcome in Figure 4 showing that our conclusions are effective.

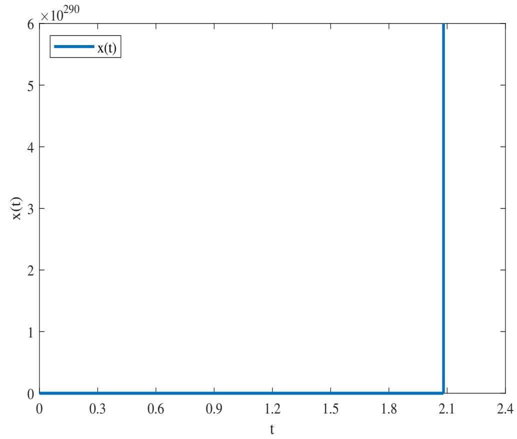

Figure 1. The trajectories of System (5.1) without impulsive control.

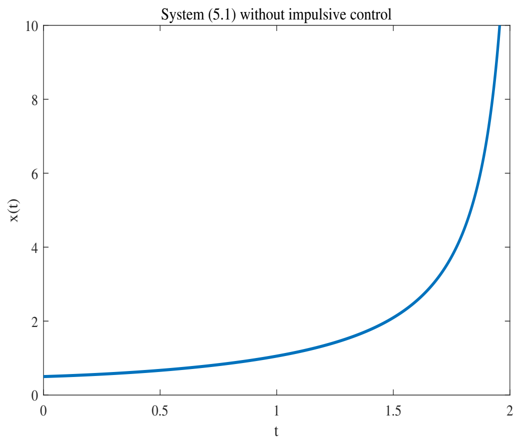

Figure 2. The trajectories of System (5.1) without impulsive control.

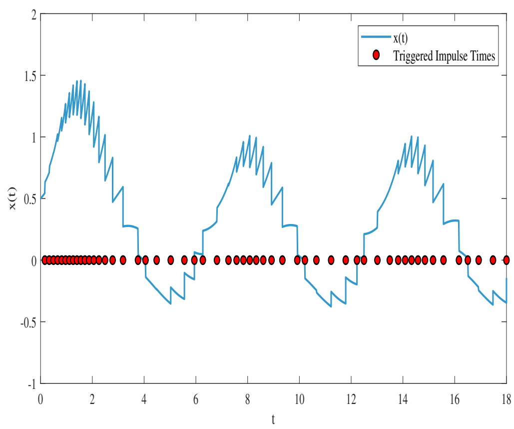

Figure 3. The trajectories of System (5.1) with STM (5.2) with $\omega = 1.1$.

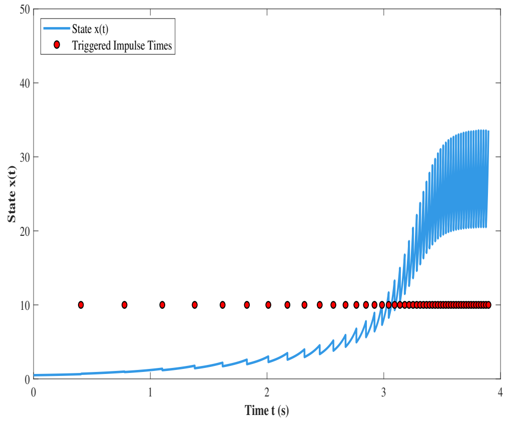

Figure 4. The trajectories of System (5.1) with STM (5.2) with $\omega = 1.8$.

Example 2. *Consider the impulsive control system (4.1) with*

$$\overline{\mathcal{A}} = \begin{bmatrix} 2 & 1.1 \\ -0.1 & 2.1 \end{bmatrix}, \quad \overline{\mathcal{B}} = \begin{bmatrix} -1.61 & 0.54 \\ -1.3 & 1.6 \end{bmatrix},$$
$$\overline{\mathcal{H}} = \begin{bmatrix} 0.5 & 0.6 \\ -0.6 & 0.7 \end{bmatrix}, \quad g = \begin{bmatrix} x_1^2 - x_2^2 \\ 2x_1 x_2 \end{bmatrix}.$$

As illustrated in Figure 5, when no impulsive control is applied, System (4.1), when started from the initial state $y(0) = (-0.5,\ 2.6)^T$, fails to satisfy the ISpS property. To ensure that this system meets the ISpS criterion, we need to devise a suitable impulse gain K and a matching trigger mechanism that dictates when the impulsive control signal activates. First, it is verifiable that Assumption 1 holds true when $l = 1,\ \sigma = 2$. Next, for the STIC strategy (used to stabilize System (4.1)), we set the initial parameters as $\rho = 8$ and $d = 0.5$, following the algorithm outlined in Remark 4. Additionally, by solving the LMIs (4.3) and (4.4) via the MATLAB LMI toolbox, we can derive the result

$$P = \begin{bmatrix} 4.4403 & -0.3506 \\ -0.3506 & 3.7166 \end{bmatrix}, \quad M = \begin{bmatrix} 19.0289 & -0.0746 \\ -0.0746 & 18.1001 \end{bmatrix},$$
$$Q = \begin{bmatrix} 17.2289 & -0.0236 \\ -0.0236 & 17.2399 \end{bmatrix}, \quad W = \begin{bmatrix} -4.4403 & 0.3506 \\ 0.3506 & -3.7166 \end{bmatrix}.$$

