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Research article

Finite-time stability of fractional-order fuzzy inertial neural networks via a new finite-time inequality

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Abstract: This paper investigates the finite-time stability (FTS) of a class of fractional-order fuzzy inertial neural networks (FOFINNs) with time-varying delays. The existing finite-time inequalities for fractional-order systems pose significant theoretical and practical limitations, as they can yield unbounded control inputs when the system state approaches zero. To address this issue, this work introduces a novel finite-time inequality. By incorporating a positive constant into the inequality structure, the proposed method effectively bounds the control signal, ensuring its practical realizability. Utilizing this inequality within a Lyapunov framework alongside an order reduction method (ORM), a composite feedback controller is designed to achieve FTS. Sufficient stability conditions are derived, and an explicit, computable upper bound for the settling time is established. Numerical simulations validate the theoretical results and demonstrate the method's superiority over existing approaches.

Keywords: inertial neural network; finite-time stability; fractional order; fuzzy logics
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1. Introduction

Neural networks (NNs) have found extensive applications across a diverse range of domains, driven by their powerful ability to model complex relationships and learn from data. Since their introduction by Hopfield in 1982, when they were initially described through first-order differential equations, NNs have evolved significantly in both theory and practical implementation. They have been effectively employed for stock price forecasting [1]. In the realm of classification, NNs have demonstrated exceptional performance in categorizing data into distinct classes [2], which is particularly valuable in some areas such as medical diagnosis and text analysis. Meanwhile, to further enhance the interpretability and adaptability of NNs in handling uncertainty, fuzzy information, and complex nonlinear systems, researchers have integrated fuzzy logic with NNs, leading to the development of fuzzy neural networks (FNNs). FNNs retain the learning capabilities of NNs while incorporating the semantic reasoning mechanisms of fuzzy systems, enabling them to effectively integrate expert knowledge and handle ambiguous or uncertain information in input-output mappings. This hybrid model demonstrates unique advantages in pattern recognition, decision support, and other scenarios where high robustness and interpretability are required, thereby expanding the application scope of intelligent systems in complex real-world environments.

Moreover, when modeling the intricate relationship between magnetic flux and electrical current, researchers typically utilize integer-order NNs that incorporate inertial terms—systems commonly referred to as inertial neural networks (INNs). These networks are described by second-order differential equations, which enable a more nuanced representation of temporal dynamics in systems where inertia plays a critical role. The concept of INNs was first introduced by Babcock and Westervelt in their seminal works [3, 4], in which they highlighted how the inclusion of inertial terms significantly contributes to chaotic behaviors in certain systems. Since their inception, INNs have garnered considerable attention within the scientific community due to their potential applications and the meaningful insights they provide into complex dynamical systems. For instance, Cui et al. conducted a comprehensive study on the stability of delayed INNs subject to stochastic impulses, in which they employed the matrix measure method [5]. Moreover, criteria for ensuring the exponential stability of INNs with proportional delays have been presented in [6], where the authors applied the non-reduced order method (NROM). Within the framework of fractional dynamics, the stability of fractional-order delayed INNs was examined in [7], where the authors used an indefinite Lyapunov–Krasovskii functional.

However, in certain scenarios, traditional integer-order NNs may fail to effectively capture the complex memory characteristics and historical dependencies inherent in neuron behavior. To address this limitation, researchers have turned to fractional calculus—a tool highlighted as pivotal in the literature [8]. In contrast to integer-order calculus, fractional-order calculus offers a more nuanced approach, one that enables the simulation of atypical diffusion processes—an ability crucial for understanding the dynamic behavior of NNs. Building upon the aforementioned developments, to further integrate the advantages of fuzzy logic, inertial dynamics, and fractional calculus, researchers have introduced the concept of fractional-order fuzzy inertial neural networks (FOFINNs). This hybrid architecture uniquely combines the semantic reasoning capabilities and uncertainty-handling abilities of fuzzy systems, the capacity to model oscillatory and momentum-driven dynamics via inertial terms, and the proficiency of fractional-order operators in capturing long-term memory and hereditary properties. The resulting model demonstrates exceptional fidelity and robustness in describing systems characterized by nonlinearity, ambiguity, and complex temporal dependencies. Consequently, FOFINNs find significant application potential across diverse advanced fields, including the forecasting and denoising of financial or geophysical signals with memory [9], the modeling of biological processes and diagnostic inference in biomedical engineering, and the designing of secure communication systems based on their rich chaotic dynamics [10].

Besides, stability is widely regarded as one of the most critical properties of NNs, as it plays a pivotal role in determining their effectiveness and reliability in various applications [11]. Furthermore, stability contributes to the convergence of the learning process, allowing the network to effectively adapt to new information over time without succumbing to chaotic or erratic behavior. Therefore, understanding and ensuring the stability of NNs is fundamental to advancing their use in fields such as artificial intelligence, robotics, and data analysis, and researchers have proposed various control methods to ensure system stability [12–14]. Recent research has yielded intriguing results regarding FOFINNs [15, 16]. For instance, the study presented in [15] investigates the stability of FOINNs characterized by delay-dependent impulses, employing Lyapunov–Krasovskii functionals as a framework for stability analysis. It is noteworthy that the aforementioned literatures [15,16] establish stability conditions through the design of various controllers. However, in practical applications, our objective is to attain stability in the model we have developed as swiftly as possible. To address this pressing need, the concept of finite-time stability (FTS) has gain more attention in recent years [17–20]. For acheving FTS of systems, many control methods have been proposed, such as adaptive control [21, 22], sliding mode control [23, 24], event-trigger control [25, 26], and so on. The design of those controllers is inherently dependent on FTS inequalities, a cornerstone in the theoretical foundation of FTS research. Given this central role, studies on FTS of NNs leveraging such inequalities have yielded a series of results. Some results about FTS of fractional-order NNs based on finite-time inequalities have been obtained in [27–29]. In [27] and [28], several FTS inequalities containing positive exponential terms are used; however, the proof of these inequalities is based on a controversial fractional order inequality, which makes the results controversial. Additionally, Li et al. [29] and Du et al. [30] proposed two fractional-order finite-time inequalities with nonnegative terms. However, the existence of a negative exponential term will make the control input approach infinity gradually when the system state approaches zero, which is difficult to realize. Therefore, the finite-time inequality techniques for fractional-order systems remain to be improved.

Drawing from the preceding discussions, the principal contributions and findings of this paper can be delineated as follows:

1) By introducing a positive constant term into the designed inequality, the inherent defect that the controller value tends to zero and increases unbounded with the state of the system is overcome when the control input is designed by using the existing inequality in [29, 30], which lays a strict foundation for theory and engineering applications.
2) A feedback controller incorporating multiple mechanisms—including linear stabilization, nonlinear finite-time convergence, and compensation for time delays and fuzzy logic—is constructed. This controller features a clear structure and can systematically handle the hybrid dynamics of such complex networks. The controller designed in this paper is also suitable for the system in [5, 6, 16].
3) Based on the new inequality and the order reduction method, Lyapunov theory is employed to rigorously prove the FTS of the closed-loop system. An explicit expression for the settling time is derived, clarifying its analytical relationship with system parameters and the fractional order.

Finally, we outline the organization of this paper. The NN models, relevant definitions, assumptions, and lemmas are pointed in Section 2. In Section 3, we examine the FTS of the delayed FOFINN through a feedback controller, and the settling time is obtained by finite-time inequality. An example in Section 4 validates the effectiveness of the thoretical results.

