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Research article

Stabilization of stochastic systems: event-triggered impulsive control

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Abstract: This paper systematically establishes criteria for ensuring r-th moment asymptotic stability and r-th moment exponential stability in stochastic systems via an event-triggered impulsive control (ETIC) strategy, considering both scenarios with and without impulsive delay. By constructing appropriate Lyapunov functions and applying stochastic analysis methods, the intrinsic relationships among event-triggered parameters, impulsive control (IC) strength, and system stability are established. Moreover, this paper thoroughly investigates how the selection of event-triggered parameters, the length of the inspection interval, and the magnitude of impulsive delay influence the convergence rate of the system. Finally, two numerical examples are presented to verify the effectiveness of the proposed ETIC method, one of which focuses on the consensus problem of stochastic multi-agent systems.

Keywords: asymptotic stability; exponential stability; event-triggered impulsive control; impulsive delay; stochastic systems
Mathematics Subject Classification: 93E03

1. Introduction

In practical applications, the design and implementation of control signals play a crucial role in maintaining the desired state of complex systems. In recent years, various control strategies have been proposed by researchers to address the nonlinearity, uncertainty, and external disturbances of the system. Among them, common control methods include sliding mode control, PID control, and adaptive control (see, e.g., [1–3]). Unlike the above continuous control methods, impulsive control (IC), as a typical discontinuous control strategy, features a simple structure and strong robustness. It has demonstrated favorable control performance and engineering feasibility in complex dynamical systems such as fractional-order systems, multi-agent systems, and inertial systems (see, e.g., [4–7]).

However, current research on IC is primarily focused on time-based IC strategies, in which the triggered instants of impulsive signals are typically pre-specified or known in advance.

Such approaches lack flexibility and may result in unnecessary control inputs, thereby causing a waste of resources. To overcome this limitation, researchers have introduced event-triggered mechanisms (see, e.g., [8–11]) into the IC framework and proposed event-triggered impulsive control (ETIC) strategies (see, e.g., [12, 13]). Compared with conventional IC, ETIC can dynamically adjust the impulsive instants based on the system state, thereby enabling adaptive adjustment of the impulsive sequence and significantly reducing ineffective control behavior. In [14], two ETIC mechanisms were designed to effectively reduce communication and energy consumption, thereby achieving quasi-synchronization among nodes in heterogeneous dynamic networks. In [15], based on sliding variables, sufficient conditions were derived to ensure the robust stability of the considered IC system, and the effects of external disturbances and uncertainties on system stability were revealed.

Notably, the research results presented in the above literature mainly focus on deterministic systems, while research on ETIC in stochastic systems remains relatively limited. In practice, due to uncertainties such as component performance fluctuations, modeling errors, and external environmental disturbances, modern engineering dynamic systems are inevitably subjected to stochastic perturbations. Therefore, it is of great theoretical value and engineering significance to study the stability and control strategy of ETIC in stochastic systems in practical applications (see, e.g., [16–19]). In [20], the concept of average impulsive delay was introduced to characterize the impact of stochastic impulsive delays on the overall dynamic behavior of the system, and criteria for the p-th moment exponential stability were established. In [21], based on a three-level ETIC strategy, the input-to-state stability of a stochastic system under external disturbances was analyzed. Compared with conventional ETIC mechanisms, the proposed three-level strategy achieved faster convergence of system states and exhibited stronger robustness and adaptability. Therefore, in order to further improve the control performance of the system, the influence of stochastic factors must be fully considered in the control design to better adapt to the behavior of dynamic systems in complex uncertain environments.

As research continues to advance, scholars have gradually recognized that impulsive delays are an unavoidable phenomenon in IC, and such impulsive delays may significantly affect the stability of the system (see, e.g., [22–27]). For example, in [28], sufficient conditions for achieving finite-time stability in systems with either stable or unstable impulses were provided based on Lyapunov stability theory. It was pointed out that impulsive delay interfered with finite-time stability and thus affected the estimation of the settling time. In [29], input-to-state stability of deterministic systems with and without impulsive delays was established under impulsive control. However, most existing studies are based on the assumption that the impulsive delay does not exceed the interval between adjacent impulses. Nevertheless, studies on the impact of impulsive delay that exceeds this interval on the system remain limited. In response to this issue, it is of significant theoretical and practical value to further explore the design of control methods under conditions of large impulsive delay.

Based on the above discussion, this paper proposes a discriminant criterion to guarantee the stability of the system for a class of stochastic systems combined with an ETIC strategy. The main contributions are as follows:

(i) Stability criteria are established to ensure that the system achieves r-th moment asymptotic stability (r-AS) and r-th moment exponential stability (r-ES), with or without impulsive delay. In particular, for the case with impulsive delay, this paper breaks through the strict assumption adopted in [30, 31] that the impulsive delay must be smaller than the adjacent impulse interval, allows the delay length to exceed the inter-impulse interval, breaks the strict limitation on impulsive delay in existing theories, and enhances the practical applicability of the model.

(*ii*) The potential relationships among event-triggered parameters, impulse strength, and system stability are systematically established. Furthermore, the effects of event-triggered parameters, inspection interval, and impulsive delay on the convergence rate of the system are analyzed. The results indicate that smaller values of these parameters lead to faster convergence.

The structure of this paper is organized as follows: Section 2 presents the necessary definitions, lemmas, and assumptions. Section 3 provides the main theoretical results. Section 4 introduces some applications of the theoretical results. Section 5 verifies the effectiveness of the ETIC strategy by using two simulation examples. Finally, Section 6 concludes the paper.

Notations: $\mathbb{N}^+$ and $\mathbb{N}$ signify the sets of positive integers and non-negative integers, respectively; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional and $n \times m$-dimensional real spaces, respectively; $P < 0$ ($P > 0$) and $P \leq 0$ ($P \geq 0$) denote that the matrix P is negative definite (positive definite) and semi-negative definite (semi-positive definite), respectively; I denotes the identity matrix of appropriate dimensions; $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent the maximum and minimum eigenvalues of the matrix A, respectively; $\lceil x \rceil$ denotes the smallest integer greater than or equal to x; $*$ stands for a symmetric block within a symmetric matrix; $PC([-h, 0], \mathbb{R}^n)$ represents the set of all piecewise right continuous functions ϕ with its norm $\phi = \sup_{-h\leq\theta\leq 0} \|\phi(\theta)\|$ from $[-h, 0]$ to $\mathbb{R}^n$. Let $PC^r_{\mathcal{F}_0}([-h, 0], \mathbb{R}^n)$ be the set of bounded $\mathcal{F}_0$-measurable, $PC([-h, 0], \mathbb{R}^n)$ valued variables, while $PC^r_{\mathcal{F}_0}([-h, 0], \mathbb{R}^n)$ be the set of $\mathcal{F}_0$-measurable, $PC([-h, 0], \mathbb{R}^n)$ valued variables with $\sup_{-h\leq\theta\leq 0} \mathbb{E}\,\|\phi(\theta)\|^r < \infty$.

2. Model description

Consider the nonlinear stochastic system given below:

$$\begin{cases} dx(t) = f(t, x(t))dt + g(t, x(t))dB(t), & t \geq t_0, \\ x(t_0) = x_0, \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the system state; $t_0 \geq 0$ is the initial time; $B(t)$ is an n-dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathbb{P})$.

