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Research article

Stability of nonlinear systems with multi-delayed random impulses: Average estimation and delay approach

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Abstract: This study focuses on analyzing the exponential stability in the pth moment for a class of random impulsive delayed nonlinear systems (RIDNSs) influenced by multiple randomly delayed impulses. The underlying continuous-time dynamics are governed by random delay differential equations subjected to stochastic perturbations modeled as second-order moment processes. To establish new stability conditions, we employ tools such as the average random impulsive estimation (ARIE), average impulsive delay (AID), average delay time (ADT), and Lyapunov-based techniques. The developed criteria are applicable not only to inherently stable or unstable systems, but also to scenarios where impulses with stabilizing and destabilizing effects appear simultaneously. Several illustrative examples are included to verify the applicability and advantages of the proposed approach.

Keywords: random impulsive delayed nonlinear systems; exponential stability; average random impulsive estimation; average-delay impulse
Mathematics Subject Classification: 34K50, 90B15, 93D20

1. Introduction

Impulsive systems, which consist of both continuous and discrete dynamics, have been widely studied since they can describe sudden changes in system states driven by time or events. These systems are typically used to model processes that experience instantaneous jumps, making them applicable in various fields such as the control theory, biology, and economics. In impulsive systems, the continuous dynamics govern the behavior between impulses, while the discrete dynamics dictate the instantaneous changes during impulses. Stability analysis of such systems has garnered significant attention, as impulsive effects can either stabilize or destabilize a system depending on their nature [1, 2].

In real-world applications, systems are often subject to random disturbances. Depending on the type of noise, these random disturbances can be categorized into stochastic differential equations (SDEs) and random differential equations (RDEs). SDEs typically describe systems influenced by continuous white noise, where disturbances affect the system in a probabilistic manner with an infinite variance. On the other hand, RDEs deal with systems affected by more general types of random disturbances, often described by finite variance processes. The presence of such random disturbances adds complexity to the analysis of impulsive systems, as they introduce unpredictability in both the timing and magnitude of impulses. Moreover, the stability analysis of RDEs was established by Wu [3] in 2015. After then, many results have been achieved in recent years, as can be seen in references [4–6] and the references therein.

Another important factor in the behaviors of impulsive systems is time delay. Delays are inherent in many systems due to factors such as communication lags or processing times. Time delays can appear in both the continuous dynamics and the impulsive effects, impacting the overall stability of the system. In many cases, time delays can destabilize a system or make it more difficult to control, as the delayed feedback can cause the system to react to outdated information. A multitude of scholars have recently investigated the dynamical behavior with time delay in depth; see [7–10] and the references therein.

The behaviors of impulsive systems are mainly affected by two key factors: the frequency of impulses (impulsive density) and their impact on the system's stability (impulsive intensity). Traditionally, research has focused on a uniform impulsive intensity, yet incorporating multiple impulsive intensities enhances modeling flexibility, especially in practical applications. For example, [11] addressed input-to-state stability in deterministic nonlinear delay systems subject to several impulses, while [12] examined exponential stability in stochastic delay systems with multiple periodic impulses. Despite these advances, much of the existing literature still concentrates on deterministic impulses, thereby restricting the applicability of impulsive dynamical systems in broader scientific and engineering contexts [13]. In response, there has been growing interest in analyzing systems influenced by random impulses. Notably, almost sure stability conditions for nonlinear systems with random impulses were presented in [14], and synchronization as well as tracking control for multi-agent systems under random impulsive effects were investigated in [15] and [16], respectively. Furthermore, [17] studied the input-to-state stability of deterministic systems with multiple random impulses. Other studies include [18], which considered the pth moment exponential stability in stochastic delay systems with several random impulses, and [19], which established global asymptotic and mth moment exponential stability results for RIDNSs. The noise-to-state stability of such systems was explored in [20]. Nevertheless, stability analysis for RIDNSs with multiple random impulses—particularly when those impulses are delayed—remains largely underdeveloped. To the best of our knowledge, no existing work has yet established rigorous stability criteria for RIDNSs subjected to multiple random delayed impulses. Moreover, the imposed assumptions and conditions in our framework can be interpreted in terms of stabilizing and destabilizing mechanisms: stability may result either from stabilizing impulses acting on an unstable continuous flow or from a stable flow compensating destabilizing impulses. Similar issues have also been addressed in the deterministic setting. In particular, Dashkovskiy et al. [21] studied the compensation of one unstable dynamics by the stabilizing properties of the other one under a two-sided dwell-time condition. More recent works by the same line of research [22, 23] further extended these results by establishing dwell-time criteria that guarantee stability even when both the continuous dynamics and the impulsive dynamics are unstable. These conditions restrict the maximal duration of instability and ensure sufficiently frequent stabilizing actions, thereby preventing instability from dominating the system. A comparison with such deterministic results highlights that our contribution significantly extends these ideas to the stochastic framework with delayed impulses, thereby broadening the applicability of impulsive dynamical systems.

Motivated by the aforementioned analysis, this work focuses on examining the exponential stability of RIDNSs subjected to multiple delayed random impulses. The system under consideration features both time-varying delays in its continuous dynamics and discrete delays in its impulsive effects. These dual types of delays, combined with the randomness and variability of impulsive intensities, contribute to the complexity and difficulty of the problem. The main contributions of this study can be summarized as follows:

(1) In the discrete dynamics, the impulsive intensity is represented by a series of random variables, offering a more realistic and challenging scenario than the deterministic cases [5,10,24]. Moreover, the interaction of multiple types of random impulses on the random nonlinear systems provides a broader framework than the single impulse cases. Additionally, random impulsive intensity $\mathbb{E}\mu_k = \eta_k$ only has a finite number [6, 19, 20], whereas in our case, there are infinitely many.

(2) We establish the pth moment exponential stability of RIDNSs with multiple random delayed impulses by using average random impulsive estimation (ARIE), average impulsive delay (AID), average dwell time (ADT), and the Lyapunov method. Moreover, the criterion is comprehensive, as it applies to both stable and unstable original systems while also accounting for the simultaneous presence of stabilizing and destabilizing impulses. This result generalizes the findings in [19].

