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Research article

Event-triggered impulsive control for exponential stabilization of fractional-order differential system

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Abstract: This paper presents a novel event-triggered control strategy for fractional-order systems. The analysis begins with an investigation of the stability of impulsive fractional differential equations using Lyapunov function methods. Based on this framework, impulsive control schemes both with and without delay are designed to be triggered by discrete events. The proposed strategies ensure exponential stability of all system states while rigorously avoiding Zeno behavior. The effectiveness and practical relevance of the approach are demonstrated through numerical simulations applied to chaotic financial systems.

Keywords: fractional derivatives; Lyapunov functions; event-triggered impulsive control; exponential stabilization; financial chaotic systems
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1. Introduction

In recent years, fractional differential equations have garnered increased attention. This is due to their myriad applications in various disciplines. These equations are an extension of the traditional concept of differential equations and are especially used to study the behavior of systems with complex and nonlinear structure. The non-locality and dependence of these equations on state history have made them the most suitable for modeling many phenomena in many fields such as engineering [39, 52], biology [4, 32, 34], and finance [6, 11, 24, 25, 43]. On the other hand, over the past few years, the event-triggered impulsive control (ETIC) method has attracted increasing attention. This method is characterized by its ability to adapt and respond quickly to unexpected and dramatic changes in an event [19, 37]. This type of control relies on limiting intervention to only predefined intermittent times to affect the system. This approach effectively reduces the cost of monitoring, as well as the storage requirements for control-related information [35, 52], and numerous findings documented in [9, 17, 30, 45, 48, 52] have emerged from studies conducted in this area.

In light of these benefits, research on ETIC has expanded beyond ordinary differential equations in the past decade to include the study of stability and analysis of impulsive equations [26, 28, 29], stochastic equations [21, 31, 52], and partial differential equations [8, 10, 12]. However, limited research has been conducted on ETIC for fractional-order systems. Recently, in [20], the ETIC method was used to address the problem of initial value sensitivity of chaotic systems. The study investigated the control of chaotic systems and derived necessary criteria for achieving global impulsive synchronization through the application of stability theory. Using the ETIC method, [50] explored the chaotic synchronization of fractional-order differential systems and the potential benefits of impulsive control methods. In [15], an observer-based event-triggered control scheme for uncertain fractional-order systems was developed, and some conditions for system stability using LMIs were derived.

More recently, in [51], the exponential stabilization of fractional-order dynamical systems using ETIC was explored. This involved a demonstration of synchronization within fractional-order chaotic systems. In [40], the impulsive synchronization of fractional-order coupled neural networks using the ETIC method was studied. In addition, some less-conservative fractional-order dependent sufficient conditions to guarantee the synchronization of fractional-order coupled neural networks were obtained based on the Kronecker product, stability theory, and Lyapunov function construction. Lately, [46] focused on the problem of finite-time synchronization in fractional-order reaction-diffusion complex networks. A hybrid controller, consisting of an event-triggered controller and an impulsive controller, was formulated to achieve global finite-time synchronization within the network under consideration.

In [3], delayed impulsive control for fractional-order systems was investigated using ETIC. By integrating the event-triggering function into the transformed fractional-order delayed impulsive control system, the event-triggered impulsive sequence was initiated while taking into account the effects of time delays in the impulses. In [22], an event-triggered stabilizing state feedback controller was designed for nonlinear fractional-order interconnected systems, and the closed-loop systems were shown to be asymptotically stable.

