Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(8): 19033–19057.
DOI: 10.3934/math.2025851
Received: 19 July 2025
Revised: 07 August 2025
Accepted: 11 August 2025
Published: 21 August 2025

Research article

Reliability assessment through group acceptance sampling under the Darna distribution

Amer Ibrahim Al-Omari[1], **Ehab M. Almetwally**[2], **Harsh Tripathi**[3] **and Ahmad A. Hanandeh**[4,*]

[1] Department of Mathematics, Faculty of Science, Al Al-Bayt University, Mafraq 25113, Jordan
[2] Department of Mathematics and Statistics, Faculty of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh 11432, Saudi Arabia
[3] Symbiosis Statistical Institute, Symbiosis International (Deemed University), Pune, India
[4] Department of Mathematics, Faculty of Science, Islamic University of Madinah, Madinah 42351, Saudi Arabia

* **Correspondence:** Email: hanandeh@iu.edu.sa; Tel: +962791542383.

Abstract: In this study, a group acceptance sampling plan was proposed when the lifetime of an item follows the Darna distribution (DD). The mean served as a quality parameter to determine the design parameters, including the acceptance number and minimum group size, under a specified test termination time and consumer risk. The operating characteristic values were presented graphically and in tabular form. The minimum group size and operating characteristic values were obtained for various values of the distribution parameters, and the results were illustrated with an example. To illustrate the applicability of the proposed plan, two real-life data sets of failure times in minutes and weeks were analyzed as practical examples. It is preferable to choose higher t/μ_0 (μ_0 is a given mean value) values to minimize the required number of groups and, hence, reduce the overall cost and inspection effort. In addition, choosing suitable values of r (r is the size of the group) and t/μ_0 ensures a balance between the inspection effort and the risk of the producer.

Keywords: acceptance sampling inspection plan; consumer's risk; Darna distribution; operating characteristic function; producer's risk; truncated life test
Mathematics Subject Classification: 62D05, 62N05, 62P30

1. Introduction

Acceptance sampling plans are essential tools in statistical quality control (SQC) designed to determine whether a batch or lot of products should be accepted or rejected based on the inspection of a sample. These plans are widely applied in manufacturing, reliability analysis, and life-testing scenarios, particularly when full inspection is impractical, time-consuming, or prohibitively expensive. In the context of an increasingly competitive and quality-driven marketplace, the success and survival of industries often depend on their ability to consistently deliver superior products.

SQC encompasses a broad set of methodologies aimed at ensuring the production of high-quality goods through robust process control and effective decision-making strategies. Among the key branches of SQC is statistical product control, within which acceptance sampling plans (ASPs) play a vital role. ASPs provide practical techniques for assessing product quality and making informed decisions regarding the acceptance or rejection of production lots.

The literature on ASPs identifies two main types: variable acceptance sampling plans and attribute acceptance sampling plans. Attribute ASPs have been extensively studied and extended over the years, giving rise to several notable schemes such as the single acceptance sampling plan (SASP), double acceptance sampling plan, group acceptance sampling plan (GASP), chain acceptance sampling plan, and skip-lot sampling plan, among others. Each of these plans offers unique advantages suited to different operational contexts and quality control requirements.

Several authors have developed a variable ASP, for example: [1], who introduced an ASP based on life tests assuming the gamma distribution, followed by [2], who extended this approach to normal and log-normal distributions. Later, [3] suggested a group acceptance sampling plan (GASP) for the Weibull distribution, which was followed by contributions such as [4] for the generalized exponential distribution and [5], who developed an ASP for the Birnbaum–Saunders distribution. [6] proposed ASP for the Marshall-Olkin extended exponential distribution, while [7] introduced a GASP using weighted binomial models for inverse Rayleigh and log-logistic distributions under truncated life tests. [8] developed a variable single ASP based on the Pareto distribution. In subsequent years, [9] introduced a double ASP for the Burr-type X distribution. [10] suggested a multi-product inventory model incorporating destructive testing acceptance sampling and inflation. In 2020, [11] developed an inspection plan based on the generalized half-normal distribution. [12] introduced an ASP for the power Lomax distribution and [13] proposed a repetitive ASP for the skew-generalized inverse Weibull distribution. Moreover, [14] suggested a group ASP based on a new compounded three-parameter Weibull model, and [15] proposed an ASP using hypergeometric theory for finite populations under the Q-Weibull distribution. [16] presented a new GASP for an extended exponential model, and [17] introduced a double ASP using the Weibull distribution under indeterminacy. Recent advancements include [18], who suggested an ASP for the odd exponential-logarithmic Fréchet distribution. Most recently, [19] proposed a sequential inspection ASP for the Burr-XII distribution, while [20] considered ASP for the Birnbaum–Saunders lifetime distribution using the intervened Poisson distribution.

Traditional acceptance sampling focuses on individual items and is typically employed when only one unit can be examined at a time. In such single sampling plans, product inspection and sample testing often require considerable time and resources. This becomes particularly challenging when testing is expensive, time-consuming, or destructive. To address these limitations, a GASP has been developed as a more efficient alternative. In a GASP, items are divided into groups, and decisions regarding lot acceptance are based on the collective performance of each group rather than individual items. This approach enables multiple items to be tested simultaneously using a single testing device, significantly reducing testing time, cost, and labor. Moreover, GASPs enhance inspection accuracy, energy efficiency, and risk management, while ensuring that products meet stringent quality standards before reaching consumers. Due to their practical advantages and flexibility under various probability distributions and censoring schemes, GASPs have gained substantial attention in recent years, particularly in the context of time-truncated life testing experiments.

The primary objectives of this study are as follows:

- Establish a sampling inspection methodology based on truncated life tests for the Darna distribution (DD).
- Identify the optimal number of groups necessary for the proposed GASP under selected parameters of the DD.
- Compute the operating characteristic values for different acceptance numbers (c). Evaluate the probability of acceptance (P^*) and its relationship with the ratio of actual mean lifetime to the specified mean lifetime (μ/μ_0).
- Demonstrate the effectiveness of the proposed plan using two real failure time data sets.

To the best of our knowledge, no prior study develops a group acceptance sampling plan for the DD, which is a versatile model capable of capturing a wide variety of failure behaviors due to its flexible structural properties. Previous works on GASP design have focused on lifetime models including the generalized Rayleigh [21], Marshall–Olkin Kumaraswamy exponential [22], Weibull [23], Pareto [24], and others, each offering varying degrees of fit and complexity. However, the statistical properties and sampling plan parameters for a GASP under the Darna model have not been investigated, despite its demonstrable flexibility and applicability to reliability problems where conventional models prove inadequate. Our study fills this important gap by establishing the GASP framework for the DD, detailing the derivation of minimum group sizes, operating characteristic (OC) curves, and illustrating its performance via real data application. This work thereby extends the acceptance sampling design toolkit for reliability practitioners, especially in cases where heterogeneous or atypical failure patterns are observed in the field.

The remainder of this article is organized as follows: Section 2 discusses the significance of the DD and presents its main properties. In Section 3, we develop the GASP based on the DD and outline its implementation procedure. Section 3 also provides detailed descriptions of all the presented tables, while Section 4 demonstrates the application of the proposed plan using real data. Finally, Section 5 offers concluding remarks and insights on the developed GASP.

2. Darna distribution

This section provides a description of the DD which was introduced by [25]. The DD is a special mixture of well-known distributions, $Exp(\frac{\theta}{\lambda})$ and $Gamma(3, \frac{\theta}{\lambda})$ with a mixing factor $\frac{2\lambda^2}{2\lambda^2+\theta^2}$. The DD is quite flexible and may be well-suited to many real data. [25] has derived several properties of the DD and also, they have discussed the methods of estimating the DD parameters. The probability density function (PDF) and cumulative distribution function (CDF) of two parameters DD are respectively given by:

$$f(x;\lambda,\theta) = \frac{\theta}{2\lambda^2+\theta^2}\left(2\lambda + \frac{\theta^4 x^2}{2\lambda^3}\right)e^{\frac{-\theta x}{\lambda}}; \quad x>0, \lambda>0, \theta>0, \theta>\lambda, \theta\neq\lambda, \tag{2.1}$$

and

$$F(x;\lambda,\theta) = 1 - \left(\frac{4\lambda^4 + 2\lambda^2\theta^2 + \theta^4x^2 + 2\lambda\theta^3x}{2\lambda^2(2\lambda^2+\theta^2)}\right)e^{\frac{-\theta x}{\lambda}}; \quad x > 0, \lambda > 0, \theta > 0, \tag{2.2}$$

where λ and θ are the shape and scale parameters of the DD, respectively.

The mean of the DD is defined as:

$$\mu = E(X) = \frac{\lambda}{2\theta}\frac{(4\lambda^2+6\theta^2)}{(2\lambda^2+\theta^2)}. \tag{2.3}$$

Some plots of the DD PDF are given in Figure 1, which reveals that the distribution has many shapes including decreasing and increasing-decreasing.

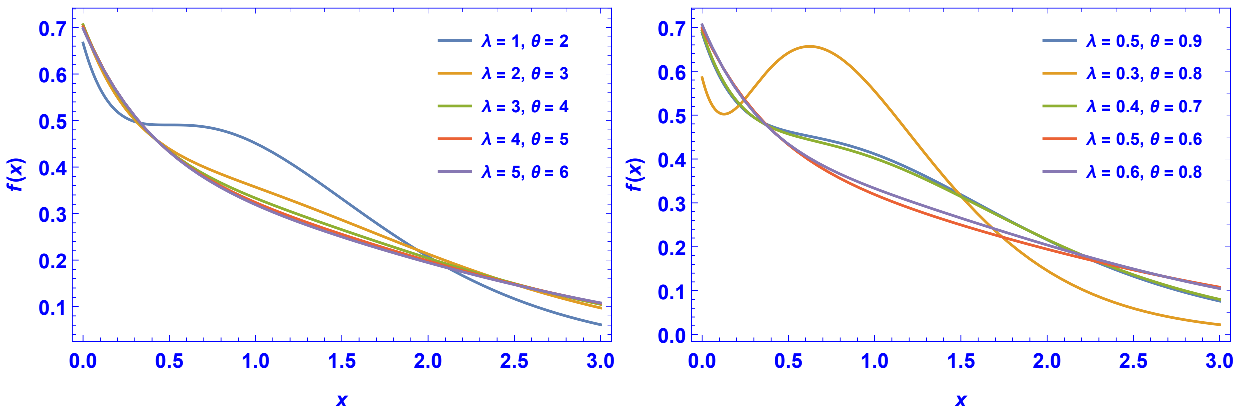


Figure 1. Plots of the DD PDF for some values of λ and θ.

