



Research article

Efficient low-light image enhancement via balancing polynomials-based adaptive contrast control using coefficient inequalities of a subclass of bi-univalent functions

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Abstract: In this research work, we introduced a novel subclass of bi-univalent functions associated with balancing polynomials. For this class, we derived coefficient estimates $|\alpha_2|$, $|\alpha_3|$ along with Fekete–Szegő inequalities. Motivated by these theoretical findings, we proposed a new model based on balancing polynomials. The proposed low-light enhancement model consists of gamma correction and adaptive polynomial components, which make the image brighter without losing details. To avoid over-enhancement, a control mechanism based on the contrast improvement index (CII) was adopted to ensure that the performance is the same in different lighting situations. According to tests on a number of low-light images, the proposed method had higher entropy with controlled contrast than other traditional methods. By maintaining a consistent balance between detail enhancement and natural visual appearance, the technique validated its effectiveness and appropriateness for practical image-enhancement applications.

Keywords: bi-univalent functions; balancing polynomials; Fekete–Szegő inequalities; low-light enhancement; gamma correction; entropy; contrast improvement index (CII)

Mathematics Subject Classification: Primary 30C45; Secondary 30C50, 33C45

1. Introduction

Let $\mathcal{A}$ denote the class of all normalized analytic functions. The functions $f(z)$ have been represented as

$$f(z) = z + \sum_{j=2}^{\infty} \alpha_j z^j \quad (z \in \mathbb{D}), \quad (1.1)$$

where $\mathbb{D}$ is the collection of complex numbers z satisfying $|z| < 1$. We characterize $\mathcal{S}$ as the univalent function subclass of $\mathcal{A}$. If $h_1, h_2 \in \mathcal{A}$, then we say $h_1(z) < h_2(z)$ if there exists an analytic function $\zeta : \mathbb{D} \rightarrow \mathbb{D}$ such that $\zeta(0) = 0$ and $h_1(z) = h_2(\zeta(z))$. A function $f(z) \in \mathcal{S}$ is said to be bi-univalent if the inverse mapping $f^{-1}(w)$ admits an analytic continuation in the region $|w| < 1$ of the w -plane. The symbol σ represents the collection of all bi-univalent functions in $\mathbb{D}$. If $f^{-1}(w) = g(w)$, then we have

$$g(w) = w + \sum_{j=2}^{\infty} b_j w^j \quad (w \in \mathbb{D}).$$

Further, we have

$$g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots \quad (w \in \mathbb{D}).$$

In 1967, Lewin [12] introduced the class of bi-univalent functions and established that $|\alpha_2| < 1.51$ for functions in this class. Brannan [4] and Clunie later improved this result to $|\alpha_2| \leq \sqrt{2}$. A substantial body of literature exists regarding the estimations of the initial coefficients of bi-univalent functions (refer to [2, 11, 20]).

The theory of bi-univalent functions has been the focus of much research owing to its applications to the coefficient estimation and geometric function theory. The use of special polynomial structures in defining subclasses of analytic and bi-univalent functions has grown rapidly in recent years, leading to several important developments [2, 15, 19]. Balancing numbers and their associated polynomial families [15, 17, 19] possess rich algebraic properties and have been extensively investigated in the literature [3, 5, 7, 8]. Motivated by these studies, balancing polynomials have recently been employed to construct new subclasses of Sakaguchi-type and Bazilevič-type bi-univalent functions, and to derive meaningful coefficient inequalities [16, 17]. These advances suggest that balancing polynomials provide a fruitful framework for further theoretical investigations and practical applications. Inspired by this observation, the present work introduces a new subclass of bi-univalent functions related to balancing polynomials and their applications to low-light image-enhancement.

The following recurrence relations define balancing polynomials [16] for any integer $n \geq 2$ and $y \in \mathbb{C}$:

$$\mathcal{B}_n(y) = 6y\mathcal{B}_{n-1}(y) - \mathcal{B}_{n-2}(y) \quad (1.2)$$

with the initial values

$$\mathcal{B}_0(y) = 0, \quad \mathcal{B}_1(y) = 1.$$

Remark 1. If we assume $n = 2$ and $n = 3$ in Eq (1.2), then we obtain the polynomials:

$$\mathcal{B}_2(y) = 6y, \quad \mathcal{B}_3(y) = 36y^2 - 1.$$

In addition, balancing polynomials can be generated using the generating function in the same manner as other number polynomials.

Lemma 1.1. The recurrence relation for balancing polynomials is given by

$$\mathcal{B}(y, z) = \sum_{n=0}^{\infty} \mathcal{B}_n(y) z^n = \frac{z}{1 - 6yz + z^2}, \quad z \in \mathbb{U}.$$

Determining sharp bounds for $|\alpha_3 - \varrho \alpha_2^2|$ within a compact family of functions is the Fekete–Szegő inequality problem. In the theory of geometric functions, the Schwarzian derivative is represented by the functional when $\varrho = 1$.

