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Research article

A novel unified solver technique for nonlinear partial differential equations with application to the stochastic δ-nonlinear Schrödinger equation

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Abstract: A novel and unified solver technique is developed for handling a wide class of nonlinear systems of partial differential equations (NPDEs) that can be systematically reduced to the standard diffusing form with cubic nonlinearity. This canonical structure represents a broad spectrum of nonlinear evolution equations arising in nonlinear optics, superfluids, plasma physics, and quantum field theory. The proposed solver provides a robust analytical framework that efficiently transforms complex NPDEs into solvable ordinary differential forms by applying a proper wave transformation. Its adaptability allows for accurate extraction of solitary, periodic, and stochastic wave solutions under diverse boundary conditions. The solver is primarily used to study the stochastic δ-nonlinear Schrödinger equation (δ-NLSE), which incorporates random fluctuations from Brownian motion into nonlinear dispersive dynamics with δ-type localized perturbations. This application highlights the solver's ability to handle deterministic and stochastic nonlinearities, providing detailed insights into how noise and localized singularities affect the stability and propagation of nonlinear waves in complicated physical mediums. This work presents, for the first time, several analytical solutions to the δ-NLSE with Brownian noise. The results highlight the accuracy and efficiency of the proposed approach, emphasizing its applicability to address other intricate models in the natural sciences.

Keywords: Brownian motion process; novel solver technique; nonlinear wave propagation; δ-NLSE; stochastic analysis
Mathematics Subject Classification: 78A10, 35C07, 60H15, 35Q55, 35R60

1. Introduction

Nonlinear partial differential equations (NPDEs) serve an important role in describing many natural and scientific events [1, 2]. Numerous fields, including biology, superfluids, plasma physics, chemical engineering, optical fibers, and materials science, heavily rely on these equations [3, 4]. Nonlinearity causes complex and diverse behaviors that linear theories cannot account for, such as wave steepening, shock production, pattern generation, turbulence, and soliton propagation [5, 6]. With the growing availability of computational tools such as Maple and Mathematica, there is considerable interest in investigating realistic traveling wave solutions for NPDEs, as these solutions provide insights into the essential features of complex systems [7, 8].

The study of solitary waves is primarily motivated by their stability and widespread applications in science and engineering. These waves keep their structure and speed, even after clashing with other waves, making them ideal transporters of energy and information [9]. In fluid dynamics, they account for enduring surface and interior waves; in plasma physics, they represent ion-acoustic and Langmuir structures; and in optics, they provide soliton transmission in fiber communication systems, allowing for high-speed data transfer with low loss. In addition to their physical uses, solitary waves provide important mathematical insights into nonlinear partial differential equations, focusing on the link between nonlinearity and dispersion. They are an important field of study because of their ability to link theoretical ideas with real-world applications, offering both fundamental knowledge and cutting-edge technical developments [10].

Consider the NPDE for $q(x, t)$ in the form

$$R(q, q_x, q_t, q_{xx}, q_{xt}, q_{tt}, ...) = 0. \tag{1.1}$$

Using the wave transformation

$$q(x, t) = Q(\zeta), \qquad \zeta = F(x - wt), \tag{1.2}$$

Equation (1.1) is transformed into the following ordinary differential equation (ODE):

$$G(Q, Q', Q'', Q''', ...) = 0. \tag{1.3}$$

Many nonlinear partial differential equations (NPDEs) that describe wave propagation in dispersive and nonlinear media can be systematically transformed into a canonical diffusive equation with cubic nonlinearity, $L\,Q''(\zeta) + M\,Q^3(\zeta) + N\,Q(\zeta) = 0$ [11–14]. By introducing an appropriate wave transformation, complex evolution equations, which often involve multiple spatial and temporal dimensions, can be reduced to this tractable ordinary differential form. This reduction preserves the essential physical mechanisms, namely the delicate balance between dispersion (or diffusion) and nonlinearity. The resulting canonical equation has long served as a fundamental model for analyzing stationary wave profiles, solitary structures, and soliton dynamics. Studying this reduced form not only simplifies the mathematical treatment of NPDEs but also provides detailed insights into the qualitative behavior, amplitude, and stability of nonlinear waves across a broad range of physical systems, including nonlinear optics, plasma physics, and fluid dynamics. Moreover, this framework establishes an unifying analytical foundation, allowing diverse nonlinear models to be systematically investigated within a single coherent approach.

