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Research article

A novel self-adaptive nonlinear grey Bernoulli model for forecasting China's industrial electricity consumption

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Abstract: The rigidity of subjectively preset adjustable parameters in the existing NGBM(1,1) model family restricts its ability to handle complex nonlinear time series. To address this issue, this paper proposed a self-adaptive nonlinear grey Bernoulli model [SANGBM(1,1)] with enhanced predictive capabilities. Three key innovations were introduced. First, hyperparameterized functions were developed to dynamically optimize adjustable parameters, overcoming the rigidity of subjectively preset hyperparameters in the existing NGBM(1,1) model family and improving adaptability. Second, the derived implicit time response formula of the SANGBM(1,1) model fundamentally resolved the jump error inherent in the traditional NGBM(1,1) model. Third, based on the systematic deconstruction of the global sensitivity mechanism of the SANGBM(1,1) model's hyperparameters, a novel data-driven model structure selection algorithm that integrates the time series rolling cross-validation method with the firefly algorithm was designed to enhance generalization performance. Empirical results demonstrate the feasibility and effectiveness of the proposed model. Additionally, China's industrial electricity consumption for the next four years (2023–2026) was predicted, offering valuable references for formulating effective industrial electricity development planning.

Keywords: nonlinear grey Bernoulli model; industrial electricity consumption; grey system theory; grey prediction model
Mathematics Subject Classification: 65Q10, 62M10

1. Introduction

Electricity is a core energy source that supports modern industrial development. Due to the rapid economic growth that China has experienced in recent decades, the country's industrial electricity consumption has increased dramatically. According to official statistics, China's industrial sector consumed 5741.3 billion kWh of electricity in 2022, accounting for 64.98% of the country's total power usage. This figure highlights the industry's substantial dependence on electrical resources. Given that electricity cannot be stored in large quantities, maintaining a dynamic balance between the supply and demand of electricity is crucial for the operational efficiency of the power grid. Overestimating industrial electricity demand results in the wasteful use of power resources, whereas underestimating it can bring industrial production to a halt. Consequently, precise prediction of industrial electricity consumption holds paramount importance for enhancing the operational efficiency of the power grid. At present, China's industrial sector is undergoing a pivotal phase of structural transformation and upgrading, marked by notable uncertainties. This situation makes it difficult for outdated statistics to accurately reflect the latest evolutionary trends in current industrial electricity consumption. Moreover, industrial electricity usage displays pronounced nonlinear patterns. Consequently, precisely predicting industrial electricity consumption in China presents an exceptionally demanding challenge. In light of this, the core objective of this study is to develop an adaptive nonlinear grey Bernoulli model applicable to scenarios with limited data and insufficient information, aiming to capture the latest evolution patterns of China's industrial electricity consumption and provide scientific decision-making support for the power sector.

Accurately predicting electricity consumption is a challenging task, as it is influenced by a multitude of external factors [1]. To tackle this, academics have successively developed a large number of electricity consumption forecasting models, which can be summarized into three categories: Time series models, econometric approaches, and artificial intelligence (AI) prediction methods.

Time series models are extensively employed in electricity consumption forecasting owing to their low computational complexity and excellent short-term predictive capabilities. For instance, Pappas et al. [2] applied the ARMA model to estimate the electricity demand load in Greece, while Ma çaira et al. [3] used Pegel's exponential smoothing techniques to estimate the residential electricity consumption in Brazil. However, the drawbacks of these methods are also obvious, such as failing to take external factors into account and requiring high-quality data to guarantee prediction accuracy.

Compared with time series forecasting models, econometric methods have been widely used in the field of electricity demand forecasting because they incorporate more external factors into the model, significantly improving their interpretability and prediction accuracy. For instance, Apadula and his team [4] employed the regression prediction model to investigate the impact of weather conditions on monthly electricity demand in Italy. On this basis, they constructed a novel multiple regression prediction model to project monthly electricity consumption, achieving high modeling accuracy. Satre-Meloy [5] utilized optimized regression models to predict household electricity consumption in California. Nevertheless, a significant limitation of these approaches is that the selection of explanatory variables relies on expert knowledge, and the selection process is somewhat subjective.

In addition to the aforementioned electricity demand forecasting models, AI methods have been extensively utilized in electricity consumption forecasting in recent years. These methods overcome the reliance of traditional approaches on prior knowledge and possess powerful nonlinear forecasting capabilities. For instance, Azadeh et al. [6] employed an integrated algorithm based on the ANN model to predict Iran's monthly electricity consumption. Similarly, Günay [7] successfully applied an artificial neural network to estimate Turkey's annual total electricity consumption, and the prediction results of the artificial neural network surpassed the official predictions and exhibited a remarkably high level of modeling accuracy. In another study, Dudek [8] utilized a univariate neural network model for short-term load forecasting and compared it with other popular time series forecasting methods. Empirical results proved the validity of the new proposed model. However, the biggest drawback of these methods is that they require large-scale data to ensure their prediction accuracy and have a black-box nature.

In summary, as shown in Table 1, the aforementioned models share a common limitation: their predictive precision is excessively dependent on the patterns and quality of historical data. However, China's industrial structure is currently undergoing a critical period of transformation and upgrading. Consequently, data accurately reflecting the latest trends in China's industrial electricity demand are very scarce, restricting the applicability of these methods.

Table 1. Comparative analysis of major methods for electricity consumption forecasting.

Method category	Representative models	Advantages	Limitations
Time series methods	ARMA [2], ETS [3]	Simple implementation with a clear mathematical foundation.	1. Limited capability in capturing complex nonlinear patterns. 2. Sensitivity to abrupt changes or outliers. 3. High demands on data quality. 4. Failure to consider external factors.
Econometric methods	Regression prediction model [4]	High interpretability.	1. Dependence on stringent exogeneity conditions. 2. Limited efficacy in high-frequency data modeling. 3. A reliance on domain expertise for feature specification.
Artificial intelligence methods	ANN [6]	Strong nonlinear fitting capability.	1. Require massive historical datasets for training. 2. Limited interpretability of inner mechanisms. 3. High overfitting risk.

Professor Deng [9] pioneered the grey prediction model in the 1980s, offering a new approach for predicting uncertain systems associated with data scarcity. Compared with the aforementioned forecasting methods, the grey prediction model can reliably reveal the inherent evolutionary trends in uncertain systems with just a small dataset (a minimum of four data points). This high adaptability to small sample sizes has garnered widespread attention, leading to extensive application in electricity consumption forecasting. For example, Ding et al. [1] proposed an optimized grey prediction model incorporating dynamic initial conditions and a rolling mechanism and conducted a forecast of China's electricity consumption, achieving satisfactory results. Zheng et al. [10] proposed an unbiased grey Bernoulli model to forecast China's hydroelectricity consumption, and its prediction accuracy significantly outperformed the traditional NGBM(1,1) model. Liu et al. [11] developed a conformable fractional unbiased grey model, demonstrating superior accuracy in forecasting China's hydroelectricity consumption compared to existing grey models. Yang et al. [12] developed a novel structural adaptive discrete grey Bernoulli model [DHGBM(1,1)], incorporating Hausdorff fractional-order accumulation and nonlinear dynamic structures, for forecasting China's electricity generation. The model demonstrated high prediction accuracy and robustness. Guo et al. [13] employed a grey model enhanced by the unified new-information priority accumulating generation operator to forecast wind electricity generation across different regional hierarchies. The new model significantly outperformed its competitors, such as ARIMA, BPNN, and LSTM, achieving satisfactory forecasting results. Nevertheless, these grey models share a common flaw: The reliance on the unrealistic modeling assumption that "some adjustable parameters are known", resulting in a rigid structure that compromises adaptability to increasingly complex industrial electricity consumption forecasting, thus requiring further improvements.

Beyond electricity demand forecasting, grey prediction models have also been successfully applied in fields such as tourism demand prediction [14,15], energy demand estimation [16,17], and traffic flow projection [18,19], owing to their exceptional predictive performance with small samples and poor information. Notably, as a nonlinear extension of the traditional GM(1,1) model, the NGBM(1,1) model demonstrates significant advantages over the GM(1,1) model in addressing nonlinear problems. Since it was first proposed by Chen [20] in 2008, this model has rapidly gained widespread attention in academia. Nevertheless, empirical studies reveal that when forecasting complex uncertain systems, its robustness requires further enhancement. To address this limitation, current research on enhancing the NGBM(1,1) model primarily focuses on the following three key directions:

(1) Model structure optimization. The fixed structure limits the ability of NGBM(1,1) models to deal with complex uncertain systems [21]. Traditional NGBM(1,1) models usually preset adjustable parameters such as the background value, initial condition, accumulated generating operator, and grey action quantity before modeling, which restricts the flexibility of the model. To address this limitation, scholars have sought to optimize adjustable parameters by introducing parameterized functions, such as background value [22–25], initial condition [26–29], accumulated generating operator [30–33], and grey action quantity [34]. To further enhance the NGBM(1,1) model's capability in capturing complex dynamic characteristics, Ma et al. [35] proposed a time-delayed fractional grey Bernoulli model with independent fractional orders (TDF-GBM). Empirical results demonstrate that this model significantly outperforms other competing models. However, existing studies have only optimized partial adjustable parameters while presetting others, thus partially limiting model flexibility. To overcome these constraints, the current paper aims to establish a unified optimization framework that incorporates all adjustable parameters within the NGBM(1,1) model, thereby significantly enhancing its adaptability.

(2) Hyperparameter estimation. Hyperparameters are the key factors that determine the prediction accuracy of the NGBM(1,1) model. Existing research predominantly explores the optimal hyperparameters with the help of computer programs [36,37] or heuristic algorithms, including the particle swarm algorithm [18,26,31], genetic algorithm [38,39], and whale optimization algorithm [40]. However, these approaches often neglect the potential overfitting problem in NGBM(1,1) models. Model overfitting typically stems from excessive complexity [11], yet determining appropriate model complexity a priori remains challenging given real-world system intricacies. Zhou [41] demonstrated that developing a scientific hyperparameter tuning mechanism can effectively mitigate overfitting risks. A fundamental tension exists in NGBM(1,1) optimization: While optimizing more hyperparameters enhances model flexibility, it simultaneously elevates model complexity. This raises an important research question: Does this enhanced flexibility thus increase the overfitting risk of the NGBM(1,1)

model? Unfortunately, this critical question has received insufficient attention in current NGBM(1,1) research, and no systematic hyperparameter optimization framework has been developed to address this challenge.

