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Research article

Improved accumulative-error event-triggered control for discrete-time linear systems with random time delays

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Abstract: A stability problem under event-triggered control for discrete linear systems with random time delays is considered in this paper. First, this paper improves an accumulative error-based event-triggered approach, which can switch the event-triggered parameter between the two-modes over time. Second, a new closed-loop discrete Markov switching system equation is derived by decoupling the accumulative error term, and the time delays are equal to the current Markov modes when the event-triggered scheme is met. Third, the co-designing problems for control gains and event-triggered parameters in the system are solved. Finally, the stochastic stability of the system is guaranteed through random analysis techniques and a free-weighting matrix method, and the effectiveness of the approach is demonstrated by using numerical examples.

Keywords: discrete-time linear systems; event-triggered control; Markov chain; free-weight matrix method; co-designing control
Mathematics Subject Classification: 34D20, 60J20, 93D30, 93B40

1. Introduction

Network control systems (NCSs), with the rapid development of science and technology, have attracted extensive attention due to their high reliability, ease of remote operation, and flexibility in installation and maintenance [1, 2]. However, with the rise of big data, it has brought some challenges in the current situation of limited network bandwidth, such as data packet loss and time delays [3]. The time delays mainly occurs when data is exchanged in the transmission network, affecting the stability of the system to some extent, and such delays are often time-varying and even random [4]. Therefore, with the frequent exchange of information in communication networks, it is necessary to design a method that can stably and selectively transmit information control under communication time delays. So, the following discussions and studies are conducted.

On the one hand, in the field of time delays theory, a time-segmented Lyapunov functional method was proposed in [5], but it does not account for the randomness of delays. Then, in [6], the delays were modeled as random time delays obeying different specific probability distributions, including the sensor-to-controller and the controller-to-actuator. However, as the research progresses, considering delays separately will gradually add some unnecessary solving steps, so the two kinds of delays have been co-designed in [7] and obey the same probability distribution. In addition, from the perspective of stochastic modeling, The Markov process is often used by many researchers as the simulation part of the random process because its assumptions can conform to many real processes, and the model calculation is simple and easy to implement. In [8], the random time delays in NCSs are simulated as a Markov jump process including two modes. In parallel, the sliding control problem of switch systems was studied in [9–11], and in [12, 13], the Markov jumps neural networks were studied. However, due to excessive useless data in the transfer process, the reduction of transmission efficiency is also a problem worth exploring under time delays, which led to the study presented in this paper.

On the other hand, in traditional periodic sampling, the sampler operates at fixed intervals regardless of the system's actual needs [14], causing unnecessary data transmission. Therefore, event-triggered control (ETC) is proposed as an effective way to improve the utilization rate of data and relieve transmission pressure [15]. Its main idea is that measurement values are only transmitted when the change in system status exceeds a specified threshold during the current sampling [16]. In general, the ETC can reduce the network communication load and reduce the computation of the controller to a certain extent, so many scientists have studied the design of ETC in different systems. In [15, 17], ETC was considered for the design and analysis of continuous-time systems, and for discrete-time systems, it was considered in [18,19]. However, ETC requires continuous monitoring to check whether the threshold is reached, so self-triggering scheme was proposed in [20, 21], and the complexity of implementation is lower than ETC. In [22,23], the average response cycle for event-triggered schemes is typically longer than that for self-triggered schemes, but the design of a self-triggered controller requires more requirements regarding the system structure. Therefore, in general, ETC still has great research value. In the field of ETC, ETC based on integration was proposed in [24], and Kwon et al. [25] applied this method to chaotic Lur'e network systems. Recently, Zhang et al. [26] proposed a new event-triggered scheme based on state error accumulation, which can greatly reduce Zeno behavior. In [27], an event-triggered scheme based on state error accumulation in discrete-time case was proposed based on [26]. However, the effects of time delays are not considered in [26, 27]. By introducing an improved triggering mechanism, the robustness and resource utilization efficiency of the system can be enhanced. This has important practical significance and broad application prospects for actual engineering systems, such as industrial automation, medicine, and intelligent transportation, which are highly dependent on communication networks. It also demonstrates the theoretical and practical value of in-depth research on this issue.

With the research progresses, NCSs considering time delays under the premise of adding event-triggered also began to emerge. In [28], an ETC system model considering time delays was established. In [29], the ETC for T-S fuzzy systems was proposed under random time delays, which are designed as a Markov process. Zhang [30] analyzed the feasibility of dynamic ETC of NCSs with random time delays, but the shortcoming is that the delays do not affect the system when the ETC does not reach the threshold. This issue is also considered in the design of the present work.

To sum up, the scheme based on discrete ETC appears to be more practical [15], and the ETC of discrete linear systems is also an emerging research topic [31–34]. In event-triggered discrete control systems, random time delays are commonly encountered in practical scenarios such as NCSs and remote measurement. Their unpredictability poses significant challenges to system stability and performance. On the one hand, random time delays may cause controllers to receive outdated information, reducing control accuracy and causing system instability. On the other hand, it also increases the complexity of event-triggered modeling and analysis, making it difficult for traditional ETC strategies to address randomness effectively. Therefore, it is particularly important to design an ETC that can dynamically adapt to random time delays. In this paper, random time delays are simulated as Markov processes. Furthermore, to better conform to the definition of event-triggered schema, Markov modes and time delays are only correlated at the time of triggering. In terms of the design of ETC, to prevent unnecessary triggering when state fluctuations are small, an improved accumulated-error-based triggering mechanism is introduced. Specifically, a two-mode switching condition has been designed, and made the triggering rule to change adaptively over time. Finally, by determining the system stability interval and recording the trigger time interval, a numerical example and a practical example are used to demonstrate the effectiveness and superiority of the control method in the paper.