Next, we focus on investigating an ETIC strategy to stabilize system (2.1). Specifically, we aim to design an ETIC strategy that guarantees the stability of the following system:

$$\begin{cases} dx(t) = f(t, x(t))dt + g(t, x(t))dB(t), & t \neq t_s, \quad t \geq t_0, \\ x(t_s) = I_s(x(t_s - h(t_s))), & s \in \mathbb{N}^+, \\ x(t+\theta) = \phi_0, & \theta \in [-\hbar, 0], \end{cases} \tag{2.2}$$

where $\{t_s, s \in \mathbb{N}^+\}$ denotes the sequence of impulse times determined by ETIC. Function $h(t) \in [0, \hbar]$ denotes impulsive delay, where $\hbar > 0$ is a given constant. Let $f, g : [t_0 - \hbar, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$ satisfy the Lipschitz condition $\|f(t, x(t)) - f(t, y(t))\| \vee \|g(t, x(t)) - g(t, y(t))\| \leq L\|x(t) - y(t)\|$, with $L > 0$. If $f(t, 0) = 0, g(t, 0) = 0$, and the impulse functions satisfy $I_s(t, 0) = 0$, then system (2.2) admits the trivial solution $x(t) \equiv 0$.

Lemma 2.1. *[32] Let $Q > 0$ be a real symmetric positive definite matrix and $\xi > 0$ be a scalar. Then, the following inequality holds:*

$$W_1^T W_2 + W_2^T W_1 \leq \xi W_1^T Q W_1 + \xi^{-1} W_2^T Q^{-1} W_2.$$

Assumption 2.1. *[33] There exist integers m_s and m with $0 \le m_s \le m$ such that*

$$t_{s-m_s-1} \le t_s - h(t_s) < t_{s-m_s}, \quad \forall m, s \in \mathbb{N}^+.$$

For convenience, when the index $i \le 0$, we define $t_i = t_0 + \frac{i\hbar}{m}$.

Remark 2.1. *Assumption 2.1 characterizes the IC behavior in system (2.2). When the controller is delay-free, the IC input depends solely on the current state $x(t_s)$. However, when delay is present, the controller must use state information from an earlier time, due to both the delay effect and the boundedness of the delay function $h(t)$. To accurately describe the temporal location of this delayed state, we introduce the interval constraint $t_{s-m_s-1} \le t_s - h(t_s) < t_{s-m_s}$, which ensures that the delayed state lies between two known historical impulse instants. This assumption has wide practical relevance in engineering applications. For instance, in smart grid systems, the process of generating and executing control commands is typically subject to delays arising from state measurement, data transmission, and information processing. As a result, the controller usually relies on system state information prior to the current instant when generating control inputs such as voltage regulation, frequency control, or active/reactive power allocation. This delayed state naturally falls within the defined time interval. A similar modeling approach can also be found in [23, 33].*

Definition 2.1. *[34] Let $x(t)$ be the solution process of system* (2.2) *with the initial state $\phi_0 \in PC^r_{\mathcal{F}_0}([-\hbar, 0], \mathbb{R}^n)$. For a given constant $r > 0$, system* (2.2) *is termed*
(i) *r-AS, if the solution satisfies*

$$\lim_{t\to\infty} \mathbb{E}[\|x(t)\|^r] = 0;$$

(ii) *r-ES, if there exist positive constants C, λ, such that*

$$\mathbb{E}[\|x(t)\|^r] \le Ce^{-\lambda(t-t_0)}\mathbb{E}[\|\phi_0\|^r_\hbar], \quad t \ge t_0,$$

where $\|\phi_0\|_\hbar = \sup_{\theta\in[t_0-\hbar,t_0]} \phi(\theta)$. And if $r = 2$, it is said to be exponentially stable in the mean square.

Definition 2.2. *[35] Function $W(t, x) : [t_0 - \hbar, \infty) \times \mathbb{R}^n \to [0, \infty) \in \mathcal{V}_0$, if*
(i) *$W(t, x)$ is continuous on each set $[t_s, t_{s+1}) \times \mathbb{R}^n$ and satisfies $\lim_{(t,y)\to(t_s^-,x)} W(t, y) = W(t_s^-, x)$;*
(ii) *$W(t, x)$ is continuously twice differentiable in x and once differentiable in t on each set $[t_{s-1}, t_s) \times \mathbb{R}^n$, $s \in \mathbb{N}^+$.*

Let $W(t, x) \in \mathcal{V}_0$, and define an operator $\mathcal{L}$ associated with system (2.2) *by*

$$\mathcal{L}W(t, x) \doteq \frac{\partial W(t, x)}{\partial t} + \frac{\partial W(t, x)}{\partial x} f(t, x) + \frac{1}{2} tr[g^T(t, x)\frac{\partial^2 W(t, x)}{\partial x^2} g(t, x)].$$

Now, we design an ETIC strategy given by

$$\begin{cases} t^*_{s+1} = \inf\{t > t_s + \delta : W(t, x) \ge a_{s+1}W(t_s, x)\}, \\ t^\diamond_{s+1} = t_s + \Delta_{s+1}, \\ t_{s+1} = \min\{t^*_{s+1}, t^\diamond_{s+1}\}, \end{cases} \tag{2.3}$$

where function $W(t, x) \in \mathcal{V}_0$. The event-triggered parameter sequence $\{a_s\}_{s\in\mathbb{N}^+}$, which satisfies $a_s > 1$ for all s, is introduced to adjust the sensitivity of the triggered threshold. To prevent performance degradation caused by system (2.2) not receiving control inputs for an extended period, an inspection interval sequence $\{\Delta_s\}_{s\in\mathbb{N}^+}$ is introduced, which satisfies $\Delta = \sup_{s\in\mathbb{N}^+}\{\Delta_s\} > 0$. Moreover, to effectively avoid Zeno-behavior (i.e., an infinite number of triggered events occurring in a finite time), a minimum waiting time $\delta > 0$ is introduced in the ETIC strategy to ensure that the time between any two successive triggered instants has a strictly positive lower bound.

3. Main results

3.1. Without impulsive delay in ETIC

In this subsection, we focus on the analysis of IC inputs without delay and derive stability criteria for r-AS and r-ES of system (2.2).

Theorem 3.1. *Suppose there exists a function $W(t, x(t)) \in \mathcal{V}_0$ and constants $r, \alpha, b_1, b_2 > 0$, $0 < \beta < 1$ such that the following conditions hold:*

(L_1) $b_1 \|x(t)\|^r \leq W(t, x(t)) \leq b_2 \|x(t)\|^r$,
(L_2) $\mathcal{L}W(t, x(t)) \leq \alpha W(t, x(t)),\ t \neq t_s$,
(L_3) $W(t_s, I_s(x(t_s))) \leq \beta W(t_s - h(t_s), x(t_s - h(t_s))),\ s \in \mathbb{N}^+$,
(L_4) $0 < \delta < \frac{\ln a_{s+1}}{\alpha},\ s \in \mathbb{N}$,
(L_5) $\beta^s \prod_{i=1}^{s} a_i \to 0,\ s \to \infty$,

then system (2.2) *is r-AS under $h(t_s) \equiv 0$.*

Furthermore, condition (L_5) is revised as follows: If there exists a constant $\gamma < 1$ such that

(L_6) $\beta^s \prod_{i=1}^{s} a_i \leq \gamma^s,\ s \in \mathbb{N}^+$,

then system (2.2) *is r-ES under $h(t_s) \equiv 0$, with a convergence rate of $\lambda = -\frac{\ln\gamma}{\Delta}$.*

Proof. For clarity and convenience, we denote $W(t) := W(t, x(t))$. In the following, we will prove

$$\mathbb{E}[W(t)] \leq a_{s+1}\mathbb{E}[W(t_s)] \tag{3.1}$$

holds for all $t \in [t_s, t_{s+1}),\ s \in \mathbb{N}$ under ETIC (2.3).

