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Research article

Adaptive fuzzy fault-tolerant formation control for nonlinear multi-agent systems under cyber-physical threats

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Abstract: This article investigated the fuzzy adaptive fault-tolerant resilient formation control issue for uncertain nonlinear multi-agent systems (MASs) with immeasurable states and under denial-of-service (DoS) attacks. Fuzzy logic systems were utilized to model unknown agents, and a fuzzy state estimator was formulated to reconstruct the agents' unknown states. A distributed resilient formation estimator was proposed to obtain the unknown leader information estimation and its high-order derivatives under DoS attacks. Based on the designed fuzzy state and resilient formation estimators, a fault-tolerant fuzzy output-feedback adaptive resilient formation control scheme was developed via the backstepping control methodology. It was proven that the developed fault-tolerant fuzzy resilient formation control scheme can guarantee that the controlled nonlinear MASs were stable and formation tracking errors converged even under unknown states, actuator faults, and DoS attacks. Finally, the proposed fuzzy adaptive fault-tolerant resilient formation control method was applied to marine surface vehicles; the simulation results and comparisons showed the effectiveness of the presented control methodology.

Keywords: multi-agent systems (MASs); resilient fault-tolerant control; fuzzy adaptive observer; denial-of-service (DoS) attacks; distributed estimation

1. Introduction

Multi-agent systems (MASs) are widely used in applications such as unmanned aerial vehicles (UAVs), marine vessels, sensor networks, and smart transportation. These systems rely on cooperation among agents to achieve coordinated objectives such as formation control and task execution. However, real-world MASs are often subjected to practical challenges that severely their performance, including: (i) actuator faults that degrade control effectiveness, (ii) denial-of-service (DoS) attacks that block communication, and (iii) unmeasurable system states, which make full-state feedback control inapplicable. These problems are not only practically critical but also theoretically challenging. Many existing works assume ideal conditions such as full-state availability, absence of actuator faults, or uninterrupted communication. Such assumptions limit their applicability in adversarial or uncertain environments. Moreover, separate solutions for estimation, attack mitigation, or fault-tolerance exist, but very few provide an integrated control strategy that ensures resilient performance under all these conditions. To address these issues, recent works have focused on resilient control devices of UAVs; nevertheless, most of them suppose full-state non-disability, or, in other words, do not implement fault-tolerance devices. Besides, the integration into secure state estimation considering the intermittent communication or event-based updates has not been studied extensively. More recently, there has been a growing interest in resilient control of nonlinear systems with uncertainties and cyber-physical threats. To satisfy the convergence specifications within a finite time, Lv et al. [1] developed a neural network prescribed-time observer for uncertain pure-feedback nonlinear systems, guaranteeing fast stabilization through output feedback. Gong and Wang explored the issue of optimization in distributed MASs [2] through designing a second-order heterogeneous robust adaptive strategy. Real-life situations, resembling cybersecurity issues, like spoofing in wearable devices, were also considered by Han et al. [3] in voice authentication systems. Liu et al. [4] developed resilient formation tracking controllers in swarm systems against malicious data deception attacks so that the performance can be similar. Zhou et al. [5] addressed the problem of class imbalance and dependence in intrusion detection relying upon the structure of HIDIM refers to a High-Dimensional Intrusion Detection Model, which has provided a defense approach at the network level. An event-based fuzzy control with the DoS attacks has been proposed by Zhang et al. [6] to improve unmanned vehicle DoS attacks against communication failures. Tang et al. [7] introduced a low-rate DoS attack, a traffic state-based mitigation scheme architecture in the SDN. Software-Defined Networking (SDN) is a networking architecture that separates the control plane from the data plane. This decoupling makes the network programmable, centrally manageable, and more flexible in responding to changing conditions or cyber threats such as DoS attacks. On the same note, Liu et al. [8] designed a robust secure control strategy of heterogeneous swarm systems during coordinated cyber-attacks. Liu et al. [9] discussed the development of nanoscale systems and introduced high-precision ultra-large-scale AFM systems, revealing the significance of a sound physical model of the system. Atomic Force Microscopy (AFM) is a nanotechnology device based on a sharp probe to provide high-resolution and 3D topographic images. They are commonly used in nanotechnology, materials science, and biology to measure structures at an atomic scale. Wu et al. [10] considered fault-tolerant neural control schemes of nonlinear MASs under actuator faults and addressed output-feedback consensus. Li et al. [11] also suggested adaptive fuzzy output-feedback secure consensus protocols in DoS attacks. Ding et al. [12] developed an adaptive memory event-triggered output feedback finite-time lane-keeping control strategy for autonomous heavy trucks with roll prevention, demonstrating enhanced robustness and performance for nonlinear vehicle dynamics. Shi et al. [13] proposed a hypergraph-based Q-learning approach in the public goods game. Zhang et al. [14] addressed the issue of nonlinearity and hysteresis with the help of adaptive pseudoinverse control, which can be applied to elastomer actuators. The balance between code estimation error and gain performance under SCER attacks was explored by Li et al. [15], and Zhou et al. [16] proposed a hybrid neural network and physics-based estimator based on little driving data. Sensor Code Estimation and Replay (SCER) attacks are cyber-physical attacks that use estimated sensor data to devise a manipulated or replayed sensor data to compromise control accuracy and stability. They take advantage of coding vulnerabilities in sensors to impair performance. Han and Jin [17] integrated fuzzy adaptive and event-triggered impulsive control of nonlinear MASs; the use of the two approaches resulted in the finite-time regulation. Nie et al. [18] enhanced finite-time control sliding mode control over fuzzy topologies in the context of improved consensus in the presence of attack. Chen et al. [19] developed a range-only distributed formation controller using contracting estimators and barrier functions. Yue et al. [20] came up with a standard DDoS detection procedure through IQR and DFFCNN in SDN setups. A Distributed Denial-of-Service (DDoS) is a massive cyber menace where an online system floods the target with unmanageable traffic, receiving the victims so that the resources are neutralised and the services are disrupted. A statistical measure that helps to identify anomalies in such patterns of attack, the Interquartile Range (IQR) is a measure of the spread between the upper and lower quartiles of data that highlights outliers. This is complemented by the fact that Deep Feedforward Convolutional Neural Networks (DFFCNNs) constitute a machine learning method that is quite effective in intrusion detection systems since it can automatically learn hierarchical features in the input data, thereby being apt to recognize complex and evolving cyber-attack signatures. Mousavian and Atrianfar [21] dealt with resilient containment control in the presence of multiple DoS and disturbances with an adaptive event-triggered approach. Su et al. [22] studied the leader-follower consensus control of DoS attacks in the nonlinear MASs and iterative learning. Ju et al. [23] presented a quantized predefined-time control approach for heavy-lift launch vehicles considering actuator faults and rate gyro malfunctions, effectively enhancing fault tolerance and ensuring stability within a predefined convergence time.

Prescribed exact-performance fuzzy robust controller technology is proposed by Wang et al. [24] in which manipulators are addressed and uncertainty is handled in a flexible way. Scholars probed event-based model-free adaptive control of MASs under attack interruptions established by Xiong et al. [25]. Li et al. [26] enhanced disturbance rejection in the electro-hydrostatic actuators through reinforcement learning. Ru et al. [27] got a hybrid-attack-secure bipartite protocol, WTOD. The WTOD protocol is a hybrid attack-resistant bipartite control strategy to increase the resilience of multi-agent and cyber-physical systems. It gives protection against a combination of threats like deception and denial-of-service attacks, and it also ensures reliability in the face of adversary-controlled coordination and communications. Xu et al. [28]covered the topic of energy-latency trade-offs in AR offloading to blockchain in UAV networks. Augmented Reality (AR) refers to technology to overlays the real world with digital information to promote user interaction. Under networks, AR offloading redirects the heavy workloads to cloud or edge computers to enhance performance and reduce latency. Meng et al. [29], in addition, provided an adaptive fixed-time stabilization approach in uncertain systems. QWen et al. [30] became the first to find a solution that suggested a composite observer-based controller to deal with deception attacks that had unknown control directions. In consensus control, Zhu et al. [31] addressed the DoS and delay impacts with the help of HDWM. The Hybrid Delay-Weighted Mechanism (HDWM) is a control scheme to be used against DoS attacks and delays of communication. It scales the performance of the system based on a weighting factor among delays and disruptions, and a stable and resilient system through multi-agent networks. As stated, Yu et al. [32] introduced a non-fragile DoS bipartite tracking control strategy based on a sampled-data approach. The contour tracking using time-varying internal models was discussed by Cao and Zhang [33]. By designing a local stabilization approach to microgrids in DoS attacks based on resilient adaptive control, Hong and Kim [34] came up with a local stabilization technique for microgrids subject to DoS attacks and Lyapunov functionals. It was observed that Lyapunov functions could only be used to study the stability of systems with time-varying delays, Wang et al. [35]. Gong et al. [36] presented a fractional-order optimization method in unbalanced MAS networks having disturbance rejection. Abdelhamid and Mohamed [37] developed strong fuzzy adaptive fault-tolerant controllers on second-order nonlinear systems. Bounemeur and Chemachema [38] considered the Nussbaum-type functions in an abstract concept of fuzzy set to process sensor and actuator faults that distributed resilient observer-based resistance to actuator failures, as well as DoS attacks that were reported by Deng and Wen [39] in heterogeneous MASs. Li et al. [40] addressed event-based formation control based on a mixed self and event-triggered mechanism. Chen et al. [41] suggested resilient adaptive H_∞ control of MASs about sensor and sensor faults. Lastly, Hu et al. [42] introduced finite-time formation–circumnavigation switching control using intermittent strategies, while Zhou and Tong [43] focused on fuzzy adaptive formation control for MASs under DoS conditions. During the past few years, repetitive learning (RL) strategies began to gain constant embrace as a representative of the design of fuzzy control systems to address the complex nonlinear dynamics. As an example, Shen et al. [44] introduced a fuzzy weight-based reinforcement learning to nonlinear servo systems that have significant improvement toward the control performance with a limited number of communication resources within the predefined-time based event-triggered event-based predefined-time firing algorithm in event-based tracking control. The interaction of fuzzy logic and reinforcement learning allowed them to develop more flexibility and accuracy in uncertain situations. Likewise, Wang et al. [45] combined a typical reinforcement Q-learning process to acquire the fuzzy H_∞ control of the discrete-time nonlinear Markov jump systems. This method was able to solve the questions of uncertainties in the system and the stochastic jumps, and was able to provide a good control solution in the cybernetic area. The combination of the fuzzy control theory and the RL-based optimization procedures, through these works as a whole, notes the possibilities of achieving control of nonlinearities and uncertainties of real-time control systems. Driven by these developments, the current research paper aims at the development of a finite-time fuzzy secure control system of nonlinear multi-agent systems in the presence of actuator faults and deception attacks.

