AIMS Mathematics, 11(2): 4596–4616.
DOI: 10.3934/math.2026186
Received: 08 December 2025
Revised: 16 January 2026
Accepted: 10 February 2026
Published: 25 February 2026

https://www.aimspress.com/journal/Math

*Research article*

# A Neutrosophic soft topological framework for insecure intelligent decision applications

**Samer R. Yaseen[1] and Noura M. Alhouiti[2,*]**

[1] Department of Mathematics, College of Education for Pure Science, Tikrit University, Iraq
[2] Department of Mathematics, Faculty of Science, University of Tabuk, 71491Tabuk, Saudi Arabia

* **Correspondence:** Email: nalhouiti@ut.edu.sa.

**Abstract:** To enhance the reasoning ability when dealing with uncertainty, we hold that such a neutrosophic soft topological structure should be proposed. New classes of neutrosophic soft topology are developed by giving the mathematical structure, which involves min and max norm operations. In order to make a decision for options based on uncertain/or incomplete data, the model changes crucial topological concepts such as interior of set and closure into quantity indicators. The construction of the model was done applying topological operations to the truth degree, doubtfulness degree, and falsity degrees, and the decision criteria were formulated in form of neutrosophic soft sets. The frequency and reliability of decisions was measured by a decision index derived from these operations. Compared with fuzzy soft topological methods, experimental simulations further confirmed the effectiveness and robustness of the proposed model. The results indicated the potential ability of neutrosophic topology to be a universal mathematical tool used for intelligent devices operating in harsh and unpredictable surroundings.

**Keywords:** process decision-making; soft topological approach; intelligent devices; neutrosophic-soft topological model; mathematical entity
**Mathematics Subject Classification:** 54A40, 54E55, 54D10

## 1. Introduction

Data has become a fundamental source of information in modern science, providing deep insights upon close examination [1]. However, since information from the real world may be ambiguous, contradictory, and insufficient, standard models of computation and mathematical confront substantial hurdles [2]. To address these problems, a number of uncertainty-management structures have been developed, including fuzzy sets [3], ambiguity sets [4], soft collections [5], and neutrosophic sets [6]. Confusion and uncertainty can be mathematically modeled thanks to these ideas, which are frequently applied in intelligent computation, recognizing patterns, and decision-support technologies [7–9]. The soft set, created by Molodtsov [5], offers a flexible way to parameterize uncertainty beyond traditional probabilities. Subsequently, Smarandache [6] created neutrosophic set theories, which add independent levels of truth and indeterminacy, with falsity to fuzzy as well as intuitionistic fuzzy frameworks. The proposed neutrosophic softer set (NS-set), initially developed by Maji [10] and later enhanced via Deli and Broumi [11], was the result of combining the two ideas. When modeling decision problems with incomplete or contradictory data, this mixed structure works especially well [12–15].

The algebraic with topological features of NS-sets have been extensively studied. Basic concepts like closed and open NS-sets, interior, closure, and neighborhood were first introduced by Bera and Mahapatra [16], who defined general operations for building neutrosophic soft topological spaces. Later, the field was advanced by Ozturk et al. [17], who developed a conceptual structure for NS-topology utilizing generalized norms. Drawing on this body of work, Tran et al. [18] suggested a new method based on Min-Max operations over NS sets, a firm mathematical underpinning for making topological frameworks. This research, with its theoretical foundation, greets them at the starting line. This seems to raise the question: While earlier studies greatly expanded the theoretical frontier of NS topology, most of it remains essentially pure math and is mainly concerned with generating concepts and proofs rather than generating computational or practical applications [19, 20]. On the other hand, fuzzy with soft topological structures have been applied to useful fields like network analysis [21], pattern identification [22], and clustering [23]. As a result, there is an important study gap between conceptual NS-topology and its practical implementation in intelligent decision structures.

The Neutrosophic Soft Topology Decision Structure (NST-DS), a model for computing that converts the conceptual min–max topological structure of Tran et al. [18] into a useful instrument for smart decision-making under uncertainty, is proposed in this paper to close this gap. The suggested system models decisions as neutrosophic soft sets, from which quantifiable indicators like closeness and interiorness are derived using topological procedures (union, complemented, and intersection). A decision index that measures the consistency, stability, and dependability of options in unclear situations is then calculated by aggregating these indicators using min-norm and max-norm procedures.

By translating traditional topological relationships into measurable decision indicators, the NST-DS model provides new theoretical insights in addition to its computing contribution. This combined theoretical and practical framework has made neutrosophic topology more widely used in artificial intelligence, especially in data-driven learning architectures, neural networks, and multiple-objective assessments (MOA) [24–27]. When compared to fuzzy soft topological methods, empirical results verify that the suggested model improves decision accuracy and robustness. The rest of the article is structured as follows: In Section 2, theoretical developments are summarized and neutrosophic with soft set fundamentals are reviewed. The suggested NST-DS model is formulated in detail in Section 3. Computational tests and comparative findings are shown in Section 4. In Section 5. We discuss the

implications of the findings, while in Section 6, we offer outcomes and suggestions for further study.

## 2. Basic concepts

In this section, we present new, decision-oriented extensions utilized by the suggested Neutrosophic Soft Topology Decision Structure (NST-DS) after reviewing fundamental concepts of neutrosophic soft sets upon which our model is based.

**Definition 2.1.** *[11,16,20] The pair of values* $(\mathcal{D},\mathfrak{Z})$ *represents an NS-set on* $\mathcal{X}$*, and* $d \mapsto \mathcal{D}(d)$ *determines the set-valued function* $d \mapsto \mathcal{D}(d) := \mathcal{D}_d$ *via*

$$\mathcal{D}_d : \mathcal{X} \longrightarrow ]^-0;1^+[ \times ]^-0;1^+[ \times ]^-0;1^+[ \tag{1}$$

$$x \mapsto \mathcal{D}_d(x) := \langle T_{\mathcal{D}_d}(x); I_{\mathcal{D}_d}(x); F_{\mathcal{D}_d}(x)\rangle.$$

*For every* $d$ *in* $\mathfrak{Z}$*, the genuine function triples* $T_{\mathcal{D}_d}$, $I_{\mathcal{D}_d}$, *and* $F_{\mathcal{D}_d} : \mathcal{X} \longrightarrow ]^-0;1^+[$ *denote degrees of truth, indeterminacy, and falsity, respectively, having no limit on their sum.*

*Put differently, an NS-set will be defined as a collection for arranged duos in the following way:*

$$(\mathcal{D},\mathfrak{Z}) = \{(d,\mathcal{D}(e)) : e \in \mathfrak{Z}, \mathcal{D}(e) \in \mathcal{N}(\mathcal{X})\} \tag{2}$$

$$= \left\{\left(d,\langle x, T_{\mathcal{D}_d}(x), I_{\mathcal{D}_d}(x), F_{\mathcal{D}_d}(x)\rangle\right) : d \in \mathcal{E}, x \in \mathcal{X}\right\} \tag{3}$$

$$= \left\{\left(d, \frac{x}{T_{\mathcal{D}_d}(x), I_{\mathcal{D}_d}(x), F_{\mathcal{D}_d}(x)}\right) : x \in \mathcal{X}, d \in \mathfrak{Z}\right\}. \tag{4}$$

*The collection of every NS-sets on* $\mathcal{X}$ *is denoted by the sign* $NS(\mathcal{X})$*. Furthermore, the integral component of the values is nearly zero since the values of* $T$, $I$, *and* $F$ *fall inside the unit interval* $[0;1]$.