If $t \in [t_s, t_s + \delta)$, from Itô's formula [36] and Fubini's theorem [37], it can be derived

$$\begin{aligned}\mathbb{E}[W(t)] &= \mathbb{E}[W(t_s)] + \int_{t_s}^{t} \mathbb{E}[\mathcal{L}W(\theta)]d\theta \\ &\leq \mathbb{E}[W(t_s)] + \alpha \int_{t_s}^{t} \mathbb{E}[W(\theta)]d\theta,\end{aligned} \tag{3.2}$$

by Gronwall's inequality [36] and (L_4), one obtains

$$\mathbb{E}[W(t)] \leq \mathbb{E}[W(t_s)]e^{\alpha(t-t_s)} \leq \mathbb{E}[W(t_s)]e^{\alpha\delta} \leq a_{s+1}\mathbb{E}[W(t_s)]. \tag{3.3}$$

If $t \in [t_s + \delta, t_{s+1})$, from ETIC (2.3), we obtain

$$\mathbb{E}[W(t)] \leq a_{s+1}\mathbb{E}[W(t_s)].$$

Thus, we can derive that inequality (3.1) holds for all $t \in [t_{s-1}, t_s)$, $s \in \mathbb{N}^+$.

Next, we establish the stability of system (2.2) under ETIC (2.3). For $t = t_s$, $s \in \mathbb{N}^+$, we have

$$\mathbb{E}[W(t_s)] \leq \beta\mathbb{E}[W(t_s^-)] \leq \beta a_s\mathbb{E}[W(t_{s-1})] \leq \cdots \leq \left(\beta^s \prod_{i=1}^{s} a_i\right)\mathbb{E}[W(t_0)]. \tag{3.4}$$

Combining (3.1), (3.4), and (L_1), one has

$$b_1\mathbb{E}[\|x(t)\|^r] \leq \mathbb{E}[W(t)] \leq a_{s+1}\left(\beta^s \prod_{i=1}^{s} a_i\right)\mathbb{E}[W(t_0)] \leq b_2 a_{s+1}\left(\beta^s \prod_{i=1}^{s} a_i\right)\mathbb{E}[\|x_0\|^r].$$

Further, we obtain

$$\mathbb{E}[\|x(t)\|^r] \leq \frac{b_2 a_{s+1}}{b_1}\left(\beta^s \prod_{i=1}^{s} a_i\right)\mathbb{E}[\|x_0\|^r].$$

If condition (L_5) holds, we can get $\lim_{s\to\infty}\mathbb{E}[\|x(t)\|^r] = 0$. As a result, system (2.2) under $h(t_s) \equiv 0$ is r-AS.

In addition, it can be obtained from ETIC (2.3) that $t_s - t_{s-1} \leq \Delta$, $s \in \mathbb{N}^+$, which implies that

$$t_s - t_0 \leq s\Delta. \tag{3.5}$$

Therefore, when $t \in [t_s, t_{s+1})$, $s \in \mathbb{N}$, from (3.3)–(3.5) and (L_6), it holds that

$$\begin{aligned}
\mathbb{E}[W(t)] &\leq e^{\alpha(t-t_s)}\mathbb{E}[W(t_s)] \\
&\leq e^{\alpha(t-t_s)}\left(\beta^s \prod_{i=1}^{s} a_i\right)\mathbb{E}[W(t_0)] \\
&\leq e^{\alpha(t-t_s)}e^{\frac{\ln\gamma}{\Delta}(t_s-t_0)}\mathbb{E}[W(t_0)] \\
&\leq e^{\alpha\Delta-\ln\gamma}e^{\frac{\ln\gamma}{\Delta}(t-t_0)}\mathbb{E}[W(t_0)].
\end{aligned}$$

By condition (L_1), one gets

$$\mathbb{E}[\|x(t)\|^r] \leq Ce^{-\lambda(t-t_0)}\mathbb{E}[\|x_0\|^r],$$

where $C = \frac{b_2}{b_1}e^{\alpha\Delta-\ln\gamma}$ and $\lambda = -\frac{\ln\gamma}{\Delta}$. Thus, system (2.2) is r-ES under $h(t_k) \equiv 0$, with a convergence rate of $\lambda = -\frac{\ln\gamma}{\Delta}$. □

Remark 3.1. *Under the condition of $h(t_k) \equiv 0$, Theorem 3.1 provides sufficient conditions for system* (2.2) *to achieve r-AS and r-ES under the action of ETIC* (2.4). *This result not only offers a theoretical foundation for the stability analysis of system* (2.2) *but also serves as a reference for the selection of control parameters in ETIC* (2.4). *Specifically, smaller event-triggered parameters a_s or shorter inspection intervals Δ_s lead to more frequent triggering of control events, thereby increasing the frequency of IC inputs and accelerating the stabilization process of system* (2.2). *Moreover, it can be observed that small α allows larger δ in the avoidance of Zeno solutions. In addition, if $\beta a_i < 1 - \varepsilon$ for an arbitrarily small $\varepsilon > 0$, then conditions (L_5) and (L_6) are automatically satisfied.*

3.2. With impulsive delay in ETIC

In this subsection, we focus on the analysis of IC inputs with delay and derive stability criteria for r-AS and r-ES of system (2.2).

Theorem 3.2. *Under conditions* $(L_1)-(L_3)$ *of Theorem 3.1, if there holds*

(L_7) $\beta^{l_s}\prod_{i=1}^{l_s} a_{s_i} \to 0, \quad l_s \to \infty,$

where $l_s = \min\{l \in \mathbb{N}^+ : \sum_{i=0}^{l-1}(m_{s_i-1}+1) \geq s\}$ *and* $s_0 = s+1, s_1 = s-m_s, s_{i+1} = s_i - 1 - m_{s_i-1}, i \in \{1, 2, \cdots, l_s-1\}$. m_s *for* $s \in \mathbb{N}^+$ *is defined with reference to Assumption 2.1. Then system* (2.2) *is r-AS under* $h(t_s) \neq 0$.

Furthermore, condition (L_7) *is revised as follows: If there exists a constant* $\gamma < 1$ *such that*

(L_8) $\beta^{l_s}\prod_{i=1}^{l_s} a_{s_i} \leq \gamma^{l_s}, \quad l_s \in \mathbb{N}^+,$

then system (2.2) *is r-ES under* $h(t_s) \neq 0$*, with a convergence rate of* $\lambda = -\frac{\ln\gamma}{(m+1)\Delta}$.

Proof. When the impulsive delay exists in ETIC (2.3), there holds

$$\mathbb{E}[W(t_s)] \leq \beta\mathbb{E}[W(t_s - h(t_s))], \quad s \in \mathbb{N}^+. \tag{3.6}$$

According to Assumption 2.1, there exist integers m_s and m, with $0 \leq m_s \leq m$, such that

$$t_{s-m_s-1} \leq t_s - h(t_s) < t_{s-m_s}, \quad s \in \mathbb{N}^+. \tag{3.7}$$

Therefore, by (3.6), (3.7), and ETIC (2.3), we obtain

$$\mathbb{E}[W(t_s)] \leq \beta a_{s-m_s}\mathbb{E}[W(t_{s-m_s-1})],$$

where $a_s = 1$ for $s \leq 0$.