The hazard rate function (HF) is a most valuable property to justify the model selection question. For the DD, the HF reliability function (RF) is given by:

$$HRF(x,\lambda,\theta) = \frac{2\theta\lambda^2\left(2\lambda + \frac{\theta^4x^2}{2\lambda^3}\right)}{2\theta^2\lambda^2 + 4\lambda^4 + \theta^4x^2 + 2\theta^3\lambda x},$$

and

$$RF(x,\lambda,\theta) = \frac{e^{-\frac{\theta x}{\lambda}}\left(2\theta^2\lambda^2 + 4\lambda^4 + \theta^4x^2 + 2\theta^3\lambda x\right)}{2\lambda^2\left(\theta^2+2\lambda^2\right)}.$$

Figure 2 shows the hazard rate and reliability function plots for some values of λ and θ.

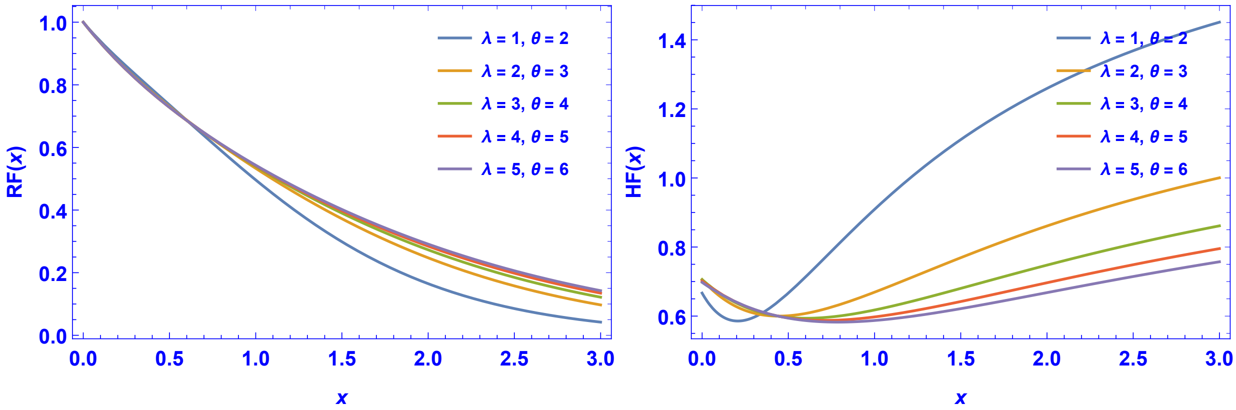


Figure 2. Plots of the hazard rate and reliability functions for some parameter values.

The DD generalizes both exponential and gamma distributions, and is able to represent increasing, decreasing, and bathtub hazard rates. This flexibility allows the DD to closely mimic real-world failure behavior, making it a suitable candidate for truncated life tests.

Due to its importance, several researchers have investigated the DD, proposed its modifications, and applied it in various studies. For instance, [26] introduced an improved attribute chain sampling plan based on the DD. [27] proposed the power DD under a right-censoring scheme. Additionally, [28] suggested the length-biased DD to better model lifetime data in specific contexts.

3. Suggested GASP

In this section, we present the mathematical formulation and procedure of the proposed GASP. In general, the GASP depends on the group size r and the number of groups g. The procedure under a time-truncated life test scheme is outlined as follows:

- Select the number of groups g and assign r items to each group, resulting in a total sample size of $n = rg$.
- Specify an acceptance number c, which denotes the maximum allowable number of failures within each group.
- Conduct the life test on the g groups simultaneously, up to a predetermined termination time t_0.
- Accept the lot if the total number of failures across all groups does not exceed c; otherwise, reject the lot.

3.1. Minimum number of groups

The binomial distribution is a useful tool for developing the GASP when sampling from a large lot. To determine the appropriate number of groups g, the acceptance number c, and the termination time t_0, specific values must be provided. The minimum number of groups can be determined using the following inequality:

$$\left(\sum_{i=0}^{c}\binom{r}{i}p^{i}(1-p)^{(r-i)}\right)^{g} \leq 1-P^{*}, \tag{3.1}$$

where p is the probability that an item will fail before or up to time t_0, i.e., p is the CDF of the DD given in Eq (2.1) and p can be modified to be written in terms of the termination ratio ($a = \frac{t_0}{\mu_0}$) and quality ratio ($b = \frac{\mu}{\mu_0}$), where P^* is the confidence level and $\beta = 1 - P^*$.

Equation (3.1) ensures the producer's risk (Type I error) is controlled; it is derived by considering the probability that no more than c failures occur in each group, each governed by the DD CDF, and using the binomial structure over g groups. For the determination of g, just replace the value of p defined in Eq (2.2) in the inequality given in Eq (3.1) for the fixed value of group size r.

The minimum number of group values are calculated for $(\lambda, \theta) = (0.5,0.5)$, (1,1.5), (1.5,1) corresponding to group size $r = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$, $t/\mu_0 = 0.75$, 1.25, 1.75, 2.25, 2.5, 3, 3.5, 4, and acceptance number $c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$. The obtained minimum number of groups that satisfies Eq (3.1) for specified $P^* = 0.75, 0.90, 0.95, 0.99$ are recorded in Tables 1–3 for the combinations ($\theta = 0.5$,$\lambda = 0.5$), ($\theta = 1$,$\lambda = 1.5$), ($\theta = 1.5$,$\lambda = 1$), respectively.

Table 1. Minimum number of groups for the proposed plan when $\theta = 0.5$, $\lambda = 0.5$.

P^*	r	c	t/μ_0							
			0.75	1.25	1.75	2.25	2.5	3	3.5	4
0.75	2	0	1	1	1	1	1	1	1	1
	3	1	2	1	1	1	1	1	1	1
	4	2	4	2	1	1	1	1	1	1
	5	3	6	2	1	1	1	1	1	1
	6	4	11	3	2	1	1	1	1	1
	7	5	18	4	2	1	1	1	1	1
	8	6	31	5	2	1	1	1	1	1
	9	7	54	7	3	2	1	1	1	1
	10	8	94	9	3	2	1	1	1	1
	11	9	167	12	3	2	1	1	1	1
	12	10	296	16	4	2	2	1	1	1
0.90	2	0	2	1	1	1	1	1	1	1
	3	1	4	2	1	1	1	1	1	1
	4	2	6	3	2	1	1	1	1	1
	5	3	10	4	2	1	1	1	1	1
	6	4	17	5	2	2	1	1	1	1
	7	5	29	6	3	2	2	1	1	1
	8	6	51	8	3	2	2	1	1	1
	9	7	89	11	4	2	2	1	1	1
	10	8	156	15	5	2	2	1	1	1
	11	9	276	20	5	3	2	1	1	1
	12	10	492	27	6	3	2	2	1	1
0.95	2	0	3	2	1	1	1	1	1	1
	3	1	4	2	2	1	1	1	1	1
	4	2	8	3	2	2	1	1	1	1
	5	3	13	5	3	2	2	1	1	1
	6	4	22	6	3	2	2	1	1	1
	7	5	38	8	4	2	2	1	1	1
	8	6	66	11	4	2	2	1	1	1
	9	7	116	14	5	3	2	2	1	1
	10	8	203	19	6	3	2	2	1	1
	11	9	359	26	7	3	3	2	1	1
	12	10	640	35	8	4	3	2	1	1
0.99	2	0	4	2	2	2	1	1	1	1
	3	1	7	4	2	2	2	1	1	1
	4	2	11	5	3	2	2	2	1	1
	5	3	20	7	4	2	2	2	1	1
	6	4	34	9	4	3	2	2	1	1
	7	5	58	12	5	3	3	2	2	1
	8	6	102	16	6	3	3	2	2	1
	9	7	177	22	7	4	3	2	2	1
	10	8	312	29	9	4	3	2	2	1
	11	9	552	39	10	5	4	2	2	1
	12	10	983	53	12	5	4	3	2	2

Table 2. Minimum number of groups for the proposed plan when $\lambda = 1$, $\theta = 1.5$.