Ensuring resilient formation control of nonlinear MASs in the presence of actuator faults, DoS attacks, and unmeasurable states is critical for applications such as autonomous marine vehicles and UAVs. Most existing works in the literature address only partial aspects of this problem. For example, Liu et al. [4] and Zhang et al. [6] studied formation tracking under deception and DoS attacks, but assumed full-state availability and did not address actuator faults. Li et al. [11] and Liu et al. [12] incorporated DoS or False Data Injection (FDI) attacks with output-feedback control, but did not integrate state estimation or distributed formation under communication constraints. FDI threats are the modification of sensor or communications data to deceive the controllers. By injecting falsified measurements, the FDI attacks perturb stability, performance, and may lead to unsafe decisions in multi-agent or networked systems. Deng and Wen [39] proposed a resilient observer-based control method for heterogeneous MASs, but without fuzzy approximation or adaptive feedback mechanisms. Motivated by these gaps, this paper presents a fuzzy adaptive fault-tolerant formation control framework for uncertain nonlinear MASs with immeasurable states, actuator faults, and DoS attacks. The key contributions of this study are:

- A fuzzy logic-based state estimator is designed to reconstruct unknown agent states with adaptive learning under fault and uncertainty conditions.
- A distributed resilient formation estimator is developed to estimate the leader's state and its derivatives even under DoS attacks.
- A fault-tolerant fuzzy adaptive output-feedback controller is constructed using the backstepping method, ensuring convergence despite cyber-physical disruptions.
- A composite Lyapunov function proves the global stability and boundedness of tracking errors and parameter estimates.
- Simulation experiments on a network of marine vehicles validate the proposed control scheme, showing improved performance compared with recent methods in the presence of DoS attacks and actuator failures.

The layout of the subsequent content is structured as follows:

Section 1, an introduction to our article. In Section 2, the paper introduces fundamental assumptions and system model nonlinear MASs with actuator faults and uncertainties, establishes a fault-tolerant adaptive communication topology, DoS fault tolerance, and fault-tolerant fuzzy control. In Section 3, we define the state estimator and distributed estimation for uncertain systems. In Section 4, the concept of fault-tolerant adaptive output feedback formation control under uncertainties is discussed. Section 5 presents numerical simulations that validate the effectiveness and robustness of the proposed control strategy.

2. Fundamental concepts and assumptions

2.1. System modeling and configuration

The full system dynamics for each agent i in an (MAS), accounting for control inputs, faults, uncertainties, and formation tracking, are outlined as follows. For each agent i in the MAS, the state dynamics are:

$$
\begin{aligned}
\dot{\tilde{x}}_{i,q} &= \tilde{x}_{i,q+1} + h_{i,q}(\tilde{x}_{i,q}) + \delta_{i,q}, \quad q = 1, \ldots, n-1 \\
\dot{\tilde{x}}_{i,n} &= (1+\alpha_i)v_i(t) + h_{i,n}(\tilde{x}_{i,n}) + \gamma_i \varepsilon_i(t) + \delta_{i,n},
\end{aligned} \tag{2.1}
$$

where: $\tilde{x}_{i,q}$ are the state variables of agent i, $h_{i,q}(\tilde{x}_{i,q})$ is a nonlinear function of these states, $\delta_{i,q}$ is the bounded uncertainty, satisfying $\|\delta_{i,q}\| \leq \delta_i$, $v_i(t)$ represents the control input, $(1+\alpha_i)$ accounts for the actuator's efficiency, $0 < \underline{\alpha} < \alpha_i < \overline{\alpha} \leq 1$ where $\underline{\alpha}$ and $\overline{\alpha}$ are known constants representing the lower and upper bounds of actuator efficiency degradation, α_i represents the actual unknown fault level for agent i,γ_i denotes a fault indicator with $\gamma_i \in \{0, 1\}$,and $\epsilon_i(t)$ is the fault term, bounded by $|\varepsilon_i(t)| \leq \varepsilon_i$.

The output y_i for each agent i is given by:

$$y_i = \tilde{x}_{i,1}. \tag{2.2}$$

As for each agent, this output y_i for control and formation tracking is employed as the measurable output. The virtual leader, providing the desired trajectory, follows the dynamics:

$$\begin{aligned} \dot{\eta}_{\text{ref}} &= A\eta_{\text{ref}}, \\ y_{\text{ref}} &= C\eta_{\text{ref}}, \end{aligned} \tag{2.3}$$

where η_{ref} is the state of the virtual leader, y_{ref} is the reference output of the leader. Let $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{1\times n}$ be known constant matrices.The formation error e_i for each agent i, relative to the desired trajectory, is defined as:

$$e_i = y_i - Cs_i - y_{\text{ref}}, \tag{2.4}$$

where s_i is the desired formation position for each agent i.

Remark 2.1. Some of the assumptions in this paper, including bounded actuator faults and Lipschitz continuity of nonlinear system dynamics, as well as knowledge of DoS timing constraints, are common in resilient control literature and can be found in other papers required to provide rigorous stability guarantees through Lyapunov-based analysis. In that case, however, we realize that such assumptions can limit the usage of the proposed method to the case of less uncertain or adversarial conditions. The upcoming studies might find mechanisms to loosen these assumptions with tantamount levels of probabilistic DoS models, unknown fault profiles, or data-driven systems.

Assumption 2.2. [31] The fault term $\varepsilon_i(t)$ for each agent i is bounded, meaning there exists a known constant $\varepsilon_i > 0$ such that

$$|\varepsilon_i(t)| \leq \varepsilon_i, \quad \forall t \geq 0. \tag{2.5}$$

This ensures that $\varepsilon_i(t)$ does not grow unbounded over time. The fault term $\varepsilon_i(t)$ is assumed to have no sudden jumps or discontinuities, implying that it varies smoothly and predictably.

Definition 2.3. [32] For the MAS described by (2.1) and the leader system (2.3) on an undirected graph $\bar{\Phi}$, the desired formation is achieved if, despite the presence of faults and uncertainties, the formation errors

$$e_i = y_i - Cs_i - y_{\text{ref}}$$

converge close to zero.

2.2. Fault-tolerant adaptive topology

The system is represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of agents, and $\mathcal{E}$ consists of edges, each represented as a pair (i, j), signifying a connection between agents i and j. The adjacency matrix $\mathcal{A} \in \mathbb{R}^{N\times N}$ encodes these connections, where $\mathcal{A}_{ij} = 1$ if there is an edge between agents i and j, and $\mathcal{A}_{ij} = 0$ otherwise. Since the graph is undirected, $\mathcal{A}$ is symmetric, meaning $\mathcal{A}_{ij} = \mathcal{A}_{ji}$.The degree matrix $\mathbf{D}$ is a diagonal matrix where each element $\mathbf{D}_i$ represents the number of connections (degree) of agent i, calculated as $\mathbf{D}_i = \sum_{j=1}^{N} \mathcal{A}_{ij}$. Therefore, the degree matrix is $\mathbf{D} = \text{diag}(\mathbf{D}_1, \mathbf{D}_2, \ldots, \mathbf{D}_N)$.

The Laplacian matrix $\mathcal{L}$ is computed by subtracting the adjacency matrix from the degree matrix:

$$\mathcal{L} = \mathbf{D} - \mathcal{A}. \tag{2.6}$$

This matrix captures the network's connectivity structure. The communication matrix $\mathbf{\Delta}$ is also a diagonal matrix, where each element λ_i indicates whether the leader can send information to agent i. If $\lambda_i = 1$, the leader can communicate with agent i; if $\lambda_i = 0$, communication is not possible. Thus, the communication matrix is $\mathbf{\Delta} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$. The updated network topology incorporating the leader's communication restrictions is represented by the matrix $\mathcal{H}$, which is the sum of the Laplacian matrix and the communication matrix:

$$\mathcal{H} = \mathcal{L} + \mathbf{\Delta}. \tag{2.7}$$

This matrix reflects both the agent's connectivity and the communication restrictions imposed by the leader.

Remark 2.4. The proposed control framework implicitly relies on spectral graph theory through the use of the Laplacian matrix. The spectral properties of $L(t)$ characterize the connectivity and interaction strength among agents, which is essential for analyzing formation stability under dynamic topologies. During DoS attacks, the time-varying nature of $L(t)$ captures link failures and restorations, enabling resilient control design and stability analysis.

Assumption 2.5. The network graph $\mathcal{G}$ is connected, and the leader has communication access to at least one agent. This is captured in the matrix $\mathcal{H} = \mathcal{L} + \mathbf{\Delta}$, where $\mathbf{\Delta} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ indicates the communication links between the leader and the agent.

2.3. DoS fault tolerance

This equation models the behavior of agent i. The term $M_i\xi_i(t)$ represents the internal dynamics of the agent, while $N_i v_i(t)$ corresponds to the control input or the interactions with other agents. The term $P_i\zeta_i(t)$ accounts for external disturbances or faults influencing the agent's behavior.

$$\dot{\xi}_i(t) = M_i\xi_i(t) + N_i v_i(t) + P_i\zeta_i(t). \tag{2.8}$$

The intervals during which a DoS attack affects the communication link between agents i and j. The attack starts at time ψ_{ij}^p and continues for a duration of τ_{ij}^p, temporarily disabling communication between the agents.

$$\Pi(i, j)_p = [\psi_{ij}^p, \psi_{ij}^p + \tau_{ij}^p). \tag{2.9}$$

Time-varying adjacency matrix $A(t)$ of the network. When the system is not under a DoS attack (i.e., $t \in \Xi_N(t)$), the communication between agents i and j is active, represented by A_{ij}. If a DoS attack occurs (i.e., $t \in \Xi_D(t)$), the link between agents is disrupted, and the corresponding matrix entry becomes zero.

$$A(t) = \begin{cases} A_{ij}, & \text{if } t \in \Xi_N(t) \\ 0, & \text{if } t \in \Xi_D(t) \end{cases} \tag{2.10}$$

- $\Xi^N(t)$: the set of time intervals during which the network is not under a DoS attack, i.e., normal communication is active.
- $\Xi^D(t)$: the set of time intervals during which the network experiences a DoS attack, and communication is interrupted.

The Laplacian matrix $L(t)$ at any given time t. It is defined as the difference between the in-degree matrix $\Theta(t)$ and the adjacency matrix $A(t)$. The matrix $\Theta(t)$ reflects the number of neighbors each agent has, and $L(t)$ encodes the network's topology. A change in $A(t)$ due to DoS attacks leads to modifications in $L(t)$, which impacts the system's overall dynamics.

$$L(t) = \Theta(t) - A(t). \tag{2.11}$$

This equation represents the control input $v_i(t)$ for agent i. It is composed of two parts: $K_i\xi_i(t)$, which is the local feedback applied to the state of the agent, and $L_i\xi_\ell(t)$, which is the influence of the leader's state $\xi_\ell(t)$ on the agent. The control law is designed to guide the agent's state toward its desired trajectory, ensuring coordination even in the presence of network disruptions caused by DoS attacks.

$$v_i(t) = K_i\xi_i(t) + L_i\xi_\ell(t). \tag{2.12}$$

Lemma 2.6. Let the system state be represented by $\tilde{x}(t)$, which is subject to both faults and (DoS) attacks. Define $V(\tilde{x}(t))$ as a Lyapunov function that captures the system's stability. Assume there are constants $\alpha > 0, \beta > 0$, and $\gamma > 0$ such that the following conditions hold:

(i) During intervals without DoS attacks, the rate of change of $V(\tilde{x}(t))$ is bounded by:

$$\dot{V}(\tilde{x}(t)) < -\alpha V(\tilde{x}(t)) + \gamma, \tag{2.13}$$

ensuring that the Lyapunov function decays exponentially and the system stabilizes.

