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Research article

The dynamic behavior of Van der Waals gas system based on new extended (G'/G)-expansion method

Yulan Wang*

Wuxi Vocational College of Science and Technology, Xinxi Road 8, Wuxi 214028, China

* **Correspondence:** Email: zhmlwxcstu@126.com; Tel: +8619384119219; Fax: +86051085343558.

Abstract: In this study, I theoretically explored different aspects of the (1+1)-dimensional Van der Waals (VDW) gas system in the viscosity capillarity sense. To find the physical nature of the described system, a couple of new reliable integrating techniques such as the extended (N, R) expansion method and the new extended (G'/G)-expansion method were used, and a variety of soliton solutions were successfully derived. To enhance our understanding and visually represent these solutions, I created 2D and 3D plots, thereby utilizing appropriate parameter values that met the constraint conditions. Furthermore, I analyzed chaotic behavior, thereby observing its presence in the perturbed dynamical system and obtaining positive results that support chaotic motion. My computational analysis confirmed the effectiveness and versatility of my techniques in addressing nonlinear problems in mathematical science and engineering.

Keywords: Van der Waals gas system; chaotic analysis; extended (N, R)-expansion method; extended (G'/G)-expansion method; soliton dynamics
Mathematics Subject Classification: 34C14, 37M05

1. Introduction

Nonlinear partial differential equations (NLPDEs) are used in many scientific domains to help model complex phenomena that represent the problems we face on a daily basis. Nonlinear models are increasingly used in a variety of fields, such as optics [1], engineering science and economics [2], finance [3], and machine learning [4]. Since analytical solutions reveal the physical characteristics of nonlinear systems, finding precise solutions to NLPDEs is an essential field of study [5–9]. The unified

Riccati equation expansion method [10], the direct Hirota bilinear method [11], the simplified Hirota method [12,13], the Painlevé approach [14], the (G'/G) expansion method [15], and the Lie symmetry approach [16–18] are a few of the many successful strategies that have been used to determine the analytical results of NLPDEs.

In physics and mathematics, a wave pulse that maintains its shape while moving at a constant speed is referred to as a soliton. In a medium, dispersive processes cancel out to form a soliton formation. In physical systems, soliton solutions are a type of nonlinear dispersive partial differential equations. The soliton phenomenon was first recorded in 1834 by John Scott Russell, who saw a lone wave in Scotland's Union Canal. In recent decades, this field of study has grown in importance. Solitons are produced by a number of well-known NLPDEs, such as the nonlinear Schrodinger equation [19], the Hietarinta equation [20], the KdV equation [21], the Gerdgikov Ivanov equation [22], the Biswas Miovic equation [23], and the Fokas system [24]. Some new and important developments for investigating the soliton solutions for nonlinear partial differential equations using some analytical methods [25,26].

In this paper, I concentrate on the equations in [27] governing the longitudinal isothermal motion of a compressible fluid (specifically, a Van der Waals gas system) in Lagrangian coordinates within a (1+1)-dimensional structure:

$$\begin{aligned} &q_t + P_x(s) = 0, \\ &s_t - q_x = 0. \end{aligned} \tag{1}$$

Here, q, s, and p represent the velocity, specific volume, and pressure, respectively.

The regularization process applied to Eq (1) involves incorporating viscosity and capillarity effects:

$$\begin{aligned} &q_t + P_x(s) = \mu q_{xx} - \omega\mu^2 s_{xxx}, \\ &s_t - q_x = 0. \end{aligned} \tag{2}$$

The symbol $\mu > 0$ represents the viscosity, while $\omega\mu^2$ represents the coefficient of interfacial capillarity; μ and ω both are positive constants. The function $P(s)$ represents a Van der Waals-like structure:

$$\begin{aligned} &q_t + P_x(s) = 0, \\ &s_t - q_x = 0. \end{aligned} \tag{3}$$

Given the finite size of the gas molecules and intermolecular interactions, the Van der Waals equation of state is a phenomenological model that explains the behavior of real gases. Even though the Van der Waals equation was originally developed for a three-dimensional system, studying simplified versions in lower dimensions can often provide valuable insights into how gases behave in confined geometries. In order to derive precise solutions, Arnous et al. [27] suggested a model that correlated an exterior Schwarzschild vacuum with an interior space-time geometry. For heavy pulsars, the model's physical viability was examined and contrasted with observational data from the Neutron Star Interior Composition Explorer. Additionally, Malaver and Kasmaei [28] suggested a model that expressed the physical variables in terms of polynomial and elementary functions and employed an exterior Schwarzschild vacuum and interior space-time geometry to obtain exact solutions in the Einstein-Maxwell system. In the case of heavy pulsars, the model's viability was examined and contrasted with observational data from the Neutron Star Interior Composition Explorer and yielded physically plausible expressions for the stellar object's mass, density, and radial pressure. Kipgen and Singh [29] analyzed the isothermal Euler equations for Van der Waals gas with dust particles in the presence of a flux approximation by applying the Riemann problem to concentration and cavitation phenomena.