A stochastic process serves as a mathematical framework for representing systems that change over time due to random influences [15]. In contrast to deterministic models, which predict future states based solely on present conditions, stochastic processes account for uncertainty and random variations, rendering them crucial for effectively characterizing numerous real-world phenomena across fields including biology, engineering, neuroscience, physics, economics, and more. Stochastic nonlinear partial differential equations (SNPDEs) are an extension of traditional nonlinear partial differential equations that incorporate random influences into system dynamics. These equations provide a more accurate representation of complex physical, biological, and engineering processes by modeling the development of systems with nonlinearity, spatial-temporal variation, and random uncertainty [16, 17]. Their creation and analysis improve modeling in a variety of domains, including biology, economics, and physics, in addition to enhancing theoretical understanding. Perturbation variables are added to the equations to introduce disturbances and random noise effects in order to adequately depict uncertainties and fluctuations in optical systems. The Brownian motion process is a typical illustration of the martingale and Markov features, which are fundamental ideas in stochastic processes. Random factors in the input data can occur as a result of inaccuracies in recorded measurements and empirical observations [18, 19].

The nonlinear Schrödinger equation (NLSE) is a key NPDE that characterizes the progression of complex wave amplitudes in weakly nonlinear dispersive environments. This equation balances dispersion and nonlinearity [20, 21]. This balance produces a variety of behaviors, such as solitary waves, rogue waves, modulation instability, and collapse events. It is particularly important in Bose-Einstein condensates, superfluids, astrophysics, collapsing wave instability, ocean waves colliding, optics, and so forth. The improved algebraic technique has been used to solve the Schrödinger-Hirota equation in order to study the propagation of solitons and chaos [22]. The dynamics of localized wave packets in nonlinear dispersive mediums subject to point-like external interactions are captured mathematically by the deterministic complex cubic NLSE with a repulsive δ-potential (δ-NLSE) [23]. It is given as follows [24]:

$$iq_t + \frac{1}{2}q_{xx} - \gamma \mid q \mid^2 q - \alpha\delta\, q = 0, \quad i = \sqrt{-1}, \tag{1.4}$$

where $\gamma, \alpha \in \mathbb{R} - \{0\}$, and δ is the Dirac measure at the origin [25]. Equation (1.4) illustrates the resonant nonlinear propagation of light via optical wave guides with localized defects [25, 26]. The delta potential is attractive when $\alpha < 0$ and repulsive when $\alpha > 0$ [26]. Soliton stability for Eq (1.4) was investigated in [25]. The behavior of the flow was introduced by the authors in [27] using Eq (1.5). This study investigates the stochastic version of model (1.4) using Brownian motion process, which is presented as follows:

$$iq_t + \frac{1}{2}q_{xx} - \gamma \mid q \mid^2 q - \alpha\delta\, q + \sigma\Xi_t\, q = 0, \tag{1.5}$$

where σ indicates the noise level, and Ξ_t represents the time derivative of the Brownian motion process $\{\Xi(t)\}_{t\geq 0}$.

Deterministic models are unable to capture the inherent noise and randomness found in both natural and artificial environments in realistic physical systems, such as imperfect optical fibers, Bose-Einstein condensates interacting with impurities, or fluid interfaces subject to environmental fluctuations. By including stochastic terms, these models give a more realistic picture of physical reality, illustrating how noise may modify, stabilize, or destabilize solitons and other coherent structures. This study seeks to introduce a novel solver for the first time for nonlinear partial differential equation systems with application to the stochastic δ-nonlinear Schrödinger equation in the Stratonovich sense. We provide innovative optical soliton solutions for the stochastic Schrödinger equation featuring a repulsive δ-potential by employing the unified solver method. This approach delivers critical solutions with customizable physical characteristics. It also significantly reduces the time required to obtain solutions compared with existing methods. To our knowledge, the proposed unified solver method has never been employed in any previous study to solve the δ-NLSE caused by multiplicative noise through the Brownian motion process. We examine how the fiber nonlinear term γ and the noise term σ affect the behavior of solutions. Additionally, stochastic analysis aids in the identification of long-term statistical behavior, predicted amplitudes, and probabilistic bounds, all of which are crucial for the design of reliable systems in wave-based technologies, quantum transport, and nonlinear optics. Therefore, the novelty does not lie merely in producing alternative expressions of known solitons, but in providing a broader class of solutions, including analytically tractable stochastic wave structures, obtained through a generalizable and unified analytical methodology.