(ii) During DoS attack intervals, the rate of change is bounded by:

$$\dot{V}(\tilde{x}(t)) < \beta V(\tilde{x}(t)) + \gamma, \tag{2.14}$$

which allows some destabilizing effects but still limits the damage due to the attack.

Given these conditions, the system's stability is guaranteed by the following inequality:

$$V(\tilde{x}(t)) \leq e^{-\alpha|\Xi_N(0,t)|}e^{\beta|\Xi_D(0,t)|}V(\tilde{x}(0)) + \frac{\gamma}{\alpha} + \frac{1}{\alpha}\sup_t \|\delta(t)\|. \tag{2.15}$$

This inequality indicates that, despite the presence of DoS attacks, faults, and uncertainties, the system remains stable as long as the perturbations $\delta(t)$ are controlled within certain bounds over time.

2.4. Fault-tolerant fuzzy control

The dynamics of nonlinear and uncertain systems using fuzzy logic systems and MASs 2.1 ensure that the approximation error becomes arbitrarily small with the proper selection of fuzzy system parameters.

Lemma 2.7. [32] Let $\tilde{x}_i(q) \in \mathbb{R}^n$ be the state variables of agent i at time q, and let $h_i(q, \tilde{x}_i(q))$ be a continuous, nonlinear function representing the system dynamics, subject to bounded uncertainty $\delta_i(q)$. Suppose $h_i(q, \tilde{x}_i(q))$ is Lipschitz continuous in $\tilde{x}_i(q)$ with a known Lipschitz constant L_h, i.e.,

$$\|h_i(q, \tilde{x}_i(q)) - h_i(q, \tilde{x}_i(q)')\| \leq L_h \|\tilde{x}_i(q) - \tilde{x}_i(q)'\|, \quad \forall \tilde{x}_i(q), \tilde{x}_i(q)' \in \mathbb{R}^n. \tag{2.16}$$

For any constant $\epsilon^* > 0$, there exists a fuzzy logic system (FLS) $h_i(q, \tilde{x}_i(q)) \approx w^T \varphi_i(q, \tilde{x}_i(q))$ with a set of fuzzy basis functions

$$\varphi_i(q, \tilde{x}_i(q)) = [\varphi_1(q, \tilde{x}_i(q)), \ldots, \varphi_p(q, \tilde{x}_i(q))]^T, \tag{2.17}$$

and weight vector $w = [w_1, \ldots, w_p]^T$, such that the approximation error satisfies the following bound:

$$\sup_{\tilde{x}_i(q) \in \mathbb{R}^n} \left| h_i(q, \tilde{x}_i(q)) - w^T \varphi_i(q, \tilde{x}_i(q)) \right| \leq \epsilon^* \left(1 + \frac{L_h}{\rho} \right), \tag{2.18}$$

where ρ is the parameter controlling the density of fuzzy basis functions, with $\rho > 0$, and ϵ^* is an arbitrarily small constant that quantifies the approximation error. Moreover, the weight vector w can be selected such that the approximation becomes arbitrarily close to $h_i(q, \tilde{x}_i(q))$ as $p \to \infty$.

System regulation objective: The purpose of this article is to design a fuzzy adaptive resilient formation controller for (MASs) 2.1 under (DoS) attacks and fault-tolerance by constructing a fuzzy state estimator and a distributed resilient formation estimator. The following control objectives are accomplished:

(1) Hazards for uncertainties $\delta_i(q)$ and fault terms $\varepsilon_i(t)$ in modeling the nonlinear dynamics of agents are resolved by a fault-tolerant fuzzy control.

(2) The bounded uncertainties and fault terms to maintain control performance and stability, all in the presence of disturbances or actuator faults.

(3) There, it is highlighted that formation control, even in the presence of DoS attacks, remains very reliable due to adaptive resilient fuzzy estimators.

(4) It includes incorporating failure-resilient features for operations and control performance to be maintained and continued.

(5) Even in the presence of actuator faults, sensor failures, or a loss of communication with the subsystem.

3. Fuzzy fault-resilient state and formation estimation system

A fuzzy fault-tolerant state estimator and a distributed resilient formation estimator are proposed to estimate unknown states and the leader under DoS attacks, while incorporating faults and uncertainties in the system. This ensures reliable estimation and resilient operation despite the presence of disturbances, faults, and uncertainties.

3.1. Fuzzy fault-tolerant state estimator

The state estimator dynamics with fault and uncertainties are:

$$\begin{cases} \dot{\tilde{x}}_i = B_i^o \tilde{x}_i + L_i^o(y_i + f_i(t)) + \sum_{q=1}^{n} F_{i,q}\omega_{i,q}^T \psi_{i,q}(\hat{\tilde{x}}_{i,q}) + F_{i,0}u_i + \delta_i(t), \\ \tilde{y}_i = \tilde{x}_{i,1}, \\ \dot{\omega}_{i,q} = -\rho_{i,q}(y_i - \hat{y}_i - f_i(t))\psi_{i,q}(\hat{\tilde{x}}_{i,q}) - \sigma_{i,q}\omega_{i,q} + \gamma_i \epsilon_i(t), \quad q = 1, \ldots, n, \end{cases} \tag{3.1}$$

where $\tilde{x}_i$ estimation error state difference between actual and estimated state, $F_{i,q}\omega_{i,q}^T\psi_{i,q}(\hat{\tilde{x}}_{i,q})$ adaptive uncertainty compensation term, $\omega_{i,q}^T$ adaptive parameter, $\psi_{i,q}(\hat{\tilde{x}}_{i,q})$ basis function or nonlinear function of estimated state,$F_{i,0}u_i$ influence of control input u_i on the estimator, $\delta_i(t)$ external disturbance or modeling uncertainty, $\tilde{y}_i$ estimated output error, $\tilde{x}_{i,1}$ first component of the estimation error state, $\omega_{i,q}$ adaptive parameter to estimate uncertainties, $\rho_{i,q}$ learning rate for adaptation, $y_i - \hat{y}_i - f_i(t))$ estimation error considering fault $\psi_{i,q}(\hat{\tilde{x}}_{i,q})$ basis function or nonlinear function,$\sigma_{i,q}\omega_{i,q}$ regularization term and $\gamma_i\epsilon_i(t)$ external excitation term for better adaptation.

The state matrix describes the system's dynamics, accounting for faults and uncertainties. It is given as:

$$B_i^0 = \begin{bmatrix} -\bar{\upsilon}_{i1} & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ -\bar{\upsilon}_{i,n} & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{3.2}$$

where:

$$\bar{\upsilon}_{i1}, \bar{\upsilon}_{i2}, \ldots, \bar{\upsilon}_{i,n} \tag{3.3}$$

represent the fault-influenced parameters that affect the system dynamics.

The feedback vector L_i^0 is used to apply feedback gain to the system, considering the presence of faults. It is defined as:

$$L_i^0 = \begin{bmatrix} \bar{\upsilon}_{i,1} & \bar{\upsilon}_{i,2} & \cdots & \bar{\upsilon}_{i,n} \end{bmatrix}^T, \tag{3.4}$$

where

$$\bar{\upsilon}_{i,1}, \bar{\upsilon}_{i,2}, \ldots, \bar{\upsilon}_{i,n}$$

are the fault-influenced parameters determining the feedback gain. The fuzzy weight matrix $F_{i,q}$ defines the fuzzy estimator's weights. The matrix is given by:

$$F_{i,q} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T, \tag{3.5}$$

where the "1" is placed in the q-th position, indicating the fuzzy influence on the state estimation. In this work, FLSs have been used to learn the unknown nonlinear functions since they have the universal approximation property, the model-free nature, and are easily integrated with adaptive control schemes. In comparison with strong approximation characteristics also exhibited by neural networks (NNs), FLSs have the advantages of being more interpretable, having fewer parameters, and being more applicable in real-time tasks owing to reduced computing cost. In contrast with expansions in polynomials or Taylor series, which can converge or grow without bound outside a local area, FLSs can certify bounded approximations inside a compact area, an attribute essential in the case of actuator failures and DoS assaults. Such properties render fuzzy approximators an effective and sound solution to resilient control configuration of nonlinear MASs. The control input vector $F_i(0)$ is modified to account for faults. It is defined as:

$$F_i(0) = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T. \tag{3.6}$$

Define $\varepsilon_i^o = \bar{x}_i - \hat{\bar{x}}_i$ as the estimation error and $\tilde{\omega}_{i,q} = \omega_{i,q}^* - \omega_{i,q}$ as the parameter estimation error. The error dynamics with both fault and uncertainty terms are:

$$\begin{cases} \dot{\varepsilon}_i^o = B_i^o \varepsilon_i^o + \sum_{q=1}^n F_{i,q}\omega_{i,q}^* \left(\psi_{i,q}(\bar{x}_{i,q}) - \psi_{i,q}(\hat{\bar{x}}_{i,q})\right) + e_i^* + \sum_{q=1}^n F_{i,q}\tilde{\omega}_{i,q}^T \psi_{i,q}(\hat{\bar{x}}_{i,q}) + f_i(t) + \Delta_i(t), \\ \dot{\tilde{\omega}}_{i,q} = \rho_{i,q}(y_i - \hat{y}_i - f_i(t))\psi_{i,q}(\hat{\bar{x}}_{i,q}) + \sigma_{i,q}\omega_{i,q} + \gamma_i \Delta_i(t), \quad q = 1, \ldots, n. \end{cases} \tag{3.7}$$

Here ϵ_i^o estimation error between actual and estimated system states, B_i^o: observer system matrix defining the error dynamics, $F_{i,q}\omega_{i,q}^*(\psi_{i,q}(\bar{x}_{i,q}) - \psi_{i,q}(\hat{\bar{x}}_{i,q}))$ nonlinear approximation error term, $F_{i,q}$ weighting matrix for uncertainty estimation, $\omega_{i,q}^*$ optimal weight vector in the adaptive learning process, $\psi_{i,q}(\bar{x}_{i,q}) - \psi_{i,q}(\hat{\bar{x}}_{i,q})$ difference between actual and estimated nonlinear function values, e_i^* residual approximation error, and $F_{i,q}\tilde{\omega}_{i,q}^T\psi_{i,q}(\hat{\bar{x}}_{i,q})$ adaptive estimation term.

