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Research article

Aperiodic intermittent control for discrete-time systems

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Abstract: In this manuscript, time-triggered and event-triggered aperiodic intermittent controls are proposed for asymptotic stabilization of unstable discrete-time systems. The time-triggered aperiodic intermittent controls (T-APICs) are designed respectively by imposing conditions on the average dwell-time and the minimum of the active control width. For event-triggered aperiodic intermittent controls (E-APICs), when the norm of the state violates the defined inequality, the control is triggered. For one proposed E-APIC mechanism, it is demanded that the control length should be bigger than a given constant. Then for another E-APIC mechanism, we impose a condition related to the state on the control length. By relaxing the constraints on the control gain matrix, the third E-APIC mechanism is proposed. For these control plans, the control continuously updates during the active control time interval. Then we propose another E-APIC for asymptotic stabilization of the discussed discrete-time system by using the concept of input -to-state stability (ISS) and imposing the conditions on the norm of the state. In order to exemplify the effectiveness of the proposed aperiodic intermittent control mechanisms, asymptotic stabilizations of three examples are discussed by the proposed theorems.

Keywords: asymptotic stability; input-to-state stability; asymptotic stabilization; time-triggered aperiodic intermittent control; event-triggered aperiodic intermittent control
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1. Introduction

In this paper, we consider a discrete-time dynamic system

$$x(k+1) = Ax(k) + f(x(k)), \tag{1.1}$$

where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, a Lipschitz continuous function $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ with $f(0) = 0$. Given an initial condition $x_0 = x(0)$, we denote the solution of system (1.1) via $x(k, x_0)$. For the convenience, $x(k, x(0))$ is shorted as $x(k)$.

We assume that the origin of system (1.1) is not asymptotically stable in this manuscript. The aim of this paper is to design T-APIC and E-APIC mechanisms to asymptotically stabilize system (1.1).

For an unstable dynamic system, researchers are interested in designing control which can stabilize this unstable dynamic system. There have been many useful research results about control designs in references such as [1, 9, 29, 31]. Among these strategies, researchers have paid more attention to intermittent control (IC) since it is more convenient and efficient to apply in practical dynamic systems. IC is different from continuous control (such as [29, 30, 34]) and impulsive control (such as [27, 33]). IC fundamentally includes two parts. One part is the active control time interval for which the control is applied on the considered system, and the other part is the inactive control time interval for which there is no control imposed on the system. Since we do not need to impose control on the system all the time, it is more efficient than continuous control. On the other hand, the control width for intermittent control is longer than impulsive control. Then the expected character, such as stability, can last for a longer period. Because of these advantages, IC is widely used to synchronize or stabilize chaotic systems in [18–20] and complex systems in references such as [23, 25, 26]. A linear coupled network in [5] was synchronized via the proposed aperiodically intermittent control. In [6], an aperiodically intermittent control was used to synchronize complex networks with multiple disturbances. An aperiodic intermittent control was utilized to synchronize inertial memristive neural networks with delays in finite time in [4]. The authors in [3] investigated finite-time synchronization of spatiotemporal Cohen-Grossberg neural networks via designing aperiodic intermittent controls. The authors in [2] considered how to synchronize spatiotemporal fuzzy neural networks in finite time via the designed aperiodic intermittent control. However, there are few results about intermittent control for stabilization of discrete-time dynamic systems. This is why we try to design intermittent control to stabilize the discrete-time system (1.1).

IC is classified into two types of intermittent control, i.e., time-triggered intermittent control and event-triggered intermittent control (EIC). As their names show, time-triggered intermittent control is designed according to a time scheme and Lyapunov stability conditions. Event-triggered intermittent control is designed based on the constraints imposed on the state of the considered system. Time-triggered intermittent control can be periodic or aperiodic. In paper [22], the authors exponentially stabilized delayed networks by designing time-triggered periodic intermittent control (T-PIC). In [28], a practical iterative learning control was studied for industrial robot manipulators. In [7], the intermittent iterative learning controller was designed for robot manipulators with packet dropouts. Furthermore, in [21], two methods of designing time-triggered aperiodic intermittent control (T-APIC) for stabilization of general linear systems were proposed by mixed Lyapunov functions. For more results about T-APIC and T-PIC, you can refer to [8, 13–16]. Since EIC is designed based on the state of the considered system, EIC is generally aperiodic. Then we call EIC event-triggered aperiodic intermittent control (E-APIC). In paper [17], E-APIC was designed to study the filter of networked systems. The authors designed event-triggered control for input-to-state-stabilization of discrete-time systems by the concept of ISS and Lyapunov-like functions in [9]. The authors in [34] proposed event-triggered control for stabilization of discrete-time systems by the concept of ISS and Lyapunov-like functions. In paper [13], the authors stabilized continuous-time dynamic systems via designing E-APIC. In [12], dynamic systems are input-to-state-stabilized by E-APIC. The authors further investigated E-APIC for stabilization of dynamic systems with delays in [11]. However, most of these papers are about stabilization of continuous-time dynamic systems. As far as we know, there is little research about aperiodic intermittent control for stabilization of discrete-time dynamic systems. Many practical phenomena, such as automation, can be described by discrete-time dynamic systems. Besides, the proposed aperiodic intermittent control for continuous-time systems may not be suitable for the corresponding discrete-time systems since the solutions for discrete-time systems are not continuous. Therefore, we are interested in designing aperiodic intermittent control to stabilize the discrete-time dynamic system (1.1).

Inspired by the above amazing results about control design, we firstly investigate T-APIC for asymptotic stabilization of system (1.1) by imposing the constraints on the average dwell-time and the norm of the state (see Theorem 1). The constraints on the average dwell-time ensure the norm of the state decreases after a fixed time. Then, by imposing the constraints on the control width and the time interval without control, another T-APIC is designed for asymptotic stabilization for system (1.1) (see Theorem 2). For Theorem 2, the norm of the state does not decrease as the time goes. It is worthy to point out that this property make it easy to implement this intermittent control mechanism. Then based on the results of the above references, we further propose E-APIC for asymptotic stabilization of system (1.1). For the proposed E-APIC mechanisms, the control is triggered by the norm of the states violating the defined inequality. The control width is the time interval during which the norm of the state satisfies the introduced inequality. Specifically, for Theorem 5 it is required that there exists a positive constant $\bar{k}$ such that the inequality (4.13) holds for $k \geq \bar{k}$. According to our knowledge, there are no similar results in references. These designed controls have to update continuously as they are implemented during the active control time interval. Furthermore, integrating the idea of [34] into intermittent control, we propose another E-APIC mechanism for stabilization of system (1.1) by using the concept of ISS and imposing the conditions on the norm of the state. In order to exemplify the effectiveness of the proposed aperiodic intermittent control mechanisms, stabilizations of three examples are discussed by the proposed theorems.

The main contribution of this manuscript is listed as follows.

(1) Theorems 1-2 state two sets of T-APIC for system (1.1) by imposing the conditions on the control starting time and control length.
(2) Theorem 3 shows us how to stabilize system (1.1) via E-APIC with constraints on the control starting time related to the state and the control width. The control width is not directly decided by the supposed conditions on the state.
(3) Based on Theorem 3, in Theorem 4 the control width is given by the requirements on the state.
(4) Compared with Theorem 4, Theorem 5 relaxes assumptions on the control gain matrix.
(5) Using the concept of ISS, another set of E-APIC in Theorem 6 is proposed to asymptotically stabilize system (1.1) via imposing constraints on the state. The control only updates when the control is triggered.

The rest of this manuscript is organized as follows. In section 2, the definition of asymptotic stability is reviewed, since it will be used to design aperiodic intermittent control. Then we present the problems discussed in this paper and give some assumptions utilized later. In section 3, we are going to propose T-APIC schemes to asymptotically stabilize system (1.1). In section 4, the E-APIC schemes for asymptotic stabilization of system (1.1) are proposed by imposing the constraints on the norm of the state and updating the control continuously during the control interval. Then in section 5, by integrating the concept of ISS into intermittent control, another E-APIC mechanism is designed to asymptotically stabilize system (1.1). In section 6, to show the effectiveness of the designed control mechanisms, three numerical examples are asymptotically stabilized via the proposed theorems. In section 7, some brief remarks are presented.

2. Problems and assumptions

In this manuscript, $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{N}$ and $\mathbb{N}_+$ are for real numbers, nonnegative numbers, integers and nonnegative integers, respectively. The n-dimensional real vector space is denoted by $\mathbb{R}^n$. For a vector $x \in \mathbb{R}^n$, the Euclidean norm is represented by $|x|$.