2. Main results

In this section, we present a new subclass of the class σ by means of the operator $\Omega_{v,t}$ involving the parameters (v, t) . The conditions are established using the concept of subordination in connection with the generating function of balancing polynomials.

Unless otherwise specified, we make the following assumptions throughout this paper:

$$\lambda_n = 1 + (n-1)(v+t) + (n^2+1)vt, \quad n \in \mathbb{N}, \quad (2.1)$$

where $v \geq 0$ and $t \in [0, 1]$. Furthermore,

$$\lambda_{n+1} - \lambda_n = (1 + (2n+1)t)v + t \geq 0,$$

which implies that $\lambda_n \in \mathbb{R}$, $\lambda_n \geq 1$, and $\{\lambda_n\}_{n \geq 1}$ is a non-decreasing sequence.

For every $k \in \mathcal{A}$, we define

$$\Omega_{v,t}(k(z)) := (1-v)(1-t) \frac{k(z)}{z} + (t+v(1+t))k'(z) + vt(zk''(z) - 2). \quad (2.2)$$

If

$$k(z) = z + \sum_{j=2}^{\infty} \beta_j z^j,$$

then

$$\Omega_{v,t}(k(z)) = 1 + \sum_{j=2}^{\infty} \lambda_j \beta_j z^{j-1}. \quad (2.3)$$

By using balancing polynomials together with the concept of subordination, we define the subclasses of σ .

Definition 1. A function $f \in \sigma$, defined by (1.1), is said to belong to the class $\mathcal{A}_{\sigma}^{(v,t)}(y, z)$ if

$$\Omega_{v,t}(f(z)) < \frac{\mathcal{B}(y, z)}{z} = I(y, z) \quad (z \in \mathbb{D}) \quad (2.4)$$

and

$$\Omega_{v,t}(g(z)) < \frac{\mathcal{B}(y, z)}{z} = I(y, z) \quad (z \in \mathbb{D}), \quad (2.5)$$

where $g(w) = f^{-1}(w) = w + \sum_{j=2}^{\infty} b_j w^j$.

Example 1. If $f \in \sigma$, defined by (1.1), belongs to the class $\mathcal{A}_\sigma^{(v,0)}(y, z) = \mathcal{D}_\sigma^{(v,0)}$, where $t = 0$ and $v \geq 1$, then

$$\Omega_{v,0}(f(z)) < \frac{\mathcal{B}(y, z)}{z} = I(y, z) \quad (z \in \mathbb{D})$$

and

$$\Omega_{v,0}(g(w)) < \frac{\mathcal{B}(y, w)}{w} = I(y, w) \quad (w \in \mathbb{D}),$$

where $g = f^{-1}$.

Example 2. If $f \in \sigma$, defined by (1.1), belongs to the class $\mathcal{A}_\sigma^{(1,0)}(y, z) = \mathcal{D}_\sigma^{(1,0)}(y, z) = \mathcal{H}_\sigma^{(1,0)}(y, z)$, where $t = 0$ and $v = 1$, then

$$\Omega_{1,0}(f(z)) < \frac{\mathcal{B}(y, z)}{z} = I(y, z) \quad (z \in \mathbb{D})$$

and

$$\Omega_{1,0}(g(z)) < \frac{\mathcal{B}(y, w)}{w} = I(y, w) \quad (z \in \mathbb{D}),$$

where $g = f^{-1}$.

3. The coefficient estimates

In this section, we derive the coefficient bounds for $|\alpha_2|$ and $|\alpha_3|$ for functions belonging to the class $\mathcal{A}_\sigma^{(v,t)}(y, z)$.

Theorem 1. If $f(z)$, defined by (1.1), belongs to the class $\mathcal{A}_\sigma^{(v,t)}(y, z)$, then

$$|\alpha_2| \leq \frac{|6y| \sqrt{|6y|}}{\sqrt{|36\lambda_3 y^2 - \lambda_2^2(36y^2 - 1)|}}, \quad y^2 \neq -\frac{\lambda_2^2}{36(\lambda_3 - \lambda_2^2)} \quad (3.1)$$

and

$$|\alpha_3| \leq 6 \left(\frac{|y|}{|\lambda_3|} + \frac{6|y|^2}{|\lambda_2|^2} \right).$$

Proof. Since $f \in \mathcal{A}_\sigma^{(v,t)}(y, z)$, there exist suitable analytic functions such that there exist analytic functions $l : \mathbb{D} \rightarrow \mathbb{D}$ and $m : \mathbb{D} \rightarrow \mathbb{D}$, given by (2.4) and (2.5), such that

$$\Omega_{v,t}(f(z)) = I(y, l(z))$$

and

$$\Omega_{v,t}(g(w)) = I(y, m(w)).$$

Define the functions $l(z)$ and $m(w)$ as

$$l(z) = l_1 z + l_2 z^2 + l_3 z^3 + \cdots \quad (3.2)$$

and

$$m(w) = m_1 w + m_2 w^2 + m_3 w^3 + \cdots \quad (3.3)$$

with $l(0) = m(0) = 0$, $|l(z)| < 1$, $|m(w)| < 1$ for all $z, w \in \mathbb{D}$.