Thus, my motivation is focused on the search for soliton solutions in the model mentioned above. I plan to accomplish this by utilizing eveloped and trustworthy methods, such as the extended (N, R) expansion method and the new extended G'/G-Expansion Method. These approaches promise to answer open-ended questions from earlier research [30,31]. The extended (N, R) expansion method is primarily used to find analytical solutions for the Van der Waals equation. My main goal is to simplify the equation while maintaining its essential properties. This standardization procedure opens the door to a precise solution of the equation and provides important information about the behavior of the system that the partial differential equation (PDE) describes. Furthermore, the new extended (G'/G)-expansion method [32] is used to obtain the exact soliton solutions of the system under consideration. This approach provides a broad range of solutions, including singular and kink soliton solutions. Additionally, I analyze chaotic behavior, thus observing its presence in the perturbed dynamical system and obtaining positive results that support chaotic motion. My computational analysis confirms the effectiveness and versatility of my techniques in addressing nonlinear problems in mathematical science and engineering.

The structure of the paper is as follows: I begin a mathematical analysis of the model in Section 2, with the goal of obtaining the ordinary differential equation (ODE); an in-depth discussion of the extended (N, R)-expansion method and the new extended (G'/G)-expansion method is covered in Section 3; in Section 4, I derive the solutions using discussed techniques along with some graphical illustrations; in Section 5, I detect the chaotic behavior of the governing system underlying the perturbed dynamical system; in Section 6, I conduct the stability analysis of the governing system with a stability region; and in Section 7, I summarize my findings and contributions.

The novelty of the work includes the use of two different methodologies, the extended (N, R) expansion method and the new extended (G'/G)-expansion method, to offer a variety of novel analytical performances for the (1+1)-dimensional Van Der Waals gas system. This analytical simplification makes it possible to use the provided methods to derive trigonometric, rational, and hyperbolic solutions. Two and three-dimensional phase profiles are used to illustrate these chaotic behaviors.

2. Mathematical analysis

I begin by using the following wave transformations:

$$\begin{aligned} q(x,t) &= u(\xi), \\ s(x,t) &= V(\xi). \end{aligned} \tag{4}$$

Here, $\xi = k(x - ct)$, and k denotes the wave width, and c represents the wave number. Then,

$$\begin{aligned} &-cu' + k^2\mu^2\omega V^{(3)} - k\mu u'' + (V - V^3) = 0, \\ &cV' + u' = 0. \end{aligned} \tag{5}$$

Integrating the above system with zero integration constants yields the following:

$$\begin{aligned} &-cu + k^2\mu^2\omega V'' - k\mu u' + (V - V^3) = 0, \\ &cV + u = 0. \end{aligned} \tag{6}$$

Combining the above system into a single equation with $u = -cV$ yields the following:

$$(c^2+1)V+ck\mu V'+k^2\mu^2\omega V''-V^3=0. \tag{7}$$

3. Proposed analytical techniques

Two techniques, namely the extended (N,R) expansion method and the new extended G'/G-expansion method [33], are explained in the undermentioned subsections.

3.1. The extended (N, R)-expansion method

Suppose that the transformed ODE has the solution of the following form:

$$V(\xi)=e_0+\sum_{i=0}^{N}(e_i N^i(\xi)+\varepsilon_i N^{i-1}(\xi)R^i(\xi)). \tag{8}$$

Here, $N=\dfrac{\theta'}{\theta}$ and $R=\dfrac{v'}{v}$ satisfy $\theta'=-\theta v$ and $v'=\lambda-v^2$, respectively. The ODEs in θ and v possess the following types of solutions:

- For $\lambda>0$,

$$\theta=\pm\sec h[\sqrt{\lambda}(\xi+\xi_0)], v=\sqrt{\lambda}\tanh[\sqrt{\lambda}(\xi+\xi_0)]. \tag{9}$$

- For $\lambda>0$,

$$\theta=\pm\csc h[\sqrt{\lambda}(\xi+\xi_0)], v=\sqrt{\lambda}\coth[\sqrt{\lambda}(\xi+\xi_0)]. \tag{10}$$

- For $\lambda=0$,

$$\theta=\frac{1}{\xi+\xi_0}, v=\frac{1}{\xi+\xi_0}. \tag{11}$$

- For $\lambda<0$,

$$\theta=\pm\sec[\sqrt{-\lambda}(\xi+\xi_0)], v=-\sqrt{-\lambda}\tanh[\sqrt{-\lambda}(\xi+\xi_0)]. \tag{12}$$

- For $\lambda<0$,

$$\theta=\pm\csc h[\sqrt{-\lambda}(\xi+\xi_0)], v=\sqrt{-\lambda}\cot[\sqrt{-\lambda}(\xi+\xi_0)]. \tag{13}$$

The equation in N or R is obtained by substituting Eq (8) into the transformed ODE. A system of algebraic equations can be constructed by equating the coefficients of various powers of variable to zero. The values of the arbitrary constants can be determined by solving the obtained system of equations.