P^*	r	c	t/μ_0							
			0.75	1.25	1.75	2.25	2.5	3	3.5	4
0.75	2	0	2	1	1	1	1	1	1	1
	3	1	3	1	1	1	1	1	1	1
	4	2	5	2	1	1	1	1	1	1
	5	3	8	3	1	1	1	1	1	1
	6	4	15	3	2	1	1	1	1	1
	7	5	28	4	2	1	1	1	1	1
	8	6	53	6	2	1	1	1	1	1
	9	7	99	8	2	1	1	1	1	1
	10	8	189	11	3	1	1	1	1	1
	11	9	363	14	3	2	1	1	1	1
	12	10	702	19	4	2	1	1	1	1
0.90	2	0	2	1	1	1	1	1	1	1
	3	1	4	2	1	1	1	1	1	1
	4	2	8	3	2	1	1	1	1	1
	5	3	14	4	2	1	1	1	1	1
	6	4	25	5	2	1	1	1	1	1
	7	5	46	7	3	2	1	1	1	1
	8	6	87	9	3	2	1	1	1	1
	9	7	165	13	4	2	2	1	1	1
	10	8	314	17	4	2	2	1	1	1
	11	9	603	23	5	2	2	1	1	1
	12	10	1166	32	6	2	2	1	1	1
0.95	2	0	3	2	1	1	1	1	1	1
	3	1	5	3	2	1	1	1	1	1
	4	2	10	4	2	1	1	1	1	1
	5	3	18	5	2	2	1	1	1	1
	6	4	32	7	3	2	1	1	1	1
	7	5	60	9	3	2	2	1	1	1
	8	6	113	12	4	2	2	1	1	1
	9	7	214	16	5	2	2	1	1	1
	10	8	408	22	5	2	2	1	1	1
	11	9	784	30	6	3	2	1	1	1
	12	10	1517	41	7	3	2	1	1	1
0.99	2	0	4	2	2	1	1	1	1	1
	3	1	8	4	2	2	2	1	1	1
	4	2	15	5	3	2	2	1	1	1
	5	3	27	7	3	2	2	1	1	1
	6	4	50	10	4	2	2	2	1	1
	7	5	92	13	5	3	2	2	1	1
	8	6	174	18	6	3	2	2	1	1
	9	7	329	25	7	3	3	2	1	1
	10	8	627	34	8	4	3	2	1	1
	11	9	1205	46	9	4	3	2	1	1
	12	10	2332	63	11	4	3	2	2	1

Table 3. Minimum number of groups for the proposed plan when $\lambda = 1.5$, $\theta = 1$.

P^*	r	c	t/μ_0							
			0.75	1.25	1.75	2.25	2.5	3	3.5	4
0.75	2	0	1	1	1	1	1	1	1	1
	3	1	2	1	1	1	1	1	1	1
	4	2	3	2	1	1	1	1	1	1
	5	3	6	2	1	1	1	1	1	1
	6	4	9	3	2	1	1	1	1	1
	7	5	15	4	2	1	1	1	1	1
	8	6	25	5	2	1	1	1	1	1
	9	7	41	6	2	2	1	1	1	1
	10	8	70	8	3	2	1	1	1	1
	11	9	119	10	3	2	2	1	1	1
	12	10	203	13	4	2	2	1	1	1
0.90	2	0	2	1	1	1	1	1	1	1
	3	1	3	2	1	1	1	1	1	1
	4	2	5	3	2	1	1	1	1	1
	5	3	9	3	2	2	1	1	1	1
	6	4	15	4	2	2	1	1	1	1
	7	5	24	6	3	2	2	1	1	1
	8	6	40	7	3	2	2	1	1	1
	9	7	68	10	4	2	2	1	1	1
	10	8	115	13	4	2	2	2	1	1
	11	9	197	17	5	3	2	2	1	1
	12	10	337	22	6	3	2	2	2	1
0.95	2	0	2	2	1	1	1	1	1	1
	3	1	4	2	2	1	1	1	1	1
	4	2	7	3	2	2	1	1	1	1
	5	3	11	4	2	2	2	1	1	1
	6	4	19	6	3	2	2	1	1	1
	7	5	31	7	3	2	2	2	1	1
	8	6	52	9	4	3	2	2	1	1
	9	7	88	12	5	3	2	2	1	1
	10	8	150	16	6	3	2	2	1	1
	11	9	256	22	7	3	3	2	1	1
	12	10	439	28	8	4	3	2	2	1
0.99	2	0	3	2	2	2	1	1	1	1
	3	1	6	3	2	2	2	1	1	1
	4	2	10	5	3	2	2	2	1	1
	5	3	17	6	4	3	2	2	2	1
	6	4	29	8	4	3	2	2	2	1
	7	5	48	11	5	3	3	2	2	1
	8	6	80	14	6	4	3	2	2	2
	9	7	135	19	7	4	3	2	2	2
	10	8	230	25	8	4	4	3	2	2
	11	9	393	33	10	5	4	3	2	2
	12	10	674	43	12	5	4	3	2	2

3.1.1. Description of Tables 1–3

In this section, we discussed the computed tables of the proposed plan for the DD. We will illustrate the results in Table 1, and the same thing can be concluded based on Tables 1–3. From Table 1, we can get the following observations:

- As the ratio t/μ_0 increases, the minimum number of groups required significantly decreases.

- For small values of t/μ_0 (e.g., 0.75), a large number of groups is needed, while for larger values of t/μ_0 (e.g., 2.25 and above), the minimum number of groups tends to stabilize at 1, indicating that increasing the test time improves the plan efficiency.

- For a fixed P^* and t/μ_0, increasing r (number of items in a group) reduces the number of groups required. However, after a certain value of r (depending on t/μ_0), the number of groups required becomes 1 and does not further reduce.

- When the consumer's confidence level (P^*) is 0.75: For $t/\mu_0 = 0.75$ with $r = 2$ and $r = 8$, only 1 and 31 groups are required, showing the sensitivity of the plan to t/μ_0 when the test time is low. But as t/μ_0 increases beyond 2.5, only 1 group is sufficient for all values of r considered.

- The acceptance number c increases with r, indicating that higher sample sizes in a group allow for higher permissible failures. However, despite increasing c, the number of groups can still be high if t/μ_0 is low.

Figure 3 illustrate the behavior of the suggested GASP in terms of the minimum number of groups for $P^* = 0.75, 0.90, 0.95, 0.99$, $r = 2, 4, ..., 12$ with $\theta = 0.5$ and $\lambda = 0.5$. It is clear that all curves are decreasing for all values of P^*.

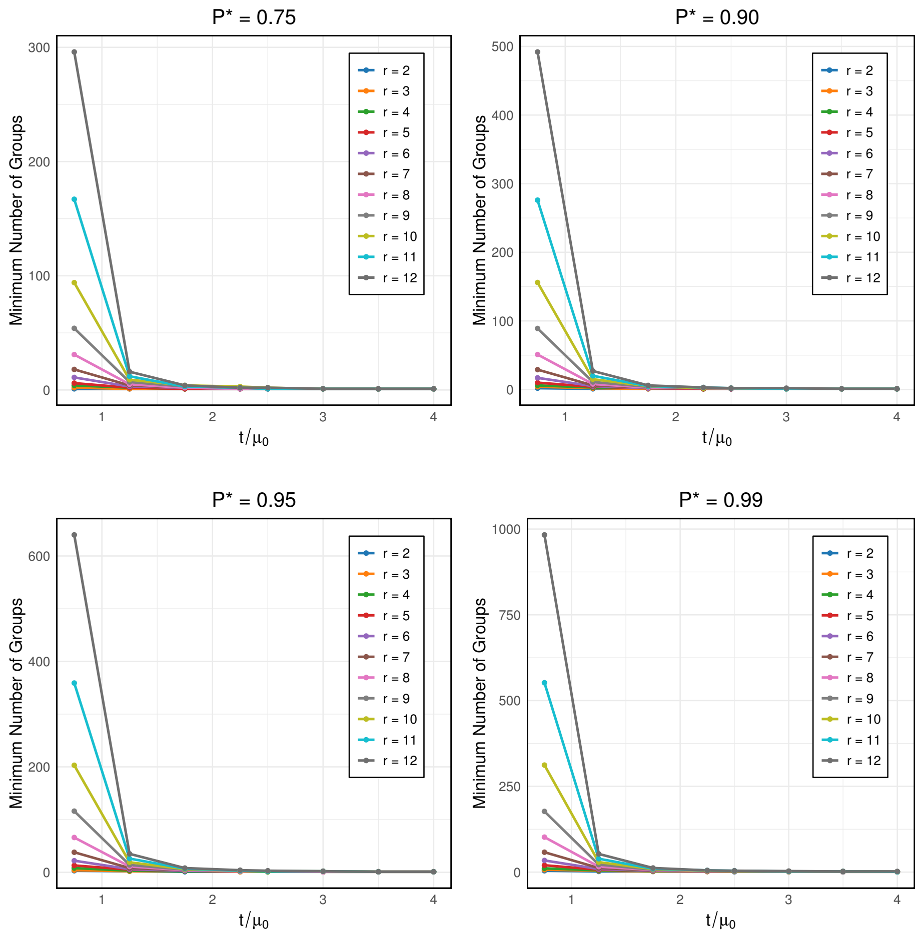


Figure 3. Minimum number of groups for $P^* = 0.75, 0.90, 0.95, 0.99$, $r = 2, 4, ..., 12$ with $\theta = 0.5$ and $\lambda = 0.5$.

3.2. Operating characteristic of the sampling plan

The acceptance probability depends on how much the given mean value μ_0 deviates from its true value μ. This relationship is described by the operational characteristic function (OC) of the sampling plan. Once the minimum number of groups g is determined, the researcher may focus on estimating the probability of accepting a lot when the product quality is exceptionally high. The product is deemed satisfactory if $\mu \geq \mu_0 \left(\frac{\mu}{\mu_0} \geq 1\right)$, indicating that the quality measure a surpasses the threshold b, which represents the minimum acceptable standard. The OC is given by:

$$L(P) = \left(\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{(r-i)}\right)^g. \tag{3.2}$$

The OC values are calculated for (λ, θ) =(0.5,0.5), (1,1.5), (1.5,1) corresponding to group size $r = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$, $t/\mu_0 = 0.75,\ 1.25,\ 1.75,\ 2.25,\ 2.5,\ 3,\ 3.5,\ 4$, and acceptance number $c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, with $P^* = 0.75, 0.90, 0.95, 0.99$ recorded in Tables 4–7 for the combinations ($\theta = 0.5$,$\lambda = 0.5$), ($\theta = 1$,$\lambda = 1.5$), ($\theta = 1.5$,$\lambda = 1$), respectively.

