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Research article

Research on the propagation mechanism of traffic congestion warning information with random interference

Huining Yan[1], Meihui Song[2], Hua Li[3,*] and Qiubai Sun[4]

[1] School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, China
[2] School of Economics and Management, University of Science and Technology Beijing, Beijing, China
[3] School of Business Administration, University of Science and Technology Liaoning, Anshan, Liaoning, China
[4] Asset Company, University of Science and Technology Liaoning, Anshan, Liaoning, China

* **Correspondence:** Email: lh1@ustl.edu.cn.

Abstract: In social systems, information dissemination is affected by environmental factors. Moreover, positive information can promote social development. Therefore, a stochastic SEIR model was proposed to study the mechanism of traffic congestion warning information dissemination. In this article, we proved the existence of global positive solutions and the extinction of information, and proposed sufficient conditions for stationary distributions. Based on the Hamiltonian function, an optimal stochastic control strategy for the random propagation model was proposed. The numerical simulation results indicated that the theoretical results could be validated and compared with deterministic models. Adding random interference could promote the propagation of congestion information, it could promote the propagation of information, and better control traffic congestion. The volatility of congestion warning information propagation became more apparent with the increase of random disturbance intensity. By controlling random parameters, the propagation of congestion warning information could be effectively controlled, thereby controlling congestion. Moreover, the propagation effect of the proposed optimal stochastic control strategy was better than that of the random model, which verified the effectiveness of the proposed optimization control strategy.

Keywords: congestion warning information; stochastic SEIR model; random parametric perturbations; optimal stochastic control strategy; traffic congestion
Mathematics Subject Classification: 60H10, 92D30, 93E20

1. Introduction

In modern cities, traffic congestion is not only an engineering problem, but also an information management issue. With the development of intelligent transportation systems (ITS) and mobile Internet, the real-time transmission of traffic information has had a profound impact on drivers' route choice, travel decision-making, and the overall road network efficiency. However, the accuracy, dissemination speed, and coverage of information may alleviate congestion or exacerbate traffic flow fluctuations due to misleading or information overload [1]. Therefore, studying the dissemination mechanism of congestion information is of great significance for optimizing traffic management and enhancing road network resilience.

In depth research on the dissemination mechanism, influencing factors, and regulatory effects of congestion information on traffic behavior not only helps to build a more intelligent traffic management system, but also provides a new idea for urban governance to guide traffic flow with information flow. Ahmad, M proposed an intelligent transportation system that conveys danger warnings and imminent traffic congestion information to all vehicles within its coverage area [2]. Yilmaz implemented traffic congestion control at the route level through a route guidance system that provides proactive warnings or suggestions to nearby or en route drivers [3]. Sha proposed that an efficient and accurate traffic information transmission system is key support for real-time monitoring of road network operation status, scientific analysis of traffic situation, and rapid implementation of emergency response [4]. Alobeidyeen et al. developed an Information Network Flow Model (INFM) to study its correlation with the evolution of traffic congestion on the transportation network under discrete timestamps [5]. Ning et al. innovatively constructed the architecture of traffic early warning information distribution system and designed a collaborative transmission scheme of vehicle early warning information based on the reverse path selection mechanism [6]. Cai et al. proposed that the dissemination of Traffic Accident Information will greatly contribute to reducing fuel consumption and congestion in the future Internet of Vehicle environment [7]. Xie et al. dynamically send current and expected traffic status information to users through advanced travel information systems to minimize congestion and enhance road network capacity [8].

In terms of methods, based on the high similarity in diffusion mechanisms between traffic congestion information dissemination and virus transmission, information dissemination can be studied using infectious disease models. Saberi proposed to simulate the process of infectious disease transmission among populations to study the spread and dissipation of urban congestion [9]. Zhang developed a new SIR propagation model that considers the impact of receivers on information propagation [10]. Qin and Li established a network-based SEIR model to examine how self-protection awareness affects disease transmission [11]. She proposed the SIS propagation model research on the collaborative evolution mechanism between epidemic information and public opinion on social networks [12]. Based on the SEIR model, Nian proposed measures that can effectively regulate the process of public opinion dissemination [13].

In terms of information dissemination, scholars have considered the impact of random factors on the dissemination mechanism. Ma introduced random factors to verify the characteristics and mechanisms of uncertain information propagation [14]. Zhou proposed a rumor propagation model that is based on information intervention and considers the decay of information over time [15]. Li et al. proposed an algorithm for generating graph structured data to alleviate the excessive smoothing of node information in deep network propagation [16]. Di Crescenzo proposed a growth model for random false news dissemination [17]. Myilsamy proposed a nonlinear rumor propagation model and randomly studied the rumor propagation problem in homogeneous networks [18]. Nian and Zhang analyzed the driving mechanism of opinion leaders on the spread of public opinion and proposed an efficient online public opinion intervention strategy based on this [19]. Nian et al. also defined three ways in which information propagates in the hierarchical structure of the network and constructed a hierarchical network information propagation model to further explore the laws of information propagation in the network [20]. Regarding the dissemination of positive information, Nyawa et al. proposed that the dissemination of reliable information can eliminate people's hesitation about getting vaccinated [21]. Al-Oraiqat et al. demonstrated the effectiveness of using the positive information dissemination model of "opinion leaders" and improved the efficiency of information dissemination [22]. Moreover, to study random information transmission in transportation networks, Ravi et al. proposed that Internet services use a random model to transmit information to vehicles and predict their traffic [23]. Lu proposed multiple dynamic evolution equations to elucidate the interactions between traffic flow, information creation, and dissemination, achieving information exchange between vehicles [24]. Xie et al. proposed the development and analysis of an interruption network evaluation method in a random traffic environment to promote information dissemination in the transportation network [25]. Zheng proposed an optimization model based on arterial signal information control stochastic simulation with traffic safety and efficiency as dual objectives [26].

The abovementioned scholars studied the influence of random factors on the dissemination of public sentiment and uncertain information, but few researchers have considered the influence of random factors on dissemination of positive information, such as congestion warning information. In the real world, not all information is unfavorable, and some positive information can promote social development. When severe traffic congestion occurs, effective dissemination of congestion warning information can help each individual take proactive measures. The impact of dynamic connections leads to random changes. Introducing random factors can cause the simulation of congestion warning information dissemination model to be more realistic. In view of this, a stochastic SEIR model is proposed to analyze the mechanism by which environmental factors affect the dissemination of traffic congestion warning information. The existence of global positive solutions is proved, the extinction and persistence of congestion information are verified, and key parameters are selected as control variables. The effectiveness of the proposed theorem is verified through numerical simulations and compared with that of deterministic models. The subsequent content of the paper is: In Section 2, we propose the stochastic congestion information propagation model constructed on the basis of model assumptions. In Section 3, we prove the existence of global positive solutions, the extinction of information, and propose sufficient conditions for stationary distributions. In Section 4, we propose an optimal stochastic control strategy for congestion information dissemination. We systematically study the impact of random noise intensity on information propagation dynamics through numerical simulation experiments, evaluate the regulatory performance of the optimal stochastic control strategy, and compare and analyze the propagation characteristics with deterministic models in Section 5. Finally, in Section 6, we summarize the content of the article.