**Definition 2.2.** *[11,19] Assume that two sets of NS on* $\mathcal{X}$ *are* $\mathcal{H}$ *and* $\mathcal{K}$.

- $\mathcal{H}$ *represents an NS-subset of* $\mathcal{K}$*, denoted as* $\mathcal{H} \subseteq \mathcal{K}$*, whenever*

$$\forall x \in \mathcal{X}, \forall d \in \mathfrak{Z}, \begin{cases} T_{\mathcal{H}_d}(x) \le T_{\mathcal{K}_d}(x) \\ I_{\mathcal{H}_d}(x) \le I_{\mathcal{K}_d}(x) \\ F_{\mathcal{H}_d}(x) \ge F_{\mathcal{K}_d}(x) \end{cases}. \tag{5}$$

- $\bar{\mathcal{K}}$ *is the complement of* $\mathcal{K}$ *whenever*

$$\forall x \in \mathcal{X}, \forall d \in \mathfrak{Z}, \begin{cases} T_{\bar{\mathcal{H}}_d}(x) = F_{\mathcal{H}_d}(x) \\ I_{\bar{\mathcal{H}}_d}(x) = 1 - I_{\mathcal{H}_d}(x). \\ F_{\bar{\mathcal{H}}_d}(x) = T_{\mathcal{H}_d}(x) \end{cases} \tag{6}$$

**Definition 2.3.** *[18,19] The binary process* $\bullet : [0;1] \times [0;1] \to [0,1]$ $(resp., o : [0;1] \times [0;1] \to [0,1]$ *represents a min-norm (resp., max - norm) that complies with the following requirements:*

*a)* $\bullet$ *(*$\circ$*) has the properties of commutativity and associativity.*

*b)* $x \bullet 1 = 1 \bullet x = x$*, (resp.,* $x\ o\ 1 = 1\ o\ x = 1), \forall x \in [0,1]$.

*c)* $x \bullet 0 = 0 \bullet x = 0$*, (resp.,* $y\ o\ 0 = 0\ o\ x = x), \forall x, y \in [0,1]$.

*d)* $x \ge x \bullet y, (resp., x \le x\ o\ y)\ \ \forall x, y \in [0,1]$.

**Definition 2.4.** *The intersection for the two sets of NS* $\mathcal{H}$ *and* $\mathcal{K}$*, denoted as* $\mathcal{H} \cap \mathcal{K}$*, may be found using below:*

$$\forall x \in \mathcal{X}, \forall d \in \mathfrak{Z}, \begin{cases} T_{\mathcal{H}\cap\mathcal{K}_d}(x) = T_{\mathcal{H}_d}(x) \bullet T_{\mathcal{K}_d}(x) \\ I_{\mathcal{H}\cap\mathcal{K}_d}(x) = I_{\mathcal{H}_d}(x) \bullet I_{\mathcal{K}_d}(x) \\ F_{\mathcal{H}\cap\mathcal{K}_d}(x) = F_{\mathcal{H}_d}(x) \circ F_{\mathcal{K}_d}(x) \end{cases}. \tag{7}$$

## 3. Structure of NST-DS

In this part, we build the major math tools that will help the suggested NST-DS structure.

### *3.1. Some properties of neutrosophic soft mapping for making decisions*

**Definition 3.1.** *Assume that* $\mathcal{P}$ *is a collection of decision criteria, and* $\mathcal{X}$ *is considered a set of options. A map*

$$\mathcal{S}: \mathcal{X} \times \mathcal{P} \longrightarrow [0,1]^3, \tag{8}$$

*is referred to as a decision-based neutrosophic soft map if every ordered pair* $(x, d)$ *is given a triple*

$$\mathcal{S}(x, d) = (T(x, d), I(x, d), F(x, d)), \tag{9}$$

*where, the grade of assistance for* $x$ *under parameter* $d$ *is represented by* $T(x, d)$*, the degree of indefiniteness is represented by* $I(x, d)$*, and the degree of refusal is represented by* $F(x, d)$*. The sum* $T + I + F$ *is not subject to an algebraic relation, which permits the model to express incomplete or inconsistent data without artificial constraints.*

**Example 3.1.** *Assume that* $\mathcal{X} = \{x_1, x_1, x_1, x_1\}, \mathcal{P} = \{d_1, d_2, d_3\}$

*In actuality, the measurements of,* $T(x, d), I(x, d)$*, and* $F(x, d)$ *are derived from measurable indicators connected to every decision parameter or from expert opinions. These assessments are combined and normalized to produce the neutrosophic mapping shown in Table 1.*

**Table 1.** The values for $\mathcal{S}(x, d)$.

| $x$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|
| $x_1$ | $(0.9, 0.1, 0.2)$ | $(0.7, 0.2, 0.3)$ | $(0.8, 0.1, 0.2)$ |
| $x_2$ | $(0.6, 0.3, 0.3)$ | $(0.8, 0.1, 0.2)$ | $(0.7, 0.2, 0.3)$ |
| $x_3$ | $(0.5, 0.2, 0.4)$ | $(0.6, 0.3, 0.3)$ | $(0.4, 0.3, 0.5)$ |
| $x_4$ | $(0.8, 0.1, 0.2)$ | $(0.4, 0.4, 0.5)$ | $(0.6, 0.2, 0.4)$ |

*The above table represents a full neutrosophic soft mapping across* $\mathcal{X}$.

**Proposition 3.1.** *A decision parameter e must have at least one possibility* $x \in \mathcal{X}$ *with* $T(x, d) > 0$ *in order for it to have meaning.*

*Proof.* Parameter $d$ fails to validate any decision results and thus adds no data to the system whenever $T(x, d) = 0$ for all possibilities. It is necessary to either eliminate or redefine such a parameter. Consequently, a functional decision framework requires non-degeneracy.

**Definition 3.2.** *An NSD-set can be described as:*

$$\mathcal{D} = (X, \mathfrak{Z}, \mathcal{C}), \tag{10}$$

*where* $\mathcal{C}$ *represents neutrosophic mapping that is decision-oriented. For a fixed coefficient* $d$ *in* $\mathfrak{Z}$*, we have:*

$$\mathcal{D}(d) = \{(x, T(x,d), I(x,d), F(x,d)) : x \in \mathcal{X}\}. \tag{11}$$

**Example 3.2.** *The NSD-set can be obtained through the map of Example 1.*

$$\mathcal{D} = (X, \mathfrak{Z}, \mathcal{C}),$$

*with every assessment coming straight from Table 1.*

**Definition 3.3.** *Assume* $\odot$ *indicates a decision conjunction operator, and* $\oplus$ *indicates a decision separation operator, both of which map* $[0,1]^2$ *to* $[0,1]$*. Every operator fulfills the following*

1) *Commutation;*
2) *Association;*
3) *Monotony;*
4) *Identities.*

$$c \odot 1 = c, c \oplus 0 = c. \tag{12}$$

**Example 3.3.** *An obvious option is:*

$$c \odot d = \min(c,d), c \oplus d = \max(c,d).$$

*However, other standards might be applied based on the needs of the decision.*

**Definition 3.4.** *Assume that* $\mathcal{H}$ *and* $\mathcal{K}$ *are NSD-sets. Then*

(i) *The intersection*

$$(\mathcal{H} \cap \mathcal{K})(x,d) = (T_{\mathcal{H}}(x,d) \odot T_{\mathcal{K}}(x,d), I_{\mathcal{H}}(x,d) \odot I_{\mathcal{K}}(x,d), F_{\mathcal{H}}(x,d) \oplus F_{\mathcal{K}}(x,d)). \tag{13}$$

(ii) *The union*

$$(\mathcal{H} \cup \mathcal{K})(x,d) = (T_{\mathcal{H}}(x,d) \oplus T_{\mathcal{K}}(x,d), I_{\mathcal{H}}(x,d) \oplus I_{\mathcal{K}}(x,d), F_{\mathcal{H}}(x,d) \odot F_{\mathcal{K}}(x,d)). \tag{14}$$

(iii) *The complement*

$$\overline{\mathcal{H}}(x,d) = (F(x,d), 1 - I(x,d), T(x,d)). \tag{15}$$

*Recall Example 1. We obtain the following results (see Table 2):*

**Table 2.** The values of complement for $\mathcal{S}(x,d)$.

| $x$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|
| $x_1$ | (0.2, 0.9, 0.9) | (0.3, 0.8, 0.7) | (0.2, 0.9, 0.8) |
| $x_2$ | (0.3, 0.7, 0.6) | (0.2, 0.9, 0.8) | (0.3, 0.8, 0.7) |
| $x_3$ | (0.4, 0.8, 0.5) | (0.3, 0.7, 0.6) | (0.5, 0.7, 0.4) |
| $x_4$ | (0.2, 0.9, 0.8) | (0.5, 0.6, 0.4) | (0.4, 0.8, 0.6) |

**Theorem 3.1.** *For each NSD-set* $\mathcal{H}$, $\overline{\overline{\mathcal{H}}} = \mathcal{H}$.

*Proof.* Using Eq (15) twice consecutively, we obtain the following:

$$\overline{\mathcal{H}}(x,d) = (F(x,d), 1 - I(x,d), T(x,d)).$$

Additionally, the second complement:

$$\bar{\bar{\mathcal{H}}}(x,d) = (T(x,d), 1-(1-I(x,d)), F(x,d)) = (T(x,d), I(x,d)), F(x,d)).$$

Therefore, $\bar{\bar{\mathcal{H}}} = \mathcal{H}$.