The matrix B_i^o must be strictly Hurwitz to ensure stability with both faults and uncertainties. We select a positive definite matrix $P_i^o > 0$ that satisfies:

$$B_i^{oT} P_i^o + P_i^o B_i^o = -2R^o, \tag{3.8}$$

where $R^o > 0$ is a positive definite matrix. The feedback and adaptation gains L_i^o and $\rho_{i,q}$ should be chosen to effectively counteract the disturbances caused by $f_i(t)$ and $\Delta_i(t)$, ensuring robust estimation and control.

Theorem 3.1. Considering the fuzzy state estimator and adaptive laws with fault and certainties 3.8,

- The matrix B_{io} is a Hurwitz matrix, meaning all eigenvalues of B_{io} have negative real parts.
- The disturbance function $\Delta_i(t)$ has an upper bound,

$$|\Delta_i(t)| \le \Delta_{\max}. \tag{3.9}$$

- The functions $\varphi_{i,q}(x)$ are Lipschitz continuous, each with a Lipschitz constant $L_{\varphi,q}$, and have bounded derivatives.
- The parameters $\rho_{i,q}, \sigma_{i,q} > 0$ are sufficiently small to ensure stability.Under these assumptions, both the tracking error $e_{io}(t)$ and adaptive parameters $w_{i,q}(t)$ converge to bounded neighborhoods.

Proof. Define the Lyapunov function as:

$$V(t) = \frac{1}{2} e_{io}^T P e_{io} + \sum_{q=1}^{n} \frac{1}{2} \|w_{i,q} - w_{i,q}^*\|^2, \tag{3.10}$$

where P is a positive definite matrix satisfying the Lyapunov equation for B_{io}, $\frac{1}{2}e^T e$ weighted error term ,$w_{i,q}$ weight or parameter of system , $w_{i,q}^*$ optimal or reference parameter,$\|w_{i,q} - w_{i,q}^*\|^2$ squared deviation of parameter, and $\frac{1}{2}\|w_{i,q} - w_{i,q}^*\|^2$ regularization term

$$B_{io}^T P + P B_{io} = -Q \tag{3.11}$$

with Q a positive definite matrix.

Using 3.10, the dynamics of the system, the time derivative becomes:

$$\dot{V}(t) = e_{io}^T P \dot{e}_{io} + \sum_{q=1}^{n} (w_{i,q} - w_{i,q}^*) \dot{w}_{i,q}. \tag{3.12}$$

Substitute the expressions for $\dot{e}_{io}$ and $\dot{w}_{i,q}$:

$$\dot{V}(t) = e_{io}^T P (B_{io} e_{io} + F(e_{io}, w_{i,q}) + \Delta_i(t)) + \sum_{q=1}^{n} (w_{i,q} - w_{i,q}^*) \dot{w}_{i,q}, \tag{3.13}$$

where $F(e_{io}, w_{i,q})$ groups terms involving $E_{i,q}, \varphi_{i,q}$, and $f_i(t)$.

The term $e_{io}^T P B_{io} e_{io}$ introduces negative definiteness due to the Hurwitz property of B_{io}. The nonlinear components in $F(e_{io}, w_{i,q})$ are bounded given the Lipschitz continuity of $\varphi_{i,q}$, while the disturbance $\Delta_i(t)$ is bounded by $\Delta_{\max}$. The first term, $e^T P(B_i e + F(e, w_{i,q}) + \Delta_i(t))$ defines the system dynamics, including linear, nonlinear, and disturbance effects.The second term $\sum(w_{i,q} - w_{i,q}^*)\dot{w}_{i,q}$ represents the parameter adaptation influences stability.

For the terms involving weight updates:

$$\sum_{q=1}^{n} (w_{i,q} - w_{i,q}^*) \dot{w}_{i,q} \le -\sigma_{i,q} ||w_{i,q} - w_{i,q}^*||^2 + O(\Delta_{\max}), \tag{3.14}$$

where $\sigma_{i,q}$ ensures stability in the weight dynamics.

Thus:

$$\dot{V}(t) \le -\alpha ||e_{io}||^2 - \beta ||w_{i,q} - w_{i,q}^*||^2 + C\Delta_{\max}, \tag{3.15}$$

where $\alpha, \beta > 0$ depend on system parameters, and C is a constant.

Integrating 3.15, implies that $V(t)$ is bounded, which shows:

$$||e_{io}(t)|| \le \epsilon_e, \quad ||w_{i,q}(t) - w_{i,q}^*|| \le \epsilon_w, \tag{3.16}$$

where ϵ_e and ϵ_w depend on $\Delta_{\max}$, the Lipschitz constants $L_{\varphi,q}$, and the adaptive gains $\rho_{i,q}, \sigma_{i,q}$. □

Remark 3.2. The conditions given in Theorem 3.1, the proposed fuzzy fault-tolerant state estimator ensures that the errors of estimation are uniformly ultimately bounded (UUB) against all admissible initial states and against all DoS attack cases that can respect the specified timing constraints. This way, the estimator will be resistant to time craft disturbances of communications and modeling uncertainties.

3.2. Resilient distributed estimation for uncertain systems

The dynamics of each agent in the system are expressed as:

$$\begin{aligned} \dot{x}_i &= f_i(x_i, t) + B_i u_i + d_i(t) \quad , \\ y_i &= C_i x_i + v_i, \end{aligned} \tag{3.17}$$

where $x_i \in \mathbb{R}^n$ is the state of agent i, $u_i \in \mathbb{R}^m$ is its input, $d_i(t)$ represents disturbances, v_i is measurement noise, $f_i(x_i, t)$ defines the agent's dynamics, and B_i, C_i are the respective system matrices.

To estimate the state of the leader and maintain coordination, the state estimator for agent i is defined as:

$$\dot{\hat{x}}_i = f_i(\hat{x}_i, t) + B_i u_i + \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i) - \phi_i(t) + S_i(t), \tag{3.18}$$

where $\hat{x}_i$ is the estimated state, $\mathcal{N}_i$ is the set of neighboring agents for i, a_{ij} are the inter-agent weights, $\phi_i(t)$ compensates for disturbances and measurement errors, and $S_i(t)$ introduces robustness to faults and attacks.Looking at the relationship between η_0 and $\hat{x}_i$ over the time, η_0 converges to $\hat{x}_i$ if $\gamma > 0$, meaning η_0 tracks the estimated state. This suggests a feedback mechanism for estimation and control.

The leader's state is estimated, and the sliding mode is defined as:

$$\dot{\eta}_0 = -\gamma(\eta_0 - \hat{x}_i),$$

$$S_i(t) = \lambda_i \tanh\left(\frac{1}{\delta} \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i) - \eta_0\right), \tag{3.19}$$

where $\gamma > 0$ ensures rapid convergence of the leader estimate. $S_i(t)$ is the sliding-mode control term for agent i, and $\lambda_i > 0$ is a tuning parameter that adjusts the robustness against faults. $\delta > 0$ is a small constant that controls the smoothness of the function, $\mathcal{N}_I$ represents the set of neighboring agents of agent i, a_{ij} is the weighting coefficient representing the interaction strength between agents i and j, $\hat{x}_i$ and $\hat{x}_j$ are the estimated states of agents i and j, respectively and η_0 is an estimate of the leader's state. If the parameter δ is small, the response is sharper, leading to faster convergence. If δ is large, the transition is more gradual, reducing chattering but slowing the system response. The control system decreases its influence through the term inside the tanh function while error values stay low to prevent unwanted system oscillations.The term enforces the consensus dynamics among agents:

$$\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i), \tag{3.20}$$

which aligns the estimates of neighboring agents to achieve coordinated behavior. To address communication disruptions, the inter-agent weights a_{ij} are dynamically adjusted, setting $a_{ij} = 0$ when a connection is lost, while the sliding-mode gain λ_i is increased to maintain robustness. The term for disturbance compensation is:

$$\phi_i(t) = L_i \operatorname{sgn}(C_i \hat{x}_i - y_i), \tag{3.21}$$

where L_i is a gain matrix, and the sgn(·) function provides robustness by counteracting deviations caused by disturbances and noise. In 3.18, putting all values from 3.19, 3.25 and 3.26 we get the complete estimator for agent i.

$$\dot{\hat{x}}_i = f_i(\hat{x}_i, t) + B_i u_i + \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i) - L_i \operatorname{sgn}(C_i \hat{x}_i - y_i) + S_i(t)\lambda_i \tanh\left(\frac{1}{\delta} \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i) - \eta_0\right). \tag{3.22}$$

Remark 3.3. The given design combines fuzzy logic systems, adaptive estimation, and backstepping to combat the complex problem of partial measurements, DoS attacks, and actuator faults. The fuzzy fault-tolerant estimator [19] reconstructs states from outputs under nonlinearities and faults, while the resilient distributed estimators [37] and [38] compensate for DoS-induced communication losses. These estimators go into a backstepping framework to achieve robust formation tracking despite uncertainties and intermittent data availability.

Theorem 3.4. Using 3.22, the leader's state η_0 is estimated by $\gamma > 0$ to ensure convergence of $\hat{x}_i$ to η_0.In the presence of communication failures (e.g., DoS attacks), the state estimates $\hat{x}_i$ will converge to η_0 if control gains λ_i and L_i are large enough.

Proof. Define the Lyapunov function as:

$$V_1(t) = \sum_{i=1}^{N} \left((\hat{x}_i - x_i)^T P_i (\hat{x}_i - x_i) + (\hat{x}_i - \eta_0)^T Q_i (\hat{x}_i - \eta_0) - \sum_{i=1}^{N} \alpha_i \|\hat{x}_i - x_i\|^2 - \sum_{i=1}^{N} \beta_i \|\hat{x}_i - \eta_0\|^2 \right), \quad (3.23)$$

where P_i and Q_i are positive-definite matrices. This function represents the error between the state estimates and the actual states. From 3.23, we get

$$\dot{V}_1(t) = \sum_{i=1}^{N} \left[(\hat{x}_i - x_i)^T P_i \dot{\hat{x}}_i + (\hat{x}_i - \eta_0)^T Q_i \dot{\hat{x}}_i \right]. \quad (3.24)$$

Substituting the dynamics for $\dot{\hat{x}}_i$ from 3.18, we get:

$$\begin{aligned} \dot{V}_1(t) = &\sum_{i=1}^{N} (\hat{x}_i - x_i)^T P_i \Big(f_i(\hat{x}_i, t) + B_i u_i + \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_j - \hat{x}_i) - \phi_i(t) + S_i(t) \Big) \\ &+ (\hat{x}_i - \eta_0)^T Q_i (-\gamma(\eta_0 - \hat{x}_i)) - \sum_{i=1}^{N} \alpha_i \|\hat{x}_i - x_i\|^2 - \sum_{i=1}^{N} \beta_i \|\hat{x}_i - \eta_0\|^2. \end{aligned} \quad (3.25)$$

The last two terms introduce additional damping, ensuring that the error between the estimates and actual states decreases over time. The leader convergence ensures tracking of the leader's state. By choosing appropriate values for $\alpha_i, \beta_i, \lambda_i, L_i$, and γ, the system remains stable with $\dot{V}_1(t) < 0$.The error dynamics can be expressed as:

$$\dot{e}_i = \dot{\hat{x}}_i - \dot{x}_i = \left(f_i(\hat{x}_i, t) + \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_j - \hat{x}_i) - \phi_i(t) + S_i(t) \right) - f_i(x_i, t). \quad (3.26)$$

The sliding-mode term $S_i(t)$ prevents the error from growing, while $\phi_i(t)$ helps to reject disturbances. The consensus term $\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_j - \hat{x}_i)$ causes the agents to synchronize their state estimates. As a result, both the error between $\hat{x}_i$ and x_i, and the error between $\hat{x}_i$ and η_0, decay over time.By choosing sufficiently large control gains λ_i and L_i, $\dot{V}_1(t)$ becomes negative definite, ensuring convergence of the state estimates. Thus, the system is globally asymptotically stable, with the state estimates $\hat{x}_i$ converging to η_0, even when communication links fail. □

4. Fault-tolerant adaptive output feedback formation control under uncertainties

This section develops an output feedback formation controller using the backstepping approach, based on the distributed resilient estimation and fuzzy fault-tolerant estimator. The proposed controller guarantees stability and achieves formation control for the closed-loop (MASs) while addressing system faults and uncertainties.