2. Materials and methods

The node status includes four types: The ignorant S refers to the group that has not yet received congestion information, the hesitant E refers to the group of people who, upon receiving congestion information, exhibit two states: One is a positive attitude towards congestion information and a preference for faster driving modes; the other party holds a negative attitude toward congestion information and tends to choose driving modes with shorter distances. The two groups are collectively referred to as hesitant individuals. The distributor I refers to the group of people who receive congestion information and spread it through social media, and the immune R is not interested in and no longer accepts congestion information.

The SEIR propagation model is shown in Figure 1.

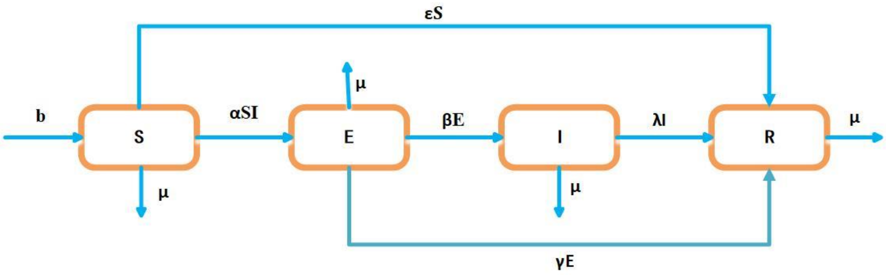


Figure 1. The SEIR propagation model.

The parameter definitions of the SEIR model are as follows:

(1) Among them, the time-varying increment of network nodes is b, and each type of user may exit the social network for any reason; that is, the population migration rate is μ.

(2) When the ignorant receive congestion information, different actors will have different attitudes toward the congestion information, hesitate whether to spread the congestion information, and become hesitant with a probability of α. In addition, ignorant individuals who receive traffic congestion information and are not interested in it or do not involve themselves will transform into immune individuals with a probability of ε.

(3) Some hesitant individuals hold a positive attitude toward congestion information and are willing to spread it through social media, with a probability of β becoming distributors. Some hesitant individuals hold a negative attitude towards traffic congestion information and believe that traffic congestion is not related to individuals. They will directly transform into immune individuals with a probability of γ.

(4) Some information distributors gradually lose their willingness to spread over time and become immune with an λ probability.

In addition, environmental noise characterizes the social uncertainty and disturbance factors inherent in the information dissemination system. It is unscientific to ignore the impact of random environmental noise fluctuations when studying congestion warning information. The incorporation of environmental noise into the deterministic model of information dissemination can better represent the propagation mode of congestion warning information in real traffic congestion situations. Due to the random disturbance α and β of environmental noise, the contact rate between individuals and information disseminators exhibits random fluctuations, as well as the dissemination rate of becoming a disseminator of congestion warning information. The parameter representation of random interference is as follows (Eq (1))

$$\alpha \to \alpha + \sigma_1 B_1(t), \beta \to \beta + \sigma_2 B_2(t). \tag{1}$$

B_i is an independent standard Brownian motion, and $\sigma_i^2 > 0 (i = 1, 2)$ represents the intensity of $B_i (i = 1, 2)$. In this work, B_1 and B_2 indicate that there is no mutual influence between α and β. In traffic congestion scenarios, the efficiency of information dissemination mainly depends on the degree of cognitive adoption of individuals and is not related to the frequency of physical contact, and the two are independent of each other.

On this basis, a stochastic $SEIR$ model is constructed:

$$\begin{cases} dS(t) = \left[b - \alpha SI - \varepsilon S - \mu S\right]dt - \sigma_1 SIdB_1(t), \\ dE(t) = \left[\alpha SI - \beta E - \gamma E - \mu E\right]dt + \sigma_1 SIdB_1(t) - \sigma_2 EdB_2(t), \\ dI(t) = \left[\beta E - \lambda I - \mu I\right]dt + \sigma_2 EdB_2(t), \\ dR(t) = [\varepsilon S + \gamma E + \lambda I - \mu R]dt. \end{cases} \tag{2}$$

3. Results

3.1. Analysis of the existence of global positive solutions

Let $(\Omega, \{F\}_{t\geq 0}, P)$ be a complete probability space, where the domain stream $\{F\}_{t\geq 0}$ satisfies the usual condition (the measure satisfies the axioms of monotonicity, continuity, and completeness). It can also be expressed as follows: $R_+^3 = \{(x_1, x_2, x_3) | x_i > 0, i = 1,2,3\}$.

The completeness analysis of the dynamic behavior of stochastic systems is based on the existence of global solutions. Moreover, according to the actual situation, the dynamic model of coupled state transfer needs to take positive values. The random system (Eq (2)) can be proven globally positive using Theorem 3.1.

Theorem 3.1. Under any initial condition $(S(t), E(t), I(t))$, $t \geq 0$, Eq (2) has a probability of 1 that a unique global solution $(S(t), E(t), I(t))$ exists. For any $t \geq 0$, $(S(t), E(t), I(t)) \in R_+^3, (S(t), E(t), I(t)) \in R_+^3 a.s.$

Proof. For any $(S(t), E(t), I(t)) \in R_+^3$, the coefficients of stochastic system Eq (2) satisfy the Lipschitz continuity condition within the local rang. Therefore, random system Eq (2) has a strong solution with unique orbits (S, E, I) on a local interval that holds for any $t \in [0, \tau_e)$, where τ_e is a certain blasting moment [27]. The proof of global existence reduces to verifying $\tau_e = \infty a.s.$ the following stopping time criterion: $\tau^+ = \inf\{S(t) \geq 0 or E(t) \geq 0 or I(t) \geq 0, t \in [0, \tau_e)\}$.

If $\inf \varnothing = \infty$ is specified, providing that $\tau^+ \leq \tau_e$ can be proven, then $\tau^+ = \infty a.s.$ can be proven, and $\tau_e = \infty a.s.$ and $(S(t), E(t), I(t)) \in R_+^3 a.s.$ will also be proven. Otherwise, when $\tau^+ < \infty$, there will be a time when $T > 0$ makes $P(\tau^+ < T) > 0$.

We define C^2 function $V : \mathbb{R}_+^3 \to \mathbb{R}_+^3$: $V(x) = \ln SEI$ (Eq (3)).

$$V(t) = \ln SEI = \ln S + \ln E + \ln I. \tag{3}$$

Because for any $\omega \in (\tau^+ < T), t \in [0, \tau^+)$, using the $It\hat{o}$ formula, we obtain the following (Eq (4)):

$$\begin{aligned} dV &= \frac{dS}{S} + \frac{dE}{E} + \frac{dI}{I} - \frac{(dS)^2}{2S^2} - \frac{(dE)^2}{2E^2} - \frac{(dI)^2}{2I^2}, \\ &= \left(\frac{b}{S} - \alpha I - \varepsilon - \mu - \frac{1}{2}\sigma_1^2 I^2\right)dt + \left(\frac{\alpha SI}{E} - \beta - \gamma - \mu - \frac{1}{2}\sigma_1^2 \frac{S^2 I^2}{E^2} - \frac{1}{2}\sigma_2^2\right)dt \\ &\quad + \left(\frac{\beta E}{I} - \lambda - \mu - \frac{1}{2}\sigma_2^2 \frac{E^2}{I^2}\right)dt - \sigma_1 I dB_1 + \frac{\sigma_1 SI}{E} dB_1 - \sigma_2 dB_2 + \frac{\sigma_2 E}{I} dB_2, \\ &= \left(\frac{b}{S} + \frac{\alpha SI}{E} + \frac{\beta E}{I} - \alpha I - \varepsilon - \beta - \gamma - \lambda - 3\mu - \frac{1}{2}\sigma_1^2 I^2 - \frac{1}{2}\sigma_1^2 \frac{S^2 I^2}{E^2} - \frac{1}{2}\sigma_2^2 - \frac{1}{2}\sigma_2^2 \frac{E^2}{I^2}\right)dt \\ &\quad - \left(\sigma_1 I - \frac{\sigma_1 SI}{E}\right)dB_1 - \left(\sigma_2 - \frac{\sigma_2 E}{I}\right)dB_2. \end{aligned} \tag{4}$$

