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Research article

Noise-induced modulation of solitary waves in the nonlinear Maccari system

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Abstract: This work investigates the stochastic dynamics of the nonlinear Maccari system by incorporating Brownian motion into its evolution equations, modeling random fluctuations relevant to physical settings such as optical fibers, plasma waves, and fluid interfaces, where noise significantly influences wave propagation. To obtain solitary wave solutions under stochastic effects, we use the refined Riccati–Bernoulli subsidiary ordinary differential equation (RB sub-ODE) method to reduce the stochastic partial differential equations (PDEs) to a manageable system of ordinary differential equations. This methodology enables the construction of a diverse family of stochastic wave solutions, including rational profiles, periodic structures, and solitary waves with noise-dependent modulation. The obtained solutions reveal significant influences of stochastic forcing on amplitude, phase, stability, and interaction dynamics. The results demonstrate that even weak noise can induce substantial modulation, phase drift, and deformation of coherent structures in the Maccari system. To illustrate the propagation of solitary waves for the stochastic nonlinear Maccari system, 2D charts of selected solutions are created using Matlab. Overall, this work offers a robust framework for analyzing noise-driven wave phenomena and highlights the effectiveness of the RB sub-ODE technique in deriving exact stochastic solutions for complex integrable models.

Keywords: Brownian motion process; noise-driven modulation; stochastic Maccari model; solitary wave solutions; rational and periodic waves
Mathematics Subject Classification: 35C07, 35Q51, 60H15, 35R60, 37K40

1. Introduction

Nonlinear partial differential equations (NPDEs) play a crucial role in modern mathematics and physics, providing a framework for understanding complex natural processes [1–3]. They are crucial for comprehending realistic systems ranging from fluid flow and optical pulse propagation to plasma dynamics and biological processes because their nonlinear nature produces rich dynamics such as solitons, turbulence, pattern generation, shocks, and multiscale interactions. The difficulty of solving

NPDEs has sparked the creation of sophisticated mathematical methods, including symmetry analysis, inverse scattering, perturbation approaches, and current computational solvers, which provide detailed insights into the qualitative and quantitative behavior of nonlinear waves and fields. As a result, nonlinear partial differential equations (NPDEs) continue to be a fundamental component of modern mathematical modeling and theoretical physics [4–6].

The capacity of solitary wave solutions to retain shape, speed, and localized structure over extended distances and during interactions makes them an exceptional class of nonlinear wave phenomena [7]. Solitary waves, which naturally arise in many nonlinear partial differential equations, capture the equilibrium between dispersion and nonlinearity, enabling the emergence of stable, particle-like waveforms in a variety of physical systems, including optical fibers, shallow water waves, and plasmas. They are important mathematical objects in the study of nonlinear dynamics because of their coherence and robustness, and they frequently appear as soliton-like structures in more generic settings or as solitons in integrable models. Solitary waves' capacity to withstand external pushing, collisions, and perturbations demonstrates both their inherent stability and the profound symmetries present in the governing equations. As a result, solitary wave solutions are essential building blocks for comprehending intricate wave interactions in nature and technology, as well as offering deep insight into the qualitative behavior of nonlinear systems [8, 9].

Stochastic partial differential equations (SPDEs) are an effective mathematical framework for describing systems in which deterministic dynamics interact with intrinsic randomness, including the effects of noise, uncertainty, and changing environments [10]. SPDEs, which incorporate stochastic processes such as the Brownian motion process, provide a more realistic explanation of complicated physical, biological, and technical phenomena, including turbulent fluid flows, random wave propagation, climate dynamics, and population dispersal [11]. Their solutions frequently display complex features such as stochastic modulation, unpredictable attractors, noise-induced transitions, and improved or suppressed stability, demonstrating how uncertainty fundamentally influences system evolution. SPDEs are much more difficult to analyze mathematically than deterministic PDEs, necessitating sophisticated methods from probability theory, functional analysis, and numerical stochastic calculus [12, 13]. As a result, SPDEs have a fundamental place in modern applied mathematics, giving crucial insight into the interaction of nonlinearity, unpredictability, and spatial-temporal dynamics.

The Maccari system represents a significant integrable model that encapsulates the complex interactions between dispersive wave elements and gradually changing mean fields within nonlinear media [14–16]. Initially developed as an evolution system, it integrates short-wave and long-wave dynamics by achieving a harmonious equilibrium of dispersion, nonlinearity, and intercomponent interaction. This system acts as a flexible framework for the examination of multiscale wave phenomena, especially in the fields of nonlinear optics, fluid dynamics, and plasma physics, where the energy transfer between rapid oscillatory modes and foundational background fields is of paramount importance. Its integrability allows for the use of sophisticated analytical methods including the inverse scattering transform, Hirota bilinear approaches, and Darboux transformations, which aid in the production of accurate solutions such as solitons, breathers, and interaction states. As a result, the Maccari system is an important mathematical structure for comprehending complex coupled-wave phenomena and furthering modern nonlinear research. Recent research has focused on developing precise solutions, such as multi-solitons, breathers, explosive and rogue wave solutions [17, 18].