It is evident that if

$$|l(z)| = |l_1 z + l_2 z^2 + l_3 z^3 + \cdots| \leq 1 \quad (z \in \mathbb{D})$$

and

$$|m(w)| = |m_1 w + m_2 w^2 + m_3 w^3 + \cdots| \leq 1 \quad (w \in \mathbb{D}),$$

then

$$|l_j| \leq 1, \quad |m_j| \leq 1, \quad j = 1, 2, \dots$$

for $j = 1, 2, \dots$

Since

$$1 + \sum_{j=2}^{\infty} \lambda_j \alpha_j z^{j-1} = 1 + \mathcal{B}_2(y) l_1 z + [\mathcal{B}_2(y) l_2 + \mathcal{B}_3(y) l_1^2] z^2 + \cdots \quad (3.4)$$

and

$$1 + \sum_{j=2}^{\infty} \lambda_j \beta_j w^{j-1} = 1 + \mathcal{B}_2(y) m_1 w + [\mathcal{B}_2(y) m_2 + \mathcal{B}_3(y) m_1^2] w^2 + \cdots, \quad (3.5)$$

from (3.4) and (3.5), we get the following expressions:

$$\lambda_2 \alpha_2 = \mathcal{B}_2(y) l_1, \quad (3.6)$$

$$\lambda_3 \alpha_3 = \mathcal{B}_2(y) l_2 + \mathcal{B}_3(y) l_1^2, \quad (3.7)$$

$$-\lambda_2 \alpha_2 = \mathcal{B}_2(y) m_1, \quad (3.8)$$

$$\lambda_3 (2\alpha_2^2 - \alpha_3) = \mathcal{B}_2(y) m_2 + \mathcal{B}_3(y) m_1^2. \quad (3.9)$$

From (3.6) and (3.8), we can easily see that

$$l_1 = -m_1. \quad (3.10)$$

Further squaring and adding (3.6) and (3.8), we get

$$2\lambda_2^2 \alpha_2^2 = \mathcal{B}_2^2(y) (l_1^2 + m_1^2). \quad (3.11)$$

Upon adding (3.7) and (3.9), we get

$$\lambda_3 \alpha_3 + \lambda_3 (2\alpha_2^2 - \alpha_3) = \mathcal{B}_2(y) (l_2 + m_2) + \mathcal{B}_3(y) (l_1^2 + m_1^2). \quad (3.12)$$

From the above equations, we obtain

$$2 \left[\lambda_3 \mathcal{B}_2^2(y) - \lambda_2^2 \mathcal{B}_3(y) \right] \alpha_2^2 = \mathcal{B}_2^3(y) (l_2 + m_2), \quad (3.13)$$

which implies

$$|\alpha_2| \leq \frac{|6y| \sqrt{|6y|}}{\sqrt{|36\lambda_3 y^2 - \lambda_2^2 (36y^2 - 1)|}}.$$

Upon subtracting (3.9) from (3.7), we get

$$\lambda_3 \alpha_3 - \lambda_3 (2\alpha_2^2 - \alpha_3) = \mathcal{B}_2(y) (l_2 - m_2) + \mathcal{B}_3(y) (l_1^2 - m_1^2). \quad (3.14)$$

Then, by using (3.10) and (3.11) in (3.14), we get

$$\alpha_3 = \frac{\mathcal{B}_2(y)(l_2 - m_2)}{2\lambda_3} + \frac{\mathcal{B}_2^2(y)(l_1^2 + m_1^2)}{2\lambda_2^2}. \quad (3.15)$$

By taking the modulus and applying (3.10), we get the desired bound for $|\alpha_3|$:

$$|\alpha_3| \leq 6 \left(\frac{|y|}{|\lambda_3|} + \frac{6|y|^2}{|\lambda_2|^2} \right). \quad (3.16)$$

Corollary 1. If $f(z)$, defined by (1.1), belongs to the class $\mathcal{A}_\sigma^{(v,0)}(y, z) = \mathcal{D}_\sigma^{(v,0)}(y, z)$, then

$$|\alpha_2| \leq \frac{|6y| \sqrt{|6y|}}{\sqrt{|(1 - 36y^2)v^2 + 2v + 1|}}, \quad y^2 \neq \frac{(1 + v)^2}{36v^2}$$

and

$$|\alpha_3| \leq 6 \left(\frac{|y|}{|1 + 2v|} + \frac{6|y|^2}{|1 + v|^2} \right).$$

Corollary 2. If $f(z)$, defined by (1.1), belongs to the class $\mathcal{A}_\sigma^{(1,0)}(y, z) = \mathcal{D}_\sigma^{(1,0)}(y, z) = \mathcal{H}_\sigma^{(1,0)}(y, z)$, then

$$|\alpha_2| \leq \frac{|6y| \sqrt{|6y|}}{\sqrt{|4 - 36y^2|}}, \quad y^2 \neq \frac{1}{9}$$

and

$$|\alpha_3| \leq 2|y| + 9|y|^2.$$

4. Fekete–Szegő inequalities

In this section, we derive sharp bounds for $|\alpha_3 - \varrho\alpha_2^2|$ and obtain a compact coefficient inequality.