3.2. The new extended (G'/G)-expansion method

Let us assume that the transformed ODE has a solution of the following form:

$$V(\xi)=\sum_{i=0}^{N}e_i(\Theta+X(\xi))^i+\sum_{i=1}^{N}\varepsilon_i(\Theta+X(\xi))^{-i}. \tag{14}$$

Here, $X(\xi)=\dfrac{G'}{G}$, e_i and ε_i are arbitrary constants to be found later, Θ is an undetermined function independent of ξ, N can be determined by the homogenous balancing principle, and $X(\xi)$ satisfies the following ODE:

$$G''(\xi)+\lambda G'(\xi)+hG(\xi)=0. \tag{15}$$

Here, λ and h are arbitrary constants to be determined later. Equation (15) possesses the following types of solutions:

- For $\Delta=A=\lambda^2-4h>0$, then

$$X_1=\frac{\sqrt{A}}{2}\coth[\Delta+\frac{\sqrt{A}}{2}\xi]-\frac{\lambda}{2}. \tag{16}$$

- For $\Delta=A=\lambda^2-4h>0$, then

$$X_2=\frac{\sqrt{A}}{2}\tanh[\Delta+\frac{\sqrt{A}}{2}\xi]-\frac{\lambda}{2}. \tag{17}$$

- For $\Delta=A=\lambda^2-4h<0$, then

$$X_3=\frac{\sqrt{-A}}{2}\cot[\Delta-\frac{\sqrt{-A}}{2}\xi]-\frac{\lambda}{2}. \tag{18}$$

- For $\Delta=A=\lambda^2-4h<0$, then

$$X_4=\frac{\sqrt{-A}}{2}\tan[\Delta-\frac{\sqrt{-A}}{2}\xi]-\frac{\lambda}{2}. \tag{19}$$

- For $\Delta=A=\lambda^2-4h=0$, then

$$X_5=\frac{\Theta}{\Theta+\Delta\xi}. \tag{20}$$

I substitute Eqs (14) and (15) into the transformed ODE and put the coefficients of X^i to equal zero; this yields a system of equations. The values of the arbitrary constants can be determined by solving the obtained system of equations.

The following section will provide the exact solutions of the proposed model by the applications of these two methods.

4. Extraction of solutions using the proposed techniques

The proposed methods are used in this section to obtain the analytic solution for the proposed problem.

4.1. The extended (N, R)-expansion method

According to this method, the solution of Eq (13) can be written as follows:

$$V(\xi)=e_0+e_1N(\xi)+\varepsilon_1R(\xi). \tag{21}$$

By inserting Eq (21) in Eq (7) and following the solution procedure of this method, I obtain the following families of solutions:

Set 1.

$$e_0 = \frac{1}{\sqrt{2(2-9\omega)}}, e_1 = -\frac{1}{\sqrt{2(2-9\omega)\lambda}}, \varepsilon_1 = \frac{1}{\sqrt{2(2-9\omega)\lambda}},$$
$$c = \frac{3\sqrt{\omega}}{\sqrt{2-9\omega}}, k = \frac{1}{2\mu\sqrt{(2-9\omega)\lambda\omega}}. \tag{22}$$

Set 2.

$$e_0 = \frac{1}{\sqrt{2(2-9\omega)}}, e_1 = -\frac{1}{\sqrt{2(2-9\omega)\lambda}}, \varepsilon_1 = 0, c = \frac{3\sqrt{\omega}}{\sqrt{2-9\omega}},$$
$$k = \frac{1}{2\mu\sqrt{(2-9\omega)\lambda\omega}}. \tag{23}$$

Set 1 yields the following solutions:
If $\lambda > 0$, then

$$s_1(x,t) = \frac{1+\coth[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}, q_1(x,t) = -c(\frac{1+\coth[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}). \tag{24}$$

If $\lambda > 0$, then

$$s_2(x,t) = \frac{1+\tanh[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}, q_2(x,t) = -c(\frac{1+\tanh[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}). \tag{25}$$

If $\lambda < 0$, then

$$s_3(x,t) = \frac{1}{\sqrt{4-18\omega}} + \frac{\sqrt{\lambda(2-9\omega)}\cot[\sqrt{-\lambda}(\xi+\xi_0)]}{\sqrt{-\lambda}(4-18\omega)},$$
$$q_3(x,t) = -c(\frac{1}{\sqrt{4-18\omega}} + \frac{\sqrt{\lambda(2-9\omega)}\cot[\sqrt{-\lambda}(\xi+\xi_0)]}{\sqrt{-\lambda}(4-18\omega)}). \tag{26}$$

If $\lambda < 0$, then

$$s_4(x,t) = \frac{1-t\tan[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}, q_4(x,t) = -c(\frac{1-t\tan[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}). \tag{27}$$

Set 2 yields the following solutions:
If $\lambda > 0$, then

$$s_5(x,t) = \frac{1-\tanh[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}, q_5(x,t) = -c(\frac{1-\tanh[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}). \tag{28}$$

If $\lambda > 0$, then

$$s_6(x,t)=\frac{1-\coth[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}},q_6(x,t)=-c(\frac{1-\coth[\sqrt{\lambda}(\xi+\xi_0)]}{\sqrt{4-18\omega}}). \tag{29}$$

If $\lambda<0$, then

$$\begin{aligned}s_7(x,t)&=\frac{1}{\sqrt{4-18\omega}}+\frac{\sqrt{-\lambda}\tan[\sqrt{-\lambda}(\xi+\xi_0)]}{\sqrt{\lambda(4-18\omega)}},\\ q_7(x,t)&=-c(\frac{1}{\sqrt{4-18\omega}}+\frac{\sqrt{-\lambda}\tan[\sqrt{-\lambda}(\xi+\xi_0)]}{\sqrt{\lambda(4-18\omega)}}).\end{aligned} \tag{30}$$