A sophisticated mathematical framework that combines integrable multicomponent wave dynamics with stochastic perturbations to describe the effects of noise and uncertainty in complicated physical media is the stochastic nonlinear Maccari system [19]. The two interacting short-wave components and the corresponding long-wave field in this extended model are affected by random fluctuations brought about by stochastic processes such as the Brownian motion process or white-noise forcing in addition to nonlinear dispersion and cross-component coupling. This stochastic extension is crucial for simulating realistic settings where noise significantly impacts coherence, resonance interactions, and the stability of the wave structures, such as optical fiber systems with amplifier noise, plasma waves in turbulent regimes, and fluid interfaces subject to random external disturbances. Rich dynamical phenomena that are not possible in a fully deterministic system are produced by the existence of stochasticity, which drastically changes soliton behavior, causes random modulation, and modulates the long-wave–short-wave energy transfer. Consequently, the stochastic nonlinear Maccari system provides a potent analytical and computational tool for investigating the interaction between realistic stochastic fluctuations and nonlinear integrable dynamics in multiwave physical systems. In this study, we investigate the nonlinear Maccari system in the context of a Brownian motion process, which is defined as follows:

$$\begin{aligned} &iQ_t + Q_{xx} + \Phi\, Q + \sigma Q\frac{d\Psi}{dt} = 0\,,\\ &i\,W_t + W_{xx} + \Phi\, W + \sigma W\frac{d\Psi}{dt} = 0,\\ &\Phi_{\,t} + \Phi_{\,y} + (|\,Q + W\,|^2)_x = 0, \end{aligned} \tag{1.1}$$

where $Q = Q(x, y, t)$ and $W = W(x, y, t)$ are complex functions, and $\Phi = \Phi(x, y, t)$ is a real function. The time derivative of the Brownian motion process $d\Psi/dt$ describes the noise influence that characterizes the time derivative of the Brownian motion process $\{\Psi(t)\}_{t\geq 0}$.

The works in [14–16, 19] and related literature construct and classify exact solutions of the deterministic Maccari system (e.g., solitons, breathers, rogue waves) where all field functions are deterministic functions of (x,y,t). In contrast, our framework extends these constructions to a stochastic setting where the random phase induces nontrivial temporal randomness. Numerous innovative solutions have not yet been created, despite a number of research findings on Maccari's system (1.1). As part of our ongoing research, we generate multiple novel stochastic solitary waves for this model that are constrained by multiplicative noises via Brownian motion process in the Stratonovich sense using the RB sub-ODE technique. Finding traveling wave solutions to NPDEs is very simple and efficient with the Riccati–Bernoulli subsidiary ordinary differential equation (RB sub-ODE) technique [20]. This technique uses physical properties to produce a number of essential types of solitary waves. Significant applications in hydrodynamics, optical fibers, and plasma physics are made possible by the stochastic solutions offered [21, 22]. This proposed approach to solving the stochastic Maccari system has never been taken into consideration, as far as we are aware.

The current article's layout is explained as follows. Section 2 summarizes the RB sub-ODE approach. Section 3 provides the ODE form of the nonlinear Maccari system (1.1). Important stochastic solutions to the stochastic Maccari system are presented in Section 4. The solutions found are described in Section 5. For appropriate values of the physical parameters, several two-dimensional graphs of the solutions have been produced. Section 6 offers a summary of the study findings along with suggestions for more research.

2. Methodology description

This part presents a shortened version of the RB sub-ODE technique [20]. Consider the NPDEs for $q(x, y, t)$ that take the form

$$\mathfrak{G}(q, q_t, q_x, q_y, q_{tt}, q_{xt}, q_{xx}, ...) = 0, \tag{2.1}$$

which are transformed into an ODE

$$\mathfrak{H}(q, q', q'', q''', ...) = 0 \tag{2.2}$$

via the transformation $\zeta = x + y + vt$. The solution of Eq (2.2) satisfies

$$q' = \alpha q^{2-k} + \beta q + \gamma q^k, \tag{2.3}$$

where α, β, γ, and k are constants. Equation (2.3) gives

$$q'' = \alpha\beta(3-k)q^{2-k} + \alpha^2(2-k)q^{3-2k} + k\gamma^2 q^{2k-1} + \beta\gamma(k+1)q^k + (2\alpha\gamma + \beta^2)q. \tag{2.4}$$