Notations: $\mathbb{R}$ is the set that includes all real numbers, and $\mathbb{R}^+ = \{x|x \in \mathbb{R}, x > 0\}$. $\mathbb{R}^{n\times m}$ refers to the space of $n \times m$-dimensional matrices with real values. $\mathbb{R}^n = \mathbb{R}^{n\times 1}$. $\mathbb{Z}^+ = \{x \in \mathbb{Z}|x > 0\}$, where

$\mathbb{Z}$ represents all integers. Fuzzy AND and fuzzy OR are, respectively, represented by $\bigvee$ and $\bigwedge$. In addition, we introduce the following convenient notations: $\sum_{\upsilon} = \sum_{\upsilon=1}^{n}$, $\bigwedge_{\upsilon} = \bigwedge_{\upsilon=1}^{n}$, and $\bigvee_{\upsilon} = \bigvee_{\upsilon=1}^{n}$.

2. Preliminaries

In this part, we primarily introduce some fundamental concepts.

Definition 2.1. *[31] For any sufficiently smooth function $y(t)$, the Caputo fractional derivative of order $\mu \in (0,1)$ is given by*

$$ {}_{t_0}^{C}D^{\mu} y(t) = \frac{1}{\Gamma(1-\mu)} \int_{t_0}^{t} \frac{y'(\gamma)}{(t-\gamma)^{\mu}} d\gamma, \tag{2.1} $$

where

$$ \Gamma(t) = \int_{0}^{\infty} e^{-\gamma} \gamma^{t-1} d\gamma. \tag{2.2} $$

Definition 2.2. *[31] If $y(t)$ is a integrable function, the μ-order Riemann–Liouville integral of function $y(t)$ is given by*

$$ {}_{t_0}^{C}D^{-\mu} y(t) = I_{t_0}^{\mu} y(t) = \frac{1}{\Gamma(\mu)} \int_{t_0}^{t} \frac{y(\gamma)}{(t-\gamma)^{1-\mu}} d\gamma. \tag{2.3} $$

Lemma 2.1. *[32] If $\mu \in (0,1)$, $\gamma \geq 1$, and $y(t) \in \mathbb{R}$, then*

$$ {}_{t_0}^{C}D^{\mu} y^{\gamma}(t) \leq \gamma y^{\gamma-1}(t) {}_{t_0}^{C}D^{\mu} y(t) \tag{2.4} $$

holds.

Lemma 2.2. *[33] Assume ${}_{t_0}^{C}D^{\mu} y(t)$ is integrable,*

$$ I_{t_0}^{\mu} {}_{t_0}^{C}D^{\mu} y(t) = y(t) - y(t_0), \tag{2.5} $$

where $\mu \in (0,1)$.

Lemma 2.3. *[34] For any constants a_1, a_2 and continous functions $y(t)$, we have*

$$ {}_{t_0}^{C}D^{\mu} [a_1 y_1(t) + a_2] = a_1 {}_{t_0}^{C}D^{\mu} y_1(t). \tag{2.6} $$

Lemma 2.4. *[34] For any continuous functions $y : [0, +\infty) \to \mathbb{R}$ and any $\mu \in (0,1)$, we have*

$$ {}_{t_0}^{C}D^{\mu} |y(t)| \leq sgn(y(t)) {}_{t_0}^{C}D^{\mu} y(t). \tag{2.7} $$

Lemma 2.5. *[35] Assume $y(t)$ is a differentiable function. If for any $\mu \in (0,1)$ and $a, b \in \mathbb{R}$,*

$$ {}_{t_0}^{C}D^{\mu} y(t) \leq aV(t) + b, \ \forall t \geq t_0, \tag{2.8} $$

then

$$ V(t) \leq (V(t_0) + \frac{b}{a}) E_{\mu}(a(t-t_0)^{\mu}) - \frac{b}{a}. \tag{2.9} $$

Lemma 2.6. *[36] Let $\theta_1, \ldots, \theta_n \geq 0$, $\bar{\kappa} > 1$, and $\underline{\kappa} \in (0,1)$, then*

$$\sum_{\upsilon} \theta_{\upsilon}^{\bar{\kappa}} \geq n^{1-\bar{\kappa}} \Big(\sum_{\upsilon} \theta_{\upsilon}\Big)^{\bar{\kappa}}, \quad \sum_{\upsilon} \theta_{\upsilon}^{\underline{\kappa}} \geq \Big(\sum_{\upsilon} \theta_{\upsilon}\Big)^{\underline{\kappa}}. \tag{2.10}$$

Consider the FOFINNs shown below:

$$\begin{aligned} {}_{t_0}^{C}D^{2\mu}\sigma_{\upsilon}(t) =& \Big\{-a_{\upsilon}{}_{t_0}^{C}D^{\mu}\sigma_{\upsilon}(t) - b_{\upsilon}\sigma_{\upsilon}(t) + \sum_{s} c_{\upsilon s}\gamma_s(\sigma_s(t)) + \sum_{s} z_{\upsilon s}\gamma_s(\sigma_s(t-\omega_s(t))) \\ &+ \bigwedge_{s} e_{\upsilon s}\gamma_s(\sigma_s(t-\omega_s(t))) + \bigvee_{s} k_{\upsilon s}\gamma_s(\sigma_s(t-\omega_s(t)))\Big\},\ \mu \in (0, \frac{1}{2}). \end{aligned} \tag{2.11}$$

Take $\bar{\sigma}_{\upsilon}(t) = \epsilon_{\upsilon}{}_{t_0}^{C}D^{\mu}\sigma_{\upsilon}(t) + \xi_{\upsilon}\sigma_{\upsilon}(t)$, $\epsilon_{\upsilon}, \xi_{\upsilon} \in \mathbb{R}^+$, and we have

$$\begin{cases} {}_{t_0}^{C}D^{\mu}\sigma_{\upsilon}(t) = \Big\{\dfrac{1}{\epsilon_{\upsilon}}\bar{\sigma}_{\upsilon}(t) - \dfrac{\xi_{\upsilon}}{\epsilon_{\upsilon}}\sigma_{\upsilon}(t)\Big\}, \\ {}_{t_0}^{C}D^{\mu}\bar{\sigma}_{\upsilon}(t) = \Big\{(\dfrac{\xi_{\upsilon}}{\epsilon_{\upsilon}} - a_{\upsilon})\bar{\sigma}_{\upsilon}(t) + (a_{\upsilon}\xi_{\upsilon} - b_{\upsilon}\epsilon_{\upsilon} - \dfrac{\xi_{\upsilon}^2}{\epsilon_{\upsilon}})\sigma_{\upsilon}(t) + \epsilon_{\upsilon}\Big[\sum_{s} c_{\upsilon s}\gamma_s(\sigma_s(t)) \\ \qquad + \sum_{s} z_{\upsilon s}\gamma_s(\sigma_s(t-\omega_s(t))) + \bigwedge_{s} e_{\upsilon s}\gamma_s(\sigma_s(t-\omega_s(t))) \\ \qquad + \bigvee_{s} k_{\upsilon s}\gamma_s(\sigma_s(t-\omega_s(t)))\Big]\Big\}, \\ \sigma_{\upsilon}(\hat{s}) = \hat{\phi}(\hat{s}),\ \bar{\sigma}_{\upsilon}(\hat{s}) = \hat{\psi}(\hat{s}), \hat{s} \in (-P, t_0], \end{cases} \tag{2.12}$$

where a_{υ} and b_{υ} are all positive constants, and $\omega_s(t) \in [0, P]$ is the time-varying delay function that occures in s-th neuron. $c_{\upsilon s}$, $z_{\upsilon s}$, $m_{\upsilon s}$, and $\lambda_{\upsilon s}$ denote the weight coefficients between the connnections of υ-th and s-th neurons. $e_{\upsilon s}$ and $k_{\upsilon s}$ are the weights of the fuzzy feedback MIN or MAX template. $\gamma_{\upsilon}(\cdot)$ is the transfer function. $\hat{\phi}(\hat{s})$ and $\hat{\psi}(\hat{s})$ are two continuous functions.