3.2.1. Description of Tables 4–7

Here, are some observations based on the OC values presented in tables:

- The OC values increase with increasing μ/μ_0; that is, for each combination of P^*, t/μ_0, and g, the OC value increases as the ratio μ/μ_0 increases from 2 to 9. This is expected because as the process mean increases relative to the standard (i.e., the process improves), the probability of accepting the lot increases. For example, $\theta = 0.5, \alpha = 0.5$, $P^* = 0.90$, $t/\mu_0 = 0.75$, $r = 5$, and $c = 3$, and then the required number of groups is 10, i.e., the total sample size is 50 and the experimenter will allocate 5 items to 10 groups and run the experiment simultaneously for all 10 groups. If number of failed units in each of the 10 groups is less than or equal to 3, then the experimenter will accept the lot with a certain specified mean. The OC value for this considered example from Table 4 is 0.9557225 when μ/μ_0 is 4, i.e., the probability of accepting the lot is 0.9557225 corresponding to $g = 10$, $P^* = 0.90$, and $c = 3$.

- Higher P^* implies a stricter plan, whereas P^* increases from 0.75 to 0.99, and the OC values for lower μ/μ_0 ratios generally decrease. This means the sampling plan becomes more stringent, allowing fewer bad lots to be accepted. For a fixed P^*, as t/μ_0 increases (i.e., more time is allowed), the number of groups g needed decreases. This makes sense because longer observation time allows for better estimation with fewer groups.

- For larger μ/μ_0 (e.g., 8 and 9), OC values across all P^* levels become very close to 1, implying that almost all lots are accepted when the process quality is good.

- Plans with lower g are more economical. For instance, when $P^* = 0.75$ and $t/\mu_0 = 1.75$, only 1 group is needed, yet OC values are still quite high at higher μ/μ_0 values. This balance between sample size and detection power is crucial in industry. The increase in OC from $\mu/\mu_0 = 8$ to 9 is relatively small (e.g., from 0.9951 to 0.9967 in some cases), showing diminishing improvement in acceptance probability at high quality levels.

Figure 4 represents the OC plots for $P^* = 0.75, 0.90, 0.95, 0.99$, $r = 5$, and $c = 3$ for a given P^* with $\theta = 0.5$, $\lambda = 0.5$. The results in the figure support the above concluded remarks.

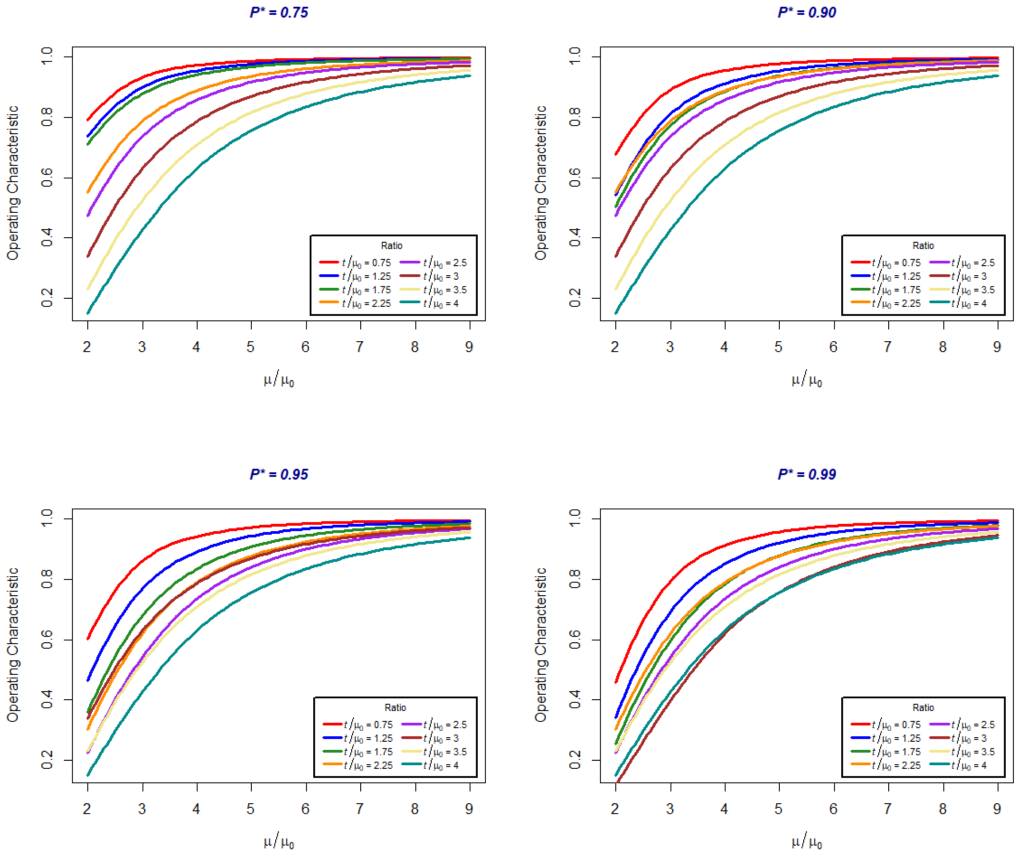


Figure 4. The OC plots for $P^* = 0.75, 0.90, 0.95, 0.99$, $r = 5$ and $c = 3$ for a given P^* with $\theta = 0.5$, $\lambda = 0.5$.

Table 4. OC values of the sampling plan of $r = 5$ and $c = 3$ for a given P^* with $\theta = 0.5$, $\lambda = 0.5$.

P^*	t/μ_0	g	μ/μ_0							
			2	3	4	5	6	7	8	9
0.75	0.75	6	0.7911667	0.9332902	0.9731933	0.9873162	0.9932604	0.9960991	0.9975895	0.9984317
	1.25	2	0.7356031	0.9001411	0.9553026	0.9772498	0.9872634	0.9923339	0.9951170	0.9967452
	1.75	1	0.7099375	0.8789491	0.9415559	0.9685304	0.9816134	0.9885594	0.9925173	0.9949038
	2.25	1	0.5503537	0.7871649	0.8890795	0.9370098	0.9617112	0.9754338	0.9835343	0.9885594
	2.5	1	0.4739482	0.7361749	0.8576731	0.9172062	0.9487577	0.9666559	0.9773958	0.9841470
	3.0	1	0.3376654	0.6298953	0.7871649	0.8705954	0.9172062	0.9447004	0.9617112	0.9726732
	3.5	1	0.2292169	0.5244268	0.7099375	0.8164088	0.8789491	0.9172062	0.9415559	0.9576117
	4.0	1	0.1491217	0.4257407	0.6298953	0.7568502	0.8351765	0.8847814	0.9172062	0.9390509
0.90	0.75	10	0.6767786	0.8913081	0.9557225	0.9789499	0.9887926	0.9935069	0.9959858	0.9973876
	1.25	4	0.5411119	0.8102540	0.9126030	0.9550171	0.9746890	0.9847266	0.9902578	0.9935011
	1.75	2	0.5040113	0.7725516	0.8865276	0.9380512	0.9635649	0.9772498	0.9850906	0.9898336
	2.25	1	0.5503537	0.7871649	0.8890795	0.9370098	0.9617112	0.9754338	0.9835343	0.9885594
	2.5	1	0.4739482	0.7361749	0.8576731	0.9172062	0.9487577	0.9666559	0.9773958	0.9841470
	3.0	1	0.3376654	0.6298953	0.7871649	0.8705954	0.9172062	0.9447004	0.9617112	0.9726732
	3.5	1	0.2292169	0.5244268	0.7099375	0.8164088	0.8789491	0.9172062	0.9415559	0.9576117
	4.0	1	0.1491217	0.4257407	0.6298953	0.7568502	0.8351765	0.8847814	0.9172062	0.9390509
0.95	0.75	13	0.6019780	0.8610656	0.9428256	0.9727217	0.9854550	0.9915672	0.9947846	0.9966052
	1.25	5	0.4640971	0.7687347	0.8919743	0.9440912	0.9684620	0.9809449	0.9878371	0.9918830
	1.75	3	0.3578165	0.6790335	0.8347153	0.9085311	0.9458482	0.9660695	0.9777194	0.9847892
	2.25	2	0.3028892	0.6196286	0.7904623	0.8779873	0.9248884	0.9514711	0.9673398	0.9772498
	2.5	2	0.2246269	0.5419534	0.7356031	0.8412672	0.9001411	0.9344237	0.9553026	0.9685454
	3.0	1	0.3376654	0.6298953	0.7871649	0.8705954	0.9172062	0.9447004	0.9617112	0.9726732
	3.5	1	0.2292169	0.5244268	0.7099375	0.8164088	0.8789491	0.9172062	0.9415559	0.9576117
	4.0	1	0.1491217	0.4257407	0.6298953	0.7568502	0.8351765	0.8847814	0.9172062	0.9390509
0.99	0.75	20	0.4580293	0.7944302	0.9134056	0.9583429	0.9777109	0.9870560	0.9919876	0.9947820
	1.25	7	0.3413913	0.6919697	0.8521054	0.9226129	0.9561271	0.9734249	0.9830135	0.9886546
	1.75	4	0.2540273	0.5968360	0.7859312	0.8799400	0.9284573	0.9550171	0.9704035	0.9797706
	2.25	2	0.3028892	0.6196286	0.7904623	0.8779873	0.9248884	0.9514711	0.9673398	0.9772498
	2.5	2	0.2246269	0.5419534	0.7356031	0.8412672	0.9001411	0.9344237	0.9553026	0.9685454
	3.0	2	0.1140179	0.3967681	0.6196286	0.7579364	0.8412672	0.8924588	0.9248884	0.9460931
	3.5	1	0.2292169	0.5244268	0.7099375	0.8164088	0.8789491	0.9172062	0.9415559	0.9576117
	4.0	1	0.1491217	0.4257407	0.6298953	0.7568502	0.8351765	0.8847814	0.9172062	0.9390509

Table 5. OC values of the sampling plan of $r = 5$ and $c = 3$ for a given P^* with $\theta = 1.5$, $\lambda = 1$.