Define $\Lambda = \beta^2 - 4\alpha\gamma$; the following are the solutions to Eq (2.3):

(1) For $k \neq 1, \beta = 0, \gamma = 0$,

$$q(\zeta) = (\alpha(k-1)(\zeta + \nu))^{\frac{1}{k-1}} . \tag{2.5}$$

(2) For $k \neq 1, \beta \neq 0, \gamma = 0$,

$$q(\zeta) = \left(\frac{-\alpha}{\beta} + \nu e^{\beta(k-1)\zeta}\right)^{\frac{1}{k-1}} . \tag{2.6}$$

(3) For $k \neq 1, \alpha \neq 0, \Lambda < 0$,

$$q(\zeta) = \left(\frac{-\beta}{2\alpha} + \frac{\sqrt{-\Lambda}}{2\alpha}\tan\left(\frac{(1-k)\sqrt{-\Lambda}}{2}(\zeta + \nu)\right)\right)^{\frac{1}{1-k}} , \tag{2.7}$$

$$q(\zeta) = \left(\frac{-\beta}{2\alpha} - \frac{\sqrt{-\Lambda}}{2\alpha}\cot\left(\frac{(1-k)\sqrt{-\Lambda}}{2}(\zeta + \nu)\right)\right)^{\frac{1}{1-k}} . \tag{2.8}$$

(4) For $k \neq 1, \alpha \neq 0, \Lambda > 0$,

$$q(\zeta) = \left(\frac{-\beta}{2\alpha} - \frac{\sqrt{\Lambda}}{2\alpha}\tanh\left(\frac{(1-k)\sqrt{\Lambda}}{2}(\zeta + \nu)\right)\right)^{\frac{1}{1-k}} , \tag{2.9}$$

$$q(\zeta) = \left(\frac{-\beta}{2\alpha} - \frac{\sqrt{\Lambda}}{2a}\coth\left(\frac{(1-k)\sqrt{\Lambda}}{2}(\zeta + \nu)\right)\right)^{\frac{1}{1-k}} . \tag{2.10}$$

(5) For $k \neq 1, \alpha \neq 0, \Lambda = 0$,

$$q(\zeta) = \left(\frac{1}{\alpha(k-1)(\zeta + \nu)} - \frac{\beta}{2\alpha}\right)^{\frac{1}{1-k}} . \tag{2.11}$$

(6) For $k = 1$,

$$q(\zeta) = \nu e^{(\alpha+\beta+\gamma)\zeta} . \tag{2.12}$$

3. Mathematical analysis

Now, we make use of the wave transformation

$$\begin{aligned} &Q(x,y,t) = e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))}q(\zeta);\, W(x,y,t) = e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))}w(\zeta);\\ &\Phi(x,y,t) = \Phi(\zeta), \zeta = x+\delta y-2ct, \end{aligned} \tag{3.1}$$

where c, μ, λ, and δ are constants. Then, Eq (1.1) is transferred into

$$\begin{aligned} &q'' - (\lambda + c^2)q + q\Phi = 0\,,\\ &w'' - (\lambda + c^2)w + \Phi\, w = 0\,,\\ &(\delta - 2c)\Phi' + ((q+w)^2)' = 0\,. \end{aligned} \tag{3.2}$$

Upon integrating the third equation of Eq (3.2) and assuming a zero integration constant, we obtain

$$\Phi = \left(\frac{-1}{\delta - 2c}\right)(q+w)^2\,. \tag{3.3}$$

By inserting Eq (3.3) into the first equation of (3.2), we obtain

$$\begin{aligned} &q'' - \frac{1}{\delta-2c}q(q+w)^2 - (\lambda+c^2)q = 0\,,\\ &w'' - \frac{1}{\delta-2c}w(q+w)^2 - (\lambda+c^2)w = 0\,. \end{aligned} \tag{3.4}$$

Although we are unable to directly solve these equations, we employ the following straightforward relations between w and q:

$$w = r\,q\,, \tag{3.5}$$

where r is an arbitrary constant. By inserting Eq (3.5) into Eqs (3.4), we obtain

$$(\delta - 2c)q'' - (1+r)^2\,q^3 - (\delta-2c)(\lambda+c^2)q = 0. \tag{3.6}$$

4. The stochastic solutions

We produce the stochastic solutions of the nonlinear Maccari system (1.1). For this purpose, we put Eq (2.4) into Eq (3.6), which gives

$$\begin{aligned} &(\delta-2c)\left(\alpha\beta(3-k)q^{2-k}+\alpha^2(2-k)q^{3-2k}+k\gamma^2q^{2k-1}+\beta\gamma(k+1)q^k+(2\alpha\gamma+\beta^2)q\right)\\ &-(1+r)^2\,q^3 - (\delta-2c)(\lambda+c^2)\,q = 0. \end{aligned} \tag{4.1}$$