Remark 2.1. *In order to simplify the analysis of the system, the order reduction method is often employed, leveraging a key property of integer-order derivatives—their commutativity $\frac{d}{dt}(\frac{d}{dt}f(t)) = \frac{d^2}{dt^2}f(t)$. This property is particularly useful when dealing with integer-order INN models. However, in fractional-order systems, such a property only holds under specific conditions [37]. Specifically, the composition property of fractional differential operators, expressed as ${}_{t_0}^{C}D^{\mu_1}{}_{t_0}^{C}D^{\mu_2}f(t) = {}_{t_0}^{C}D^{\mu_1+\mu_2}f(t)$, requires that the sum of the orders satisfies $\mu_1 + \mu_2 < 1$.*

Remark 2.2. *In the process of analysis, the order reduction method (ORM) is mainly used in this paper. At present, in order to analyze the dynamic properties of inertial neural network models, two methods are proposed: first is ORM, and the second is NROM. Compared to the NROM, the ORM simplifies higher-order differential equations to lower-order ones, easing system analysis, though it requires extra parameters for stability verification. The NROM, with results from [38–41], directly analyzes INN properties using a clear Lyapunov functional. This approach involves complex derivations without simplification, leading to more intricate stability conditions.*

We assume that the function $\gamma_s(\cdot)$ satisfies the following condition.

Assumption 2.1. *Activation $\gamma_s(\cdot)$ satisfies that*

$$|\gamma_s(\wp_1) - \gamma_s(\wp_2)| \leq \phi_s|\wp_1 - \wp_2|,\ \phi_s \in \mathbb{R}^+,$$

where $\wp_1,\ \wp_2 \in \mathbb{R}$.

The definition of FTS is given below.

Definition 2.3. *The system described by Eq (2.12) can achieve FTS if it satisfies that $\lim_{t\to\mathbb{T}(y_0)} y(t) = 0$ and $y(t) = 0$ for all $t > \mathbb{T}(y_0)$, where $\mathbb{T}(y_0)$ is referred to as the settling time.*

Lemma 2.7. *[42] Consider v and s as two states of the FOFINN system described by (2.12). Then,*

$$\Big|\bigwedge_s e_{\upsilon s}\gamma_s(\wp_1) - \bigwedge_s e_{\upsilon s}\gamma_s(\wp_2)\Big| \leq \sum_s |e_{\upsilon s}||\gamma_s(\wp_1) - \gamma_s(\wp_2)|,$$
$$\Big|\bigvee_s k_{\upsilon s}\gamma_s(\wp_1) - \bigvee_s k_{\upsilon s}\gamma_s(\wp_2)\Big| \leq \sum_s |k_{\upsilon s}||\gamma_s(\wp_1) - \gamma_s(\wp_2)|.$$

3. Main results

3.1. Finite-time inequality

In this section, we will present our main results on the FTS of FOFINN model (2.12) based on the ORM. Before that, we first give the following lemma, which will be used in the subsequent analysis.

Lemma 3.1. *For any $\mu \in (0, 1)$, if there is a function $V : \mathbb{R} \to \mathbb{R}_0^+$ such that*

$$ {}_{t_0}^{C}D^{\mu}V(t) \leq -aV(t) - b(V(t) + \psi)^{1-\eta} + c, \tag{3.1}$$

where $a, b, c > 0$, and $2 \geq \eta \geq 1$, then we have

$$V(t) \in \Omega = \left\{V(t)\middle|V(t) \leq \frac{c}{a}\right\},\ for\ t \geq \mathbb{T}_1, \tag{3.2}$$

where $\mathbb{T}_1$ is defined as follows:

$$\mathbb{T}_1 = t_0 + \left[\frac{\Gamma(1+\mu)}{b\eta}\Big(\tilde{V}^{\eta}(t_0) - (\frac{c}{a} + \psi)^{\eta}\Big)\right]^{\frac{1}{\mu}}. \tag{3.3}$$

Proof. Define set $\Omega = \left\{V(t)\middle|V(t) \leq \frac{c}{a}\right\}$. When $V(t) \notin \Omega$, we have

$$ {}_{t_0}^{C}D^{\mu}V(t) \leq -b(V(t) + \psi)^{1-\eta}. \tag{3.4}$$

Let $\tilde{V}(t) = V(t) + \psi$. From Lemmas 2.1 and 2.3, we have

$$ {}_{t_0}^{C}D^{\mu}\tilde{V}^{\eta}(t) \leq \eta\tilde{V}^{\eta-1}(t){}_{t_0}^{C}D^{\mu}\tilde{V}(t) = \eta\tilde{V}^{\eta-1}(t){}_{t_0}^{C}D^{\mu}V(t) \leq -b\eta. \tag{3.5}$$

Furthermore, by Lemma 2.2, we can obtain

$$\tilde{V}(t) \leq \Big(\tilde{V}^{\eta}(t_0) - \frac{b\eta}{\Gamma(1+\mu)}(t-t_0)^{\mu}\Big)^{\frac{1}{\eta}}. \tag{3.6}$$

Define $\mathscr{Q}(t) = \tilde{V}^{\eta}(t_0) - \frac{b\eta}{\Gamma(1+\mu)}(t-t_0)^{\mu}$. We can easily observe that $\mathscr{Q}(t)$ is is a decreasing function for the variable t. Hence, there must exists a $\mathbb{T}_1$ such that $\mathscr{Q}(\mathbb{T}_1) = \frac{c}{a}$, and

$$\begin{aligned} V(t) \leq & \Big(\tilde{V}^{\eta}(t_0) - \frac{b\eta}{\Gamma(1+\mu)}(t-t_0)^{\mu}\Big)^{\frac{1}{\eta}} - \psi \\ \leq & \Big(\tilde{V}^{\eta}(t_0) - \frac{b\eta}{\Gamma(1+\mu)}(\mathbb{T}_1-t_0)^{\mu}\Big)^{\frac{1}{\eta}} - \psi = \frac{c}{a}, \end{aligned} \tag{3.7}$$

where

$$\mathbb{T}_1 = t_0 + \Big[\frac{\Gamma(1+\mu)}{b\eta}\Big(\tilde{V}^{\eta}(t_0) - (\frac{c}{a}+\psi)^{\eta}\Big)\Big]^{\frac{1}{\mu}}. \tag{3.8}$$

In addition, when $V(t_0) \in \Omega$, we have

$${}_{t_0}^{C}D^{\mu}V(t) \leq -aV(t) + c. \tag{3.9}$$

From Lemma 2.5, we have

$$V(t) \leq (V(t_0) - \frac{c}{a})E_{\mu}(-a(t-t_0)^{\mu}) + \frac{c}{a} \leq \frac{c}{a}. \tag{3.10}$$

