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Research article

Pythagorean multi-fuzzy N-soft sets: a novel hybrid model for group decision making and its applications in distance education

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Abstract: The rapid development of distance education has led to a pressing need for practical evaluation of learning platforms, especially in situations where decision-making involves multiple evaluators and uncertain information. To address this need, we introduce a new hybrid model called Pythagorean Multi-Fuzzy N-Soft Sets (PMFNSS). This model is designed to improve group decision-making under complex and ambiguous conditions. The proposed PMFNSS model was built by combining Pythagorean Fuzzy N-Soft Sets (PFNSS) and Multi-Fuzzy N-Soft Sets (MFNSS). First, we formally defined the concept of PMFNSS. Next, we proposed a group decision-making algorithm based on the framework. Finally, to validate its applicability, the model was used in a real-life case study on the evaluation of digital learning platforms in distance education. The results indicated that PMFNSS can serve as a multi-attribute group decision-making tool. Its application to distance education offers a systematic approach to evaluating and optimizing digital learning platforms for resource efficiency and sustainability.

Keywords: Pythagorean multi-fuzzy n-soft sets; n-soft sets; fuzzy sets; hybrid model; distance education
Mathematics Subject Classification: 03E72, 03E75

1. Introduction

The rapid development of digital technology has driven the growth of distance education. Distance education has the potential to ensure educational continuity in unforeseen situations, in addition to its advantages such as flexibility in time and space and the option of self-paced learning [1]. While offering high flexibility and accessibility, distance education also poses significant challenges in evaluating the quality of the learning process. One influential factor is the learning platform used. Decisions regarding platform selection and improvement often involve multiple attributes and stakeholders, thus requiring evaluation methods that can address uncertainty and assessor subjectivity.

Several decision-making methods have been developed to represent uncertain information. In 1965, Zadeh [2] proposed the concept of fuzzy set to describe information in decision-making problems. Atanassov [3] presented the idea of intuitionistic fuzzy set (IFS), where the elements of an IFS satisfy the condition: $0 \leq \mu + v \leq 1$, where μ is the degree of membership and v is the degree of non-membership. However, there is a drawback to IFS-based decision-making because the sum of the degrees of membership and non-membership can exceed 1. Therefore, Yager [4] suggested the idea of Pythagorean fuzzy set (PFS) where each element of the PFS satisfies the condition: $0 \leq \mu^2 + v^2 \leq 1$ with $\mu, v \in [0,1]$.

Furthermore, in 1999, Molodtsov [5] developed the concept of soft sets as a parameterization tool to handle uncertainty. Maji et al. [6] demonstrated some concepts, operations, and examples of soft sets and provided applications to solve decision-making problems. Additionally, several hybrid models have been developed to address uncertainty problems. Moreover, Maji et al. [7] combined soft sets with fuzzy sets, introducing a hybrid model of fuzzy soft sets. Majumdar and Samanta [8] proposed the concept of generalized fuzzy soft sets and used it for decision-making problems. Yang et al. [9] demonstrated the concept of interval-valued fuzzy soft sets, which is a combination of interval-valued fuzzy sets and soft sets. Peng and Gard [10] proposed a new algorithm for interval-valued fuzzy soft sets in emergency decision-making based on WDBA and CODAS with a new information measure. Jiang et al. [11] constructed intuitionistic fuzzy soft sets. Several other researchers integrate soft sets, such as possibility fuzzy soft sets [12], Pythagorean fuzzy soft sets [13], generalization of IFS [14], hesitant fuzzy soft sets [15], soft fuzzy rough sets [16], and IFSS for multi-group decision making [17,18].

In addition, several multi-group decision-making models that utilize the Pythagorean concept have been proposed by Asif et al. [19], who applied the Hamacher aggregation operator to PFS sets in multi-attribute decision-making problems, and by Palanikumar & Kausar [20], who discussed the use of aggregation operators in Pythagorean fuzzy. Tahir et al. [21] studied Pythagorean soft sets and hypersoft sets in the context of uncertainty-based decision-making.

The models mostly work on binary estimation (0 or 1) or real numbers between 0 and 1. In 2018, Fatimah et al. [22] extended soft sets under a non-binary evaluation environment, known as N-soft sets. Mahmood et al. [23] proposed the concept of fuzzy N-soft sets from the integration of fuzzy sets with N-soft sets, and Sebastian and Ramakrishnan [24] extended fuzzy sets into multi-fuzzy sets. From this idea, Fatimah and Alcantud [25] combined the concept of multi-fuzzy sets with N-soft sets, resulting in a new theory called multi-fuzzy N-soft sets (MFNSS). This theory is also a generalization of multi-fuzzy soft sets introduced by Yang et al. [26]. MFNSS can be used as a decision-making tool for multi-criteria. However, MFNSS has not considered non-membership variables in decision-making. On the other hand, Akram et al. [27] introduced intuitionistic fuzzy N-soft sets (IFNS) that accommodate membership and non-membership variables. To address the shortcomings of IFNS, Zhang et al. [28] integrated PFS with N-soft sets and proposed a theory called Pythagorean fuzzy N-soft sets (PFNSS). However, research on PFNSS that accommodates multi-attribute groups is still scarce. In general, a comparison of various decision-making models is presented in Table 1.

Table 1. Comparison of various decision-making models.