Putting $k = 1$, Eq (4.1) becomes

$$(\delta-2c)\left(2\alpha\beta q+\alpha^2q+\gamma^2q+2\beta\gamma q+(2\alpha\gamma+\beta^2)q\right) - (1+r)^2\,q^3 - (\delta-2c)(\lambda+c^2)\,q = 0. \tag{4.2}$$

Setting each coefficient of $q^i (i = 1, 3)$ to zero yields

$$(\delta - 2c)\left(2\alpha\beta+\alpha^2+\gamma^2+2\beta\gamma+2\alpha\gamma+\beta^2\right) - (\delta - 2c)(\lambda + c^2) = 0. \tag{4.3}$$

Hence,

$$\alpha + \beta + \gamma = \pm\sqrt{\lambda + c^2}$$

and

$$r = -1.$$

Therefore, the solutions of (3.6) are

Family I:

$$q_{1,2}(\zeta) = \nu e^{\pm\sqrt{\lambda+c^2}\,\zeta},\ \lambda + c^2 > 0. \tag{4.4}$$

Consequently, the solutions to Eq (1.1) are

$$Q_{1,2}(x, y, t) = \nu e^{\pm\sqrt{\lambda+c^2}\,(x+\delta y-2ct)}\, e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \lambda + c^2 > 0. \tag{4.5}$$

$$W_{1,2}(x, y, t) = -\nu e^{\pm\sqrt{\lambda+c^2}\,(x+\delta y-2ct)}\, e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \lambda + c^2 > 0. \tag{4.6}$$

$$\Phi_{1,2}(x, y, t) = 0. \tag{4.7}$$

To generate additional families of solutions, we set $k = 0$, whereby Eq (4.1) reduces to

$$(\delta - 2c)\left(3\alpha\beta q^2+2\alpha^2 q^3+\beta\gamma+(2\alpha\gamma+\beta^2)q\right) - (1 + r)^2\, q^3 - (\delta - 2c)(\lambda + c^2)\, q = 0. \tag{4.8}$$

Setting all coefficients of $q^i (i = 0, 1, 2, 3)$ to zero yields

$$(\delta - 2c)\,\beta\gamma = 0\,,$$

$$(2\alpha\gamma + \beta^2) - (\lambda + c^2) = 0\,,$$

$$(\delta - 2c)\,\alpha\beta = 0\,,$$

$$2\,\alpha^2(\delta - 2c) - (1 + r)^2 = 0\,.$$

Resolving these equations yields

$$\beta = 0\,,$$

$$\alpha\gamma = \frac{\lambda + c^2}{2},$$

$$\alpha = \pm\sqrt{\frac{(1 + r)^2}{2(\delta - 2c)}}.$$

Therefore, the solutions of (3.6) are

Family II: For $\lambda = -c^2$ and defining $\Xi = (1+r)^2/2(\delta - 2c)$, we have

$$q_{3,4}(\zeta) = \left(\mp\sqrt{\Xi}\,(\zeta+\nu)\right)^{-1}, \delta > 2c. \tag{4.9}$$

Consequently, the solutions to Eq (1.1) are

$$Q_{3,4}(x,y,t) = \left(\mp\sqrt{\Xi}\,(x+\delta y-2ct+\nu)\right)^{-1} e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta > 2c. \tag{4.10}$$

$$W_{3,4}(x,y,t) = r\left(\mp\sqrt{\Xi}\,(x+\delta y-2ct+\nu)\right)^{-1} e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta > 2c. \tag{4.11}$$

$$\Phi_{3,4}(x,y,t) = -2\,(x+\delta y-2ct+\nu)^{-2}. \tag{4.12}$$

For the next two families, we define $\Upsilon = (\delta-2c)(\lambda+c^2)/(1+r)^2$ and $\Gamma = (\lambda+c^2)/2$.