Thus, we can obtain that when the original value belongs to the residual set, the function will not exceed the residual set. The proof is complete. □

Lemma 3.2. *For any $\mu \in (0,1)$, if there is a function $V : \mathbb{R} \to \mathbb{R}_0^+$ such that*

$${}_{t_0}^{C}D^{\mu}V(t) \leq -aV(t) - b(V(t)+\psi)^{1-\eta}, \tag{3.11}$$

where $a, b > 0$, $2 \geq \eta \geq 1$. Then there must exist a $\mathbb{T}_2$ such that $\lim_{t\to\mathbb{T}_2} V(t) = 0$ and $V(t) = 0, \forall t \geq \mathbb{T}_2$, where $\mathbb{T}_2$ is defined as follows:

$$\mathbb{T}_2 = t_0 + \Big[\frac{\Gamma(1+\mu)}{b\eta}\Big(\tilde{V}^{\eta}(t_0) - \psi^{\eta}\Big)\Big]^{\frac{1}{\mu}}. \tag{3.12}$$

Proof. From (3.11), we have

$${}_{t_0}^{C}D^{\mu}V(t) \leq -aV(t) - b(V(t)+\psi)^{1-\eta} \leq -b(V(t)+\psi)^{1-\eta}. \tag{3.13}$$

Similar with (3.4)–(3.7), we can obtain that

$$V(t) \leq \Big(\tilde{V}^{\eta}(t_0) - \frac{b\eta}{\Gamma(1+\mu)}(\mathbb{T}_1-t_0)^{\mu}\Big)^{\frac{1}{\eta}} - \psi. \tag{3.14}$$

There must exists a $\mathbb{T}_2$ such that $\tilde{V}^{\eta}(t_0) - \frac{b\eta}{\Gamma(1+\mu)}(\mathbb{T}_2-t_0)^{\mu} = \psi^{\eta}$, where

$$\mathbb{T}_2 = t_0 + \Big[\frac{\Gamma(1+\mu)}{b\eta}\Big(\tilde{V}^{\eta}(t_0) - \psi^{\eta}\Big)\Big]^{\frac{1}{\mu}}. \tag{3.15}$$

□

Corollary 3.1. *For $\mu = 1$, if there is a function $V : \mathbb{R} \to \mathbb{R}_0^+$ such that*

$$\dot{V}(t) \leq -aV(t) - b(V(t) + \psi)^{1-\eta} + c, \tag{3.16}$$

where $a, b, c > 0$, $2 \geq \eta \geq 1$, then we have

$$V(t) \in \Omega = \left\{ V(t) \middle| V(t) \leq \frac{c}{a} \right\} for\ t \geq \mathbb{T}_1, \tag{3.17}$$

where $\bar{\mathbb{T}}_1$ is defined as follows:

$$\bar{\mathbb{T}}_1 = t_0 + \frac{1}{b\eta}\Big(\tilde{V}^{\eta}(t_0) - (\frac{c}{a} + \psi)^{\eta} \Big). \tag{3.18}$$

Corollary 3.2. *For $\mu = 1$, if there is a function $V : \mathbb{R} \to \mathbb{R}_0^+$ such that*

$$\dot{V}(t) \leq -aV(t) - b(V(t) + \psi)^{1-\eta}, \tag{3.19}$$

where $a, b > 0$, $2 \geq \eta \geq 1$, then there must exist a $\mathbb{T}_2$ such that $\lim_{t \to \mathbb{T}_2} V(t) = 0$ and $V(t) = 0, \forall t \geq \mathbb{T}_2$, where $\mathbb{T}_2$ is defined as follows:

$$\bar{\mathbb{T}}_2 = t_0 + \frac{\Gamma(1+\mu)}{b\eta}\Big(\tilde{V}^{\eta}(t_0) - \psi^{\eta} \Big). \tag{3.20}$$

Remark 3.1. *In [29, Lemma 8] and [30, Lemma 9], the FTS of fractional-order systems is investigated. The finite-time inequalities established in these works contain nonnegative terms, which may lead to a scenario where the associated control term grows unbounded as the system state approaches zero, posing both theoretical and practical challenges. To address this issue, we introduce a positive constant into the negative exponential term. This modification ensures an upper bound on the value of the negative exponential term even when the system state converges to zero, thereby preventing the control term from tending to infinity.*

3.2. *Finite-time stabilization of FOFINN* (2.12)

We first give the following controlled FOFINN system:

$$\begin{cases} {}_{t_0}^{C}D^{\mu}\sigma_{\upsilon}(t) = \Big\{ \frac{1}{\epsilon_{\upsilon}}\bar{\sigma}_{\upsilon}(t) - \frac{\xi_{\upsilon}}{\epsilon_{\upsilon}}\sigma_{\upsilon}(t) \Big\} + u_{\upsilon}, \\ {}_{t_0}^{C}D^{\mu}\bar{\sigma}_{\upsilon}(t) = \Big\{ (\frac{\xi_{\upsilon}}{\epsilon_{\upsilon}} - a_{\upsilon})\bar{\sigma}_{\upsilon}(t) + (a_{\upsilon}\xi_{\upsilon} - b_{\upsilon}\epsilon_{\upsilon} - \frac{\xi_{\upsilon}^2}{\epsilon_{\upsilon}})\sigma_{\upsilon}(t) + \epsilon_{\upsilon}\Big[\sum_{s} c_{\upsilon s}\gamma_s(\sigma_s(t)) \\ \qquad + \sum_{s} z_{\upsilon s}\gamma_s(\sigma_s(t - \omega_s(t))) + \bigwedge_{s} e_{\upsilon s}\gamma_s(\sigma_s(t - \omega_s(t))) \\ \qquad + \bigvee_{s} k_{\upsilon s}\gamma_s(\sigma_s(t - \omega_s(t)))\Big]\Big\} + v_{\upsilon}. \end{cases} \tag{3.21}$$

For ease of reference, we introduce some convenient notations: $\sigma_{\upsilon}(t) = \sigma_{\upsilon}$, $\bar{\sigma}_{\upsilon}(t) = \bar{\sigma}_{\upsilon}$, $\sigma_{\upsilon}(t - \omega_{\upsilon}(t)) = \sigma_{\upsilon}^{\omega}$, $\bar{\sigma}_{\upsilon}(t - \omega_{\upsilon}(t)) = \bar{\sigma}_{\upsilon}^{\omega}$, $u_{\upsilon} = u_{\upsilon}(t)$, and $v_{\upsilon} = v_{\upsilon}(t)$.