By iterating the inequality, one can derive

$$\begin{aligned}\mathbb{E}[W(t_s)] &\leq (\beta a_{t_{s-m_s}})(\beta a_{s-m_s-1-m_{s-m_s-1}})\mathbb{E}[W(t_{s-m_s-1-m_{s-m_s-1}-1})],\\ &\leq \left(\beta^{l_s}\prod_{i=1}^{l_s} a_{s_i}\right)\mathbb{E}[W(\phi_0)].\end{aligned} \tag{3.8}$$

According to Eqs (3.1), (3.8), and (L_1), one may deduce that

$$\mathbb{E}[\|x(t)\|^r] \leq \frac{b_2 a_{s+1}}{b_1}\left(\beta^{l_s}\prod_{i=1}^{l_s} a_{s_i}\right)\mathbb{E}[\|\phi_0\|_{\hbar}^r].$$

If condition (L_7) holds, we can get $\lim_{s\to\infty}\mathbb{E}[\|x(t)\|^r] = 0$. As a result, system (2.2) under $h(t_s) \neq 0$ is r-AS.

Besides, based on Eqs (3.3), (3.8), and (L_8), one can derive

$$\begin{aligned}\mathbb{E}[W(t)] &\leq e^{\alpha(t-t_s)}\left(\beta^{l_s}\prod_{i=1}^{l_s} a_{s_i}\right)\mathbb{E}[W(\phi_0)]\\ &\leq e^{\alpha(t-t_s)}e^{\frac{\ln\gamma}{(m+1)\Delta}(t_s-t_0)}\mathbb{E}[W(\phi_0)]\\ &\leq e^{\alpha\Delta-\frac{\ln\gamma}{m+1}}e^{\frac{\ln\gamma}{(m+1)\Delta}(t-t_0)}\mathbb{E}[W(\phi_0)].\end{aligned}$$

By condition (L_1), it holds

$$\mathbb{E}[\|x(t)\|^r] \le Ce^{\lambda(t-t_0)}\mathbb{E}[\|\phi_0\|_{\hbar}^r],$$

where $C = \frac{b_2}{b_1}e^{\alpha\Delta-\frac{\ln\gamma}{m+1}}$ and $\lambda = -\frac{\ln\gamma}{(m+1)\Delta}$. Thus, system (2.2) is r-ES under $h(t_s) \neq 0$, with a convergence rate of $\lambda = -\frac{\ln\gamma}{(m+1)\Delta}$. □

Corollary 3.1. *Under conditions (L_1)–(L_4), suppose that $m_s \equiv m$ for all $s \in \mathbb{N}^+$. If the following condition also holds:*

(L_9) $\beta^{\lceil\frac{s}{m+1}\rceil}\prod_{i=1}^{\lceil\frac{s}{m+1}\rceil} a_{s-m-(i-1)(m+1)} \to 0,\ s \to \infty,$

then system (2.2) is r-AS under $h(t_s) \neq 0$.

Moreover, If condition (L_9) is replaced with the following condition:

(L_{10}) $\beta^{\lceil\frac{s}{m+1}\rceil}\prod_{i=1}^{\lceil\frac{s}{m+1}\rceil} a_{s-m-(i-1)(m+1)} \le \gamma^{\lceil\frac{s}{m+1}\rceil},\ s \in \mathbb{N}^+,$

then system (2.2) is r-ES under $h(t_s) \neq 0$, with a convergence rate of $\lambda = -\frac{\ln\gamma}{(m+1)\Delta}$.

Proof. When $m_s \equiv m$ for all $s \in \mathbb{N}^+$, from Theorem 3.2, one has

$$\begin{aligned} l_s &= \min\{l \in \mathbb{N}^+ : \sum_{i=0}^{l-1}(m+1) \ge s\} \\ &= \min\{l \in \mathbb{N}^+ : l \ge \frac{s}{m+1}\} \\ &= \lceil\frac{s}{m+1}\rceil, \end{aligned} \tag{3.9}$$

and

$$s_i = s - m - (i-1)(m+1), \quad i \in \{1, 2, \cdots, \lceil\frac{s}{m+1}\rceil\}. \tag{3.10}$$

Therefore, conditions (L_7) and (L_8) in Theorem 3.2 are reduced to conditions (L_9) and (L_{10}), respectively, and thus Corollary 3.1 holds. □

Remark 3.2. *In the case of $m_s = 0$, the impulsive delay satisfies $t_{s-1} \le t_s - h(t_s) \le t_s$, which is consistent with the assumptions on impulsive delays in [30,31]. Moreover, the delay model proposed in this paper can be regarded as an extension of the approach in [23] to the context of stochastic systems. Therefore, the impulsive delay model proposed in this paper is capable of covering a wider range of delay types, thereby demonstrating stronger robustness and practical applicability.*

Remark 3.3. *As shown in Theorem 3.2 and Corollary 3.1, the exponential convergence rate of system (2.2) gradually decreases as the parameter m increases. The underlying reason for this phenomenon is that a larger value of m implies a greater impulsive delay introduced in the controller, and the increase in impulsive delay negatively affects the convergence performance and stability of system (2.2).*

Remark 3.4. *It is worth noting that in Theorem 3.1 (the case without impulsive delay), the initial condition is given at a single point $x(t_0) = x_0$, and thus the term $\mathbb{E}[\|x_0\|^r]$ is used. In contrast, Theorem 3.2 considers impulsive delay, requiring an initial function ϕ_0 defined over $[-\hbar, 0]$. Therefore, we adopt the history norm $\mathbb{E}[\|\phi_0\|_{\hbar}^r]$, where $\|\phi_0\|_{\hbar} = \sup_{\theta\in[-\hbar,0]}\|\phi_0(\theta)\|$, to reflect the dependence on the entire past state trajectory.*

4. Applications

In this section, the results from Section 3 are applied to impulsive stochastic systems, which are represented as follows:

$$\begin{cases} dx(t) = (Ax(t) + BF(x(t)))dt + (Cx(t) + DG(x(t)))dB(t), & t \in [t_s, t_{s+1}), \\ x(t_s) = Kx(t_s - h(t_s)), & s \in \mathbb{N}^+, \\ x(t+\theta) = \phi_0, & \theta \in [-\hbar, 0], \end{cases} \tag{4.1}$$

where real matrices $A, B, C, D \in \mathbb{R}^{n\times n}$ and the impulsive control gain matrix $K \in \mathbb{R}^{n\times n}$. Functions $F, G : \mathbb{R}^n \to \mathbb{R}^n$ satisfy the global Lipschitz condition with Lipschitz matrices L_F and L_G, respectively.

Theorem 4.1. *Suppose that there exist constants $\alpha, \delta, m > 0$, $0 < \beta < 1$, and $\{a_s\}_{s\in\mathbb{N}^+} > 1$, as well as positive definite $n \times n$ matrices P, Q_1, Q_2, such that the following inequality holds:*

$$\begin{bmatrix} \Lambda & PB & L_G^T D^T P \\ * & -Q_1 & 0 \\ * & * & -Q_2 \end{bmatrix} \le 0, \tag{4.2}$$

$$\begin{bmatrix} \beta P & K^T P \\ * & -P \end{bmatrix} \le 0, \tag{4.3}$$

$$0 < \delta < \frac{\ln a_{s+1}}{\alpha}, \quad s \in \mathbb{N}, \tag{4.4}$$

where $\Lambda = PA + A^T P + L_F^T Q_1 L_F + C^T PC + L_G^T D^T PDL_G + C^T Q_2 C - \alpha P$.