The study is structured according to its pursuits. Section 2 introduces a brief description for the new extended hyperbolic function method. Section 3 introduces the novel solver technique for the Duffing equation $L\,Q''(\zeta) + M\,Q^3(\zeta) + N\,Q(\zeta) = 0$. Section 4 introduces the novel stochastic optical solitary waves associated with the model (1.5). The physical analysis of the gathered results is described in Section 5. Section 6 illustrates how the noise term and the coefficient parameters affect the behavior of solutions. Section 7 provides concluding remarks on the studied equation.

2. New extended hyperbolic function method

Here, we illustrate a reduced version of the new extended hyperbolic function methodology as shown in [28]. This method assumes that the solution of Eq (1.3) is given as follows:

$$Q(\zeta) = \sum_{i=0}^{J} A_i G^i(\zeta), \quad A_i \neq 0, \tag{2.1}$$

where A_0 and A_i are constants to be determined. By balancing the highest derivative and the nonlinear term in Eq (1.3), we can find the value of the constant J.

The function $G(\zeta)$ satisfies the following equation;

$$G'(\zeta) = G(\zeta)\sqrt{\lambda + \mu G^2(\zeta)}, \tag{2.2}$$

where λ and μ are constants. The function $G(\zeta)$ takes the following forms:

- If $\lambda > 0$ and $\mu < 0$, then the solutions are

$$G(\zeta) = \sqrt{\frac{\lambda}{-\mu}}sech\left(\sqrt{\lambda}(\zeta + C)\right),$$

  where C is an arbitrary constant.
- If $\lambda > 0$ and $\mu > 0$, then the solutions are

$$G(\zeta) = -\sqrt{\frac{\lambda}{\mu}}csch\left(\sqrt{\lambda}(\zeta + C)\right).$$

- If $\lambda < 0$ and $\mu > 0$, then the solutions are

$$G(\zeta) = \sqrt{\frac{-\lambda}{\mu}}sec\left(\sqrt{-\lambda}(\zeta + C)\right),$$

$$G(\zeta) = \sqrt{\frac{-\lambda}{\mu}} csc\left(\sqrt{-\lambda}(\zeta + C)\right).$$

- If $\lambda > 0$ and $\mu = 0$, then the solutions are

$$G(\zeta) = exp\left(\sqrt{\lambda}(\zeta + C)\right).$$

- If $\lambda = 0$ and $\mu > 0$, then the solutions are

$$G(\zeta) = \pm\frac{1}{\sqrt{\mu}\,(\zeta + C)}.$$

3. The novel solver technique

Here, we introduce a novel solver for the Duffing equation

$$L\,Q''(\zeta) + M\,Q^3(\zeta) + N\,Q(\zeta) = 0, \tag{3.1}$$

where L, M, and N are arbitrary constants. The motivation for studying Eq (3.1) arises from its significance in explaining the balance of dispersion and nonlinearity in various physical systems. This equation is a simplified version of numerous nonlinear evolution equations, including the nonlinear Schrödinger equation, Langmuir ion sound model, Korteweg-de Vries equation, and Ginzburg-Landau model, produced by the traveling wave transformation. The analysis of this equation is motivated by its ability to capture the essential dynamics of localized wave structures. The second derivative term, $L\,Q''(\zeta)$, models dispersive or diffusive effects, and the nonlinear term, $M\,Q^3(\zeta)$, accounts for self-interaction or intensity-dependent responses of the medium. Understanding this equilibrium allows us to characterize stable solitary waves, pulse propagation, and coherent structures found in nonlinear optics, plasma physics, and fluid dynamics. Thus, investigating this equation not only broadens our theoretical understanding of nonlinear dynamics but also directly contributes to the design and control of physical systems influenced by similar wave mechanisms.