Family III: For $\Gamma > 0$, we have

$$q_{5,6}(\zeta) = \pm\sqrt{\Upsilon}\tan\left(\sqrt{\Gamma}\,(\zeta+\nu)\right),\ \delta > 2c \tag{4.13}$$

and

$$q_{7,8}(\zeta) = \pm\sqrt{\Upsilon}\cot\left(\sqrt{\Gamma}\,(\zeta+\nu)\right),\ \delta > 2c. \tag{4.14}$$

Consequently, the solutions to Eq (1.1) are

$$Q_{5,6}(x,y,t) = \pm\sqrt{\Upsilon}\tan\left(\sqrt{\Gamma}\,(x+\delta y-2ct+\nu)\right)e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta > 2c, \tag{4.15}$$

$$W_{5,6}(x,y,t) = \pm r\sqrt{\Upsilon}\tan\left(\sqrt{\Gamma}\,(x+\delta y-2ct+\nu)\right)e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta > 2c, \tag{4.16}$$

$$\Phi_{5,6}(x,y,t) = -2\Gamma\tan^2\left(\sqrt{\Gamma}\,(x+\delta y-2ct+\nu)\right), \tag{4.17}$$

and

$$Q_{7,8}(x,y,t) = \pm\sqrt{\Upsilon}\cot\left(\sqrt{\Gamma}\,(x+\delta y-2ct+\nu)\right)e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta > 2c, \tag{4.18}$$

$$W_{7,8}(x,y,t) = \pm r\sqrt{\Upsilon}\cot\left(\sqrt{\Gamma}\,(x+\delta y-2ct+\nu)\right)e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta > 2c, \tag{4.19}$$

$$\Phi_{7,8}(x,y,t) = -2\Gamma\cot^2\left(\sqrt{\Gamma}\,(x+\delta y-2ct+\nu)\right). \tag{4.20}$$

Family IV: For $\Gamma < 0$, we have

$$q_{9,10}(\zeta) = \pm\sqrt{-\Upsilon}\tanh\left(\sqrt{-\Gamma}\,(\zeta+\nu)\right),\ \delta > 2c \tag{4.21}$$

and

$$q_{11,12}(\zeta) = \pm\sqrt{-\Upsilon}\coth\left(\sqrt{-\Gamma}\,(\zeta+\nu)\right),\ \delta > 2c. \tag{4.22}$$

Consequently, the solutions to Eq (1.1) are

$$Q_{9,10}(x,y,t) = \pm\sqrt{-\Upsilon}\tanh\left(\sqrt{-\Gamma}\,(x+\delta y-2ct+\nu)\right)e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta > 2c, \tag{4.23}$$

$$W_{9,10}(x,y,t) = \pm r\sqrt{-\Upsilon}\tanh\Big(\sqrt{-\Gamma}\,(x+\delta y-2ct+\nu)\Big)\,e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta>2c, \tag{4.24}$$

$$\Phi_{9,10}(x,y,t) = 2\Gamma\tanh^2\Big(\sqrt{-\Gamma}\,(x+\delta y-2ct+\nu)\Big), \tag{4.25}$$

and

$$Q_{11,12}(x,y,t) = \pm\sqrt{-\Upsilon}\coth\Big(\sqrt{-\Gamma}\,(x+\delta y-2ct+\nu)\Big)\,e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta>2c, \tag{4.26}$$

$$W_{11,12}(x,y,t) = \pm r\sqrt{-\Upsilon}\coth\Big(\sqrt{-\Gamma}\,(x+\delta y-2ct+\nu)\Big)\,e^{i(cx+\mu y+\lambda t+\sigma\Psi(t))},\ \delta>2c, \tag{4.27}$$

$$\Phi_{11,12}(x,y,t) = 2\Gamma\coth^2\Big(\sqrt{-\Gamma}\,(x+\delta y-2ct+\nu)\Big). \tag{4.28}$$

5. Outcome analysis and discussion

The RB sub-ODE technique provides a powerful and versatile framework for obtaining solitary wave solutions of broad classes of nonlinear partial differential equations by reducing complex NPDEs to simpler, solvable ODEs without losing the essential nonlinear dynamics. This approach systematically generates solitary, periodic, and other coherent wave patterns across integrable and nonintegrable systems, accommodating polynomial, rational, and nonpolynomial nonlinearities, as well as stochastic disturbances. Applying the proposed method to the stochastic Maccari system under Brownian motion yields robust exponential, hyperbolic, trigonometric, and rational solutions, revealing the intricate interplay of nonlinearity, dispersion, and randomness. The wave transformation and subsequent reduction are valid because Stratonovich calculus preserves the ordinary chain rule. Although deterministic solutions offer insight, only stochastic solutions capture the effects of real-world noise in systems such as ocean dynamics, nonlinear optics, and plasma physics, providing a foundation to understand soliton stability, wave collapse, vortex dynamics, and rare extreme events such as rogue waves. By advancing both the mathematical theory of nonlinear stochastic dynamics and the predictive power of models in uncertain environments, the RB sub-ODE method proves essential for exploring complex physical phenomena.