Model	Membership Degree	Non-membership Degree	Hesitation / Uncertainty	Parameter Handling	Multi-Expert Capability	Utility
Fuzzy Soft Set (FSS)	√	-	implicit	Soft parameters	-	simple problems, low ambiguity
Intuitionistic Fuzzy Soft Set (IFSS)	√, notation μ	√, notation v with $0 \leq \mu + v \leq 1$	limited $\pi = 1 - \mu - v$	Soft parameters	-	moderate support and rejection
Pythagorean Fuzzy Soft Set (PFSS)	√, notation μ	v, notation v with $0 \leq \mu^2 + v^2 \leq 1$	limited $\pi^2 = 1 - \mu^2 - v^2$	Soft parameters	-	high ambiguity and doubt problems
N-Soft Set (NSS)	√	-	-	N-level soft parameters	-	multilevel qualitative evaluation
Fuzzy N-Soft Set (FNSS)	√	-	implicit	N-level soft parameters	-	fuzziness + parameter level
Pythagorean Fuzzy N-Soft Set (PFNSS)	√, notation μ	√, notation v with $0 \leq \mu^2 + v^2 \leq 1$	limited $\pi^2 = 1 - \mu^2 - v^2$	N-level soft parameters	-	high ambiguity problems and multilevel qualitative data
Multi-Fuzzy N-Soft Set (MFNSS)	√	-	explicit	N-level soft parameters	√	involves complex group decisions

Therefore, we propose a new hybrid model called Pythagorean Multi-Fuzzy N-Soft Sets (PMFNSS). PMFNSS is obtained by integrating PFNSS and MFNSS, which enables it to accommodate fuzzy multi-attribute group decision-making problems and consider membership and non-membership variables of PMFNSS elements. To validate this model, we provide a case study on distance education related to multi-attribute group decision-making problems on a learning platform. The major contributions of this study are: Introducing the formal definition and basic properties of PMFNSS, developing a PMFNSS-based decision algorithm for multi-attribute group decision-making problems, and providing an example of the model's application to a case study of a distance learning platform evaluation using real data.

This article is organized as follows: In section 2, we provide a brief review of the concepts of PFS, NSS, PPFNSS, and MFNSS. In section 3, we introduce PMFNSS and PMFNSS-based decision-making algorithms. In section 4, we provide an example of the application of PMFNSS to real-world data, specifically a case study of evaluating a learning platform in distance education. Section 5 contains conclusions and future research directions that can be developed based on the results of this article.

2. Preliminaries

This section contains several concepts related to Pythagorean fuzzy sets, N-soft sets, Pythagorean fuzzy N-soft sets, and multi-fuzzy N-soft sets.

2.1. Pythagorean fuzzy sets (PFS)

Definition 2.1. [4,28] *Suppose U is the universe of discourse. A Pythagorean fuzzy set is an object that has the following form:*

$$A = \{(u, \mu_A(u), v_A(u)) | u \in U\},$$

where $\mu_A: U \to [0,1]$ *represents the degree of membership,* $v_A: U \to [0,1]$ *represents the degree of non-membership of the element* $u \in U$ *to set A, and for any* $u \in U$*, it holds that* $0 \leq (\mu_A(u))^2 + (v_A(u))^2 \leq 1$ *where* $\mu_A \in [0,1]$ *and* $v_A \in [0,1]$*. The degree of uncertainty is given as* $\pi_A(u) = \sqrt{1 - (\mu_A(u))^2 - (v_A(u))^2}$*. Furthermore,* $\alpha = (\mu_\alpha, v_\alpha)$ *is called the Pythagorean Fuzzy Number (PFN).*

The use of Pythagorean fuzzy numbers (PFNs) improves the accuracy and flexibility of group decision-making processes. PFNs provide decision-makers with greater freedom to express their preferences by enabling a wider range of degrees of membership and non-membership, as long as the sum of the squares is less than or equal to one (unlike intuitionistic fuzzy numbers, which require the sum of degrees of membership and non-membership to be less than or equal to one). This increased flexibility in expressing uncertainty minimizes information loss during the decision-making process, as the opinions expressed can be more accurately represented.

Definition 2.2. [4,28] *Let* $\alpha = (\mu_\alpha, v_\alpha)$*,* $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$*, and* $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$ *is any three PFNs over U then:*

(i) $\alpha^C = (v_A, \mu_A)$*;*

(ii) $\alpha_1 \cup \alpha_2 = (max(\mu_{\alpha_1}, \mu_{\alpha_2}), min(v_{\alpha_1}, v_{\alpha_2}))$*;*

(iii) $\alpha_1 \cap \alpha_2 = (min(\mu_{\alpha_1}, \mu_{\alpha_2}), max(v_{\alpha_1}, v_{\alpha_2}))$*;*

(iv) $\alpha_1 \geq \alpha_2$ *if only if* $\mu_{\alpha_1} \geq \mu_{\alpha_2}$ *and* $v_{\alpha_1} \leq v_{\alpha_2}$*;*

(v) $\alpha_1 = \alpha_2$ *if only if* $\mu_{\alpha_1} = \mu_{\alpha_2}$ *and* $v_{\alpha_1} = v_{\alpha_2}$*.*

Definition 2.3. [28] *Let* $\alpha = (\mu_\alpha, v_\alpha)$ *is any PFN over U. The score function and accuracy function of* α *are defined respectively as follows:*

$$S(\alpha) = \mu_\alpha^2 - v_\alpha^2 \text{ and } Q(\alpha) = \mu_\alpha^2 + v_\alpha^2,$$

where $S(\alpha) \in [-1, 1]$ *and* $Q(\alpha) \in [0, 1]$. *For any two PFNs* α_1 *and* α_2,

(i) *if* $S(\alpha_1) < S(\alpha_2)$ *then* $\alpha_1 < \alpha_2$;

(ii) *if* $S(\alpha_1) > S(\alpha_2)$ *then* $\alpha_1 > \alpha_2$;

(iii) *if* $S(\alpha_1) = S(\alpha_2)$, *then*

(a) *if* $Q(\alpha_1) > Q(\alpha_2)$, *then* $\alpha_1 > \alpha_2$;

(b) *if* $Q(\alpha_1) = Q(\alpha_2)$, *then* $\alpha_1 \sim \alpha_2$.