Due to the fact that non-integer-order fractional derivatives deviate from Leibniz's theorem and the chain rule [41, 42], there are notable difficulties in the transition of ETIC design techniques from integer to fractional order systems. The present study aims to contribute to overcoming this obstacle by leveraging our previous research explored in the context of the damped wave equation in [8] and in [9] for nonlinear continuous systems, attempting to modify them for applicability in fractional-order equations. The primary findings of this study can be succinctly outlined in the following manner: First, the impulsive instants are defined by a trigger function that depends on a given Lyapunov function of the systems and predefined trigger conditions. Second, measurements of the Lyapunov function are taken at predetermined instants, and the updated controller is modified only when the event condition is violated. This means that there is no need to measure the Lyapunov function outside of these instants. This further reduces the communication overhead and the frequency of controller updates compared to the ETIC approach discussed in [15, 40, 46, 50, 51]. Finally, ETIC is used to select the intervention instants to act on the system. This guarantees not only the exponential convergence of the system, but also the absence of the Zeno phenomenon, which generally remains a real challenge when using the

ETIC method.

The paper's structure is outlined as follows: In Section 2, definitions and results of differential calculus and fractional differentiation are given. In Section 3, the exponential stability of fractional-order impulsive systems is proven, the ETIC method is used to ensure the exponential stability of fractional-order systems, and the delay that may accompany the implementation of the intervention decision to affect the system is subsequently taken into account. In Section 4, an illustrative simulation is performed to authenticate the efficiency of the proposed ETIC approach and its success in exponentially stabilizing the system under consideration. Thereafter, concluding remarks and future works are given in Section 5.

2. Preliminaries

In this section, we present several definitions pertaining to fractional calculus, recall some important results to be used later, and formulate our problem.

2.1. Caputo fractional-order derivative

Definition 2.1. *The Euler Gamma function is defined for any real $s > 0$ by*

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt. \tag{2.1}$$

Definition 2.2. *Let $\alpha \in (0,1)$ and let $t_0, T \in \mathbb{R}$ with $t_0 < T$. Let $x : [t_0, T] \to \mathbb{R}$ be an absolutely continuous function. Then, the Caputo fractional derivative of order α of x at time $t \in (t_0, T]$ is defined by*

$${}^C D_{t_0}^{\alpha} x(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-\tau)^{-\alpha} x'(\tau)\, d\tau, \tag{2.2}$$

where $\Gamma(\cdot)$ is the Euler Gamma function. The derivative $x'(\tau)$ is understood in the classical sense.

Definition 2.3. *Let $\alpha > 0$, $\beta > 0$, and $z \in \mathbb{C}$. The one-parameter and two-parameter Mittag-Leffler functions are defined, respectively, by*

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \quad \text{for } z \in \mathbb{C}, \tag{2.3}$$

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \quad \text{for } z \in \mathbb{C}. \tag{2.4}$$

Now that we have covered the important definitions and symbols used in fractional calculus, we can move on to some basic results that will help us with the analysis that follows.

Proposition 2.1. *[49] Let $\alpha \in (0,1)$ and $t \geq t_0$. Then, for all $c > 0$, the function $t \to E_\alpha[c(t-t_0)^\alpha]$ is non-negative and is monotonically nonincreasing, and $E_\alpha[c(t-t_0)^\alpha] \geq 1$.*

Lemma 2.1. *[36] Assume that $\alpha > 0, \beta > 0$, and $c \in \mathbb{R}$. Then,*

$$\int_{t_0}^{t} E_{\alpha,\beta}(cs^\alpha) s^{\beta-1} ds = (t-t_0)^\beta E_{\alpha,\beta+1}(c(t-t_0)^\alpha). \tag{2.5}$$

In particular, if we let $\beta = \alpha$, the following equality holds:

$$\int_{t_0}^{t} E_{\alpha,\alpha}(cs^{\alpha})s^{\alpha-1}ds = (t-t_0)^{\alpha}E_{\alpha,\alpha+1}(c(t-t_0)^{\alpha}). \tag{2.6}$$

2.2. Problem statement

Consider the Caputo fractional non-autonomous system:

$${}^{c}D_{t_0}^{\alpha}x(t) = f(t, x(t)); \quad t \geq t_0 \tag{2.7}$$

$$x(t_0) = x_0, \tag{2.8}$$

where $\alpha \in (0, 1)$, $x(t) \in \mathbb{R}^n$ is the system state, and x_0 is the initial condition at the initial time t_0. In this paper, the function $f(t, x)$ is assumed to be locally Lipschitz at x with constant L, piecewise continuous at t, and satisfying $f(t, 0) = 0$ for all $t \geq t_0$.