Theorem 2. If $f(z)$, defined by (1.1), belongs to the class $\mathcal{A}_\sigma^{(v,t)}(y, z)$ and $\varrho \in \mathbb{R}$, then

$$|\alpha_3 - \varrho\alpha_2^2| \leq \begin{cases} \frac{|6y|}{|\lambda_3|}, & \text{if } 0 \leq |\varrho - 1| \leq \left| 1 - \frac{\lambda_2^2(36y^2 - 1)}{36\lambda_3y^2} \right|, \\ \frac{216|1 - \varrho||y|^3}{|36\lambda_3y^2 - \lambda_2^2(36y^2 - 1)|}, & \text{if } |\varrho - 1| \geq \left| 1 - \frac{\lambda_2^2(36y^2 - 1)}{36\lambda_3y^2} \right|. \end{cases}$$

Proof: From (3.14) and for $\varrho \in \mathbb{R}$, we have

$$\alpha_3 - \varrho\alpha_2^2 = \frac{\mathcal{B}_2(y)(l_2 - m_2)}{2\lambda_3} + (1 - \varrho)\alpha_2^2. \quad (4.1)$$

By using (3.13), we get

$$\begin{aligned} \alpha_3 - \varrho\alpha_2^2 &= \frac{\mathcal{B}_2(y)(l_2 - m_2)}{2\lambda_3} + (1 - \varrho) \left(\frac{\mathcal{B}_2^3(y)(l_2 + m_2)}{2[\lambda_3\mathcal{B}_2^2(y) - \lambda_2^2\mathcal{B}_3(y)]} \right) \\ &= \mathcal{B}_2(y) \left[\left(\frac{1}{2\lambda_3} + \Theta(\varrho, y) \right) l_2 + \left(-\frac{1}{2\lambda_3} + \Theta(\varrho, y) \right) m_2 \right], \end{aligned} \quad (4.2)$$

where

$$\Theta(\varrho, y) = \frac{(1 - \varrho)\mathcal{B}_2^2(y)}{2(\lambda_3\mathcal{B}_2^2(y) - \lambda_2^2\mathcal{B}_3(y))}.$$

Taking the modulus, we obtain

$$|\alpha_3 - \varrho\alpha_2^2| \leq \begin{cases} \frac{|6y|}{|\lambda_3|}, & \text{if } 0 \leq |\Theta(\varrho, y)| \leq \frac{1}{2|\lambda_3|}, \\ 2|6y\Theta(\varrho, y)|, & \text{if } |\Theta(\varrho, y)| \geq \frac{1}{2|\lambda_3|}. \end{cases}$$

Corollary 3. If $f(z)$, defined by (1.1), belongs to the class $\mathcal{D}_\sigma^{(v,0)}(y, z)$ and $\varrho \in \mathbb{R}$, then

$$|\alpha_3 - \varrho\alpha_2^2| \leq \begin{cases} \frac{|6y|}{|1 + 2v|}, & \text{if } 0 \leq |\varrho - 1| \leq \left|1 - \frac{(1 + v)^2(36y^2 - 1)}{36(1 + 2v)y^2}\right|, \\ \frac{216|1 - \varrho||y|^3}{|36(1 + 2v)y^2 - (1 + v)^2(36y^2 - 1)|}, & \text{if } |\varrho - 1| \geq \left|1 - \frac{(1 + v)^2(36y^2 - 1)}{36(1 + 2v)y^2}\right|. \end{cases}$$

Corollary 4. If $f(z)$, defined by (1.1), belongs to the class $\mathcal{H}_\sigma^{(1,0)}(y, z)$ and $\varrho \in \mathbb{R}$, then

$$|\alpha_3 - \varrho\alpha_2^2| \leq \begin{cases} 2|y|, & \text{if } 0 \leq |\varrho - 1| \leq \left|\frac{1 - 9y^2}{27y^2}\right|, \\ \frac{54|1 - \varrho||y|^3}{|1 - 9y^2|}, & \text{if } |\varrho - 1| \geq \left|\frac{1 - 9y^2}{27y^2}\right|. \end{cases}$$

The results established in Section 4 directly motivate the polynomial intensity transformation employed in Section 5. The derived bounds for the coefficients α_2 and α_3 constrain the nonlinear mapping, thereby ensuring smooth contrast enhancement and, preserving structural details in the enhanced image.