(3) Our results demonstrate that delays in multiple random delayed impulses can exert dual effects: they have the potential to stabilize unstable systems while also possessing the ability to destabilize already stable systems. This finding extends previous results in [5, 6, 19, 20, 24, 25], which did not consider delays in impulses.

2. Preliminaries

Notations: $PC^b_{\mathcal{F}_0}((-\infty, t_0], \mathbb{R}^n)$ denotes the family of all $\mathcal{F}_0$-measurable bounded $PC((-\infty, t_0], \mathbb{R}^n)$-valued random variables with $\sup_{s\le t_0} \mathbb{E}|\psi(s)|^p < \infty$, where $PC((-\infty, t_0], \mathbb{R}^n)$ represent the set of all piecewise right continuous function ψ with its norm $||\psi||=\sup_{s\le t_0}|\psi(s)|$ from $(-\infty, t_0]$ to $\mathbb{R}^n$. $||.||$ and $|.|$ represents the spectral norm and Euclidean norm, respectively. Let $\mathbb{Z}_+ = \{1, 2, ...\}$, $\mathbb{N} = \{0, 1, 2, ...\}$. For any matrix A, A^T and $\lambda_{max}(A)$ represent the transpose and the largest eigenvalue of A, respectively.

In this paper, we investigate the following RIDNSs with multiple random delayed impulses:

$$\begin{cases} \dot{x}(t) = f(t, x(t), x(t-\tau(t))) + g(t, x(t), x(t-\tau(t)))\xi(t), & t \neq t_k, t \geq t_0, \\ x(t_k) = I_k(x(t_k)^-, x(t_k-\theta_k)^-), & t = t_k, k \in \mathbb{Z}_+, \end{cases} \tag{2.1}$$

where $x(t)$ is the system state, which satisfies $x(t^-) = \lim_{s\to t^-} x(s)$. The initial value $x(t_0 + s) = \varphi(s) \in PC^b_{\mathcal{F}_0}((-\infty, 0], \mathbb{R}^n)$. $\tau(t) : [t_0, +\infty) \to [0, \infty]$, $\theta_k \in \mathbb{R}_+, t_{k-1} \le t_k - \theta_k \le t_k$, $\lim_{k\to+\infty} t_k = +\infty$, $k \in \mathbb{Z}_+$. We also assume that f, g fulfill the local linear growth condition and Lipschitz condition to guarantee the existence and uniqueness of the global solution for system (1) [20]. Moreover, $\xi(t) \in \mathbb{R}^m$ is a stochastic process that follows the assumption outlined below:

(A1): The stochastic process $\xi(t) \in \mathbb{R}^m$, which is independent of the initial value $\varphi(t)$, has a finite second-order moment, i.e., there exists a positive constant Θ such that

$$\sup_{t\geq t_0} \mathbb{E}|\xi(t)|^2 < \Theta^2 < \infty.$$

Remark 1. In this formulation, the time-varying delay $\tau(t)$ in the continuous flow is allowed to be unbounded, making it less conservative than existing results that assume bounded delays, such as in [6, 10, 19, 20].

Definition 2.1. For the initial value $\varphi \in PC^b_{\mathcal{F}_0}((-\infty, 0], \mathbb{R}^n)$, if there exist two positive constants $M > 0$ and $\lambda > 0$ satisfying

$$\mathbb{E}|x(t)|^p \leq Me^{-\lambda(t-t_0)} \sup_{s\leq 0} \mathbb{E}|\varphi(s)|^p,$$

then the trivial solution of system (1) is said to be pth moment exponentially stable (p-ES).

Definition 2.2. [26] Let $N(t_0, t)$ denote the number of impulses in the interval $(t_0, t]$. If there exist positive constants N_0 and T_0 such that

$$\frac{t_0 - t}{T_0} - N_0 \leq N(t_0, t) \leq \frac{t_0 - t}{T_0} + N_0.$$

Then N_0 is called the elasticity number, and T_0 is referred to as the average delay time (ADT).

Definition 2.3. [8] Let $N(t_0, t)$ represent the number of impulses in $(t_0, t]$. If there exist positive constants $\bar{\theta}$ and θ^*, such that for any sequence of impulsive instants t_k in $(t_0, t]$ the following inequality holds:

$$\bar{\theta}N(t_0, t) - \theta^* \leq \sum_{i=1}^{N(t_0,t)} \theta_i \leq \bar{\theta}N(t_0, t) + \theta^*,$$

then θ^* is called the preset value, and $\bar{\theta}$ is referred to as the average impulsive delay (AID).

Definition 2.4. [18] Let $N(t_0, t)$ represent the impulse count in $(t_0, t]$. Assume that there exist constants $\ln\bar{\mu}$ and μ^* such that for any sequence of impulsive intensities $\{\mu_k\}$ with $\mathbb{E}\mu_k < \infty$, the following inequality holds:

$$N(t_0, t)\ln\bar{\mu} - \mu^* \leq \sum_{i=1}^{N(t_0,t)} \ln\mathbb{E}\mu_i \leq N(t_0, t)\ln\bar{\mu} + \mu^*,$$

where $\mu^* \geq 0$, then the constant $\ln\bar{\mu}$ is called average random impulsive estimation (ARIE).

Definition 2.5. [27] If there exist constants $c > 0$ and $d \geq 0$ such that

$$\int_{t_1}^{t_2} f(s)ds \leq -c(t_2 - t_1) + d,\ t_2 > t_1 \geq t_0,$$

then the function $f(t)$ is called a uniformly asymptotically stable function (UASF).

3. Results

In this section, we will develop a set of sufficient conditions for the pth moment exponential stability of system (1).