The $V(t)$ implies (Eq (5))

$$dV(t) \geq L(S,E,I)dt - \sigma_1(I - \frac{SI}{E})dB_1 - \sigma_2(1 - \frac{E}{I})dB_2. \tag{5}$$

Among them (Eq (6)),

$$L(S,E,I) = -\varepsilon - \beta - \gamma - \lambda - 3\mu - \frac{1}{2}\sigma_1^2 I^2 - \frac{1}{2}\sigma_1^2 \frac{S^2 I^2}{E^2} - \frac{1}{2}\sigma_2^2 - \frac{1}{2}\sigma_2^2 \frac{E^2}{I^2}. \tag{6}$$

Thus,

$$\begin{aligned} V(t) \geq\ & V(0) + \int_0^t L(S(j),E(j),I(j))dj \\ & - \int_0^t \sigma_1(I(j) - \frac{S(j)I(j)}{E(j)})dB_1(j) \\ & - \int_0^t \sigma_2(1 - \frac{E(j)}{I(j)})dB_2(j). \end{aligned} \tag{7}$$

Since the $V(\tau^+)$ part is equal to 0, we therefore obtain

$$\lim_{t \to \tau^+} V(t) = -\infty. \tag{8}$$

In the system (Eq (7)), we obtain $t \to \tau^+$

$$\begin{aligned} -\infty \geq\ & V(0) + \int_0^t L(S(j),E(j),I(j))dj \\ & - \int_0^t \sigma_1(I(j) - \frac{S(j)I(j)}{E(j)})dB_1(j) \\ & - \int_0^t \sigma_2(1 - \frac{E(j)}{I(j)})dB_2(j) > -\infty. \end{aligned} \tag{9}$$

According to Eqs (7) and (8), Eq (9) is less than or equal to $-\infty$. Moreover, for any initial condition, $(S(0),E(0),I(0)) \in R_+^3$ and $S(j),E(j),I(j)$ in Eq (9) always lie within a forward invariant bounded set. $S(j),E(j),I(j)$ >0, and Eq (9) is greater than $-\infty$, which is contradictory. The result of Eq (9) rejects the original hypothesis $\tau^+ < \infty$; thus, we obtain $\tau^+ = \infty$. For any $t \in [0, \tau_e)$, a unique local solution $S(j),E(j),I(j)$ for the random system Eq (2) exists.

3.2. Disappearance of the congestion warning information

The conditions for the extinction of congestion information are proposed in Theorems 3.2 and 3.3. In the random SEIR model established in the article, the extinction of the congestion information requires the following conditions to be met: 1) No individuals who are skeptical of congestion warning information are present under road congestion conditions, and 2) in the state of road congestion, no distributor of the information is present. If either of the above two conditions is met, the information is not present in the social system.

First, Theorem 3.2 provides the conditions under which individuals who are skeptical of congestion warning information are not present under road congestion conditions.

Theorem 3.2. For any determined initial condition $(S(0),E(0),I(0)) \in R_+^3$, $\limsup_{t \to \infty} \frac{\ln E}{t} \leq G(\sigma_1^2, \sigma_2^2)$ is established, and $G(\sigma_1^2, \sigma_2^2) < 0$. At this point, $E(t)$ tends exponentially toward 0, with

$$G(\sigma_1^2,\sigma_2^2)=\frac{\alpha^2}{2\sigma_1^2}-\left(\beta+\gamma+\mu+\frac{1}{2}\sigma_2^2\right).$$

Proof. Using formula $It\hat{o}$ to differentiate $E(t)$ in random system Eq (2), and (Eq (10))

$$d\ln E(t)=\left[\frac{\alpha SI}{E}-(\beta+\gamma+\mu)-\frac{1}{2}\sigma_1^2\frac{S^2I^2}{E^2}-\frac{1}{2}\sigma_2^2\right]dt+\frac{\sigma_1 SI}{E}dB_1-\sigma_2 dB_2. \tag{10}$$

Thus, we obtain Eqs (11) and (12)

$$\begin{aligned}\ln E(t)=\ln E(0)+\int_0^t\left[\frac{\alpha S(j)I(j)}{E(j)}-(\beta+\gamma+\mu)-\frac{1}{2}\sigma_1^2\frac{S^2(j)I^2(j)}{E^2(j)}-\frac{1}{2}\sigma_2^2\right]dj\\+\int_0^t\frac{\sigma_1 S(j)I(j)}{E(j)}dB_1(j)-\sigma_2B_2(t).\end{aligned} \tag{11}$$

$$\Phi_1(t)=\int_0^t\frac{\sigma_1 S(j)I(j)}{E(j)}dB_1(j). \tag{12}$$

Among them, the quadratic variation of $\Phi_1(t)$ (Eq (13)) is

$$\langle\Phi_1(t)\rangle=\sigma_1^2\int_0^t\frac{S^2(j)I^2(j)}{E^2(j)}dj. \tag{13}$$

According to the exponential martingale inequality

$$P\left\{\sup_{0\le t\le k}\left[\Phi(t)-\frac{d}{2}\langle\Phi(t)\rangle\right]>\frac{d}{2}\ln k\right\}\le k^{-\frac{2}{d}}. \tag{14}$$

Equation (14) where $0<d<1$ and where k is a random integer. By using the Borel Cantelli lemma, we can know that the random integer $k_0(\omega)$ exists, such that $k>k_0$ has $\sup_{0\le t\le k}\left[\Phi(t)-\frac{d}{2}\langle\Phi(t)\rangle\right]\le\frac{2}{d}$ for almost all $\omega\in\Omega$; therefore, for all $t\in[0,k]$, we can obtain Eq (15)

$$\int_0^t\frac{\sigma_1 S(j)I(j)}{E(j)}dB_1(j)\le\frac{1}{2}d{\sigma_1}^2\int_0^t\frac{S^2(j)I^2(j)}{E^2(j)}dj+\frac{2}{d}\ln k. \tag{15}$$

Then, we can obtain Eq (16)

$$\begin{aligned}\ln E(t)\le\ln E(0)+\int_0^t\left[\frac{\alpha S(j)I(j)}{E(j)}-(\beta+\gamma+\mu)-\frac{1}{2}(1-d)\sigma_1^2\frac{S^2(j)I^2(j)}{E^2(j)}-\frac{1}{2}\sigma_2^2\right]dj\\+\frac{2}{d}\ln k-\sigma_2B_2(t).\end{aligned} \tag{16}$$

Note that,

$$\frac{\alpha S(j)I(j)}{E(j)}-\frac{1}{2}(1-d)\sigma_1^2\frac{S^2(j)I^2(j)}{E^2(j)}\le\frac{\alpha^2}{2(1-d)\sigma_1^2}. \tag{17}$$

Substituting Eq (16) into Eq (17) yields Eq (18):

$$\ln E(t)\le\ln E(0)+\left[\frac{\alpha^2}{2(1-d)\sigma_1^2}-\left(\beta+\gamma+\mu+\frac{1}{2}\sigma_2^2\right)\right]t+\frac{2}{d}\ln k-\sigma_2B_2(t). \tag{18}$$