Definition 2.4. [28] *Let* $\alpha = (\mu_\alpha, v_\alpha)$ *is any PFN over U. The rank function of* α *is defined as follows*

$$R_\alpha = \frac{1}{2} + r_\alpha \left(\frac{1}{2} - \frac{2\theta_\alpha}{\pi}\right)$$

with $r_\alpha = \sqrt{\mu_\alpha^2 + v_\alpha^2}$, $\theta_\alpha = arc\cos\frac{\mu_\alpha}{r_\alpha}$, *and* $\theta_\alpha = arc\sin\frac{v_\alpha}{r_\alpha}$. *For any two PFNs* α_1 *and* α_2, *we have*

(i) *if* $R(\alpha_1) < R(\alpha_2)$ *then* $\alpha_1 < \alpha_2$;

(ii) *if* $R(\alpha_1) > R(\alpha_2)$ *then* $\alpha_1 > \alpha_2$;

(iii) *if* $R(\alpha_1) = R(\alpha_2)$, *then* $\alpha_1 = \alpha_2$.

2.2. N-soft sets

Definition 2.5. [22] *Let U be a non-empty universal set of objects, A be a set of attributes, and* $D = \{0, 1, 2, \ldots, N-1\}$ *be a grade-ordered set with* $N \in \{2, 3, \ldots\}$. *The triple* (F, Z, N) *is called an NSS on U if* F *is a mapping* $F : Z \to 2^{U\times D}$, *with the property that for every* $z \in Z$ *there exists a unique* $(u, d_z) \in U \times D$ *such that* $(u, d_z) \in F(z), u \in U, d_z \in D$.

2.3. Pythagorean fuzzy N-soft sets (PFNSS)

Definition 2.6. [28] *Let U be a non-empty universal set of objects, A be a set of attributes, and* $Z \subseteq A$. *Let* $D = \{0,1,2, \ldots, N-1\}$ *be a grade-ordered set with* $N \in \{2,3, \ldots\}$. *The triple* (F_P, Z, N) *is called a PFNSS on U if* F_P *s a mapping* $F_P : Z \to 2^{U\times D} \times PFN$, *where* $F: Z \to 2^{U\times D}$, *and* $P: Z \to PFN$ *where representing Pythagorean fuzzy numbers i.e.,* $\mu: Z \to [0,1]$ *and* $v: Z \to [0,1]$ *for every* $z \in Z, 0 \leq \mu_z^2(u) + v_z^2(u) \leq 1$.

2.4. Multi-fuzzy N-soft sets (MFNSS)

Definition 2.7. [25] *Let U be a non-empty universal set of objects, Q be a non-empty set of attributes, and* $P \subseteq Q$. *Let* $MFS(U)^{(N,q)}$ *be a set of all q-tuples of triple of object from* $R \times [0,1]$ *indexed by U, i.e., the collection of all object as follow:* $\{\langle u, (r_1(u), \mu_1(u)), \ldots, (r_q(u), \mu_q(u))\rangle : u \in U\}$, *where* $r_k: U \to R$, *and* $\mu_k: U \to [0,1]$. *A multi-fuzzy N-soft set (MFNSS) on U is a pair* (Ψ, P) *such that* $\Psi: P \to MFS(U)^{(N,q)}$ *defined by* $\Psi(p) = \{\langle u, (r_1(u), \mu_1(u)), \ldots, (r_q(u), \mu_q(u))\rangle : u \in U\}$ *where* $p \in P$.

3. Results

In this section, we present the findings of this article, divided into two subsections. In the first subsection, we introduce the PMFNSS concept and the proposed algorithm based on it. In the second subsection, we implement the algorithm on real-world data, specifically evaluation data from learning platforms, which were conducted by three decision-making groups and involved several attributes/parameters in their assessments.

3.1. Pythagorean multi-fuzzy N-soft sets (PMFNSS)

The PMFNSS model extends the PFNSS model. The PFNSS model can handle uncertainty in group decision-making by modeling information not only as membership degrees but also as non-membership degrees and uncertainty (hesitation) simultaneously, using pairs of values (μ_{ij}, ν_{ij}) for each group. For each group, each object, and each parameter, the triple $(d_{ij}, \mu_{ij}, \nu_{ij})$ with the highest membership degree is selected. Next, the ranking function in Definition 2.4 is used to obtain the ranking order for each object, as done by Zhang et al. [28]. However, this model is limited to a small dataset.

In group decision-making (GDM), uncertainty arises from several factors, including the large number of decision-makers, subjective or vague assessments, multiple (multi-attribute) assessment criteria, and conflicting opinions (support or rejection). In this case, PFS manages uncertainty and doubt through the concepts of degrees of support and rejection and degrees of uncertainty. MFS accommodates multiple decision makers, and NSS can handle multiple criteria or parameters. PMFNSS is a combination of PFNSS and MFNSS.