Assumption 3.1. Assume that exists $\mathcal{U}(x(t), t)$: $\mathbb{R}^n \times [t_0, \infty) \to \mathbb{R}^+$, $\lambda(t) \in \mathbb{R}$, and $c_1 > 0, c_2 > 0$, $\lambda_0 > 0, \mu_k > 0, k \in \mathbb{Z}_+$ such that

(A2) $c_1|x(t)|^p \leq \mathcal{U}(x(t), t) \leq c_2 \sup_{s\leq 0} |x(t + s)|^p$;

(A3) $\mathcal{U}_t + \mathcal{U}_x f(x(t), x(t - \tau(t)), t) \leq \lambda(t)\mathcal{U}(x(t), t)$,

$\mathcal{U}_x g(x(t), x(t - \tau(t)), t) \leq \lambda_0 \mathcal{U}(x(t), t)$,

where $\mathcal{U}_t = \frac{\partial \mathcal{U}(x(t),t)}{\partial t}$, $\mathcal{U}_x = \frac{\partial \mathcal{U}(x(t),t)}{\partial x}$;

(A4) $\mathcal{U}(x(t_k), t_k) \leq \mu_k \mathcal{U}(x((t_k - \theta_k)^-), (t_k - \theta_k)^-)$, where random variable sequence $\{\mu_k\}$ is mutually independent with $\mathbb{E}\mu_k < \infty$ and also independent of $\xi(t)$;

(A5) there is a function δ such that $\mathbb{E}e^{\int_{t_0}^t \lambda_0|\xi(s)|ds} \leq e^{\delta(\lambda_0)(t-t_0)}$.

Remark 2. The conditions (A2), (A3), and (A5) in Assumption 3.1 were proposed in [19, 20]. For instance, Assumption (A5) is satisfied when $\xi(t)$ is a stochastic process with bounded moments. In our case, the condition (A4) takes into account the effect of impulsive delays, making it more general than that in [5, 6, 19, 20, 24, 25].

Lemma 3.1. Assume that Assumption 3.1 holds for all $t \in [t_0, +\infty)$, and let the number of impulses be denoted by $N(t_0, t)$. Then, we have:

$$\mathcal{U}(x(t), t) \leq \prod_{i=1}^{N(t_0,t)} \mu_i \mathcal{U}(x(t_0), t_0) \exp\left\{-\sum_{i=1}^{N(t_0,t)} \int_{t_i-\theta_i}^{t_i} \lambda(s)ds + \int_{t_0}^{t} \lambda(s) + \lambda_0|\xi(s)|ds\right\}. \tag{3.1}$$

Proof. For any $t \in [t_0, t_1)$, On the basis of (A3), we obtain

$$\begin{aligned} D^+[\mathcal{U}(x, t)] &= \mathcal{U}_t + \mathcal{U}_x(f(x(t), x(t - \tau(t)), t) + g(x(t), x(t - \tau(t)), t)\xi(t)) \\ &\leq (\lambda(t) + \lambda_0|\xi(t)|)\mathcal{U}(x, t). \end{aligned}$$

According to Gronwall inequality, we obtain

$$\mathcal{U}(x(t), t) \leq \mathcal{U}(x(t_0), t_0) \exp\left\{\int_{t_0}^{t} (\lambda(s) + \lambda_0|\xi(s)|)ds\right\} \tag{3.2}$$

When $t \in [t_1, t_2)$, According to (3.2), one has

$$\mathcal{U}(x(t), t) \leq \mathcal{U}(x(t_1), t_1) \exp\left\{\int_{t_1}^{t} (\lambda(s) + \lambda_0|\xi(s)|)ds\right\}$$

and

$$\mathcal{U}(x((t_1 - \theta_1)^-), (t_1 - \theta_1)^-) \leq \mathcal{U}(x(t_0), t_0) \exp\left\{\int_{t_0}^{t_1-\theta_1} (\lambda(s) + \lambda_0|\xi(s)|)ds\right\}.$$

From (A4), it can be verified that

$$\mathcal{U}(x(t), t) \leq \mathcal{U}(x(t_1), t_1) \exp\left\{\int_{t_1}^{t} (\lambda(s) + \lambda_0|\xi(s)|)ds\right\}$$

$$
\begin{aligned}
&\le \mu_1\mathcal{U}(x((t_1-\theta_1)^-),(t_1-\theta_1)^-)\exp\left\{\int_{t_1}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&\le \mu_1\mathcal{U}(x(t_0),t_0)\exp\left\{\int_{t_0}^{t_1-\theta_1}(\lambda(s)+\lambda_0|\xi(s)|)ds+\int_{t_1}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&= \mu_1\mathcal{U}(x(t_0),t_0)\exp\left\{-\int_{t_1-\theta_1}^{t_1}(\lambda(s)+\lambda_0|\xi(s)|)ds+\int_{t_0}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&\le \mu_1\mathcal{U}(x(t_0),t_0)\exp\left\{-\int_{t_1-\theta_1}^{t_1}\lambda(s)ds+\int_{t_0}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}.
\end{aligned}
$$

For any $t\in[t_k,t_{k+1})$, we obtain

$$
\begin{aligned}
\mathcal{U}(x(t),t) &\le \mathcal{U}(x(t_k),t_k)\exp\left\{\int_{t_k}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&\le \mu_k\mathcal{U}(x((t_k-\theta_k)^-),(t_k-\theta_k)^-)\exp\left\{\int_{t_k}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&\le \mu_k\mathcal{V}(x(t_{k-1}),t_{k-1})\exp\left\{\int_{t_{k-1}}^{t_k-\theta_k}(\lambda(s)+\lambda_0|\xi(s)|)ds+\int_{t_k}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&= \mu_k\mathcal{U}(x(t_{k-1}),t_{k-1})\exp\left\{-\int_{t_k-\theta_k}^{t_k}(\lambda(s)+\lambda_0|\xi(s)|)ds+\int_{t_{k-1}}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&\le \mu_k\mathcal{U}(x(t_{k-1}),t_{k-1})\exp\left\{-\int_{t_k-\theta_k}^{t_k}\lambda(s)ds+\int_{t_{k-1}}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}.
\end{aligned}
$$