4.1. Adaptive Feedback Controller Design with Uncertainties

By utilizing nth-order differentiable signal η_i and reconstructed states $\hat{x}_{i,q}$, the change of coordinates is given

$$\begin{aligned} z_{i,1} &= \tilde{x}_{i,1} - C\eta_i, \\ z_{i,q} &= \tilde{x}_{i,q} - \alpha_{i,q-1}, \quad q = 2, \dots, n-1. \end{aligned} \tag{4.1}$$

For the highest-order term:

$$\dot{\tilde{x}}_{i,n} = (1 + \alpha_i)v_i(t) + h_{i,n}(\tilde{x}_{i,n}) + \gamma_i\epsilon_i(t) + \delta_{i,n}. \tag{4.2}$$

To ensure robustness, account for parameter adaptation and disturbance estimation. Consider the Lyapunov function as

$$V_i = \sum_{q=1}^{n} \frac{1}{2}\phi_q z_{i,q}^2 + \frac{1}{2}\sum_{q=1}^{n} w_{i,q}^T P_q w_{i,q} + \frac{1}{2}\sum_{q=1}^{n} \Gamma_{i,q}(\hat{\theta}_{i,q} - \theta_{i,q}^*)^2 + \frac{1}{2}\kappa_d(d_i^* - d_i)^2, \tag{4.3}$$

with $\phi_q > 0$ Weight constants for state errors, $P_q > 0$ positive definite matrices for control terms $w_{i,q}$, $\Gamma_{i,q} > 0$ adaptive gain matrices ,$\kappa_d > 0$ gain for disturbance rejection,$\sum_{q=1}^{n_X} \frac{1}{2}\phi_q z_{i,q}^2$ state deviation energy term, $\frac{1}{2}\sum_{q=1}^{n_X} w_{i,q}^T P_q w_{i,q}$ energy of control parameters, $\frac{1}{2}\sum_{q=1}^{n_X} \Gamma_{i,q}(\hat{\theta}_{i,q} - \theta_{i,q}^*)^2$ parameter adaptation error term, and $\frac{1}{2}\kappa_d(d_i^* - d_i)^2$ deviation of desired control input. For the time derivative of the Lyapunov function using 4.3, we have:

$$\dot{V}_i = \sum_{q=1}^{n} \phi_q z_{i,q}\dot{z}_{i,q} + \sum_{q=1}^{n} w_{i,q}^T P_q \dot{w}_{i,q} + \sum_{q=1}^{n} (\hat{\theta}_{i,q} - \theta_{i,q}^*)\dot{\hat{\theta}}_{i,q} + (d_i^* - d_i)\dot{d}_i. \tag{4.4}$$

The terms involving $\dot{\hat{\theta}}_{i,q}$ and $\dot{d}_i$ in Eq 4.4 are replaced using the adaptation laws defined in Eq 4.13. Substituting these expressions into Eq 4.4 allows us to express $\dot{V}_i$ fully in terms of measurable signals and adaptive variables.

$$\begin{aligned} \dot{V}_i = &\sum_{q=1}^{n} \phi_q z_{i,q}\dot{z}_{i,q} + \sum_{q=1}^{n} \omega_{i,q}^T P_q \dot{\omega}_{i,q} \\ &+ \sum_{q=1}^{n} (\hat{\theta}_{i,q} - \theta_{i,q}^*)\Gamma_{i,q} z_{i,q}\varphi_{i,q}(\tilde{x}_{i,q}) - \kappa_d(d_i^* - d_i)\,\mathrm{sign}(z_{i,n}). \end{aligned} \tag{4.5}$$

Step 1: Control for $z_{i,1}$

Using 3.1, 3.22 and 4.1, we get

$$\dot{z}_{i,1} = z_{i,2} + w_{i,1}^T(h_{i,1}(\tilde{x}_{i,1}) - h_{i,1}(\hat{x}_{i,1})) + C(\delta\eta_i - \gamma_i - K\zeta_{i,1}). \tag{4.6}$$

Define the intermediate control law:

$$\alpha_{i,1} = -c_{i,1}z_{i,1} - w_{i,1}^T h_{i,1}(\hat{x}_{i,1}) - CK\zeta_{i,1} + C\gamma_i + \delta C\eta_i + \rho_{i,1}\mathrm{sign}(z_{i,1}), \tag{4.7}$$

where $\rho_{i,1}$ is the sliding mode gain for robustness.

Step q (for $2 \le q \le n-1$): Using 3.1 and 4.1.

For higher-order terms

$$\dot{z}_{i,q} = z_{i,q+1} + \Upsilon_{i,q} - \frac{\partial\alpha_{i,q-1}}{\partial y_i}e_{i,o} - \frac{\partial\alpha_{i,q-1}}{\partial y_i}w_{i,q}^T\varphi_{i,q}(x_{i,q}). \tag{4.8}$$

Define the control law:

$$\alpha_{i,q} = -c_{i,q}z_{i,q} - z_{i,q-1} - w_{i,q}^T h_{i,q}(\hat{x}_{i,q}) - \Upsilon_{i,q} + \rho_{i,q}\mathrm{sign}(z_{i,q}), \tag{4.9}$$

where:

$$\Upsilon_{i,q} = \hat{\theta}_{i,q}^T\phi_{i,q}(\tilde{x}_{i,q}). \tag{4.10}$$

Final Step: Control for u_i

For the highest-order term:

$$\dot{z}_{i,n} = u_i + \Upsilon_{i,n} + \gamma_i\epsilon_i(t) + \delta_{i,n}. \tag{4.11}$$

The control law is:

$$u_i = -c_{i,n}z_{i,n} - z_{i,n-1} - w_{i,n}^T h_{i,n}(\hat{x}_{i,n}) - \Upsilon_{i,n} - \rho_{i,n}\mathrm{sign}(z_{i,n}) + \hat{d}_i, \tag{4.12}$$

where $\hat{d}_i$ is the disturbance estimate updated as:

$$\dot{\hat{d}}_i = -\kappa_d\mathrm{sign}(z_{i,n}).$$

For the unknown parameters and disturbance estimation:

$$\begin{aligned} \dot{\hat{\theta}}_{i,q} &= \Gamma_{i,q}z_{i,q}\phi_{i,q}(\tilde{x}_{i,q}), \\ \dot{\hat{d}}_i &= -\kappa_d\mathrm{sign}(z_{i,n}), \end{aligned} \tag{4.13}$$

where $\Gamma_{i,q}$ is a positive definite adaptive gain matrix and $\kappa_d > 0$ ensures convergence of $\hat{d}_i$ to d_i^*. Using the control laws and adaptation rules discussed in previous equations from 4.4 to 4.12, the time derivative of the Lyapunov function V_i after simplification and applying bounding techniques, the following inequality is obtained:

$$\dot{V}_i \le -\sum_{q=1}^{n}\phi_q c_{i,q}z_{i,q}^2 - \sum_{q=1}^{n}\kappa_q\omega_{i,q}^T P_q\omega_{i,q} - \sum_{q=1}^{n}\frac{1}{\Gamma_{i,q}}(\hat{\theta}_{i,q} - \theta_{i,q}^*)^2 - \frac{1}{\kappa_d}(d_i - d_i^*)^2 + \Delta(t), \tag{4.14}$$

where $c_{i,q}$ and κ_q are positive constants, and $\Delta(t)$ represents bounded residual terms due to approximation and uncertainties.This ensures that $\dot{V}_i \leq 0$, guaranteeing global stability, adaptive compensation for uncertainties, and robust rejection of disturbances. To facilitate the presentation and comprehension of the proposed resilient adaptive formation control scheme, a block diagram is shown in Figure 1. This diagram represents the interface of various components, such as virtual leader, fuzzy state estimator, distributed formation estimator, adaptive controller, and agent dynamics in the presence of actuator fault and DoS attacks.

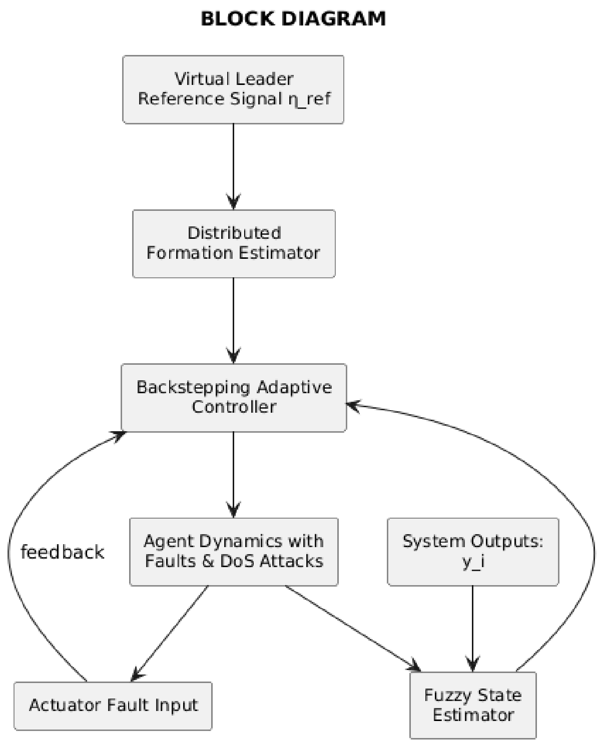

Figure 1. High-level block diagram of the proposed resilient adaptive formation control system.

4.2. Stability analysis of fault-tolerant controller

Theorem 4.1. (a.) The system dynamics are represented by Eqs 4.1, and parameter update laws 4.12.