Then, based on conditions (L_4)–(L_9) *from Theorems 3.1 and 3.2, the r-AS or r-ES of system* (4.1) *under the ETIC* (2.3) *framework can be established.*

Proof. Consider $W(x(t)) = x^T(t)Px(t)$; then it implies

$$\lambda_{\min}(P)\,\|x(t)\|^2 \le W(x(t)) \le \lambda_{\max}(P)\,\|x(t)\|^2 .$$

According to Definition 2.2, for $t \in [t_{s-1}, t_s)$, $s \in \mathbb{N}^+$, we obtain

$$\begin{aligned} \mathcal{L}W(t) =& W_t + W_x(Ax(t) + BF(x(t))) + \frac{1}{2} tr[(Cx(t) + DG(x(t)))^T W_{xx}(Cx(t) + DG(x(t)))] \\ =& 2x^T(t)P(Ax(t) + BF(x(t)) + (Cx(t) + DG(x(t)))^T P(Cx(t) + DG(x(t)) \\ =& 2x^T(t)PAx(t) + 2x^T(t)PBF(x(t)) + x^T(t)C^T PCx(t) + G^T(x(t))D^T PDG(x(t)) \\ & + G^T(x(t))D^T PCx(t) + x^T C^T PDG(x(t)). \end{aligned} \tag{4.5}$$

Applying Lemma 2.1, it is obtained that

$$\begin{cases} 2x^T(t)PBF(x(t)) \le x^T(t)PBQ_1^{-1}B^T Px(t) + x^T(t)L_F^T Q_1 L_F x(t), \\ G^T(x(t))D^T PDG(x(t)) \le x^T(t)L_G^T D^T PDL_G x(t), \\ G^T(x(t))D^T PCx(t) + x^T C^T PDG(x(t)) \le x^T(t)C^T Q_2 Cx(t) + x^T(t)L_G^T D^T PQ_2^{-1} PDL_G x(t). \end{cases} \tag{4.6}$$

By substituting (4.6) into (4.5) and using (4.2), yields

$$\mathcal{L}W(t) = x^T(t)(\Lambda + \alpha P + PBQ_1^{-1}B^T P + L_G^T D^T Q_2^{-1} PDL_G)x(t) \le \alpha W(t). \tag{4.7}$$

When $t = t_s$, $s \in \mathbb{N}^+$, one can see

$$\begin{aligned} W(t_s) &= x^T(t_s)Px(t_s) \\ &= x^T(t_s - h(t_s))K^T PKx(t_s - h(t_s)) \\ &= \beta W(t_s - h(t_s)). \end{aligned} \tag{4.8}$$

According to Theorem 3.1, when the impulsive delay $h(t_s) \equiv 0$, system (4.1) is r-AS if condition (L_5) is satisfied; if condition (L_6) holds, system (4.1) is r-ES with a convergence rate $\lambda = -\frac{\ln\gamma}{\Delta}$. Furthermore, according to Theorem 3.2, when the impulsive delay $h(t_s) \neq 0$, system (4.1) remains r-AS under condition (L_7) and becomes r-ES with a convergence rate of $\lambda = -\frac{\ln\gamma}{(m+1)\Delta}$ if condition (L_8) is satisfied. □

5. Numerical examples

Two simulation examples are provided in this section to validate the proposed theory and illustrate its effectiveness and applicability.

Example 5.1. Consider system (4.1) with $F(x(t)) = 0.6\cos(1.1t)x(t), G(x(t)) = 0.6\sin(1.1t)x(t)$, and

$$A = \begin{bmatrix} -0.8 & -1.9 & -2.1 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.9 & 0.2 & 0.3 \\ 0.2 & 0.9 & 0 \\ 0.3 & 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.3 & 0.5 & 0.3 \\ 0.3 & -0.5 & 0 \\ 0.1 & 0 & 0.3 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

Let $x_0 = [-3, 2, 3]^T$, the state trajectories of system (4.1) without IC input are shown in Figure 1, which indicates that system (4.1) is unstable.

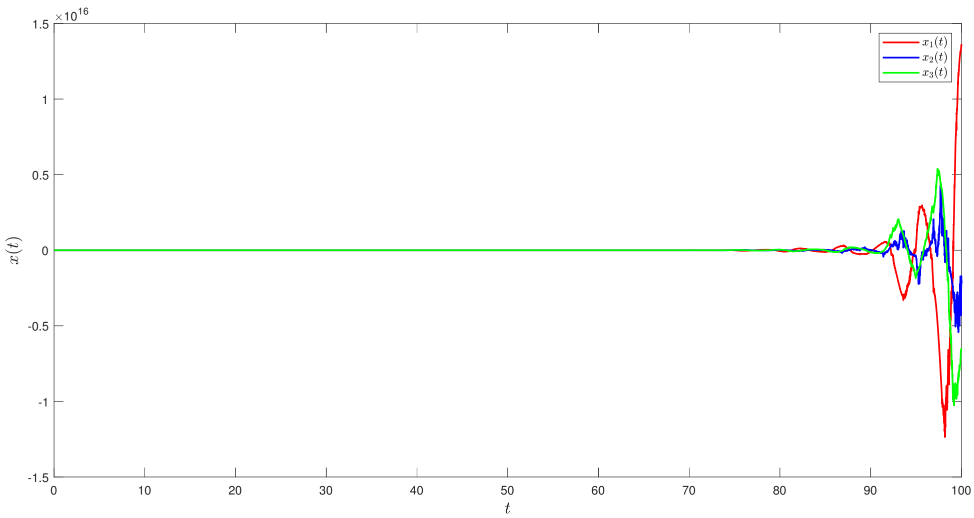


Figure 1. State trajectories of system (4.1) without ETIC.

To further analyze the r-ES of system (4.1), the ETIC (2.3) strategy is introduced with the parameters chosen as $a_s = e^{\frac{0.36(s+2)}{s+1}}$, $\Delta_s = 2 + \frac{(-1)^s}{s}$, $\delta = 0.01$, $\gamma = 0.9969$, $L_F = L_G = 0.6I_3$, $\alpha = 3.2$, $\beta = 0.5$. In addition, by solving the linear matrix inequalities (LMI) (4.2) and (4.3), the resulting matrices P and K are obtained as

$$P = \begin{bmatrix} 0.8300 & -0.1899 & 0.5520 \\ -0.1899 & 0.4771 & 0.1602 \\ 0.5520 & 0.1602 & 1.4382 \end{bmatrix}, \quad K = \begin{bmatrix} 0.69 & 0 & 0 \\ 0 & 0.67 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}.$$

According to Theorem 3.1, under the given data conditions, system (4.1) is r-ES under impulsive delay $h(t_s) \equiv 0$, with a convergence rate of $\lambda = -\frac{\ln\gamma}{\Delta} = 0.0012$. Figure 2 shows the state trajectories of system (4.1).