For $\forall t\in[t_0,+\infty)$, by iterating from 1 to $N(t_0,t)$, it can be verified that

$$
\begin{aligned}
\mathcal{U}(x(t),t) &\le \prod_{i=1}^{N(t_0,t)}\mu_i\mathcal{U}(x(t_0),t_0)\exp\left\{-\sum_{i=1}^{N(t_0,t)}\int_{t_i-\theta_i}^{t_i}(\lambda(s)+\lambda_0|\xi(s)|)ds+\int_{t_0}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}\\
&\le \prod_{i=1}^{N(t_0,t)}\mu_i\mathcal{U}(x(t_0),t_0)\exp\left\{-\sum_{i=1}^{N(t_0,t)}\int_{t_i-\theta_i}^{t_i}\lambda(s)ds+\int_{t_0}^{t}(\lambda(s)+\lambda_0|\xi(s)|)ds\right\}.
\end{aligned}
$$

Theorem 3.1. *Assume that Assumption 3.1 holds and the following conditions are true.*
(A6) $\lambda(t)$ is a UASF with parameters c and d;
(A7) $-c+\delta(\lambda_0)+\frac{\ln\bar{\mu}+c\bar{\theta}+d}{T_0}<-\lambda^*$, where $\lambda^*>0$ is a constant;
then system (1) demonstrates pth moment exponential stability.

Proof. Letting $M_1=\max\{e^{\mu^*+\ln\bar{\mu}N_0},e^{\mu^*-\ln\bar{\mu}N_0}\}$, from Definition 2.4, we have

$$
\begin{aligned}
\mathbb{E}\prod_{i=0}^{N(t_0,t)}\mu_i &= \prod_{i=0}^{N(t_0,t)}\mathbb{E}\mu_i=\exp\left\{\sum_{i=0}^{N(t_0,t)}\ln\mathbb{E}\mu_i\right\}\\
&\le \exp\{N(t_0,t)\ln\bar{\mu}+\mu^*\}\\
&= e^{\mu^*}\exp\{\ln\bar{\mu}N(t_0,t)\},
\end{aligned}
$$

this fact, together with the Definition 2.2, yields

$$\mathbb{E}\prod_{i=0}^{N(t_0,t)}\mu_i \le M_1 \exp\left\{\frac{\ln\bar{\mu}}{T_0}(t-t_0)\right\}. \tag{3.3}$$

Next, we prove

$$\exp\left\{-\sum_{i=1}^{N(t_0,t)}\int_{t_i-\theta_i}^{t_i}\lambda(s)ds+\int_{t_0}^{t}\lambda(s)ds\right\}\le M_2\exp\left\{(\frac{c\bar{\theta}+d}{T_0}-c)(t-t_0)\right\}, \tag{3.4}$$

where $M_2 = e^{(c\bar{\theta}+d)N_0+c\theta^*}$. In fact, from (A6), Definitions 2.2 and 2.3, we have

$$\begin{aligned}
&\exp\left\{-\sum_{i=1}^{N(t_0,t)}\int_{t_i-\theta_i}^{t_i}\lambda(s)ds+\int_{t_0}^{t}\lambda(s)ds\right\}\\
=\;&\exp\left\{-\sum_{i=0}^{N(t_0,t)-1}\int_{t_i}^{t_{i+1}-\theta_{i+1}}\lambda(s)ds+\int_{t_{N(t_0,t)}}^{t}\lambda(s)ds\right\}\\
\le\;&\exp\left\{\sum_{i=0}^{N(t_0,t)-1}[-c(t_{i+1}-\theta_{i+1})-t_i)]-c(t-t_{N(t_0,t)})+dN(t_0,t)\right\}\\
=\;&\exp\left\{\sum_{i=0}^{N(t_0,t)}c\theta_i-c(t-t_0)+dN(t_0,t)\right\}\\
\le\;&\exp\left\{(c\bar{\theta}+d)N(t_0,t)+c\theta^*-c(t-t_0)\right\}\\
\le\;&M_2\exp\left\{(\frac{c\bar{\theta}+d}{T_0}-c)(t-t_0)\right\}.
\end{aligned}$$

By taking the expectations of both sides of (2) in Lemma 3.1, according to (4), (5), (A5), and (A7), we have

$$\begin{aligned}
\mathbb{E}\mathcal{U}(x(t),t)\;&\le\;\mathbb{E}\prod_{i=1}^{N(t_0,t)}\mu_i\mathbb{E}\mathcal{U}(x(t_0),t_0)\exp\left\{-\sum_{i=1}^{N(t_0,t)}\int_{t_i-\theta_i}^{t_i}\lambda(s)ds+\int_{t_0}^{t}\lambda(s)ds\right\}\mathbb{E}\exp\left\{\int_{t_0}^{t}\lambda_0|\xi(s)|ds\right\}\\
&\le\;M_1\exp\left\{\frac{\ln\bar{\mu}}{T_0}(t-t_0)\right\}M_2\exp\left\{(\frac{c\bar{\theta}+d}{T_0}-c)(t-t_0)\right\}\exp\left\{\int_{t_0}^{t}\delta(\lambda_0)ds\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&=\;M_1M_2\exp\left\{(\frac{\ln\bar{\mu}+c\bar{\theta}+d}{T_0}+\delta(\lambda_0)-c)(t-t_0)\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&\le\;Me^{-\lambda^*(t-t_0)}\mathbb{E}\mathcal{U}(x(t_0),t_0),
\end{aligned}$$

where $M = M_1M_2$. According to (A2), we have

$$\mathbb{E}|x(t)|^p \le \frac{c_2}{c_1}Me^{-\lambda^*(t-t_0)}\sup_{s\le 0}\mathbb{E}|\varphi(s)|^p.$$

From Definition 2.1, it can be concluded that system (1) demonstrates exponential stability in the pth moment.

Remark 3. This criterion in Theorem 3.1 is comprehensive, as it applies to both stable and unstable original systems while also considering the coexistence of stabilizing and destabilizing impulses. In addition, the conditions of Theorem 3.1 can also be interpreted intuitively. If the continuous flow of the system is unstable, stability may still be guaranteed provided that sufficiently strong stabilizing impulses are applied. Conversely, if the impulses themselves are destabilizing, the overall system can remain stable as long as the continuous flow is sufficiently stable to compensate for the impulsive effects.