Definition 2.4. *Let $x_e \in \mathbb{R}^n$. x_e is an equilibrium point of systems (2.7) and (2.8) if $f(t, x_e) = 0, \forall t \geq t_0$.*

Remark 2.1. *We can always return to the case where $x_e = 0$ is an equilibrium point; in fact, if this is not the case and $x_e \neq 0$, by performing the change of the variable $y = x - x_e$, since the Caputo derivative of a constant is zero, we obtain the following system ${}^{c}D_{t_0}^{q}y(t) = f(t, x(t) - x_e) = g(t, y(t))$ where $y = 0$ is an equilibrium point.*

Proposition 2.2. *[27] Let $\alpha \in (0, 1)$, and $x = 0$ be an equilibrium point of systems (2.7) and (2.8). Then, for all $t \geq t_0$, the solution $x(t)$ of systems (2.7) and (2.8) satisfies*

$$\|x(t)\| \leq \|x_0\| \, E_{\alpha}(L(t-t_0)^{\alpha}). \tag{2.9}$$

Suppose a particular control $u(x(t_k))$ is chosen to influence the evolution of the state variable $x(t)$ in the dynamical system governed by Eqs (2.7) and (2.8) at discrete instances t_k, constituting an unbounded increasing sequence. The choice of this control can be precisely executed to ensure exponential convergence of $x(t)$ towards the origin. More precisely, we are specifically concerned with the construction of a sequence $(t_k, u(x(t_k)))$ with the aim of achieving exponential convergence for the solution of the subsequent system.

$$cD_{t_k}^{\alpha}x(t) = f(t, x(t)), \; t \geq t_0, t \in (t_k, t_{k+1}] \tag{2.10}$$

$$x(t_k^+) = x(t_k) + u(x(t_k)), \quad k = 1, 2, .. \tag{2.11}$$

$$x(t_0^+) = x_0. \tag{2.12}$$

Throughout this paper, we suppose that $x(t)$ is assumed to be left continuous at t_k, i.e., $x(t_k^-) := \lim\limits_{t \to t_k^-} x(t) = x(t_k)$.

Remark 2.2. *In the theory of impulsive fractional differential equations (IFDE), two main approaches are commonly adopted depending on the lower limit of the Caputo fractional derivative [1, 14].*

***(1) Fixed Lower Limit at** t_0**:** In this approach, the Caputo derivative ${}^{C}D_{t_0}^{\alpha}x(t)$ is defined with a fixed lower limit, typically the initial time t_0 (often $t_0 = 0$). This implies that the derivative accumulates memory from the entire history of the state since the beginning of the evolution. Even in the presence* *of impulses, the memory effect persists over the interval $[t_0, t]$, maintaining a global dependence on the past trajectory. This formulation is commonly used in the analysis of existence and uniqueness of solutions [2, 14], and also in stability studies for systems with global memory [38, 44].*

***(2) Variable Lower Limit at** t_k**:** Alternatively, the Caputo derivative $^{C}D^{\alpha}_{t_k}x(t)$ is reinitialized at each impulse time t_k. That is, for each interval $(t_k, t_{k+1}]$, the fractional derivative is computed from the latest impulse, effectively resetting the memory. This introduces a local memory mechanism where only the state history after the last impulse is considered, giving the system a "short-term memory" structure. This short-memory formulation has been used in several recent works focusing on stability analysis [1, 18, 47], and provides a more localized response suitable for impulsive control applications.*

In this paper, we adopt the second approach with variable lower limits at impulse times t_k, as it naturally aligns with the structure of event-triggered impulsive control and simplifies the analysis of stability between impulses.