Therefore, when $k-1 \le t \le k$, we obtain Eq (19)

$$\frac{\ln E(t)}{t} \le \frac{\ln E(0)}{t} + \frac{\alpha^2}{2(1-d)\sigma_1^2} - \left(\beta+\gamma+\mu+\frac{1}{2}\sigma_2^2\right) + \frac{2}{d}\ln k - \sigma_2 B_2(t). \tag{19}$$

According to the strong law of Brownian motion, let $k \to \infty$, and then $t \to \infty$. Thus, we know that $\limsup_{t\to\infty} \frac{B_2(t)}{t} = 0.$ We obtain Eq (20)

$$\limsup_{t\to\infty} \frac{\ln E(t)}{t} \le \frac{\alpha^2}{2(1-d)\sigma_1^2} - \left(\beta+\gamma+\mu+\frac{1}{2}\sigma_2^2\right). \tag{20}$$

Finally, let $d \to 0.$ We obtain Eq (21)

$$\limsup_{t\to\infty} \frac{\ln E}{t} \le \frac{\alpha^2}{2\sigma_1^2} - \left(\beta+\gamma+\mu+\frac{1}{2}\sigma_2^2\right). \tag{21}$$

Theorem 3.2 has been proven.

Next, Theorem 3.3 proposes the condition under which not distributor of congestion warning information exists under road congestion conditions.

Theorem 3.3. For any given initial value, $(S(0), E(0), I(0)) \in R_+^3$, $\limsup_{t\to\infty} \frac{\ln I(t)}{t} \le G(\sigma_2^2)$ holds *a.s.* Furthermore, $G(\sigma_2^2) < 0.$ Then, $I(t)$ tend to 0 exponentially *a.s.*, where $G(\sigma_2^2) = \frac{\beta^2}{2\sigma_2^2} - (\lambda+\mu)$.

At this point, $I(t)$ is exponentially approaching 0 (the proof process is consistent with that of Theorem 3.2 and is omitted here).

3.3. *Sufficient conditions for a stationary distribution*

We investigate the existence of stationary distributions of coupled states. In the deterministic model, the basic reproduction number $R_0 = \sqrt{\frac{b\alpha\beta}{\mu(\lambda+\mu)(\beta+\gamma+\mu)}}$. When the system reaches equilibrium, four state nodes in the system reach the equilibrium point $A^* = (S^*, E^*, I^*)$, where $S^* = \frac{b}{\mu R_0^{\,2} - \varepsilon}, E^* = \frac{b\mu(R_0^{\,2}-1) - b\varepsilon}{\mu R_0^{\,2}(\beta+\gamma+\mu)}, I^* = \frac{\mu R_0^{\,2} - \mu - \varepsilon}{\alpha}$.

Theorem 3.4. If $R_0 = \sqrt{\frac{b\alpha\beta}{\mu(\lambda+\mu)(\beta+\gamma+\mu)}} > 1$, Eq (2) satisfies the following conditions under given initial conditions $(S(0), E(0), I(0)) \in R_+^3$ (Eq (22)):

$$0 < \Gamma < \min(\xi_1 S^2, \xi_2 E^2, \xi_3 I^2). \tag{22}$$

Among them, the constraint condition is $\sigma_2^2 < \gamma + \mu$, we can obtain Eq (23)

$$\begin{aligned} &\Gamma = \sigma_1^2 S^{*2} I^{*2} + \frac{1}{2}\sigma_2^2 E^* + \sigma_2^2 E^{*2}, \\ &\xi_1 = \varepsilon + \mu, \\ &\xi_2 = \gamma + \mu - \sigma_2^2, \\ &\xi_3 = \lambda + \mu. \end{aligned} \tag{23}$$

For any $(S(0),E(0),I(0)) \in R_+^3$, the system has a steady-state distribution π , and its solutions are ergodic.

Proof. We can obtain Eq (24)

$$\Theta(S,E,I) = \Theta_1(E) + \Theta_2(I) + \Theta_3(S,E,I), \tag{24}$$

where (Eq (25))

$$\begin{aligned}
&\Theta_1(E) = E - E^* - E^* \ln\frac{E}{E^*},\\
&\Theta_2(I) = I - I^* - I^* \ln\frac{I}{I^*},\\
&\Theta_3(S,E,I) = \frac{1}{2}(S + E + I - S^* - E^* - I^*)^2.
\end{aligned} \tag{25}$$

First, let $A = S - S^*, B = E - E^*, C = I - I^*$, we can obtain Eq (26)

$$\begin{aligned}
L\Theta_1 &= B\left[\alpha(\frac{SI}{E} - \frac{S^*I^*}{E^*})\right] + \frac{1}{2}\sigma_1^2 S^2 I^2 + \frac{1}{2}\sigma_2^2 E^*\\
&\le \alpha ABC + \sigma_1^2 A^2 C^2 + \sigma_1^2 S^{*2} I^{*2} + \frac{1}{2}\sigma_2^2 E^*.
\end{aligned} \tag{26}$$

Next, we can calculate that (Eqs (27) and (28))

$$\begin{aligned}
L\Theta_2 &= \left[\beta E - (\lambda+\mu)I\right]\frac{\partial\Theta_2}{\partial I} + \frac{1}{2}(\sigma_2^2 E^2)\frac{\partial^2\Theta_2}{\partial I^2}\\
&= C\left[\frac{\beta E}{I} - (\lambda+\mu)\right] + \frac{1}{2}\sigma_2^2 E^2\\
&\le \beta BC + \frac{1}{2}\sigma_2^2 E^2.
\end{aligned} \tag{27}$$

$$\begin{aligned}
\Theta(S,E,I) &\le \sigma_1^2 S^{*2} I^{*2} + \frac{1}{2}\sigma_2^2 E^* + \sigma_2^2 B^2 + \sigma_2^2 E^{*2}\\
&\quad + \left[-(\varepsilon+\mu)A^2 - (\gamma+\mu)B^2 - (\lambda+\mu)C^2\right]\\
&= -(\varepsilon+\mu)A^2 - (\gamma+\mu-\sigma_2^2)B^2 - (\lambda+\mu)C^2\\
&\quad + \sigma_1^2 S^{*2} I^{*2} + \frac{1}{2}\sigma_2^2 E^* + \sigma_2^2 E^{*2}.
\end{aligned} \tag{28}$$

We obtain an ellipsoid from $0 < \Omega < \min\left\{\xi_1 S^{*2}, \xi_2 E^{*2}, \xi_3 I^{*2}\right\}$ equation (Eq (29))

$$-\xi_1 A^2 - \xi_2 B^2 - \xi_3 C^2 + \Gamma = 0. \tag{29}$$

This lies in $\mathbb{R}_+^3$. It can be proven that the stochastic system has a unique stationary distribution (Eq (30)).

$$\begin{aligned}
&\lim_{(\sigma_1,\sigma_2)\to 0} \Gamma = 0,\\
&\lim_{(\sigma_1,\sigma_2)\to 0} \xi_1 = \varepsilon + \mu > 0,\\
&\lim_{(\sigma_1,\sigma_2)\to 0} \xi_2 = \gamma + \mu > 0,\\
&\lim_{(\sigma_1,\sigma_2)\to 0} \xi_3 = \lambda + \mu.
\end{aligned} \tag{30}$$

The solution of the stochastic SEIR model oscillates randomly in the neighborhood of equilibrium point A^*. The smaller the value of σ_1,σ_2, the asymptotic deviation between deterministic systems and their randomly perturbed systems approaches zero.