Remark 4. When $\delta(\lambda_0)-c<0$, in order to maintain stability, $\ln\bar{\mu}$ should not be too large, implying that the ARIE must be controlled to avoid destabilizing effects. $\bar{\theta}$ should not be too large, meaning that AID should not be excessive, as long delays could undermine system stability. T_0 should not be too small, meaning that impulses should not occur too frequently, as frequent impulses could also destabilize the original system.

Remark 5. When $\delta(\lambda_0)-c>0$, to achieve stability, $\ln\bar{\mu}+c\bar{\theta}+d$ must be less than zero. Specifically, $\ln\bar{\mu}$, i.e., ARIE, needs to be sufficiently small to counteract the destabilizing effects. Additionally, $\bar{\theta}$, i.e., AID, should not be too large, as longer delays could exacerbate instability. ADT T_0 should be small enough, meaning that stabilizing impulses must occur frequently. A shorter T_0 ensures that the stabilizing effects of the impulses are applied regularly enough to overcome the system's inherent instability.

Corollary 3.1. *Let $\theta_k=0$ and assume that Assumption 3.1 holds.*
(A7′) $\lambda(t)+\delta(\lambda_0)+\frac{\ln\bar{\mu}}{T_a}$ is UASF with parameters c and d.
Then system (1) demonstrates pth moment exponential stability.

Proof. In the same way as demonstrated in Theorem 3.1, we have

$$\begin{aligned}
\mathbb{E}\mathcal{U}(x(t),t) &\le \prod_{i=0}^{N(t_0,t)}\mathbb{E}\mu_i\exp\left\{\int_{t_0}^{t}\lambda(s)+\delta(\lambda_0)ds\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&\le M_1\exp\left\{\frac{\ln\bar{\mu}}{T_0}(t-t_0)\right\}\exp\left\{\int_{t_0}^{t}\lambda(s)+\delta(\lambda_0)ds\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&= M_1\exp\left\{\int_{t_0}^{t}\frac{\ln\bar{\mu}}{T_0}+\lambda(s)+\delta(\lambda_0)ds\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&\le Me^{-c(t-t_0)}\mathbb{E}\mathcal{U}(x(t_0),t_0),
\end{aligned}$$

where $M=M_1e^d$. By applying (A2) and Definition 2.1, it can be concluded that system (1) demonstrates exponential stability in the pth moment.

Remark 6. In [19], the discrete part of the system is defined as $x(t_k)=I_{l(t_k)}(x(t_k)^-,t_k{}^-)$, $l(t_k)\in\{1,2,\cdots,v\}$, and the criterion for exponential stability is derived using the mode-dependent average impulsive interval. However, in our case, using the ARIE method, v can be infinite.

Corollary 3.2. *Let $\lambda(t)=\lambda$ be a constant. Assume that Assumption 3.1 holds and there exists $\lambda^*>0$ such that $\frac{\ln\bar{\mu}-\lambda\bar{\theta}}{T_0}+\lambda+\delta(\lambda_0)\le-\lambda^*$, then system (1) demonstrates pth moment exponential stability.*

Proof. According to Definition 2.3, we have

$$\exp\left\{\sum_{i=1}^{N(t_0,t)} -\lambda\theta_i\right\} \leq M_3 e^{-\lambda\bar{\theta}N(t_0,t)},$$

where $M_3 = \max\{e^{\lambda\theta^*}, e^{-\lambda\theta^*}\}$. In the same way as demonstrated in Theorem 3.1, we have

$$\begin{aligned}
\mathbb{E}\mathcal{U}(x(t),t) &\leq \prod_{i=0}^{N(t_0,t)} \mathbb{E}\mu_i \exp\left\{-\sum_{i=1}^{N(t_0,t)}(\lambda\theta_i) + (\lambda+\delta(\lambda_0))(t-t_0)\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&\leq M_1\exp\left\{\frac{\ln\bar{\mu}}{T_0}(t-t_0)\right\} M_3\exp\left\{-\lambda\bar{\theta}N(t_0,t)+\lambda+\delta(\lambda_0)(t-t_0)\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&\leq M\exp\left\{(\frac{\ln\bar{\mu}-\lambda\bar{\theta}}{T_0}+\lambda+\delta(\lambda_0))(t-t_0)\right\}\mathbb{E}\mathcal{U}(x(t_0),t_0)\\
&\leq Me^{-\lambda^*(t-t_0)}\mathbb{E}\mathcal{U}(x(t_0),t_0),
\end{aligned}$$

where $M = M_1M_3M_4$, $M_4 = \max\{e^{-\lambda\bar{\theta}N_0}, e^{\lambda\bar{\theta}N_0}\}$. By applying (A2) and Definition 2.1, it can be concluded that system (1) demonstrates exponential stability in the pth moment.

Remark 7. Through Corollary 3.3, we can see that AID has a dual effect: it can stabilize unstable systems as well as destabilize stable systems. This result extends previous results in [5,6,19,20,24,25], which did not consider delays in impulses.

Remark 8. RIDNSs have been studied in the literature [5, 6, 10, 19, 20, 24, 25]. However, in [5, 6, 19, 20, 24, 25], delays were only considered in continuous dynamics. Furthermore, in [10], the time delay in the continuous system was treated as a constant, whereas in our work, the delay is modeled as a time-varying function. In [5, 10, 24], all impulsive intensities μ_i are the same constant. The impact of multiple random impulses without delay is considered in [19, 20], where μ_i takes only a finite number of values.