In the following, some notations that will be used are mentioned, and the exponential stability of an equilibrium point of systems (2.10)–(2.12) is defined.

Notation 1. Let $r > 0$. B_r will design the ball with center 0 and radius r.

$$B_r = \{x \in \mathbb{R}^n, \|x\| < r\}. \tag{2.13}$$

Notation 2. The class of functions $V : \mathbb{R}_+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ that are locally Lipschitzian at x and continuous everywhere except perhaps at a strictly increasing and unbounded sequence of points t_k is denoted $\mathcal{V}_0$. At these points t_k, $V(t, x)$ is continuous on the left of t_k, and the right limit $V(t_k^+, x)$ exists for all $x \in \mathbb{R}^n$.

Definition 2.5. *The origin $x = 0$ of systems (2.10)–(2.12) is exponentially stable if there exist positive constants r, c, and γ such that*

$$\|x(t)\| \leq c\|x_0\|\, e^{-\gamma(t-t_0)}, \tag{2.14}$$

for all $x_0 \in B_r$ and $t \geq t_0$.

3. Main results

In this section, we present our main findings and the reasons behind them. A result of the exponential stability of fractional impulsive systems (2.10)–(2.12) is presented first.

3.1. Exponential stability

Theorem 3.1. *Consider the impulsive systems (2.10)–(2.12). Assume that there exist functions $V \in \mathcal{V}_0$, and positive constants $r, c_1, c_2, \underline{\theta}, \theta$, and $\mu \in (0, \frac{c_1}{c_2})$ such that, for any $x \in B_r$,*

$$i)\ \ c_1\|x\|^2 \leq V(t, x) \leq c_2\|x\|^2, \quad \forall t \geq 0, x \in B_r \tag{3.1}$$

$$ii)\ \ V(t_{k+1}^+, x(t_{k+1}^+)) \leq \mu\, V(t_k^+, x(t_k^+)) \tag{3.2}$$

$$iii)\ \ \underline{\theta} \leq t_{k+1} - t_k \leq \theta. \tag{3.3}$$

Then, the origin $x = 0$ is exponentially stable.

Remark 3.1. *(1) The hypothesis of a positive lower bound on inter-pulse intervals, i.e., $t_{k+1} - t_k \geq \underline{\theta}$ for some positive constant $\underline{\theta}$, is commonly employed in event-triggered strategies control to preclude Zeno behavior.*

(2) Most Lyapunov stability results for impulsive fractional-order systems, as presented in the literature [1, 2, 38, 49], rely on Lyapunov-type comparison theorems. These approaches require that the Caputo derivative $D^\alpha + V(t, x)$ of a Lyapunov function V be strictly negative on each interval $(t_k, tk + 1)$, and that the sequence $V(t_k^+, x(t_k^+))$ be non-increasing at impulse instants t_k. On the other hand, Theorem 3.1 only needs a decay term at the impulse times, that is, $V(t_{k+1}^+) \leq \mu V(t_k^+)$ with $\mu \in (0, 1)$ and a bound of the inter-pulse time. This has several advantages: it eliminates the continuous-time evaluation of the Caputo derivative of Lyapunov function, it reduces the memory demand to only discrete-time values, and it is more easily implementable, particularly in the context of control when impulses coincide with control updates. Thus, our result provides a more practical and less conservative criterion while ensuring exponential stability.

Proof. First, note that to apply hypotheses i) and ii), we must ensure that for any initial condition x_0 taken in a neighborhood B_δ close to the origin $x = 0$, the solution $x(t)$ does not leave the ball B_r for all $t \geq t_0^+$.