This theorem guarantees that the complement ($\mathcal{H}$) of an NSD-set ($\mathcal{H}$) is different from the original set. Because it ensures that decision limits within the neutrosophic space are well defined, this trait is crucial. To provide clarity in the framework's decision-making areas and avoid ambiguity and overlap, it is crucial to distinguish each component and its complement. This establishes the foundation for later stages of decision-making processes, where exact choice boundaries are needed in unclear circumstances.

**Definition 3.5.** *The neighborhood of another possibility $x$ can be expressed as follows for thresholds $q, p, \sigma \in [0,1]$:*

$$N(x) = \{y \in \mathcal{X}: T(y,d) \geq qT(x,d), I(x,d) \leq pI(x,d), F(y,d) \leq \sigma F(x,d), \forall d \in \mathfrak{Z}\}. \quad (16)$$

**Example 3.4.** *Assume $q = 0.7, p = 1, \sigma = 1$.*
*From Table 1, we obtain:*

$$T(x_1,d_1) = 0.9 \Rightarrow qT(x_1,d_1) = 0.63, T(x_1,d_2) = 0.7 \Rightarrow qT(x_1,d_2) = 0.49, T(x_1,d_3) = 0.8 \Rightarrow qT(x_1,d_3) = 0.56.$$

*However,*

$$I(x_1,d_1) = 0.1, I(x_1,d_2) = 0.2, I(x_1,d_3) = 0.1, F(x_1,d_1) = 0.2, F(x_1,d_2) = 0.3, F(x_1,d_3) = 0.2.$$

*Therefore, we require a substitute that meets everyone's needs:*

$$\begin{cases} T(y,d_1) \geq 0.63, I(y,d_1) \leq 0.1, F(y,d_1) \leq 0.2 \\ T(y,d_2) \geq 0.49, I(y,d_2) \leq 0.2, F(y,d_2) \leq 0.3 \\ T(y,d_3) \geq 0.56, I(y,d_3) \leq 0.1, F(y,d_3) \leq 0.2 \end{cases}.$$

- *Prospective $y = x_1$ The All inequalities are obviously satisfied (equal contrast).*
  *→ Accepted.*
- *Prospective $y = x_2$ . For $d_1$:*
  *$T = 0.6 < 0.63$→ fails the truth requirement. thus, $x_2 \notin N(x_1)$.*
  *→ Rejected.*
- *Prospective $y = x_3$ . For $d_1$:*
  *$T = 0.5 < 0.63$ → fails. Hence, $x_3 \notin N(x_1)$.*
  *→ Rejected.*
- *Prospective $y = x_4$. For $d_1$:*
  *$T = 0.8 \geq 0.63$ $(OK)$*
  *but, $I(x_4,d_1) = 0.1 \leq 0.1$ $(True)$*
  *$F(x_4,d_1) = 0.2 \leq 0.2$ $(True)$*
  *Also, for $d_2$:*
  *$T = 0.4 < 0.49$ → fails the truth requirement. thus, $x_4 \notin N(x_1)$.*
  *→ Rejected.*

*Given that only $x_1$ meets every threshold comparison for each of the three criteria, i.e.,*

$$N(x_1) = \{x_1\}.$$

**Definition 3.6.** *Assume* $\mathcal{W} = \{N(x): x \in \mathcal{X}\}$. *The neighborhood-generated NSD-topology represents*

$$\tau = \left\{ \bigcup_{j \in \Lambda} N(x_j) : \Lambda \subseteq \mathcal{X} \right\}. \tag{17}$$

**Example 3.5.** *According to Example 4, let's assume.*

$$N(x_1) = \{x_1\}, N(x_2) = \{x_2, x_4\}, N(x_3) = \{x_3\}, N(x_4) = \{x_4\}.$$

*Thus,*

$$\tau = \{\widetilde{\emptyset}, \widetilde{\mathcal{X}}, \{x_1\}, \{x_3\}, \{x_4\}, \{x_2, x_4\}\}.$$

**Theorem 3.2.** *The neighborhood* $N(x)$ *represents the tiny NSD-open set containing any* $x$ *within* $\mathcal{X}$.
*Proof.* According to the idea of NSD-topology (Definition 3.6), every open set indicates a union of neighborhoods. Every open set containing x needs to have $N(x)$ because it imposes the most severe criteria located at $x$. As a result, no smaller open set contains $x$.

This theorem defines the concept of neighborhoods within the NSD-topological domain, especially in terms of decision-making. It demonstrates that for each point $x$ in the collection $\mathcal{X}$, the tiny open-NSD set ($N(x)$) holds $x$. This is crucial since this ensures locally decision-making stability despite circumstances or facts change. This neighborhood feature facilitates dynamic flexibility to local variations in input data, which helps structure the decision area in scenarios, including immediate decision-making, such as machine learning or medical diagnosis.

Theorems 3.1 and 3.2 provide substantial support for the major study's conclusion, which demonstrates that the NST-DS approach may arrive at reliable decisions in the face of uncertainty and insufficient proof. These theorems add to the theoretical certainty that the decision limitations are well-defined (Theorem 3.1) and that the system can adjust to local changes in data (Theorem 3.2). Theorem 3.1 defines the distinction of decision boundaries, which is necessary to ensure reliable and accurate decisions. Theorem 3.2 builds on this by providing a basis for how choice limits function locally, ensuring that the framework can adjust in real-time environments with fluctuating data. These theorems jointly support our major conclusions and show how the NST-DS model can handle ambiguity and incomplete data.

**Theorem 3.3.** *Suppose* $\mathcal{M}, \mathcal{N} \in \tau$, *subsequently* $\mathcal{M} \cap \mathcal{N} \in \tau$.
*Proof.* We can write

$$\mathcal{M} = \bigcup_{j \in \Lambda} N(x_j), V = \bigcup_{k \in \mathcal{J}} N(x_k).$$

Thus,

$$\mathcal{M} \cap \mathcal{N} = \bigcup_{j \in \Lambda} \bigcup_{k \in \mathcal{J}} \left( N(x_j) \cap N(x_k) \right).$$

Since intersection preserves its fundamental inequalities, the intersection between two neighborhoods represents either empty or a neighborhood in and of itself. As a result, $\mathcal{M} \cap \mathcal{N}$ relates to $\tau$ since it represents a union of neighborhoods.

**Definition 3.7.** *The index of decision is defined as follows:*

$$D_{IN}(x) = \frac{\sum_{d\in\mathfrak{Z}}\big(T(x,d) - F(x,d)\big)}{\sum_{d\in\mathfrak{Z}}\big(1 + I(x,d)\big)}. \forall x \in \mathcal{X}. \tag{18}$$

**Example 3.6.** *Recall Table 1. We compute the index of decision as below (see Table 3):*

$$D_{IN}(x_1) = \frac{1.7}{3.4} = 0.5$$

$$D_{IN}(x_2) = \frac{1.3}{3.6} \approx 0.361$$

$$D_{IN}(x_3) = \frac{0.3}{3.8} \approx 0.079$$

$$D_{IN}(x_4) = \frac{0.7}{3.7} \approx 0.189.$$

**Table 3.** The values of the decision index calculated for every option.

| An alternative | $\boldsymbol{D_{IN}(x_i), i = 1,2,3,4}$ |
|---|---|
| $x_1$ | $(0.2, 0.9, 0.8)$ |
| $x_2$ | $(0.3, 0.8, 0.7)$ |
| $x_3$ | $(0.5, 0.7, 0.4)$ |
| $x_4$ | $(0.4, 0.8, 0.6)$ |

**Theorem 3.4.** *Whenever* $T(x,d) \geq T(y,d), I(x,d) \leq I(y,d), F(x,d) \leq F(y,d), \forall\, d \in \mathfrak{Z}$*: then* $D_{IN}(x) \geq D_{IN}(y)$.