5. Experimental setup and results

The effectiveness of the suggested method in improving low-light images was assessed through a number of experiments. The implementation was done on a system with a standard operating system using Python™. We used both qualitative and quantitative measures to evaluate the efficiency of the suggested model. We used publicly available low-light image datasets for the experiments. One of these was the LOL (Low-Light) dataset, found at <https://daooshee.github.io/BMVC2018website/>, which has paired low-light and normal-light images, as well as other real-world photos taken in different lighting conditions. These datasets include a wide range of environments, such as indoors, outdoors, and at nighttime, which ensures a full evaluation of the proposed method. We suggest a new way to improve low-light images in this work using adaptive contrast control and balancing polynomials.

$$T(p) = 1.08 p^\gamma + 0.028\alpha_2\mathcal{B}_2(p) + 0.010\alpha_3\mathcal{B}_3(p),$$

where p represents the normalized pixel intensity, γ controls the brightness adjustment, and $\mathcal{B}_2(p)$ and $\mathcal{B}_3(p)$ are balancing polynomials with coefficient bounds α_2 and α_3 . The mean normalized intensity is calculated by

$$\mu = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N p(i, j)$$

and $\gamma = 0.60$ for $\mu < 0.40$.

- **Images with adequate illumination levels ($\mu > 0.5$):** The pictures in this category are already sufficiently bright. So they do not need strong enhancement, as they already have structural clarity.
- **Moderately illuminated images ($0.2 \leq \mu \leq 0.5$):** The proposed method, based on balancing polynomials, uses gamma correction and adaptive polynomial components to brighten the images with moderate illumination. Also, the coefficient inequality bounds help to preserve edge sharpness while reducing the chance of saturation effects. This proposed low-light image-enhancement method leads to a visually consistent enhancement process and is perfect for images with moderate illumination.
- **Poorly illuminated images ($\mu < 0.2$):** Pictures captured under extremely low light conditions require strong enhancement. In such a case, gamma correction, together with polynomial components, is applied within the proposed model. The polynomial mapping guided by these inequality constraints improves contrast better without over-brightening dark areas. Thus, the proposed low-light enhancement model based on balancing polynomials with suitable coefficient constraints generates clearer and visually enhanced images in extremely dark conditions.

Entropy is a numerical metric used to evaluate the information content of an image. It is expressed as $H = -\sum p(i) \log p(i)$, where $p(i)$ denotes the probability distribution of pixel intensities. Low entropy values may indicate loss of important image information or excessive enhancement, whereas higher entropy values imply a wider distribution of intensity levels and richer image information.

The contrast improvement index (CII) is a statistical measure used to assess the improvement in image contrast after processing. It is defined by $\text{CII} = \frac{\Sigma_{\text{enhanced}}}{\Sigma_{\text{original}}}$, where Σ represents the standard deviation of grayscale intensity values. Standard deviation quantifies the spread of intensity values and serves as an indicator of image contrast. A CII value greater than one indicates an increase in image contrast. Consequently, the proposed low-light image-enhancement model based on balancing polynomials and suitable coefficient inequality bounds achieves an effective balance between entropy and CII values while preserving the structural details of the images.

6. Proposed algorithm

Let M_2 and M_3 denote the upper bounds for $|\alpha_2|$ and $|\alpha_3|$, respectively, obtained from the coefficient estimates established in the preceding theorems. In the implementation, the admissible coefficients are chosen adaptively as

$$\alpha_2 = (1 - \mu)M_2, \quad \alpha_3 = (1 - \mu)^2 M_3,$$

so that their magnitudes remain within the theoretically established bounds. The constants 1.08, 0.028, and 0.010 are fixed scaling factors used to balance illumination correction and polynomial contrast enhancement while limiting excessive amplification.

Algorithm 1 Proposed low-light image enhancement using balancing polynomials

Require: Input image I

Ensure: Enhanced image I_{enh}

1. Normalize the image intensities:

$$Y = \frac{I}{255}, \quad \mu = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N Y(i, j).$$

2. Compute the balancing polynomials

$$\mathcal{B}_2(Y) = 6Y, \quad \mathcal{B}_3(Y) = 36Y^2 - 1.$$

3. Determine the adaptive illumination parameter

$$\gamma = \frac{\log(0.5)}{\log(\max(\mu, \varepsilon))},$$

where $\varepsilon > 0$ is a small constant used to avoid numerical instability.

4. Let M_2 and M_3 be the theoretical upper bounds for $|\alpha_2|$ and $|\alpha_3|$, and set

$$\alpha_2 = (1 - \mu)M_2, \quad \alpha_3 = (1 - \mu)^2 M_3.$$

5. Generate the enhanced intensity mapping

$$Y_p = 1.08Y^\gamma + 0.028\alpha_2\mathcal{B}_2(Y) + 0.010\alpha_3\mathcal{B}_3(Y).$$

6. Normalize Y_p to $[0, 1]$ and perform contrast adjustment.

7. Compute the contrast improvement index:

$$\text{CII} = \frac{\sigma_{\text{enh}}}{\sigma_{\text{orig}}},$$

where σ_{enh} and σ_{orig} denote the standard deviations of the enhanced and original intensity images, respectively.