Comparison functions are very important for stability analysis. Hence, we recall the definitions of comparison functions. A continuous function $\alpha : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is a $\mathcal{K}$ function, if α is strictly increasing and satisfies $\alpha(0) = 0$. If a $\mathcal{K}$ function α is unbounded, then we call it a $\mathcal{K}_\infty$ function. A function $\beta(s,t) : \mathbb{R}_+ \times \mathbb{R}_+ \mapsto \mathbb{R}_+$ is said to be a class $\mathcal{KL}$, if $\beta(\cdot, t)$ is a $\mathcal{K}$ function for any fixed t and for each s the function $\beta(s,t)$ decreases as t increases and meets the constraint $\lim\limits_{t\to+\infty} \beta(s,t) = 0$.

First we propose sufficient constraints for asymptotic stabilization of system (1.1) via designing aperiodic intermittent control (APIC) defined as follows:

$$u(k) = \begin{cases} Dx(k), & k_i \le k < k_i + l_i, \\ 0, & k_i + l_i \le k < k_{i+1}, \end{cases} \tag{2.1}$$

where $D \in \mathbb{R}^{n\times n}$ is a control gain matrix, k_i is the time when the $i-$th APIC is imposed on system (1.1), and the i-th control width is denoted by l_i ($l_i \in \mathbb{N}_+, l_i \ge 1, i \in \mathbb{N}_+$).

When we implement an APIC in system (1.1), system (1.1) can be written as the following switched system:

$$x(k+1) = \begin{cases} Ax(k) + f(x(k)) + Dx(k), & k_i \le k < k_i + l_i, \\ Ax(k) + f(x(k)), & k_i + l_i \le k < k_{i+1}. \end{cases} \tag{2.2}$$

For system (2.2), the switching times are decided by the asymptotic stabilization constraints with APIC (2.1).

Definition 1. *Consider system* (2.2). *If there exists a function $\beta \in \mathcal{KL}$ such that for any initial condition x_0*

$$|x(k, x_0)| \le \beta(|x_0|, k), \tag{2.3}$$

then the origin of system (2.2) *is asymptotically stable, i.e., system* (1.1) *is asymptotically stabilized under the designed aperiodic intermittent control* (2.1).

In this manuscript, the key work is to search for the constraints on the parameters k_i, l_i of the proposed APIC (2.1), which asymptotically stabilize system (1.1). We investigate this problem under time-triggered APIC and event-triggered APIC schemes, respectively. In this paper, the function f satisfies the constraint as the following:

$$|f(x) - f(y)| \le L_f |x - y|, \ \ \forall x, y \in \mathbb{R}^n. \tag{2.4}$$

A1: *We assume there exist a suitable matrix D and constants* $g_1 > 1$, $1 > g_2 > 0$ *such that*

$$|Ax + f(x)| \leq g_1|x|, \ \ \forall x \in \mathbb{R}^n, \tag{2.5}$$

$$|(A + D)x + f(x)| \leq g_2|x|, \ \ \forall x \in \mathbb{R}^n. \tag{2.6}$$

3. Asymptotic stabilization by using T-APIC

In this section, we are going to design T-APIC to stabilize system (1.1) by utilizing Lyapunov functions with relaxed constraints.

Theorem 1. *If the assumption A1 holds, then system* (1.1) *is asymptotically stabilization by T-APIC* (2.1) *satisfying the following constraint* (3.1)*, i.e., there exist constants* $a \in (0, -\ln g_2)$ *and* $i^* \in \mathbb{N}_+$ *(*$i^* \geq 1$*) such that*

$$N(k) \geq \frac{k}{\frac{\ln g_1 - \ln g_2}{\ln g_1 + a}\overline{l_i}} + 1, \ \ k \in [k_i, k_{i+1}), \ i \geq i^*, \tag{3.1}$$

where $\overline{l_i}$ *is defined by* $\overline{l_i} = \frac{\sum_{m=0}^{i-1} l_m}{i}$ *and* $N(k)$ *denotes the number of the T-APIC* (2.1) *with* (3.1) *implemented in system* (1.1) *during* $[0, k)$.

Proof. For $k_i \leq k < k_i + l_i$, the T-APIC is imposed on system (1.1). Then based on the assumption (2.6), it is computed that

$$|x(k + 1)| = |(A + D)x(k) + f(x(k))| \leq g_2|x(k)|,$$

for $k_i \leq k < k_i + l_i$.

By recursion, it is derived that

$$|x(k + 1)| \leq g_2^{k+1-k_i}|x(k_i)|,$$

for $k_i \leq k < k_i + l_i$.

Thus, it is easy to get that

$$|x(k_i + l_i)| \leq g_2^{l_i}|x(k_i)|. \tag{3.2}$$

For $k_i + l_i \leq k < k_{i+1}$, there is no T-APIC implemented in system (1.1). According to the constraint (2.5), it is satisfied that

$$|x(k + 1)| = |A(x(k)) + f(x(k))| \leq g_1|x(k)|.$$

By recursion, it holds that for $k_i + l_i \leq k \leq k_{i+1}$

$$|x(k)| \leq g_1^{k-k_i-l_i}|x(k_i + l_i)|. \tag{3.3}$$

Then it follows that

$$|x(k_{i+1})| \leq g_1^{k_{i+1}-k_i-l_i}|x(k_i + l_i)|. \tag{3.4}$$

By using the inequalities (3.2) and (3.4), we have that

$$|x(k_{i+1})| \leq g_1^{k_{i+1}-k_i-l_i} g_2^{l_i} |x(k_i)| \leq g_1^{k_{i+1}-k_i} (\frac{g_2}{g_1})^{l_i} |x(k_i)| \leq g_1^{k_{i+1}} (\frac{g_2}{g_1})^{\sum_{m=0}^{i} l_m} |x_0|.$$

According to the above inequalities, it follows that

$$|x(k)| \leq \begin{cases} g_1^{k_i} g_2^{k-k_i} (\frac{g_2}{g_1})^{\sum_{m=0}^{i-1} l_m} |x_0|, \ \forall k_i \leq k < k_i + l_i, \\ g_1^{k-k_i-l_i} g_1^{k_i} g_2^{l_i} (\frac{g_2}{g_1})^{\sum_{m=0}^{i-1} l_m} |x(0)|, \ \forall k_i + l_i \leq k < k_{i+1}. \end{cases} \tag{3.5}$$

Thus it holds that

$$|x(k)| \leq g_1^k (\frac{g_2}{g_1})^{\sum_{m=0}^{i-1} l_m} |x(0)|, \ k \in [k_i, k_{i+1}].$$

From the assumption (3.1), we get that

$$N(k) \geq \frac{k}{\frac{\ln g_1 - \ln g_2}{\ln g_1 + a} \overline{l_i}}.$$

Then it follows that

$$N(k)\overline{l_i}(\ln g_1 - \ln g_2) \geq (\ln g_1 + a)k.$$

Thus, it leads to the following inequality:

$$\ln g_1 k - (\ln g_1 - \ln g_2)(\sum_{m=0}^{i-1} l_m) \leq -ak, \ k \geq k_{i^*}.$$

Based on the inequalities from (3.5), it is easy to calculate that

$$|x(t)| \leq |x_0| \exp(-ak), \ k \geq k_{i^*}. \tag{3.6}$$

Let $q = \max\{1, \exp^{(\ln g_1 + a)k_{i^*} - (\ln g_1 - \ln g_2)(\sum_{m=0}^{k_{i^*}-1} l_m)}\}$. According to the obtained inequalities (3.5) and (3.6), it is satisfied that

$$|x(t)| \leq q|x_0| \exp(-ak), \ k \geq 0,$$

with $q \geq 1$.

Therefore, system (1.1) is asymptotically stabilized by the T-APIC with the requirements (3.1). □

Remark 1. *(1) The proof of Theorem 1 is inspired by the proof of Theorem 3.1 from [13] which is concerned about a continuous-time dynamic system.*

(2) From the proof of Theorem 1, it is known that the assumption (3.1) *on the T-APIC is very important, since it makes sure that the norm of the state of system* (1.1) *is decreasing as the time k increases for $k \geq k_{i^*}$.*

(3) *For the constraint* (2.5) *of Theorem 1, since f is Lipschitz continuous, it is easy to find a constant g_1 such that the requirement* (2.5) *is satisfied.*

(4) *For the constraint* (2.6)*, given the Lipschitz constant $L_f < 1$, in order to ensure* (2.6) *holds, it is not difficult to get a matrix D satisfying $0 < |A + D| \leq g_2 - L_f$. For the case $L_f \geq 1$, we have not figured out an unified way to get a suitable control gain matrix D such that the inequality* (2.6) *holds. We will investigate how to design nonlinear control to stabilize system* (1.1).