4. Examples

Example 4.1. We first examine the following one-dimensional RIDNSs with multiple random delayed impulses:

$$\begin{cases} \dot{x}(t) = \boldsymbol{a}x(t) + \boldsymbol{b}f(x(t)) + \boldsymbol{c}f(x(t-\tau(t))) + \boldsymbol{d}x(t)\xi(t), & t \neq t_k,\\ x(t_k) = \eta_k h x(t_k-\theta_k)^-, & t = t_k. \end{cases} \tag{4.1}$$

Let $f(x(t)) = \tanh(x(t))$, $\tau(t) = \frac{1}{2}(1+\sin t) \in [0,1]$ with $\dot{\tau}(t) \leq \frac{1}{2} < 1$, and $\xi(t) = \frac{1}{2}\cos(t+T)$, where T is a uniformly distributed random variable on $[0, 2\pi]$. Since $\mathbb{E}|\xi|^2 = \frac{1}{8}$ and $\mathbb{E}e^{\int_{t_0}^{t}\lambda_0|\xi(s)|ds} = e^{\frac{\lambda_0}{\pi}(t-t_0)}$, assumptions (A1) and (A5) are satisfied with $\Theta^2 = \frac{1}{8}$ and $\delta(\lambda_0) = \frac{\lambda_0}{\pi}$.

Let the Lyapunov function

$$\mathcal{U}(t,x(t)) = \frac{1}{2}x^2(t) + 2e^3\int_{t-\tau(t)}^{t} e^{3(s-t)}x^2(s)\,ds.$$

Here, the Lyapunov function is chosen as a standard Lyapunov-Krasovskii functional for time-delay systems, where the integral term is specifically introduced to handle the time-varying delay $\tau(t)$. By the mean value theorem, there exists $a \in [t-\tau(t), t]$ such that

$$\begin{aligned}\mathcal{U}(t, x(t)) &= \frac{1}{2}x^2(t) + 2e^3 \int_{t-\tau(t)}^{t} e^{3(s-t)} x^2(s)\, ds \\ &= \frac{1}{2}x^2(t) + 2e^{3(a-t+1)}\tau(t)x^2(a) \\ &\leq \left(\frac{1}{2} + e^3\right) \sup_{s\leq 0} x^2(t+s),\end{aligned}$$

and clearly $\mathcal{U}(t, x(t)) \geq \frac{1}{2}x^2(t)$, so condition (A2) holds with $c_1 = \frac{1}{2}$, $c_2 = \frac{1}{2} + e^3$, $p = 2$.

We can estimate the derivative as follows:

$$\begin{aligned}\mathcal{U}_t + \mathcal{U}_x f &\leq x(t)\left[\boldsymbol{a}x(t) + \boldsymbol{b}f(x(t)) + \boldsymbol{c}f(x(t-\tau(t)))\right] + 2e^3x^2(t) \\ &\quad -2e^3e^{-3\tau(t)}(1-\dot{\tau}(t))x^2(t-\tau(t)) - 6e^3 \int_{t-\tau(t)}^{t} e^{3(s-t)}x^2(s)\, ds \\ &\leq \left(\boldsymbol{a} + \boldsymbol{b} + \frac{1}{4}\boldsymbol{c}^2 + 2e^3\right)x^2(t) - 6e^3 \int_{t-\tau(t)}^{t} e^{3(s-t)}x^2(s)\, ds.\end{aligned}$$

We also have

$$\mathcal{U}_x dx(t) \leq d\mathcal{U}(t, x(t)),$$

and for all $t \neq t_k$, the impulsive effect satisfies

$$\begin{aligned}\mathcal{U}(t_k, x(t_k)) &= \frac{1}{2}x^2(t_k) + 2e^3 \int_{t_k-\tau(t_k)}^{t_k} e^{3(s-t_k)}x^2(s)\, ds \\ &\leq \eta_k^2 \mathcal{U}((t_k-\theta_k)^-, x((t_k-\theta_k)^-)),\end{aligned}$$

so $\mu_k = \eta_k^2$.

Specifically, let

$$\boldsymbol{a} = -35, \quad \boldsymbol{b} = -15, \quad \boldsymbol{c} = 2, \quad \boldsymbol{d} = 0.25, \quad h = 1,$$

with $t_k = k$, $\theta_k = 0.3 + \frac{|\cos k|}{10}$, and $\eta_k = e^{X_k}$, where $X_k \sim N\left(-0.05 + \frac{|\sin k|}{100}, 0.1\right)$. Then, $\mathbb{E}\mu_k = \mathbb{E}[\eta_k^2] = e^{0.1+\frac{|\sin k|}{50}}$, so $\ln\bar{\mu} = 0.1$. From the above estimates, we have

$$\mathcal{U}_t + \mathcal{U}_x f \leq -3\mathcal{U}(t, x(t)),$$

with $\mu(t) = -3$, $c = 3$, and $d = 0$. From the inequality $\mathcal{U}_x dx(t) \leq 0.25\mathcal{U}(t, x(t))$, it follows that $\lambda_0 = 0.25$ and $\delta(\lambda_0) = \frac{1}{4\pi}$. With $T_0 = 1$ and $\bar{\theta} = 0.3$, we have

$$-c + \delta(\lambda_0) + \frac{\ln\bar{\mu} + c\bar{\theta} + d}{T_0} = -3 + \frac{1}{4\pi} + 0.1 + 0.9 = -1.975 < 0,$$

so condition (A7) holds. Therefore, all the conditions in Theorem 3.1 are satisfied, and system (4.1) is p-exponentially stable and its simulation is shown in Figure 1.

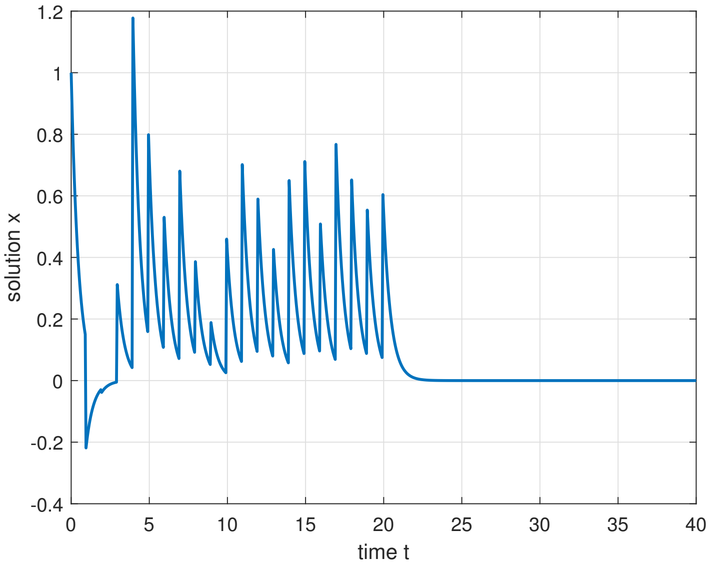

Figure 1. The behaviors of system (4.1).