4. The stochastic optimal control model

In order to promote the dissemination of positive information such as congestion information, in this section, we propose to achieve maximum coverage of congestion information in the shortest time possible while satisfying all constraints. The optimal control model transforms a complex decision problem into a mathematical framework. This provides a powerful tool to analyze the impact of model parameter changes on optimal strategies and results and to evaluate the robustness of strategies.

In order to more accurately reflect the maximum size of the group that is aware of and may participate in the dissemination of congestion information, the research object is selected as $E(t)+I(t)$, and the optimal stochastic control strategy is proposed.

Convert α,β in the model into $\alpha(t),\beta(t)$.

The objective function is represented as Eq (31):

$$J(E,I)=\int_0^{t_f}\left[E(t)+I(t)-\frac{c_1}{2}\alpha^2(t)-\frac{c_2}{2}\beta^2(t)\right]dt, \tag{31}$$

satisfy the following state system as Eq (32):

$$\begin{cases} dS(t)=\left[b-\alpha(t)SI-\varepsilon S-\mu S\right]dt-\sigma_1 SIdB_1(t), \\ dE(t)=\left[\alpha(t)SI-\beta(t)E-\gamma E-\mu E\right]dt+\sigma_1 SIdB_1(t)-\sigma_2 EdB_2(t), \\ dI(t)=\left[\beta(t)E-\lambda I-\mu I\right]dt+\sigma_2 EdB_2(t). \end{cases} \tag{32}$$

Under initial constraint conditions (Eq (33)):

$$S(0)=S_0, E(0)=E_0, I(0)=I_0, \tag{33}$$

where (Eq (34))

$$\alpha(t),\beta(t)\in U\Delta=\left\{(\alpha(t),\beta(t))measurable, 0\le\alpha(t),\beta(t)\le 1,\forall t\in\left[0,t_f\right]\right\}. \tag{34}$$

Theorem 4.1. The existence of optimal control for $(\alpha^*,\beta^*)\in Z$ enables us to establish the objective functional (Eq (35)):

$$M(\alpha^*,\beta^*)=\max\left\{M(\alpha,\beta):(\alpha,\beta)\in Z\right\}. \tag{35}$$

Proof. Let $X(t)=(S(t),E(t),I(t))^T$ and obtain Eq (36)

$$N(t,Y(t),\alpha(t),\beta(t))=E+I-\frac{c_1}{2}\alpha^2(t)-\frac{c_2}{2}\beta^2(t). \tag{36}$$

The existence of optimal control requires the following five conditions to be met:

(1) The control domain and state domain have non emptiness.

(2) Control the range of variable values to form a closed convex set.

(3) The linear functions in the state and control variables determine the right-hand side of the state system.

(4) The integrand of the objective functional is convex on Z.

(5) There exist constants $d_1, d_2 > 0$ and $\rho > 1$ such that the integrand of the objective functional satisfied Eqs (37) and (38)

$$-N(t, Y(t), \alpha(t), \beta(t)) \geq d_1(|\alpha|^2 + |\beta|^2)^{\rho^2} - d_2. \tag{37}$$

$$S(t) \leq B, E(t) \leq \alpha(t)SI, I(t) \leq \beta(t)E. \tag{38}$$

Next, for any $t \geq 0$, a positive constant L (Eq (39)),

$$-N(t, Y(t), \alpha(t), \beta(t)) = \frac{c_1}{2}\alpha^2(t) + \frac{c_2}{2}\beta^2(t) - E - I \geq d_1(|\alpha|^2 + |\beta|^2)^{\rho^2} - 2L. \tag{39}$$

Let $d_1 = \min\left\{\frac{c_1}{2}, \frac{c_2}{2}\right\}, d_2 = 2L$, and $\rho = 2$ meet condition (5). Therefore, the optimal stochastic control strategy has been achieved.

In the dissemination of congestion information, the introduction of a companion system with cross-sectional equations introduces a dynamic and forward-looking "value evaluation system" for information dissemination strategies, enabling control strategies to intelligently judge the real-time value of information and make optimal resource allocation decisions at critical moments based on this. In real traffic management, the value of congestion information is high at the beginning of peak hours. Timely notification can prevent a large number of vehicles from rushing into congestion points, so maximum resources should be invested in dissemination. As time goes by, the value of this information will continuously decay through the accompanying equation, so it is necessary to quickly reduce investment and shift resources to more important information. Theorem 4.2 proposes corresponding mathematical conditions for this.

Theorem 4.2. There exist adjoint variables $\varpi_1, \varpi_2, \varpi_3$ that satisfy Eq (40)

$$\begin{cases} d\varpi_1(t) = \left[\alpha(t)(\varpi_1 - \varpi_2)I + \sigma_1(\lambda_1 - \lambda_2)I + \varpi_1(\varepsilon + \mu)\right]dt - \lambda_1 dW_1, \\ d\varpi_2(t) = \left[1 + \beta(t)(\varpi_2 - \varpi_3) + \varpi_2(\gamma + \mu) + \sigma_2(\lambda_2 - \lambda_3)\right]dt + \lambda_2 dW_1 - \lambda_2 dW_2, \\ d\varpi_3(t) = \left[1 + \alpha(t)(\varpi_1 - \varpi_2)S + \varpi_3(\lambda + \mu) + \sigma_1(\lambda_1 - \lambda_2)S\right]dt + \lambda_3 dW_2. \end{cases} \tag{40}$$

The boundary conditions need to meet Eq (41):

$$\varpi_1(t_g) = \varpi_2(t_g) = \varpi_3(t_g) = 0. \tag{41}$$

In addition, we obtain Eq (42)

$$\begin{aligned} \alpha^*(t) &= \min\left\{1, \max\left\{0, \frac{(\varpi_1 - \varpi_2)SI}{c_1}\right\}\right\}, \\ \beta^*(t) &= \min\left\{1, \max\left\{0, \frac{(\varpi_2 - \varpi_3)E}{c_2}\right\}\right\}. \end{aligned} \tag{42}$$

Proof. By constructing Hamiltonian functions, we obtain Eq (43)

$$\begin{aligned} H = & -E - I + \frac{c_1}{2}\alpha^2(t) + \frac{c_2}{2}\beta^2(t) + \varpi_1\left[b - \alpha(t)SI - \varepsilon S - \mu S\right] \\ & + \varpi_2\left[\alpha(t)SI - \beta(t)E - \gamma E - \mu E\right] + \varpi_3\left[\beta(t)E - \lambda I - \mu I\right] - \lambda_1\sigma_1 SI \\ & + \lambda_2(\sigma_1 SI - \sigma_2 E) + \lambda_3\sigma_2 E. \end{aligned} \tag{43}$$

Using the Pontryagin maximum principle, the adjoint system is described by the following differential Eq (44):

$$\frac{d\varpi_1}{dt} = -\frac{\partial H}{\partial S}, \frac{d\varpi_2}{dt} = -\frac{\partial H}{\partial E}, \frac{d\varpi_3}{dt} = -\frac{\partial H}{\partial I}, \tag{44}$$

and the boundary conditions are (Eq (45))

$$\varpi_1(t_g) = \varpi_2(t_g) = \varpi_3(t_g) = 0. \tag{45}$$

Approach (Eq (46))

$$\begin{aligned} &\frac{\partial H}{\partial \alpha} = c_1\alpha - \varpi_1 SI + \varpi_2 SI = 0, \\ &\frac{\partial H}{\partial \beta} = c_2\beta - \varpi_2 E + \varpi_3 E = 0. \end{aligned} \tag{46}$$