8. **if** $\text{CII} > 4.2$ **then**

9. Blend the enhanced image with the original image:

$$Y_p \leftarrow \lambda Y_p + (1 - \lambda)Y, \quad 0 < \lambda < 1.$$

10. **end if**

11. Apply edge-preserving filtering and convert the normalized image back to the original intensity range.

12. **return** I_{enh}
-

The threshold $CII = 4.2$ is an empirically selected operating parameter to avoid over-enhancement. When $CII > 4.2$, blending is applied to preserve the natural appearance of the enhanced image.

7. Quantitative analysis

The quantitative measure of the enhanced images observed in Figure 1 proves the effectiveness of the proposed method based on balancing polynomials with suitable coefficient constraints. For all ten sample images, the proposed model significantly improves the structural information content, as reflected by higher entropy values. The enhanced images exhibit visual clarity, improved object visibility, and moderate illumination compared with the original dark images. In most cases, the proposed method provides us with stronger enhancement and balanced contrast than DA (Dehazing Algorithm), FOIET (Fractional-Order Image Enhancement Technique), and IFA (Improved Filtering Algorithm). Ref. [14], Ref. [1], and Ref. [18]. Though HSA (Histogram-Specific Algorithm) entropy values are high for some images, we can observe that these images may suffer from over-brightness and an unnatural look. But our proposed enhanced images preserve natural appearance and structural details. The visual quality of dark images remains stable for all ten test images. The CII values obtained, prove that the contrast enhancement is effective without unnecessary amplification. Therefore, the obtained numerical results and visual observations confirm that the proposed model based on balancing polynomials with suitable coefficient constraints is far better for low-light image enhancement in comparison with other conventional methods.

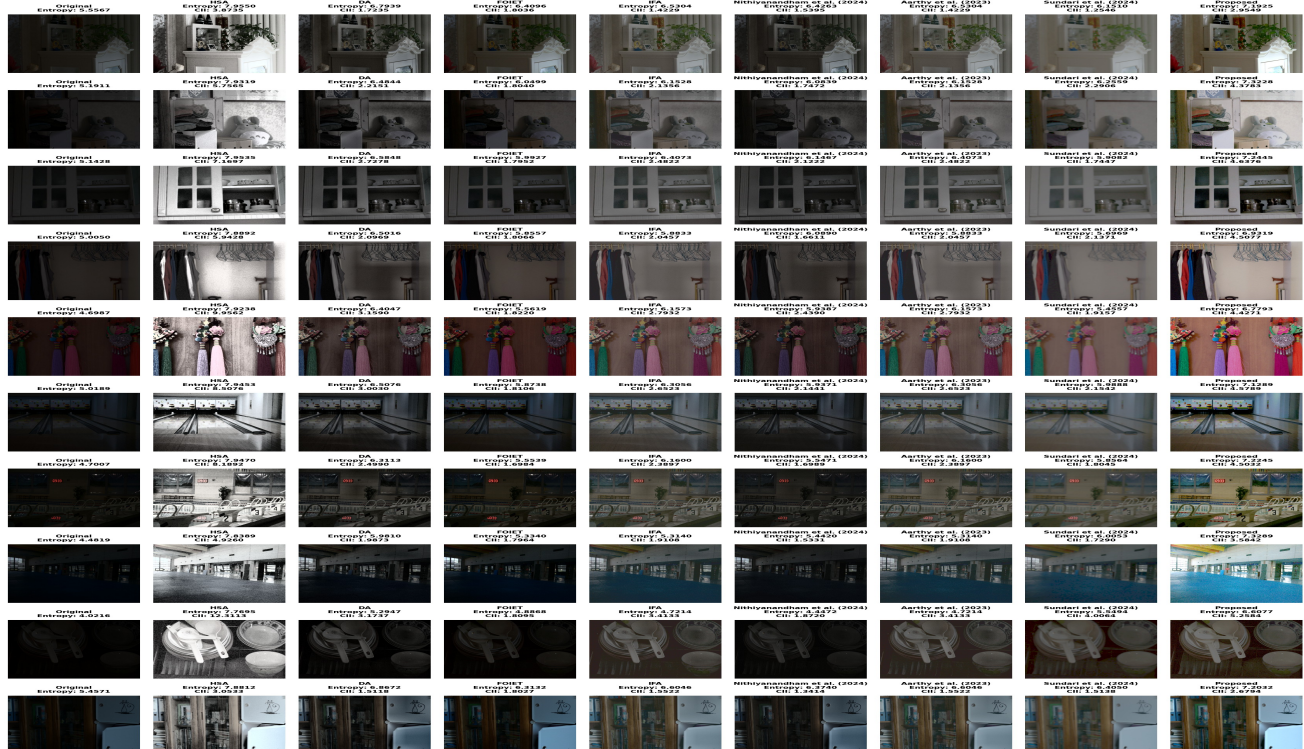


Figure 1. Comparison of entropy and contrast improvement index (CII) values for different low-light image-enhancement methods with the proposed method based on balancing polynomials with suitable coefficient constraints.