Example 4.2. We examine the following two-dimensional RIDNSs with multiple random delayed impulses:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bf(x(t)) + Cf(x(t-\tau(t)))dt + Dx(t)\xi(t), \ t \neq t_k, \\ x(t_k) = \eta_k H x(t_k - \theta_k)^-, \ t = t_k. \end{cases} \tag{4.2}$$

Let us assume $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t))^T = (\tanh(x_1(t)), \tanh(x_2(t)))^T$, $\tau(t) = \frac{1}{2}(1+\sin t) \in [0,1]$ with $\dot{\tau}(t) \leq \frac{1}{2} < 1$, $\xi(t) = \frac{1}{2}\cos(t+T)$, here T is a uniformly distributed random variable on $[0, 2\pi]$. Since $\mathbb{E}|\xi|^2 = \frac{1}{8}$, $\mathbb{E}e^{\int_{t_0}^{t} \lambda_0 |\xi(s)| ds} = e^{\frac{\lambda_0}{\pi}(t-t_0)}$, thus (A1) and (A5) holds with $\Theta^2 = \frac{1}{8}$, $\delta(\lambda_0) = \frac{\lambda_0}{\pi}$.

Let the Lyapunov function

$$\mathcal{U}(t, x(t)) = \frac{1}{2}x(t)^T x(t) + 2e^3 \int_{t-\tau(t)}^{t} e^{3(s-t)} x(s)^T x(s) ds.$$

A constant $a \in [t-\tau(t), t]$ exists by the mean value theorem, such that

$$\begin{aligned} \mathcal{U}(t, x(t)) &= \frac{1}{2}x(t)^T x(t) + 2e^3 \int_{t-\tau(t)}^{t} e^{3(s-t)} x(s)^T x(s) ds \\ &= \frac{1}{2}x(t)^T x(t) + 2e^{3(a-t+1)}\tau(t) x(a)^T x(a) \\ &\leq \frac{1}{2}\sup_{s\leq 0} x(t+s)^T x(t+s) + 2e^3 \tau(t) \sup_{s\leq 0} x(t+s)^T x(t+s) \\ &\leq (\frac{1}{2} + e^3) \sup_{s\leq 0} x(t+s)^T x(t+s). \end{aligned}$$

Due to $\mathcal{U}(t, x(t)) \geq \frac{1}{2}x(t)^T x(t)$, then condition (A2) is satisfied with $c_1 = \frac{1}{2}$, $c_2 = \frac{1}{2} + e^3$, $p = 2$. On the other hand, we can get

$$\mathcal{U}_t + \mathcal{U}_x f = \mathcal{U}_t + \mathcal{U}_x[Ax(t) + Bf(x(t)) + cf(x(t-\tau(t)))]$$

$$
\begin{aligned}
&\leq x(t)^T[Ax(t) + Bf(x(t)) + cf(x(t-\tau(t)))] + 2e^3x(t)^Tx(t) \\
&- 2e^3e^{-3\tau(t)}(1-\dot{\tau(t)})x(t-\tau(t))^Tx(t-\tau(t)) - 6e^3\int_{t-\tau(t)}^{t} e^{3(s-t)}x(s)^Tx(s)ds \\
&\leq x(t)^T[A + B + 2e^3I]x(t) + x(t)^TCx(t-\tau(t)) \qquad (4.3)\\
&- x(t-\tau(t))^Tx(t-\tau(t)) - 6e^3\int_{t-\tau(t)}^{t} e^{3(s-t)}x(s)^Tx(s)ds \\
&\leq x(t)^T[A + B + \frac{1}{4}C^TC + 2e^3I]x(t) - 6e^3\int_{t-\tau(t)}^{t} e^{3(s-t)}x(s)^Tx(s)ds.
\end{aligned}
$$

We can further derive

$$
\mathcal{U}_xDx(t) \leq \lambda_{max}(D)\mathcal{U}(t, x(t)). \qquad (4.4)
$$

Regarding condition (A4), for all $t \neq t_k$, we have

$$
\begin{aligned}
\mathcal{U}(t_k, x(t_k)) &= \frac{1}{2}x(t_k)^Tx(t_k) + 2e^3\int_{t_k-\tau(t_k)}^{t} e^{3(s-t_k)}x(s)^Tx(s)ds \\
&\leq \eta_k^2\mathcal{U}((t_k-\theta_k)^-, x((t_k-\theta_k)^-)),
\end{aligned}
$$

thus, $\mu_k = \eta_k^2$.
Case 1. Let

$$
A = \begin{bmatrix} -15 & 0 \\ 1 & -15 \end{bmatrix}, \quad B = \begin{bmatrix} -9 & 0 \\ 2 & -9 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.25 & 0 \\ 1 & 0.2 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
$$

Letting $t_k = k$, $\theta_k = 0.3 + \frac{|\cos k|}{10}$, and $\eta_k = e^{X_k}$, where X_k follows a normal distribution defined as: $X_k \sim N(-0.05 + \frac{|sink|}{100}, 0.1)$. Thus $\mathbb{E}\mu_k = \mathbb{E}\eta_k^2 = e^{2X_k} = e^{0.1+\frac{|sink|}{50}}$, then $\ln\bar{\mu} = 0.1$.

From (4.3), we have $\mathcal{U}_t + \mathcal{U}_xf \leq -3\mathcal{U}(t, x(t))$, that is $\mu(t) = -3$; thus $c = 3$, $d = 0$. From (4.4), we have $\mathcal{U}_xDx(t) \leq \frac{1}{4}\mathcal{U}(t, x(t))$, $\lambda_0 = \frac{1}{4}$. Thus (A3) and (A6) hold. Additionally, $\delta(\lambda_0) = \frac{1}{4\pi}$; this indicates that the continuous system is stable. We have $T_0 = 1$, $\bar{\theta} = 0.3$. Thus

$$
-c + \delta(\lambda_0) + \frac{\ln\bar{\mu} + c\bar{\theta} + d}{T_0} = -2.92 < 0,
$$

this indicates that (A7) holds. Thus, the condition in Theorem 3.1 is satisfied, which implies that system (2) is p-ES, and its simulation is shown in Figure 2.