In summary, the optimal stochastic control strategy of the system can be characterized as (Eq (47)):

$$\begin{aligned} &\alpha^*(t) = \min\left\{1, \max\left\{0, \frac{(\varpi_1 - \varpi_2)SI}{c_1}\right\}\right\}, \\ &\beta^*(t) = \min\left\{1, \max\left\{0, \frac{(\varpi_2 - \varpi_3)E}{c_2}\right\}\right\}. \end{aligned} \tag{47}$$

Remark 4.1. The complete description of the optimal control system includes the state equation constrained by initial Eq (32) and the adjoint equation with cross-sectional Eq (41), and its coupled system can be expressed as (Eqs (48) and (49)):

$$\begin{cases} dS(t) = \left[b - \min\left\{1, \max\left\{0, \frac{(\varpi_1 - \varpi_2)SI}{c_1}\right\}\right\} SI - \varepsilon S - \mu S \right] dt - \sigma_1 SI dB_1(t), \\ dE(t) = \begin{bmatrix} \min\left\{1, \max\left\{0, \frac{(\varpi_1 - \varpi_2)SI}{c_1}\right\}\right\} SI \\ -\min\left\{1, \max\left\{0, \frac{(\varpi_2 - \varpi_3)E}{c_2}\right\}\right\} E - \gamma E - \mu E \end{bmatrix} dt + \sigma_1 SI dB_1(t) - \sigma_2 E dB_2(t), \\ dI(t) = \left[\min\left\{1, \max\left\{0, \frac{(\varpi_2 - \varpi_3)E}{c_2}\right\}\right\} E - \lambda I - \mu I \right] dt + \sigma_2 E dB_2(t), \\ d\varpi_1(t) = \begin{bmatrix} \min\left\{1, \max\left\{0, \frac{(\varpi_1 - \varpi_2)SI}{c_1}\right\}\right\} (\varpi_1 - \varpi_2) I \\ +\sigma_1(\lambda_1 - \lambda_2) I + \varpi_1(\varepsilon + \mu) \end{bmatrix} dt - \lambda_1 dW_1(t), \\ d\varpi_2(t) = \begin{bmatrix} 1 + \min\left\{1, \max\left\{0, \frac{(\varpi_2 - \varpi_3)E}{c_2}\right\}\right\} (\varpi_2 - \varpi_3) \\ +\varpi_2(\gamma + \mu) + \sigma_2(\lambda_2 - \lambda_3) \end{bmatrix} dt + \lambda_2 dW_1(t) - \lambda_2 dW_2(t), \\ d\varpi_3(t) = \begin{bmatrix} 1 + \min\left\{1, \max\left\{0, \frac{(\varpi_1 - \varpi_2)SI}{c_1}\right\}\right\} (\varpi_1 - \varpi_2) S \\ +\varpi_3(\lambda + \mu) + \sigma_1(\lambda_1 - \lambda_2) S \end{bmatrix} dt + \lambda_3 dW_2(t), \end{cases} \tag{48}$$

and

$$\varpi_1(t_g) = \varpi_2(t_g) = \varpi_3(t_g) = 0. \tag{49}$$

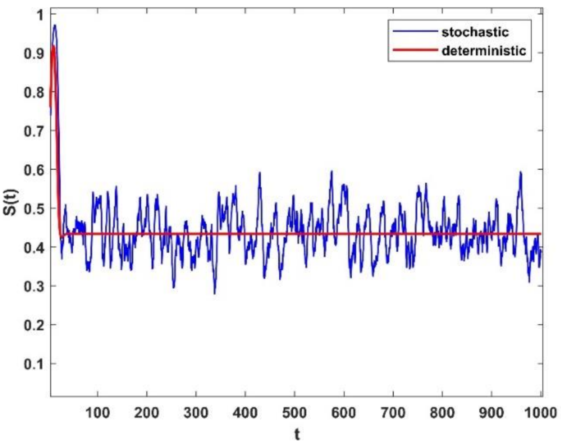
5. Numerical simulations

To investigate the regulatory effect of environmental factors on congestion information diffusion under a deterministic framework and the impact of random disturbances on group behavior evolution, system parameters need to meet the basic threshold conditions of information propagation, that is, the basic reproduction number needs to comply with the requirements of the fundamental laws of propagation dynamics $R_0 > 1$. First, the density change trends of each population in stochastic systems and deterministic systems are compared. Second, we observe the impact of different disturbance intensities on dissemination of congestion information in random systems. The effectiveness of optimal stochastic control strategy proposed is then verified. Finally, by comparing the optimal control effects under different noise intensities, the regulatory effect of control strategies on propagation dynamics of congestion warning information is quantitatively analyzed.

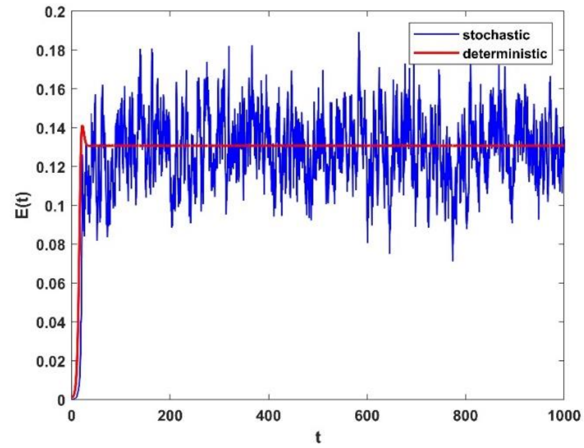
In real life, people hold different attitudes toward congestion information based on their personality, values, job nature, etc., so it is difficult to obtain real data on the conversion between groups. Thus, we aim to investigate the observation of deterministic and stochastic systems, as well as the changes in each set of features during information propagation as long as the system parameters meet the threshold conditions for information propagation, namely the basic reproduction number $R_0 > 1$. Therefore, drawing on the research parameter settings of Kang Sida et al. [28], the parameters are set as follows:

$$b = 3, \alpha = 0.5, \mu = 0.3, \varepsilon = 0.3, \beta = 0.3, \gamma = 0.7, \lambda = 0.2 .$$

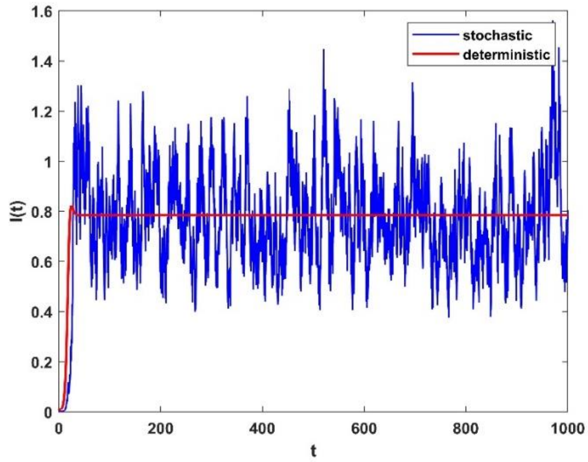
After calculation, $R_0 = \sqrt{\dfrac{b\alpha\beta}{\mu(\lambda+\mu)(\beta+\gamma+\mu)}} \approx 1.52 > 1$ meets the threshold condition for information dissemination. Figure 2 compares the trends of each population over time in deterministic and stochastic systems when the interference intensity is $\sigma_i = 0.001(i = 1, 2)$. Figures 2(a)–(c) respectively show the temporal trends of the ignorant, hesitant, and distributor in deterministic and stochastic systems. It can be seen that the solution of a deterministic system converges to a positive equilibrium point, while the solution of a stochastic system oscillates randomly near the positive equilibrium point. Moreover, in the system with added stochastic disturbances, the dissemination of congestion information is superior to deterministic systems, indicating that random environmental interference promotes information dissemination. When information spreads in the system, the solution of the random system is in an unstable state, and the density distribution of each group shows a stable and ergodic fluctuation over time, and its statistical characteristics approach the actual observed distribution.