If $\lambda<0$, then

$$\begin{aligned}s_8(x,t)&=\frac{1}{\sqrt{4-18\omega}}+\frac{\sqrt{-\lambda}\cot[\sqrt{-\lambda}(\xi+\xi_0)]}{\sqrt{\lambda(4-18\omega)}},\\ q_8(x,t)&=-c(\frac{1}{\sqrt{4-18\omega}}+\frac{\sqrt{-\lambda}\cot[\sqrt{-\lambda}(\xi+\xi_0)]}{\sqrt{\lambda(4-18\omega)}}).\end{aligned} \tag{31}$$

4.2. The new extended G′/G-expansion method

According to the proposed technique, the solution takes the following form:

$$V(\xi)=e_0+e_1N(\xi)+\varepsilon_1R(\xi). \tag{32}$$

Moreover, Sets 3 and 4 have been retrieved by the application of proposed method as follows:

Set 3.

$$\begin{aligned}&e_0=\frac{(\lambda-2\Theta)\sqrt{\lambda^2-4h}}{\sqrt{4-18\omega}\sqrt{\lambda^2-4h}},e_1=\frac{\sqrt{2}}{\sqrt{(2-9\omega)(\lambda^2-4h)}},\varepsilon_1=0,\\ &c=\frac{3\sqrt{\omega}}{\sqrt{2-9\omega}},k=-\frac{1}{\mu\sqrt{(2-9\omega)(\lambda^2-4h)\omega}}.\end{aligned} \tag{33}$$

Set 4.

$$\begin{aligned}&e_0=-\frac{(2\Theta-\lambda)+\sqrt{\lambda^2-4h}}{\sqrt{4-18\omega}\sqrt{\lambda^2-4h}},e_1=0,\varepsilon_1=\frac{\sqrt{2}(h+\Theta)(\Theta-\lambda)}{\sqrt{(2-9\omega)(\lambda^2-4h)}},\\ &c=-\frac{3\sqrt{\omega}}{\sqrt{2-9\omega}},k=\frac{\mu}{\sqrt{(2-9\omega)(\lambda^2-4h)\omega}}.\end{aligned} \tag{34}$$

The solutions that correspond to Set 3 are given below:
If $\lambda^2-4h>0$, then

$$s_9(x,t)=\frac{\lambda-2\Theta+\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+\frac{\sqrt{2}(\Theta-\lambda/2+\sqrt{\lambda^2-4h}\coth[\Delta+\xi\sqrt{\lambda^2-4h}/2]/2}{\sqrt{(\lambda^2-4h)(2-9\omega)}}, \tag{35}$$
$$q_9(x,t)=-cs_9(x,t).$$

If $\lambda^2-4h>0$, then

$$s_{10}(x,t)=\frac{\lambda-2\Theta+\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+\frac{\sqrt{2}(\Theta-\lambda/2+\sqrt{\lambda^2-4h}\tanh[\Delta+\xi\sqrt{\lambda^2-4h}/2]/2}{\sqrt{(\lambda^2-4h)(2-9\omega)}}, \tag{36}$$
$$q_{10}(x,t)=-cs_{10}(x,t).$$

If $\lambda^2-4h<0$, then

$$s_{11}(x,t)=\frac{\lambda-2\Theta+\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+\frac{\sqrt{2}(\Theta-\lambda/2+\sqrt{4h-\lambda^2}\cot[\Delta-\xi\sqrt{4h-\lambda^2}/2]/2}{\sqrt{(\lambda^2-4h)(2-9\omega)}}, \tag{37}$$
$$q_{11}(x,t)=-cs_{11}(x,t).$$

If $\lambda^2-4h<0$, then

$$s_{12}(x,t)=\frac{\lambda-2\Theta+\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+\frac{\sqrt{2}(\Theta-\lambda/2+\sqrt{\lambda^2-4h}\tan[\Delta-\xi\sqrt{\lambda^2-4h}/2]/2}{\sqrt{(\lambda^2-4h)(2-9\omega)}}, \tag{38}$$
$$q_{12}(x,t)=-cs_{12}(x,t).$$

The solutions that correspond to Set 4 are given below:
If $\lambda^2-4h>0$, then

$$\begin{aligned}s_{13}(x,t)&=\frac{\lambda-2\Theta-\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+\\&\frac{2\sqrt{2}(h+\Theta)(\Theta-\lambda)}{\sqrt{(\lambda^2-4h)(2-9\omega)}(2\Theta-\lambda+\sqrt{\lambda^2-4h}\coth[\Delta+\xi\sqrt{\lambda^2-4h}/2])},\\q_{13}(x,t)&=-cs_{13}(x,t).\end{aligned} \tag{39}$$