(3) Model mechanism improvement. Although the traditional NGBM(1,1) model demonstrates remarkable efficacy in the realm of nonlinear prediction, it still fails to fundamentally address its jump error between parameter estimation and actual prediction, which affects the robustness of its prediction performance. To mitigate this issue, researchers have proposed several targeted solutions. Zheng's team [10] put forward a new type of unbiased NGBM(1,1) model, whereas Wu's group [40] presented an innovative FDNGBM(1,1) model by introducing the idea of discrete grey modeling into the traditional NGBM(1,1) model. Furthermore, Wu et al. [42] developed a novel conformable fractional-order grey Bernoulli model by optimizing linear parameters and establishing mathematical identities, which effectively solves the jump error inherent in the traditional NGBM(1,1) model. Motivated by current research advances, this research presents an implicit time-response formula that fundamentally eliminates jump errors in the conventional NGBM (1,1) model.

To address the aforementioned limitations in the existing NGBM(1,1) model family, this study introduces the innovative SANGBM(1,1) model, with its primary innovations encapsulated in the following four dimensions:

(1) A unified optimization framework incorporating all adjustable parameters of the NGBM(1,1) model is formulated, effectively addressing the limitation of subjectively preset adjustable parameters in the traditional NGBM(1,1) model and its modified versions while substantially enhancing model adaptability.

(2) An innovative data-driven model structure selection algorithm is devised, combining the time-series rolling cross-validation method with the firefly algorithm to improve generalization capability. This enables the SANGBM(1,1) model to autonomously adapt its structure to data characteristics, significantly mitigating overfitting risks.

(3) The implicit time-response equation of the proposed model is deduced, eliminating the intrinsic jump error between parameter estimation and actual prediction in the conventional NGBM(1,1) model.

(4) The SANGBM(1,1) model is applied to predict China's industrial electricity consumption from 2023 to 2026, and targeted policy recommendations are provided.

The remaining content of this paper is organized as follows: Section 2 elaborates on the modeling methodology of the self-adaptive nonlinear grey Bernoulli model [SANGBM(1,1)]. Section 3 evaluates the proposed SANGBM(1,1) model's accuracy using two real-world cases. Section 4 explores the practical application of the SANGBM(1,1) model in predicting industrial electricity consumption in China. Finally, key conclusions and future research directions are presented in Section 5.

2. Methodology

2.1. Traditional NGBM(1,1) model

Definition 2.1. Let the original observation data sequence be denoted as $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(m)\right)$, $m \geq 4$. The sequence $X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(m)\right)$ can be computed by performing a first-order accumulation on $X^{(0)}$, where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \quad, k = 1,2,\cdots,m. \tag{1}$$

Definition2.2. Assuming $X^{(0)}$ and $X^{(1)}$ are as stated in Definition 2.1, the differential expression of the conventional NGBM(1,1) model is presented as follows:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b\left(x^{(1)}(t)\right)^n. \tag{2}$$

In Eq (2), parameter n is referred to as the power exponent, with $n \neq 1$.

By performing integration on both sides of Eq (2) across the range from $k-1$ to k, we can derive the NGBM(1,1) model's difference version.

$$x^{(0)}(k) + az^{(1)}(k) = b[z^{(1)}(k)]^n, \tag{3}$$

where $z^{(1)}(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1)), k = 2,3,\cdots,m.$ is the background value. The structural parameter vector $[\hat{a}, \hat{b}]$ can be estimated through the ordinary least squares (OLS) method, which is

$$[\hat{a}, \hat{b}]^T = (B^T B)^{-1} B^T Y, \tag{4}$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & [z^{(1)}(2)]^n \\ -z^{(1)}(3) & [z^{(1)}(3)]^n \\ \vdots & \vdots \\ -z^{(1)}(m) & [z^{(1)}(m)]^n \end{bmatrix}, \qquad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(m) \end{bmatrix}.$$

Under the initial value $\hat{x}^{(1)}(1) = x^{(0)}(1)$, we obtain the particular solution of the NGBM(1,1) model, as shown in Eq (5).

$$\hat{x}^{(1)}(k+1) = \left[\left(x^{(0)}(1)^{1-n} - \frac{\hat{b}}{\hat{a}}\right) e^{-\hat{a}(1-n)k} + \frac{\hat{b}}{\hat{a}}\right]^{1/(1-n)}. \tag{5}$$

By performing a reverse transformation on the predicted value $\hat{X}^{(1)}$, the simulated and predicted values $\hat{X}^{(0)} = \left(\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(m)\right)$ are derived, where

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \quad , k = 2,3,\cdots,m.$$

2.2. The novel proposed SANGBM(1,1) model

This section first elucidates the optimization methods for tunable parameters in the traditional NGBM(1,1) model. Subsequently, it systematically expounds the definition and modeling mechanism of the SANGBM(1,1) model. Building upon this foundation, the compatibility of the proposed model is analytically examined. Following this, a hyperparameter estimation method for the SANGBM(1,1) model is proposed. Finally, detailed modeling procedures are delineated.

2.2.1. Optimization of the NGBM(1,1) model tunable parameters

(1) Initial value optimization

Equation (5) indicates that the conventional NGBM(1,1) model assumes that the initial value equals $x^{(0)}(1)$. Nevertheless, based on the least-squares principle, there is no assurance that the optimal fitting curve will traverse the point $(1, x^{(0)}(1))$. As a result, the assumption that the initial value equals $x^{(0)}(1)$ lacks sufficient theoretical justification. To address this issue, the current paper suggests considering the initial condition as grey information, and it should be ascertained based on the inherent characteristics of the modeling data. Specifically, we introduce a grey correction term λ to the first element of the accumulated generating sequence, thereby obtaining an optimized initial value. The time response expression of the optimized NGBM(1,1) model is provided by Eq (6). From Eq (6), it can be concluded that the optimized initial value will pass through the best-fitting curve. Consequently, the new proposed optimization approach has a scientific theoretical basis.

$$\begin{cases} \hat{x}^{(1)}(k+1) = \left[\left(\hat{x}^{(1)}(1)^{1-n} - \frac{\hat{b}}{\hat{a}}\right)e^{-\hat{a}(1-n)k} + \frac{\hat{b}}{\hat{a}}\right]^{1/(1-n)} \\ \hat{x}^{(1)}(1) = x^{(0)}(1) + \lambda \end{cases}. \tag{6}$$

(2) Background value optimization

The conventional NGBM(1,1) model adopts a straightforward formula $z^{(1)}(k) = \frac{1}{2}(x)^{(1)}(k) + x^{(1)}(k-1)$ to compute the background value. However, this approximation deviates fundamentally from the theoretically precise integral formulation $\int_{k-1}^{k} x^{(1)}(t)dt$. This discrepancy constitutes a significant theoretical limitation in the traditional grey modeling approach, adversely affecting prediction accuracy. To address this fundamental issue while adhering to the simplicity principle of grey system theory, we propose an enhanced background value calculation method based on linear interpolation, as shown in Eq (7).

$$z^{(1)}(k) = \theta x^{(1)}(k) + (1-\theta)x^{(1)}(k-1), 0 \le \theta \le 1, \tag{7}$$

where θ denotes the background value generation coefficient. In this paper, we construct a nonlinear constrained optimization model whose objective function is to achieve the minimum value of the MAPE. The optimal background value generation coefficient θ is then determined through a time-series rolling cross-validation method combined with the firefly algorithm.

(3) Accumulated generating operator optimization

The accumulated generating operation (AGO) serves as an effective method to reduce randomness in original sequences while revealing integral characteristics inherent in chaotic raw data. Analysis of the NGBM(1,1) modeling mechanism indicates that the conventional NGBM(1,1) model usually assumes that the cumulative order is equal to 1, which goes against the principle of giving priority to new information. In consideration of the significance of new information priority and the distinctive characteristics of fractional-order accumulation, we introduce the fractional-order accumulation operation to preprocess the original modeling data. This approach enables more comprehensive extraction of integral characteristics and nonlinear components embedded within the raw data.

(4) Grey action quantity optimization

In the conventional NGBM(1,1) model, it is commonly assumed that the grey action quantity is a constant b, treating external perturbations as time-invariant. However, Yin et al. [43] demonstrated that the grey action quantity exhibits time-varying characteristics. Therefore, the unreasonable modeling assumption regarding the grey action quantity constitutes one of the primary causes of poor robustness in traditional grey prediction models. To address this limitation, we propose an enhanced formulation where the grey action quantity coefficient is represented as a full-order time power term $\sum_{i=1}^{h} b_i t^{h-i}$. Consequently, the time-varying grey action quantity is redefined as $\sum_{i=1}^{h} b_i t^{h-i} [z^{(1)}(t)]^n$. The optimal hyperparameter h is determined by using the time series rolling cross-validation method and the firefly algorithm. This optimized grey action quantity significantly enhances the model's capability to capture nonlinear patterns in the original data sequence while improving prediction accuracy and stability.

2.2.2. Establishment of the SANGBM(1,1) model

Definition 2.3. Let $X^{(0)}$ be defined according to Definition 2.1. We can obtain $X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(m)\right)$ by applying a fractional-order transformation operation on $X^{(0)}$, assume that $\binom{r-1}{0} = 1$, $\binom{k-1}{k} = 0$, then

$$x^{(r)}(k) = \sum_{i=1}^{k} \binom{k-i+r-1}{k-i} x^{(0)}(i), k = 1,2, \cdots, m. \tag{8}$$

Definition 2.4. Given $X^{(0)}$ and $X^{(r)}$ as defined in Definition 2.3, the equation

$$\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = \sum_{i=1}^{h} b_i\, t^{h-i} \left(x^{(r)}(t)\right)^n \tag{9}$$

is designated as the self-adaptive nonlinear grey Bernoulli model, referred to as the SANGBM(1,1) for short, where the parameter h is regarded as the order of the time power term $(h \in Z_{\geq 1})$, and n is identified as the power exponent, with $n \neq 1$.

Integrating both sides of Eq (9) over the interval from $k-1$ to k, the discretized formula of the SANGBM(1,1) model is derived as follows:

$$y^{(r)}(k) - y^{(r)}(k-1) + a(1-n)z^{(r)}(k) = \sum_{i=1}^{h} b_i\, (1-n) \frac{k^{h-i+1} - (k-1)^{h-i+1}}{h-i+1}, \tag{10}$$

where $y^{(r)}(k) = (x^{(r)}(k))^{1-n}, n \neq 1, z^{(r)}(k) = \theta y^{(r)}(k) + (1-\theta)y^{(r)}(k-1)$.