To measure and evaluate uncertainty, we first use the concept of Pythagorean fuzzy sets to assess uncertainty based on degree of membership and degree of non-membership, and second, the idea of N-Soft Sets to rank the uncertain data environment. The ranking indicates how strong and which a direction the assessment is, whether it is more positive or more negative, and also serves as a measure of ambiguity. Thus, the proposed method has measured and evaluated the level of ambiguity.

The formal concept for PMFNSS proposed in this article is presented in Definition 3.1.

Definition 3.1. *Let U be a non-empty universal set of objects, A be a set of attributes, and* $Z \subseteq A$. *Let* $D = \{0,1,2,\ldots,N-1\}$ *be a grade-ordered set with* $N \in \{2,3,\ldots\}$. *Let* μ *represents the degree of membership of the element* $u \in U$ *and* ν *represents the degree of non-membership where* $\mu: Z \to [0,1]$ *and* $\nu: Z \to [0,1]$ *for every* $z \in Z$ *with* $0 \le \mu_z^2(u) + \nu_z^2(u) \le 1$, *and X is a set of attributes of set Z associated with* μ *and* ν *with threshold* α *and* β. *Triple* (F_{PM}, X, N) *is called a PMFNSS on U if* F_{PM} *is a mapping* $F_{PM}: X \to 2^{U\times D} \times PFN$ *such that* $(F_{PM}, X, N) = \{(u, (d_z, \mu_z(u), \nu_z(u))): \mu_z(u) \ge \alpha_z, \nu_z(u) \le \beta_z, u \in U\}$.

Remark 3.1. *Let* $U = \{u_1, u_2, u_3\}$ *be the universal set of objects,* $P = \{p_1, p_2\}$ *be the set of attributes, and* $X = \left\{p_1^{(\alpha_1,\beta_1)}, p_2^{(\alpha_2,\beta_2)}\right\}$ *be the set of attributes associated with threshold* (α_j, β_j) *for each attribute* $p_j \in P$. *Then the PMFNSS on U over X can be written as*

$$(F_{PM}, X, N) = \left\{\left(u_i, \left(d_{ij}, \mu_{ij}(u_i), \nu_{ij}(u_i)\right)\right): \mu_{ij}(u_i) \ge \alpha_j, \nu_{ij}(u_i) \le \beta_j, u_i \in U\right\}.$$

Furthermore, given a finite number of PMFNSSs, we can define the union and intersection operations of those PMFNSSs as shown in Definition 3.2.

Definition 3.2. *Let U be a non-empty universal set of objects, A be a set of attributes, and* $A \subseteq Z$, *and* X *is a set of attributes with threshold* α *and* β *for each element attribute in Z associated with* μ *and* v. *Given* n *PMFNSSs on U, then the union of n PMFNSSs is defined as* $\bigcup_{k=1}^{n}(F_{PM_k}, X, N) = \left(\left(max(d_{ij}^{k}), \left(max(\mu_{ij}^{k}), min(v_{ij}^{k})\right)\right)\right)$ *and the intersection of n PMFNSS is defined as* $\bigcap_{k=1}^{n}(F_{PM_k}, X, N) = \left(\left(min(d_{ij}^{k}), \left(min(\mu_{ij}^{k}), max(v_{ij}^{k})\right)\right)\right)$, *where* d_{ij} *is the ranking score for object* u_i *based on parameter* z_j, μ_{ij} *is the degree of membership for object* u_i *based on parameter* z_j, *and* v_{ij} *is the degree of non-membership for object* u_i *based on parameter* z_j.

To demonstrate the effectiveness of PMFNSS, we propose a hybrid model-based decision-making algorithm for PMFNSS. The algorithm is as follows:

Algorithm 3.1.
INPUT

(1) Let $U = \{u_1, u_2, \ldots, u_n\}$ be the set of objects, $Z = \{z_1, z_2, \ldots, z_m\}$ be the set of parameters/attributes, $O = \{o_1, o_2, \ldots, o_r\}$ be the set of respondents, $G = \{g_1, g_2, \ldots, g_s\}$ be the set of respondent groups, and $D = \{0, 1, 2, \ldots, N-1\}$ be the set of ranks with $N \in \{2, 3, \ldots\}$.

(2) Input N-soft sets (F, Z, N) such that $u_i \in C, z_j \in Z$, $\exists!\ d_{ij} \in D$ for each o_l, $l \in \{1, \ldots, r\}$, and g_k, $k \in \{1, \ldots, s\}$.

OPERATIONS

(3) For each group of respondent g_k, determine PFNSS $\left(F_{P_k}, Z, N\right) = \left(d_{ij}^{g_k}, \left(\mu_{ij}, v_{ij}\right)\right)$, $k \in \{1, \ldots, s\}$.

   a) Determine the degree of uncertainty π.

   b) Calculate $v_{ij} = \sqrt{1 - \mu^2 - \pi^2}$.

(4) Define the threshold $T\left(\alpha_j, \beta_j\right)$ with $\alpha_j, \beta_j \in [0,1]$, provided that $\mu_{ij} \geq \alpha_j$ and $v_{ij} \leq \beta_j$ for each $j \in \{1, 2, \ldots, m\}$.

(5) Set the PMFNSS (F_{PM}, X, N) with $X = \left\{z_1^{(\alpha_1, \beta_1)}, z_2^{(\alpha_2, \beta_2)}, \ldots, z_m^{(\alpha_m, \beta_m)}\right\}$.

(6) Use certain operations (e.g., arithmetic mean or geometric mean) to obtain the PMFNSS table of each group.