If $\lambda^2-4h>0$, then

$$\begin{aligned}s_{14}(x,t)&=\frac{\lambda-2\Theta-\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+\\&\frac{2\sqrt{2}(h+\Theta)(\Theta-\lambda)}{\sqrt{(\lambda^2-4h)(2-9\omega)}(2\Theta-\lambda+\sqrt{\lambda^2-4h}\tanh[\Delta+\xi\sqrt{\lambda^2-4h}/2])},\\q_{14}(x,t)&=-cs_{14}(x,t).\end{aligned} \tag{40}$$

If $\lambda^2-4h<0$, then

$$s_{15}(x,t)=\frac{\lambda-2\Theta-\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+$$
$$\frac{2\sqrt{2}(h+\Theta)(\Theta-\lambda)}{\sqrt{(\lambda^2-4h)(2-9\omega)}(2\Theta-\lambda+\sqrt{4h-\lambda^2}\cot[\Delta-\xi\sqrt{4h-\lambda^2}/2])}, \tag{41}$$
$$q_{15}(x,t)=-cs_{15}(x,t).$$

If $\lambda^2-4h<0$, then

$$s_{16}(x,t)=\frac{\lambda-2\Theta-\sqrt{\lambda^2-4h}}{\sqrt{\lambda^2-4h}\sqrt{4-18\omega}}+$$
$$\frac{2\sqrt{2}(h+\Theta)(\Theta-\lambda)}{\sqrt{(\lambda^2-4h)(2-9\omega)}(2\Theta-\lambda+\sqrt{4h-\lambda^2}\tan[\Delta-\xi\sqrt{4h-\lambda^2}/2])}, \tag{42}$$
$$q_{16}(x,t)=-cs_{16}(x,t).$$

Now, I set specific parameter levels for the theoretical formulas (22) and (23) obtained, which can be analyzed to obtain a visualization of the calculation results. The dynamic 3D graph can reflect the analytical solution.

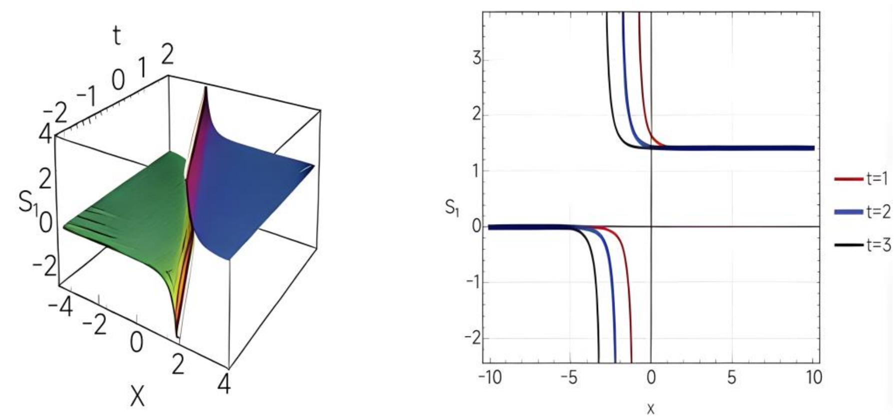

Figure 1. Singular solution to Set-1 for $\lambda=1,\ \omega=1/9$.

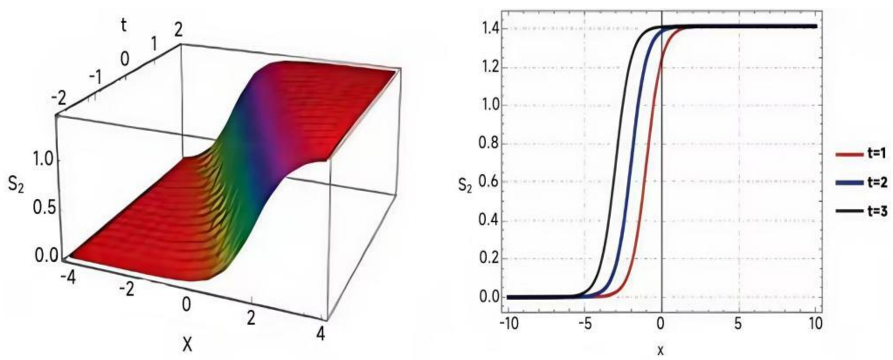

Figure 2. Kink solution to Set-1 for $\lambda=1,\ \omega=1/9$.

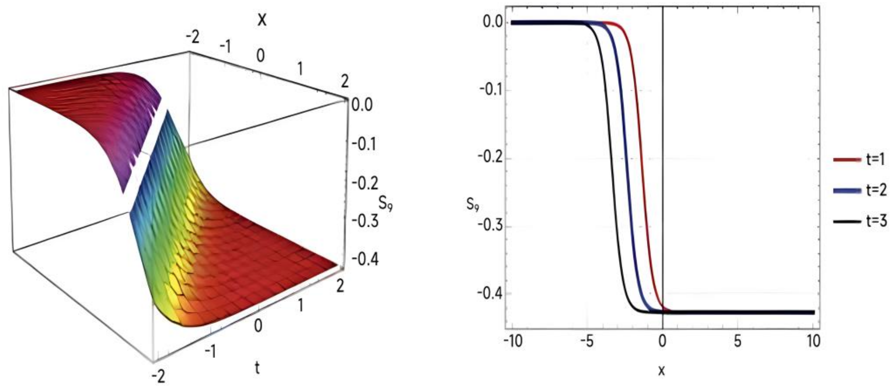

Figure 3. Kink-singular solution to Set-3 for $h=1,\ \lambda=3,\ \omega=1/9,\ \mu=\Theta=\Delta$.