The structure parameter vector $\hat{a} = [a, b_1, b_2, \cdots, b_h]^T$ is determined using the least squares method.

$$\hat{a} = [a, b_1, b_2, \cdots, b_h]^T = \left(B_r{}^T B_r\right)^{-1} B_r{}^T Y_r, \tag{11}$$

where

$$B_r = (1-n)\begin{bmatrix} -z^{(r)}(2) & \frac{2^h-1}{h} & \cdots & \frac{2^2-1}{2} & 1 \\ -z^{(r)}(3) & \frac{3^h-2^h}{h} & \cdots & \frac{3^2-2^2}{2} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -z^{(r)}(m) & \frac{m^h-(m-1)^h}{h} & \cdots & \frac{m^2-(m-1)^2}{2} & 1 \end{bmatrix},\ Y_r = \begin{bmatrix} \mathrm{y}^{(\mathrm{r})}(2) - \mathrm{y}^{(\mathrm{r})}(1) \\ \mathrm{y}^{(\mathrm{r})}(3) - \mathrm{y}^{(\mathrm{r})}(2) \\ \vdots \\ \mathrm{y}^{(\mathrm{r})}(\mathrm{m}) - \mathrm{y}^{(\mathrm{r})}(\mathrm{m}-1) \end{bmatrix}.$$

Theorem 2.1. Let $B_r, Y_r, and\ \hat{a} = [a, b_1, b_2, \cdots, b_h]^T$ as presented in Definition 2.4. The initial value is set as $\hat{x}^{(\mathrm{r})}(1) = x^{(0)}(1) + \lambda$, then the following statement is valid.

(1) The solution of the SANGBM(1,1) model is presented in Eq (12).

$$\hat{x}^{(r)}(k) = \left[m^{k-1}\left(x^{(0)}(1) + \lambda\right)^{1-n} + l \sum_{j=2}^{k} m^{(k-j)} \sum_{i=1}^{h} b_i\, (1-n) \frac{j^{h-i+1} - (j-1)^{h-i+1}}{h-i+1}\right]^{1/(1-n)}, \tag{12}$$

where

$$m = \frac{\left(1 - \frac{\mathrm{a}(1-\mathrm{n})}{2}\right)}{\left(1 + \frac{\mathrm{a}(1-\mathrm{n})}{2}\right)},\ \mathrm{l} = \frac{1}{\left(1 + \frac{\mathrm{a}(1-\mathrm{n})}{2}\right)}.$$

(2) The predicted values $\hat{X}^{(0)}$ are derived using the following formula.

$$\hat{x}^{(0)}(k) = \sum_{i=1}^{k} \binom{k-i-r-1}{k-i} \hat{x}^{(r)}(i).$$

Proof. Substituting the background value $z^{(r)}(k) \approx \theta y^{(r)}(k) + (1-\theta)y^{(r)}(k-1)$ into Eq (10), we get

$$y^{(r)}(k)-y^{(r)}(k-1)+a(1-n)\left(\theta y^{(r)}(k)+(1-\theta)y^{(r)}(k-1)\right)=\sum_{i=1}^{h}b_i\,(1-n)\frac{k^{h-i+1}-(k-1)^{h-i+1}}{h-i+1}. \quad (13)$$

Arranging Eq (13), we can obtain Eq (14) as follows:

$$y^{(r)}(k)=\frac{(1-a(1-n)(1-\theta))}{(1+\theta a(1-n))}y^{(r)}(k-1)+\frac{1}{(1+\theta a(1-n))}\sum_{i=1}^{h}b_i\,(1-n)\frac{k^{h-i+1}-(k-1)^{h-i+1}}{h-i+1}. \quad (14)$$

Let

$$m=\frac{(1-a(1-n)(1-\theta))}{(1+\theta a(1-n))},\qquad l=\frac{1}{(1+\theta a(1-n))}.$$

Then Eq (14) can be changed to Eq (15)

$$y^{(r)}(k)=my^{(r)}(k-1)+l\sum_{i=1}^{h}b_i\,(1-n)\frac{k^{h-i+1}-(k-1)^{h-i+1}}{h-i+1}. \quad (15)$$

When $k=2$

$$y^{(r)}(2)=my^{(r)}(1)+l\sum_{i=1}^{h}b_i\,(1-n)\frac{k^{h-i+1}-(k-1)^{h-i+1}}{h-i+1}.$$

When $k=3$

$$y^{(r)}(3)=my^{(r)}(2)+l\sum_{i=1}^{h}b_i\,(1-n)\frac{3^{h-i+1}-2^{h-i+1}}{h-i+1}$$
$$=m^2y^{(r)}(1)+ml\sum_{i=1}^{h}b_i\,(1-n)\frac{2^{h-i+1}-1}{h-i+1}+l\sum_{i=1}^{h}b_i\,(1-n)\frac{3^{h-i+1}-2^{h-i+1}}{h-i+1}.$$

When $k=4$

$$y^{(r)}(4)=my^{(r)}(3)+l\sum_{i=1}^{h}b_i\,(1-n)\frac{4^{h-i+1}-3^{h-i+1}}{h-i+1}$$
$$=m^3y^{(r)}(1)+\ m^2l\sum_{i=1}^{h}b_i\,(1-n)\frac{2^{h-i+1}-1}{h-i+1}$$
$$+ml\sum_{i=1}^{h}b_i\,(1-n)\frac{3^{h-i+1}-2^{h-i+1}}{h-i+1}$$
$$+l\sum_{i=1}^{h}b_i\,(1-n)\frac{4^{h-i+1}-3^{h-i+1}}{h-i+1}$$
$$\cdots\cdots\cdots$$

Equation (16) can be obtained by analogy.

$$y^{(r)}(k)=m^{k-1}y^{(r)}(1)+l\sum_{j=2}^{k}m^{(k-j)}\sum_{i=1}^{h}b_i\,(1-n)\frac{j^{h-i+1}-(j-1)^{h-i+1}}{h-i+1}. \quad (16)$$

Substituting $x^{(r)}(k)=(y^{(r)}(k))^{1/(1-n)}$ into Eq (16), we can obtain

$$\hat{x}^{(r)}(k)=\left[m^{k-1}(x^{(0)}(1))^{1-n}+l\sum_{j=2}^{k}m^{(k-j)}\sum_{i=1}^{h}b_i\,(1-n)\frac{j^{h-i+1}-(j-1)^{h-i+1}}{h-i+1}\right]^{1/(1-n)},$$

where

$$\mathrm{m}=\frac{(1-a(1-n)(1-\theta))}{(1+\theta a(1-n))},\qquad \mathrm{l}=\frac{1}{(1+\theta a(1-n))},\quad n\neq 1.$$

(3) The predicted values $\hat{X}^{(0)}$ are given below.

$$\hat{x}^{(0)}(k)=\hat{x}^{(r)}(k)-\hat{x}^{(r)}(k-1)=\sum_{i=1}^{k}\binom{k-i-r-1}{k-i}\hat{x}^{(r)}(i).$$

Proof completed.

2.2.3. Compatibility analysis of the SANGBM(1,1) model

This segment analyzes the SANGBM(1,1) model for its compatibility features. Analysis of the SANGBM(1,1) model's mechanism revealed that varying the hyperparameters (λ, r, n, θ, and h) enables its transformation into the GM(1,1) [9], NGBM(1,1) [20], FANGBM(1,1) [30], NGBM(1,1,k,c) [34], SAIGM(1,1) [44], SAIGM_FO [45], OFANGBM(1,1) [39], and GVM(1,1) [46]. Figure 1 illustrates these hierarchical relationships.

In summary, the SANGBM(1,1) model can be regarded as a general form of many established grey models, exhibiting superior adaptability and flexibility compared to benchmark methods.

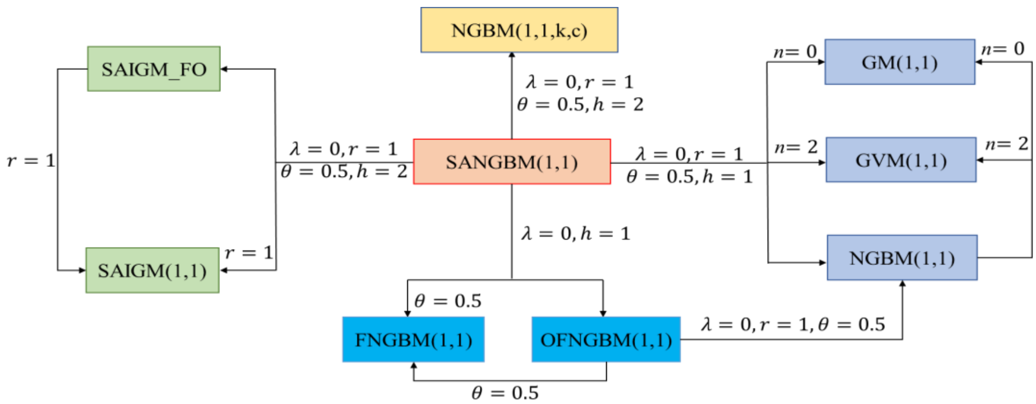

Figure 1. SANGBM(1,1) model in comparison to existing models.

2.2.4. Hyperparameter estimation for the SANGBM(1,1) model

The SANGBM(1,1) exhibits strong dependence on its hyperparameters λ, r, n, θ, and h. As demonstrated in prior studies [47], improper selection of these hyperparameters may lead to overfitting issues, especially when dealing with high-frequency time series. Once optimal hyperparameters are identified, the structural parameter vector $\hat{a} = [\mathrm{a}, b_1, b_2, \cdots, b_h]^T$ can be precisely determined using the least squares estimation. Therefore, accurate determination of hyperparameters λ, r, n, θ, and h in the SANGBM(1,1) model critically influences its predictive performance. To optimize these hyperparameters, we construct a constrained optimization model that aims to minimize the MAPE, and the parameter optimization mechanism is given by Eq (17).

$$Min\ f(\lambda、r、n、\theta、h) = \frac{1}{m-1}\sum_{k=2}^{m}\frac{\left|\hat{x}^{(0)}(k)-x^{(0)}(k)\right|}{x^{(0)}(k)},$$

$$\begin{cases} z^{(r)}(k) \approx \theta y^{(r)}(k) + (1-\theta) y^{(r)}(k-1), \\ \hat{x}^{(r)}(k) = \left[m^{k-1}\left(x^{(0)}(1)+\lambda\right)^{1-n} + l\sum_{j=2}^{k} m^{(k-j)} \sum_{i=1}^{h} b_i (1-n)\frac{j^{h-i+1}-(j-1)^{h-i+1}}{h-i+1}\right]^{1/(1-n)}, \\ m = \frac{\left(1-\frac{a(1-n)}{2}\right)}{\left(1+\frac{a(1-n)}{2}\right)}, l = \frac{1}{\left(1+\frac{a(1-n)}{2}\right)}, y^{(r)}(t) = \left(x^{(r)}(t)\right)^{1-n}, \\ x^{(r)}(k) = \sum_{i=1}^{k}\binom{k-i+r-1}{k-i} x^{(0)}(i), \\ \hat{x}^{(0)}(k) = \sum_{i=1}^{k}\binom{k-i-r-1}{k-i} \hat{x}^{(r)}(i), \\ [a, b_1, b_2, \cdots, b_h]^T = \left(B_r{}^T B_r\right)^{-1} B_r{}^T Y_r, \\ n \neq 1, k \geq 2, h \in Z_{\geq 1}. \end{cases} \quad (17)$$

Equation (17) represents a classical nonlinear optimization problem. Obtaining exact solutions for such models through traditional deterministic algorithms is computationally challenging due to inherent complexity. To tackle this issue, the well-recognized meta-heuristic algorithm, namely the firefly algorithm, is employed to ascertain the most efficacious hyperparameters. The firefly algorithm, originally developed by Yang in the realm of optimization [48], has demonstrated remarkable effectiveness in solving nonlinear programming problems [49,50]. The computational mechanism for determining the SANGBM(1,1) model's hyperparameters via the firefly algorithm is presented in the pseudocode of Algorithm 1.