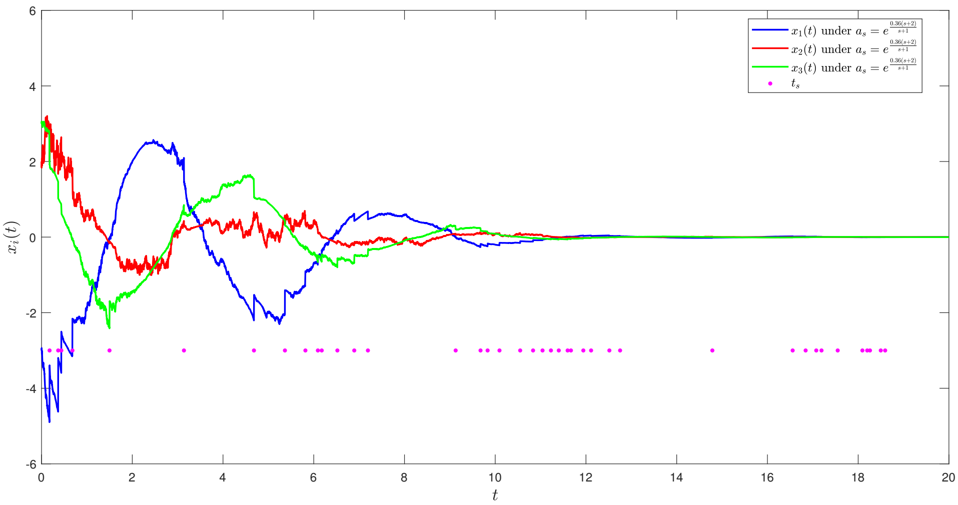
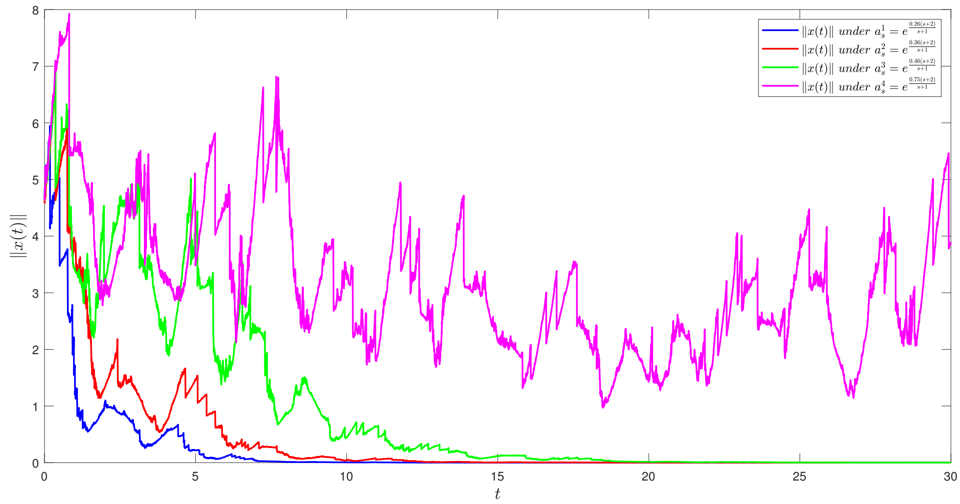


Figure 2. State trajectories of system (4.1) under impulsive delay $h(t_s) \equiv 0$.

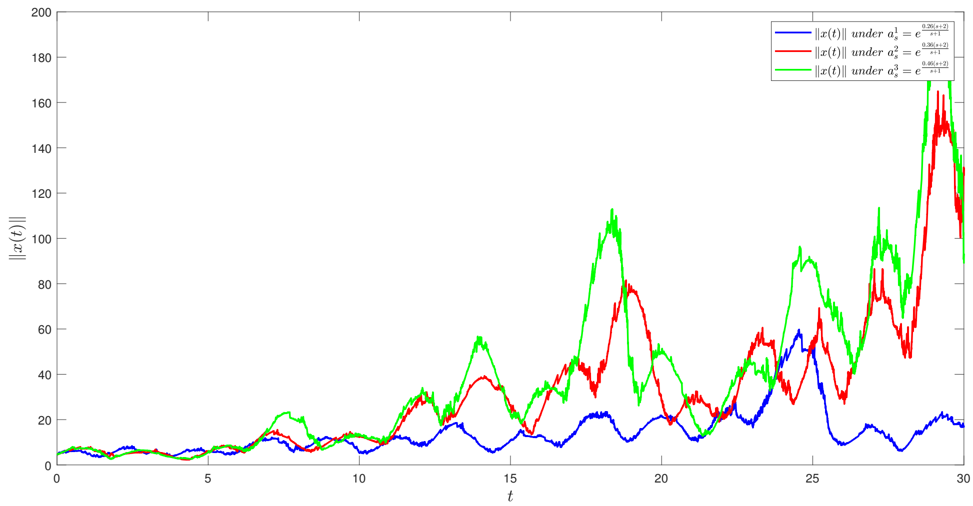
We first investigate the influence of different event-triggered parameters on the settling time of system (4.1). The inspection interval and impulsive control gain are fixed as $\Delta_s = 2 + \frac{(-1)^s}{s}$ and K, respectively. Three sets of event-triggered parameters, $a_s^1 = e^{\frac{0.26(s+2)}{s+1}}$, $a_s^2 = e^{\frac{0.36(s+2)}{s+1}}$, and $a_s^3 = e^{\frac{0.46(s+2)}{s+1}}$ are considered. Figure 3(a) illustrates the trajectories of the state norm under these different event-triggered parameters.

Next, we analyze the impact of different inspection intervals on the settling time of system (4.1). The event-triggered parameter and impulsive control gain are fixed as $a_s = e^{\frac{0.45(s+2)}{s+1}}$ and K, respectively. Three sets of inspection intervals, $\Delta_s^1 = 1.01 + \frac{(-1)^s}{s}$, $\Delta_s^2 = 1.3 + \frac{(-1)^s}{s}$, and $\Delta_s^3 = 3 + \frac{(-1)^s}{s}$ are considered. Figure 4(a) shows the corresponding state norm trajectories under different inspection intervals.

Remark 5.1. *When the event-triggered parameter is adjusted to* $a_s^4 = e^{\frac{0.75(s+2)}{s+1}}$*, such that condition* (L_5) *is no longer satisfied, system (4.1) exhibits apparent instability. In contrast, when the inspection interval is increased to* $\Delta_s^4 = \infty$*, i.e., no forced impulse is applied to system (4.1), system (4.1) remains stable, although with a slower convergence rate, which is consistent with the conclusion reported in [21]. Furthermore, if the control gain matrix is adjusted to* $K = \text{diag}\{0.9, 0.9, 0.9\}$ *such that the* *condition* (L_5) *fails, while keeping the other parameters in Figures 3(a) and 4(a) unchanged, it can be observed from Figures 3(b) and 4(b) that system (4.1) becomes unstable. These results indicate that the stability of system (4.1) is highly sensitive to the tuning of the event-triggered parameter and impulsive control gain, while exhibiting a certain degree of robustness with respect to variations in the inspection interval.*