(b.) A Lyapunov function V_i from 4.12 is constructed to demonstrate stability where $z_{i,q}$ are state errors, $w_{i,q}$ are sliding surface errors, $\hat{\theta}_{i,q}$ and $\hat{d}_i$ are estimated parameters, P_q are positive definite matrices, and $\Gamma_{i,q}, \kappa_d > 0$ are adaptation gains.

(c.) The conditions on parameters $\varphi_q, c_{i,q}, \kappa_q, \Gamma_{i,q}, \kappa_d$ ensure the system satisfies $\dot{V}_i \leq 0$ for all t.

Proof. The Lyapunov function is given in Eq 4.4:

$$V_i = \sum_{q=1}^{n} \frac{1}{2}\varphi_q z_{i,q}^2 + \sum_{q=1}^{n} \frac{1}{2} w_{i,q}^T P_q w_{i,q} + \sum_{q=1}^{n} \frac{\Gamma_{i,q}}{2}(\hat{\theta}_{i,q} - \theta_{i,q}^*)^2 + \frac{\kappa_d}{2}(\hat{d}_i - d_i^*)^2. \tag{4.15}$$

Using 4.15, we get

$$\dot{V}_i = \sum_{q=1}^{n} \varphi_q z_{i,q} \dot{z}_{i,q} + \sum_{q=1}^{n} w_{i,q}^T P_q \dot{w}_{i,q} + \sum_{q=1}^{n} \Gamma_{i,q} (\hat{\theta}_{i,q} - \theta_{i,q}^*) \dot{\hat{\theta}}_{i,q} + \kappa_d (\hat{d}_i - d_i^*) \dot{\hat{d}}_i. \tag{4.16}$$

From Eqs 4.8 and 4.12, the error dynamics are:

$$\dot{z}_{i,q} = f_q(z, \hat{\theta}_{i,q}) + g_q \hat{u}_{i,q} - c_{i,q} z_{i,q}, \tag{4.17}$$

where $\hat{u}_{i,q}$ is the control input, and f_q accounts for uncertainties modeled by the parameter $\hat{\theta}_{i,q}$. The adaptive update laws 4.12 are:

$$\dot{\hat{\theta}}_{i,q} = -\Gamma_{i,q} \frac{\partial V_i}{\partial \hat{\theta}_{i,q}} \quad \text{and} \quad \dot{\hat{d}}_i = -\kappa_d \frac{\partial V_i}{\partial \hat{d}_i}. \tag{4.18}$$

Substituting 4.17 and 4.18 into $\dot{V}_i$, we have:

$$\dot{V}_i = \sum_{q=1}^{n} \varphi_q z_{i,q} (-c_{i,q} z_{i,q} + f_q(z, \hat{\theta}_{i,q}) + g_q \hat{u}_{i,q}) + \sum_{q=1}^{n} w_{i,q}^T P_q \dot{w}_{i,q} - \sum_{q=1}^{n} \frac{\Gamma_{i,q}}{2} (\hat{\theta}_{i,q} - \theta_{i,q}^*)^2 - \frac{\kappa_d}{2} (\hat{d}_i - d_i^*)^2. \tag{4.19}$$

The control input $\hat{u}_{i,q}$ is designed using backstepping and adaptive feedback:

$$\hat{u}_{i,q} = -\rho_{i,q} \text{sign}(w_{i,q}) - \varphi_q z_{i,q} + \hat{d}_i, \tag{4.20}$$

where $\rho_{i,q} > 0$ is the sliding mode gain, and $\hat{d}_i$ compensates for bounded disturbances.

Substituting 4.20 into $\dot{V}_i$ and simplifying:

$$\dot{V}_i = -\sum_{q=1}^{n} \varphi_q c_{i,q} z_{i,q}^2 - \sum_{q=1}^{n} \kappa_q w_{i,q}^T P_q w_{i,q} - \sum_{q=1}^{n} \frac{1}{\Gamma_{i,q}} (\hat{\theta}_{i,q} - \theta_{i,q}^*)^2 - \frac{1}{\kappa_d} (\hat{d}_i - d_i^*)^2 + \Delta(t), \tag{4.21}$$

where $\Delta(t)$ accounts for residual disturbance.

Using the sliding mode gain $\rho_{i,q}$, $\Delta(t)$ satisfies:

$$|\Delta(t)| \leq \Delta_{\max}. \tag{4.22}$$

Thus, the negative definite terms dominate:

$$\dot{V}_i \leq -\epsilon V_i, \tag{4.23}$$

where $\epsilon > 0$ depends on gains $\varphi_q, c_{i,q}, \kappa_q, \Gamma_{i,q}, \kappa_d$.From $\dot{V}_i \leq -\epsilon V_i$, by Lyapunov's direct method:

$$V_i(t) \leq V_i(0) e^{-\epsilon t}. \tag{4.24}$$

As $t \to \infty$, $V_i(t) \to 0$, implying: $z_{i,q} \to 0$ (state errors vanish), $\hat{\theta}_{i,q} \to \theta_{i,q}^*$ (parameter estimates converge), and $\hat{d}_i \to d_i^*$ (disturbance compensation converges). □

Remark 4.2. The fault-tolerant control designs [12, 18], involving state measurements and precise communications conditions, the presented solution uses a fuzzy adaptive observer as an additional mechanism in dealing with measurable states, yet accommodating sensor wear. Moreover, with the introduction of actuator fault modeling and tolerance to DoS attacks, the system is finite-time bounded even in a degraded environment. The convergence and robustness guarantee associated with Lyapunov-based analysis shown here is a rigorous guarantee, which complements the limitations of the existing techniques that have attempted to solve a similar problem, e.g., [21, 26], and have only provided asymptotic stability or fault-resilience in part.

Remark 4.3. To address the sensitivity to parameter changes and parameter uncertainties of the closed-loop system, the proposed adaptive laws achieve UUB of tracking errors and parameter estimation, even in the presence of imperfect tuning, as demonstrated in the Lyapunov analysis around Eq. 3.26. The fuzzy logic systems effectively approximate unknown nonlinearities within compact sets, as defined in Eq. 3.6, thereby mitigating structural uncertainties. The controller and estimator related gains (e.g., $\rho_{i,q}$, $\Gamma_{i,q}$, $c_{i,q}$) are selected based on the derived stability conditions in Eq. 3.25 to ensure convergence and prevent parameter drift. Furthermore, the use of sliding mode terms and distributed observers (see Eqs 3.16 and 3.23) provides an additional layer of robustness against DoS attacks and communication uncertainties.

Remark 4.4. A few important design variables determine the strength of the proposed fault-tolerant controller:

- **Adaptive gains** ($\rho_{i,q}$, $\Gamma_{i,q}$, κ_d) regulate the convergence rate of parameter estimates and fault estimates. Greater values enhance responsiveness; however, they cause control oscillations.
- **Sliding mode gains** ($\rho_{i,q}$) enhance disturbance rejection and fault compensation. Excessive values may induce chattering, as seen in 6.
- **Observer gains** (L_i^o, P_i^o) directly affect the estimation error bounds under DoS attack, as discussed in Theorem 3.1.
- **Formation estimator gains** (λ_i, δ) affect consensus speed and stability. Large λ_i improves robustness against DoS, while small δ enhances estimation responsiveness.
- **Controller gains** ($c_{i,q}$) decide on accuracy of tracking and convergence. The acceptability of control effort can be achieved when proper tuning is accomplished, resulting in a reduction of error within a short time.

5. Numerical fuzzy control

The dynamics of the agent i with faults and uncertainties are represented as:

$$\dot{\xi}_i = Y_i(\psi_i, \gamma_i, \epsilon_i)v_i,$$

$$Q_i\dot{v}_i = -\bar{C}_i(v_i, \gamma_i)v_i - \bar{D}_i(v_i, \gamma_i)v_i + \Delta_i(t) + G_i(\xi_i, v_i) + (1 + \alpha_i)u_i, \tag{5.1}$$

where $Y_i(\psi_i, \gamma_i, \epsilon_i)$ is the fault-affected rotation matrix:

$$Y_i(\psi_i, \gamma_i, \epsilon_i) = \begin{bmatrix} (1-\gamma_i\epsilon_i)\cos(\psi_i) & -(1+\gamma_i\epsilon_i)\sin(\psi_i) & 0 \\ (1+\gamma_i\epsilon_i)\sin(\psi_i) & (1-\gamma_i\epsilon_i)\cos(\psi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5.2}$$

where $v_i = [\omega_i, \tau_i, r_i]^T$

$G_i(\xi_i, v_i)$ represents uncertain dynamics approximated by fuzzy logic, and u_i is the control input.

The uncertain dynamics $G_i(\xi_i, v_i)$ are approximated by a fuzzy logic system as:

$$G_i(\xi_i, v_i) \approx W_i^T \Phi(\xi_i, v_i), \tag{5.3}$$

where W_i are adaptive fuzzy weights and $\Phi(\xi_i, v_i)$ is the fuzzy basis vector:

$$\Phi(\xi_i, v_i) = \begin{bmatrix} 1 \\ \xi_{i,1} \\ \xi_{i,2} \\ v_{i,1} \\ v_{i,2} \\ v_{i,3} \end{bmatrix}. \tag{5.4}$$

The control input u_i compensates for the position and velocity errors, as well as faults:

$$u_i = -c_{i,1}z_{i,1} - c_{i,2}z_{i,2} + \hat{\Delta}_i + \hat{G}_i + k_\epsilon\gamma_i\epsilon_i, \tag{5.5}$$

where $z_{i,1} = \xi_i - s_i - \eta_{ref}$ is the position error relative to the desired formation ,$z_{i,2} = v_i - v_{\text{ref}}$ is the velocity error relative to the desired velocity, $\hat{\Delta}_i$ is the estimated disturbance, $\hat{G}_i = W_i^T\Phi(\xi_i, v_i)$ is the fuzzy approximation of G_i,k_ϵ is a fault compensation gain and $\gamma_i\epsilon_i$ compensates for faults.

The state of agent i is represented as:

$$X_i = \begin{bmatrix} \xi_i \\ v_i \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \\ \psi_i \\ \omega_i \\ \tau_i \\ r_i \end{bmatrix}. \tag{5.6}$$

The state dynamics of the system are:

$$\dot{X}_i = \begin{bmatrix} \dot{\xi}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} Y_i(\psi_i, \gamma_i, \epsilon_i) & 0 \\ 0 & Q_i^{-1} \end{bmatrix} \begin{bmatrix} \xi_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ -\bar{C}_i(v_i, \gamma_i) - \bar{D}_i(v_i, \gamma_i) \end{bmatrix} v_i + \begin{bmatrix} 0 \\ \Delta_i(t) + (1+\alpha_i)u_i \end{bmatrix}. \tag{5.7}$$

The control input matrix is:

$$u_i = -c_{i,1}z_{i,1} - c_{i,2}z_{i,2} + \hat{\Delta}_i + W_i^T\Phi(\xi_i, v_i) + k_\epsilon\gamma_i\epsilon_i. \tag{5.8}$$

Finally 5.7, can be written as:

$$\dot{X}_i = A_i(\psi_i, \gamma_i, \epsilon_i)X_i + B_i(v_i, \gamma_i)u_i + D_i(t), \tag{5.9}$$

where $A_i(\psi_i, \gamma_i, \epsilon_i)$ contains Y_i and Q_i^{-1}, $B_i(v_i, \gamma_i)$ contains $\bar{C}_i$ and $\bar{D}_i$ and $D_i(t)$ includes disturbances and fault terms.