8. Qualitative analysis

The qualitative comparison of the proposed low-light method based on balancing polynomials with suitable coefficient inequality bounds in comparison with existing techniques, such as HSA, DA, FOIET and IFA, and also with other traditional methods [1, 14, 18], is carried out to evaluate its visual performance. Existing conventional methods frequently suffer from over-brightening or unnatural color appearance, which lead to the loss of important information content.

The comparisons in Tables 1 and 2 show that the suggested method works better. Furthermore, balancing polynomials based on the coefficient inequality bounds model guarantees a smooth and bounded intensity transformation. In theory, these constraints inhibit abrupt amplification of luminance, thereby retaining edge information and ensuring a natural-looking visual appearance across varying illumination conditions.

Table 1. Comparison of entropy values for different enhancement methods.

S.No	Image	Original	HSA [10]	DA [6]	FOIET [13]	IFA [9]	Ref. [14]	Ref. [1]	Ref. [18]	Proposed
1	Image 1	5.5567	7.9550	6.7939	6.4096	6.5304	6.4263	6.5304	6.1510	7.1925
2	Image 2	5.1911	7.9319	6.4844	6.0499	6.1528	6.0839	6.1528	6.2559	7.3228
3	Image 3	5.1428	7.9535	6.5848	5.9927	6.4073	6.1467	6.4073	5.9082	7.2445
4	Image 4	5.0050	7.8892	6.5016	5.8557	5.8833	6.0890	5.8833	5.6969	6.9319
5	Image 5	4.6987	7.9238	6.4047	5.5619	6.1573	5.9387	6.1573	5.4557	6.7793
6	Image 6	5.0189	7.9453	6.5076	5.8738	6.3056	5.9371	6.3056	5.9888	7.1289
7	Image 7	4.7007	7.9470	6.3113	5.5539	6.1600	5.5471	6.1600	5.8564	7.2245
8	Image 8	4.4819	7.8389	5.9810	5.3340	5.3140	5.4420	5.3140	6.0053	7.3289
9	Image 9	4.0216	7.7695	5.2947	4.8868	4.7214	4.4472	4.7214	5.5494	6.6077
10	Image 10	5.4571	7.8812	6.8672	6.3132	6.6046	6.3740	6.6046	6.4050	7.2032

Table 2. Comparison of CII values for different enhancement methods.

S.No	Image	HSA [10]	DA [6]	FOIET [13]	IFA [9]	Ref. [14]	Ref. [1]	Ref. [18]	Proposed
1	Image 1	3.8735	1.7235	1.8036	1.4229	1.5395	1.4229	1.2546	2.9549
2	Image 2	5.7565	2.2151	1.8040	2.1356	1.7472	2.1356	2.2906	4.3783
3	Image 3	7.1697	2.7278	1.7952	2.4822	2.1222	2.4822	1.7447	4.6376
4	Image 4	5.9428	2.0969	1.8066	2.0457	1.6611	2.0457	2.1371	4.5077
5	Image 5	9.9562	3.1590	1.8220	2.7932	2.4390	2.7932	1.9157	4.4271
6	Image 6	8.5076	3.0030	1.8106	2.6523	2.1441	2.6523	2.1542	4.5789
7	Image 7	8.1892	2.4990	1.6984	2.3897	1.6989	2.3897	1.8045	4.5032
8	Image 8	4.9260	1.9873	1.7964	1.9108	1.5331	1.9108	1.7290	3.5842
9	Image 9	12.3113	3.1737	1.8095	3.4133	1.8720	3.4133	4.0064	5.2584
10	Image 10	3.0533	1.5118	1.8027	1.5522	1.3414	1.5522	1.5138	2.6794

Remark

Although HSA entropy values are high, their corresponding CII values are extremely high, which leads to over-enhancement and the loss of some structural details. The proposed model acquires entropy values ranging from 6.6077 to 7.3289, and CII values range between 2.6794 and 5.2584, indicating strong contrast improvement while preserving structural details. Figure 2 shows the comparison with DA, FOIET, IFA, Ref. [14], Ref. [1], and Ref. [18], the proposed framework based on balancing polynomials with suitable coefficient inequality bounds consistently acquires better entropy values and higher contrast enhancement for almost all images. Therefore, the balancing polynomials-based model maintains a balance between structural detail preservation, effective contrast enhancement, and better visual quality.