Understanding the behavior of nonlinear wave interactions under random external perturbations is crucially dependent on the study of stochastic solutions for the Maccari system. In physical terms, this noise-induced phase modulation may result in decoherence effects, random alterations in phase-sensitive interference patterns, and stochastic resonance phenomena when interacting with other external field elements that cannot be adequately represented by deterministic solutions alone. The intrinsic uncertainties found in optical fibers, plasma settings, shallow water systems, and other physical media where fluctuations cannot be disregarded are captured by the stochastic Maccari system by adding noise through Brownian motion. Stochastic solutions show how soliton amplitude, phase, velocity, and stability are modulated by random forcing, resulting in phenomena such as noise-induced coherence, random scattering, and stochastic resonance. By predicting wave profile probability distributions and measuring the probability of extreme events, these methods also offer profound insights into the resilience of coherent structures. Applications range widely, from replicating uncertainty in fluid interfaces to modeling turbulent energy transfer in plasma physics to creating noise-resilient optical communication links. In order to connect deterministic theory with realistic, noisy physical circumstances, stochastic assessments of the Maccari system have become crucial.

We employed simulation visualization tools to illustrate and analyze the dynamical behavior of the derived stochastic solitary wave solutions. Figure 1 presents the two-dimensional profile of the

rational wave solution $Q_3(x, y, t)$ in the absence of stochastic perturbations ($\sigma = 0$). The plot shows that the solution represents a highly localized rational wave with blow-up type behavior at the center and vanishing amplitude at infinity. Figure 2 depicts the two-dimensional profile of the periodic traveling wave exhibiting singular spikes corresponding to $Q_5(x, y, t)$ for $\sigma = 0$. Between the spikes, the amplitude remains very small and close to zero. Figure 3 displays the 2D profile of the periodic wave solution $Q_9(x, y, t)$ for several values of t and $\sigma = 0$. This plot illustrates how the solution varies spatially for different times, highlighting the periodicity in x and the time-dependent phase shift, which is characteristic of traveling or oscillatory wave solutions in nonlinear systems. Figure 4 shows the corresponding stationary soliton wave solution $\Phi_9(x, y, t)$ for distinct values of t with phase shift. Finally, Figures 5 and 6 depict the 2D profile of the exponential wave solution of $Q_1(x, y, t)$ and 2D periodic wave profile of $Q_9(x, y, t)$, respectively, for different values of the noise term σ. These figures demonstrate how the stochastic solution evolves with respect to the spatial variable x and the noise intensity σ, highlighting a clear periodic structure with a consistent phase shift and no reversal in propagation direction. The inclusion of a stochastic forcing term in nonlinear wave models introduces an inherent degree of randomness, which can significantly alter the qualitative and quantitative behavior of solitary waves, including optical and hydrodynamic solitons.

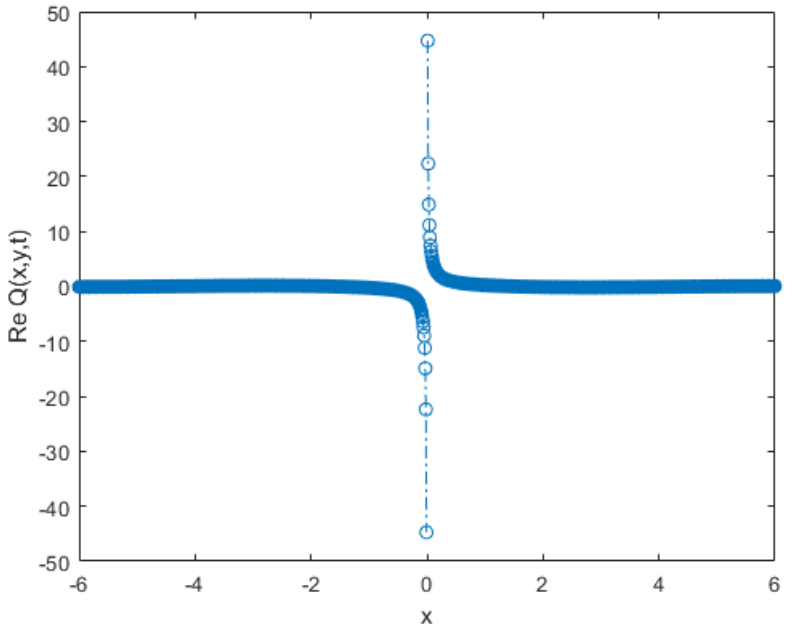

Figure 1. 2D profile of rational wave solution $Q_3(x, y, t)$.