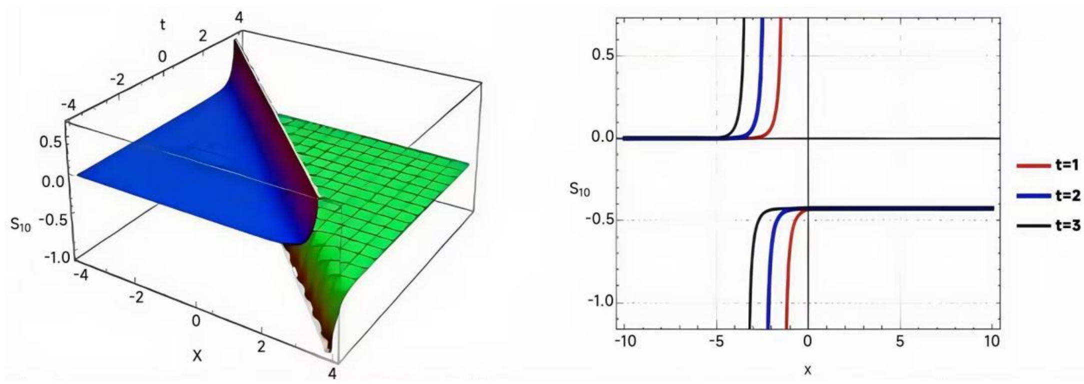

Figure 4. Kink solution to Set-3 for $h=1,\ \lambda=3,\ \omega=1/9,\ \mu=\Theta=\Delta$.

Figures 1–4 illustrate these results under the specified parameter values. The significance of solitary wave solutions lies in their ability to elucidate the underlying dynamics of complex physical systems.

5. Chaotic behavior of the governing system

In this analysis, I have investigated the chaotic behavior of the governing system using Eq (7). Using the Givens transformation, I convert Eq (7) into a planar dynamical system and add a perturbation term to disturb the periodic motion of the system and to obtain fruitful results of chaotic behavior [34]. Consequently, the perturbed dynamical system of (7) subject to an external periodic force can be written as follows:

$$\begin{cases} dV/d\xi = S, \\ dS/d\xi = k_1V^3 - k_2V - k_3S + f_0\cos(\phi), \\ d\phi/d\xi = \omega. \end{cases} \tag{43}$$

Here, $\phi=\omega\xi$. In system (43), the parameters denoted by f_o and w correspond to the frequency and magnitude of the external force applied, respectively. To understand these phenomena, some tools available in the literature [35,36] have been utilized to show the presence of the chaotic behavior of the model underlying the perturbed dynamical system (43). These tools include the phase portrait, the Poincaré map, the time series, and the Lyapunov exponent. The presence of chaotic behavior in the perturbed dynamical system is observed at the initial condition (0.5, 0, 0.5) and using other suitable parameter values, as shown in Figures 5–8.

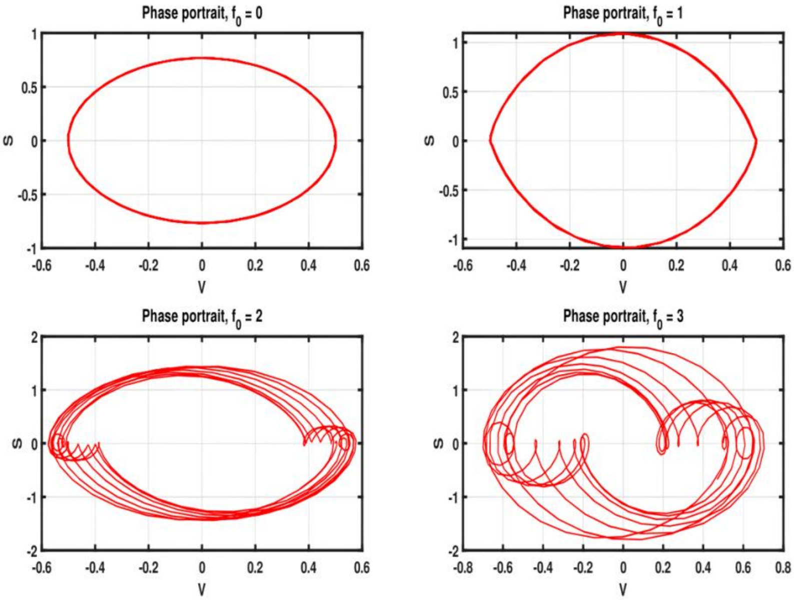

Figure 5. Phase portraits of the dynamical system (43) using the initial condition (0.5, 0, 0.5), various values of f_0, and other suitable parameters $k_1 = 1.2,\ k_2 = 2.5,\ k_3 = -0.001,\ \omega = 4.5$.