Let us take the scenario where $c = 4$. Accordingly, select $\omega = 1.3$, $\varepsilon = 4$, and $\tau = 0.01$. Following Theorem 4.1, when paired with STM (4.5), System (4.1) satisfies the ISpS property. The impulse gain matrix K is constructed as

$$K = \begin{bmatrix} -1.5018 & -0.4556 \\ -0.4858 & -1.0782 \end{bmatrix}.$$

For simulation purposes, set the input as $w(t) = (0.8\sin t, 0.8\cos t)^T$. The state trajectory of System (4.1) under this setup is illustrated in Figure 6. As illustrated in Figure 7, setting $\varepsilon = 0.1$ accelerates the system's convergence to a steady state.

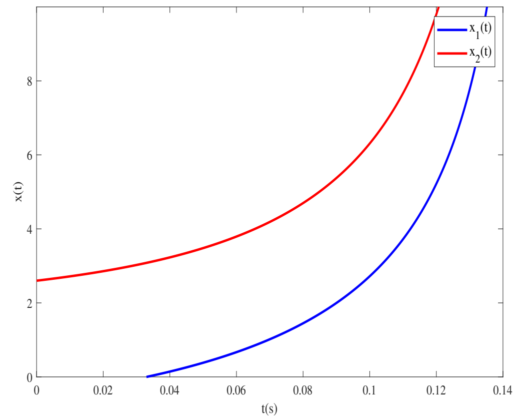

Figure 5. The trajectories of System (4.1) without impulsive control.

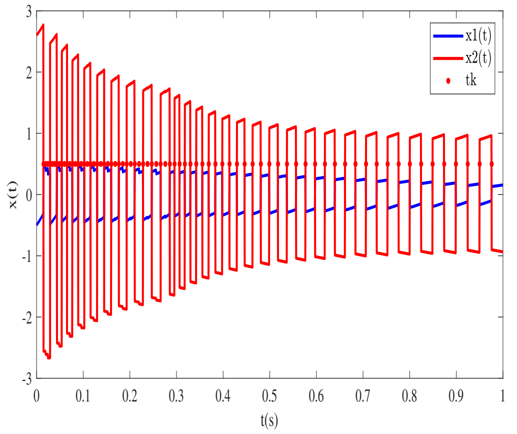

Figure 6. The trajectories of System (4.1) with STM (4.5) [$\varepsilon = 4$].

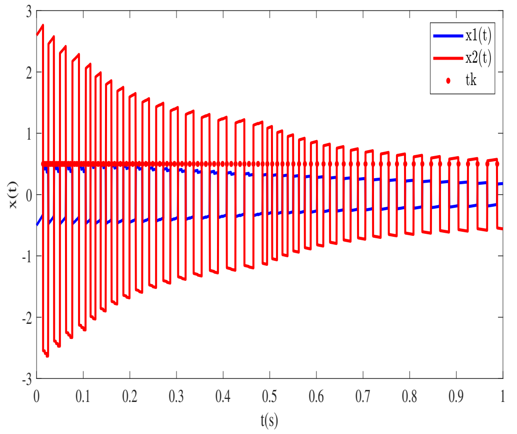

Figure 7. The trajectories of System (4.1) with STM (4.5) [$\varepsilon = 0.1$].

6. Conclusions

In this paper, we focus on a class of nonlinear systems subject to both external disturbances and time-delay impulses and investigate the issues associated with the STIC scheme. Based on the STM proposed herein, we derive sufficient conditions that ensure the system satisfies the ISpS property, while eliminating the occurrence of Zeno behavior. Admittedly, despite the attractive features of this STIC strategy, it still has limitations in certain specific application scenarios; the properly designed STM requires access to real-time information of external inputs to function effectively. On this basis, subsequent research will focus on optimizing the existing STM to make it applicable to a wider range of general scenarios. In addition, more complex influencing factors such as stochastic processes will be taken into account in future on the STIC problem. Such efforts may include extending self-triggered impulsive control to coupled cooperative memristive neural networks [35] and investigating the system-switching problem using the looped-functional method under self-triggered impulsive control [36].

Author contributions

Yuan Cai: Writing-original draft; Biwen Li: Supervision, Writing-review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this paper.

Conflict of interest

The authors declare that there are no conflicts of interest.

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