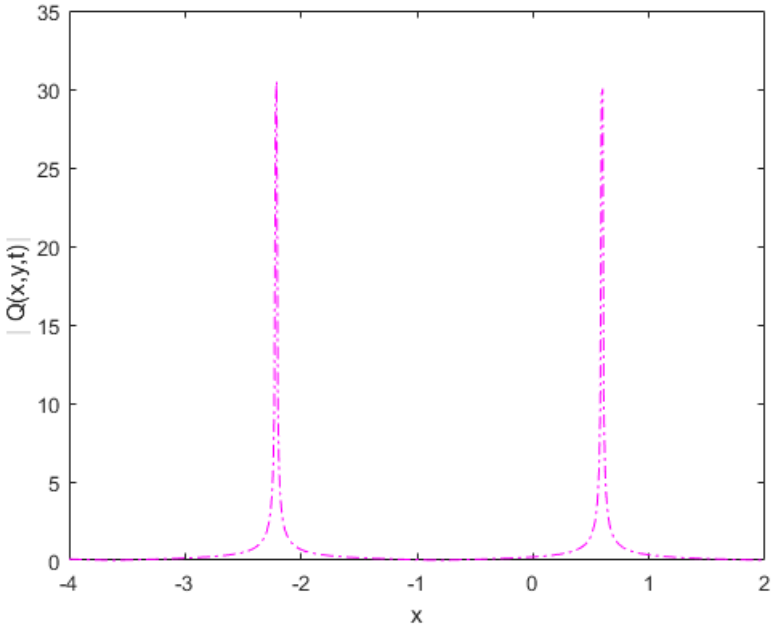

Figure 2. 2D profile of periodic wave with two localized peaks of $Q_5(x, y, t)$.

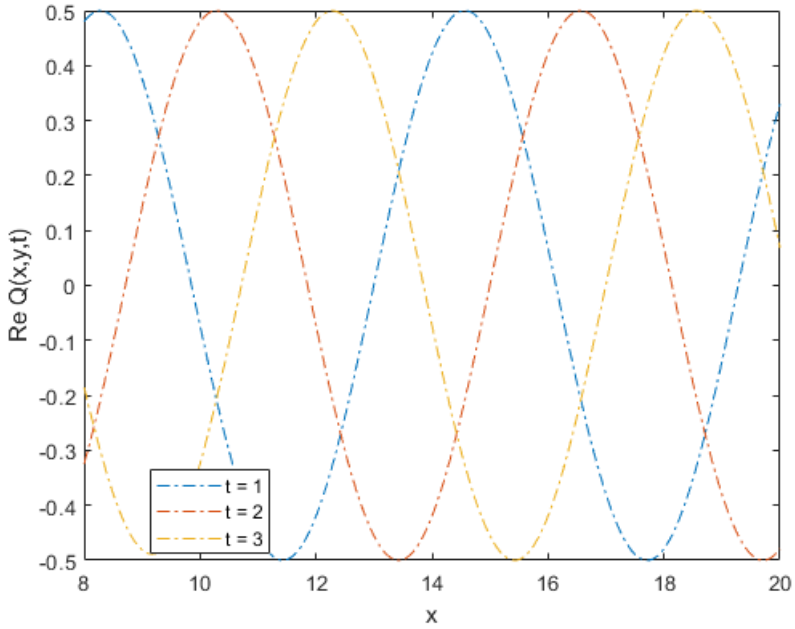

Figure 3. 2D profile of periodic wave solution of $Q_9(x, y, t)$ with distinct values of t.

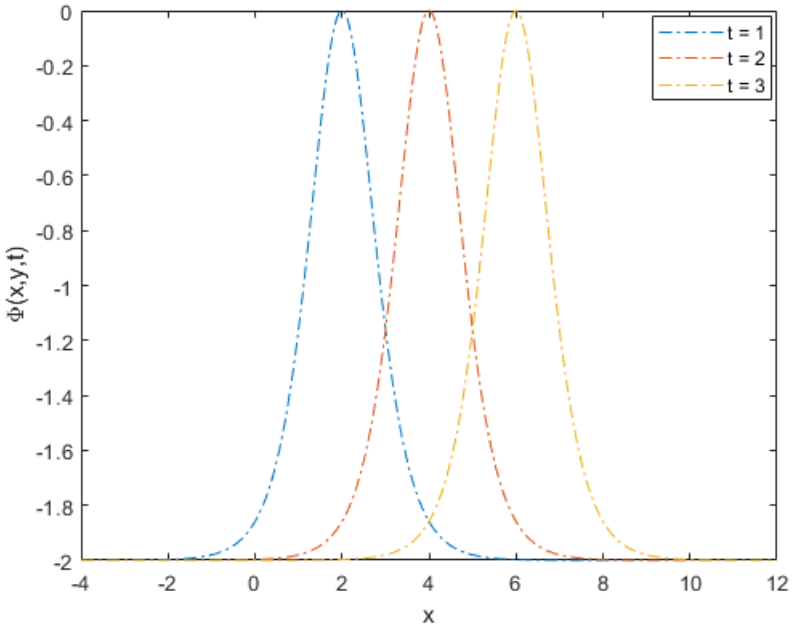

Figure 4. 2D profile of periodic wave solution of $\Phi_9(x, y, t)$ with distinct values of t.