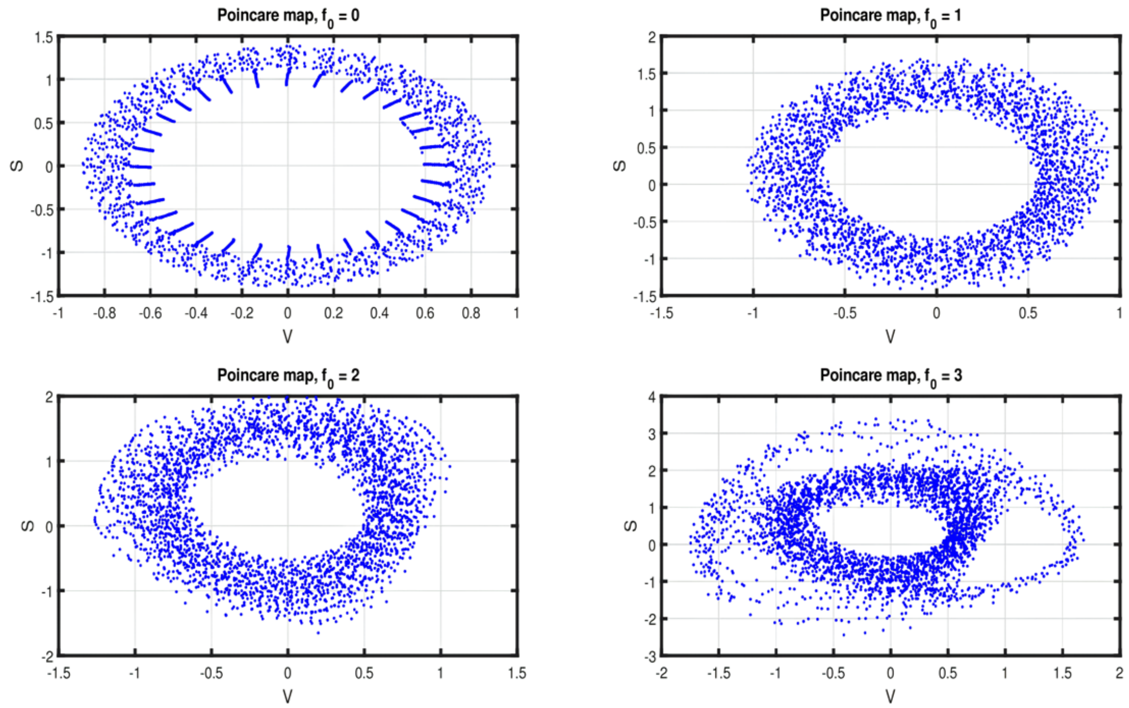

Figure 6. Poincaré maps of the dynamical system (43) using the initial condition (0.5, 0, 0.5), various values of f_0, and other suitable parameters $k_1 = 1.2,\ k_2 = 2.5,\ k_3 = -0.001,\ \omega = 4.5$.

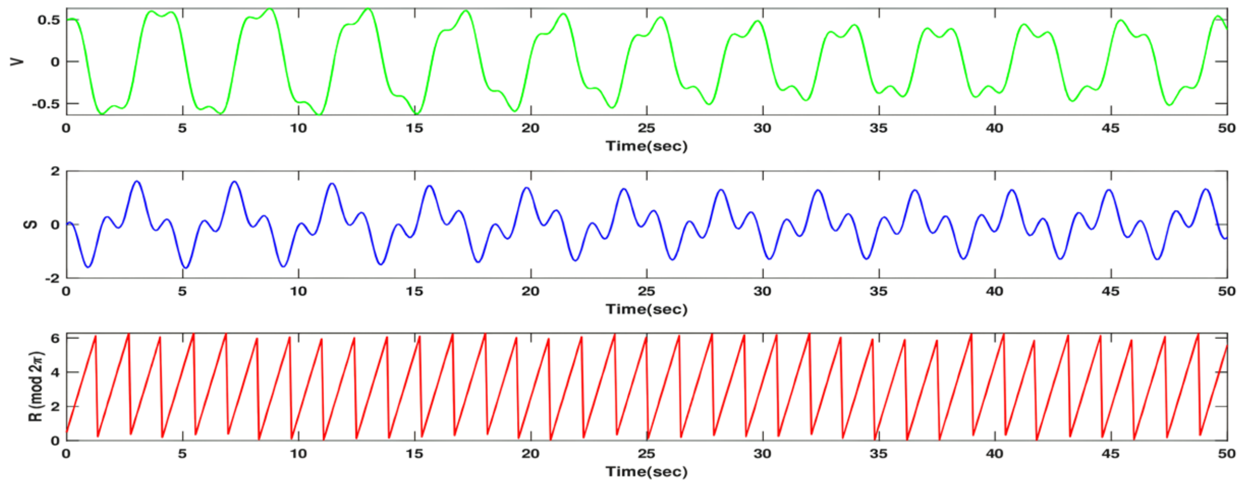

Figure 7. Chaotic behavior of the dynamical system (43) via a time series using the initial condition (0.5, 0, 0.5), various values of f_0, and other suitable parameters $k_1 = 1.2,\ k_2 = 2.5,\ k_3 = -0.001,\ \omega = 4.5,\ f_0 = 2.5$.