Taking the balance between Q'' and Q^3, we obtain that the value of $J = 1$. Hence, Eq (2.1) becomes

$$Q(\zeta) = A_0 + A_1 G(\zeta), \quad A_1 \neq 0. \tag{3.2}$$

We now substitute Eq (3.2) along with Eq (2.2) into Eq (3.1) to form an algebraic equation. Taking the coefficients of the function G^i, $\forall i$ leads to a system of equations whose solutions are given in the following families.

Family 1: For $\frac{N}{M} < 0$, $\frac{N}{L} < 0$, the solutions are

$$Q_{1,2}(\zeta) = \pm\sqrt{\frac{-2N}{M}} sech\left(\sqrt{\frac{-N}{L}}(\zeta + C)\right). \tag{3.3}$$

Family 2: For $\frac{N}{M} > 0$, $\frac{N}{L} < 0$, the solutions are

$$Q_{3,4}(\zeta) = \pm\sqrt{\frac{2N}{M}} csch\left(\sqrt{\frac{-N}{L}}(\zeta + C)\right). \tag{3.4}$$

Family 3: For $\frac{N}{M} < 0$, $\frac{N}{L} > 0$, the solutions are

$$Q_{5,6}(\zeta) = \pm\sqrt{\frac{-2N}{M}}sec\left(\sqrt{\frac{N}{L}}(\zeta + C)\right). \tag{3.5}$$

Family 4: For $\frac{N}{M} < 0$, $\frac{N}{L} > 0$, the solutions are

$$Q_{7,8}(\zeta) = \pm\sqrt{\frac{-2N}{M}}csc\left(\sqrt{\frac{N}{L}}(\zeta + C)\right). \tag{3.6}$$

Family 5: For $N = 0$, $\frac{L}{M} < 0$, the solutions are

$$Q_{9,10}(\zeta) = \pm\sqrt{\frac{-2L}{M}}\frac{1}{\zeta + C}. \tag{3.7}$$

4. The stochastic solutions for the δ-NLSE

Using the wave transformation

$$q(x,t) = Q(\zeta)e^{i(p\,x+rt+\sigma\Xi(t))}, \quad \zeta = \beta(x - w\,t), \tag{4.1}$$

Equation (1.5) is converted into Eq (3.1) from the real part, where $L = \beta^2, M = -2\gamma$, $N = -(2\alpha\delta + 2\,r + p^2)$, and $p = w$ from the imaginary part. Applying the obvious unified solver, we get
Family 1: For $\gamma < 0$, $2\alpha\delta + 2\,r + p^2 > 0$, the solutions are

$$Q_{1,2}(\zeta) = \pm\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\gamma}}sech\left(\sqrt{\frac{2\alpha\delta + 2\,r + p^2}{\beta^2}}(\zeta + C)\right). \tag{4.2}$$

Thus, the solutions of Eq (1.5) are

$$q_{1,2}(x,t) = \pm\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\gamma}}sech\left(\sqrt{\frac{2\alpha\delta + 2\,r + p^2}{\beta^2}}(\beta(x - w\,t) + C)\right)e^{i(p\,x+rt+\sigma\Xi(t))}. \tag{4.3}$$

Family 2: For $\gamma > 0$, $2\alpha\delta + 2\,r + p^2 > 0$, the solutions are

$$Q_{3,4}(\zeta) = \pm\sqrt{\frac{2\alpha\delta + 2\,r + p^2}{\gamma}}csch\left(\sqrt{\frac{2\alpha\delta + 2\,r + p^2}{\beta^2}}(\zeta + C)\right). \tag{4.4}$$

Thus, the solutions of Eq (1.5) are

$$q_{3,4}(x,t) = \pm\sqrt{\frac{2\alpha\delta + 2\,r + p^2}{\gamma}}csch\left(\sqrt{\frac{2\alpha\delta + 2\,r + p^2}{\beta^2}}(\beta(x - w\,t) + C)\right)e^{i(p\,x+rt+\sigma\Xi(t))}. \tag{4.5}$$

Family 3: For $\gamma > 0$, $2\alpha\delta + 2\,r + p^2 < 0$, the solutions are

$$Q_{5,6}(\zeta) = \pm\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\gamma}}sec\left(\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\beta^2}}(\zeta + C)\right). \tag{4.6}$$