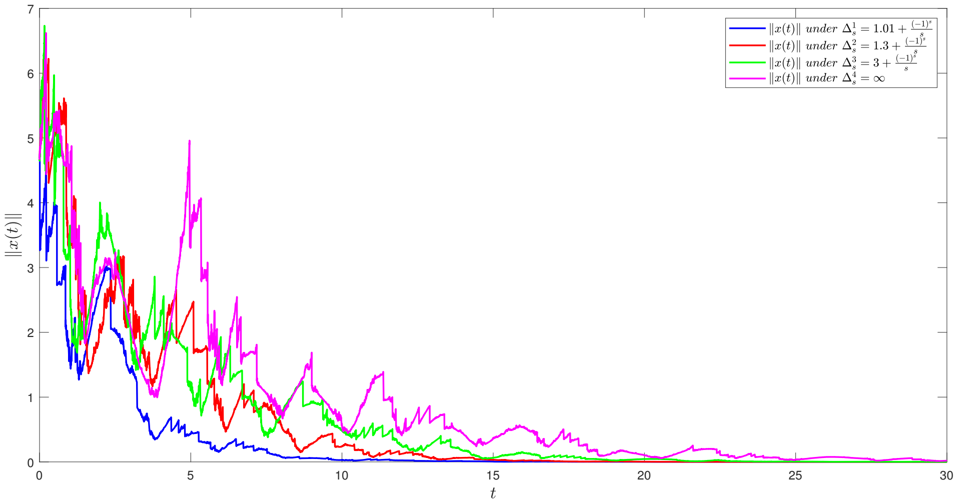
(a)



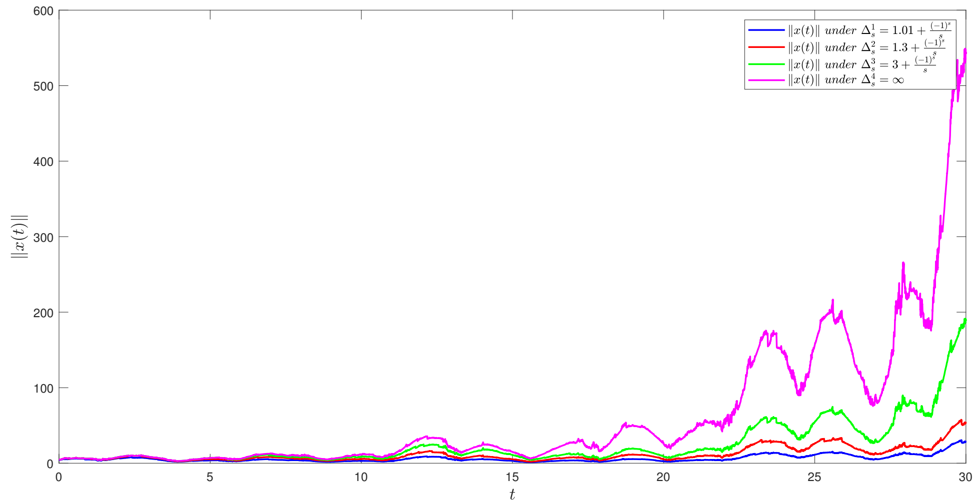
(b)

Figure 3. State norm of system (4.1) under different a_s and K.

Remark 5.2. *Figures 3(a) and 4(a) clearly illustrate the dynamic evolution of the state norm of system (4.1) under the ETIC (2.3) strategy. The simulation results indicate that smaller event-triggered parameters and shorter inspection intervals can accelerate the stabilization process. This* *demonstrates that increasing the triggered frequency contributes to improving the convergence rate of the system.*



(a)



(b)

Figure 4. State norm of system (4.1) under different Δ_s and K.

Finally, we investigate the influence of impulsive delay on the stability of system (4.1). The impulsive delay is defined as $h(t) = t - t_{s-m} - 0.01|sin(t)|$, where $m \in \{0, 1, 2\}$. The event-triggered parameter and inspection interval are selected as $a_s = e^{\frac{0.4(s+2)}{s+1}}$ and $\Delta_s = 1.5 + \frac{(-1)^s}{s}$, respectively. According to Corollary 3.1, system (4.1) remains r-ES under these conditions. Figure 5 shows the trajectories of the state norm under different impulsive delays.

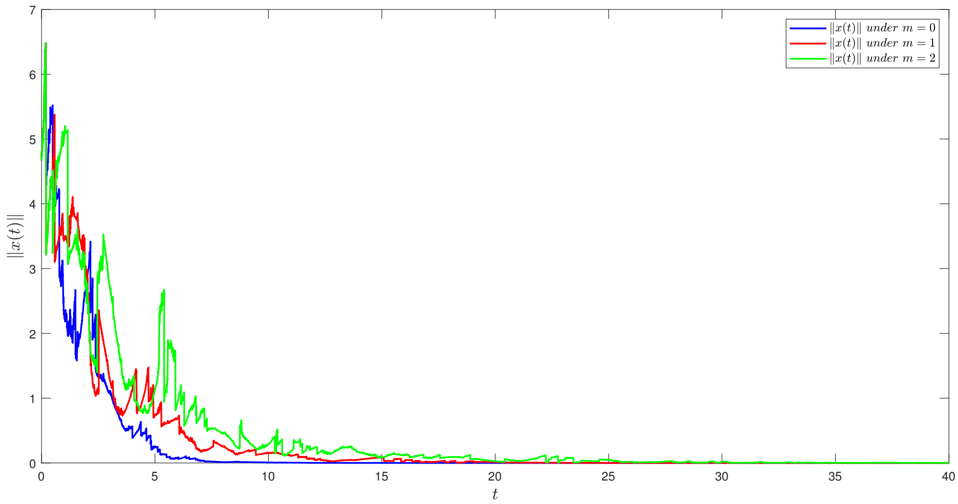


Figure 5. State norm of system (4.1) under different $h(t_s)$.

Example 5.2. Consider the following the leader state:

$$dx_0(t) = Ax_0(t)dt + \mu x_0(t)dB(t), \tag{5.1}$$

where $\mu = 0.85$ denotes the intensity of the stochastic disturbance. Accordingly, the following multi-agent system is considered:

$$dx_i(t) = (Ax_i(t) + \sum_{j=1}^{5} a_{ij}(x_j(t) - x_i(t)) + d_i(x_0(t) - x_i(t)))dt + \mu x_i(t)dB(t), \tag{5.2}$$

where $i = 1, 2, 3, 4, 5$, and the coefficient matrix A is defined as

$$A = \begin{bmatrix} -2 & 2.5 & 0 \\ -1 & -1 & 1 \\ 0 & -1 & 0 \end{bmatrix}.$$

Consider the communication topology of a leader-following multi-agent system as depicted in Figure 6. The leader adjacency matrix $\mathcal{D} = [d_i]$, the weighted adjacency matrix $\mathcal{A} = [a_{ij}]$, and the Laplacian matrix $\mathcal{L}$ corresponding to Figure 6 are given as follows:

$$\mathcal{D} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} 1.5 & 0 & 0 & 0 & -1.5 \\ 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1.2 & 1.2 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

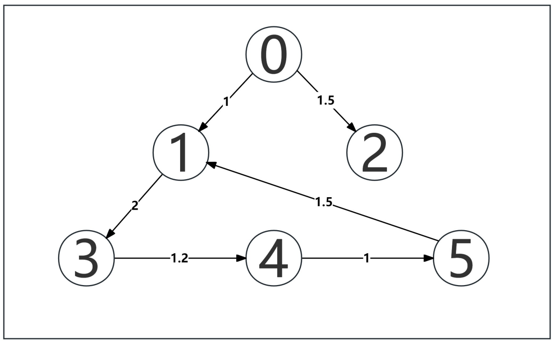


Figure 6. Topology of a leader-following multi-agent system.