Algorithm 1. Solution to optimal hyperparameters $\lambda, r, n, \theta,$ and h.

Input: Original observations;
Output: Optimal hyperparameters $\lambda, r, n, \theta,$ and h;
1: Initialize parameters in the firefly algorithm: Population size N, maximum generation T, maximum attractiveness β_0, step factor α, light absorption coefficient γ.
2: while t < T do
3: for i = 1 to N do
4: for j = 1 to N do
5: if $I_j > I_i$ then
6: Update the position of firefly i according to the following equation: $x_i(t+1) = x_i(t) + \beta_0 e^{-\gamma r_{ij}^2}\left(x_j(t) - x_i(t)\right) + \alpha\left(rand - \frac{1}{2}\right)$, where r_{ij} is the distance between the i-th firefly and the j-th firefly, and rand represents a random number within the interval [0,1].
7: end if
8: Evaluate new solutions and update the light intensity
9: end for
10: end for
11: Rank fireflies and find the current best firefly.
12: t = t + 1
13: end while
14: Obtain the optimal hyperparameters $\lambda, r, n, \theta,$ and h.

To mitigate overfitting and enhance model generalization capability, a novel data-driven model structure selection algorithm, which integrates a time series rolling cross-validation method with the firefly algorithm, is designed for optimal hyperparameter estimation of the SANGBM (1,1) model. Figure 2 outlines the procedure of the SANGBM(1,1) model.

2.2.5. Modeling steps of the SANGBM(1,1) model

The modeling procedure for the SANGBM(1,1) model comprises the following key phases:

Step 1. Data acquisition. Determine the raw dataset $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(m))$.

Step 2. Dataset partitioning. Following the time-series rolling cross-validation method, first divide the raw data sequence into training and testing sets, and then subdivide the training set into multiple training subsets and their corresponding validation subsets.

Step 3. Data preprocessing. Apply fractional-order accumulation to obtain the accumulated sequence $X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(m)\right)$.

Step 4. Model structural parameter vector estimation. Compute the structural parameter vector

$\hat{a} = [a, b_1, b_2, \cdots, b_h]^T$ using Eq (11).

Step 5. Hyperparameter optimization. Optimize the hyperparameters of the SANGBM(1,1) model using the firefly algorithm with the time-series rolling cross-validation method.

Step 6. Model structure selection. Determine the SANGBM(1,1) model structure based on the hyperparameter computation results.

Step7. Generalization assessment. Evaluate the generalization capability of the SANGBM(1,1) model on the test set.

Step 8. Make predictions. Predict future values using Theorem 2.1.

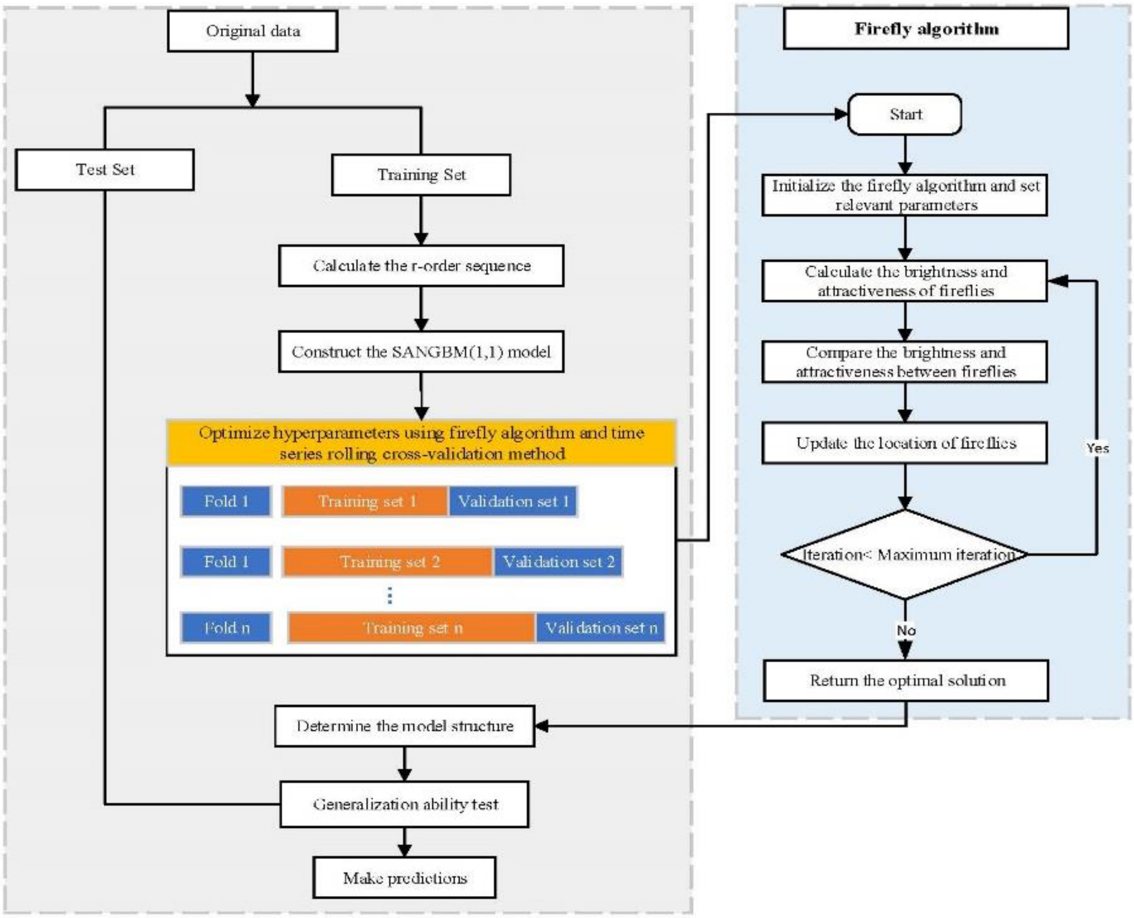

Figure 2. Flowchart of the SANGBM(1,1) model.

3. Validation of the SANGBM(1,1) model

In this section, the effectiveness of the new proposed SANGBM(1,1) model is validated using two real-world cases: Residential electricity consumption in China and hydroelectricity generation in the United States. The proposed model is comparatively analyzed against benchmark models, including the NGBM(1,1) model [20], FANGBM(1,1) model [30], HNBGM(1,1) model [51], artificial intelligence models (SVR model) [52], and statistical models (ARIMA model) [53]. Furthermore, to comprehensively assess the computational efficiency and performance of the employed firefly algorithm, we selected the particle swarm optimization algorithm (PSO) [54] and genetic algorithm (GA) [55] as benchmark algorithms, with a specific focus on comparing convergence performance. To ensure fairness, the population size and maximum number of iterations were uniformly set to 50 for all algorithms. Using a Monte Carlo simulation, each algorithm was executed for 100 independent runs, recording the following metrics: Average MAPE, standard deviation of MAPE values (computed across all 100 runs), and average convergence speed (average number of iterations to convergence).

3.1. Model performance indicators

To objectively and fairly evaluate the effectiveness of the newly introduced SANGBM(1,1) model, we employ three well-established evaluation criteria: MAPE, RMSE, and MAE. The definition of these metrics is given by Eqs (18)–(20).

$$MAPE = \frac{1}{n-1}\sum_{k=2}^{n}\left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| * 100\%. \tag{18}$$

$$RMSE = \sqrt{\frac{1}{n-1}\sum_{k=2}^{n}\left(x^{(0)}(k) - \hat{x}^{(0)}(k)\right)^2}. \tag{19}$$

$$MAE = \frac{1}{n-1}\sum_{k=2}^{n}\left| x^{(0)}(k) - \hat{x}^{(0)}(k)\right|. \tag{20}$$

Within the aforementioned metrics, $x^{(0)}(k)$ stands for the real-world observed value, while $\hat{x}^{(0)}(k)$ represents the associated predicted value.

3.2. Case 1: Forecasting China's residential electricity consumption

This section uses data on China's residential electricity consumption (2012–2022) to validate the effectiveness of the SANGBM(1,1) model, with all data sourced from the official website of the National Bureau of Statistics of China. The modeling dataset is partitioned into a training set (2012–2020) and a test set (2021–2022). Following the time series rolling cross-validation method, the training set is further divided into four distinct training subsets and corresponding validation subsets for hyperparameter optimization.

Global sensitivity analysis of SANGBM(1,1) model hyperparameters was first conducted using the Sobol method with MAPE as the objective function. Sensitivity indices were calculated in Python 3.9 via the SALib library, and the results are presented in Table 2. As shown in Table 2, significant interaction effects exist among the hyperparameters of the SANGBM(1,1) model. Therefore, multi-parameter collaborative optimization is an efficient method for identifying optimal hyperparameters.

Table 2. Hyperparameter sensitivity analysis of the SANGBM(1,1) model in case 1.