The main contributions of this paper are the following three aspects:

1. An improved accumulative-error-based ETC strategy is proposed which incorporates two-modes switching mechanism that allows the system to adaptively change triggering modes over time.
2. To handle the complexity caused by random time delays, a novel closed-loop discrete-time Markov switching system model is established by simplifying the accumulative-error term.
3. Unlike existing approaches, the proposed method explicitly models the variation of random time delays within the data transmission process when the event-triggered controller is inactive.

The structure of this paper is as follows: In Section 2, we describe the research subject, classification of ETC systems, and model design for closed-loop systems. Section 3 focuses on the stability analysis and controller design of the proposed approach. In Section 4, numerical simulation studies, including both mathematical examples and practical examples, are carried out to demonstrate the effectiveness and superiority of the method. Finally, Section 5 summarizes the entire paper.

2. Problem formulation

2.1. Plant

Consider the NCSs in Figure 1, and assume that the model of the plant is a discrete-time linear system as follows:

$$x(k+1) = Ax(k) + Bu(k), \tag{2.1}$$

where A and B are known real matrices, $x(k) \in R^n$ is the state of the plant, and x_0 is the initial condition. Under the condition of being event-triggered, let $\{k_1, ..., k_t\}$ be the time series of the triggering time where t denotes the number of triggering times, so the system control input will be updated in the following form:

$$u(k) = Kx(k_t), k_t \le k \le k_{t+1}, \tag{2.2}$$

where K is the feedback gain to be designed later,$u(k) \in R^m$ is the input of the plant, and thus Eq (2.1) is written as follows:

$$x(k+1) = Ax(k) + BKx(k_t). \tag{2.3}$$

Due to network bandwidth limitations, in Figure 1 there are inevitable time delays in the signal transmission process. In NCSs, the delays at present are usually related to the delays that occurred previously, so we consider the random time delays $\tau_{k_t} \in [1, \tilde{\tau}]$ obeying a Markov chain. The random variable $r(k) = i$ denotes a discrete-time Markov chain whose value is from a finite set $M = \{1, 2, 3, \cdots, N\}(N \ge \tilde{\tau})$, it has the transition probabilities as follows:

$$P_r\{r(k+1) = j|r(k) = i\} = \pi_{ij}, \tag{2.4}$$

where π_{ij} indicates the element in the i-th row and j-th column of the π matrix, and $0 \le \pi_{ij} \le 1$, $\forall i, j \in M$, $\sum\limits_{j=1}^{N} \pi_{ij} = 1$.

The relationship between time delays τ_{k_t} and the random variable $r(k)$ is as follows:

$$\tau_{k_t} = \begin{cases} r(k) & \text{if the event} - \text{triggered scheme satisfies} \\ r(k_t) & \text{else} \end{cases} \tag{2.5}$$

where $k_t \le k < k_{t+1}$.

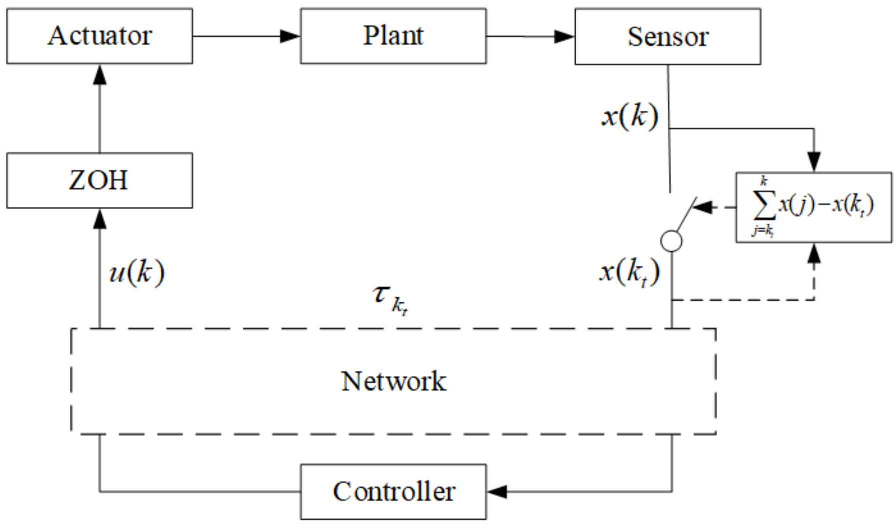

Figure 1. Event-triggered networked control systems with time delays.

Remark 1. This is different from the random time delays designed in [30]. When ETC is not satisfied, the time delays does not vary at any time, because no packet is transmitted to the controller. Specifically, in Eq (2.5), the time delays are assumed to depend on the current Markov mode only when ETC is satisfied. At the end of the paper, the trajectory of the size of the time delays are described by simulation.

2.2. Event-triggered scheme

Denoting $x(k)$ as the currently sampled data packet and $x(k_t)$ as the latest transmitted data packet, in general [23, 29, 30], $x(k)$ can be transmitted only when the following conditions are satisfied:

$$e^T(k)\Omega e(k) - \varepsilon x^T(k_t)\Omega x(k_t) > 0, \tag{2.6}$$

where $\Omega > 0$ is a weighted matrix, $e(k)$ represents the error between $x(k_t)$ and $x(k)$, i.e., $e(k) = x(k) - x(k_t)$ and the trigger threshold $\varepsilon \in [0, 1)$ is a given scalar parameter. In particular, when $\varepsilon = 0$, it is a periodic sampling.