Thus, the solutions of Eq (1.5) are

$$q_{5,6}(x,t) = \pm\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\gamma}}sec\left(\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\beta^2}}(\beta(x - w\,t) + C)\right)e^{i(p\,x+rt+\sigma\Xi(t))}. \tag{4.7}$$

Family 4: For $\gamma > 0$, $2\alpha\delta + 2\,r + p^2 < 0$, the solutions are

$$Q_{7,8}(\zeta) = \pm\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\gamma}}csc\left(\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\beta^2}}(\zeta + C)\right). \tag{4.8}$$

Thus, the solutions of Eq (1.5) are

$$q_{7,8}(x,t) = \pm\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\gamma}}csc\left(\sqrt{\frac{-(2\alpha\delta + 2\,r + p^2)}{\beta^2}}(\beta(x - w\,t) + C)\right)e^{i(p\,x+rt+\sigma\Xi(t))}. \tag{4.9}$$

Family 5: For $2\alpha\delta + 2\,r + p^2 = 0$, $\gamma > 0$ the solutions are

$$Q_{9,10}(\zeta) = \pm\sqrt{\frac{\beta^2}{\gamma}}\frac{1}{\zeta + C}. \tag{4.10}$$

Thus, the solutions of Eq (1.5) are

$$q_{9,10}(x,t) = \pm\sqrt{\frac{\beta^2}{\gamma}}\frac{1}{\beta(x - w\,t) + C}\,e^{i(p\,x+rt+\sigma\Xi(t))}. \tag{4.11}$$

5. Results and discussion

We introduce the novel unified solver for the Duffing equation $L\,Q''(\zeta)+M\,Q^3(\zeta)+N\,Q(\zeta) = 0$ based on the new extended hyperbolic function method. This solver technique has a number of advantages over previous complex approaches, such as avoiding complex and time-consuming computations and producing accurate results by using physical factors. Additionally, it can be used as a box solver or pre-built function to answer a variety of systems and equations found in applied science. This solver is simple, dependable, and durable. Additionally, mathematicians, physicists, and engineers can illustrate some complex events in practical settings thanks to this methodology.

In this work, we implement the unified solver technique to solve the stochastic δ-NLSE through the Brownian motion process in the Stratonovich sense. This section is crucial for understanding results and clarifying their physical meaning in mathematical physics and nonlinear optics. This offers a comprehensive grasp of how the findings progress various domains while accounting for system dynamics and broader ramifications. The study of optical stochastic solutions for the complex δ-NLSE is the work's new contribution. When stochastic perturbations are added using Brownian motion, the model captures the random fluctuations found in realistic physical systems such as optical fibers, Bose-Einstein condensates, and plasma channels, where environmental noise and microscopic irregularities cannot be ignored. In order to capture the effects of random fluctuations in nonlinear optical systems, it is crucial to use optical stochastic solutions via the Brownian motion process. These vital solutions provide a realistic depiction of system dynamics under noise, which is necessary to comprehend real-world events in optics and other fields.

The majority of prominent publications have examined the δ-NLSE within a deterministic framework. In contrast to that approach, we investigate this model under a stochastic framework, influenced by multiplicative noise through the Brownian motion process. Brownian motion $\Xi(t)$ represents a crucial stochastic process. This process facilitates the transformation of the δ-NLSE with multiplicative noise into a standard nonlinear ordinary differential equation. We utilize the improved unified solver technique in order to derive several distinct stochastic solutions for the δ-NLSE, resulting in various applications. The solutions obtained elucidate numerous intriguing physical phenomena within these fields, specifically:

1. The resulting solutions describe how optical solitons develop and propagate in fibers and waveguides, taking into account random fluctuations and localized disturbances. This directly relates to signal transmission stability and the creation of durable optical communication systems.
2. The solutions shed light on the behavior of nonlinear plasma waves under stochastic perturbations, such as amplitude modulation, stability, and interactions between isolated structures. This can help with the modeling of ion-acoustic waves and magnetohydrodynamic events.
3. The solutions can characterize localized wave structures in shallow water or superfluid systems, including the impact of stochastic forcing or localized impurities on wave amplitude, stability, and long-term dynamics.
4. The stochastic δ-NLSE solutions demonstrate how noise and localized nonlinearities impact wave packet dynamics in Bose-Einstein condensates and other quantum systems with nonlinear interactions.