Index	First-order sensitivity index	Total-order sensitivity	Coupling sensitivity index
Power exponent n	0.02166701	0.65983004	0.63816303
Cumulative generation operator order r	0.0550309	0.17081595	0.11578505
Initial value correction factor λ	0.00017594	0.00018698	0.00001104
Time power term order h	0.06189067	1.00806893	0.94617826
Background value correction coefficient θ	0.0187944	0.54397749	0.52518309

Figure 3 compares the average MAPE, MAPE standard deviation, and convergence speed of the firefly algorithm, genetic algorithm, and particle swarm optimization algorithm across all folds using

China's residential electricity consumption dataset. As shown in Figure 3, the genetic algorithm and the firefly algorithm significantly outperform the particle swarm optimization algorithm in both average MAPE and MAPE standard deviation. Notably, although the firefly algorithm exhibits slower convergence than the genetic algorithm and particle swarm optimization algorithm in folds 1–3, it demonstrates superior performance in both average MAPE and MAPE standard deviation across these folds, surpassing the genetic algorithm and particle swarm optimization algorithm. This indicates that the firefly algorithm maintains robust convergence accuracy on China's residential electricity consumption dataset, establishing it as a promising approach for determining optimal prediction parameters.

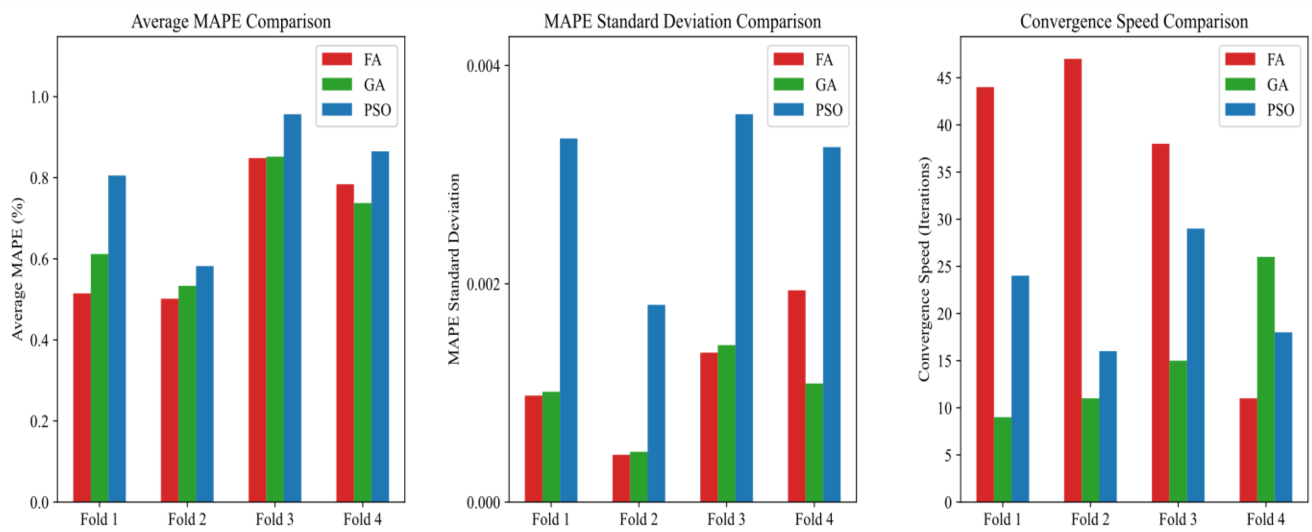

Figure 3. Convergence performance comparison of algorithms across folds in Case 1.

Based on these findings, we employed the firefly algorithm to optimize the SANGBM(1,1) hyperparameters and generate predictions. The corresponding results and errors are presented in Table 3, while Figure 4 visually compares the prediction errors between the SANGBM(1,1) model and benchmark models across training and test datasets. As demonstrated in Table 3 and Figure 4, the SANGBM(1,1) model significantly outperforms benchmark models on both the training and test sets, demonstrating substantial potential for practical applications involving uncertain systems with limited data and poor information.

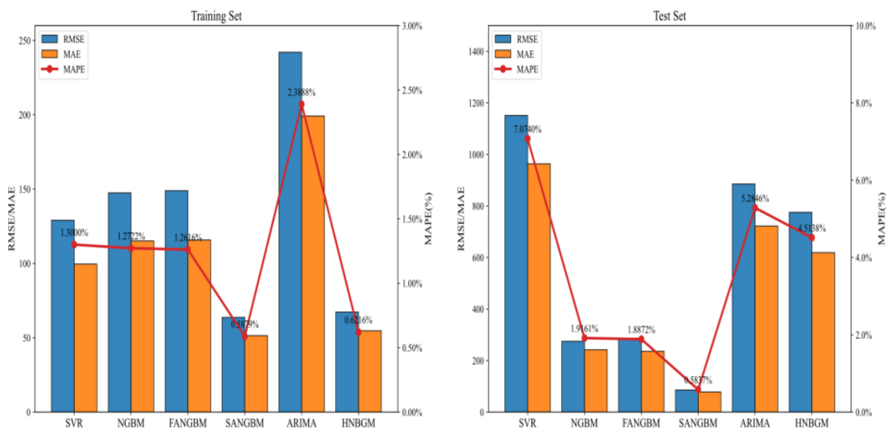

Figure 4. Errors among six models in Case 1.

Table 3. Predicted values and error comparisons for models in Case 1.

Year	Actual data	SVR(Kernel=rbf)	NGBM(1,1)	FANGBM(1,1)	SANGBM(1,1)	ARIMA	HNBGM(1,1)
			$n=-0.1158$	$r=1.1355$ $n=0.0181$	$n=1.5123$ $r=0.7037$ $\lambda=67.7616$ $\theta=0.4884$ $h=4$	$p=2$ $d=1$ $q=1$	$\gamma=-0.0194$ $r=0.3274$ $\lambda=0.2502$ $\theta=2.8714$
2012	6219.0	6387.1558	—	—	—	—	—
2013	6989.2	6744.7484	6988.6260	6988.8751	7050.4278	6890.6136	6951.0477
2014	7176.1	7190.0599	7276.8748	7253.2032	7175.1155	7639.8118	7173.1782
2015	7565.2	7734.6493	7746.9205	7720.6328	7594.2539	7794.7778	7649.6235
2016	8420.6	8379.8778	8329.3598	8306.8856	8344.654	8201.7384	8367.4268
2017	9071.6	9109.9699	9006.263	8990.8468	9187.6595	9084.3975	9157.9117
2018	10057.6	9889.0118	9773.6929	9767.2753	9977.1818	9715.9348	9931.9483
2019	10637.2	10663.4316	10633.4337	10637.2564	10683.7448	10736.5716	10676.4634
2020	11396.5	11369.7725	11590.3196	11605.1722	11395.2152	11267.5206	11404.3512
MAPE		1.3000%	1.2722%	1.2616%	**0.5879%**	2.3888%	0.6216%
RMSE		129.0546	147.4599	148.9081	**63.7051**	242.0650	67.3240
MAE		99.6284	115.1424	115.7977	**51.4399**	199.1939	54.7186
2021	12278.9	11945.8525	12651.1939	12677.5233	12322.4999	12068.6025	12127.1664
2022	13936.0	12342.3373	13824.4677	13862.4242	13822.7852	12701.7492	12850.1131
MAPE		7.0740%	1.9161%	1.8872%	**0.5837%**	5.2846%	4.5138%
RMSE		1151.2344	274.8110	286.6304	**85.7862**	885.3248	775.2978
MAE		963.3551	241.9131	236.0996	**78.4074**	722.2741	618.8103

3.3. Case 2: Forecasting U.S. hydroelectricity generation

In this section, U.S. hydroelectricity generation data from the 74th edition of the Statistical Review of World Energy is utilized to validate the effectiveness of the new proposed SANGBM(1,1) model. Initially, the data from 2014 to 2024 is divided into a training set and a testing set. The training set covers data from 2014 to 2022, while the testing set covers data from 2023 to 2024. Additionally, the training set is subdivided into four different training subsets and corresponding validation subsets to determine the optimal hyperparameters based on the time series rolling cross-validation method.

Global sensitivity analysis of the hyperparameters in the SANGBM(1,1) model was conducted using the Sobol method, with MAPE serving as the objective function. The sensitivity indices calculated through the SALib library on Python 3.9 are presented in Table 4. The results demonstrate significant interaction effects among the model's hyperparameters. Therefore, implementing a multi-parameter collaborative optimization strategy represents an effective approach for determining optimal hyperparameters.

Figure 5 presents a comparative performance evaluation of the firefly algorithm, genetic algorithm, and particle swarm optimization algorithm across folds of the U.S. hydroelectricity generation dataset. Monte Carlo simulation results reveal distinct performance characteristics among the algorithms on the U.S. hydroelectricity generation dataset. While the particle swarm optimization algorithm demonstrates exceptional convergence speed in fold 1 and fold 4, its MAPE standard deviation in these folds is higher than that of both the genetic algorithm and firefly algorithm, indicating stability limitations. Although the genetic algorithm achieves faster convergence in folds 2–3, its average MAPE in these folds is relatively high, suggesting that it may have converged to local optima during rapid optimization. Conversely, although the firefly algorithm exhibits slightly slower convergence speed than both the genetic algorithm and particle swarm optimization algorithm across all folds, it achieves superior average MAPE in folds 2–4. Notably, in folds 3–4, the firefly algorithm outperforms its competitors in both average MAPE and MAPE standard deviation, demonstrating robust global search capability and stability. Consequently, the firefly algorithm was adopted to derive optimal hyperparameters for U.S. hydroelectricity generation forecasting.

Table 4. Hyperparameter sensitivity analysis of the SANGBM(1,1) model in case 2.

Index	First-order sensitivity index	Total-order sensitivity index	Coupling sensitivity index
Power exponent n	0.06705045	0.28618247	0.21913202
Cumulative generation operator order r	0.05308815	0.28618261	0.23309446
Initial value correction factor λ	0.00232706	0.04336095	0.04103389
Time power term order h	0.31790885	0.78917022	0.47126137
Background value correction coefficient θ	0.19882491	0.67643144	0.47760653

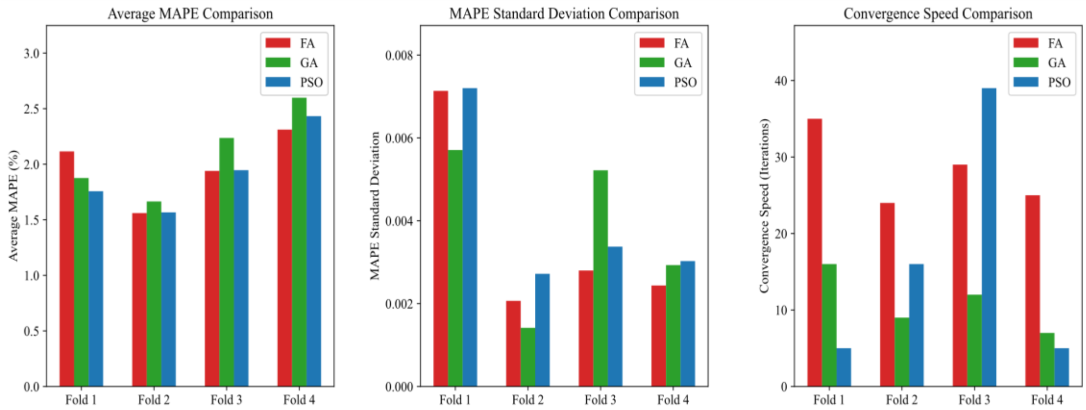

Figure 5. Convergence performance comparison of algorithms across folds in Case 2.