However, if the system state fluctuation is small, especially when the system enters stabilization, Eq (2.6) will be difficult to satisfy, and the event-triggered controller will enter into a non-operational state for a long period. Therefore, this paper solves the problem by introducing the following accumulative error in [27], which can be seen in detail in [27].

$$\sum_{j=k_t}^{k} e^T(j)\Omega \sum_{j=k_t}^{k} e(j) - \varepsilon x^T(k_t)\Omega x(k_t) > 0. \tag{2.7}$$

In this paper, when considering random time delays, the event-triggered scheme (2.7) will be improved in the following form:

$$\sum_{j=k_t}^{k} e^T(j)\Omega_i \sum_{j=k_t}^{k} e(j) - \varepsilon x^T(k_t)\Omega_i x(k_t) > 0, \tag{2.8}$$

where the value of the weighting matrix Ω_i is switched by the value of $r(k) = i$, which represents the Markov chain (2.4). In Figure 1, during the data packet transmission process, ETC (2.8) is designed. When the system state fluctuation is small, this enables the transmission of useful information for the system by setting a certain threshold, to a certain extent, it also reduces the data bandwidth.

Remark 2. The text differs from [27] in the following two respects: In terms of event-triggered matrix design, to accommodate the varying effects of random time delays, this paper proposes an enhanced event-triggered matrix(Ω_i) that adaptively adjusts the triggering condition based on delay characteristics. In terms of handling accumulation terms $\sum_{j=k_t}^{k} e(j)$, [27] mainly focuses on how to solve Eq (2.7) by using a new closed-loop function, but this paper applies Eq (2.7) to NCSs and proves the stability of the system by the L-K functional, and a new closed-loop system is proposed by using decoupling the accumulative term $\sum_{j=k_t}^{k} e(j)$, whose details are explained below (Subsection 2.3).

It is well known that Zeno behavior is one of the unavoidable problems in ETC research, but in discrete ETC, the worst circumstances are sending data at every moment k for the discrete event-triggered, which seems like a time-triggered scheme. Therefore, even though the discrete-time ETC occurs under the Zeno phenomenon, the worst-case scenario is simply losing the ability to reduce the data transmission frequency, which does not have an impact on the system's stability.

2.3. Closed-loop system modeling analysis

By considering the random time delays, the trigger signal arrives at the controller at a random and irregular time each time. With the aid of ZOH, the design of the controller will be rewritten as follows:

$$u(k) = K_{\tau_{k_t}} x(k_t), \tag{2.9}$$

where the value of the weighting matrix $K_{\tau_{k_t}}$ is switched by the value of τ_{k_t}. Thus, the closed-loop system (2.3) could be rewritten as follows:

$$x(k+1) = Ax(k) + BK_{\tau_{k_t}} x(k_t), \tag{2.10}$$

where $k_t + \tau_{k_t} \leq k \leq k_{t+1} + \tau_{k_{t+1}} - 1$.

Remark 3. Different from the conventional Markov jump system, in [3, 29], the system matrices A and B also have multiple modes according to the difference of $r(k)$. In this paper, consider a method of designing different controller gains based on the time delays to better incorporate the nature of ETC. This paper discusses the classification between $r(k)$ and τ_{k_t}, as in Eq (2.5). It is intuitive to say that because of $\{k_1, k_2, \cdots, k_{t-1}, k_t\} \subseteq \{1, 2, \cdots, k-1, k\}(t \leq k, k_{t-1} < k_t)$, then $\{\tau_{k_1}, \tau_{k_2}, \cdots, \tau_{k_t}\} \subseteq \{r(1), r(2), \cdots, r(k)\}$. As far as authors concerned, there is no such study prior to the current paper.

As described in Remark 3, for the term $\sum_{j=k_t}^{k} e(j)$ of Eq (2.7), in [27], a closed-loop functional approach was proposed to solve it. However, this paper proposes a new form of closed-loop system (2.11) by combining $\sum_{j=k_t}^{k} e(j)$ with Eq (2.10) as follows ($k_t + \tau_{k_t} \leq k \leq k_{t+1} + \tau_{k_{t+1}} - 1$):

$$x(k+1) = Ax(k) + BK_s \Upsilon(k)(s = \tau_{k_t}), \tag{2.11}$$

where $\Upsilon(k) = (\Theta_1(k) - \Theta_2(k))$, with $\Theta_1(k) = \frac{1}{k-k_t+1} \sum_{j=k_t}^{k} x(j)$ and $\Theta_2(k) = \frac{1}{k-k_t+1} \sum_{j=k_t}^{k} e(j)$. The brief proof of Eq (2.11) is as follows:

The terms in $\sum_{j=k_t}^{k} e(j)$ can be transformed by equivalent transformations as follows:

$$\sum_{j=k_t}^{k} e(j) = \sum_{j=k_t}^{k} x(j) - (k - k_t + 1)x(k_t). \tag{2.12}$$

The transformation of Eq (2.12), we get:

$$x(k_t) = \frac{1}{k - k_t + 1} \sum_{j=k_t}^{k} x(j) - \frac{1}{k - k_t + 1} \sum_{j=k_t}^{k} e(j). \tag{2.13}$$

This completes the proof.

From the paper [3, 16, 22, 28, 30], a function is defined as:

$$\tau(k) = k - k_t, k \in [k_t + \tau_{k_t}, k_{t+1} + \tau_{k_{t+1}} - 1], \tag{2.14}$$

and, obviously,

$$\tau_{k_t} \leq \tau(k) \leq (k_{t+1} - k_t) + \tau_{k_{t+1}} - 1 \leq 1 + \tilde{\tau} = \tau_m, \tag{2.15}$$

where $\tilde{\tau}$ is the maximum value of τ_{k_t}. According to Eq (2.8), the trigger condition is not met with $k \in [k_t + \tau_{k_t}, k_{t+1} + \tau_{k_{t+1}} - 1]$, so by Eqs (2.13) and (2.14), (2.8) can be rewritten as follows:

$$(\tau(k) + 1)^2 \Theta_2^T(k) \Omega_i \Theta_2(k) \leq \varepsilon \Upsilon^T(k) \Omega_i \Upsilon(k). \tag{2.16}$$