The discussion (2) of Remark 1 explains that the norm of the state of system (1.1) under the T-APIC from Theorem 1 decreases as k ($k \geq k_i^*$) increases. Based on the above work, the relaxed constraints on T-APIC which asymptotically stabilize system (1.1) are proposed.

Theorem 2. *If the assumption A1 holds and there exist positive integers $\tau, T \in \mathbb{N}_+$ satisfying the following requirements,*

$$g_1^T g_2^\tau = \xi < 1, \tag{3.7}$$

then system (1.1) *is asymptotically stabilized by T-APIC* (2.1) *with the constraint* (3.8).

$$\frac{l_i}{k_{i+1} - k_i - l_i} \geq \frac{\tau}{T}, \quad i \in \mathbb{N}_+. \tag{3.8}$$

Proof. According to the constraint (2.6), for $k_i \leq k < k_i + l_i$ the control is applied to system (1.1). Then we derive that

$$|x(k+1)| \leq g_2 |x(k)|, \; k_i \leq k < k_i + l_i.$$

Then it is deduced that

$$|x(k+1)| \leq g_2^{k-k_i} |x(k_i)|, \; k_i \leq k < k_i + l_i.$$

It is attained that

$$|x(k_i + l_i)| \leq g_2^{l_i} |x(k_i)|. \tag{3.9}$$

For $k_i + l_i \leq k < k_{i+1}$, by utilizing the assumption (2.5), it is obtained that

$$|x(k)| \leq g_1^{k-k_i-l_i} |x(k_i + l_i)|.$$

Based on the inequality (3.9), it holds that for $k \in [k_i + l_i, k_{i+1})$

$$\begin{aligned} |x(k)| &\leq g_1^{k-k_i-l_i} g_2^{l_i} |x(k_i)| \\ &\leq g_1^T g_2^\tau |x(k_i)|. \end{aligned} \tag{3.10}$$

Since the requirements (3.7) and (3.8) are satisfied, we attain that

$$|x(k_i)| \leq \xi |x(k_{i-1})| \leq \xi^i |x_0|, \forall i \in \mathbb{N}_+. \tag{3.11}$$

Using the inequalities (3.9), (3.10) and (3.11), it is attained that

$$|x(k)| \leq \begin{cases} \xi^i g_2^{k-k_i} |x_0|, \; k \in [k_i, k_i + l_i), \\ \xi^i g_1^{k-k_i-l_i} g_2^{l_i} |x_0|, \; k \in [k_i + l_i, k_{i+1}). \end{cases}$$

Therefore, it is satisfied that for $k \in [k_i, k_{i+1})$

$$\begin{aligned}|x(k)| &\leq \xi^i g_1^{T_M}|x_0| \leq \xi^i g_1^{T_M}|x_0| \\ &\leq \xi^{\frac{k}{T_M+\tau_m}} g_1^{T_M}|x_0|,\end{aligned}$$

where $T_M = \max\{k_{i+1} - k_i - l_i, i \in \mathbb{N}_+\}$, $\tau_m = \max\{l_i, i \in \mathbb{N}_+\}$.

Therefore, we conclude that system (1.1) is asymptotically stabilized by the T-APIC (2.1) satisfying the assumptions $A1$ and (3.8). □

Remark 2. *(1) Compared with the constraints on the T-APIC from Theorem 1, from the proof of Theorem 2, it is clear that the constraint* (3.8) *ensures the inequality* (3.11) *holds for all* $i \in \mathbb{N}_+$*. However, the norm of the state of system* (1.1) *with the T-APIC from Theorem 2 is not decreasing as the time* k *increases. From this point of view, the constraint on the T-APIC from Theorem 2 is relaxed.*

(2) A suitable gain matrix D *can be searched by the method described in (4) of Remark 1.*

(3) The constraints (3.7) *and* (3.8) *display the relationship between the maximal inactive control time interval and the minimal active control width of the proposed T-APIC.*

The above T-APIC schemes are not designed based on the state of system (1.1). Hence, in the following, we will design E-APIC for asymptotic stabilization of system (1.1).

4. Asymptotic stabilization by E-APIC

In this section, E-APIC schemes will be designed to asymptotically stabilize system (1.1).

Given positive integers $l_i \geq 1$ ($i \in \mathbb{N}_+$), a sufficient large integer χ and a positive constant $\delta > 1$, we define a set

$$\theta_i(s, s+\chi) = \{k : s+\chi \geq k > s+l_i, |x(k)| > \delta|x(s+l_i)|, s \in \mathbb{N}_+\}. \tag{4.1}$$

Based on θ_i, we design an E-APIC plan according to δ and a sufficient large integer χ as follows:

$$u(k) = \begin{cases} Dx(k), & k_i \leq k < k_i + l_i, \\ 0, & k_i + l_i \leq k < k_{i+1}, \end{cases} \tag{4.2}$$

and k_{i+1} is decided by

$$k_{i+1} = \begin{cases} \min\{k : k \in \theta_i(k_i, k_i+\chi)\}, & \text{if } \theta_i(k_i, k_i+\chi) \neq \emptyset, \\ k_i + \chi, & \text{if } \theta_i(k_i, k_i+\chi) = \emptyset, \end{cases} \tag{4.3}$$

and the active control length l_i satisfies that

$$-\frac{\ln(g_1\delta)}{\ln g_2} < \tau \leq l_i, \tag{4.4}$$

where τ is a positive integer.

For system (1.1) under the E-APIC (4.2) with the constraints (4.3) and (4.4), the following results are derived.

Theorem 3. *If the assumption* $A1$ *and the inequality* (4.4) *hold, then system* (1.1) *is asymptotically stabilized via the specified E-APIC* (4.2) *with the constraints* (4.3) *and* (4.1).

Proof. Based on the designed E-APIC mechanism, for $k \in [k_i + l_i, k_{i+1})$, the control $u(k)$ is not implemented in system (1.1). Thus by the constraint (2.5), we get that for $k \in [k_i + l_i, k_{i+1})$

$$|x(k)| \leq g_1^{k-k_i-l_i}|x(k_i + l_i)|.$$

Hence it holds that

$$|x(k_{i+1})| \leq g_1^{k_{i+1}-k_i-l_i}|x(k_i + l_i)|. \tag{4.5}$$

For the case $\theta_i(k_i, k_i + \chi) \neq \emptyset$, the inequalities $k_{i+1} - k_i \leq \chi$ and $|x(k_{i+1})| \geq \delta|x(k_i + l_i)|$ are satisfied. Since the inequality (4.5) holds, we have that

$$\delta|x(k_i + l_i)| \leq g_1^{k_{i+1}-k_i-l_i}|x(k_i + l_i)|.$$

Thus it is satisfied that

$$\frac{\ln \delta}{\ln g_1} + l_i \leq k_{i+1} - k_i. \tag{4.6}$$

For the case $\theta_i(k_i, k_i + \chi) = \emptyset$, we have $k_{i+1} - k_i = \chi$. Based on (4.4) and (4.6), we attain that

$$1 < \frac{\ln \delta}{\ln g_1} + l_i \leq k_{i+1} - k_i \leq \chi. \tag{4.7}$$

Under the constraints of (4.6) and (4.7), for the designed E-APIC (4.2) it is clear that $k_{i+1} - k_i \geq 2$. Especially if $\delta > g_1$ holds, then $k_{i+1} - k_i \geq 3$. It means that system (2.2) does not switch every time.

By utilizing the given requirements, for $k \in [k_i, k_i + l_i)$ the control is imposed on system (1.1). Then for $k \in [k_i, k_i + l_i]$ we have

$$|x(k)| \leq g_2^{k-k_i}|x(k_i)|.$$

Hence it fulfills that

$$|x(k_i + l_i)| \leq g_2^{l_i}|x(k_i)|. \tag{4.8}$$

According to the constraints (2.5), (4.1) and (4.8), it holds that

$$|x(k_{i+1})| \overset{(2.5)}{\leq} g_1|x(k_{i+1} - 1)| \overset{(4.1)}{\leq} g_1\delta|x(k_i + l_i)| \overset{(4.8)}{\leq} g_1\delta g_2^{l_i}|x(k_i)|. \tag{4.9}$$

For $k \in [k_i + l_i, k_{i+1}]$, based on the assumptions (4.1), (4.2) and the inequality (4.9), we get that

$$|x(k)| \leq g_1\delta|x(k_i + l_i)| \leq \delta g_1|x(k_i)|.$$

Therefore, because of the inequalities (4.8) and (4.9), it is satisfied that for $k \in [k_i, k_{i+1}]$,

$$\begin{aligned} |x(k)| &\leq g_1\delta|x(k_i)| \leq g_1\delta(g_1\delta)^i g_2^{\sum_{m=0}^{i-1} l_m}|x_0| \\ &\leq g_1\delta(g_1\delta g_2^{\tau})^i|x_0| \leq g_1\delta\xi^{\frac{k}{\chi}}|x_0|, \end{aligned}$$

where $0 < \xi = g_1\delta g_2^{\tau} < 1$ derived from the constraint (4.4).