We employ the symbolic software Matlab to generate visual representations of particular stochastic solutions via Brownian motion process. The selected solution is visually represented through two-dimensional graphs created using Matlab software to illustrate its behavior. Figure 1 shows the 2D stationary soliton wave of solution $q_1(x,t)$ with $\alpha = 1.8, \beta = 1.4, \delta = 1, r = 1.3, w = 1.5, \gamma = -1.2, C = 0$. This is a classic representation of a localized, stable soliton solution in a nonlinear wave equation, highlighting its peak amplitude and rapid decay away from the center. Figure 2 shows the 2D singular wave of solution $q_3(x,t)$ with $\alpha = 1.2, \beta = 1.4, \delta = 1, r = 1.1, w = 1.2, \gamma = 3.2, C = 0$. This solution has a significant localized singularity at $x = 0$, with the amplitude rapidly diverging towards both positive and negative infinity. Away from the singularity, the solution decays rapidly to zero, indicating that the unique structure is highly localized. Figure 3 shows the explosive pulse wave of solution $q_7(x,t)$ with $\alpha = -1.2, \beta = 0.4, \delta = 1, r = -1.1, w = 0.6, \gamma = 3.2, C = 1$. This solution has a strong, isolated peak near $x = -2.5$. The amplitude rapidly climbs to a considerable value, indicating explosive behavior typical of highly nonlinear dynamics. Away from the peak, the solution rapidly decays to near zero, indicating that the explosive feature is very concentrated in space. This profile depicts a pulse-like structure with large amplitude, which can be linked to phenomena such as rogue waves or localized energy concentration in nonlinear dispersive systems.

Future studies could concentrate on improving the analytical approach that was previously stated by adding more intricate models with greater dimensions and different types of nonlinearity. Another major advancement would be the experimental validation of these theoretical models, especially with regard to real-world applications in biological tissues, sophisticated ultrasound technology, realistic fluid dynamics, and medical diagnostics. Investigating the connections between stochastic effects and physical characteristics may improve our comprehension of wave events in many media and pave the way for the creation of increasingly complex models in the future. Engineering and technological domains that require the manipulation and control of wave behavior may benefit from such advancements.

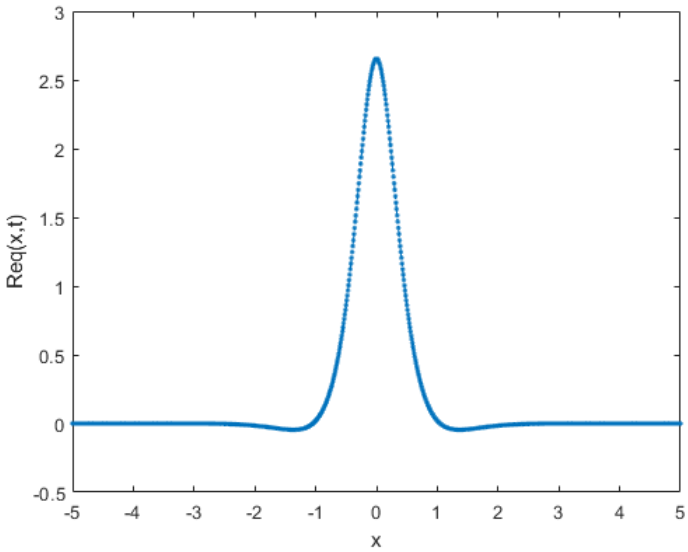

Figure 1. 2D stationary soliton wave solution $q_1(x,t)$.