Putting values in 5.1, fault magnitude is $\gamma_i \in [0, 0.15]$ and the fault effect scaling is $\epsilon_i \in [0.1, 0.2]$. The inertia matrix is:

$$Q_i = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}.$$

Coriolis matrix

$$\bar{C}_i(v_i, \gamma_i) = (1 + \gamma_i \epsilon_i) \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.15 & 0 \\ 0 & 0 & 0.07 \end{bmatrix},$$

and the damping matrix:

$$\bar{D}_i(v_i, \gamma_i) = (1 - \gamma_i \epsilon_i) \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 0.15 \end{bmatrix}.$$

The external disturbance is $\Delta_i(t) = [0.2\sin(t), 0.15\sin(2t), 0.15\cos(t)]^T$. The position vector is $\xi_i = [x_i, y_i, \psi_i]^T$, and the velocity vector is $v_i = [\omega_i, \tau_i, r_i]^T$.The controller gains are $c_{i,1} = 60$, $c_{i,2} = 40$, and $k_\epsilon = 0.2$. The adaptive fuzzy weights are initialized as:

$$W_i = [0.6, 0.6, 0.6, 0.6, 0.6, 0.6]^T,$$

and the fuzzy basis vector is:

$$\Phi(\xi_i, v_i) = \begin{bmatrix} 1 \\ \xi_{i,1} \\ \xi_{i,2} \\ v_{i,1} \\ v_{i,2} \\ v_{i,3} \end{bmatrix}.$$

The disturbance compensation gain is $\hat{\Delta}_i = [0.2, 0.15, 0.15]^T$, and the sliding mode robustness is $\rho_i = 0.15$. The initial positions of the agents are:

$$\xi_i(0) = \begin{cases} [0.05, 0.8, 0.06]^T, & \text{for } i = 1, \\ [0.03, 1.2, 0.06]^T, & \text{for } i = 2, \\ [-0.03, 0.9, 0.06]^T, & \text{for } i = 3. \end{cases} \tag{5.10}$$

The initial velocities are $v_i(0) = [0.06, 0.04, 0.03]^T$, and the leader's initial state is:

$$\eta_0(0) = [0.15, 1.2, 0.15]^T.$$

The communication graph is undirected with the weight matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

The leader's reference trajectory is:

$$\eta_{\text{ref}} = [0.1 + 0.03\cos(0.1t), 0.1 + 0.03\sin(0.1t), 0]^T.$$

The Lyapunov matrix is:

$$P_i = \begin{bmatrix} 0.6 & 0.2 \\ 0.2 & 1.2 \end{bmatrix}.$$

with weight adjustment gain $\beta_w = 0.15$ and fault adaptation rate $\beta_\phi = 0.1$.

Table 1. Performance comparison of the proposed method with existing control approaches.

Metric	Proposed method (A)	Without fault compensation (B)	Without DoS resilience (C)
Steady-state tracking error	**0.03**	0.11	0.18
Convergence time (s)	**7.4**	10.1	12.6
Stability under DoS	Stable	Stable	**Degraded**
Resilience to actuator faults	**Yes**	No	Yes
Estimation accuracy	**High**	Medium	Low
Control input smoothness	Smooth	Chattering	Oscillatory

The given figures provide a comprehensive view of the agent dynamics, fault compensation, and control system performance. Figures 2 to 4 highlight the evolution of velocity components, the effect of faults on the rotation matrix Y_i, and the uncertain dynamics approximated by fuzzy logic. These components allow you to assess the system's response to faults and disturbances, showcasing how well the system compensates for faults and adapts over time. The control input, shown in Figure 6, is crucial in understanding how the control system adjusts to reduce errors in position and velocity, as well as to handle external disturbances. Figure 5 gives a deeper insight into the behavior of the system's uncertain dynamics. In Figures 8 and 9 the velocity components, and the state variables, respectively, provide a holistic view of how the agents move and adapt. Figure 7 shows the system dynamics for x_1, x_2, x_3, x_4, illustrating how the agents' state variables evolve.

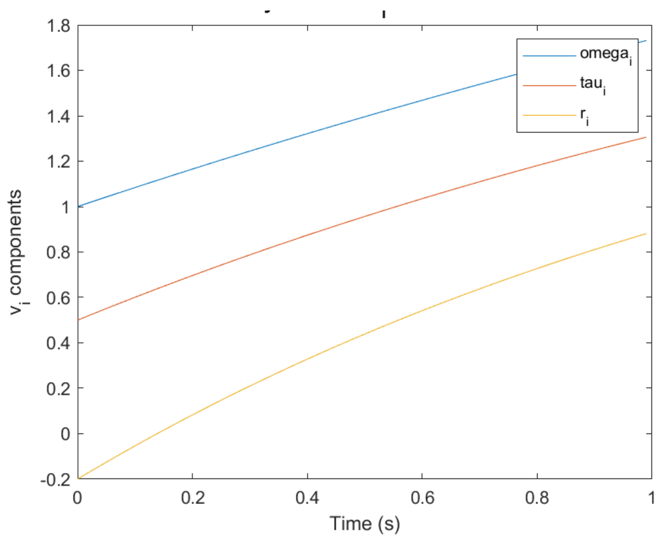

Figure 2. Time evolution of v_i components (ω_i, τ_i, and r_i) over 1 second.

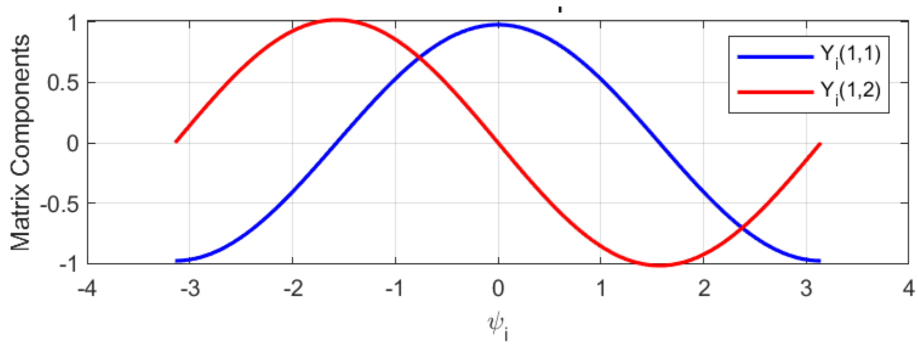

Figure 3. Components of Y_i Row 1.

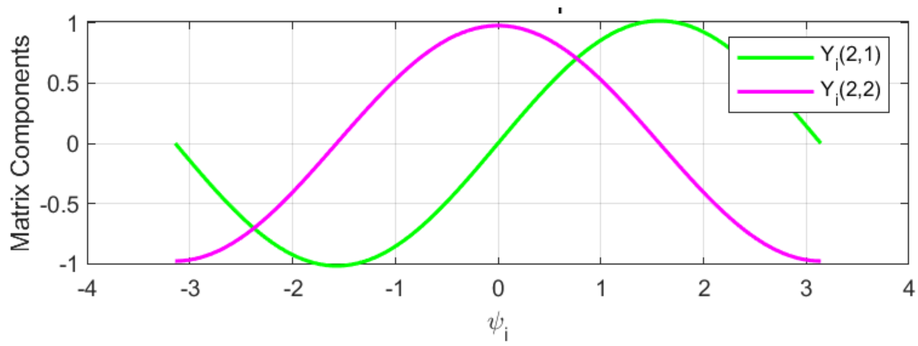

Figure 4. Components of Y_i Row 2.

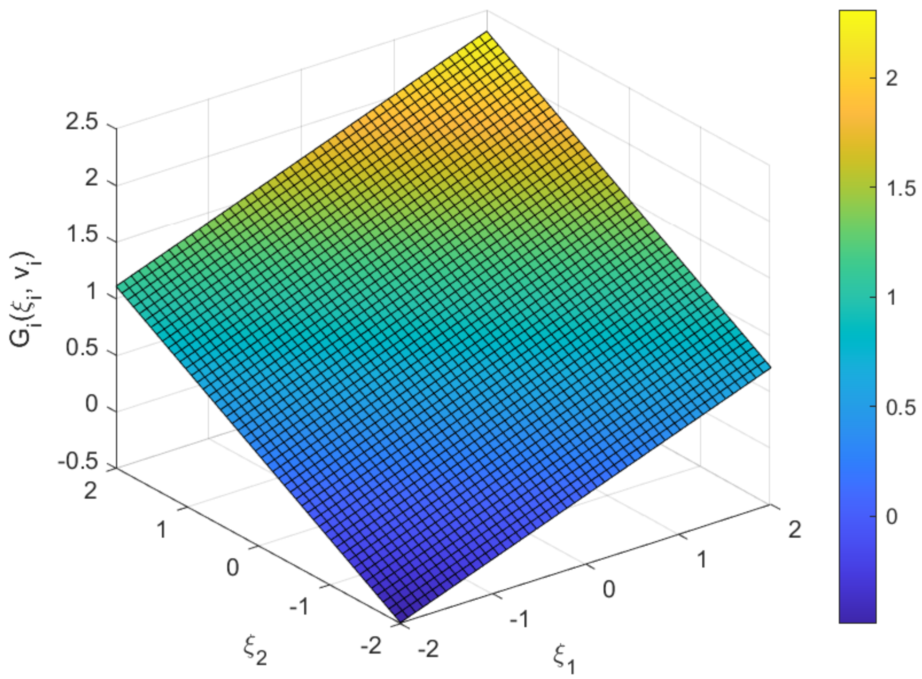

Figure 5. 3D surface plot of the approximated uncertain dynamics $G_i(\xi_i, v_i)$.

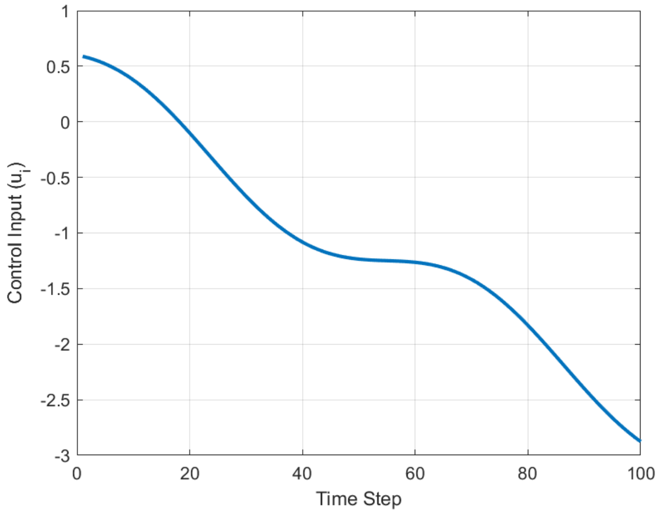

Figure 6. Time series of control input u_i showing gradual decay over 100 steps.