Thus we claim that system (1.1) is asymptotically stabilized via the specified E-APIC (4.2) with the constraints (4.3) and (4.4). □

Remark 3. *(1) Compared with Theorem 1 and Theorem 2, in Theorem 3 k_i is decided by* (4.3) *related to the state of the considered system.*

(2) The constraint (4.4) *ensures that the inequality* $|x(k_{i+1})| \leq \xi|x(k_i)|$ *holds with* $1 > \xi > 0$. *Hence, the norm of the state* $x(k)$ *does not decrease as the time* k *increases.*

In Theorem 3 the width of the control imposed on system (1.1) satisfies the constraint (4.4). It is evident that the width of the control is given in advance. Thus in the following we are going to see how to stabilize system (1.1) by designing an E-APIC plan with the width of control given based on the state.

Given a positive constant $1 > \lambda > 0$, we now consider an E-APIC plan defined by (4.2) and (4.3) with the width of control specified by

$$l_i = \begin{cases} \min\{\omega \in \mathbb{N}_+, \omega \geq 1 : |x(\omega)| \leq \lambda|x_0|\}, \ i = 0, \\ \min\{\omega \in \mathbb{N}_+, \omega \geq 1 : |x(k_i + \omega)| \leq \lambda|x(k_{i-1} + l_{i-1})|\}, \ i \neq 0. \end{cases} \tag{4.10}$$

According to the assumptions (4.10), it is evident that $l_i \geq 1$ $(i \in \mathbb{N}_+)$ and for $i \neq 0$, $|x(k_i + l_i - 1)| > \lambda|x(k_{i-1} + l_{i-1})|$ and $|x(k_i + l_i)| \leq \lambda|x(k_{i-1} + l_{i-1})|$.

Under the above defined E-APIC (4.2), (4.3) with the control length (4.10), the main result is described as the following.

Theorem 4. *If the assumption A1 holds, then system* (1.1) *is asymptotically stabilized via the specified E-APIC* (4.2) *with the constraints* (4.3) *and* (4.10).

Proof. Via the similar deduction of (4.7), it is obtained that

$$1 < \frac{\ln \delta}{\ln g_1} + l_i \leq k_{i+1} - k_i \leq \chi.$$

Hence the inequality $k_{i+1} - k_i \geq 2$ holds. If $\delta > g_1$, then $k_{i+1} - k_i \geq 3$ and system (2.2) does not switch every time.

For $k \in [k_i, k_i + l_i]$, the control is applied to system (1.1). Via the inequality (4.9), it is satisfied that

$$|x(k)| \leq g_2^{k-k_i}|x(k_i)| \leq g_2^{k-k_i} g_1 \delta |x(k_{i-1} + l_{i-1})|.$$

Then

$$|x(k_i + l_i - 1)| \leq g_2^{l_i - 1} g_1 \delta |x(k_{i-1} + l_{i-1})|.$$

However, it holds that

$$|x(k_i + l_i - 1)| \overset{(4.10)}{>} \lambda|x(k_{i-1} + l_{i-1})|.$$

Thus

$$\lambda|x(k_{i-1} + l_{i-1})| < g_2^{l_i - 1} g_1 \delta |x(k_{i-1} + l_{i-1})|. \tag{4.11}$$

Therefore, the width of the control satisfies

$$1 \leq l_i < \frac{\ln \lambda - \ln(g_1 \delta)}{\ln g_2} + 1, \ i \geq 1.$$

Based on the above analysis, we get that for $k \in [k_i + l_i, k_{i+1}](i \geq 1)$,

$$|x(k)| \leq g_1\delta|x(k_i + l_i)| \leq g_1\delta\lambda|x(k_{i-1} + l_{i-1})|,$$

and for $k \in [k_i, k_i + l_i](i \geq 1)$,

$$|x(k)| \leq |x(k_i)| \leq g_1\delta|x(k_{i-1} + l_{i-1})|.$$

Then for $k \in [k_i, k_{i+1}](i \geq 1)$

$$\begin{aligned}|x(k)| &\leq g_1\delta|x(k_{i-1} + l_{i-1})| \leq g_1\delta\lambda^{i-1}|x(k_0 + l_0)| \\ &\leq g_1\delta\lambda^i|x(k_0)| = g_1\delta\lambda^i|x_0|.\end{aligned}$$

For $k \in [0, l_0]$, it is clear that

$$|x(k)| \leq \lambda|x_0|.$$

For $k \in [l_0, k_1]$, we have that

$$|x(k)| \leq g_1\delta|x(l_0)| \leq g_1\delta\lambda|x_0| \leq g_1\delta|x_0|.$$

Hence for $k \geq 0, k \in [k_i, k_{i+1}]$, it holds that

$$|x(k)| \leq \frac{g_1\delta}{\lambda}\lambda^i|x_0| \leq \frac{g_1\delta}{\lambda}\lambda^{\frac{k}{\chi}}|x_0|.$$

Therefore, system (1.1) is asymptotically stabilized by the given E-APIC (4.2) with (4.3) and (4.10). □

Remark 4. *(1) From the assumptions of Theorem 4, it is known that the width of the control and the starting time when the control is implemented in system* (1.1) *are in relationship with the state.*
(2) Under the constraints of Theorem 3, it holds that $|x(k_i)| \leq \xi|x(k_{i-1})|$ *with* $0 < \xi < 1$. *Under the assumptions of Theorem 4, it is satisfied that* $|x(k_{i+1} + l_{i+1})| \leq \lambda|x(k_i + l_i)|$ *with* $0 < \lambda < 1$.

Based on the above results, we are going to relax the constraint (2.6). For system (4.12)

$$x(k + 1) = (A + D)x(k) + f(x(k)),\ x \in \mathbb{R}^n, \tag{4.12}$$

we assume that there exist a control gain matrix D and constants $\bar{k} > 1(\bar{k} \in \mathbb{N}_+)$, $g_3 > 0$ and $0 < g_2 < 1$ such that

$$|(A + D)x(k) + f(x(k))| \leq g_2|x(k)|, \text{for } k \geq \bar{k},\ x(0) \in \mathbb{R}^n, \tag{4.13}$$

$$|(A + D)x + f(x)| \leq g_3|x|, \text{for } x \in \mathbb{R}^n. \tag{4.14}$$

Based on the above assumptions, how to stabilize system (1.1) is described via the following theorem.

Theorem 5. *If the constraints* (2.5)*,* (4.13) *and* (4.14) *hold, then system* (1.1) *is asymptotically stabilized via the specified E-APIC* (4.2) *with* (4.3) *and the width of control decided by*

$$l_i = \min\{\omega \in \mathbb{N}_+, \omega \geq 1 : |x(k_i + \omega)| \leq \lambda |x(k_i)|\}, \;\; i \in \mathbb{N}_+, \tag{4.15}$$

with $\lambda g_1 \delta < 1$.

Proof. The proof is similar to the proof of Theorem 4. For $k \in [0, \bar{k}]$, by utilizing the conditions (4.13), (4.14) and (4.15), we have that

$$|x(k)| \leq g_3^{\bar{k}} |x(k_i)|. \tag{4.16}$$

And it holds that

$$|x(k_i + l_i)| \leq \lambda |x(k_i)|. \tag{4.17}$$

For $k \in [k_i + l_i, k_{i+1}]$, based on the constraint (4.2), it holds that

$$|x(k)| \leq g_1 \delta |x(k_i + l_i)|.$$

Then it is satisfied that

$$|x(k_{i+1})| \leq g_1 \delta \lambda |x(k_i)|. \tag{4.18}$$

Thus for $k \in [k_i, k_{i+1}]$, we get that

$$|x(k)| \leq A |x(k_i)| \leq A (g_1 \delta \lambda)^i |x_0|,$$

where $A = \max\{g_1 \delta, g_3^{\bar{k}}\}$. Since the inequality $g_1 \delta \lambda < 1$ holds, we conclude system (1.1) is stabilized by the given control design. □

Remark 5. *(1) Compared with the constraint* (2.6)*, it is evident that the condition* (4.13) *is more relaxed, since it is required that* (4.13) *holds for* $k \geq \bar{k} > 1$*,* $x(0) \in \mathbb{R}^n$*.*

(2) The efficiency of Theorem 5 is demonstrated via stabilizing a chaotic system in Section 6.