Definition 1. [15] The system is stochastically stable if the following conditions hold for any initial values $(x(0), r(0))$:

$$E\{\sum_{k=0}^{\infty} \|x(k)\|^2 | x(0), r(0)\} < \infty. \tag{2.17}$$

3. Main results

3.1. Stability analysis

Theorem 1. For the given parameters $0 \leq \varepsilon < 1$, $\tau_m > 0$, and gains $K_s(s = \tau_{k_t})$, the closed-loop discrete system (2.11) is stochastically stable if there are real matrices $P_i > 0$, $Q > 0$, $R > 0$, $S > 0$, $\Omega_i > 0(i = r(k))$, X and Y of suitable dimensions satisfying that $\begin{bmatrix} S & X \\ * & R \end{bmatrix} \geq 0$, $\begin{bmatrix} S & Y \\ * & R \end{bmatrix} \geq 0$, and for $i \in M$,

$$\Xi_i = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ * & -P_i & 0 \\ * & * & -\tau_m R \end{bmatrix} < 0, \tag{3.1}$$

$$\sum_{j=1}^{\tilde{\tau}} \pi_{ij} P_j \leq P_i, \tag{3.2}$$

where

$$\begin{aligned} \Gamma_{11} &= \Phi_i + \zeta + \zeta^T + \tau_m S, \\ \Gamma_{12} &= [\ P_i A, 0, P_i B K_s, -P_i B K_s\ ]^T, \\ \Gamma_{13} &= [\ R(A - I), 0, RBK_s, -RBK_s\ ]^T, \\ \zeta &= [\ X, -Y, Y - X, X - Y\ ], \\ \Phi_i &= \begin{bmatrix} -P_i + Q & 0 & 0 & 0 \\ * & -Q & 0 & 0 \\ * & * & (\varepsilon - 1)\Omega_i & \varepsilon\Omega_i \\ * & * & * & -\Omega_i \end{bmatrix}. \end{aligned}$$

Proof. Consider the Lyapunov functional [3, 30]:

$$\begin{aligned} V(k) &= V_1(k) + V_2(k) + V_3(k) \\ &= x^T(k) P_i x(k) + \sum_{l=k-\tau_m}^{k-1} x^T(l) Q x(l) + \sum_{s=-\tau_m}^{-1} \sum_{l=k+s}^{k-1} \xi^T(l) R \xi(l), \end{aligned} \tag{3.3}$$

where $\xi(l) = x(l + 1) - x(l)$.

Let $E(\cdot)$ denote the mathematical expectation of the random process. Then, $r(k) = i$, $r(k + 1) = j$ is given by Eq (2.4):

$$\begin{aligned} E\{\Delta V(k)\} &= E\{V(x(k + 1), r(k + 1))|x(k), r(k)\} - V(x(k), r(k)) \\ &= x^T(k + 1) \sum_{j=1}^{N} \pi_{ij} P_j x(k + 1) - x^T(k) P_i x(k) + x^T(k) Q x(k) \\ &\quad - x^T(k - \tau_m) Q x(k - \tau_m) + \tau_m \xi^T(k) R \xi(k) - \tau_m \sum_{q=k-\tau_m}^{k-1} \xi^T(l) R \xi(l), \end{aligned} \tag{3.4}$$

from Eq (2.16), we have:

$$\Theta_2^T(k)\Omega_i\Theta_2(k) \le (\tau(k)+1)^2\Theta_2^T(k)\Omega_i\Theta_2(k) \le \varepsilon\Upsilon^T(k)\Omega_i\Upsilon(k), \tag{3.5}$$

and then

$$\varepsilon\Upsilon^T(k)\Omega_i\Upsilon(k) - \Theta_2^T(k)\Omega_i\Theta_2(k) \ge 0, \tag{3.6}$$

and $k \in [k_t, k_{t+1})$.

Combining the Eq (3.6) and letting $\sum_{j=1}^{\tilde{\tau}} \pi_{ij}P_j \le P_i$ follow the solution of system (2.11), Eq (3.4) become:

$$\begin{aligned} E\{\Delta V(k)\} \le {} & [Ax(k) + BK_s\Upsilon(k)]^T P_i[Ax(k) + BK_s\Upsilon(k)] \\ & - x^T(k)P_ix(k) + x^T(k)Qx(k) - x^T(k-\tau_m)Qx(k-\tau_m) \\ & + \tau_m\xi^T(k)R\xi(k) - \tau_m \sum_{q=k-\tau_m}^{k-1} \xi^T(l)R\xi(l) + \varepsilon\Upsilon^T(k)\Omega_i\Upsilon(k) - \Theta_2^T(k)\Omega_i\Theta_2(k). \end{aligned} \tag{3.7}$$

Introducing a free power matrix inspired by [30], we get:

$$0 = x(k) - x(k-\tau(k)) - \sum_{l=k-\tau(k)}^{k-1} \xi(l), \tag{3.8}$$

$$0 = x(k-\tau(k)) - x(k-\tau_m) - \sum_{l=k-\tau_m}^{k-\tau(k)} \xi(l), \tag{3.9}$$

$$0 = \tau_m\eta_1^T(k)S\eta_1(k) - \sum_{l=k-\tau_m}^{k-1} \eta_1^T(l)S\eta_1(l), \tag{3.10}$$

where $\eta_1(k) = \begin{bmatrix} x^T(k), x^T(k-\tau_m), \Theta_1(k), \Theta_2(k) \end{bmatrix}^T$. However, based on Eqs (2.11), (2.13), (2.14) in this paper, Eqs (3.8) and (3.9) will be improved as follows:

$$0 = x(k) - \Upsilon(k) - \sum_{l=k-\tau(k)}^{k-1} \xi(l), \tag{3.11}$$

$$0 = \Upsilon(k) - x(k-\tau_m) - \sum_{l=k-\tau_m}^{k-\tau(k)} \xi(l). \tag{3.12}$$