(7) Because the results of the operations (arithmetic mean or geometric mean) of the ranking degrees form a set of decimal numbers, whereas in Definition 3.1, the grade-ordered set is defined in the form of integers, namely $D = \{0,1,2, \ldots, N-1\}$ with $N \in \{2,3, \ldots\}$, then determine the ranking for each group of respondentsg_k, with $d_{ij}^{g_k} = \begin{cases} \left\lceil d_{ij}^{g_k} \right\rceil, \ if \ d_{ij}^{g_k} \geq \left\lfloor d_{ij}^{g_k} \right\rfloor + 0{,}5 \\ \left\lfloor d_{ij}^{g_k} \right\rfloor, \ if \ d_{ij}^{g_k} < \left\lfloor d_{ij}^{g_k} \right\rfloor + 0{,}5 \end{cases}$

(8) Determine the combined and intersection PMFNSS using $\bigcup_{k=1}^{s}(F_{PMk}^{T}, X, N) = \left(\left(max(d_{ij}^{k}), \left(max(\mu_{ij}^{k}), min(v_{ij}^{k})\right)\right)\right)$ and $\bigcap_{k=1}^{s}(F_{PMk}^{T}, X, N) = \left(\left(\min(d_{ij}^{g_k}), \left(\min(\mu_{ij}^{g_k}), \max(v_{ij}^{g_k})\right)\right)\right)$.

(9) Calculate $H_i = \left(\sum_{j=1}^{m} d_{ij}, \sum_{j=1}^{m} R_{z_{ij}}\right)$ for each index i with $(z_i, d_{ij}) \in F(z)$ and $R_z = \frac{1}{2} + r_z\left(\frac{1}{2} - \frac{2\theta_z}{\pi}\right)$ for each PMFNSS operation (union or intersection).

(10) Calculate $H = \max H_i$ where $i = 1,2,\ldots,n$ for each PMFNSS operation.

OUTPUT

(11) Alternative decisions are obtained based on the ranking of the H values.

3.2. Application PMFNSS in distance education

In distance education, several learning platforms are used. To evaluate the use of these platforms, PMFNSS, combined with the proposed algorithm, can be applied to this problem. Suppose one is given three distance learning platforms $U = \{u_1, u_2, u_3\}$ where u_1 is a face-to-face tutorial, u_2 is a webinar tutorial, and u_3 is an online tutorial via Learning Management System (LMS). Then, the tutor, as the decision-maker, provides an assessment of each learning platform. Tutors are divided into three groups based on generation, namely generation X, generation Y, and generation Z. Furthermore, let Z be the set of attributes "digital platform evaluation in distance education", and $A \subseteq Z$, where a_1 is the effectiveness of material delivery, a_2 is the effectiveness of competency achievement by students, and a_3 is the ease of implementing the tutorial. Each assessment level is represented numerically as $D = \{0, 1, 2, 3, 4, 5\}$, indicating that the higher the score given, the higher the assessment of the intended component. Since there is an element of uncertainty π in decision making, in this article, π is limited to no more than 0.1. Therefore, the degree of non-membership v can be determined by

$$v_{ij} = \sqrt{1 - \mu^2 - \pi^2}. \quad (1)$$

The choice of $\pi = 0.1$ is motivated by the concept of error in statistics, which indicates a tolerable error in statistics. In decision-making, particularly in this article, the uncertainty value is assumed to be no greater than 0.1, meaning that the smaller the uncertainty value, the better the information used in decision-making. This aligns with Possolo & Iyer [29], who state that the associated uncertainty must be small enough to make the measured value acceptable.

The PMFNSS model can integrate concepts of membership and non-membership from the data. Decision makers can determine the level of support (degree of membership μ) and the level of rejection (degree of non-membership v) of a statement or opinion proposed with the condition $0 \leq \mu^2 + v^2 \leq 1$. For example, in selecting a distance learning platform, decision-makers are divided into three groups based on generation or age. Each generation has different opinions regarding the learning platform. In the problem of selecting a distance learning platform, we are given the following data (see Tables 2–7):

Table 2. Assesment levels d and μ scores of Generation X.

d	a_1			a_2			a_3		
	u_1	u_2	u_3	u_1	u_2	u_3	u_1	u_2	u_3
0	0.00	0.08	0.08	0.00	0.08	0.15	0.00	0.08	0.08
1	0.00	0.08	0.15	0.00	0.00	0.00	0.00	0.00	0.15
2	0.00	0.08	0.15	0.00	0.15	0.15	0.00	0.15	0.08
3	0.08	0.15	0.08	0.08	0.15	0.15	0.23	0.08	0.15
4	0.08	0.23	0.23	0.08	0.23	0.23	0.15	0.38	0.23
5	0.85	0.38	0.31	0.85	0.38	0.31	0.62	0.31	0.31

Table 3. Assesment levels d and μ scores of Generation Y.

d	a_1			a_2			a_3		
	u_1	u_2	u_3	u_1	u_2	u_3	u_1	u_2	u_3
0	0.00	0.02	0.02	0.00	0.02	0.02	0.00	0.02	0.02
1	0.00	0.02	0.05	0.00	0.02	0.07	0.00	0.02	0.02
2	0.00	0.02	0.05	0.00	0.00	0.02	0.00	0.00	0.02
3	0.00	0.07	0.07	0.02	0.14	0.09	0.07	0.02	0.09
4	0.05	0.23	0.21	0.09	0.23	0.23	0.05	0.21	0.12
5	0.95	0.63	0.60	0.88	0.58	0.56	0.88	0.72	0.72

Table 4. Assessment levels d and μ scores of Generation Z.

d	a_1			a_2			a_3		
	u_1	u_2	u_3	u_1	u_2	u_3	u_1	u_2	u_3
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.15	0.15	0.15	0.15	0.15	0.15	0.08	0.08	0.08
2	0.00	0.00	0.08	0.08	0.23	0.08	0.15	0.15	0.15
3	0.15	0.31	0.23	0.15	0.23	0.38	0.08	0.38	0.31
4	0.15	0.31	0.38	0.46	0.38	0.23	0.15	0.31	0.23
5	0.54	0.23	0.15	0.15	0.00	0.15	0.54	0.08	0.23

Using formula (1), the degree of non-membership (v) of each group is obtained.