Theorem 3.1. *Assuming Assumption 2.1 holds, the FOFINN* (2.12) *is considered to be finite-time stable when using the following controller:*

$$\begin{cases} u_\upsilon = -\rho_{\upsilon 1}\sigma_\upsilon - \rho_{\upsilon 2} sign(\sigma_\upsilon)(|\sigma_\upsilon| + \psi)^{1-\ell} - \wp_{\nu 1}, \\ v_\upsilon = -\aleph_{\upsilon 1}\bar{\sigma}_\upsilon - sign(\bar{\sigma}_\upsilon)\sum_s \bar{\vartheta}_{\upsilon s}|\sigma_s^\omega| - \aleph_{\upsilon 2} sign(\bar{\sigma}_\upsilon)(|\bar{\sigma}_\upsilon| + \psi)^{1-\ell} - \wp_{\nu 2}, \\ \qquad \upsilon,\ s \in \{1, \ldots, n\}, \end{cases} \tag{3.22}$$

where $2 \geq \ell \geq 1$, *and* $\bar{\vartheta}_{\upsilon s} \geq \epsilon_\upsilon(|z_{\upsilon s}| + |e_{\upsilon s}| + |k_{\upsilon s}|)\phi_s$. ψ, $\rho_{\upsilon 2}, \aleph_{\upsilon 2}$, $\wp_1$, *and* $\wp_2$ *are positive constants. Besides, the following conditions are satisfied:*

$$\aleph_{\upsilon 1} \geq \frac{1}{\epsilon_\upsilon} + \frac{\xi_\upsilon}{\epsilon_\upsilon} - a_\upsilon, \tag{3.23}$$

$$\rho_{\upsilon 1} \geq -\frac{\xi_\upsilon}{\epsilon_\upsilon} + |a_\upsilon \xi_\upsilon - b_\upsilon \epsilon_\upsilon - \frac{\xi_\upsilon}{\epsilon_\upsilon}| + \sum_s \epsilon_s |c_{s\upsilon}| L_i. \tag{3.24}$$

Thus, the settling time is estimated by

$$\mathbb{T}_1 = t_0 + \left[\frac{\Gamma(1+\mu)}{\mathcal{B}\ell}\Big(\tilde{V}^\ell(t_0) - (\frac{\mathcal{C}}{\mathcal{A}} + \psi)^\ell\Big)\right]^{\frac{1}{\mu}}, \tag{3.25}$$

where $\tilde{V}(t_0) = \sum_\upsilon |\sigma_\upsilon(t_0)| + \sum_\upsilon |\bar{\sigma}_\upsilon(t_0)|$ *and* $\omega = \{V(t)|V(t) \leq \frac{\mathcal{A}}{\mathcal{C}}\}$.

Proof. Define $V = V(\sigma_\upsilon, \bar{\sigma}_\upsilon) = \sum_\upsilon |\sigma_\upsilon| + \sum_\upsilon |\bar{\sigma}_\upsilon|$. Hence

$$\begin{aligned} {}_{t_0}^{C}D^\mu V \leq & \sum_\upsilon sign(\sigma_\upsilon) {}_{t_0}^{C}D^\mu \sigma_\upsilon + \sum_\upsilon sign(\bar{\sigma}_\upsilon) {}_{t_0}^{C}D^\mu \bar{\sigma}_\upsilon \\ \leq & \sum_\upsilon sign(\sigma_\upsilon)\Big\{\frac{1}{\epsilon_\upsilon}\bar{\sigma}_\upsilon - \frac{\xi_\upsilon}{\epsilon_\upsilon}\sigma_\upsilon - \rho_{\upsilon 1}\sigma_\upsilon - \rho_{\upsilon 2} sign(\sigma_\upsilon)(|\sigma_\upsilon| + \psi)^{1-\ell} + \wp_{\nu 1}\Big\} \\ & + \sum_\upsilon sign(\bar{\sigma}_\upsilon)\Big\{(\frac{\xi_\upsilon}{\epsilon_\upsilon} - a_\upsilon)\bar{\sigma}_\upsilon + (a_\upsilon \xi_\upsilon - b_\upsilon \epsilon_\upsilon - \frac{\xi_\upsilon^2}{\epsilon_\upsilon})\sigma_\upsilon + \epsilon_\upsilon\Big[\sum_s c_{\upsilon s}\gamma_j(\sigma_j) \\ & + \sum_s z_{\upsilon s}\gamma_j(\sigma_s^\omega) + \bigwedge_s e_{\upsilon s}\gamma_j(\sigma_s^\omega) + \bigvee_s k_{\upsilon s}\gamma_j(\sigma_s^\omega)\Big] \\ & - \aleph_{\upsilon 1}\bar{\sigma}_\upsilon - sign(\bar{\sigma}_\upsilon)\sum_s \bar{\vartheta}_{\upsilon s}|\sigma_s^\omega| - \aleph_{\upsilon 2} sign(\bar{\sigma}_\upsilon)(|\bar{\sigma}_\upsilon| + \psi)^{1-\ell} + \wp_{\nu 2}\Big\}. \end{aligned} \tag{3.26}$$

Combined with Lemma 2.7, we can obtain

$$\begin{aligned} {}_{t_0}^{C}D^\mu V \leq & \sum_\upsilon \Big\{\frac{1}{\epsilon_\upsilon}|\bar{\sigma}_\upsilon| - \frac{\xi_\upsilon}{\epsilon_\upsilon}|\sigma_\upsilon| - \rho_{\upsilon 1}|\sigma_\upsilon| - \rho_{\upsilon 2}|\sigma_\upsilon|^{1-\ell} - \wp_{\nu 1}\Big\} \\ & + \sum_\upsilon \Big\{(\frac{\xi_\upsilon}{\epsilon_\upsilon} - a_\upsilon)|\bar{\sigma}_\upsilon| + |a_\upsilon \xi_\upsilon - b_\upsilon \epsilon_\upsilon - \frac{\xi_\upsilon^2}{\epsilon_\upsilon}||\sigma_\upsilon| \\ & + \epsilon_\upsilon\Big[\sum_s |c_{\upsilon s}|\phi_s|\sigma_j| + \sum_s |z_{\upsilon s}|\phi_s|\sigma_s^\omega| + \sum_s |e_{\upsilon s}|\phi_s|\sigma_s^\omega| + \sum_s |k_{\upsilon s}|\phi_s|\sigma_s^\omega|\Big] \end{aligned}$$

$$- \aleph_{\upsilon 1}|\bar{\sigma}_{\upsilon}| - \sum_{s} \bar{\vartheta}_{\upsilon s}|\sigma_s^{\omega}| - \aleph_{\upsilon 2}(|\bar{\sigma}_{\upsilon}| + \psi)^{1-\ell} - \wp_{\nu 2}\Big\}. \tag{3.27}$$