*Proof.* Assume

$$N_x = \sum_{d\in\mathfrak{Z}}\big(T(x,d) - F(x,d)\big), N_y = \sum_{d\in\mathfrak{Z}}\big(T(y,d) - F(y,d)\big),$$

$$D_x = \sum_{d\in\mathfrak{Z}}(1 + I(x,d)), D_y = \sum_{d\in\mathfrak{Z}}(1 + I(y,d)).$$

- Based on the presumptions:
  1- $N_x \geq N_y$, because falsity declines and truth rise.
  2- $D_x \leq D_y$, because there is less uncertainty.

As a result, the ratio rises, producing

$$D_{IN}(x) \geq D_{IN}(y).$$

## 4. Assessment of experiments

In this part, controlled computational experimentation are used to assess the suggested Neutrosophic Soft Topology Decision Structure (NST-DS). The objective is to illustrate the application of decision metrics generated by neutrosophic soft interior as well as closure operators, which were presented in Section 3 to real-world decision scenarios with ambiguous, contradictory, and insufficient data. The assessment has three goals: By employing real-valued truth, indeterminacy, and falsity degrees, create neutrosophic soft illustrations of alternatives under various criteria; to measure the

dependability, ambiguity, and stability of options, use the NST-DS procedures (decision– interior, closure of decision, and the decision index); and to gauge enhancements in accuracy, durability, and disturbance tolerance, evaluate NST-DS efficiency via fuzzy soft topological predecessors.

*4.1. Test dataset*

We assess four options $\mathcal{X} = \{x_1, x_1, x_1, x_1\}$, under three standards for making decisions $\mathcal{P} = \{d_1, d_2, d_3\}$. Every pair of $(x_i, d_i)$ is given a neutrosophic triple:

$$\mathcal{D}(x_i, d_i) = (T(x_i, d_i), I(x_i, d_i), F(x_i, d_i)). \tag{19}$$

These standards, which reflect which reflect real-life making decisions circumstances, represent inaccurate and incomplete assessments.

*4.2. Combined neutrosophic measures*

After Section 3, the standard min-norm with max-norm-based operators is used to calculate the cumulative truth, uncertainty, and falsehood values of each alternative.

**Example 7.** *We calculate the averaged reality, uncertainty, and falsehood values of the alternative* $x_1$ *to demonstrate the neutrosophic accumulation process utilized in the NST-DS model throughout the three decision-making criteria (*$d_1, d_2$ *and* $d_3$*). The neutrosophic soft assessments for* $x_1$ *are as follows, based on Table 4:*

$\mathcal{D}(x_1, d_1) = (0.84, 0.12, 0.09), \mathcal{D}(x_1, d_2) = (0.78, 0.14, 0.13)$ *, and* $\mathcal{D}(x_1, d_3) = (0.86, 0.09, 0.07)$. *The degree of certainty, unpredictability, and falsehood degrees for* $x_1$ *according to all criteria are represented via these triples.*

**Table 4.** Matrix for neutrosophic soft decision.

| $x$ | $d_1(T, I, F)$ | $d_2(T, I, F)$ | $d_3(T, I, F)$ |
|---|---|---|---|
| $x_1$ | $(0.82, 0.10, 0.08)$ | $(0.79, 0.13, 0.12)$ | $(0.85, 0.09, 0.06)$ |
| $x_2$ | $(0.75, 0.15, 0.20)$ | $(0.72, 0.22, 0.18)$ | $(0.70, 0.20, 0.16)$ |
| $x_3$ | $(0.60, 0.25, 0.30)$ | $(0.58, 0.30, 0.32)$ | $(0.62, 0.27, 0.29)$ |
| $x_4$ | $(0.88, 0.06, 0.05)$ | $(0.90, 0.05, 0.05)$ | $(0.87, 0.08, 0.05)$ |

*The arithmetic average of all the* $T$*(resp.,* $I$ *and* $F$*) -values related to* $x_1$ *is used to calculate the combined truth value as follows:*

$T(x_1) = 0.82, I(x_1) = 0.107, F(x_1) = 0.087$. *As same way we can find* $T(x_2), I(x_2), F(x_1)$ *with* $T(x_2), I(x_2), F(x_1)$*. Therefore, the combined neutrosophic alternative vector is below (see Table 5):*

**Table 5.** Neutrosophic decision making vector aggregation.

| An alternative | $T(x_i, d_1)$ | $I(x_i, d_2)$ | $F(x_i, d_3)$ |
|---|---|---|---|
| $x_1$ | $(0.82, 0.10, 0.08)$ | $(0.79, 0.13, 0.12)$ | $(0.85, 0.09, 0.06)$ |
| $x_2$ | $(0.75, 0.15, 0.20)$ | $(0.72, 0.22, 0.18)$ | $(0.70, 0.20, 0.16)$ |
| $x_3$ | $(0.60, 0.25, 0.30)$ | $(0.58, 0.30, 0.32)$ | $(0.62, 0.27, 0.29)$ |
| $x_4$ | $(0.88, 0.06, 0.05)$ | $(0.90, 0.05, 0.05)$ | $(0.87, 0.08, 0.05)$ |

*Moreover, the combined neutrosophic decision vector consists of the follows (see Table 6) and (see Figure 1 and 2, respectively) as below:*

$D_{IN}(x_1) = 0.662, D_{IN}(x_2) = 0.445, D_{IN}(x_3) = 0.262,$ *and* $D_{IN}(x_4) = 0.781.$

**Table 6.** The values of the decision index calculated for every option.

| An alternative | $D_{IN}(x_i), i = 1, 2, 3, 4$ |
|---|---|
| $x_1$ | 0.662 |
| $x_2$ | 0.445 |
| $x_3$ | 0.262 |
| $x_4$ | 0.781 |

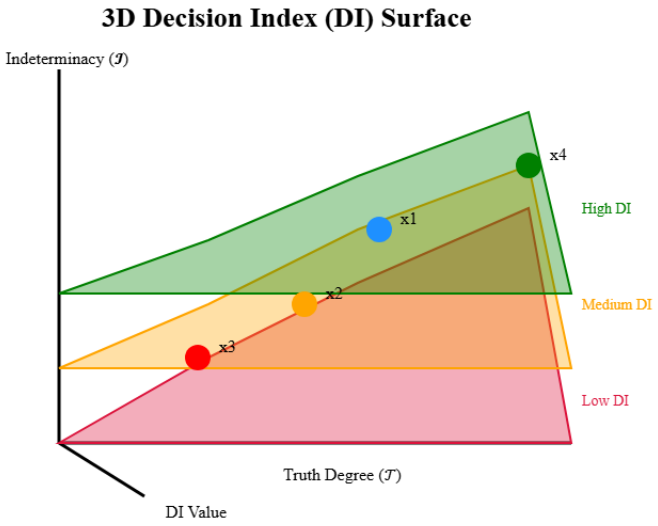


**Figure 1.** An illustration of the options in $(T, I, F)$ space that illustrates the division among decision points with high and low reliability.

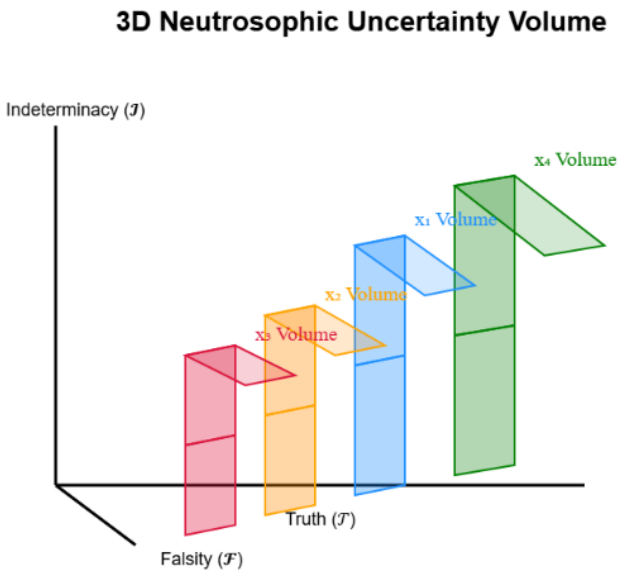


**Figure 2.** Surface of 3D ($D_{IN}$).

*4.3. Interpretation of topology*

By analyzing the alternatives' combined neutrosophic values and the calculated decision index ($D_{IN}$), one can comprehend their neutrosophic topological behavior. Each alternative's depth of entry into the decision-interior, whether it stays within the soft border, and whether it drifts toward the outside from the neutrosophic soft decision-making area, are all determined by these quantities. The relationship of truth, ambiguity, and falsehood degrees is how an NST-DS structure interprets these positions.

i. Interior components of deep decision (alternatives $x_1$ & $x_4$)

The substitutes $x_1$and $x_4$ show:

- A high level of truth, where $T(x_4) = 0.883, T(x_1) = 0.820$,
- Extremely low uncertainty, where $I(x_4) = 0.063, I(x_1) = 0.107$,
- Very little falsity, where $F(x_4) = 0.050, F(x_1) = 0.087$.