P^*	t/μ_0	g	μ/μ_0							
			2	3	4	5	6	7	8	9
0.75	0.75	8	0.8324639	0.9498441	0.9801912	0.9906624	0.9950354	0.9971206	0.9982163	0.9988367
	1.25	3	0.7319402	0.9097027	0.9615936	0.9808885	0.9894074	0.9936509	0.9959610	0.9973072
	1.75	1	0.7646462	0.9182988	0.9641350	0.9816188	0.9895426	0.9935885	0.9958409	0.9971801
	2.25	1	0.5900788	0.8396780	0.9261776	0.9610459	0.9773400	0.9858479	0.9906731	0.9935885
	2.5	1	0.5001442	0.7909527	0.9012083	0.9471369	0.9689466	0.9804525	0.9870304	0.9910318
	3.0	1	0.3359557	0.6798702	0.8396780	0.9116751	0.9471369	0.9662483	0.9773400	0.9841693
	3.5	1	0.2082050	0.5598869	0.7646462	0.8660705	0.9182988	0.9471369	0.9641350	0.9747208
	4.0	1	0.1204877	0.4422671	0.6798702	0.8111061	0.8823675	0.9228556	0.9471369	0.9624377
0.9	0.75	14	0.7255034	0.9138851	0.9655925	0.9837165	0.9913282	0.9949664	0.9968807	0.9979651
	1.25	4	0.6596306	0.8814533	0.9491222	0.9745995	0.9859016	0.9915436	0.9946183	0.9964113
	1.75	2	0.5846838	0.8432727	0.9295563	0.9635755	0.9791946	0.9872180	0.9916992	0.9943682
	2.25	1	0.5900788	0.8396780	0.9261776	0.9610459	0.9773400	0.9858479	0.9906731	0.9935885
	2.5	1	0.5001442	0.7909527	0.9012083	0.9471369	0.9689466	0.9804525	0.9870304	0.9910318
	3.0	1	0.3359557	0.6798702	0.8396780	0.9116751	0.9471369	0.9662483	0.9773400	0.9841693
	3.5	1	0.2082050	0.5598869	0.7646462	0.8660705	0.9182988	0.9471369	0.9641350	0.9747208
	4.0	1	0.1204877	0.4422671	0.6798702	0.8111061	0.8823675	0.9228556	0.9471369	0.9624377
0.95	0.75	18	0.6619454	0.8906719	0.9559810	0.9791129	0.9888643	0.9935329	0.9959912	0.9973844
	1.25	5	0.5944646	0.8540812	0.9368124	0.9683508	0.9824082	0.9894406	0.9932774	0.9955161
	1.75	2	0.5846838	0.8432727	0.9295563	0.9635755	0.9791946	0.9872180	0.9916992	0.9943682
	2.25	2	0.3481930	0.7050591	0.8578049	0.9236092	0.9551935	0.9718960	0.9814331	0.9872180
	2.5	1	0.5001442	0.7909527	0.9012083	0.9471369	0.9689466	0.9804525	0.9870304	0.9910318
	3.0	1	0.3359557	0.6798702	0.8396780	0.9116751	0.9471369	0.9662483	0.9773400	0.9841693
	3.5	1	0.2082050	0.5598869	0.7646462	0.8660705	0.9182988	0.9471369	0.9641350	0.9747208
	4.0	1	0.1204877	0.4422671	0.6798702	0.8111061	0.8823675	0.9228556	0.9471369	0.9624377
0.99	0.75	27	0.5385590	0.8405753	0.9347035	0.9688336	0.9833431	0.9903151	0.9939929	0.9960792
	1.25	7	0.4828101	0.8018605	0.9126698	0.9559734	0.9754584	0.9852482	0.9906010	0.9937282
	1.75	3	0.4470762	0.7743764	0.8962178	0.9458638	0.9689548	0.9808885	0.9875746	0.9915643
	2.25	2	0.3481930	0.7050591	0.8578049	0.9236092	0.9551935	0.9718960	0.9814331	0.9872180
	2.5	2	0.2501442	0.6256062	0.8121764	0.8970684	0.9388575	0.9612871	0.9742290	0.9821440
	3.0	1	0.3359557	0.6798702	0.8396780	0.9116751	0.9471369	0.9662483	0.9773400	0.9841693
	3.5	1	0.2082050	0.5598869	0.7646462	0.8660705	0.9182988	0.9471369	0.9641350	0.9747208
	4.0	1	0.1204877	0.4422671	0.6798702	0.8111061	0.8823675	0.9228556	0.9471369	0.9624377

Table 6. OC values of the sampling plan of $r = 5$, and $c = 3$ for a given P^* with $\theta = 1$, $\lambda = 1.5$.

P^*	μ/μ_0	g	t/μ_0							
			2	3	4	5	6	7	8	9
0.75	0.75	6	0.7643871	0.9253802	0.9704252	0.9861663	0.9927153	0.9958133	0.9974276	0.9983341
	1.25	2	0.7010514	0.8858973	0.9493302	0.9744811	0.9858530	0.9915570	0.9946607	0.9964629
	1.75	1	0.6772197	0.8612518	0.9328982	0.9640964	0.9791935	0.9871581	0.9916629	0.9943595
	2.25	1	0.5164053	0.7594942	0.8727367	0.9276498	0.9562077	0.9720723	0.9814015	0.9871581
	2.5	1	0.4433104	0.7048300	0.8372881	0.9048574	0.9412212	0.9619285	0.9743358	0.9821050
	3.0	1	0.3179355	0.5950859	0.7594942	0.8518170	0.9048574	0.9365310	0.9562077	0.9688846
	3.5	1	0.2213085	0.4913440	0.6772197	0.7914511	0.8612518	0.9048574	0.9328982	0.9514639
	4.0	1	0.1503255	0.3983022	0.5950859	0.7268340	0.8121917	0.8678582	0.9048574	0.9300056
0.9	0.75	9	0.6682978	0.8901851	0.9559675	0.9793213	0.9890928	0.9937265	0.9961438	0.9975022
	1.25	3	0.5869820	0.8338253	0.9249663	0.9619668	0.9788548	0.9873623	0.9920018	0.9946990
	1.75	2	0.4586265	0.7417547	0.8702990	0.9294818	0.9588200	0.9744811	0.9833952	0.9887507
	2.25	2	0.2666744	0.5768315	0.7616694	0.8605342	0.9143331	0.9449246	0.9631489	0.9744811
	2.5	1	0.4433104	0.7048300	0.8372881	0.9048574	0.9412212	0.9619285	0.9743358	0.9821050
	3.0	1	0.3179355	0.5950859	0.7594942	0.8518170	0.9048574	0.9365310	0.9562077	0.9688846
	3.5	1	0.2213085	0.4913440	0.6772197	0.7914511	0.8612518	0.9048574	0.9328982	0.9514639
	4.0	1	0.1503255	0.3983022	0.5950859	0.7268340	0.8121917	0.8678582	0.9048574	0.9300056
0.95	0.75	11	0.6110468	0.8674685	0.9464489	0.9747845	0.9866852	0.9923378	0.9952889	0.9969480
	1.25	4	0.4914731	0.7848140	0.9012278	0.9496133	0.9719062	0.9831853	0.9893500	0.9929382
	1.75	2	0.4586265	0.7417547	0.8702990	0.9294818	0.9588200	0.9744811	0.9833952	0.9887507
	2.25	2	0.2666744	0.5768315	0.7616694	0.8605342	0.9143331	0.9449246	0.9631489	0.9744811
	2.5	2	0.1965241	0.4967854	0.7010514	0.8187669	0.8858973	0.9253065	0.9493302	0.9645302
	3.0	1	0.3179355	0.5950859	0.7594942	0.8518170	0.9048574	0.9365310	0.9562077	0.9688846
	3.5	1	0.2213085	0.4913440	0.6772197	0.7914511	0.8612518	0.9048574	0.9328982	0.9514639
	4.0	1	0.1503255	0.3983022	0.5950859	0.7268340	0.8121917	0.8678582	0.9048574	0.9300056
0.99	0.75	17	0.4670762	0.8027382	0.9184579	0.9612996	0.9794974	0.9881832	0.9927286	0.9952871
	1.25	6	0.3445479	0.6952647	0.8555627	0.9253802	0.9581567	0.9748843	0.9840676	0.9894261
	1.75	4	0.2103382	0.5502000	0.7574203	0.8639365	0.9193358	0.9496133	0.9670662	0.9776280
	2.25	3	0.1377121	0.4381002	0.6647368	0.7982745	0.8742923	0.9185351	0.9452357	0.9619668
	2.5	2	0.1965241	0.4967854	0.7010514	0.8187669	0.8858973	0.9253065	0.9493302	0.9645302
	3.0	2	0.1010830	0.3541272	0.5768315	0.7255922	0.8187669	0.8770903	0.9143331	0.9387374
	3.5	2	0.04897747	0.24141892	0.45862646	0.62639485	0.74175469	0.81876687	0.87029899	0.90528351
	4.0	1	0.1503255	0.3983022	0.5950859	0.7268340	0.8121917	0.8678582	0.9048574	0.9300056

The proposed methodology for group acceptance sampling under the DD relies on several key assumptions and limitations that must be clearly addressed. One of the main assumptions is that the lifetime data follows the DD, which could limit its applicability if the data does not fit this distribution well. The method assumes right-censored data, and while it works for truncated life tests, it may not be suitable for other types of censoring, such as interval or progressive censoring, without further extensions. The precision of the estimations may also be compromised in cases with very low failure probabilities or small sample sizes. In such scenarios, grouping the data appropriately is crucial for improving the reliability of the results. Therefore, the authors must discuss these constraints, particularly in terms of the type of censoring, distributional fit, and sample size requirements, to ensure the methodology's broader applicability and reliability in various real-world contexts.