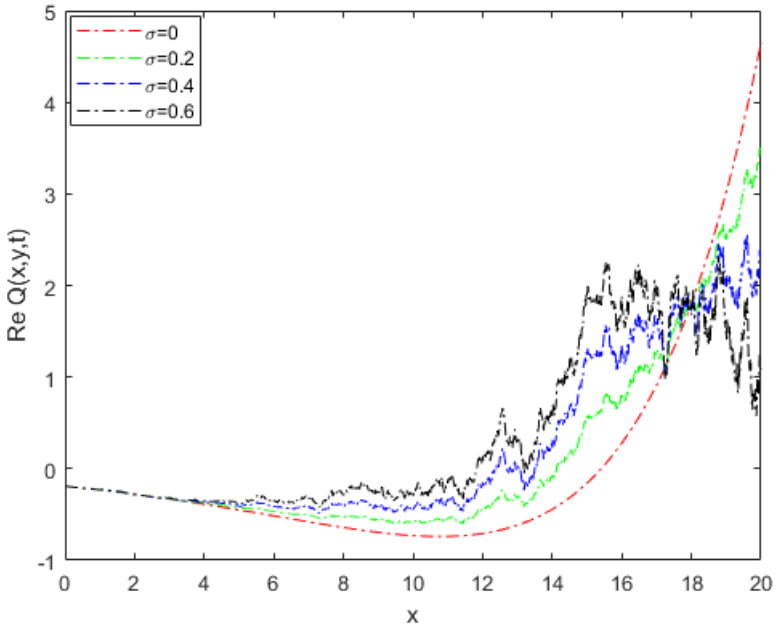

Figure 5. 2D profile of exponential wave solution of $Q_1(x, y, t)$ with distinct values of σ.

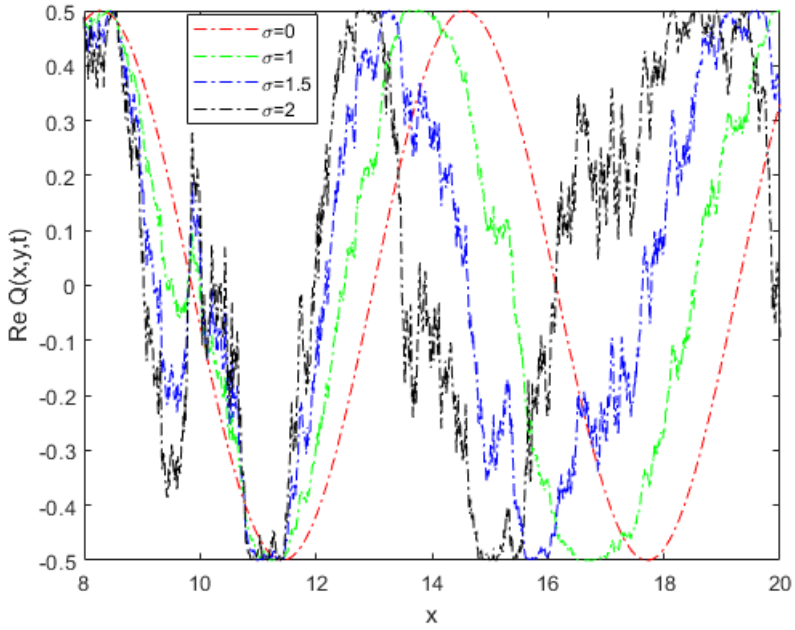

Figure 6. 2D profile of periodic wave solution of $Q_9(x, y, t)$ with distinct values of σ.

6. Conclusions

In this study, we have conducted an analytical investigation into the stochastic dynamics of the nonlinear Maccari system by integrating Brownian motion into its nonlinear evolution. Employing the RB sub-ODE technique, we have derived a comprehensive family of exact stochastic wave solutions, which include rational, periodic, and solitary wave structures, all of which display modulation dependent on noise. The produced results reveal that stochastic forcing, even at low levels, has a significant impact on amplitude, phase, stability, and interaction dynamics, resulting in modulation, phase drift, and deformation of coherent structures. These results offer valuable insights into noise-driven wave phenomena within nonlinear systems and highlight the robustness and efficacy of the RB sub-ODE method in generating exact solutions for complex integrable models. In summary, this research establishes a robust analytical framework for the examination of stochastic effects in nonlinear wave propagation and paves the way for future studies into noise-induced dynamics in other integrable systems.

Use of Generative-AI tools declaration

The author declares that he has not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The author declares that he has no conflict of interest.

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