Table 5. Assesment levels d and v scores of Generation X.

d	a_1			a_2			a_3		
	u_1	u_2	u_3	u_1	u_2	u_3	u_1	u_2	u_3
0	0.99	0.95	0.95	0.99	0.95	0.92	0.99	0.95	0.95
1	0.99	0.95	0.92	0.99	0.99	0.99	0.99	0.95	0.92
2	0.99	0.95	0.92	0.99	0.92	0.92	0.99	0.95	0.92
3	0.95	0.92	0.95	0.95	0.92	0.92	0.95	0.92	0.95
4	0.95	0.87	0.87	0.95	0.87	0.87	0.95	0.87	0.87
5	0.37	0.78	0.82	0.37	0.78	0.82	0.37	0.78	0.82

Table 6. Assesment levels d and v scores of Generation Y.

d	a_1			a_2			a_3		
	u_1	u_2	u_3	u_1	u_2	u_3	u_1	u_2	u_3
0	0.99	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98
1	0.99	0.98	0.97	0.99	0.98	0.96	0.99	0.98	0.97
2	0.99	0.98	0.97	0.99	0.99	0.98	0.99	0.98	0.97
3	0.99	0.96	0.96	0.98	0.92	0.95	0.99	0.96	0.96
4	0.97	0.87	0.88	0.95	0.87	0.87	0.97	0.87	0.88
5	0.20	0.60	0.62	0.33	0.64	0.66	0.20	0.60	0.62

Table 7. Assessment levels d and v scores of Generation Z.

d	a_1			a_2			a_3		
	u_1	u_2	u_3	u_1	u_2	u_3	u_1	u_2	u_3
0	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
1	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
2	0.99	0.99	0.95	0.95	0.87	0.95	0.99	0.99	0.95
3	0.92	0.82	0.87	0.92	0.87	0.78	0.92	0.82	0.87
4	0.92	0.82	0.78	0.73	0.78	0.87	0.92	0.82	0.78
5	0.67	0.87	0.92	0.92	0.99	0.92	0.67	0.87	0.92

From these tables, the PFNSS table and PMFNSS table can be obtained as explained in subsections 3.2.1 and 3.2.2.

3.2.1. Application PFNSS

Before we discuss the PMFNSS application, we first show the PFNSS application for the previously mentioned problem regarding the selection of a distance learning. First, we identify the triples $(d_{ij}, \mu_{ij}, v_{ij})$ from Tables 2 and 5 that have $\mu_{ij} > 0$, yielding Table 8. Similarly, for Tables 9 and 10, the μ value is identified successively from Tables 3 and 6, as well as from Tables 4 and 7. Next, Tables 8–10 present the tabular representation of PFNSS for each group (generation) of decision makers.

Table 8. Tabular representation of $(F_P, Z, 6)$-soft set of generation X.

	a_1	a_2	a_3
u_1	(3, 0.08, 0.99), (4, 0.08, 0.99), (5, 0.85, 0.52)	(3, 0.08, 0.99), (4, 0.08, 0.99), (5, 0.85, 0.52)	(3, 0.23, 0.97), (4, 0.15, 0.98), (5, 0.62, 0.78)
u_2	(0. 0.08, 0.99), (1, 0.08, 0.99), (2, 0.08, 0.99), (3, 0.15, 0.98), (4, 0.23, 0.97), (5, 0.38, 0.92)	(0. 0.08, 0.99), (2, 0.15, 0.98), (3, 0.15, 0.98), (4, 0.23, 0.97), (5, 0.38, 0.92)	(0. 0.08, 0.99), (2, 0.15, 0.98), (3, 0.08, 0.99), (4, 0.38, 0.92), (5, 0.3, 0.95)
u_3	(0. 0.08, 0.99), (1, 0.15, 0.98), (2, 0.15, 0.98), (3, 0.08, 0.99), (4, 0.23, 0.97), (5, 0.31, 0.95)	(0. 0.15, 0.98), (2, 0.15, 0.98), (3, 0.15, 0.98), (4, 0.23, 0.97), (5, 0.31, 0.95)	(0, 0.08, 0.99), (1, 0.15, 0.98), (2, 0.08, 0.99), (3, 0.15, 0.98), (4, 0.23, 0.97), (5, 0.31, 0.95)

Table 9. Tabular representation of $(F_P, Z, 6)$-soft set of generation Y.