Based on these findings, the firefly algorithm was utilized to optimize hyperparameters of the SANGBM(1,1) model. Corresponding results and error metrics are summarized in Table 5, with Figure 6 providing a visual comparison of prediction errors between the SANGBM(1,1) and benchmark models across training and testing phases. As indicated by both Table 5 and Figure 6, the SANGBM(1,1) model achieves significantly higher accuracy than benchmark models on both training and test datasets. This conclusion is fully consistent with the findings from Case 1, further validating the robustness of the SANGBM(1,1) model. This performance advantage underscores the substantial practical utility of the SANGBM(1,1) model in uncertain systems characterized by data scarcity and limited information availability.

Table 5. Predicted values and error comparisons for models in Case 2.

Year	Actual data	SVR(Kernel=rbf)	NGBM(1,1)	FANGBM(1,1)	SANGBM(1,1)	ARIMA	HNBGM(1,1)
			$n = 0.4185$	$r = 0.4017$	$n = -2.4396$	$p = 2$	$\gamma = 2.0540$
				$n = 1.6232$	$r = 0.9018$	$d = 1$	$r = 0.2091$
					$\lambda = 0.0187$	$q = 1$	$\lambda = 0.8519$
					$\theta = 0.5125$		$\theta = 0.6379$
					$h = 4$		
2014	255.75	253.9048	—	—	—	—	—
2015	246.45	263.2723	243.2889	246.5091	246.9813	256.6335	243.2082
2016	263.76	273.5429	273.5796	270.3366	258.2917	246.3923	268.1852
2017	296.81	282.4494	287.0509	286.5450	297.9929	270.9311	288.4180
2018	289.51	287.6587	289.5045	290.8576	295.9968	295.1995	294.1405
2019	285.47	287.3103	284.6248	285.6293	284.4739	278.0635	287.3771
2020	282.78	280.4705	274.9058	274.6631	271.5976	297.6158	274.2037
2021	248.96	267.3800	262.0900	261.1628	259.3618	274.5612	259.8249
2022	251.27	249.4248	247.4146	247.2177	248.2257	244.9539	247.0841
MAPE		2.9254%	2.2730%	1.9852%	**1.8276%**	5.2549%	2.1439%
RMSE		10.2371	7.5136	6.9073	**6.2965**	16.1156	6.4684
MAE		7.6753	6.0562	5.3475	**4.9117**	14.1599	5.7780
2023	241.4	228.8442	231.7631	233.9622	238.2147	269.6529	236.9324
2024	238.7	208.2602	215.7646	221.8944	229.2290	261.9501	229.3006
MAPE		8.9768%	6.8003%	5.0608%	**2.6436%**	10.7220%	2.8942%
RMSE		23.2834	17.5913	12.9951	**7.0656**	25.8727	7.3589
MAE		21.4978	16.2862	12.1217	**6.3282**	25.7515	6.9335

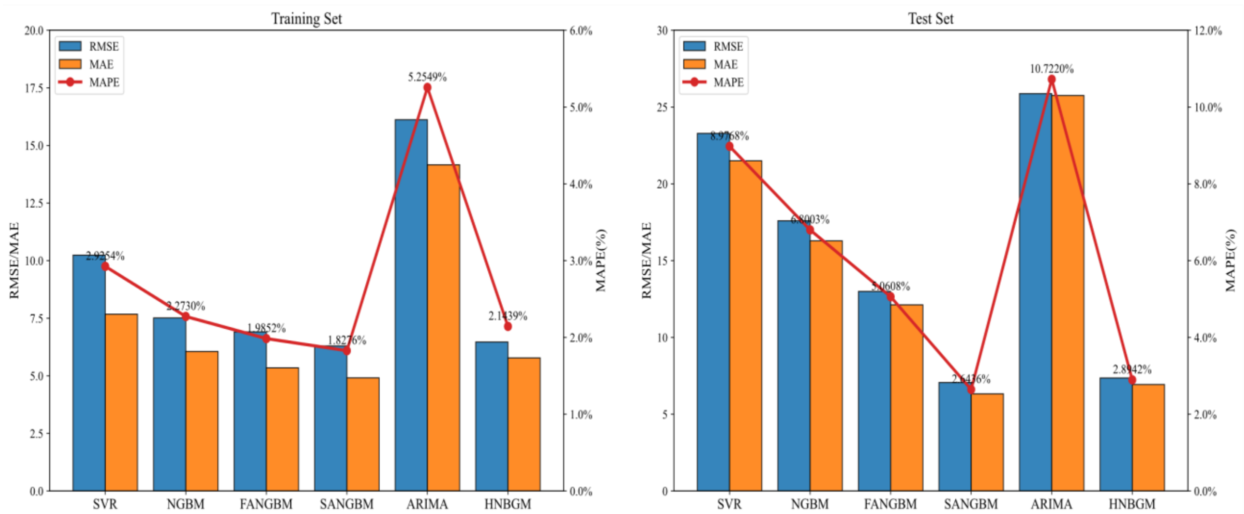

Figure 6. Errors among the six models in Case 2.

In summary, it is evident from Cases 1 and 2 that the newly proposed SANGBM(1,1) model consistently outperforms the NGBM(1,1) model and its variants, as well as the SVR and ARIMA models. Notably, no universally optimal algorithm exists among the firefly algorithm, genetic algorithm, and particle swarm optimization algorithm. Under different datasets, each algorithm has its advantages and disadvantages; a specific algorithm may yield impressive results for one problem but prove disappointing for another. Synthesizing findings from the two cases, we ultimately adopt the SANGBM(1,1) model for forecasting China's industrial electricity consumption and employ the firefly algorithm to determine the model's optimal hyperparameters.

4. Applications

4.1. Experimental design

This study utilizes China's industrial electricity consumption dataset sourced from the official website of the National Bureau of Statistics of China (available from: https://data.stats.gov.cn/english/easyquery.htm?cn=C01). Figure 7 illustrates the sectoral breakdown of electricity consumption from 2012 to 2022, highlighting that industrial electricity consumption consistently makes up the predominant portion of China's overall electricity consumption. This dominant position stresses the necessity of an accurate prediction of industrial electricity consumption. Nevertheless, considerable fluctuations in its growth rates present major modeling challenges. To address this, we employ the newly introduced SANGBM(1,1) model to conduct projections regarding China's industrial electricity consumption. To comprehensively and fairly evaluate the SANGBM(1,1) model's projection efficacy, we introduce three classic metrics, namely MAPE, RMSE, and MAE, to assess its predictive effectiveness, and compare its predictive performance with benchmark models, including the NGBM(1,1) model [20], FANGBM(1,1) model [30], HNBGM(1,1) [51], artificial intelligence models (SVR model) [52], and statistical models (ARIMA model [53]). Additionally, for model performance evaluation, the collected modeling dataset is partitioned into a training set and a test set. The training set covers the period from 2012 to 2020, whereas the test set spans 2021 to 2022. Furthermore, the training set is subdivided into four distinct training subsets and corresponding validation subsets to determine the optimal hyperparameters based on the time series rolling cross-validation method.

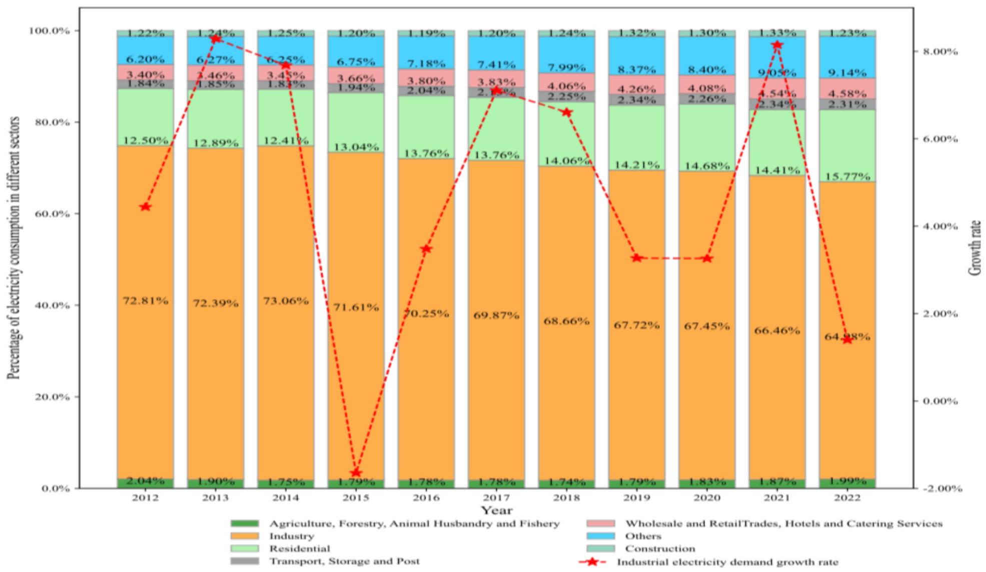

Figure 7. Proportion of electricity consumption in different sectors and growth rate of industrial electricity consumption from 2012 to 2022.

4.2. Global sensitivity indices for the hyperparameters of the SANGBM(1,1) model

In this section, the comprehensive sensitivity of the hyperparameters within the SANGBM(1,1) model is evaluated using the Sobol method, where the MAPE is employed as the objective function. The ranges for these hyperparameters are determined by considering the SANGBM(1,1) model's mechanisms and data characteristics, as presented in Table 6. Global sensitivity indices for the hyperparameters of the SANGBM(1,1) model were computed in Python 3.9 using the SALib library, and the results are displayed in Table 7.

Table 6. Range of hyperparameters of the SANGBM(1,1) model.