5. Asymptotic stabilization based on E-APIC and ISS

In the above, it is clear that APIC schemes from Theorem 1–5 continuously update during the control time interval. However, in this section, we design an E-APIC as follows

$$u(k) = \begin{cases} Dx(k_i), \; k_i \leq k < k_i + l_i, \\ 0, \; k_i + l_i \leq k < k_{i+1}, \end{cases} \tag{5.1}$$

where k_i is defined by (4.1) and (4.3) given in Section 4.

For $k \in [k_i, k_i + l_i)$, the control is applied to system (1.1), i.e.,

$$x(k+1) = (A + D)x(k) + f(x(k)) + De(k), \tag{5.2}$$

where $e(k) = x(k_i) - x(k)$.

The E-APIC (5.1) is designed based on the concept of ISS. The details of ISS for discrete-time systems are discussed in [32].

We assume that under the control gain matrix D system (5.3) is ISS as regards the measure error $e(k)$,

$$x(k+1) = (A + D)x(k) + f(x(k)) + De(k), \tag{5.3}$$

and there exist positive constants $0 < \mu_1 < 1$ and μ_2 such that the norm of the state satisfies

$$|x(k+1)| - |x(k)| \leq -(1 - \mu_1)|x(k)| + \mu_2|e(k)|. \tag{5.4}$$

Remark 6. *(1) Then linear constraint* (5.4) *on the state makes it is easy to prove the following Theorem 6 and stabilize an illustrative example discussed in Section 6.6.*

Based on the assumed inequality (5.4), for system (5.2) it holds that

$$|x(k+1)| \leq \mu_1|x(k)| + \mu_2|e(k)|, \tag{5.5}$$

for $k \in [k_i, k_i + l_i)$.

Then we define the control length l_i as the following:

$$l_i = \max\{\omega \in \mathbb{N}_+, \omega \geq 0 : |e(k_i + \omega)| \leq \rho|x(k_i + \omega)|\}, \tag{5.6}$$

where $e(k_i + \omega) = x(k_i) - x(k_i + \omega)$, the constant ρ satisfying

$$\mu_1 + \mu_2\rho < 1. \tag{5.7}$$

Now we state the main outcome of this section.

Theorem 6. *For system* (1.1)*, if there exist a suitable matrix D, positive constants* $0 < \mu_1 < 1$ *and* μ_2 *such that system* (5.3) *is ISS and the inequality* (2.5) *holds, then system* (1.1) *is asymptotically stabilized via the designed E-APIC* (5.1) *with* (4.3), (5.6), (5.7) *and the following constraint* (5.8).

There exist constants $0 < a < 1$ *and* i_* *such that for all* $i > i_*$

$$\frac{\ln a - \ln(g_1\delta)}{\ln(\mu_1 + \mu_2\rho)} < \overline{l_i}, \tag{5.8}$$

where $\overline{l_i} = \frac{\sum_{j=0}^{i-1} l_j}{i}$.

Proof. Using the similar analysis as the proof of Theorem 4, we derive that

$$1 < \frac{\ln\delta}{\ln g_1} + l_i \leq k_{i+1} - k_i \leq \chi.$$

It is clear that $k_{i+1} - k_i \geq 2$ holds.

Under the constraints of Theorem 6, on one hand for $k \in [k_i, k_i + l_i]$, we have that

$$|x(k)| \leq (\mu_1 + \mu_2\rho)|x(k-1)|.$$

Thus it follows that

$$|x(k_i + l_i)| \leq (\mu_1 + \mu_2\rho)^{l_i}|x(k_i)|. \tag{5.9}$$

On the other hand, for $k \in [k_i + l_i, k_{i+1}]$ it holds that

$$|x(k)| \leq g_1\delta|x(k_i + l_i)| \leq g_1\delta(\mu_1 + \mu_2\rho)^{l_i}|x(k_i)|.$$

Then

$$|x(k_{i+1})| \leq g_1\delta|x(k_i + l_i)| \leq g_1\delta(\mu_1 + \mu_2\rho)^{l_i}|x(k_i)|. \tag{5.10}$$

Based on the above derivation and the condition on the variable ρ, we deduce that for $k \in [k_i, k_{i+1}]$

$$\begin{aligned} |x(k)| &\leq g_1\delta|x(k_i)| \leq g_1\delta(g_1\delta)^i(\mu_1 + \mu_2\rho)^{\sum_{j=0}^{i-1} l_j}|x(k_0)| \\ &\leq g_1\delta((\mu_1 + \mu_2\rho)^{\overline{l_i}}g_1\delta)^i|x_0|. \end{aligned}$$

According to the constraint (5.8), for $k \in [k_i, k_{i+1})$ $(i \geq i_*)$ it is obtained that

$$|x(k)| \leq g_1\delta a^i|x_0| \leq g_1\delta a^{\frac{k}{\chi}}|x_0|.$$

For $i < i_*$, $k \in [k_i, k_{i+1}]$, we have

$$\begin{aligned} |x(k)| &\leq g_1\delta|x(k_i)| \leq g_1\delta(g_1\delta)^i(\mu_1 + \mu_2\rho)^{\sum_{j=0}^{i-1} l_j}|x_0| \\ &\leq g_1\delta(\frac{g_1\delta}{a})^{i_*}a^{\frac{k}{\chi}}|x_0|. \end{aligned}$$

Then it holds that for $k \in [k_i, k_{i+1}), i \in \mathbb{N}_+$,

$$|x(k)| \leq qa^{\frac{k}{\chi}}|x_0|, \tag{5.11}$$

where $q = g_1\delta(\frac{g_1\delta}{a})^{i_*}$.

According to Definition 1, system (1.1) is asymptotically stabilized. □

Remark 7. *(1) For Theorem 6, if* $(\mu_1 + \mu_2\rho)g_1\delta < 1$ *holds, then* i_* *can be* 0.

(2) Under the given constraints of Theorem 6, for $i > i_*$ *we have* $|x(k_{i+1})| < (\mu_1 + \mu_2\rho)g_1\delta|x(k_i)|$.

(3) Compared with the APIC mechanisms from Sections 3 and 4, the control plan (5.1) *with* (4.3) *and* (5.6) *only updates at the time when the control is triggered.*

(4) If we could not find a suitable ρ *such that the inequality* (5.7) *hold, then it is better to use* $u(x) = Dx$ *(Theorems 1–4) to asymptotically stabilize the considered system.*

6. Illustrative examples

In this section, three numerical examples are discussed to illustrate the efficiency of the obtained theorems.

For Theorems 1–4, we consider the following example described by

$$x(k+1) = Ax(k) + f(x(k)), \tag{6.1}$$

where $x = (x_1, x_2)^\top \in \mathbb{R}^2$, $A = \begin{pmatrix} 1 & 0 \\ 0 & \frac{3}{2} \end{pmatrix}$, $f(x(k)) = (\frac{1}{2+x_2^2}x_2, \frac{1}{2+x_1^2}x_1)^\top$.

Now based on the obtained Theorems 1–4, we are going to design intermittent control for asymptotic stabilization of system (6.1).

For system (6.1), it is easy to obtain that

$$|Ax + f(x)| \leq g_1|x|, \text{ with } g_1 = 2, \text{ for } x \in \mathbb{R}^n.$$

In order to ensure the inequality (2.6) holds, we choose the matrix D as

$$D = \begin{pmatrix} -\frac{4}{5} & 0 \\ 0 & -\frac{13}{10} \end{pmatrix}. \tag{6.2}$$

Then the inequality (2.6) holds with $g_2 = \frac{7}{10}$, i.e.,

$$|Ax + Dx + f(x)| \leq g_2|x|, \text{ with } g_2 = \frac{7}{10} \text{ for } x \in \mathbb{R}^n.$$

6.1. Control design based on Theorem 1

Now we consider how to design T-APIC which asymptotically stabilizes system (6.1) based on Theorem 1.

For the variable a, it is required that $a = 0.15$. Then we have that for $k > k_{i^*}$

$$N(k) \geq 0.7555k + 1.$$

From this inequality, in order to find a suitable k_{i^*} for (3.1), we have to require that there should at least be 9 control times during $[k_i, k_{i+1}] = 10$. It is checked that the inequality (3.1) holds with $k_{i^*} = 0$. Based on Theorem 1, under the given T-APIC from Theorem 1, system (6.1) is asymptotically stabilized. In order to show this conclusion, we plot the Figures 1 and 2 of the state of system (6.1) with the initial condition $x_0 = (4, 5)^\top$ and T-APIC satisfying the constraints of Theorem 1.