	a_1	a_2	a_3
u_1	(4, 0.05, 0.99), (5, 0.95, 0.30)	(3, 0.02, 0.99), (4, 0.09, 0.99), (5, 0.88, 0.46)	(3, 0.07, 0.99), (4, 0.05, 0.99), (5, 0.88, 0.46)
u_2	(0. 0.02, 0.99), (1, 0.02, 0.99), (2, 0.02, 0.99), (3, 0.07, 0.99), (4, 0.23, 0.97), (5, 0.63, 0.77)	(0. 0.02, 0.99), (1, 0.02, 0.99), (3, 0.14, 0.99), (4, 0.23, 0.97), (5, 0.58, 0.81)	(0. 0.02, 0.99), (1, 0.02, 0.99), (3, 0.02, 0.99), (4, 0.21, 0.97), (5, 0.72, 0.69)
u_3	(0. 0.02, 0.99), (1, 0.05, 0.99), (2, 0.05, 0.99), (3, 0.07, 0.99), (4, 0.21, 0.97), (5, 0.60. 0.79)	(0. 0.02, 0.99), (1, 0.07, 0.99), (2, 0.02, 0.99), (3, 0.09, 0.99), (4, 0.23, 0.97), (5, 0.56, 0.82)	(0. 0.02, 0.99), (1, 0.02, 0.99), (2, 0.02, 0.99), (3, 0.09, 0.99), (4, 0.12, 0.99), (5, 0.72, 0.69)

Table 10. Tabular representation of $(F_P, Z, 6)$-soft set of generation Z.

	a_1	a_2	a_3
u_1	(1, 0.15, 0.98), (3, 0.15, 0.98), (4, 0.15, 0.98), (5, 0.54, 0.84)	(1, 0.15, 0.98), (2, 0.08, 0.99), (3, 0.15, 0.98), (4, 0.46, 0.88), (5, 0.15, 0.98)	(1, 0.08, 0.99), (2, 0.15, 0.98), (3, 0.08, 0.99), (4, 0.15, 0.98), (5, 0.54, 0.84)
u_2	(1, 0.15, 0.98), (3, 0.31, 0.95), (4, 0.31, 0.95), (5, 0.23, 0.97)	(1, 0.15, 0.98), (2, 0.23, 0.97), (3, 0.23, 0.97), (4, 0.38, 0.92)	(1, 0.08, 0.99), (2, 0.15, 0.98), (3, 0.38, 0.92), (4, 0.31, 0.95), (5, 0.08, 0.99)
u_3	(1, 0.15, 0.98), (2, 0.08, 0.99), (3, 0.23, 0.97), (4, 0.38, 0.92), (5, 0.15, 0.98)	(1, 0.15, 0.98), (2, 0.08, 0.99), (3, 0.38, 0.92), (4, 0.23, 0.97), (5, 0.15, 0.98)	(1, 0.08, 0.99), (2, 0.15, 0.98), (3, 0.31, 0.95), (4, 0.23, 0.97), (5, 0.23, 0.97)

Next, select the highest μ_{ij} value from each object u_i and parameter a_j in each group to obtain Tables 11–13.

Table 11. Tabular representation of PFNSS of generation X.

	a_1	a_2	a_3
u_1	(5, 0.85, 0.52)	(5, 0.85, 0.52)	(5, 0.62, 0.78)
u_2	(5, 0.38, 0.92)	(5, 0.38, 0.92)	(4, 0.38, 0.92)
u_3	(5, 0.31, 0.95)	(5, 0.31, 0.95)	(5, 0.31, 0.95)

Table 12. Tabular representation of PFNSS of generation Y.

	a_1	a_2	a_3
u_1	(5, 0.95, 0.30)	(5, 0.88, 0.46)	(5, 0.88, 0.46)
u_2	(5, 0.63, 0.77)	(5, 0.58, 0.81)	(5, 0.72, 0.69)
u_3	(5, 0.60. 0.79)	(5, 0.56, 0.82)	(5, 0.72, 0.69)

Table 13. Tabular representation of PFNSS of generation Z.

	a_1	a_2	a_3
u_1	(5, 0.54, 0.84)	(4, 0.46, 0.88)	(5, 0.54, 0.84)
u_2	(4, 0.31, 0.95)	(4, 0.38, 0.92)	(3, 0.38, 0.92)
u_3	(4, 0.38, 0.92)	(3, 0.38, 0.92)	(3, 0.31, 0.95)

Next, from each group, using the combined concept of PFNSS, the following Table 14 was obtained.

Table 14. Tabular representation of $(F_P, Z, 6)$-soft set of generation Z.

	a_1	a_2	a_3	H_i	Rank
u_1	(5, 0.95, 0.28)	(5, 0.88, 0.46)	(5, 0.88, 0.46)	2,2036	1
u_2	(5, 0.63, 0.77)	(5, 0.58, 0.81)	(5, 0.72, 0.69)	1,3479	3
u_3	(5, 0.60, 0.79)	(5, 0.56, 0.82)	(5, 0.72, 0.69)	1,3109	2

Based on the evaluation of three factors (content quality, ease of access, and student interaction) for objects u_1, u_2, and u_3, respectively, it was found that u1 is the best platform based on the PFNSS assessment.

3.2.2. Application PMFNSS

From the three PFNSSs, threshold parameters for μ and v, namely α and β, are determined. In this case, set $X = \left\{a_1^{(0.31,\ 0.95)}, a_2^{(0.22,\ 0.97)}, a_3^{(0.16,\ 0.98)}\right\}$. To obtain the PMFNSS, we use the arithmetic mean and the geometric mean to compare the final results based on the proposed algorithms of the two techniques. To apply PMFNSS, from Tables 8–10 , we use steps (6) and (7) to obtain Tables 15–20.