From Lemma 2.6, we can get

$$\begin{aligned}
{}_{t_0}^{C}D^{\mu}V \leq & \sum_{\upsilon}\Big\{\frac{1}{\epsilon_{\upsilon}} + \frac{\xi_{\upsilon}}{\epsilon_{\upsilon}} - a_{\upsilon} - \aleph_{\upsilon 1}\Big\}|\bar{\sigma}_{\upsilon}| + \sum_{\upsilon}\Big\{-\frac{\xi_{\upsilon}}{\epsilon_{\upsilon}} - \rho_{\upsilon 1} + |a_{\upsilon}\xi_{\upsilon} - b_{\upsilon}\epsilon_{\upsilon} - \frac{\xi_{\upsilon}}{\epsilon_{\upsilon}}| \\
& + \sum_{s}\epsilon_s|c_{s\upsilon}|L_i\Big\}|\sigma_{\upsilon}| + \sum_{\upsilon}\Big\{\sum_{s}\Big[\epsilon_{\upsilon}\Big(|z_{\upsilon s}| + |e_{\upsilon s}| + |k_{\upsilon s}|\Big)\phi_s - \bar{\mu}_{ij}\Big]\Big\}|\sigma_s^{\omega}| \\
& - \sum_{\upsilon}\rho_{\upsilon 2}(|\sigma_{\upsilon}| + \psi)^{1-\ell} - \sum_{\upsilon}\aleph_{\upsilon 2}(|\bar{\sigma}_{\upsilon}| + \psi)^{1-\ell} + \sum_{\upsilon}(\wp_{\nu 1} + \wp_{\nu 2}) \\
\leq & - \mathcal{A}V - \sum_{\upsilon}\rho_{\upsilon 2}(|\sigma_{\upsilon}| + \psi)^{1-\ell} - \sum_{\upsilon}\aleph_{\upsilon 2}(|\bar{\sigma}_{\upsilon}| + \psi)^{1-\ell} + \sum_{\upsilon}(\wp_{\nu 1} + \wp_{\nu 2}) \\
\leq & - \mathcal{A}V - \min_{\upsilon\in\mathbb{R}^+}\{\rho_{\upsilon 2}, \aleph_{\upsilon 2}\}\Big[\sum_{\upsilon}(|\sigma_{\upsilon}| + |\bar{\sigma}_{\upsilon}| + 2\psi)\Big]^{1-\ell} + \sum_{\upsilon}(\wp_{\nu 1} + \wp_{\nu 2}) \\
\leq & - \mathcal{A}V - \mathcal{B}(V(t) + 2N\psi)^{1-\ell} + C,
\end{aligned} \tag{3.28}$$

where $\mathcal{A} = \min\{\aleph_{\upsilon 1} - \frac{1}{\epsilon_{\upsilon}} - \frac{\xi_{\upsilon}}{\epsilon_{\upsilon}} + a_{\upsilon}, \frac{\xi_{\upsilon}}{\epsilon_{\upsilon}} + \rho_{\upsilon 1} - |a_{\upsilon}\xi_{\upsilon} - b_{\upsilon}\epsilon_{\upsilon} - \frac{\xi_{\upsilon}}{\epsilon_{\upsilon}}| - \sum_s \epsilon_s|c_{s\upsilon}|L_i\}$, $\mathcal{B} = \min_{\upsilon\in\mathbb{R}^+}\{\rho_{\upsilon 2}, \aleph_{\upsilon 2}\}$, and $C = \sum_{\upsilon}(\wp_{\nu 1} + \wp_{\nu 2})$. Then, FTS is achieved by ultlizing Lemma 3.1. □

Corollary 3.3. *Assume $\mu = 1$ and Assumption 2.1 holds, then the FOFINN* (2.12) *is considered to be finite-time stable when using the controller 3.22. The settling time is estimated by*

$$\mathbb{T}_1 = t_0 + \frac{1}{\mathcal{B}\ell}\Big(\tilde{V}^{\ell}(t_0) - (\frac{C}{\mathcal{A}} + \psi)^{\ell}\Big), \tag{3.29}$$

where $\tilde{V}(t_0) = V(t_0) + 2N\psi$ and the residual set is defined as $\omega = \{V(t)|V(t) \leq \frac{C}{\mathcal{A}}\}$.

Theorem 3.2. *Assuming Assumption 2.1 and conditions* (3.23)–(3.24) *hold, the FOFINN* (2.12) *is considered to be finite-time stable when using the controller* (3.22) *with $\wp_{\nu 1} = 0$ and $\wp_{\nu 2} = 0$.*

The settling time is estimated by

$$\mathbb{T}_1 = t_0 + \Big[\frac{\Gamma(1+\mu)}{\mathcal{B}\ell}\Big(\tilde{V}^{\ell}(t_0) - \psi^{\ell}\Big)\Big]^{\frac{1}{\mu}}, \tag{3.30}$$

where $\tilde{V}(t_0) = V(t_0) + 2N\psi$.

Remark 3.2. *In controller* (3.22)*, terms $-\rho_{\upsilon 1}\sigma_{\upsilon}(t)$ and $-\aleph_{\upsilon 1}\bar{\sigma}_{\upsilon}(t)$ are used to overcome the linear growth term in the system* (2.12)*; terms $-\rho_{\upsilon 2}sign(\sigma_{\upsilon}(t))(|\sigma_{\upsilon}(t)| + \psi)^{1-\ell}$ and $-\aleph_{\upsilon 2}sign(\bar{\sigma}_{\upsilon}(t))(|\bar{\sigma}_{\upsilon}(t)| + \psi)^{1-\ell}$ are designed to dominate the nonlinear part of system* (2.12) *to ensure FTS; term $-sign(\bar{\sigma}_{\upsilon})\sum_s \bar{\vartheta}_{\upsilon s}|\sigma_s^{\omega}|$ is used to handle the effect of delay terms and fuzzy logics in the system; terms $\wp_{\nu 1}$ and $\wp_{\nu 2}$ are introduced to handle the nonnegative term in the established fractional-order finite-time inequality, ensuring that the settling time can be accurately estimated even in the presence of such terms.*

4. Examples

In this part, we present numerical examples to validate our results via Simulink and FOTF toolbox [43].

Example 4.1. *Consider system* (3.21) *with* $\mu = 0.49$, $\epsilon_1 = \epsilon_2 = \xi_1 = \xi_2 = 1$, $a_1 = 2.9080$, $a_2 = 0.8252$, $b_1 = 1.3790$, $b_2 = 1.0582$, $c_{11} = -0.4686, c_{12} = 1.0984, c_{21} = -0.2725, c_{22} = -0.2779$, $z_{11} = 0.7015, z_{12} = -0.3538, z_{21} = -2.0518, z_{22} = -0.8236$, $e_{11} = -1.5771, e_{12} = 0.2820, e_{21} = 0.5080, e_{22} = 0.0335$, $k_{11} = -1.3337, k_{12} = 0.3502, k_{21} = 1.1275, k_{22} = -0.2991$, *and* $\omega_v(t) = \frac{\exp(\sin(t))+1}{\exp(\sin(t))}$. *Figure 1 shows the dynamic behaviour of FOINN* (2.12)*. Without the controller, we can clearly observe that FOFINN cannot achieve practical FTS. From Theorem 3.1, we have* $\bar{\vartheta}_{11} \geq 3.1623$, $\bar{\vartheta}_{12} \geq 0.9860$, $\bar{\vartheta}_{21} \geq 3.6873$, *and* $\bar{\vartheta}_{22} \geq 1.1562$. *Select* $\rho_{11} = 12$, $\rho_{21} = 13$, $\aleph_{11} = 14$, $\aleph_{21} = 22$, $\rho_{12} = 12$, $\rho_{22} = 13$, $\aleph_{12} = 14$, $\aleph_{22} = 10$, $\wp_{11} = 1.2, \wp_{12} = 1.5, \wp_{21} = 2.1, \wp_{22} = 1.2$, $\bar{\vartheta}_{vs} = [21, 13; 12, 11]$, *and* $\ell = 1.8$. *Then we can obtain* $\mathcal{A} = 11.3907$, $\mathcal{B} = 10$, *and* $C = 6$. *After verification, the conditions* (3.23) *and* (3.24) *are satisfied. Set* $\psi = 1$. *According to Theorem 3.1, the settling time can be estimated as* $\mathbb{T}_1 = 1.9173$ *under the initial values* $\sigma_1(t_0) = 1.2, \sigma_2(t_0) = -1.3, \sigma_3(t_0) = -2.5$, *and* $\sigma_4(t_0) = 1.4$. *The trajectories of system* (3.21) *with controller* (3.22) *are shown in Figure 2. It can be observed that the FOFINN* (2.12) *achieves PFTS under the designed controller before* $\mathbb{T}_1$.