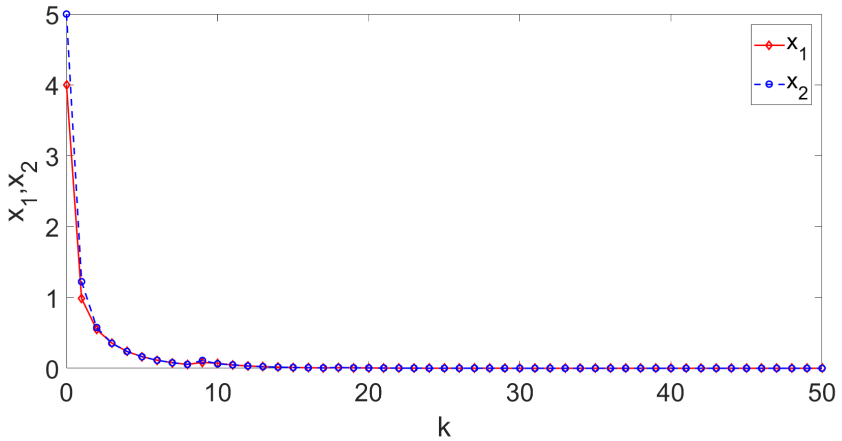

Figure 1. The trajectory of the state of system (6.1) with $u(x) = Dx$ and the initial condition $x_0 = (4, 5)^\top$.

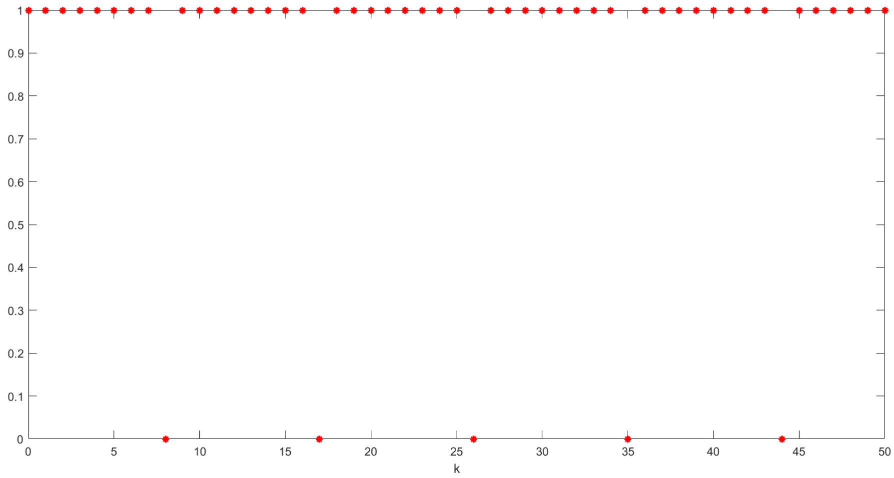

Figure 2. For system (6.1), '1' means the control is imposed, '0' means the control is not imposed.

6.2. *Control design based on Theorem 2*

In this section, we still use the matrix D given by (6.2). In order to make sure the inequality (3.7) fulfills, we let $\tau = 2$, $T = 1$. Then it holds that $g_1 g_2^2 = \xi = \frac{49}{50}$, $\frac{l_i}{k_{i+1}-k_i-l_i} \geq 2$. Based on Theorem (2), under the given intermittent control from Theorem 2, system (6.1) is asymptotically stabilized. To show the conclusion, we plot the trajectory of the state of system (6.1) with $l_i = 2$, $k_{i+1} - k_i = 3$, $u(x) = Dx$ and the initial condition $x_0 = (4, 5)^\top$ (see Figure 3). Figure 4 shows when the control is imposed on system (6.1).

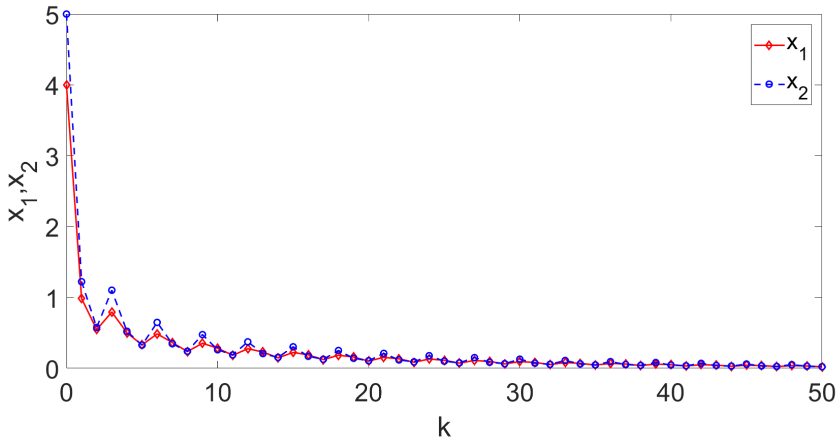

Figure 3. The trajectory of the state of system (6.1) with $l_i = 2$, $k_{i+1} - k_i = 3$, $u(x) = Dx$ and the initial condition $x_0 = (4, 5)^\top$.

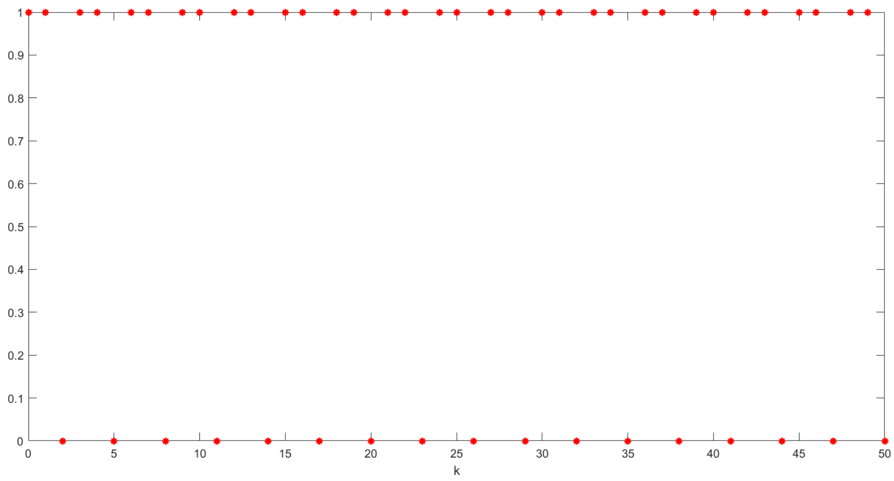

Figure 4. For system (6.1), '1' means the control is imposed, '0' means the control is not imposed.

Remark 8. *According to Theorem 2, under the constraints of Theorem 2, there are many choices for* l_i *and* $k_{i+1} - k_i$. *Here in order to make the simulation simple, we let* $l_i = 2$, $k_{i+1} - k_i = 3$.

6.3. *Control design based on Theorem 3*

The control gain matrix (6.2) is still utilized in this section. Hence for $x \in \mathbb{R}^n$ we have

$$|Ax + f(x)| \leq g_1|x|,\ g_1 = 2,$$
$$|Ax + Dx + f(x)| \leq g_2|x|,\ g_2 = \frac{7}{10}.$$

By letting $\delta = 5$, the variables τ, l_i from Theorem 3 satisfy $\tau \geq 7$, $l_i \geq 7$. Under the conditions of Theorem 3, we can stabilize system (6.1) under the given control (4.2) with (4.3) illustrated by Figure 5. Figure 6 shows when the control is imposed on system (6.1).

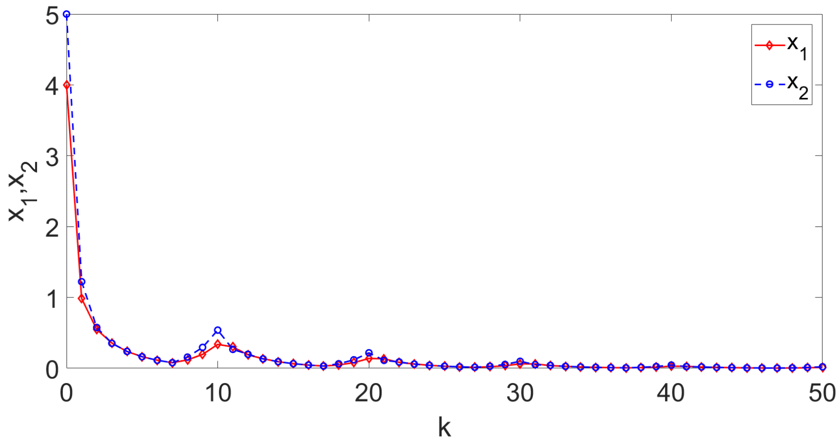

Figure 5. The trajectory of the state of system (6.1) with $l_i = 6$, $u(x) = Dx$ and the initial condition $x_0 = (4, 5)^\top$.