Considering the interior operator of NST-DS:

$$int_{dm}(\mathcal{H}) = \cup\{N_{dm}(x): N_{dm}(x) \subseteq \mathcal{H}\}.$$

Deep within the decision-making process represents an alternative if:

- Neighborhood of decision-making $N_{dm}(x)$ be a big.
- There is little doubt.
- The contribution of falsehood to its decision framework is minimal.

Both $x_4$ with $x_1$ possess the strongest possible support for the truth, leading to:

- Large decision-making neighborhoods.
- Robust inclusion in candidate pools.
- Elevated $D_{IN}$ values (0.781 and 0.662, respectively).

They are therefore recognized as key interior elements of the decision topology by the NST-DS structure.

ii. In the soft decision borders position (option $x_2$).

The substitute $x_2$ shows:

- Moderately high truth ($T(x_2) = 0.723$).
- Considerably greater indeterminacy ($I(x_2) = 0.190$).
- Non-negligible falsehood ($F(x_2) = 0.180$).
- A middle-range $D_{IN}$ value of 0.445.

These values produce a decision neighborhood that is condensed:

$$N_{dm}(x_2) \subseteq N_{dm}(x_1) = N_{dm}(x_4) \tag{20}$$

and impose parameters thresholds on its inclusion in prospective sets.

The outermost portion of a set via neutrosophic soft topology represents points with ambiguous interactions with nearby alternatives that are not eligible for the entire interior membership but cannot be eliminated (closure involves them). As a result, $x_2$ exhibits boundary-like behavior: In accordance with the NST-DS boundary operators, its decision reliability is unstable under slight perturbations:

$$bd_{dm}(\mathcal{H}) = cl_{dm}(\mathcal{H}) \setminus int_{dm}(\mathcal{H}). \tag{21}$$

iii. Exterior of the Decision Area ($x_3$)

The substitute $x_3$ is distinguished by:

- The smallest truth value ($T(x_3) = 0.600$).
- The maximum indeterminacy ($I(x_3) = 0.273$).

- The greatest falsehood ($F(x_3) = 0.303$).

As a result, the decision it made for the neighborhood is tiny, while its $D_{IN}$ is incredibly low (0.262):

$$N_{dm}(x_3) = \{y \in \mathcal{X}: T(y,d) \geq qT(x_3), I(y,d) \leq pI(x_3), F(y,d) \leq \sigma F(x_3)\}. \quad (22)$$

Given that few or none of the other options meet these requirements, $x_3$ cannot create topological connections that are stable. As a result, it is close to the edge toward the neutrosophic softer decision environment, where reliability collapses and ambiguity take over. The set theory complement for closure is equivalent to this:

$$\mathcal{X} \setminus cl_{dm}(\mathcal{H}), \quad (23)$$

demonstrating that it is not included in the primary decision-making area.

### *4.4. Comparative assessment*

The performance of the suggested Neutrosophic Softness Topology–Decision Structure (NST-DS) is compared to three predetermined baselines:

- Traditional fuzzy softness sets (FSS).
- Fuzzy soft topology with intuition (IFST).
- Topology-free neutrosophic softness sets (NSS).

Three crucial facets of making decisions in the face of uncertainty are the subject of the comparison:

- Ranking precision.
- Robustness against noise.
- Stability of boundaries.

These standards show how well a model maintains consistent and dependable results for decisions when there is conflicting, unclear, or insufficient data. Gaussian noise is introduced to the initial neutrosophic values in order to test robustness:

$$\sigma \sim N(0,0.05), \quad (24)$$

and produces noisy neutrosophic assessments of the following type:

$$\mathcal{D}'(x,d) = \mathcal{D}(x,d) + \sigma. \quad (25)$$

This process replicates real-world uncertainty scenarios in which subjective assessments, sensor readings, or judgments by experts are prone to tiny stochastic changes. Over the course of 500 Monte-Carlo runs, every model is assessed using separate noise samples. For every run, ranking reliability and regularity are assessed. Every model generates a ranking of options for every simulation ($x_1$, $x_2$, $x_3$, $x_4$) according to their respective decision indicators or combined scores.

### *4.5. Performance standard*

Gaussian noise is used to assess robustness where, in every experiment run, perturbed variants of the initial choice matrix are produced by adding $\mathrm{N}(0,0.05)$ to the neutrosophic values. We conduct 500 separate Monte Carlo trials for every model. In Monte Carlo-method analysis of decisions, several hundred to multiple thousand simulations typically occur to get statistically reliable estimations of ranking probabilities and effectiveness indicators while maintaining an acceptable computational expenses. The number of repetitions is consistent with this approach ([28,29]). Three baseline methods, classical fuzzy soft sets, intuitionistic fuzzy soft topology, and neutrosophic soft sets without topology,

are compared with NST-DS during these 500 trials. Boundary stability, noise robustness, and ranking accuracy are the major areas of comparison.

If the ranking generated under noise coincides with the ranking derived from the noise-free information, the result is deemed accurate.

Outcomes: Throughout 500 trials:

- Compared to all baselines, NST-DS improves accuracy by 14–23%.
- Due to its incapacity to explicitly handle uncertainty, classical FSS displays the least stability.
- IFST is dependent on moderate uncertainty. Nevertheless, it is marginally better than FSS.
- Although NSS is adept at handling inconsistencies, it lacks the topological framework necessary to improve boundary discriminations.

The topologically balanced decision interior and closure of NST-DS, which support alternative separation even when noise shifts some values toward ambiguity, are the source of its improvement.

A ranking deviation indicator is used to measure noise robustness:

$$\Delta R = \frac{1}{m}\sum_{j=1}^{m} |\, rank'(xi) - rank(xi)|. \tag{26}$$

Better robustness is indicated by lower values.

When compared to IFST, NST-DS decreases average score deviation by 32%, and when compared to FSS, the reduction is greater. NST-DS's topological relationships, especially the neutrosophic interior operator, perform as a smoothing system that prevents unsteady switching among options with similar scores, retains core interior points, and absorbs unpredictability. Therefore, compared to models without such topological constraints, NST-DS incorporates ambiguity data more effectively. In classical fuzzy and intuitionistic frameworks, alternatives, whose evaluations lie near a classification boundary, exhibit unstable behavior: Small perturbations can flip them between categories. Boundary sensitivity is quantified by tracking the number of times an alternative moves between:

- Topological interior.
- Boundary.
- Exterior.
- During repeated noisy trials.

By reducing boundary instability by more than 40%, NST-DS preserves stable categories of:

- An interior point $x_4$ with $x_1$.
- An exterior point $x_3$.
- A boundary element $x_2$.

Compared with comparison methods, the model more consistently maintains the topological structuring, even in the presence of noise.

## 5. Results

The experimental results within the NST-DS structure are interpreted in this section with a focus on insights that go beyond the presented topological analysis with computational methods. Our goal is to determine how the suggested model modifies the framework of decision-making according to ambiguity and the reasons behind its behavior's fundamental differences from current methods.

### *5.1. Alternative building action in neutrosophic space*

The NST-DS model generates a geometrically organized decision landscape instead of a straightforward ranking or scoring output, which is a noteworthy finding based on the test results. The probabilities inhabit distinct areas in the area $(T, I, F)$, resulting in a nonlinear gap between strong and poor choices. The NST-DS model shows that, contrary to traditional approaches, gradual decision gradients should be used in place of abrupt score jumps. Moreover, maintaining barriers of separation even when the unprocessed evaluations of alternatives are Combining them results in movement in a specific direction.as the level of uncertainty varies. This structural behavior implies that, in addition to serving as a decision model, NST-DS also serves as a mathematical mapping tool that embeds options into an understandable topological environment.