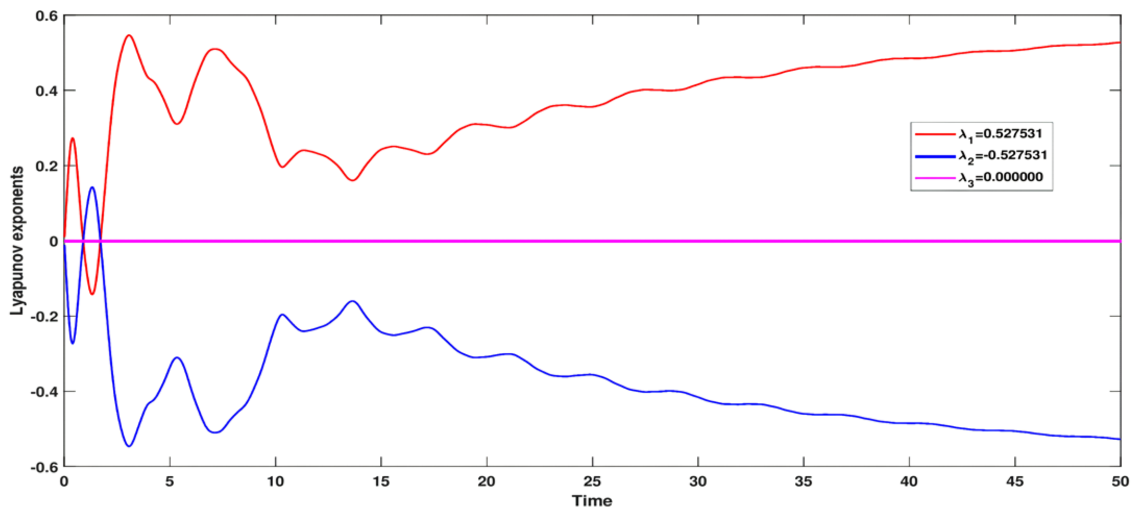

Figure 8. Chaotic behavior of the dynamical system (43) via the Lyapunov exponent using the initial condition (0.5, 0, 0.5), various values of f_0, and other suitable parameters $k_1 = 1.2,\ k_2 = 2.5,\ k_3 = -0.001,\ \omega = 4.5,\ f_0 = 2.5$.

It can be seen from Figure 6 that the function value of the system's conserved quantity I_N is constant near 0.3456. Thus, the conserved quantity-preserving numerical method constructed by discrete gradient will not introduce artificial dissipation with the growth of time and has better tracking ability.

6. Discussion

In this section, I present the results obtained through the application of the extended (N, R) expansion method and the new extended (G'/G)-expansion method. The general form of solitary wave solutions, thereby encompassing kink, singular, and kink-singular solutions, was derived. To enhance the understanding of these solutions, various graphical representations, including two-dimensional and three-dimensional plots, were employed with carefully chosen parameter values and constraint conditions. Figures 1–4 illustrate these results under the specified parameter values. The significance of solitary wave solutions lies in their ability to elucidate the underlying dynamics of complex physical systems.

Furthermore, I studied the chaotic behavior of the model underlying the perturbed dynamical system at the initial condition (0.5, 0, 0.5) alongside other suitable parameter values of $k_1,\ k_2,\ k_3,\ f_0,\ \omega$. Some tools were used to detect the presence of the chaotic behavior of the model and obtain fruitful results of chaotic behavior, as shown in Figures 5–8. Therefore, these findings contribute valuable insights for a deeper comprehension of the studied phenomena. Notably, the results are novel and provide useful contributions to the dynamics of nonlinear waves and the soliton theory for the examined equation.

7. Conclusions

In conclusion, I theoretically explored different aspects of the (1+1)-dimensional van der Waals gas system. The exact solutions of the system were obtained using two reliable approaches, namely the extended (N, R)-expansion method and the new extended (G'/G)-expansion method, which successfully derived a variety of soliton dynamics such as the kink-type and singular wave solutions by varying the values of the parameters. To enhance our understanding and visually represent these solutions, 2D and 3D plots were created, thereby utilizing appropriate parameter values that met the constraint conditions. Additionally, the chaotic behavior was analyzed, where the presence of chaos was observed in the perturbed dynamical system; positive results supporting chaotic motion were obtained using some chaos detecting tools such as the phase portrait, the Poincaré map, a time series, and the Lyapunov exponent. I studied the chaotic behavior of the model underlying the perturbed dynamical system at the initial condition (0.5, 0, 0.5) alongside other suitable parameter values of $k_1,\ k_2,\ k_3,\ f_0,\ \omega$. Some tools were used to detect the presence of the chaotic behavior of the model and obtain fruitful results of chaotic behavior.

The computational analysis confirmed the effectiveness and versatility of the techniques in addressing nonlinear problems in mathematical science and engineering. Prospects for the future include investigating alternative analytical methods and deriving generalized solutions such as multiple solutions and Jacobi elliptic function solutions. In addition, one can pursue breathers, rogue waves, N-soliton solutions, and other fascinating ideas. Furthermore, I can use the (1+1)- dimensional van der Waals gas system to describe the fluid motion rule in microchannel gas cooling and thermal management, and the solution of the van der Waals gas system can be compared with the analytical method and numerical algorithms.

Use of Generative-AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tool in the creation of this article.

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Conflict of interest

The author declares that there is no conflict of interest.

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