4. Real data applications

This section examines the practical use of the proposed group acceptance sampling plans, highlighting their role in enhancing quality control efficiency and reducing inspection costs across two data sets. These applications illustrate how ASP supports producers in maintaining quality standards, shortening inspection times, and achieving a balanced approach to risk and quality assurance. The two data sets are:

Data I: The following observations represent the failure times in minutes for a sample of 15 electronic component in accelerated life test presented in [29], and it is given by 1.4, 5.1, 6.3, 10.8, 12.1, 18.5, 19.7, 22.2, 23, 30.6, 37.3, 46.3, 53.9, 59.8, 66.2.

Data II: The data set represents the failure times (in weeks) of 50 components and, also, this mentioned data was studied by [30] in studying the Marshall-Olkin exponential power distribution and its generalization. The data are given as follows: 0.013, 0.065, 0.111, 0.111, 0.613, 0.309, 0.426, 0.535, 0.684, 0.747, 0.997, 1.284, 1.304, 1.647, 1.829, 2.336, 2.838, 3.269, 3.997, 3.981, 4.52, 4.789, 4.849, 5.202, 5.291, 5.349, 5.911, 6.018, 6.427, 6.456, 6.572, 7.023, 7.087, 7.291, 7.787, 8.596, 9.388, 10.261, 10.731, 11.658, 13.006, 13.388, 13.842, 17.152, 17.283, 19.418, 23.471, 24.777, 32.795, 48.105.

The descriptive summary for the two sets is given in Table 7. It can be seen that both sets are positive and the true population means are 41.8 and 10.043 for Data I and II, respectively.

Table 7. Descriptive summary of the considered data sets.

Data set	Minimum	Q_1	Median	Mean	Q_3	Maximum	CS	CK
I	1.400	11.450	22.200	27.550	41.800	66.200	0.5660	2.0596
II	0.013	1.390	5.320	7.831	10.043	48.105	2.3105	9.4268

The histogram-density, CDFs, and P-P plots for both data set I and III are presented in Figures 5 and 6, respectively. The shape of the hazard rate function for a dataset is determined using a graphical approach based on the total time on test (TTT) method [31] for a given sample size n, plotting $T(i/n)$ against i/n gives the empirical TTT plot, where $T(i/n)$ is

$$T(i/n) = \frac{\sum_{j=1}^{i} X_{(j)} + (n-i)X_{(i)}}{\sum_{j=1}^{n} X_{(j)}},$$

with $X_{(i)}$ as the i-th sample order statistic. Figure 7 involves the box, TTT, and violin plots of the DD based on data Set I and III.

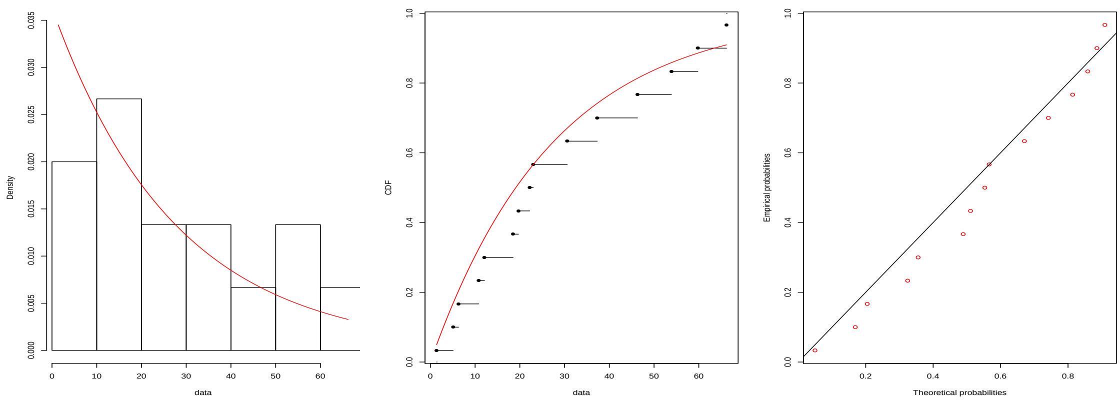


Figure 5. Histogram-density, CDFs, and P-P plots for data set I.

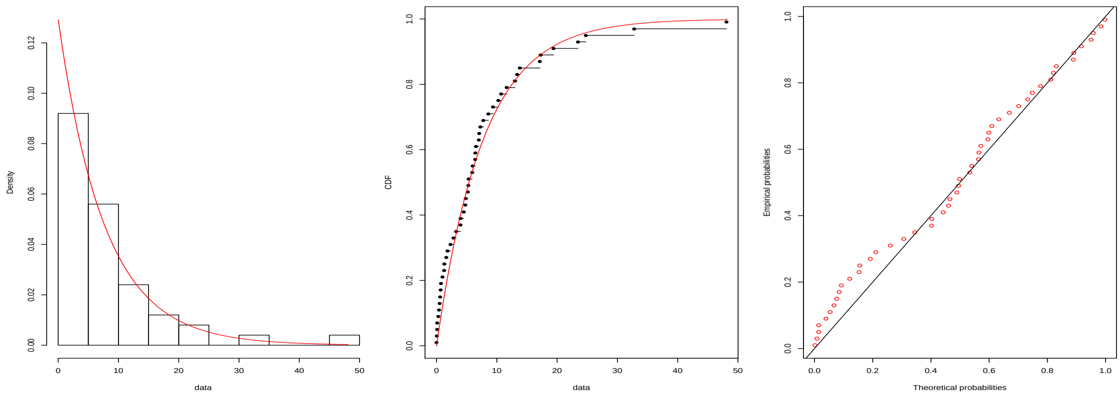
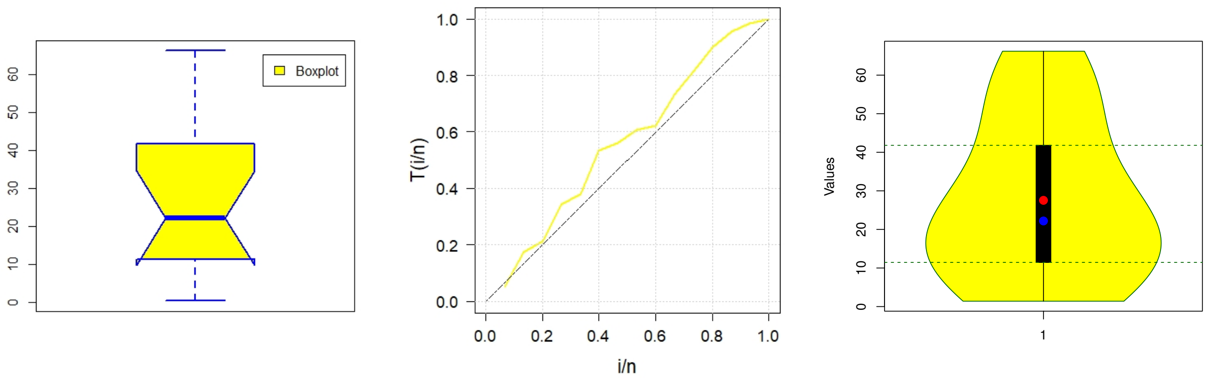
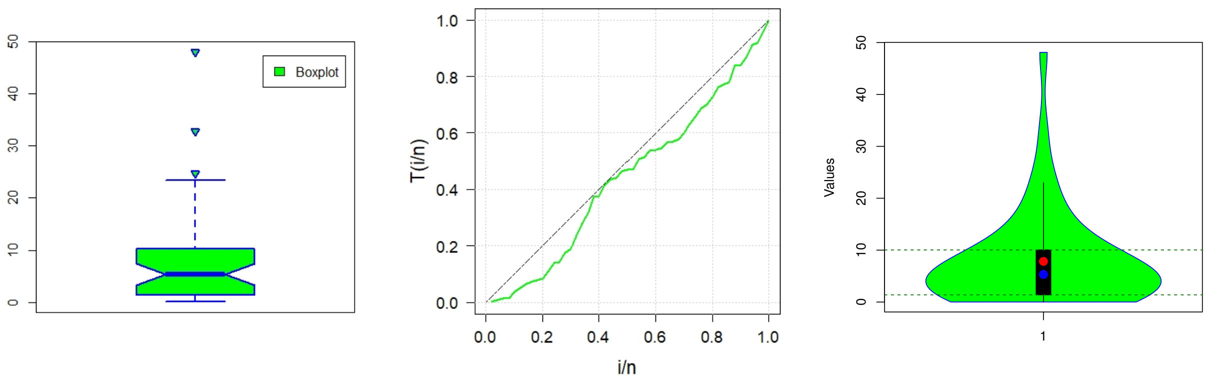


Figure 6. Histogram-density, CDFs, and P-P plots for the data set II.



(a) Box-Data I (b) TTT-Data I (c) Violin-Data I



(d) Box-Data II (e) TTT-Data II (f) Violin-Data II

Figure 7. The box, TTT, and Violin plots of the DD based on data Set I and III.

The distribution parameters are estimated using the maximum likelihood method, along with the calculation of negative maximized log-likelihood (MLL), Akaike information criterion (AIC), Consistent Akaike information criterion (CAIC), Bayesian information criterion (BIC), Hannan-Quinn information criterion (HQIC), and Kolmogorov-Smirnov (KS) test statistics. For statistical quality, the best-fitting distribution is indicated by the lowest value of K-S, AIC, BIC, CAIC, and HQIC.