(a)



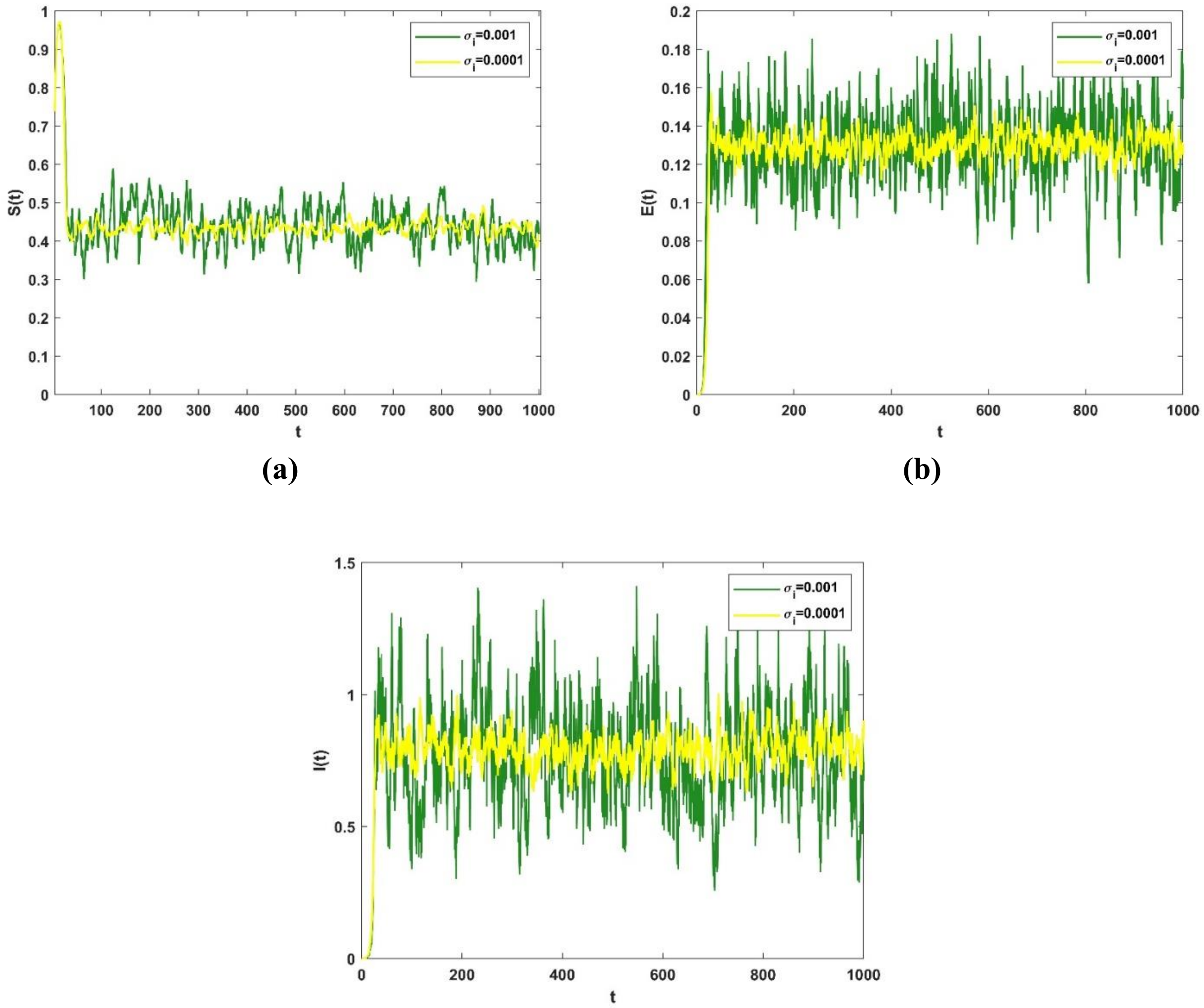
(b)



(c)

Figure 2. Comparing the trends of (a) S, (b) E, and (c) I density changes in various populations between deterministic and stochastic systems.

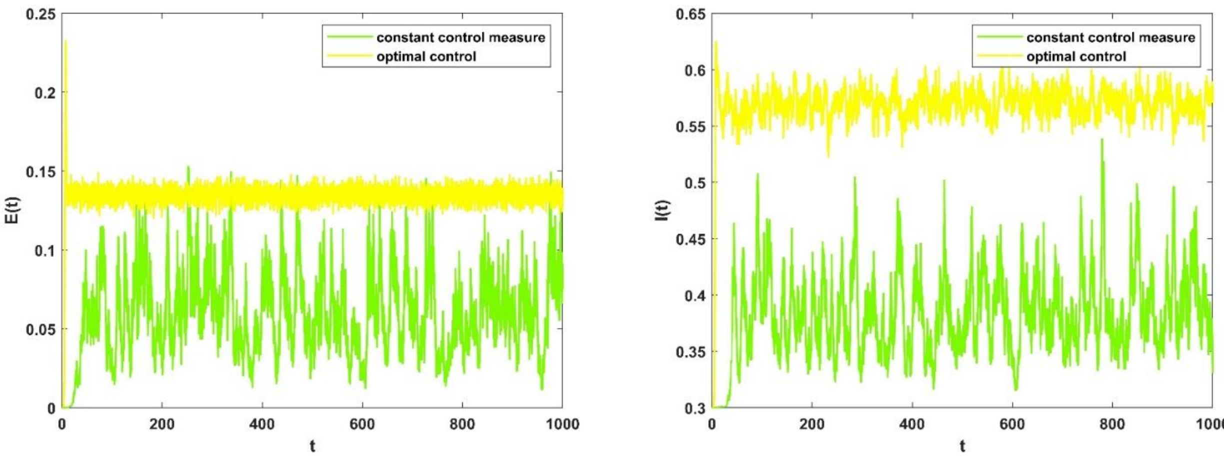
To compare the system response on the propagation of congestion warning information under perturbation parameter gradient, we simulate trend charts of congestion information propagation over time for stochastic systems with disturbance intensities of $\sigma_i (i = 1, 2) = 0.001$ *and* 0.0001 and conduct a combined analysis. Figures 3(a)–(c) show the temporal trends of the ignorant, hesitant, and distributor under different levels of interference intensity, respectively. It can be seen that regardless of the group, the fluctuation of congestion warning information dissemination tends to flatten with the decrease of interference intensity, and congestion information exhibits stronger propagation and penetration in strong noise environments. Therefore, the speed of congestion information dissemination can be controlled by controlling the degree of interference in the system.