From (2.9), for all $t \in (t_k, t_{k+1}]$, we obtain

$$\|x(t)\| \leq \|x(t_k^+)\| E_\alpha(L(t_k - t)^\alpha). \tag{3.4}$$

Since $t - t_k \leq t_{k+1} - t_k \leq \theta$, Proposition 2.1 yields, for all $t \in (t_k, t_{k+1}]$,

$$\begin{aligned} \|x(t)\| &\leq \|x(t_k^+)\| E_\alpha(L\theta^\alpha), \\ &=: c_\alpha \|x(t_k^+)\|, \end{aligned} \tag{3.5}$$

where $c_\alpha = E_\alpha(L\theta^\alpha)$. Under assumptions *i)-ii)*, for every $k \geq 1$, we further obtain

$$\|x(t_k^+)\| \leq \sqrt{\frac{c_2}{c_1}\mu}\, \|x(t_{k-1}^+)\|. \tag{3.6}$$

Iterating (3.6) gives the explicit bound

$$\|x(t_k^+)\| \ \leq \ \Big(\sqrt{\frac{c_2}{c_1}\mu}\Big)^k \|x_0\|, \qquad k \geq 1. \tag{3.7}$$

As $\mu \in (0, \frac{c_1}{c_2})$, combining (3.5) and (3.7) yields for all $t \in (t_k, t_{k+1}]$,

$$\|x(t)\| \leq c_\alpha \|x_0\|. \tag{3.8}$$

In addition, it follows from (3.7), $\frac{c_2}{c_1}\mu \leq 1$, and the fact that $c_\alpha > 1$ that for $t = t_k^+$ we have

$$\begin{aligned} \|x(t_k^+)\| &\leq \|x_0\| \\ &\leq c_\alpha \|x_0\|. \end{aligned} \tag{3.9}$$

Then, for all $t \geq t_0$, we obtain

$$\|x(t)\| \leq c_\alpha \|x_0\|. \tag{3.10}$$

By choosing $\delta \leq \frac{r}{c_\alpha}$, we ensure that if $\|x_0\| \leq \delta$, then $\|x(t)\| \leq r, \forall t \geq t_0^+$. This implies for all $x_0 \in B_\delta$ that the solution $x(t) \in B_r, \forall t \geq t_0^+$.
Now, (3.5) and (3.7) give $\forall t \in (t_k, t_{k+1}]$

$$\|x(t)\| \leq c_\alpha \Big(\sqrt{\frac{c_2}{c_1}\mu} \Big)^k \|x_0\|. \tag{3.11}$$

Furthermore, it yields from assumption *iii)* that for $t \in (t_k, t_{k+1}]$,

$$0 < t - t_0 \leq (k+1)\theta, \tag{3.12}$$

which gives

$$k \geq \frac{t - t_0}{\theta} - 1. \tag{3.13}$$

Using (3.11) and (3.13) gives us

$$\|x(t)\| \leq c \, \|x_0\| \, e^{-\gamma(t-t_0)}, \tag{3.14}$$

where $c = c_\alpha \sqrt{\dfrac{c_1}{\mu c_2}}$ and $\gamma = -\dfrac{\ln\big(\frac{\mu c_2}{c_1}\big)}{2\theta}$. This demonstrates the exponential stability of impulsive systems (2.10)–(2.12), and achieves the proof of Theorem 3.1. □

3.2. Exponential stability under ETIC

In this section, Theorem 3.1 is employed for the purpose of developing an event-triggered control with the aim of achieving exponential stability of systems (2.7) and (2.8). The strategy is based on the construction of a sequence t_k of intervention times to influence the system using values of $u(x(t_k))$ inserted into the values of $x(t_k^+)$ in order to bring systems (2.10)–(2.12) exponentially closer to zero. More precisely, let $\delta_1, \ldots, \delta_p, p$, be positive constants, and $0 < \lambda < 1$. The, starting at time $t = t_0$ from point x_0, assuming that t_k has been constructed, to construct t_{k+1}, we examine the following set E_k:

$$E_k = \{\delta_i, 1 \leq i \leq p, \|x(t_k + \delta_i)\| \geq \lambda \|x(t_k^+)\|\}. \tag{3.15}$$