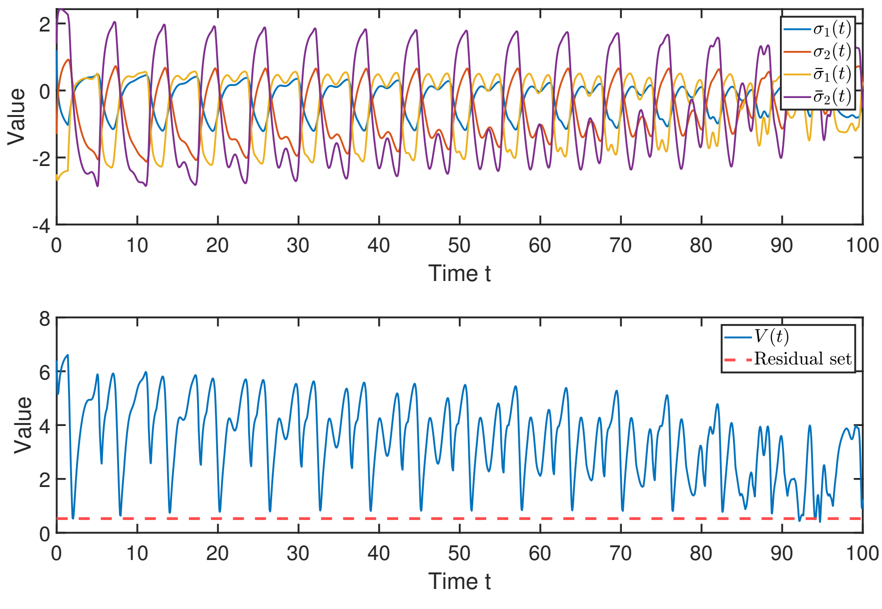

Figure 1. Trajectories of system (3.21) without control input.

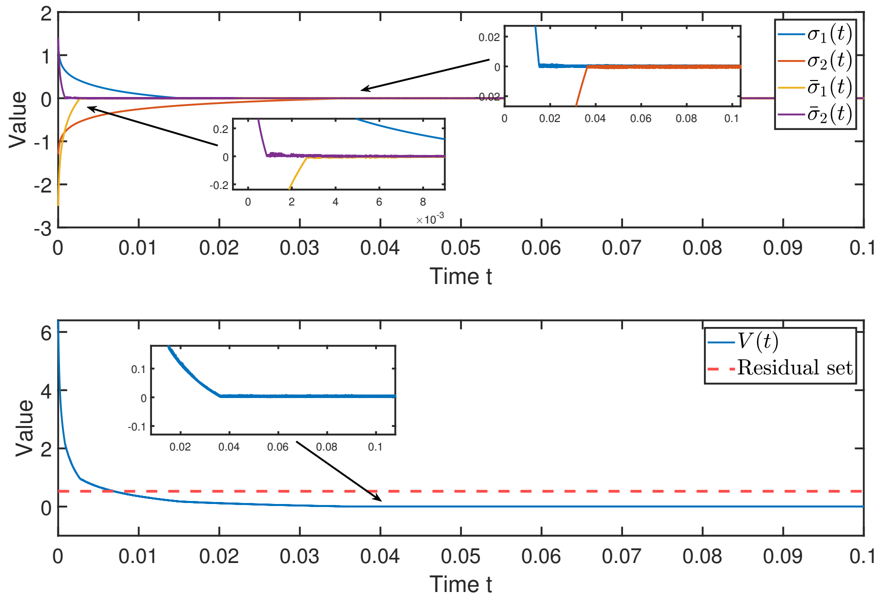

Figure 2. Trajectories of system (3.21) with controller (3.22).

Under the same parameters, the trajectories of system (3.21) with controller (3.22) with $\wp_1 = 0$ and $\wp_2 = 0$ are shown in Figure 3, which illustrates that the FOFINN (2.12) can also achieve FTS under the designed controller where settling time is estimated by $\mathbb{T}_2 = 1.9358$. In Figure 4, we illustrate the changes of settling time $\mathbb{T}_1$ with fractional order μ and parameter ψ. It can be observed that both $\mathbb{T}_1$ decrease as the fractional order μ increases, and $\mathbb{T}_1$ increases as the parameter ψ increases. Besides, from (3.3) and (3.12), we can easily obtain that $\mathbb{T}_1$ and $\mathbb{T}_2$ have the same changing trend with fractional order μ and parameter ψ.

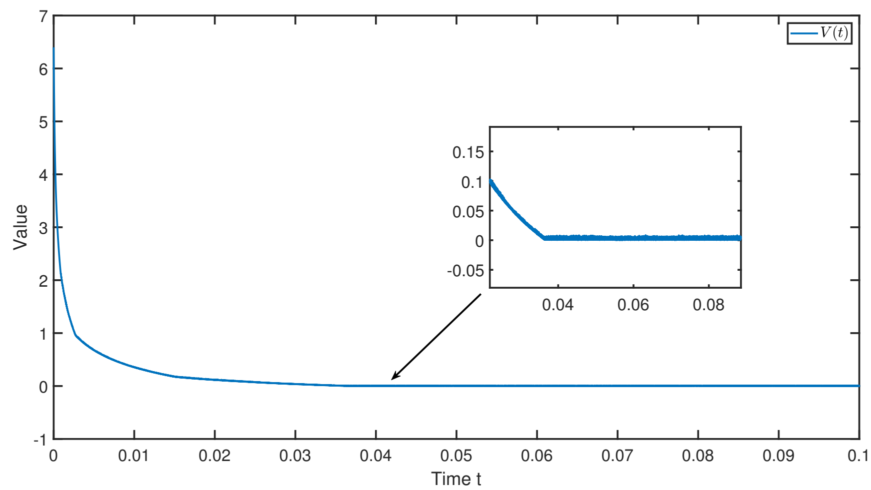

Figure 3. Trajectories of system (3.21) under controller (3.22) with $\wp_{v1} = 0$ and $\wp_{v2} = 0$.

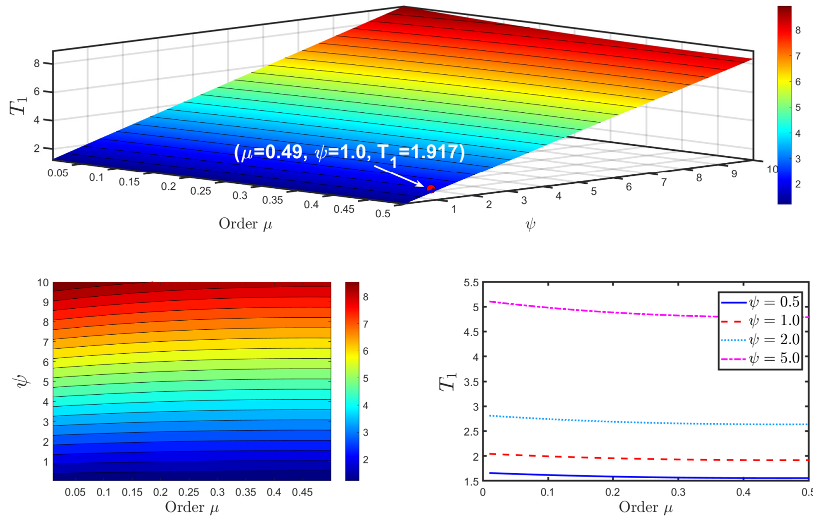

Figure 4. Value of $\mathbb{T}_1$ changes with fractional order μ and parameter ψ.