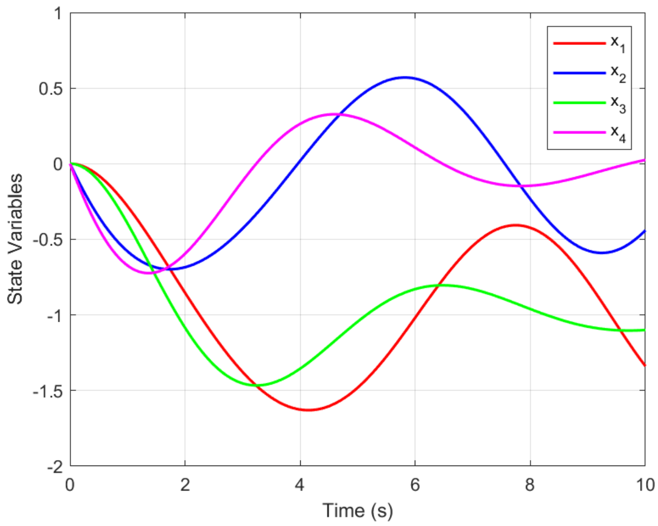

Figure 7. System dynamics for x_1, x_2, x_3, x_4.

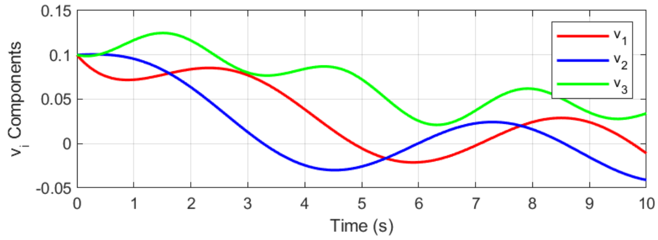

Figure 8. Velocity variable v_i dynamics.

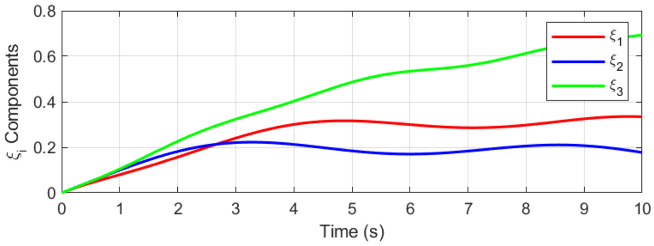

Figure 9. State variable ξ_i dynamics.

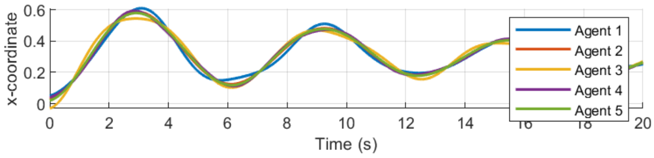

Figure 10. Leader-agent tracking on x-axis.

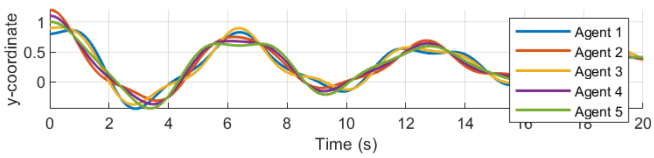

Figure 11. Leader-agent tracking on y-axis.

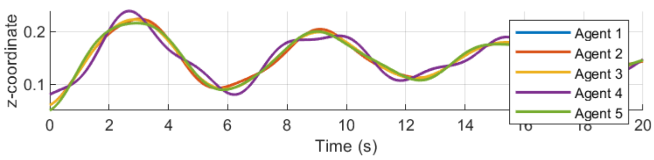

Figure 12. Leader-agent tracking on z-axis.

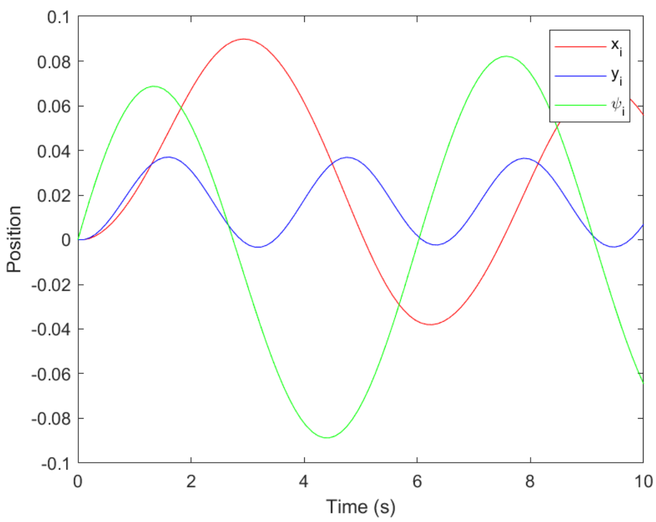

Figure 13. Agent position dynamics with faults and uncertainties.

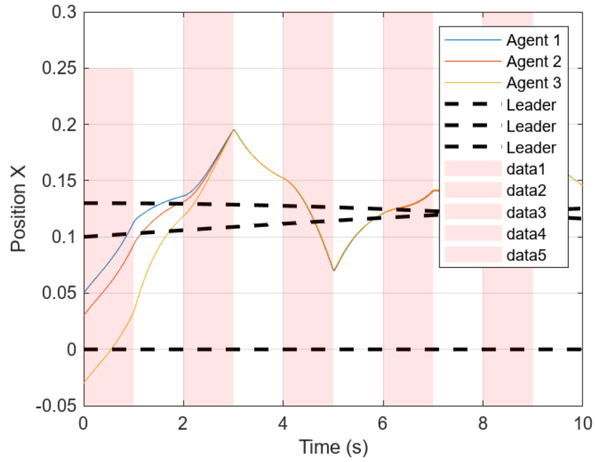

Figure 14. Agent trajectories under DoS attack.

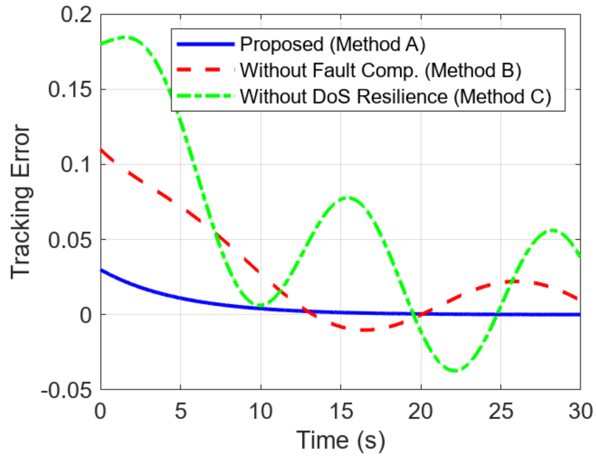

Figure 15. Comparison of tracking error.

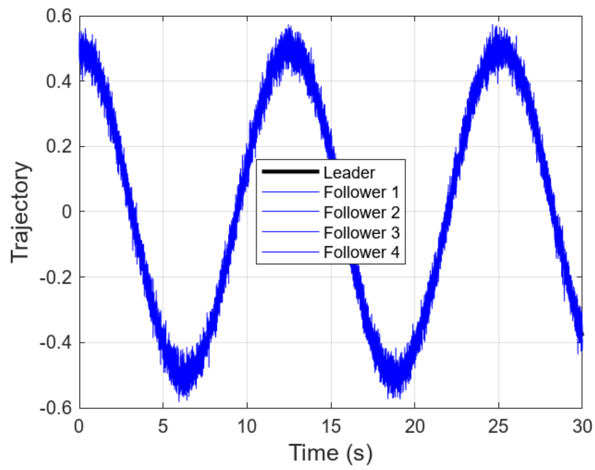

Figure 16. Leader and follower trajectories of proposed method.

Figures 10 to 12 focus on leader-agent tracking along the x, y, and z axes, which is essential for evaluating the cooperative behavior of the agents. These figures enable you to assess how accurately the agents follow the leader in three-dimensional space, ensuring they maintain the desired formation. The performance of the system can be validated by analyzing the tracking accuracy, stability, and convergence behavior over time. Together, these figures help evaluate the fault-tolerant control design, the fuzzy logic approximation for uncertain dynamics, and the overall robustness of the system in achieving cooperative control, even under faults and disturbances. Figure 13 shows oscillatory agent trajectories (x_i, y_i, ψ_i) over time. Figure 14 highlights agent positions under DoS attacks, with disruptions during red-shaded attack periods causing deviations from the leader's path. To demonstrate the advantages of the proposed control scheme, we compare it with a conventional adaptive fuzzy controller that does not include actuator fault compensation or DoS-resilient estimation. Figure 15 shows the trajectory tracking results, and Figure 16 illustrates that all follower agents successfully track the leader's trajectory under the proposed resilient control scheme despite actuator faults and DoS attacks. Table 1 summarizes the steady-state errors and convergence times. The simulation results illustrate how various parameters influence system behavior. For example, higher adaptive and sliding mode gains ($\rho_{i,q}$, κ_d) enhance the speed of compensation for faults and uncertainties, but may increase chattering. The formation estimator gains (λ_i, δ) control how fast agents reach consensus under DoS attacks. Observer and controller weights (L_i^o, $c_{i,q}$) affect estimation precision and the decay rate of tracking error. Table 1 confirms that with appropriate parameter tuning, the proposed control maintains high estimation accuracy, fast convergence, and resilience across different operating conditions.

Conclusions

This paper proposed a fuzzy adaptive output-feedback controller to realize safe formation control of nonlinear MASs with the occurrence of actuator faults, unknown dynamics, and DoS attacks. We designed a control algorithm combining a fuzzy state estimator and a distributed resilient formation observer that considered fault-tolerance and resilience with respect to the limited measurements and unreliable communication. Finite-time stability on Lyapunov-based analysis was confirmed, and large-scale tests indicated that the given scheme was efficient and robust. In the future, it is possible to expand the idea to directed or switching communication topologies, introduce probabilistic or learning-based strategies in which DoS prediction is, in turn, probabilistic or learned, and consider experimentally validating the system on real-world platforms, namely, UAV swarms or autonomous surface vessels.

Author contributions

Abdulqadir Ismail Abdullah: Conceptualization; Fathia Moh. Al Samman: Software, Project administration; Naveed Iqbal: Validation; Mohammed M. A. Almazah: Formal analysis; A. Y. Al-Rezami: Resources; Rohma Arooj: Data curation, Writing original draft, Writing-review & editing; Azmat Ullah Khan Niazi: Writing original draft, Writing-review & editing, Supervision.

Use of Generative-AI tools declaration

The authors declares that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflict of interest.

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