For Eq (3.7), we have

$$\begin{aligned} E\{\Delta V(k)\} \le {} & [Ax(k) + BK_s(\Upsilon(k))]^T P_i[Ax(k) + BK_i(\Upsilon(k))] - x^T(k)P_ix(k) + x^T(k)Qx(k) \\ & - x^T(k-\tau_m)Qx(k-\tau_m) + \tau_m\xi^T(k)R\xi(k) - \tau_m \sum_{q=k-\tau_m}^{k-1} \xi^T(l)R\xi(l) + \varepsilon\Upsilon^T(k)\Omega\Upsilon(k) \\ & - \Theta_2^T(k)\Omega\Theta_2(k) + 2\eta_1^T(k)X[x(k) - \Upsilon(k) - \sum_{l=k-\tau(k)}^{k-1} \xi(l)] + 2\eta_1^T(k)Y[\Upsilon(k) - x(k-\tau_m) \end{aligned}$$

$$-\sum_{l=k-\tau_m}^{k-\tau(k)}\xi(l)] + \tau_m\eta_1^T(k)S\eta_1(k) - \sum_{l=k-\tau_m}^{k-1}\eta_1^T(l)S\eta_1(l), \tag{3.13}$$

and the Eq (3.13) can be simplified to

$$E\{\Delta V(k)\} \le \eta_1^T(k)\Xi_i\eta_1(k) - \sum_{l=k-\tau(k)}^{k-1}\eta_2^T(l)Z_1\eta_2(l) - \sum_{l=k-\tau_m}^{k-\tau(k)}\eta_2^T(l)Z_2\eta_2(l), \tag{3.14}$$

where $\eta_2(k) = [\eta_1(k), \xi(k)]$, $Z_1 = \begin{bmatrix} S & X \\ * & R \end{bmatrix}, Z_2 = \begin{bmatrix} S & Y \\ * & R \end{bmatrix}$. By the Schur complement, if Theorem 1 is satisfied, $E\{\Delta V(k)\} \le 0$ will be ensured. Eq (3.14) can be changed to

$$\begin{aligned} E\{\Delta V(k)\} \le \eta_1^T(k)\Xi_i\eta_1(k) - \sum_{l=k-\tau(k)}^{k-1}\eta_2^T(l)Z_1\eta_2(l) \\ - \sum_{l=k-\tau_m}^{k-\tau(k)}\eta_2^T(l)Z_2\eta_2(l) \le \eta_1^T(k)\Xi_i\eta_1(k), \end{aligned} \tag{3.15}$$

because of $\Xi_i < 0$. Thus,

$$\begin{aligned} E\{\Delta V(k)\} &= E\{V(x(k+1), r(k))|x(k), r(k)\} - V(x(k), r(k)) \\ &\le -\lambda x^T(k)x(k), \end{aligned} \tag{3.16}$$

where $\lambda = \inf\{\sigma_{\min}(-\Xi_i)\}$, and $\sigma_{\min}(-\Xi_i)$ indicates the minimal eigenvalues of $-\Xi_i$. From Eq (3.16), for $T > 1$ the following conclusions can be drawn:

$$E\{V(x(T+1), r(T+1))\} - E\{V(x(0), r(0))\} \le -\lambda\sum_{k=0}^{T}E\{x^T(k)x(k)\}, \tag{3.17}$$

and then

$$\begin{aligned} \sum_{k=0}^{T}E\{x^T(k)x(k)\} &\le \frac{1}{\lambda}(E\{V(x(0), r(0))\} - E\{V(x(T+1), r(T+1))\}) \\ &\le \frac{1}{\lambda}E\{V(x(0), \tau_0)\} \le \infty. \end{aligned} \tag{3.18}$$

Therefore, system (2.11) is stochastically stable according to Definition 1.

3.2. Controller design

Theorem 2. For the given parameters $0 \le \varepsilon < 1$, $\tau_m > 0$, the discrete system (2.11) is stochastically stable with ETC (2.8) if there are real matrices $W_i > 0$, $\tilde{V} > 0$, $\tilde{Q} > 0$, $\tilde{S} > 0$ and $\tilde{\Omega}_i > 0 (i = r(k))$ and $\tilde{X}$, $\tilde{Y}$ of suitable dimensions such that $\begin{bmatrix} S & \tilde{X} \\ * & 2W_i - V \end{bmatrix} \ge 0$, $\begin{bmatrix} S & \tilde{Y} \\ * & 2W_i - V \end{bmatrix} \ge 0$, and for appropriate dimensions a matrix $\tilde{K}_s$ such that

$$\tilde{\Xi}_i = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ * & -W_i & 0 \\ * & * & -\tau_m\tilde{V} \end{bmatrix} < 0, \tag{3.19}$$

$$\begin{bmatrix} (\pi_{ii}-1)W_i & \alpha_i \\ * & \beta_i \end{bmatrix} \leq 0, \tag{3.20}$$

where

$$\begin{aligned}
\Gamma_{11} &= \tilde{\Phi}_i + \tilde{\zeta} + \tilde{\zeta}^T + \tau_m \tilde{S}, \\
\Gamma_{12} &= \begin{bmatrix} AW_i, 0, B\tilde{K}_s, -B\tilde{K}_s \end{bmatrix}^T, \\
\Gamma_{13} &= \begin{bmatrix} (A-I)W_i, 0, B\tilde{K}_s, -B\tilde{K}_s \end{bmatrix}^T, \\
\zeta &= \begin{bmatrix} \tilde{X}, -\tilde{Y}, \tilde{Y}-\tilde{X}, \tilde{X}-\tilde{Y} \end{bmatrix}, \\
\alpha_i &= \begin{bmatrix} W_i\sqrt{\pi_{i1}}, \cdots, W_i\sqrt{\pi_{i,i-1}}, W_i\sqrt{\pi_{i,i+1}}, \cdots, W_i\sqrt{\pi_{i,\tilde{\tau}}} \end{bmatrix}, \\
\beta_i &= diag\{-W_1, \cdots, -W_{i-1}, -W_{i+1}, \cdots, -W_{\tilde{\tau}}\}, \\
\Phi_i &= \begin{bmatrix} -W_i+\tilde{Q} & 0 & 0 & 0 \\ * & -\tilde{Q} & 0 & 0 \\ * & * & (\varepsilon-1)\tilde{\Omega}_i & \varepsilon\tilde{\Omega}_i \\ * & * & * & -\tilde{\Omega}_i \end{bmatrix}.
\end{aligned}$$