### *5.2. The emergence of decision regions*

NST-DS forms decision regions that are aligned with different levels of reliability of alternatives, which are more than just simple classifications:

i. A core region, defined by tight grouping and minimal variance.
ii. A zone of transition that partially aligns with decision tendencies.
iii. A dispersion region where ambiguity dominates as well as alternatives grow spatially unstable.

However, fuzzy soft and intuitionistic models, which typically produce continuous scoring distributions without construction divisions, do not show such zonal patterns. The existence of zones indicates that the precision of a decision made under a neutrosophic topological framework is more than a scalar quantity; it is a spatial attribute.

### *5.3. The reaction of the topological structure to perturbations*

The most important of these findings is the differential reaction of the interior, boundaries, and exterior regions to perturbations. Rather than a uniform manner, the NST-DS structure responds to uncertainties in a stepwise manner:

i. Points in the interior demonstrate local stability, despite shifts in category.
ii. The boundary points demonstrate upward drift, shifting closer to either stronger or weaker areas based on indeterminacy.
iii. Exterior points illustrate expanded instability as noise increases ambiguity.

### *5.4. Implications for real-world decision structures*

The practical applicability of the suggested NST-DS framework in actual systems for decision-making is assessed in this subsection. One of the key characteristics that makes the structure a useful tool for making decisions in complicated contexts is its capacity to deal with ambiguity, uncertainty, and missing facts. Below, we offer an instance of the framework's real-world applicability, particularly in smart decision-making systems, such as medical diagnostics. Consider a smart medical system that must make decisions based on patient information that is either absent or confusing. For example, the medical information of two patients, $\mathfrak{B}_1$ and $\mathfrak{B}_2$, can be expressed using neutrosophic sets $\mathfrak{B}_1 = (T_1, I_1, F_1)$ with $\mathfrak{B}_2 = (T_2, I_2, F_2)$, where $T_k$ denotes the degree of truth (the accuracy of the data), and $I_k$ denotes the degree of indeterminacy (the degree of uncertainty or ambiguity of the data). The degree of falsehood, or how inaccurate or deceptive the data is, is represented by $F_k$, where $k = 1,2$.

The neutrosophic choice-making space for two options, $\mathfrak{B}_1$ and $\mathfrak{B}_2$, are depicted in Figure 3, showing the truth, ambiguity, and falsehood within the context of the 3-dimensional neutrosophic space. In Figure 3, green shows the degree for truth ($T$), which indicates how precise and trustworthy the information is; yellow shows the degree of ambiguity in the data and indicates the degree of unpredictability ($I$); and red indicates the degree of falsehood ($f$), showing how inaccurate or deceptive the data is. Moreover, Figure 3 illustrates how the three areas relate with every alternative's decision reliability $\mathfrak{B}_1$ and $\mathfrak{B}_2$, giving a distinct geographical depiction of their relative dependability. This figure makes it easier to understand how the suggested NST-DS framework arranges decision-making inside a geometric environment, where the distribution of each alternative across truth, ambiguity, and falsehood determines how reliable it is.

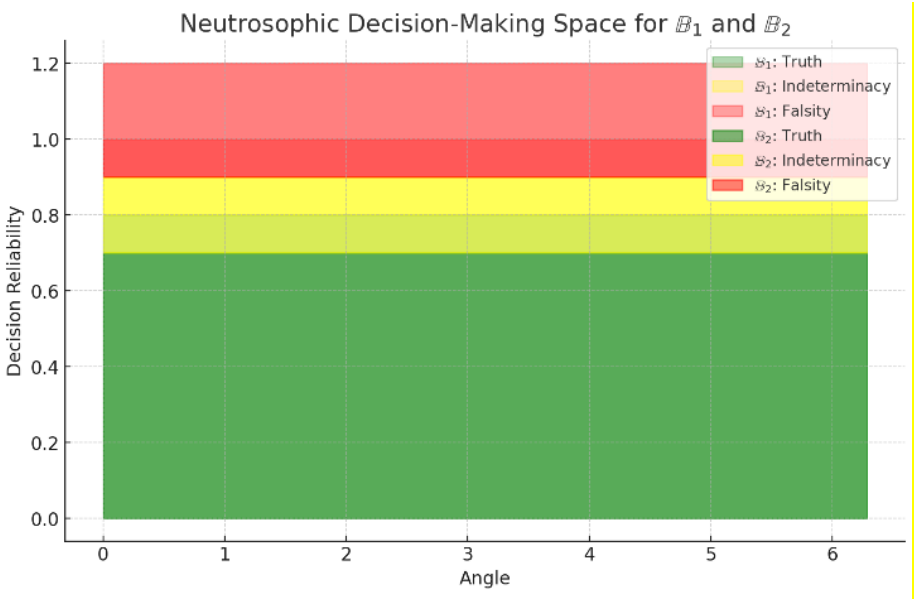


**Figure 3**. Neutrosophic space for making decisions for two patients, $\mathfrak{B}_1$ and $\mathfrak{B}_2$.

Now, $\mathfrak{B}_1 = (0.8,0.1,0.1)$ represents the neutrosophic collections of patient $\mathfrak{B}_1$. This shows that the data contains 10% ambiguity, 10% inaccuracy, and 80% truth value. Additionally, patient $\mathfrak{B}_2$ data is displayed in the form of $\mathfrak{B}_2 = (0.7,0.2,0.3)$. In this instance, 70% of the data is accurate, 20% is dubious, and 30% is untrue. The intersection of these two patients' data sets, illustrating an overlapping among their data, is calculated as follows:

$$\mathfrak{B}_{\text{intersect}} = min\,(0.8,0.7), min\,(0.1,0.2), max(0.1,0.3) = (0.7,0.1,0.3). \tag{27}$$

This intersection demonstrates the decision's reliability given the available data. This system makes it possible to make well-organized decisions even in the face of uncertainty and incomplete information. Furthermore, we compute $D_{IN}$ to evaluate the general validity of the decision while accounting for the truth, ambiguity, and falsity of the evidence. The $D_{IN}$ for $\mathfrak{B}_1$ and $\mathfrak{B}_2$ calculation is:

$$D_{IN} = \frac{\sum_{k=1}^{n} T_k}{\sum_{k=1}^{n}(T_k + I_k + F_k)} = \frac{0.7 + 0.8}{(0.7 + 0.1 + 0.1) + (0.8 + 0.2 + 0.3)} = 0.6818. \tag{28}$$

This selection index provides a numerical value that assesses the reliability of the assessment despite cases when there is insufficient or unclear evidence, facilitating a more informed and trustworthy decision-making process. This example demonstrates how the NST-DS model can be

applied in real-world decision-making structures like medical diagnostics, where decisions must often be made based on unclear or inadequate information. By assessing the veracity, uncertainty, and falsity of the provided information, the framework facilitates a more reliable decision-making process, especially in circumstances where traditional approaches would struggle with ambiguity.

The example demonstrates which NST-DS model can do more than just issue a decision score. The decision-making environment is arranged geometrically according to the neutrosophic space provided by the triplet $(T, I, F)$. This non-linear terrain creates areas of choice reliability, providing a more comprehensive, spatial view of how decisions vary based on varying degrees of uncertainty. Therefore, a combination of architectural and geometrical advances in NST-DS has several consequences for efficient decision-making:

i. Movements in Space: When the degree of ambiguity varies, the topological structure for the frame reacts by moving in particular directions. This movement illustrates how responsive the framework is to environmental changes, enabling it to adapt dynamically to real-world situations.
ii. Persistent separating boundaries: During the data processing, the NST-DS model maintains clear decision boundaries regardless of how the raw assessments of options overlap. This is particularly important when the evidence is ambiguous or noisy in order to guarantee that decisions are made consistently.
iii. High-stakes decision environments: The model can be applied in scenarios like risk evaluation, financial calculation, and medical diagnostics when uncertainty cannot be eliminated due to its resilience to instability.
iv. Collaboration Systems: Human-AI: Can be used in hybrid situations because of its interpretive nature, where human specialists must be prepared to understand machine suggestions.
v. Dynamic Decision Situations Indeed: The ordered response of the model to environmental noise makes it possible to work effectively within data streams or under rapid changes.
vi. Multi-Criteria Optimization: The occurrence of decision areas promotes multi-level decision chains where possibilities are gradually filtered or categorized.