We vary the fractional order to integer order. The corresponding trajectory of the system without the controller is given in Figure 5. Similarly, we can observe that the FOFINN (2.12) cannot achieve PFTS without the controller. Under the same parameters, the trajectory of system (3.21) with controller (3.22) is shown in Figure 6, which illustrates that the FOFINN (2.12) achieves PFTS under the designed controller.

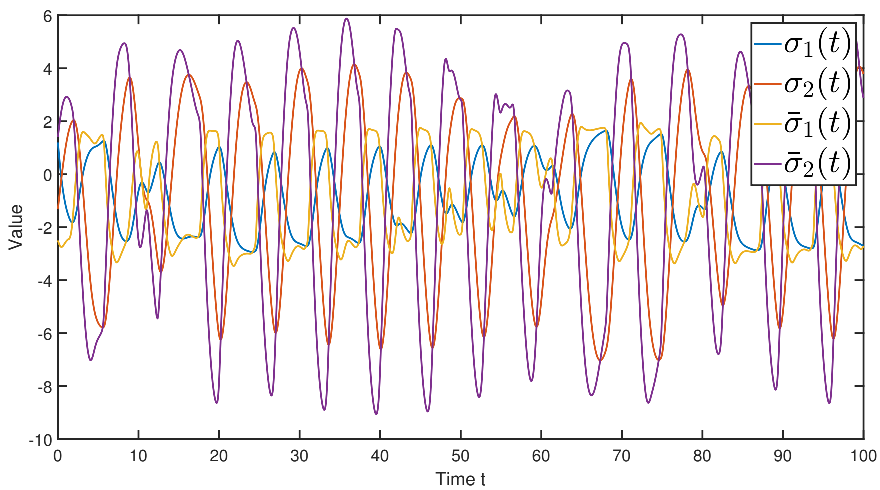

Figure 5. Trajectories of system (3.21) without control input when $\mu = 1$.

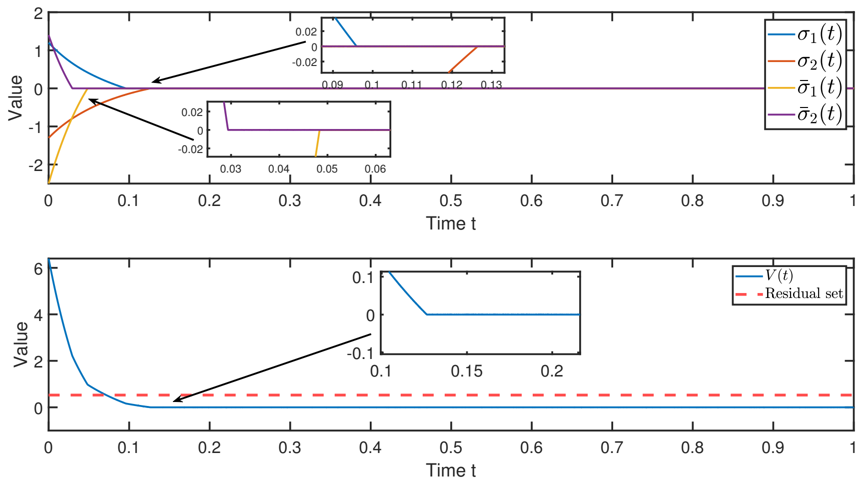

Figure 6. Trajectories of system (3.21) with control input when $\mu = 1$.

Remark 4.1. *The existing fractional-order finite-time inequalities and corresponding controllers are summarized in Table 1. We can find that although the finite-time inequalities established in [29, Lemma 8] and [30, Lemma 9] also contain nonnegative terms, in the controller, their use may lead to a scenario where the associated control term grows unbounded as the system state approaches zero, posing both theoretical and practical challenges. To address this issue, we introduce a positive constant into the negative exponential term. This modification ensures an upper bound on the value of the negative exponential term even when the system state converges to zero, thereby preventing the control term from tending to infinity.*

Table 1. The existing fractional-order finite-time inequalities and corresponding controllers.

	Fractional order inequality	Controller
Li et al. [29]	${}_{t_0}^{C}D^{\mu}V(t) \leq -aV^{-\eta}(t)$	$u_\upsilon(t) = -\frac{\xi_\upsilon e_\upsilon(t)\|e(t)\|_1}{\varsigma+\|e(t)\|_1} - \varrho_\upsilon sign(e_\upsilon(t)) - \frac{sign(e_\upsilon(t))\sigma}{\|e(t)\|_1}$, $\|e\|_1 \neq 0$
Du et al. [30]	${}_{t_0}^{C}D^{\mu}V(t) \leq -aV^{-\eta}(t) - bV(t) + c$	$u_\upsilon(t) = -\frac{\xi_\upsilon e_\upsilon(t)}{\lvert e_\upsilon(t)\rvert_1}\lvert e_\upsilon(t)\rvert^{\ell} - \varpi e_\upsilon(t) - \frac{\sigma}{\lvert e_\upsilon(t)\rvert_1}$, $\lvert e_\upsilon(t)\rvert_1 \neq 0$
This paper	${}_{t_0}^{C}D^{\mu}V(t) \leq -aV(t) - b(V(t)+\psi)^{1-\eta} + c$ ${}_{t_0}^{C}D^{\mu}V(t) \leq -aV(t) - b(V(t)+\psi)^{1-\eta}$	$u_\upsilon(t) = -\rho_{\upsilon 1}\sigma_\upsilon(t) - \rho_{\upsilon 2} sign(\sigma_\upsilon(t))(\lvert\sigma_\upsilon(t)\rvert + \psi)^{1-\ell} - \wp_{\upsilon 1}$

5. Conclusions

In this paper, the FTS of FOFINNs with time-varying delays is systematically studied. The key innovation lies in the proposed finite-time inequality with improved structure. By introducing normal numbers into the exponential term, the theoretical defect that the control input may tend to infinity when the state approaches zero in traditional methods is effectively solved. Based on this, a compound feedback controller integrating linear stabilization, nonlinear convergence, time delay, and fuzzy compensation is designed. By combining Lyapunov theory and ORM, the FTS of closed-loop system is strictly proved, and the explicit estimation formula of stability time is given. Finally, the effectiveness and superiority of the proposed method are verified by numerical simulation, which provides a theoretical framework for dealing with complex NN control problems with memory, fuzzy, and inertia characteristics. Future research can be further extended to robust control of systems with random delays, mixed delays, and external disturbances; explore low communication cost control strategies based on event triggering or quantization; promote the application verification of this method in practical scenarios such as financial time series prediction and biomedical signal processing; and extend the theoretical framework to complex-valued, high-order or reaction-diffusion systems to deepen the finite-time behavior analysis and synthesis of complex neurodynamic systems.

Author contributions

Tiecheng Zhang: Investigation, formal analysis, visualization; Wenxiang Fang: Writing – original draft, simulation; Liyan Wang: Resources, writing – review & editing; Yonglong Lu: Conceptualization, methodology. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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