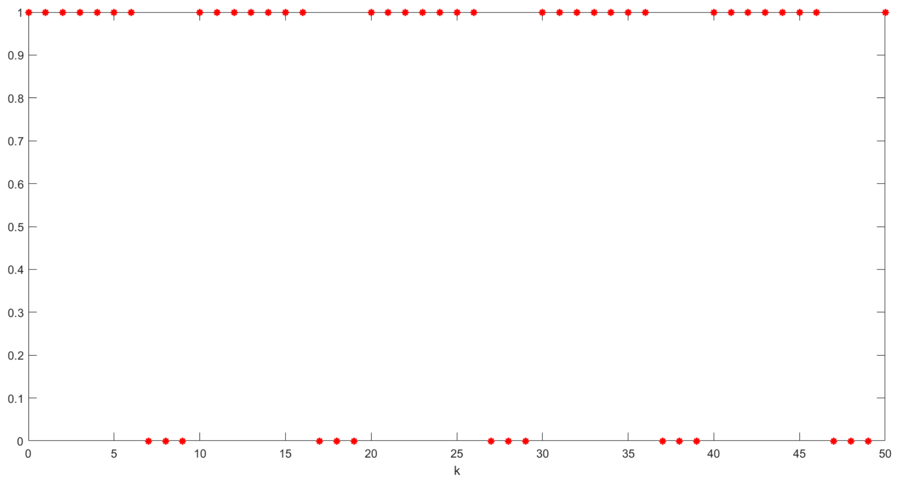

Figure 6. For system (6.1), '1' means that the control is imposed, '0' means that the control is not imposed.

Remark 9. *Actually, according to Theorem 3, under the constraints of Theorem 3, there are many choice for l_i under the requirement $l_i \geq 7$. In order to make the simulation simple, we let $l_i = 7$. Then according to Theorem 3, system* (6) *is asymptotically stabilized under the designed E-APIC from Theorem 3.*

6.4. Control design based on Theorem 4

We still use the control gain matrix (6.2) and for $x \in \mathbb{R}^n$

$$\begin{aligned} |Ax + f(x)| &\leq g_1|x|,\ g_1 = 2, \\ |Ax + Dx + f(x)| &\leq g_2|x|,\ g_2 = \frac{7}{10}. \end{aligned}$$

According to the assumptions of Theorem 4, k_i is decided by (4.3) and l_i is specified by (4.10). The variables λ, δ satisfy $\lambda = \frac{1}{3}$, $\delta = 5$. Then it follows that system (6.1) is asymptotically stabilized via the designed intermittent control (4.2) with (4.3) and (4.10). Figure 7 is presented to demonstrate the results. Figure 8 shows when the control is imposed on system (6.1).

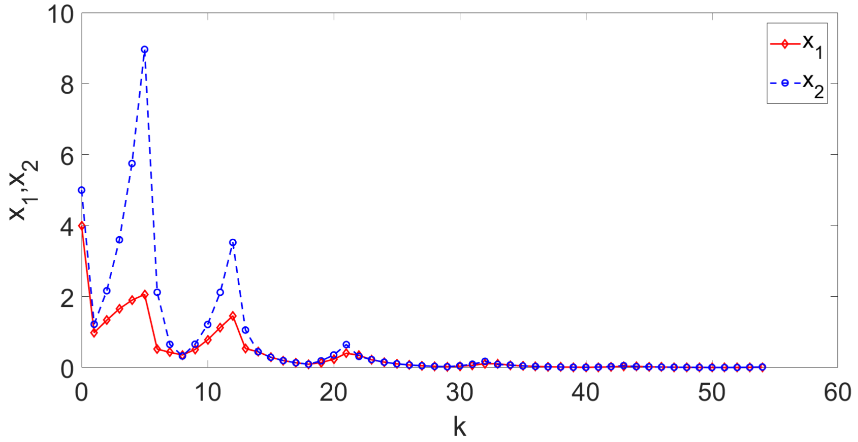

Figure 7. The trajectory of the state of system (6.1) with $u(x) = Dx$ and the initial condition $x_0 = (4, 5)^\top$.

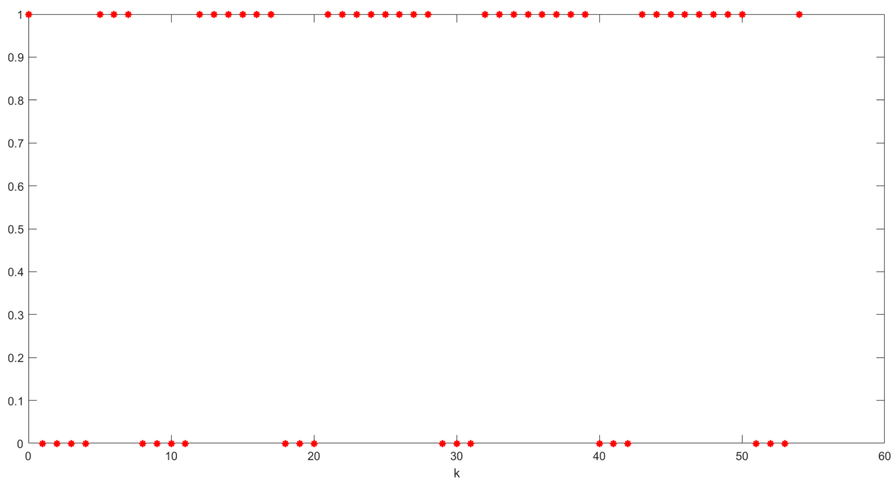

Figure 8. For system (6.1), '1' means that the control is imposed, '0' means that the control is not imposed.

In the following, we will show the efficiency of Theorem 5. Since it is required that the inequality (4.13) holds, we are not going to study system (6). We will investigate how to stabilize a chaotic system by Theorem 5.

6.5. Control design based on Theorem 5

In this section, we are going to stabilize the discrete-time Lorenz system (6.3) via Theorem 5.

$$x(k + 1) = Ax(k) + f(x(k)), \tag{6.3}$$

where $x = (x_1, x_2, x_3)^\top \in \mathbb{R}^n$,

$$A = \begin{pmatrix} -9 & 10 & 0 \\ 28 & 0 & 0 \\ 0 & 0 & -\frac{5}{3} \end{pmatrix},$$

and

$$f(x(k)) = \begin{pmatrix} 0 \\ -x_1(k)x_3(k) \\ x_1(k)x_2(k) \end{pmatrix}.$$

In order to ensure the conditions of Theorem 5 hold, the control gain matrix D is chosen as the following.

$$D = \begin{pmatrix} 9 & -10 & 0 \\ -28 & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{23}{12} \end{pmatrix}. \tag{6.4}$$

Then it is checked that for $k \geq \overline{k} = 2$ the constraint (4.13) holds with $g_2 = \frac{1}{4}$. The conditions (2.5) and(4.14) are satisfied with $g_1 = 22$, $g_3 = \frac{5}{4}$. For Theorem 5, λ and δ are chosen as $\lambda = \frac{1}{30}$ and $\delta = 2$. Then it is clear that $g_1\lambda\delta < 1$ holds. Thus based on Theorem 5, the Lorenz system (6.3) is asymptotically stabilized by the specified E-APIC (4.2) with (4.3) and the width of control related to state specified by (4.15) and the control gain matrix D defined by (6.4). In order to show the effectiveness of this result, figures of the trajectory of the state and control times are plotted (see Figures 9 and 10).

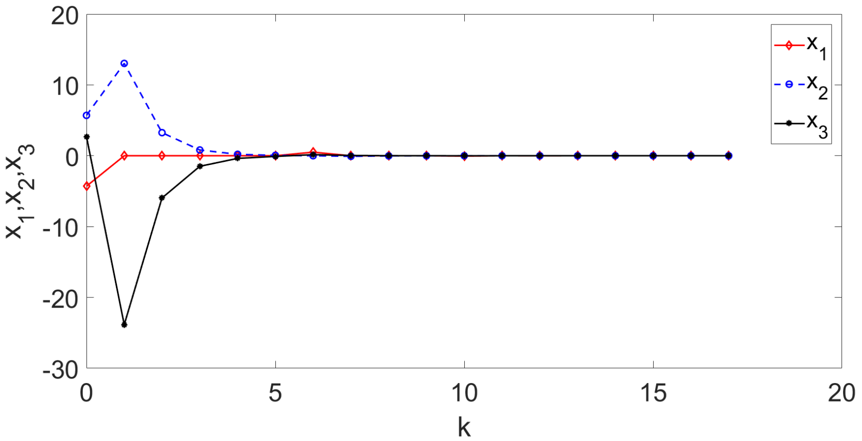

Figure 9. The trajectory of the state of system (6.3) with $u(x) = Dx$ and the initial condition $x_0 = (-4.3, 5.7, 2.7)^\top$.