(a) **(b)** **(c)**

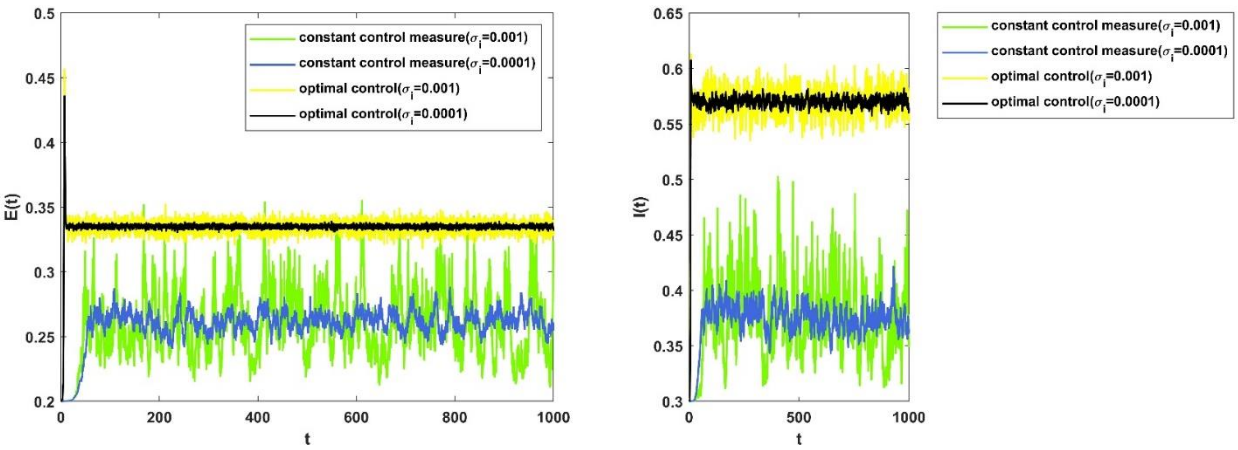
Figure 3. Density variation trends of different groups (a) S, (b) E, and (c) I under different levels of interference ($\sigma_i (i=1,2)=0.001$ and 0.0001).

Finally, to verify the effectiveness of the proposed control strategy, under the condition of fixing other system parameters, the control effect of optimal stochastic control strategy on dynamic evolution of population density E and I are quantitatively analyzed by adjusting the random parameters α and β. Figures 4(a),(b) respectively compare the temporal trends of the hesitant and distributors under constant control and optimal control strategies. It can be seen that when the disturbance intensity is $\sigma_i = 0.001$, population density E and I under the optimal stochastic control strategy of random parameters α and β is better than that under constant control measures, which verifies the global optimality of the control law. The values of the parameters are further changed, and the density change trends of groups E and I under optimal control and constant control under different disturbance intensities ($\sigma_i (i=1,2)=0.001$ and 0.0001) are compared, as shown in Figures 5(a),(b). Experimental verification shows that the proposed strategy has superior performance independent of parameters. Moreover, adopting the optimal stochastic control strategy can suppress congestion warning information fluctuation during propagation. In addition, theoretical and numerical simulations show that random disturbance can reduce the phase transition threshold of information propagation, which can be used to optimize congestion control.



(a) **(b)**

Figure 4. Density variation trends (a) E and (b) I under constant control measure and optimal control when $\sigma_i(i=1,2)=0.001$.



(a) **(b)**

Figure 5. Density variation trends (a) E and (b) I of different disturbance intensities under constant control measures and optimal control.

6. Conclusions

In this article, we constructed a congestion information propagation model that considers random factors. The model uses Gaussian white noise to represent the two parameters, confirming the existence of positive global solutions and verifying the extinction and persistence of the congestion warning information distribution. The global optimal control law of the random model was proposed, and the propagation dynamics of congestion information were simulated and reproduced using Matlab. System response characteristics of disturbance intensity gradient on the propagation of congestion information were compared, and the effectiveness of the proposed optimal stochastic control strategy was verified.

The following conclusions can be drawn from our research.

(1) The dissemination of congestion information is filled with many unpredictable random factors. Ignoring these factors can lead to overly idealized models. Moreover, actively introducing random white noise interference into the model is not to increase complexity, but to make the model more realistic, robust, and use randomness to improve the effectiveness of congestion information dissemination and better control traffic congestion.

(2) As the noise level in the environment where the congestion information dissemination system is located significantly increases, its dynamic behavior will exhibit more intense and nonlinear fluctuation characteristics. Communicators can intentionally introduce or amplify noise to achieve the goal of expanding the scope of congestion information dissemination. By controlling random parameters, the propagation of information can be effectively controlled, thereby achieving intelligent suppression and control of traffic congestion phenomena at the macro level.

(3) Under the constructed optimal random control strategy framework, the system dynamically generates the optimal stochastic control strategy by sensing the dynamic state of congestion information propagation in real time. Compared with the uncontrolled benchmark random propagation model, the congestion information propagation system using this strategy exhibits multi-dimensional performance indicators, such as higher peak propagation scale, shorter burst response time, and more stable sustained propagation trend, verifying the effectiveness of the proposed random control strategy in improving the efficiency of congestion information propagation.

(4) We successfully transferred the stochastic SEIR model from the field of epidemiology to the field of traffic information dissemination, providing a new perspective for understanding driver behavior and combining theoretical models with engineering practice. This model enhances the capability of traffic management departments from passive information dissemination to active management of the information dissemination process. For example, the model can predict how long it will take for a new congestion message to reach a sufficient number of drivers, and the traffic management center can determine the lead time based on this; timely and effective dissemination of information is crucial for traffic recovery in emergency situations such as traffic accidents and severe weather; and the success of traffic management policies, such as new traffic restrictions, toll policies, and promotion of public transportation, largely depends on public awareness and acceptance. This model can also be a tool for policy dissemination simulation.

In the real world, information dissemination is influenced by many uncertain factors. By incorporating these uncertain factors into a deterministic model, a more accurate stochastic dissemination model can be established. For positive information, such as congestion warning messages, the randomness and complexity of social systems promote the dissemination of information, thereby controlling traffic congestion. For negative information, such as public opinion and rumors, it is possible to minimize randomness and thus limit the spread of this information.

This article also has the following research limitations: First, although the random propagation model constructed in this article considers the interference of random factors in complex social systems, we did not take into account the impact of the heterogeneity of social networks on the process of congestion warning propagation. Second, we used only Matlab for numerical simulations, but when traffic congestion occurred, parameter data such as initial values of each node could not be obtained, and could draw only on the parameter settings of other scholars. Finally, in the event of sudden traffic congestion, the dissemination of congestion information presents typical multi-scale and nonlinear dynamic characteristics. Subsequent research will be conducted from the following three aspects: (1) In the future, the impact of scale-free networks and small world networks on the dissemination of congestion warning information will be studied and compared. (2) By developing more advanced parameter estimation and uncertainty quantification methods, the problem of parameter setting can be solved. (3) We will focus on constructing a nonlinear dynamic model of congestion information propagation under the coupling of individual psychology and behavior, and systematically analyze the cascading effects caused by sudden traffic incidents.

Author contributions

Huining Yan: Writing-original draft, writing-review & editing, methodology, formal analysis, conceptualization; Hua Li: Writing–review & editing, supervision, project administration, methodology; Qiubai Sun: Writing–review & editing, supervision, methodology, investigation, formal analysis, conceptualization; Meihui Song: Writing–review & editing, supervision, methodology, formal analysis. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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