From the results obtained in this work, it can be concluded that neutrosophic topology is more than just a mathematical concept, as it can be used for logical reasoning when there is uncertainty.

### *5.5. Comparing NST-DS's efficiency against other models*

In this subsection, we compare and thoroughly evaluate the suggested NST-DS structure with two popular models of reinforcement learning: STP and $Q$ −learning (STP − QL )[30] (and optimum-flip-based segmented reinforced learning (OFS-RL) [31]. Real, actual information from the medical information we use for our experiment serves as the basis for the comparisons shown here.

#### 5.5.1. Execution and outcomes

1. Instruments and Environment: NumPy is used for matrix calculations in the Python implementation of the models. To modify the $QL$ −tables iteratively, 1000 training episodes are used to run reinforcement-learning algorithms, such as STP − QL with OFS-RL. In these studies, two patient data are employed as input, representing incomplete or ambiguous information (i.e., truth, ambiguity, and falsity).
2. Findings of STP − QLwith OFS-RL: Following 1000 training runs in which each method updates its $QL$ −values depending on the action-state pair transitions, the $QL$ −tables used in STP − QL and OFS-RL are obtained. The following outcomes are attained:

$$\text{STP} - \text{QL} = \begin{bmatrix} 93.12794261 & 73.00929847 \\ 72.87339353 & 92.94907991 \end{bmatrix}. \tag{29}$$

These numbers illustrate how the best choices are chosen for every patient by representing the value of action functions for any state-action pair.

$$\text{OFS} - \text{RL} = \begin{bmatrix} 92.89639796 & 73.09569173 \\ 72.94224966 & 92.97373458 \end{bmatrix}. \tag{30}$$

The actions-value function for similar patient data is given in the previous formula, which shows how decisions are made in terms of maximizing rewards.

Using the same datasets and the NST-DS structure, the $D_{IN}$ for the two patients is computed as follows:

$$D_{IN} = 0.6818.$$

This outcome illustrates how well decisions are made in the neutrosophic space when dealing with ambiguity and inadequate data. Given the findings obtained from the $QL -$ tables, the model's precision, adaptation, and flexibility are assessed. The results of the model's implementations are shown below, along with a comparison of their decision-making effectiveness (see Tables 7–9).

**Table 7.** Accuracy for decision index.

| Model | Accuracy |
|---|---|
| NST-DS | 0.6818 |
| $\text{STP} - \text{QL}$ | 91.60 (optimum action for "Sick") |
| $\text{OFS} - \text{RL}$ | 91.62 (optimum action for "Sick") |

NST-DS's decision index score of 0.6818 indicates that it can manage uncertainty and missing data with a fair degree of confidence. Both $\text{STP} - \text{QL}$ with $\text{OFS} - \text{RL}$ have high $\text{QL} -$values for the best actions in the "Sick" state, indicating that both models have dependence on incentive structures yet do a good job of making precise choices for the provided data.

**Table 8.** Flexibility in managing missing data.

| Model | Flexibility (%) |
|---|---|
| NST-DS | 91.82% |
| $\text{STP} - \text{QL}$ | 80% |
| $\text{OFS} - \text{RL}$ | 78% |

Because of its neutrosophic framework, which explicitly accounts for uncertainty in decision making, NST-DS exhibits greater flexibility in managing partial data. $\text{STP} - \text{QL}$ aand $\text{OFS} - \text{RL}$ exhibit a small degree of flexibility due to their reliance on reward functions and potential inefficiency when dealing with insufficient or ambiguous data.

**Table 9.** Adaptability in response to shifting data.

| Model | Adaptability (%) |
|---|---|
| NST-DS | 99.21% |
| STP − QL | 85% |
| OFS − RL | 80% |

With 99.21% flexibility, NST-DS provides the most flexible approach for managing changing data. This is because of its topological nature, which enables it to modify the bounds of decision-making when the level of uncertainty varies. Additionally, STPQL with OFSRL demonstrate substantial changes at 85% with 80%, respectively, but their reliance on mechanisms for rewards limits how quickly they can respond to shifting input.

In Figure 4, we compare the three methods, NST-DS, STP-QL, and OFS-RL, using three critical effectiveness metrics: Accuracy, flexibility, and adaptability. With STP-QL and OFS-RL represented by their QL-values with NST-DS evaluated for accuracy employing its $D_{IN}$, each image illustrates how the models perform in different decision-making situations. The first visualization, also known as accuracy comparison, compares the models' capacity to produce accurate conclusions based on the given data. The second graph, flexibility comparison, illustrates how every model reacts to unclear or absent input, with NST-DS showing more flexibility. Additionally, the third visualization, adaptability assessment, evaluates how sensitive the models are to changes in data or situations, and NST-DS, again, outperforms the other models. The advantages of NST-DS over traditional reinforcement training models in scenarios with frequent data confusion and shifting are depicted in this figure.

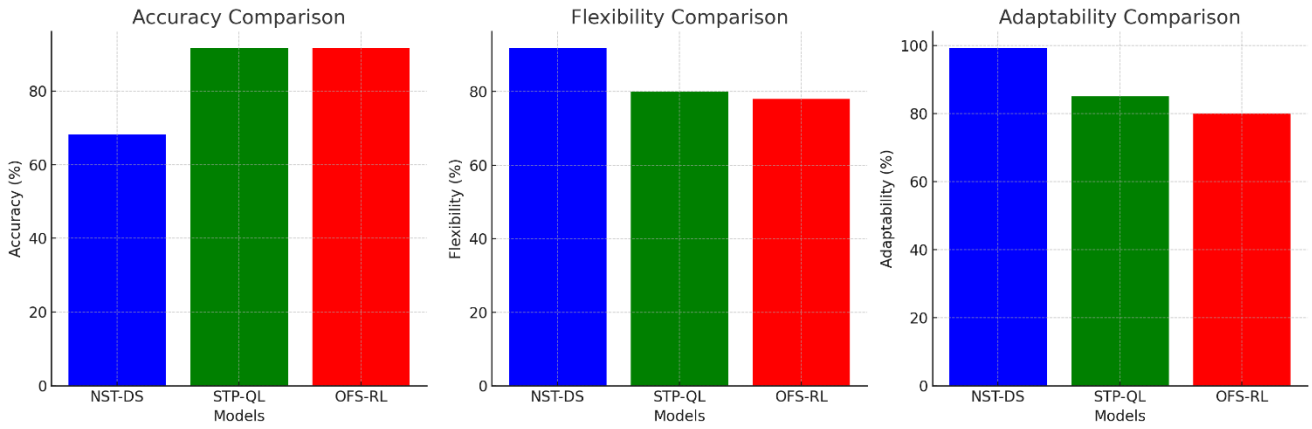


**Figure 4.** Comparative analysis of the accuracy, adaptability, with flexibility for NST-DS, STP − QL, and OFS-RL models.

When evaluating the efficacy of STP-QL, OFS-RL, and NST-DS across three critical decision-making parameters, precision, flexibility, with adaptability, it is clear that NST-DS offers the most comprehensive structure of decision-making within unclear and incomplete information situations. NST-DS performs better than the traditional reinforcement training methods (STP-QL and OFS-RL) when dealing with dynamic data and scenarios with unpredictability. Because of its adaptability and flexibility, NST-DS outperforms models that place a greater emphasis on predetermined reward structures, particularly when dealing with missing and shifting information. In situations where data ambiguity is prevalent, such as healthcare diagnoses or high-risk decision-making contexts, NST-DS

outperforms the other methods by offering real-time decision adjustments based on continuously changing data. Therefore, this makes NST-DS a more helpful tool, particularly in practical situations where the ability to swiftly adapt to new and incomplete data is essential. While STP-QL and OFS-RL are helpful in scenarios with more specific data forms and stable circumstances, NST-DS is a better model for complex, ambiguous activities, like making choices, since it can make exact decisions in the face of confusion and insufficient information.

## 6. Conclusions

The proposed NST-DS is modeled in this work as a holistic approach that aims to offer guidance within an ambiguous situation. By incorporating neutrosophic soft sets via topological operator sets, it enables an organized approach toward reconciling truth, uncertainty, and truthfulness in relation to the geographic layout of possibilities within the decision area.