Hyperparameters	n	r	λ	h	θ
Sampling range	$-2 \leq n < 1$ and $1 < n \leq 2$	0~1	-100~100	1~4	0-1

Table 7. Sensitivity analysis results for each hyperparameter of the SANGBM(1,1) model.

Index	First-order sensitivity index	Total-order sensitivity index	Coupling sensitivity index
Power exponent n	0.06949849	0.45749947	0.38800098
Cumulative generation operator order r	0.13823719	0.5973724	0.45913521
Initial value correction factor λ	0.00001582	0.00001861	0.00000279
Time power term order h	0.20651723	0.68304518	0.47652795
Background value correction coefficient θ	0.00465527	0.24946436	0.24480909

To intuitively illustrate how each hyperparameter affects the prediction accuracy of the SANGBM(1,1) model, Figure 8 visually presents a radar chart of the hyperparameter sensitivity indices for the SANGBM(1,1) model. As shown in Table 7 and Figure 8, two key findings emerge. First, the time power term order h of the SANGBM(1,1) model exhibits the most significant sensitivity, with its first-order and global sensitivity indices reaching 0.20651723 and 0.68304518, respectively. Second, there is a strong coupling between the hyperparameters of the SANGBM(1,1) model.

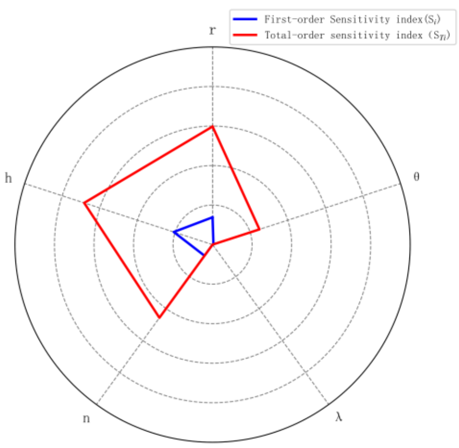

Figure 8. Radar chart of global sensitivity indices for SANGBM(1,1) model hyperparameters.

4.3. Simulating and forecasting results

Drawing from the earlier analysis, it is evident that there is a strong coupling among the hyperparameters of the SANGBM(1,1) model. Therefore, this section employs an integrated multi-parameter collaborative optimization algorithm that combines the time series cross-validation method with the firefly algorithm to determine the optimal hyperparameters for the SANGBM(1,1) model. To comprehensively evaluate the performance of the firefly algorithm, this study employs the genetic algorithm and particle swarm optimization algorithm as alternative optimizers for comparative analysis. To ensure fairness in comparison, we set the population size and maximum iterations uniformly to 50 for all algorithms. Employing Monte Carlo simulation, each algorithm was executed for 100 independent runs to record three key metrics: Average MAPE, standard deviation of MAPE values, and average convergence speed. Figure 9 illustrates the performance comparison of the firefly algorithm, genetic algorithm, and particle swarm optimization algorithm across folds on China's industrial electricity consumption dataset. Monte Carlo simulation results indicate that all three algorithms demonstrate excellent performance in all folds while exhibiting distinct characteristics. The particle swarm optimization algorithm shows outstanding convergence speed, particularly in folds 2 and 4, significantly outperforming both firefly and genetic algorithms. However, it exhibits a higher average MAPE across all folds and elevated MAPE standard deviation in folds 2–4, indicating solution instability. Genetic algorithm achieves average MAPE comparable to the firefly algorithm in folds 2–4 but shows slightly higher MAPE standard deviation in these folds, suggesting marginally inferior robustness. In contrast, although the firefly algorithm exhibits slightly slower convergence speed than the two alternative algorithms and shows no significant difference in average MAPE, it consistently achieves a slightly lower MAPE standard deviation across folds 1–4 compared to both the genetic algorithm and particle swarm optimization algorithm, demonstrating exceptional robustness. This exceptional robustness may be attributed to the unique biological inspiration mechanism of the firefly algorithm, making it particularly suitable for solving high-dimensional nonlinear optimization problems.

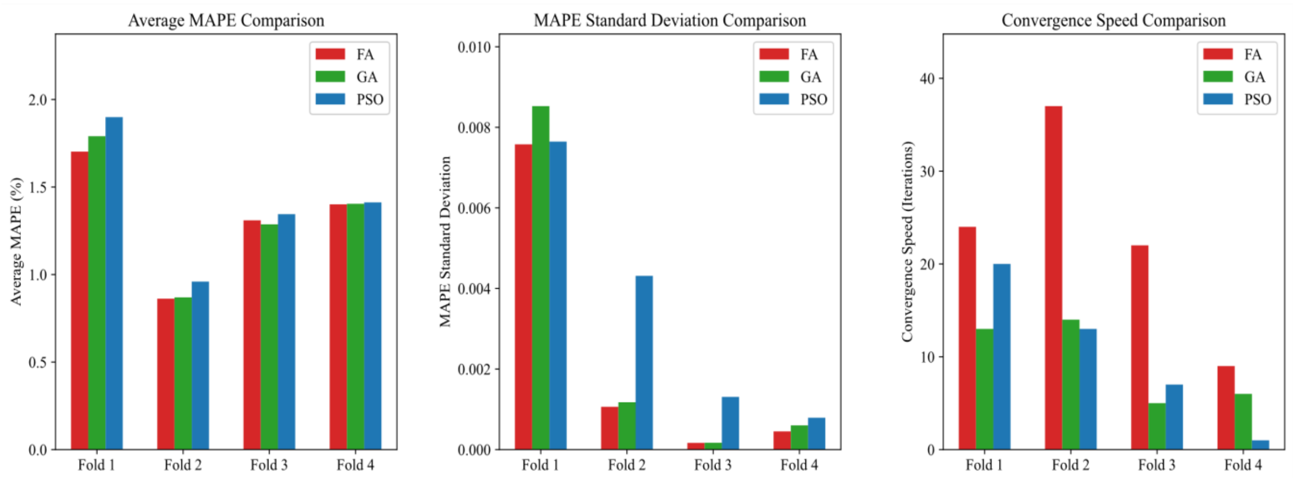

Figure 9. Convergence performance comparison of algorithms on China's industrial electricity consumption dataset.

In summary, the firefly algorithm demonstrates exceptional robustness in optimizing SANGBM(1,1) model hyperparameters for China's industrial electricity consumption forecasting, establishing it as an efficient methodology. Consequently, the firefly algorithm was utilized to optimize the hyperparameters of the SANGBM(1,1) model ($h = 2, r = 0.7011, n = 0.226, \lambda = 13.6779, \theta = 0.5213$) . By substituting the modeling data and optimal hyperparameters into the SANGBM(1,1) model, we can obtain the model's structural parameter vector $\hat{a} = [0.8751,1030.8723,4710.9148]^T$ and subsequently construct a forecasting model for China's industrial electricity consumption.

For fair comparison, the firefly algorithm is also employed to solve the optimal hyperparameters of each competing grey model. The hyperparameters of the competing grey models are systematically documented in Table 8.

4.4. Results and discussion

Table 8 displays the predictive outcomes and associated errors for the SANGBM(1,1) model alongside other competing models. Figure 10 compares the MAPE, RMSE, and MAE values of the SANGBM(1,1) model with benchmark models on the Chinese industrial electricity consumption dataset.

Initially, the overall predictive capabilities of the SANGBM(1,1) model were compared with those of the competing models. As illustrated in Table 8 and Figure 10, the SANGBM(1,1) model demonstrated high predictive accuracy on the training set, achieving the second-lowest MAPE (1.3558%), RMSE (757.7395), and MAE (608.6323) after the HNBGM(1,1) model, while significantly outperforming other competing models. This indicates that the SANGBM(1,1) model demonstrates excellent capability in capturing the inherent developmental patterns of China's industrial electricity consumption. Notably, on the test set (2021–2022), the SANGBM(1,1) model delivered superior performance with the lowest errors across all metrics: MAPE (1.3660%), RMSE (1069.3763), and MAE (773.6979), surpassing all rival models. This superior performance can be attributed to three key enhancements in the SANGBM(1,1) model. First, the SANGBM(1,1) model successfully addresses the constraints imposed by subjectively preset adjustable parameters within the current NGBM(1,1) model and its variants. Second, a new data-driven algorithm for model structure selection is constructed through the systematic integration of the time-series rolling cross-validation method with the firefly algorithm, thereby enhancing the generalization capability of the SANGBM(1,1) model. Third, the implicit time response formulation effectively overcomes the inherent jump errors in the traditional NGBM(1,1) model, thereby enhancing the stability of the SANGBM(1,1) model's predictive performance.

Subsequently, we conducted a comparative assessment of the SANGBM(1,1) model against classic nonlinear grey prediction models, utilizing three renowned error assessment criteria, as shown in Figure 10. During the training phase, the HNBGM(1,1) model exhibited significantly lower prediction errors compared to other nonlinear grey models, further confirming that adjustable parameters are a crucial factor affecting the predictive accuracy of grey prediction models [56]. Nevertheless, in the test set, the HNBGM(1,1) model demonstrated larger prediction errors compared to the SANGBM(1,1) model and other benchmark models, indicating that the HNBGM(1,1) model suffers from a certain degree of overfitting. This is mainly because traditional grey prediction model hyperparameter solving algorithms do not pay sufficient attention to the overfitting problem, which renders them susceptible to overfitting. In contrast, the newly proposed SANGBM(1,1) model in this paper demonstrates satisfactory prediction results, exhibiting lower prediction errors in both the training and test datasets compared to alternative rival models. This result indicates that the multi-parameter collaborative optimization algorithm, which integrates the time-series rolling cross-validation algorithm with the firefly algorithm as proposed in this paper, can effectively mitigate the risk of overfitting.

Table 8. Predicted values and error comparisons for models on China's industrial electricity consumption.