Proof. Let $\Lambda_1 = diag\{P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}, R^{-1}\}$ and $\Lambda_2 = diag\{P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}\}$. Therefore, pre- and post-multiplying Eq (3.1), $\begin{bmatrix} S & X \\ * & R \end{bmatrix}$, $\begin{bmatrix} S & Y \\ * & R \end{bmatrix}$, and Eq (3.2) with Λ_1,Λ_2,Λ_2, P_i^{-1}, respectively, then, by defining $W_i = P_i^{-1}$, $\tilde{V} = R^{-1}$, $\tilde{Q} = P_i^{-1}QP_i^{-1}$, $\tilde{R} = P_i^{-1}RP_i^{-1}$, $\tilde{\Omega}_i = P_i^{-1}\Omega_i P_i^{-1}$, $\tilde{S} = \Lambda_2 S \Lambda_2$, $\tilde{\zeta} = \Lambda_2\zeta\Lambda_2$, and $K_s = \tilde{K}_s W_i^{-1}$, Eq (3.19) can be obtained. In addition, from Eq (3.2), we have

$$\sum_{j=1}^{\tilde{\tau}} \pi_{ij} W_i P_j W_i \leq W_i. \tag{3.21}$$

Then, by using the Schur complement to change it, which is equivalent to Eq (3.20) we complete the proof.

Remark 4. The vector $\eta_1(k)$ is designed by decoupling $\tau(k)$. When changing the ETC, like the idea of incorporating the error term into the vector $\eta_1(k)$ in other papers [3, 9, 15, 26, 27, 29], we also aim to better reflect the accumulative term $\sum_{j=k_t}^{k} e(j)$. In addition, the influence of external disturbances on NCSs is inevitable, which greatly motivates our future research.

4. Numerical examples

Example 1. In this section, we provides a numerical example to validate the efficacy of the presented approach. For a discrete-time linear system (2.1), the corresponding coefficient matrices are given below:

$$\text{A} = \begin{bmatrix} 0.85 & 0 & 0.1 \\ 0.01 & 0.96 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{B} = \begin{bmatrix} -0.1 \\ -0.2 \\ -0.1 \end{bmatrix}.$$

the $r(k)$ has two modes, $r(k) = \{1, 2\}$, which is switched by a Markov process changing with k , and state transition matrix as follows:

$$\pi = \begin{bmatrix} 0.3 & 0.7 \\ 0.85 & 0.15 \end{bmatrix}.$$

The possible change process of the mode of $r(k)$ with time is shown in Figure 2.

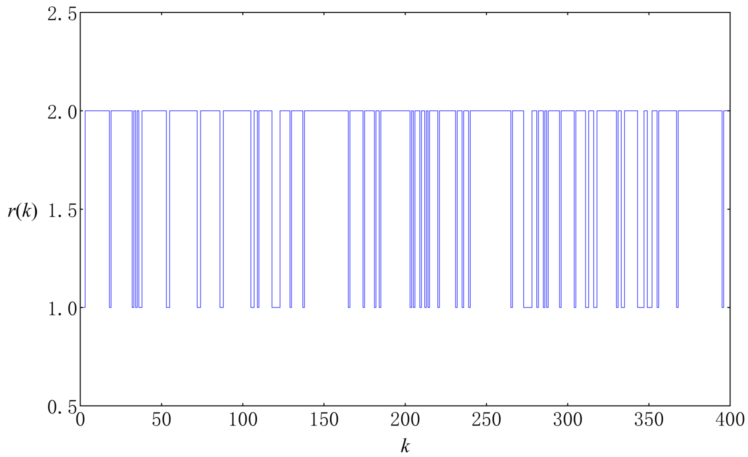

Figure 2. Parameter change of $r(k)$.

However, for simplicity, the value of random time delays τ_{k_t} for different $r(k)$ are shown in Table 1. When $\varepsilon = 0.15$ and $\tau_m = 3$ by Theorem 2, the systems control gain and the event-triggered parameter are co-designed together as:

$$K_1 = \begin{bmatrix} 0.0771 & 0.1924 & 0.1324 \end{bmatrix},$$

$$\Omega_1 = \begin{bmatrix} 0.0113 & -0.0001 & 0.0002 \\ -0.0001 & 0.0124 & -0.0003 \\ 0.0002 & -0.0003 & 0.0132 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.1672 & 0.4051 & 0.2790 \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} 0.0790 & -0.0010 & 0.0003 \\ -0.0010 & 0.0797 & -0.0021 \\ 0.0003 & -0.0021 & 0.0839 \end{bmatrix}.$$

Table 1. Delay τ_{k_t} corresponding to different $r(k)$.

	$r(k) = 1$	$r(k) = 2$
τ_{k_t}	1	2

The state response of the system with $x_0 = \begin{bmatrix} -3, -1, 2 \end{bmatrix}^T$ is shown in Figure 3, from which it can clearly be seen that system (2.1) is stochastically stable. Moreover, the time interval between two consecutive events is shown in Figure 4.