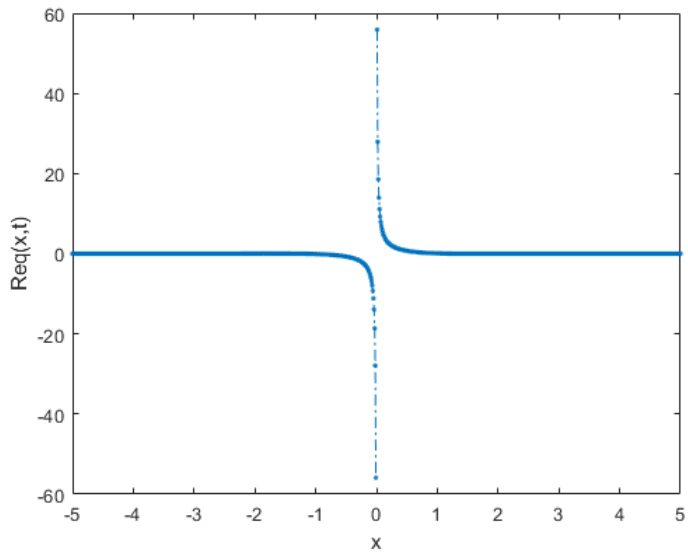

Figure 2. The 2D singular solution $q_3(x,t)$.

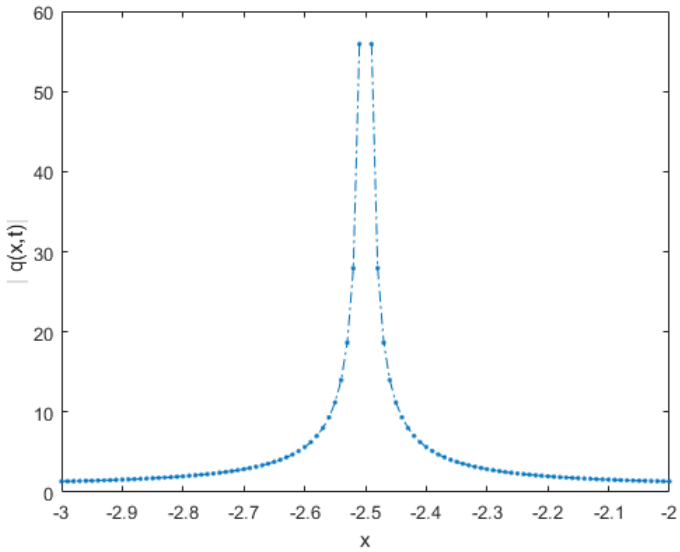

Figure 3. 2D explosive pulse wave solution $q_7(x, t)$.

6. The impact of noise and nonlinear fiber terms

We investigate the effect of the noise term σ and the fiber nonlinear term γ on the dynamic behavior of the obtained solutions. In the area of nonlinear underwater acoustics, this study is very important because it may be used to mimic actual complicated systems in materials science, quantum physics, and optics.

6.1. The effect of σ

Introducing a noise term into nonlinear wave models fundamentally alters traveling wave solutions, shifting them from deterministic, stable forms to stochastically modified structures. Depending on the system and noise strength, this might result in distortion, drift, dissipation, or even collapse. However, in certain circumstances, stochastic coherence and noise improve localization. Understanding these effects is crucial for developing reliable nonlinear wave systems in optical communication, quantum computing, and fluid engineering. Figure 4 shows how the stochastic solution $q_1(x, t)$ varies depending on the spatial variable x and noise strength σ with $\alpha = 1.8, \beta = 1.4, \delta = 1, r = 1.3, w = 1.5, \gamma = -1.2, C = 0$. At $\sigma = 0$, the soliton has near-ideal shape and trajectory with limited phase diffusion. However, as σ grows, the soliton undergoes magnified radiative losses, random positional shifts, and accelerated phase jitter, eventually leading to severe deformation or decay. This degradation is especially important in applications such as optical communications, where phase stability and shape preservation are key for signal integrity.

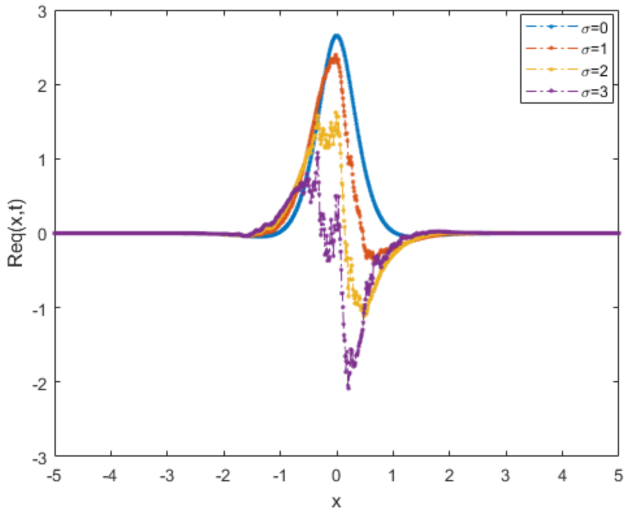

Figure 4. 2D stationary soliton wave solution $q_1(x, t)$ with different values of σ.