Define $e(t) = [e_1^T(t), \cdots, e_5^T(t)]^T$, where $e_i(t) = x_i(t) - x_0(t)$. To achieve leader-follower consensus control for systems (5.1) and (5.2), ETIC is introduced, resulting in the following error system:

$$\begin{cases} de(t) = ((I_5 \otimes \mathcal{A} - (\mathcal{L} + \mathcal{D}) \otimes I_3)e(t))dt + \mu e(t)dB(t), & t \in [t_s, t_{s+1}), \\ e(t_s) = (I_5 \otimes K)e(t_s - h(t_s)), & s \in \mathbb{N}^+, \\ e(t_0 + \theta) = \Phi_0, & \theta \in [-h, 0]. \end{cases} \tag{5.3}$$

And the triggered parameters are set as $a_s = e^{\frac{0.475(s+2)}{s+1}}$, $\Delta_s = 1.5 + \frac{(-1)^s}{s}$, $\delta = 0.1$. The corresponding ETIC is structured as shown below:

$$\begin{cases} t_{s+1}^* = \inf\left\{t > t_s + \delta : e^T(t)(I_5 \otimes P)e(t) \geq a_{s+1}e^T(t_s)(I_5 \otimes P)e(t_s)\right\}, \\ t_{s+1} = \min\left\{t_{s+1}^*,\ t_s + \Delta_{s+1}\right\}. \end{cases}$$

By selecting the parameters $\alpha = 4.2$ and $\beta = 0.49$, based on Theorem 4.1, the matrices

$$P = \begin{bmatrix} 0.1155 & 0.0305 & 0.0041 \\ 0.0305 & 0.1462 & -0.0008 \\ 0.0041 & -0.0008 & 0.1577 \end{bmatrix}, \quad K = \begin{bmatrix} 0.6912 & 0 & 0 \\ 0 & 0.6991 & 0 \\ 0 & 0 & 0.6925 \end{bmatrix}$$

are obtained by solving the LMI

$$\begin{cases} I_5 \otimes (PA + A^T P + \sigma^2 P - \alpha P) - (\mathcal{L} + \mathcal{D}) \otimes P - (\mathcal{L} + \mathcal{D})^T \otimes P \leq 0, \\ K^T PK - \beta P \leq 0. \end{cases}$$

Setting the initial conditions as $x_0(0) = [-1, 7, 13]^T$, $x_1(0) = [-1, 2, 3]^T$, $x_2(0) = [-9, 10, -5]^T$, $x_3(0) = [6, 3, 5]^T$, $x_4(0) = [3, -4, 7]^T$, and $x_5(0) = [-4, 5, 2]^T$. Figure 7 depicts the trajectories of the state norm of error system (5.3) under different impulsive delays, where the impulsive delay is defined as $h(t) = t - t_{s-m} - 0.05|\sin(t)|$, with $m \in \{0, 1, 2\}$.

Remark 5.3. *Figures 5 and 7 depict the state norm trajectories of system (4.1) and the multi-agent error system (5.3), respectively, under different impulsive delay conditions. The simulation results show that as the impulsive delay increases, the time required for the system to reach stability also increases, demonstrating that impulsive delays have a negative impact on the convergence rate. Therefore, the proper design of the impulsive delay plays a crucial role in improving system stability.*

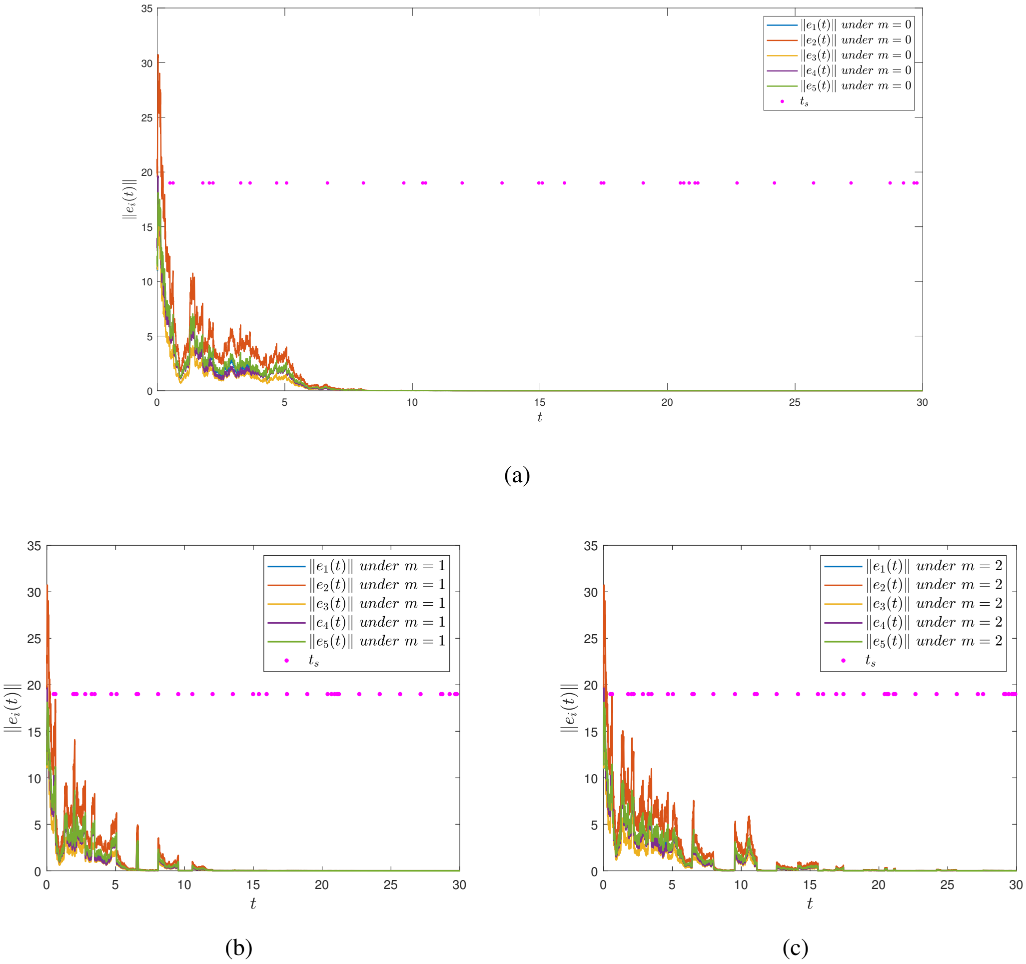


Figure 7. State norm of error system (5.3) under these different $h(t_s)$.

6. Conclusions

This paper mainly addresses the ETIC problem in stochastic systems. Based on Lyapunov functions and stochastic analysis methods, a series of sufficient conditions are established to ensure that the system achieves r-AS and r-ES under two scenarios: without impulsive delay and with impulsive delay. Meanwhile, the effects of triggered parameters and impulsive delay on system stability are analyzed. Finally, two representative numerical simulation examples are provided to verify the feasibility and effectiveness of the proposed control strategies. Future research will focus on extending these strategies to more complex system environments, such as external disturbances or switching topologies, aiming to achieve more efficient control and resource optimization.

Author contributions

Huiling Li: Writing–original draft; Jin-E Zhang: Supervision, Writing–review and editing; Ailong Wu: Writing–review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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