Table 15. $(F_{PM}, X, 6)$-soft set generation X based on the arithmetic mean.

	a_1	a_2	a_3
u_1	(5, 0.85, 0,69)	(5, 0.85, 0.52)	(4, 0.43, 0.88)
u_2	(5, 0.38, 0.92)	(5, 0.31, 0.95)	(5, 0.34, 0.94)
u_3	(5, 0.31, 0.63)	(5, 0.27, 0.96)	(5, 0.27, 0.96)

Table 16. $(F_{PM}, X, 6)$-soft set generation Y based on the arithmetic mean.

	a_1	a_2	a_3
u_1	(5, 0.95, 0.30)	(5, 0.88, 0.46)	(5, 0.88, 0.46)
u_2	(5, 0.63, 0.77)	(5, 0.41, 0.89)	(5, 0.47, 0.83)
u_3	(5, 0.60, 0.79)	(5, 0.40, 0.90)	(5, 0.72, 0.69)

Table 17. $(F_{PM}, X, 6)$-soft set generation Z based on the arithmetic mean.

	a_1	a_2	a_3
u_1	(5, 0.54, 0.84)	(4, 0.46, 0.88)	(5, 0.54, 0.84)
u_2	(4, 0.31, 0.95)	(3, 0.28, 0.95)	(4, 0.35, 0.94)
u_3	(4, 0.38, 0.92)	(4, 0.31, 0.95)	(4, 0.26, 0.96)

Table 18. $(F_{PM}, X, 6)$-soft set generation X based on the geometric mean.

	a_1	a_2	a_3
u_1	(5, 0.85, 0,66)	(5, 0.85, 0.52)	(4, 0.38, 0.87)
u_2	(5, 0.38, 0.92)	(5, 0.30, 0.94)	(5, 0.34, 0.93)
u_3	(5, 0.31, 0.54)	(5, 0.27, 0.96)	(5, 0.27, 0.96)

Table 19. $(F_{PM}, X, 6)$-soft set generation Y based on the geometric mean.

	a_1	a_2	a_3
u_1	(5, 0.95, 0.30)	(5, 0.88, 0.46)	(5, 0.88, 0.46)
u_2	(5, 0.63, 0.77)	(5, 0.37, 0.89)	(5, 0.39, 0.82)
u_3	(5, 0.60, 0.79)	(5, 0.36, 0.89)	(5, 0.72, 0.69)

Table 20. $(F_{PM}, X, 6)$-soft set generation Z based on the geometric mean.

	a_1	a_2	a_3
u_1	(5, 0.54, 0.84)	(4, 0.46, 0.88)	(5, 0.54, 0.84)
u_2	(4, 0.31, 0.95)	(3, 0.27, 0.95)	(4, 0.34, 0.93)
u_3	(4, 0.38, 0.92)	(4, 0.30, 0.94)	(4, 0.25, 0.96)

To obtain choices from the three learning platforms, u_1, u_2, and u_3, we use the combined operation as defined in Definition 3.2 and obtain the results presented in Tables 21 and 22.

Table 21. Tabular representation of value selection from the union of $(F_{PM}, X, 6)$-soft set of three generations based on the arithmetic mean.

	a_1	a_2	a_3	H_i
u_1	(5, 0.95, 0,30)	(5, 0.88, 0.46)	(5, 0.88, 0.46)	2.1877
u_2	(5, 0.63, 0.77)	(5, 0.41, 0.89)	(5, 0.47, 0.83)	1.0464
u_3	(5, 0.60, 0.63)	(5, 0.40, 0.90)	(5, 0.72, 0.69)	1.2689

Table 22. Tabular representation of value selection from the union of $(F_{PM}, X, 6)$-soft set of three generations based on the geometric mean.

	a_1	a_2	a_3	H_i
u_1	(5, 0.95, 0,30)	(5, 0.88, 0.46)	(5, 0.88, 0.46)	2.1877
u_2	(5, 0.63, 0.77)	(5, 0.37, 0.89)	(5, 0.39, 0.82)	0.9980
u_3	(5, 0.60, 0.54)	(5, 0.36, 0.89)	(5, 0.72, 0.69)	1.2920

The results show that using the arithmetic mean and geometric mean techniques causes the ranking order of the three learning platforms to remain the same, namely $u_1 > u_3 > u_2$. Therefore, face-to-face tutorials are the best choice for all three generations in this case. This demonstrates that PMFNSS can serve as a multi-criteria group decision-making tool on real-world data.

4. Conclusions

In this study, we introduce Pythagorean Multi-Fuzzy N-Soft Sets (PMFNSS) as a novel hybrid model that combines Pythagorean Fuzzy N-Soft Sets (PFNSS) and Multi-Fuzzy N-Soft Sets (MFNSS). This study provides a new mathematical framework to address the complexity of group decision-making under uncertainty, (ii) provides a systematic algorithm that can be implemented in real-world situations, and (iii) demonstrates the model's effectiveness through applications in distance education. Through a case study evaluating a digital learning platform in a distance education context, it is demonstrated that PMFNSS can represent vague information, accommodate diverse assessments from multiple evaluators, and generate group decisions. These results confirm that PMFNSS not only has theoretical contributions to the development of fuzzy soft set theory but also practical value for stakeholders in distance education. This article has the advantage of representing and facilitating an uncertain environment for complex decision-making by combining three uncertainty concepts. A limitation of this article is the complexity of processing and aggregating large amounts of data.

Further research can entail developing other variations of PMFNSS and exploring its applications in different fields such as resource management, health, and the environment. Furthermore, it is possible to extend this concept to complex multi-fuzzy N-soft sets. Thus, PMFNSS opens opportunities for further research in theoretical and applied fields.

Author contributions

Fatia Fatimah, Elin Herlinawati, Andriyansah: Conceptualization, Investigation, Methodology, Supervision, Validation, Visualization, Writing–original draft, Writing–review & editing. All authors of this article have been contributed equally. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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