To show the flexibility of the DD, the following competitors are considered:

Gamma Lindley distribution [32] (GLD) with PDF

$$f(x;\lambda,\theta)=\frac{\theta^2\left[(\lambda+\lambda\theta-\theta)x+1\right]}{\lambda(1+\theta)}\,e^{-\theta x},x,\ \lambda,\ \theta>0.$$

Area biased gamma Lindley distribution [33] (ABGLD) with PDF

$$f(x;\lambda,\theta)=\frac{\theta^4}{2\left(3\lambda(\theta+1)-2\theta\right)}\cdot x^2\cdot\left(x(\theta\lambda+\lambda-\theta)+1\right)e^{-\theta x},x>0,\ \theta>0,\ \lambda>\frac{\theta}{\theta+1}.$$

Length biased two-parameters Mirra distribution [34] (LBTPMD) with PDF

$$f(x;\lambda,\theta)=\frac{\theta^4}{3\lambda+\theta^2}\cdot x\left(1+\frac{\lambda x^2}{2}\right)e^{-\theta x},x,\ \lambda,\ \theta>0.$$

Power length biased Suja distribution [35] (PLBSD) with PDF

$$f(x;\lambda,\theta)=\frac{\lambda^6\theta\,x^{2(\theta-1)}\left(x^{4\theta}+1\right)e^{-\lambda x^{\theta}}}{\lambda^4+120},x,\ \lambda,\ \theta>0.$$

Table 8 presents the values of MLE estimators AIC, CAIC, HQIC, BIC, and K-S with corresponding p-vlaue. The results show that the DD achieves small values of these criteria, along with the highest p-value of K-S for both data sets. Therefore, it can be concluded that DD model provides a good fit to the given data sets.

Table 8. Model fitting summary of the considered data sets.

Model	Estimates	- L-L	AIC	CAIC	BIC	HQIC	K-S	P-value
Data set I								
DD	$\hat{\lambda}=2.1083,\hat{\theta}=0.0766$	64.7405	133.4811	134.4811	134.8972	133.4660	0.1558	0.8077
ABGLD	$\hat{\lambda}=0.0982,\hat{\theta}=0.1089$	67.2564	138.5129	139.5129	139.9290	138.4978	0.1863	0.6100
PLBSD	$\hat{\lambda}=1.3358,\hat{\theta}=0.4723$	108.6540	221.3080	222.3080	222.7241	221.2929	0.1568	0.7918
Data set II								
DD	$\hat{\lambda}=2.0433,\hat{\theta}=0.2666$	152.8345	309.6691	309.9244	313.4931	311.1253	0.1079	0.6556
GLD	$\hat{\lambda}=0.1133,\hat{\theta}=0.1277$	152.9031	309.8062	310.0615	313.6303	311.2624	0.1090	0.5923
ABGLD	$\hat{\lambda}=0.2770,\hat{\theta}=0.3831$	207.2416	418.4832	418.7386	422.3073	419.9395	0.2658	0.0017
LBTPMD	$\hat{\lambda}=0.0054,\hat{\theta}=0.2955$	174.3056	352.6111	352.8665	356.4352	354.0674	0.2130	0.0214
PLBSD	$\hat{\lambda}=2.4261,\hat{\theta}=0.4404$	210.8612	425.7224	425.9777	429.5464	427.1786	0.1748	0.6422

For the first data set, the lifetime follows the DD and suppose the experimenter aims to determine whether the average failure life exceeds the given mean failure life. The maximum likelihood estimates of the distribution parameters are $\lambda = 2.10832758$ and $\theta = 0.07659991$. Mean lifetime (μ_0) of the considered data set based on the estimated parameters is $\hat{\mu} = \frac{\hat{\lambda}}{2\hat{\theta}}\frac{4\hat{\lambda}^2+6\hat{\theta}^2}{2\hat{\lambda}^2+\hat{\theta}^2} = 27.5602$. Assume that the specified mean lifetime is $\mu_0 = 27.5602$, and the testing time is $t_0 = 34.45025$, which leads to the ratio $t/\mu_0 = 1.25$, with the number of testers $r = 5$. Then, from Table 9, with $P^* = 0.90$, we have the minimum number of groups as $g = 3$ and an acceptance number $c = 3$. Therefore, the process may be done by choosing $g = 3$ groups and allocating $r = 5$ testers for each group, yielding a sample size of 15 items. As the acceptance number is 3, then if no more than 3 units fail in each of the 3 groups before testing time 34.45025 is completed, the lot will be accepted and it is statistically shown that the mean failure life is longer than the determined lifetime. Otherwise, the lot is rejected. It is found that the number of failures before 34.45025 is 10 which is greater than the acceptance number $c = 3$, so the lot is rejected.

For the second data set, the lifetime follows the DD distribution and maximum likelihood estimates are $\lambda = 2.0432835$ and $\theta = 0.2666318$. Hence, the estimated mean is $\hat{\mu} = \frac{\hat{\lambda}}{2\hat{\theta}} \frac{4\hat{\lambda}^2+6\hat{\theta}^2}{2\hat{\lambda}^2+\hat{\theta}^2} = 7.792705$. Suppose the experimenter wants the termination ratio (t/μ_0) to be 1.75, and then the termination time is 13.65 when the consumer's risk is 0.10 with the number of testers $r = 8$. Then, from Table 10, the minimum number of groups is $g = 6$ with $c = 4$.

Therefore, the process may be done by choosing $g = 6$ groups and allocating $r = 8$ testers for each group, yielding a sample size of 48 items. As the acceptance number is 4, then if no more than 4 units fail in each of the 6 groups before testing time 7.792705 is completed, the lot will be accepted. Otherwise the lot is rejected. Since the number of failures is 34, the lot is rejected.

Table 9. Minimum number of groups for the proposed plan when $\hat{\theta} = 0.07659991$ and $\hat{\lambda} = 2.10832758$.

P^*	r	c	t/μ_0							
			0.75	1.25	1.75	2.25	2.5	3	3.5	4
0.75	2	0	1	1	1	1	1	1	1	1
	3	1	2	1	1	1	1	1	1	1
	4	2	4	2	1	1	1	1	1	1
	5	3	6	2	1	1	1	1	1	1
	6	4	10	3	2	1	1	1	1	1
	7	5	17	4	2	1	1	1	1	1
	8	6	28	5	2	1	1	1	1	1
0.90	2	0	2	1	1	1	1	1	1	1
	3	1	3	2	1	1	1	1	1	1
	4	2	6	3	2	1	1	1	1	1
	5	3	10	3	2	1	1	1	1	1
	6	4	16	4	2	2	1	1	1	1
	7	5	27	6	3	2	2	1	1	1
	8	6	46	7	3	2	2	1	1	1
0.95	2	0	2	2	1	1	1	1	1	1
	3	1	4	2	2	1	1	1	1	1
	4	2	7	3	2	2	1	1	1	1
	5	3	12	4	2	2	2	1	1	1
	6	4	21	6	3	2	2	1	1	1
	7	5	35	7	3	2	2	1	1	1
	8	6	60	10	4	2	2	2	1	1
0.99	2	0	4	2	2	2	1	1	1	1
	3	1	6	3	2	2	2	1	1	1
	4	2	11	5	3	2	2	2	1	1
	5	3	19	6	3	2	2	2	1	1
	6	4	32	8	4	3	2	2	2	1
	7	5	54	11	5	3	3	2	2	1
	8	6	92	14	6	3	3	2	2	1

Table 10. Minimum number of groups for the proposed plan when $\hat{\theta} = 0.2666318$ and $\hat{\lambda} = 2.0432835$.

P^*	r	c	t/μ_0							
			0.75	1.25	1.75	2.25	2.5	3	3.5	4
0.75	2	0	1	1	1	1	1	1	1	1
	3	1	2	1	1	1	1	1	1	1
	4	2	4	2	1	1	1	1	1	1
	5	3	6	2	1	1	1	1	1	1
	6	4	10	3	2	1	1	1	1	1
	7	5	16	4	2	1	1	1	1	1
	8	6	27	5	2	1	1	1	1	1
0.90	2	0	2	1	1	1	1	1	1	1
	3	1	3	2	1	1	1	1	1	1
	4	2	6	3	2	1	1	1	1	1
	5	3	9	3	2	1	1	1	1	1
	6	4	16	4	2	2	1	1	1	1
	7	5	27	6	3	2	2	1	1	1
	8	6	45	7	3	2	2	1	1	1
0.95	2	0	2	2	1	1	1	1	1	1
	3	1	4	2	2	1	1	1	1	1
	4	2	7	3	2	2	1	1	1	1
	5	3	12	4	2	2	2	1	1	1
	6	4	20	5	3	2	2	1	1	1
	7	5	35	7	3	2	2	1	1	1
	8	6	59	9	4	2	2	2	1	1
0.99	2	0	4	2	2	2	1	1	1	1
	3	1	6	3	2	2	2	1	1	1
	4	2	11	5	3	2	2	2	1	1
	5	3	18	6	3	2	2	2	1	1
	6	4	31	8	4	3	2	2	2	1
	7	5	53	11	5	3	3	2	2	1
	8	6	90	14	6	3	3	2	2	1

5. Conclusions

In summary, this study developed a comprehensive GASP tailored for the new DD under truncated life testing, addressing a tangible gap in the reliability sampling literature. The key findings demonstrate that the Darna-based GASP effectively balances producer's and consumer's risks at specified quality levels, with design parameters—such as minimum group sizes and acceptance numbers—computed to ensure the required level of discrimination between lots. OC function values further validated that the plan exhibits desirable risk properties, reinforcing its utility for practical deployment in reliability assurance scenarios.

More broadly, the proposed approach illustrates that leveraging a flexible lifetime distribution like the Darna enables the modeling of diverse hazard rate behaviors, including monotonically increasing, decreasing, and even non-monotonic (bathtub-shaped) failure rates. This versatility is consistent with the motivation behind using alternative advanced distributions in other studies, such as the exponentiated exponential, Weibull-Fréchet, and generalized gamma, where the choice of distribution directly impacts the discrimination power and parsimony of the acceptance plan.

A notable strength of the current work is its adaptability for real-world application, as demonstrated through both simulated and empirical data sets. The practical interpretation of plan parameters—specifically, how they translate to actionable lot acceptance strategies—confirms the immediate relevance for industry practitioners.