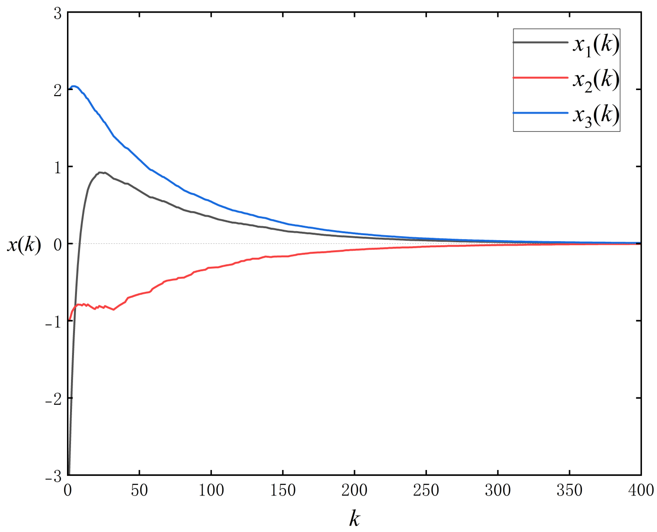

Figure 3. State responses of the system described by Example 1 with the ETC.

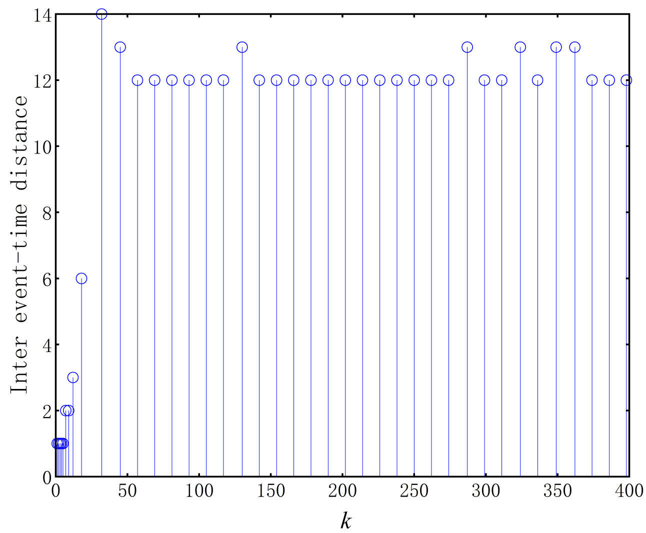

Figure 4. The inter-event time distance for Example 1.

It can clearly be seen that 41 events are triggered from Figure 4 during $1 \leq k \leq 400$, which means that

41 data packets are released. Thus, under the influence of ETC (2.8), the release rate is 10.25%, leading to a great savings of 89.25% in networked resources. The minimum inter-event time distance is 1, and the maximum inter-event time distance is 14, and all are greater than or equal to 1. Specifically, after the system reaches a steady state, the event generator does not enter hibernation, but works normally, i.e., $k \geq 250$. On the contrary, the utilization of conventional event-triggered schema in [30] leads to sporadic functioning of the event-triggered controller after the system reaches a steady state, almost entering a hibernation state by simply considering different event-triggered schemas.

In addition, Figure 5 shows the possible change in the process of time delays τ_{k_t} with k.

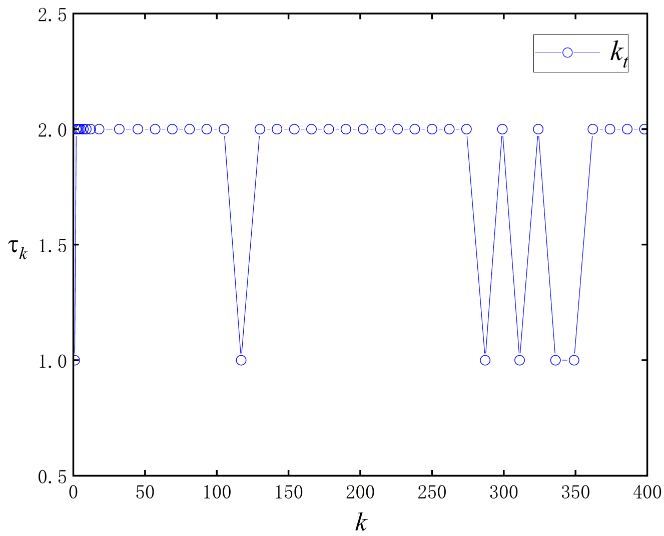

Figure 5. Parameter change of τ_{k_t} with k by Example 1.

By comparing Figures 2, 4, and 5, we can see that even though the time delays follow the Markov process with the change of $r(k)$, when the ETC has no effect, the current time delays still maintain the size of the time delays corresponding to the previous triggering time $r(k)$, which can better reflect the rigor of the ETC.

Example 2. As a physical object of the NCSs, an unstable batch reactor is a common research subject. Its purpose is to design a stable controller that enables the NCSs to tolerate a permissible range of network-induced delays and packet losses [26, 27]. Therefore, this paper considers an unstable batch reactor from [27] and discretizes the system with a sampling period of 0.0005 to obtain the corresponding discrete-time system described by Eq (2.1):

$$A = \begin{bmatrix} 1.0070 & -0.0010 & 0.0330 & -0.0278 \\ -0.0029 & 0.9788 & -0.0000 & 0.0034 \\ 0.0052 & 0.0211 & 0.9675 & 0.0288 \\ 0.0002 & 0.0211 & 0.0066 & 0.9897 \end{bmatrix}, B = \begin{bmatrix} 0.0000 & -0.0003 \\ 0.0281 & 0.0000 \\ 0.0060 & -0.0155 \\ 0.0060 & -0.0001 \end{bmatrix}.$$

Same as Example 1, applying it with $\varepsilon = 0.15$, $\tau_m = 3$, and the parameter changes of τ_{k_t} and $r(k)$, the system's control gain and the event-triggered parameter are co-designed together according to