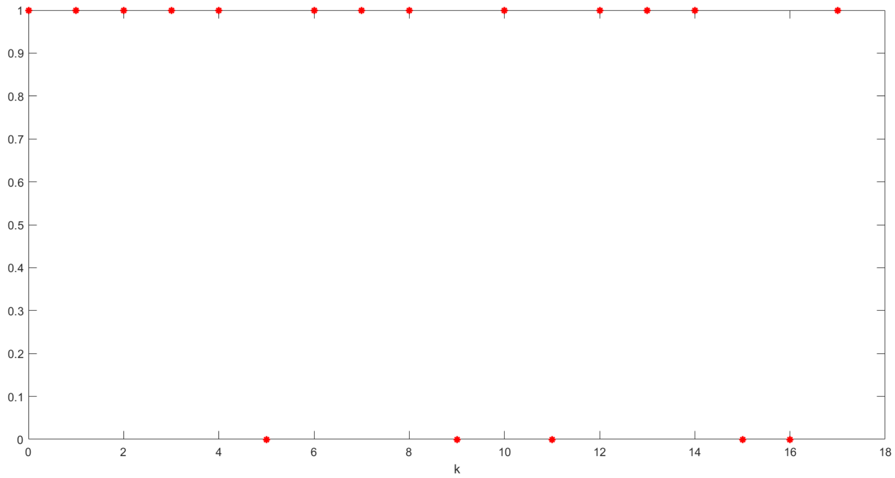

Figure 10. For system (6.3), '1' means that the control is imposed, '0' means that the control is not imposed.

Remark 10. *The above conclusion about system* (6.3) *shows that Theorem 5 can be used to stabilized chaotic systems.*

6.6. Control design based on Theorem 6

In this section, we consider

$$x(k+1) = \begin{cases} x_1 + \frac{1}{100+x_2^2} x_2, \\ 0.9x_2 + \frac{1}{100+x_1^2} x_1, \end{cases} \tag{6.5}$$

where $x = (x_1, x_2)^\top \in \mathbb{R}^2$.

It is clear that the inequality (2.5) holds with $g_1 = 1.01$.

In order to ensure the inequalities (5.3), (5.4), (5.2) and (5.7) hold, the matrix D is chosen as the following.

$$D = \begin{pmatrix} -0.1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{6.6}$$

Under the above constraints, we have

$$|x(k+1)| \leq 0.91|x(k)| + 0.1|e(k)|.$$

Under the conditions of the variables $\mu_1 = 0.91$ and $\mu_2 = 0.1$, we let $\rho = 0.7$ and $\delta = 1.01$. It is clear that $(\mu + \mu_2\rho)g_1\delta = 0.9997 < 1$. Then we can let i_* from Theorem 6 be 0. Therefore, system (6.5) is asymptotically stabilized via the E-APIC schemes from Theorem (6). In order to exhibit the results, the figures of the trajectory of the state (Figure 11) and the control imposed on system (6.5) (Figure 12) are plotted.

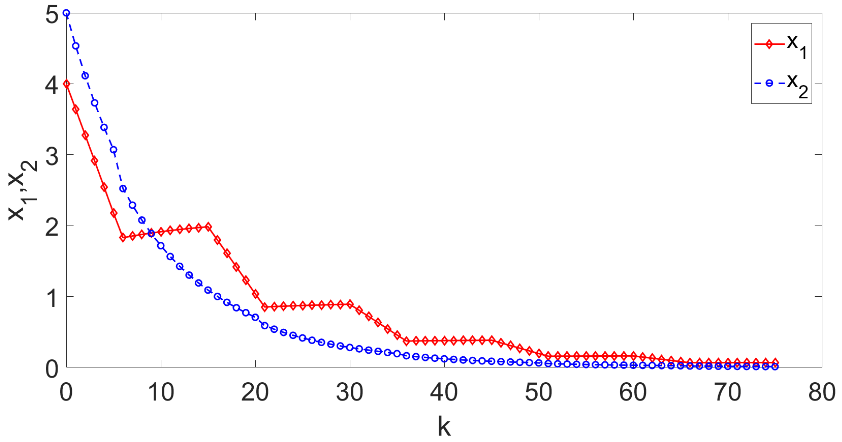

Figure 11. The trajectory of the state of system (6.5) with $u(x) = Dx$ and the initial condition $x_0 = (4, 5)^\top$.

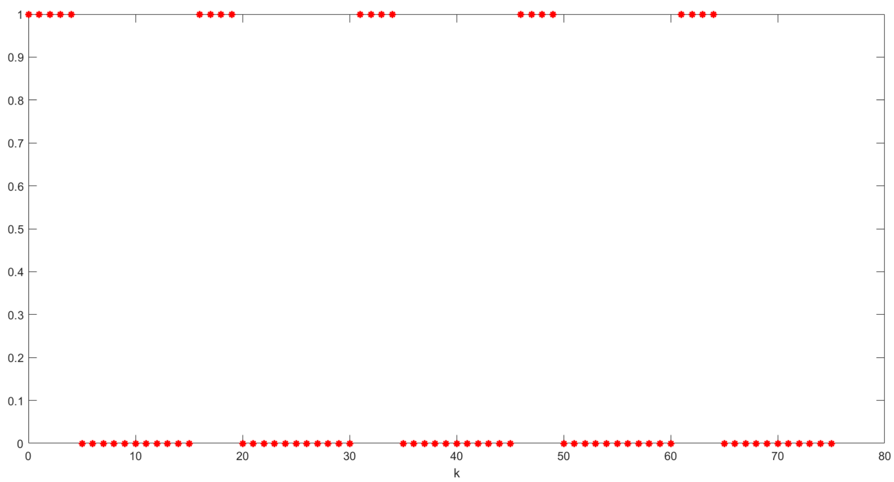

Figure 12. For system (6.5), '1', '0' mean that the control is imposed and is not imposed respectively.

Remark 11. *(1) For showing the efficiency of Theorem 6, system* (6.1) *is not considered, since we have not found out a suitable ρ such that the inequality* (5.7) *holds.*

(2) We have demonstrated that the proposed theorems can be used to asymptotically stabilize system (1.1). *Since the conditions on the theorems are not strong and it is easy to check these constraints, we believe that the methods can be utilized in practical engineering problems which we will consider in future.*

(3) Compared with results of [6], for Theorems 3–6, the control starting time is not prefixed. It is decided by the inequality imposed on the state. In Theorem 6, the control only updates when the control is triggered.

7. Conclusions

In this manuscript, T-APIC and E-APIC schemes were proposed for asymptotic stabilization of the discrete-time system (1.1). The T-APIC mechanisms were designed respectively by imposing the conditions on the average dwell-time and the control width. Under the constraints of Theorem 1, the norm of the state (2.2) is decreasing after a fixed time. Theorem 2 leads to the fact $|x(k_i)| \leq \xi|x(k_{i-1})|$ with $0 < \xi < 1$. Theorems 3–5 are about E-APIC mechanisms. For these designed control, when the norm of the state violates the defined inequality, the control is triggered. Theorem 3 demands the control length should be bigger than a given constant. Theorem 4 describes a condition related to the control length of the state. It leads to the inequality $|x(k_{i+1} + l_{i+1})| \leq \lambda|x(k_i + l_i)|$ with $0 < \lambda < 1$. A numerical example was presented to show the effectiveness of Theorems 1–4. In order to relax the condition (2.6), we demanded the inequality (4.13) holds. Then Theorem 5 was obtained based on Theorem 4. A chaotic system was asymptotically stabilized via the proposed Theorem 5. For these control plans, the control continuously updates during the control time interval. In Theorem 6, the control only updates when the control is triggered. Hence, the control mechanism from Theorem 6 was designed based on the concept of ISS. Then an numerical example was investigated to illustrate the efficiency of Theorem 6.

Author contributions

H.Li contributed the idea to this manuscript and edited the manuscript. Y.Ning was responsible for the calculation of these examples and edited the manuscript. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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