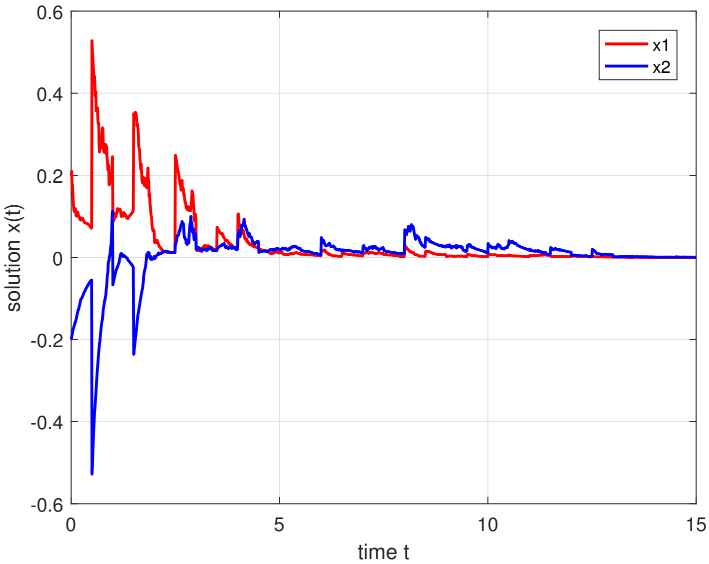

Figure 2. The behaviors of system (4.2) in Case 1.

Case 2. Let A, B, C and H same in case 1, i.e., $\lambda(t) = -3$, let $D = \begin{bmatrix} 10 & 0 \\ 1 & 2.5 \end{bmatrix}$, then from (4.4), $\mathcal{V}_x Dx(t) \leq 10\mathcal{V}(t, x(t))$, i.e., $\lambda_0 = 10$, $\delta(\lambda_0) = \frac{10}{\pi}$. This indicates that the continuous system is unstable. Let $t_k = k$, $\theta_k = 0.2 + \frac{|\sin k|}{10}$, and $\eta_k = e^{-X_k}$, where X_k follows a normal distribution defined as: $X_k \sim N(0.6 + \frac{|sink|}{100}, 0.1)$, thus $T_0 = 1$, $\bar{\theta} = 0.2$, $\mathbb{E}\mu_k = e^{-1+\frac{|sink|}{50}}$, i.e., $\ln \bar{\mu} = -1$. We have

$$-c + \delta(\lambda_0) + \frac{\ln \bar{\mu} + c\bar{\theta} + d}{T_0} = -0.215 < 0,$$

this indicates that (A7) holds. Thus, the condition in Theorem 3.1 is satisfied, which implies that system (2) is p-ES, and its simulation is shown in Figure 3.

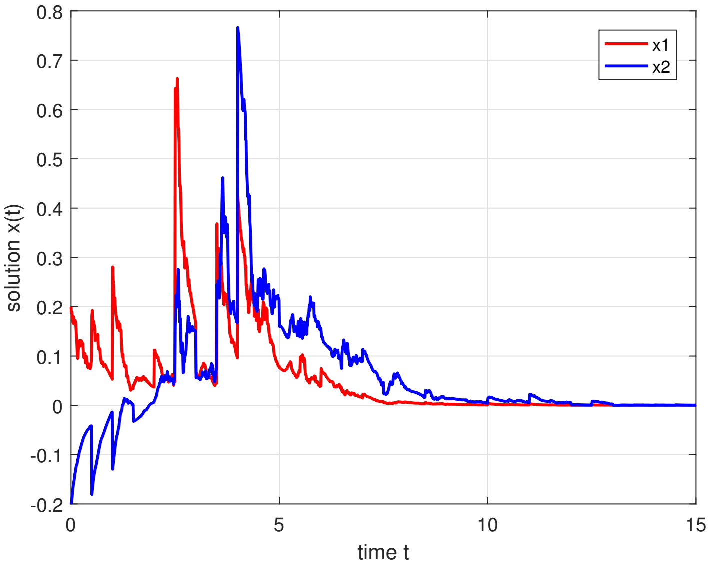

Figure 3. The behaviors of system (4.2) in Case 2.

Remark 9. This numerical simulation example verifies the correctness of our Theorem 3.1. In Case 1, $\ln \bar{\mu} = 0.1 > 0$, this indicate that when the continuous system is stable, the system remains stable even under the influence of destabilizing random delayed impulses. When the continuous system is unstable, stabilizing random delayed impulses can stabilize the system.

5. Conclusions

This paper has investigated the pth moment exponential stability of RIDNSs with multiple random delayed impulses. By employing ARIE, AID, ADT, and the Lyapunov method, we have derived novel criteria for exponential stability. These criteria are comprehensive, as they apply to both stable and unstable original systems and also account for cases where stabilizing and destabilizing impulses coexist, thereby generalizing previous findings. Our work has relaxed prior conditions, providing a broader framework. Additionally, our results indicate that delayed impulses can have dual effects: they may stabilize unstable systems or destabilize stable ones. Future work will focus on further relaxing restrictions on impulsive delays, specifically by removing the condition $t_{k-1} \leq t_k - \theta_k \leq t_k$, to enhance applicability in stability analysis. This condition currently plays a key role in preventing overlapping impulsive effects during analysis, and relaxing it would introduce new mathematical challenges in handling the interaction between multiple impulses.

Author contributions

Yao Lu: Writing and original draft preparation. Dehao Ruan: Validation and software development. Quanxin Zhu: Project administration and supervision.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

Quanxin Zhu is the Guest Editor of the special issue "Dynamics, control, optimization, and applications of nonlinear systems" for AIMS Mathematics. Quanxin Zhu was not involved in the editorial review and the decision to publish this article.

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