Theorem 2:

$$K_1 = \begin{bmatrix} 0.0407 & -1.9423 & -0.0013 & -0.4822 \\ -0.3007 & -0.6346 & -3.6837 & 0.1085 \end{bmatrix},$$

$$\Omega_1 = \begin{bmatrix} 0.0135 & -0.0000 & 0.0005 & -0.0003 \\ -0.0000 & 0.0120 & 0.0002 & 0.0001 \\ 0.0005 & 0.0002 & 0.0118 & 0.0003 \\ -0.0003 & 0.0001 & 0.0003 & 0.0131 \end{bmatrix} * 10^{-6},$$

$$K_2 = \begin{bmatrix} 0.0662 & -4.0542 & -0.0184 & -0.9564 \\ -0.4755 & -1.2771 & -4.6181 & 0.1826 \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} 0.0825 & -0.0000 & 0.0006 & -0.0006 \\ -0.0000 & 0.0768 & 0.0001 & -0.0005 \\ 0.0006 & 0.0001 & 0.0766 & 0.0002 \\ -0.0006 & -0.0005 & 0.0002 & 0.0817 \end{bmatrix} * 10^{-6}.$$

The state response of the system with $x_0 = \begin{bmatrix} 5, -1, -4, 2 \end{bmatrix}^T$ is shown in Figure 6. Clearly, the system under control is stochastically stable during $1 \leq k \leq 400$. In addition, the time interval between two consecutive events is shown in Figure 7. From Figures 6 and 7, for clarity, when using the event-triggered scheme, 49 events are triggered during $1 \leq k \leq 400$, which is a release rate of 12.25% and saves network resources by 87.75%. Before the system reaches its stable state at $k = 150$, there are 20 events triggered, which means that the event-triggered rate is 13.3%. After $k = 150$, there are 19 events triggered until $k = 400$, leading to an event-triggered rate of 7.6%. It can be seen that the event-triggered mechanism designed in this paper not only can operate reliably under random time delays, but also continues to operate normally at a low rate. This further demonstrates the advantages of the ETC (2.7). Table 2 shows that for [27], it triggers 81 events and releases 81 packets at times $1 \leq k \leq 600$, which is a release rate of 13.5%, saving network resources by 86.5%, and the system is stable at times $k \geq 300$. Therefore, this highlights the advantages offered by the event-triggered scheme (2.7) after improving.

Table 2. Release rate and stability moments.

	release rate	stability moments
[27]	13.5%	$k \geq 300$
Theorem 2	12.25%	$k \geq 150$

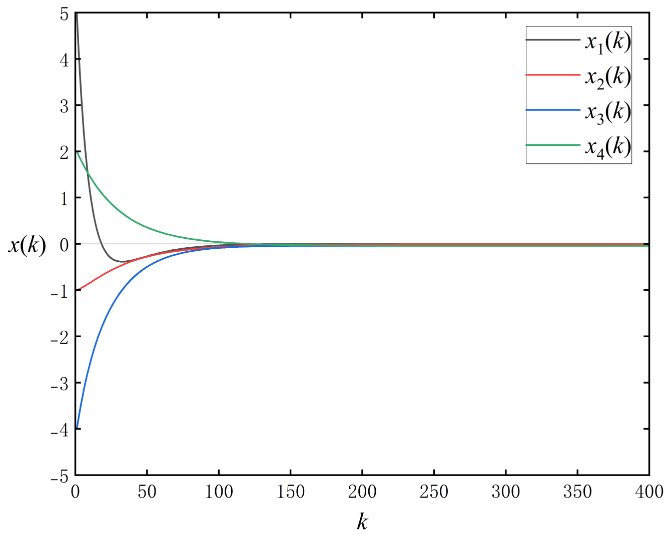

Figure 6. State responses of the system described by Example 2 with the ETC.

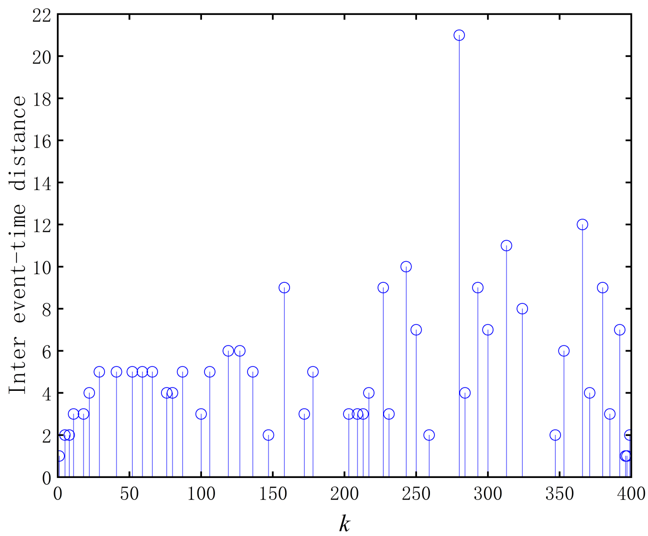

Figure 7. The inter-event time distance for Example 2.

5. Conclusions

The control problem of event-triggered discrete linear systems with random time delays is discussed in this paper. On the one hand, different from existing papers, this paper improves an ETC based on accumulative error and sets different event-triggered parameters for different Markov modes. By decoupling the accumulative error term, a new closed-loop discrete switching system equation is obtained. On the other hand, the Markov chain with the trigger delays are classified by designing, and the control gain parameters with different delays are set. Based on the Lyapunov functional and free-weight matrix method, the stochastic stability criterion of the system is obtained by co-designing appropriate control gain and event-triggered parameters. Finally, to confirm the effectiveness of the proposed method, numerical simulations are conducted, demonstrating the accuracy of the derived results.

Author contributions

Shen-Ping Xiao: Writing-review & editing, formal analysis, validation, conceptualization, funding acquisition; Zhi Fu: Writing-original draft, software, methodology, investigation; Jiang-Lin Huang: Writing-review & editing; Chang-Xin Li: Writing-review & editing, supervision. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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