Continued work in this domain can further benefit from integrating economic design criteria—minimizing inspection and testing costs while maintaining statistical rigor—as demonstrated in recent studies utilizing economic and Bayesian perspectives. Incorporating advanced simulation techniques and robust visual assessment tools for plan comparison, as embodied in modern R packages and quality control software, will facilitate broad uptake in both academic and industrial contexts. In addition, formal investigation of model misspecification effects, parameter uncertainty, and robustness under real manufacturing and service conditions, as shown in work on the exponentiated half logistic and Pareto-type distributions, will be crucial for evolving the GASP into a universally adaptable quality control strategy.

Author contributions

Amer Ibrahim Al-Omari: Conceptualization, methodology, software, investigation, validation, writing original draft; Ehab M. Almetwally: Conceptualization, funding acquisition, methodology, investigation, validation, software, writing review and editing; Harsh Tripathi: Conceptualization, methodology, investigation, validation, data curation, writing original draft; Ahmad A. Hanandeh: Conceptualization, methodology, investigation, validation, writing original draft. All authors read and approved the final manuscript.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was supported and funded by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-DDRSP2503).

Funding statement

This work was supported and funded by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-DDRSP2503).

Conflict of interest

The authors declare that they have no conflicts of interest.

References

1. S. S. Gupta, S. S. Gupta, Gamma distribution in acceptance sampling based on life tests, *J. Am. Stat. Assoc.*, **56** (1961), 942–970. https://doi.org/10.1080/01621459.1961.10482137
2. S. S. Gupta, Life test sampling plans for normal and lognormal distributions, *Technometrics*, **4** (1962), 151–175. https://doi.org/10.1080/00401706.1962.10490002
3. M. Aslam, C. H. Jun, A group acceptance sampling plan for truncated life test having Weibull distribution, *J. Appl. Stat.*, **36** (2009), 1021–1027. https://doi.org/10.1080/02664760802566788
4. M. Aslam, D. Kundu, M. Ahmad, Time truncated acceptance sampling plans for generalized exponential distribution, *J. Appl. Stat.*, **37** (2010), 555–566. https://doi.org/10.1080/02664760902769787
5. M. Aslam, C. H. Jun, M. Ahmad, New acceptance sampling plans based on life tests for Birnbaum-Saunders distributions, *J. Stat. Comput. Simul.*, **81** (2011), 461–470. https://doi.org/10.1080/00949650903418883
6. G. S. Rao, Double acceptance sampling plans based on truncated life tests for the Marshall-Olkin extended exponential distribution, *Aust. J. Stat.*, **40** (2016), 169–176. https://doi.org/10.17713/ajs.v40i3.208
7. A. Ramaswamy, P. Anburajan, Group acceptance sampling plans using weighted binomial on truncated life tests for inverse Rayleigh and log – logistic distributions, *IOSR J. Math.*, **2** (2012), 33–38. https://doi.org/10.9790/5728-0233338
8. G. Sathya Narayanan, V. Rajarathinam, A procedure for the selection of single sampling plans by variables based on Pareto distribution, *J. Qual. Reliab. Eng.*, **2013** (2013), 808741. https://doi.org/10.1155/2013/808741
9. W. Gui, X. Lu, Double acceptance sampling plan based on the Burr type X distribution under truncated life tests, *Int. J. Ind. Syst. Eng.*, **28** (2018), 319–330. https://doi.org/10.1504/IJISE.2018.089742
10. B. Maleki Vishkaei, S. T. A. Niaki, M. Farhangi, I. Mahdavi, A single-retailer multi-supplier multi-product inventory model with destructive testing acceptance sampling and inflation, *J. Ind. Prod. Eng.*, **36** (2019), 351–361. https://doi.org/10.1080/21681015.2018.1479893
11. H. Tripathi, M. Saha, V. Alha, An application of time truncated single acceptance sampling inspection plan based on generalized half-normal distribution, *Ann. Data Sci.*, **9** (2022), 1243–1255. https://doi.org/10.1007/s40745-020-00267-z
12. A. D. Al-Nasser, M. A. ul Haq, Acceptance sampling plans from a truncated life test based on the power Lomax distribution with application to manufacturing, *Stat. Trans. New Ser.*, **22** (2021), 1–13. https://doi.org/10.21307/stattrans-2021-024
13. N. Singh, G. Singh, A. Kanwar, N. Singh, Repetitive acceptance sampling plan for lifetimes following a skew-generalized inverse Weibull distribution, *Pesqui. Oper.*, **42** (2022), 1–13. https://doi.org/10.1590/0101-7438.2022.042.00249077
14. A. Algarni, Group acceptance sampling plan based on new compounded three-parameter Weibull model, *Axioms*, **11** (2022), 438. https://doi.org/10.3390/axioms11090438

15. A. Al-Nasser, B. Y. Alhroub, Acceptance sampling plans using hyper-geometric theory for finite population under Q-Weibull distribution, *Electron. J. Appl. Stat. Anal.*, **15** (2022), 352–366. https://doi.org/10.1285/i20705948v15n2p352

16. M. Imran, L. A. Al-Essa, M. H. Tahir, C. Chesneau, S. Shakoor, F. Jamal, An extended exponential model: Estimation using ranked set sampling with applications, *J. Radiat. Res. Appl. Sci.*, **16** (2023), 100642. https://doi.org/10.1016/j.jrras.2023.100642

17. A. H. AL-Marshadi, M. Aslam, A. Alharbey, Design of double acceptance sampling plan for Weibull distribution under indeterminacy, *AIMS Mathematics*, **8** (2023), 13294–13305. https://doi.org/10.3934/math.2023672

18. N. Hussain, M. H. Tahir, F. Jamal, M. Ameeq, S. Shafiq, J. T. Mendy, An acceptance sampling plan for the odd exponential-logarithmic Fréchet distribution: Applications to quality control data, *Cogent Eng.*, **11** (2024), 2304497. https://doi.org/10.1080/23311916.2024.2304497

19. M. Abiad, M. M. Rahman, O. S. Balogun, Y. A. Tashkandy, M. E. Bakr, M. Yusuf, et al., Sequential inspection sampling plan based on Burr-XII amputated life testing with numerical illustrations and industrial applications, *AIMS Mathematics*, **10** (2025), 14917–14942. https://doi.org/10.3934/math.2025669

20. P. V. Kumarasamy, T. Ramesh, A. T. Thottathil, Acceptance-rejection criteria based on Birnbaum-Saunders lifetime distribution using intervened Poisson distribution, *Qual. Reliab. Eng. Int.*, **41** (2025), 1183–1194. https://doi.org/10.1002/qre.3719

21. T. R. Tsai, S. J. Wu, Acceptance sampling based on truncated life tests for generalized Rayleigh distribution, *J. Appl. Stat.*, **33** (2006), 595–600. https://doi.org/10.1080/02664760600679700

22. A. M. Almarashi, K. Khan, C. Chesneau, F. Jamal, Group acceptance sampling plan using Marshall-Olkin Kumaraswamy exponential (MOKw-E) distribution, *Processes*, **9** (2021), 1066. https://doi.org/10.3390/pr9061066

23. M. Kim, B. J. Yum, Reliability acceptance sampling plans for the Weibull distribution under accelerated Type-I censoring, *J. Appl. Stat.*, **36** (2009), 11–20. https://doi.org/10.1080/02664760802382483

24. M. Aslam, A. R. Mughal, M. Ahmad, Z. Yab, Group acceptance sampling plans for Pareto distribution of the second kind, *J. Test. Eval.*, **38** (2010), 143–150. https://doi.org/10.1520/JTE102426

25. D. H. Shraa, A. I. Al-Omari, Darna distribution: Properties and application, *Electron. J. Appl. Stat. Anal.*, **12** (2019), 520–541. https://doi.org/10.1285/I20705948V12N2P520

26. H. Tripathi, A. I. Al-Omari, M. Saha, A. Al-Anzi, Improved attribute chain sampling plan for Darna distribution, *Comput. Syst. Sci. Eng.*, **38** (2021), 382–392. https://doi.org/10.32604/csse.2021.015624

27. A. I. Al-Omari, K. Aidi, R. AlSultan, Power Darna distribution with right censoring: Estimation, testing and applications, *Appl. Sci.*, **12** (2022), 8272. https://doi.org/10.3390/app12168272

28. D. V. Saraja, C. Subramanian, A. A. Rather, N. S. Rao, Some contributions to Darna distribution with properties and applications in medical science, *Dickensian J.*, **22**, (2022), 2057–2072.

29. J. F. Lawless, *Statistical models and methods for lifetime data*, New York: Wiley, 2002. https://doi.org/10.1002/9781118033005
30. K. K. Jose, A. Paul, Marshall-Olkin exponential power distribution and its generalization: Theory and applications, *IAPQR Trans.*, **43** (2018), 1–29. https://doi.org/10.32381/IAPQRT.2018.43.01.1
31. M. V. Aarset, How to identify a bathtub hazard rate, *IEEE Trans. Reliab.*, **36** (1987), 106–108. https://doi.org/10.1109/TR.1987.5222310
32. H. Zakerzadeh, A. Dolati, Generalized Lindley distribution, *J. Math. Ext.*, **3** (2009), 1–17.
33. M. A. Thaimer, A. I. Al-Omari, Weighted gamma Lindley distribution: Statistical properties, reliability analysis, with modeling of COVID-19 data, *J. Appl. Prob. Stat.*, **20** (2025), 1–32.
34. A. J. Alidamat, A. I. Al-Omari, The extended length biased two parameters Mirra distribution with an application to engineering data, *Adv. Math. Model. Appl.*, **6** (2021), 113–127.
35. A. I. Al-Omari, K. Alhyasat, K. Ibrahim, M. Abu Bakar, Power length-biased Suja distribution: Properties and application, *Electron. J. Appl. Stat. Anal.*, **12** (2019), 429–452. https://doi.org/10.1285/i20705948v12n2p429

AIMS Press

© 2025 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0)