Year	Actual data	SVR(Kernel=rbf)	NGBM(1,1)	FANGBM(1,1)	SANGBM(1,1)	ARIMA	HNBGM(1,1)
			$n = -0.0108$	$r = 0.0974$ $n = -8.9431$	$n = 0.226$ $r = 0.7011$ $\lambda = 13.6779$ $\theta = 0.5213$ $h = 2$	$p = 1$ $d = 2$ $q = 2$	$\gamma = 1.0257$ $r = 0.4864$ $\lambda = 0.5418$ $\theta = 2.9812$
2012	36232.2	36744.7682	—	—	—	—	—
2013	39236.9	38478.1768	39242.3784	39302.2816	39213.6872	—	39577.4732
2014	42248.7	40222.5894	40794.7745	40923.4423	40784.3742	42035.6882	40953.0930
2015	41550.0	42062.5451	42495.4753	42525.8795	42166.8175	40800.5782	41676.5049
2016	42996.9	44072.0009	44309.4321	44225.7370	43867.1924	41976.4787	43197.9476
2017	46052.8	46267.6728	46226.6313	46054.0323	45873.7798	44783.3349	45716.5720
2018	49094.9	48579.8052	48244.4822	48023.5578	48080.9126	49745.1431	48626.5044
2019	50698.3	50851.3869	50363.4834	50141.2746	50394.9818	52438.5292	51117.9763
2020	52353.4	52866.9331	52585.7143	52412.0257	52751.4838	51771.1250	52603.2137
MAPE		1.6307%	1.4997%	1.4882%	1.3558%	1.9010%	**0.9620%**
RMSE		879.4380	840.3938	842.9453	757.7395	1003.5698	**550.2261**
MAE		697.9595	663.5989	660.4477	608.6323	889.2953	**429.7308**
2021	56622.3	54401.7983	54914.1986	54840.0194	55110.3899	53695.8344	52879.7827
2022	57413.0	55276.3808	57352.6009	57429.528	57448.4856	55399.0802	52083.2320
MAPE		3.8215%	1.5609%	1.5882%	**1.3660%**	4.3381%	7.9464%
RMSE		2178.9641	1208.5649	1260.3169	**1069.3763**	2511.9787	4605.0441
MAE		2178.5605	884.2503	899.4043	**773.6979**	2470.1927	4536.1427

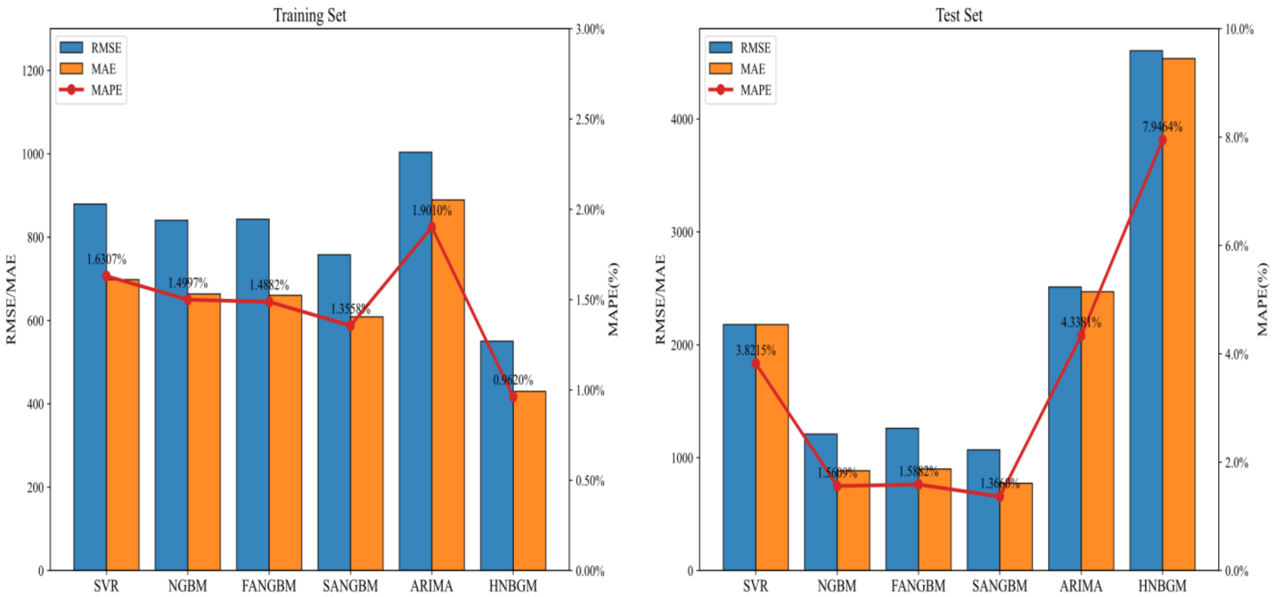

Figure 10. Errors among six models on the industrial electricity consumption dataset.

Finally, we compared the MAPE, RMSE, and MAE values of the proposed SANGBM(1,1) model against those of representative artificial intelligence models (SVR) and statistical models (ARIMA). As depicted in Figure 10, among all the models evaluated, the SVR and ARIMA models demonstrated notably higher prediction errors compared to the grey prediction models. This discrepancy is mainly because the predictive accuracy of these two models is highly sensitive to the sample size. When the modeling sample is limited, SVR suffers from compromised generalization capacity due to insufficient support vectors, while ARIMA experiences increased prediction variance stemming from biased estimation of its autocorrelation structure.

In summary, this study demonstrates that the SANGBM(1,1) model can accurately capture the inherent development patterns of China's industrial electricity consumption. Based on these findings, we employ the SANGBM(1,1) model to forecast China's industrial electricity consumption for the period 2023–2026.

4.5. Projections of China's industrial electricity consumption from 2023 to 2026

In this section, the SANGBM(1,1) model is applied to forecast the industrial electricity consumption in China spanning the years 2023 to 2026. Data spanning the period from 2012 to 2022 is utilized as the training dataset for constructing the model. Following the time-series rolling cross-validation method, the training dataset is further partitioned into five separate training subsets along with their respective validation subsets. The firefly algorithm is employed to ascertain the optimal hyperparameters ($n = 0.7287, r = 0.6479, \lambda = 55.9738, h = 3, \theta = 0.4954$). The forecasting outcomes are presented in Table 9.

Table 9. Forecasted results of industrial electricity consumption from 2023 to 2026 (unit: Billion kWh).

Year	2023	2024	2025	2026
Forecast	5996.0952	6162.4901	6292.3007	6382.3421

As shown in Table 9, China's industrial electricity consumption is projected to follow a consistent upward trend from 2023 to 2026, reaching 6382.3421 billion kWh by 2026. This trend suggests that the imbalance between supply and demand in China's industrial electricity sector will continue. These findings offer valuable insights for policymakers in tackling potential electricity shortages and facilitating sustainable socio-economic development. Based on the forecast outcomes, the following policy suggestions are put forward.

(1) Formulate explicit industrial electricity-saving plans. To address the challenge of China's industrial electricity demand growing at an annual rate of 2.69% from 2023 to 2026, the Chinese government should establish detailed energy conservation plans. These plans should define annual energy-saving targets and concrete implementation steps to improve industrial electricity efficiency at the same 2.69% annual rate. This will enable China to meet its growing industrial electricity consumption demands more sustainably.

(2) Accelerate renewable energy deployment, optimize the electricity supply structure, and accelerate the construction of a modern energy system. Continuously increase the proportion of green electricity in industrial power consumption to reduce dependence on coal-fired power.

(3) Advance market-oriented electricity pricing reform for industrial consumption by establishing a refined time-of-use pricing mechanism with dynamic zoning and launching industrial electricity futures contracts to curb price volatility, thereby forming a market-driven electricity price signaling

system. This enables industrial enterprises to proactively adjust production schedules and electricity consumption patterns based on price signals, ultimately achieving a green and low-carbon transition in industrial production.

5. Conclusions

To enhance the predictive power of the existing NGBM(1,1) model family, a novel self-adaptive nonlinear grey Bernoulli model [referred to as SANGBM (1,1)] is proposed in this paper. The model discards the unreasonable modeling assumption of subjectively preset adjustable parameters in the existing NGBM(1,1) model family and greatly improves prediction performance. In addition, we construct a model structure selection mechanism that integrates the firefly algorithm with a time series rolling cross-validation algorithm to identify the optimal hyperparameters. To validate the model's effectiveness, we first verified it using two real-world case studies: Chinese residential electricity consumption data and U.S. hydroelectricity generation data. Building on this, the proposed model was employed to simulate and forecast China's industrial electricity consumption, and its prediction accuracy was compared with that of existing nonlinear grey prediction models, as well as non-grey prediction models such as the SVR model and the ARIMA model. The following key conclusions can be drawn:

(1) The proposed SANGBM(1,1) model overcomes the rigidity of subjectively preset hyperparameters in the existing NGBM(1,1) model family by introducing adaptive hyperparameterization functions. This innovation dynamically optimizes the adjustable parameters, significantly enhancing model adaptability to capture nonlinear evolutionary patterns in complex time series while boosting predictive capability.

(2) The implicit time response equation of the SANGBM (1,1) model effectively solves the jump error inherent in the traditional NGBM(1,1) model, thereby significantly enhancing robustness.

(3) The novel model structure selection algorithm significantly improves the predictive accuracy of the SANGBM (1,1) model. Empirical validation across three critical energy datasets (Chinese industrial electricity consumption, Chinese residential electricity consumption, and U.S. hydroelectricity generation) demonstrates its superiority over competing models, including three grey forecasting models [NGBM(1,1), FANGBM(1,1), and HNBGM(1,1)], the statistical model ARIMA, and the artificial intelligence model SVR.

(4) Following rigorous validation, the proposed SANGBM(1,1) model was employed to project Chinese industrial electricity consumption over the next four years. Predictions indicate that, with the vigorous rise of emerging industries and the accelerated upgrading of traditional industries, China's industrial electricity consumption will maintain a growth trend in the short term and is expected to reach 6382.3421 billion kWh by 2026.

While the SANGBM(1,1) model has demonstrated excellent prediction performance in the field of small-sample nonlinear time series prediction, several pivotal domains still warrant further investigation.

(1) Although the SANGBM(1,1) model demonstrates relatively high accuracy in predicting nonlinear dynamic systems, it is essentially a univariate grey prediction model that does not take external factors into account during the modeling process. Therefore, developing a self-adaptive nonlinear multivariate grey prediction model will become the focus of our next work.

(2) The model structure selection algorithm proposed in this study, which combines the firefly algorithm with a time-series rolling cross-validation method, effectively enhances the generalization ability of the SANGBM(1,1) model. However, its implementation requires frequent model training and evaluation, resulting in high computational costs. Consequently, optimizing this data-driven model

structure selection algorithm to reduce iteration frequency and computational complexity will be our subsequent research direction.

(3) China is currently undergoing a critical period of industrial transformation. Within this dynamic and complex process, data scarcity significantly impacts model prediction accuracy. Therefore, developing an adaptive multi-source heterogeneous data fusion grey model represents a vital direction for future research.

Author contributions

Xiaozhong Tang: Conceptualization, software, writing–original draft, writing–review and editing, funding acquisition; Zhijun Zhu: Visualization, investigation; Xiaomei Liu: Writing–review and editing, funding acquisition; Huibin Zhan: Formal analysis, supervision. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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