6.2. *The effect of* γ

The nonlinear coefficient γ regulates the balance of dispersion and nonlinearity, directly affecting soliton stability, propagation properties, and phase evolution. Figure 5 shows that increasing the magnitude of γ increases the effective nonlinear self-focusing strength of the solution $q_1(x, t)$, resulting in a more localized soliton peak with $\alpha = 1.8, \beta = 1.4, \delta = 1, r = 1.3, w = 1.5, \gamma = -2, C = 0$. The parameter γ controls nonlinear phase accumulation, which speeds phase modulation, reinforces wave confinement, and stabilizes the stationary soliton against dispersive spreading. Conversely, weaker nonlinearity reduces phase locking, making the soliton more vulnerable to dispersion-induced broadening and possible instability when perturbed. As a result, γ functions as a tuning parameter that controls the soliton's phase coherence, which is crucial for preserving long-term stability in nonlinear wave mediums, in addition to its amplitude and width during propagation.

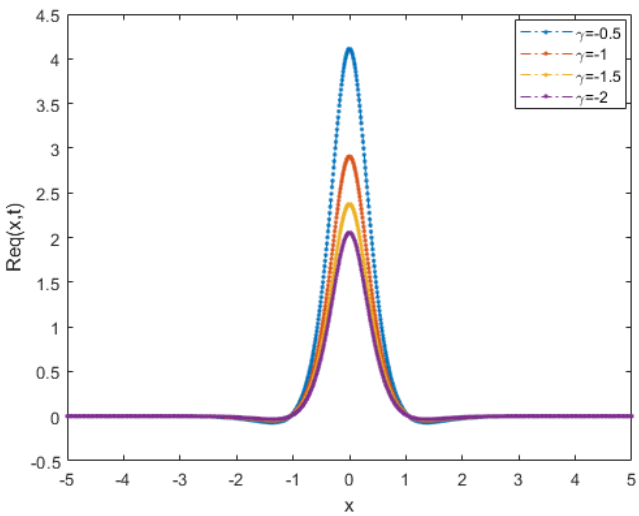

Figure 5. 2D stationary soliton wave solution $q_1(x, t)$ with different values of γ.

7. Conclusions

In this work, we created a new unified solver which can be used to handle a wide range of nonlinear systems of partial differential equation (NPDEs). These equations are reduced to model (3.1). The solver effectively transforms complex nonlinear systems into analytically tractable forms, allowing for the extraction of solutions such as solitary, periodic, and stochastic wave structures. The stochastic δ-nonlinear Schrödinger equation (δ-NLSE), in which random perturbations approximated by the Brownian motion process interact with nonlinear dispersive dynamics and δ-type localized potentials, was used to illustrate its adaptability. The derived stochastic solutions provide a greater understanding of real-world systems such as optical fibers, Bose-Einstein condensates, and plasma channels by highlighting the complex effects of noise and singular perturbations on wave propagation, stability, and soliton dynamics.

The stochastic δ–NLSE serves as a concrete, physically meaningful application demonstrating the solver's capabilities. Importantly, the proposed method is not limited to the stochastic δ–NLSE; it can be directly applied to a wide range of NPDEs that reduce to the canonical form, including other stochastic, deterministic, or higher-dimensional systems. This generality emphasizes both the novelty and practical usefulness of the solver, unifying the treatment of diverse nonlinear equations under a single analytical framework. Future research could expand this approach to higher-dimensional and multicomponent systems, increasing its usefulness in a variety of nonlinear science domains.

Author contributions

M. B. Almatrafi and Mahmoud A. E. Abdelrahman: Conceptualization, software, formal analysis, writing–original draft. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that he has no conflict of interest.

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