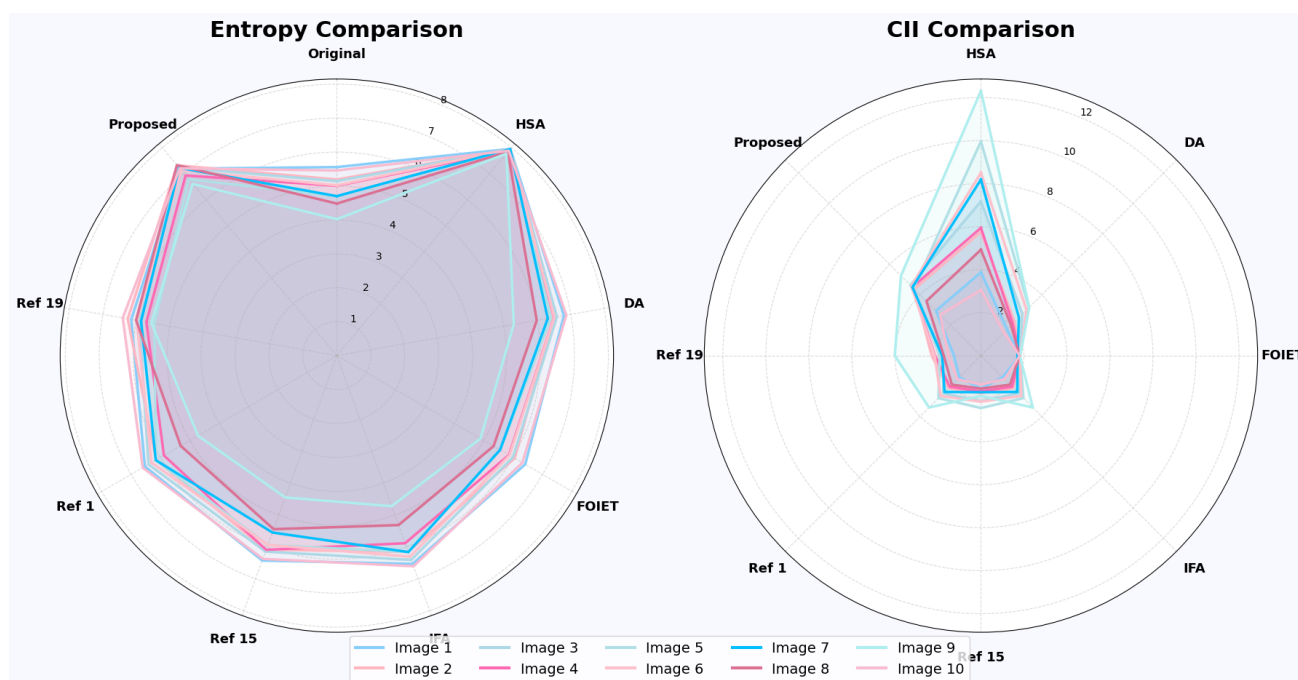


Figure 2. Radar-chart comparison of entropy and contrast improvement index (CII) values for ten test images using the original image, HSA, DA, FOIET, IFA, Ref. [15], Ref. [1], Ref. [19], and the proposed enhancement method.

9. Conclusions

In this research, we introduced a new subclass of bi-univalent functions defined using balancing polynomials. We derived the coefficient inequality bounds $|\alpha_2|$, $|\alpha_3|$ and established the corresponding Fekete–Szegő inequalities for the defined class. The obtained results inspired us to further present our low-light enhancement method, based on balancing polynomials with suitable coefficient inequality constraints. It uses gamma correction and adaptive polynomial components to make the image brighter without losing any details. Our proposed method further uses a control mechanism based on the contrast improvement index (CII) to stop over-enhancement, and it makes sure that the performance stays the same under different lighting situations. The experimental results show that the suggested approach consistently achieves higher entropy while maintaining controlled contrast levels. Overall,

the suggested approach provides a fair trade-off between detail enhancement and a natural visual appearance, making it efficient and consistent compared to existing methods [1, 14, 18].

Author contributions

Vanithakumari Balasubramaniam: Conceptualization, investigation, validation, methodology, experimentation, data interpretation and visualization, writing—original draft, Manuscript review and editing; Sibel Yalcin: Manuscript review and editing; Saravanan Gunasekar and Selvaraj Palanisamy: Conceptualization, investigation, methodology, experimentation, validation, data interpretation and visualization, manuscript review and editing, supervision. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors certify that they possess no financial interests or personal connections that could have affected the work described in this paper.

Data availability

The dataset utilized in this study is entirely available to the research community. One can find the Low-Light (LOL) dataset and other related materials at <https://daooshee.github.io/BMVC2018website/>. The repository has evaluation subsets, extra source code for benchmarking, and pairs of low-light and normal-light ground truth images. This dataset is available to everyone and provides a standard platform for comparative analysis. It has been widely used in research on improving low-light images. The fact that this resource is available ensures openness, makes it easier to reproduce the experimental results reported in this work, and makes future progress in the field easier. Researchers can use the dataset for academic and non-commercial purposes as long as they follow the rules set by the dataset authors.

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