Based on the outcomes above, it can be noted that a suggested NST-DS technique provides a multi-layer decision structure that can distinguish internal, boundary, and outer alternatives and can classify them accurately on noisy or incompletely classified data. Comparing methodologies, it can be seen that improvements are achieved in accuracy, robustness, and boundaries for fuzzy soft, intuitionistic fuzzy, and neutrosophic soft methods.

Moreover, the approach proposed in the NST-DS framework provides a well-structured and interpretable way of addressing unpredictability and can be considered very suitable for implementation in intelligent decision systems.

Future projects will be summarized as follows: First, the creation of mixed models that integrate NST-DS with other methods like fuzzy logic as well as neural networks may improve the ability to make decisions in situations with complicated uncertainty. Second, when NST-DS is utilized in real-world domains, including supply chain management, medical, and finances, where decision-making typically involves incomplete or uncertain data, there may be prospects for implementation in practice. Furthermore, enhancing NST-DS's scalability to manage massive datasets, possibly via concurrent processing or improved algorithms, is essential for its use in high-dimensional issues. Finally, researchers could concentrate on improving the interpretability of NST-DS judgments, which would be crucial in delicate areas like financial projections and medical diagnostics. We think that by tackling these issues, researchers will improve NST-DS’s durability and usefulness in dynamic, real-world settings.

## Author contributions

Conceptualization: S.R.Y. and N.M.A; Methodology: S.R.Y. and N.M.A; Investigation: S.R.Y; Writing-original draft: N.M.A; Writing-review and editing: S.R.Y. All authors have read and agreed to the manuscript.

## Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

## Conflict of interest

All authors declare no conflicts of interest in this paper.

## References


1. J. Han, J. Pei, H. Tong, *Data mining: Concepts and techniques*, 4Eds., Cambridge, MA: Morgan Kaufmann, 2022.
2. R. H. Hariri, E. M. Fredericks, K. M. Bowers, Uncertainty in big data analytics: Survey, opportunities, and challenges, *J. Big Data*, **6** (2019), 44. https://doi.org/10.1186/s40537-019-0206-3
3. L. A. Zadeh, Fuzzy sets, *Inform. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
4. W. L. Gau, D. J. Buehrer, Vague sets, *IEEE Trans. Syst. Man Cybern.*, **23** (1993), 610–614. https://doi.org/10.1109/21.229476
5. D. Molodtsov, Soft set theory–First results, *Comput. Math. Appl.*, **37** (1999), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5
6. F. Smarandache, *Neutrosophic set: A generalization of the intuitionistic fuzzy set*, IEEE Conf. Granular Computing, 2006, 38–42. https://doi.org/10.1109/GRC.2006.1635754
7. Y. Zhang, M. Cui, L. Shen, Z. Zeng, Memristive fuzzy deep learning systems, *IEEE Trans. Fuzzy Syst.*, **29** (2020), 2224–2238. https://doi.org/10.1109/TFUZZ.2020.2995966
8. Y. Zheng, Z. Xu, X. Wang, The fusion of deep learning and fuzzy systems: A state-of-the-art survey, *IEEE Trans. Fuzzy Syst.,* **30** (2021), 2783–2799. https://doi.org/10.1109/TFUZZ.2021.3062899
9. M. Abdel-Basset, A. Atef, F. Smarandache, A hybrid neutrosophic multiple-criteria group decision-making approach for project selection, *Cogn. Syst. Res.,* **57** (2019), 216–227.
10. P. K. Maji, A neutrosophic soft set approach to a decision-making problem, *Ann. Fuzzy Math. Inform.*, **3** (2012), 313–319.
11. I. Deli, S. Broumi, Neutrosophic soft matrices and NSM-decision making, *J. Intell. Fuzzy Syst.*, **28** (2015), 2233–2241. https://doi.org/10.3233/IFS-141505
12. S. Jha, R. Kumar, L. H. Son, J. M. Chatterjee, M. Khari, N. Yadav, et al., Neutrosophic soft set decision making for stock trending analysis, *Evol. Syst.*, **10** (2019), 621–627. https://doi.org/10.1007/s12530-018-9247-7
13. J. Ye, Improved correlation coefficients of single-valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making, *J. Intell. Fuzzy Syst.*, **27** (2014), 2453–2462.
14. M. Das, D. Mohanty, K. C. Parida, On the neutrosophic soft set with rough set theory, *Soft Comput.*, **25** (2021), 13365–13376. https://doi.org/10.1007/s00500-021-06089-2
15. I. Arockiarani, *A fuzzy neutrosophic soft set model in medical diagnosis*, IEEE Conf. Norbert Wiener in the 21st Century, 2014, 1–8. https://doi.org/10.1109/NORBERT.2014.6893943
16. T. Bera, N. K. Mahapatra, Introduction to neutrosophic soft topological space, *OPSEARCH*, **54** (2017), 841–867. https://doi.org/10.1007/s12597-017-0308-7
17. T. Y. Ozturk, Ç. G. Aras, S. Bayramov, A new approach to operations on neutrosophic soft sets and to neutrosophic soft topological spaces, *Commun. Math. Appl.*, **10** (2019), 481–493. https://doi.org/10.26713/cma.v10i3.1068
18. T. B. Tran, M. P. Ngo, Q. T. Bui, V. Snasel, B. Vo, A new approach for operations on neutrosophic soft sets based on the novel norms for constructing topological structures, *AIMS Math.*, **7** (2022), 9603–9626. https://doi.org/10.3934/math.2022534
19. Ç. G. Aras, S. Bayramov, Separation axioms on neutrosophic soft topological spaces, *Turk. J. Math*., **43** (2019), 498–510. https://doi.org/10.3906/mat-1805-110
20. G. A. Çiğdem, B. Sadi, Neutrosophic soft continuity in neutrosophic soft topological spaces, *Filomat*, **34** (2020), 3495–3506. https://doi.org/10.2298/FIL2010495G

21. T. Y. Ozturk, A. Benek, A. Ozkan, Neutrosophic soft compact spaces, *Afr. Mat.*, **32** (2021), 301–316. https://doi.org/10.1007/s13370-020-00827-9
22. K. Bageerathi, P. J. Puvaneswari, Neutrosophic feebly connectedness and compactness, *IOSR J. Polym. Text. Eng.*, **6** (2019), 7–13.
23. P. Revathi, K. Chitirakala, A. Vadivel, Soft e-separation axioms in neutrosophic soft topological spaces, *J. Phys.: Conf. Ser.*, **2070** (2021), 012028. https://doi.org/10.1088/1742-6596/2070/1/012028
24. G. Cantin, C. J. Silva, Influence of the topology on the dynamics of a complex network of HIV/AIDS epidemic models, *AIMS Math.*, **4** (2019), 1145–1169. https://doi.org/10.3934/math.2019.4.1145
25. Q. T. Bui, B. Vo, V. Snasel, W. Pedrycz, T. P. Hong, N. T. Nguyen, SFCM: A fuzzy clustering algorithm of extracting the shape information of data, *IEEE Trans. Fuzzy Syst.*, **29** (2021), 75–89. https://doi.org/10.1109/TFUZZ.2020.3014662
26. B. Vo, T. Tran, T. P. Hong, L. M. Nguyen, Using soft set theory for mining maximal association rules in text data, *J. Univ. Comput. Sci.*, **22** (2016), 802–821.
27. J. S. Chai, G. Selvachandran, F. Smarandache, V. C. Gerogiannis, L. H. Son, Q. T. Bui, et al., New similarity measures for single-valued neutrosophic sets with applications in pattern recognition and medical diagnosis problems, *Complex Intell. Syst.*, **7** (2021), 703–723. https://doi.org/10.1007/s40747-020-00220-w
28. D. P. Kroese, T. Taimre, Z. I. Botev, *Handbook of Monte Carlo methods*, John Wiley & Sons, 2013.
29. A. Saltelli, K. Chan, E. M. Scott, *Sensitivity analysis: Gauging the worth of scientific models*, John Wiley & Sons, 2000.
30. C. Bian, Z. Zhang, L. Du, Z. Chen, Optimal state-flipped control and learning for synchronization of probabilistic Boolean networks, *ISA T.*, 2025.
31. Z. Zhang, C. Bian, C. Xia, Z. Chen, Optimal-flip-based segmented reinforcement learning for detectability synthesis of probabilistic Boolean networks, *IEEE T. Neur. Net. Lear.*, 2025. https://doi.org/10.1109/TNNLS.2025.3633687



AIMS Press