Let $\Delta = \max\limits_{1 \leq i \leq p} \delta_i$. The next instant t_{k+1} of intervention times to control the system is then defined according to the following event:

$$\mathcal{E}_1 : \begin{cases} t_{k+1} = t_k + \min E_k \text{ and } x(t_{k+1}^+) = x(t_{k+1}) + u((x(t_{k+1}))), \text{ if } E_k \neq \emptyset, \\ t_{k+1} = t_k + \Delta \text{ and } x(t_{k+1}^+) = x(t_{k+1}), \text{ if } E_k = \emptyset. \end{cases}$$

Remark 3.2. *In most of the literature, event-triggered control strategies do not rely on pre-specified impulse instants. Instead, the sequence of impulsive times $\{t_k\}$ is generated iteratively based on an event-triggering condition. Specifically, the next impulsive instant t_{k+1} is determined as follows (see [20, 22, 26, 28–31, 40–42, 46, 48]):*

$$t_{k+1} = \inf\{t > t_k \mid F(t) \geq 0\}, \tag{3.16}$$

where $F(t)$ is a triggering function that typically measures a state-dependent error or deviation from a desired behavior. In this paper, the strategy is different, and the sequence of impulse instants $\{t_k\}$ is constructed dynamically using a triggering mechanism based on the evolution of the system state. More precisely, at each instant t_k, the system checks whether the norm of the state at certain future instants $t_k + \delta_i$ (where $\delta_i > 0$ are predefined sampling instants) exceeds a fraction $\lambda \in (0, 1)$ of the norm $\|x(t_k^+)\|$. The next pulse instant t_{k+1} is then defined as the smallest of these instants satisfying this condition. This construction is formalized in event $\mathcal{E}_1$.

This discrete triggering mechanism limits the number of verifications to a finite set of points, which is in line with the practical constraints of numerical systems. Moreover, as all δ_i are strictly positive, we always have $t_{k+1} > t_k$, which guarantees that the $\{t_k\}$ sequence is strictly increasing and rules out any Zeno-type behavior.

Now we are able to establish our second result regarding the exponential stability of the impulsive systems (2.10)–(2.12).

Theorem 3.2. *Consider systems (2.10)–(2.12), and suppose there exist $r > 0$ and $l \in (0, 1)$ such that, for all $x \in B_r$,*

$$\|x + u(x)\| \leq l\|x\|, \tag{3.17}$$

and the sequence $(t_k, u(x(t_k))$ is defined by the mechanism $\mathcal{E}_1$. Then, the origin $x = 0$ of systems (2.10)–(2.12), is exponentially stable.

Proof. The proof of the guarantee that Zeno's phenomenon does not occur is the same as in Theorem 3.1, so it has been omitted.

Let $V(t, x(t)) = \|x(t)\|^2$ for all $t \geq t_0$. It is clear that V fulfills assumptions i) and ii) of Theorem 3.1 with $c_1 = c_2 = 1$. Let us prove that V also meets the requirement iii) of the aforementioned theorem. According to the definition of t_{k+1}, if $E_k = \emptyset$,

$$\begin{aligned} \|x(t_{k+1}^+)\|\| &= \|x(t_{k+1})\| \\ &\leq \lambda\|x(t_k^+)\|. \end{aligned} \tag{3.18}$$

If $E_k \neq \emptyset$, we have

$$\begin{aligned} \|x(t_{k+1}^+)\|\| &= \|x(t_{k+1}) + u(x(t_{k+1}))\| \\ &\leq l\|x(t_{k+1})\| \\ &\leq l\|x(t_k^+)\|E_\alpha(L(t_{k+1} - t_k)^\alpha) \\ &\leq lc_\alpha\|x(t_k^+)\|. \end{aligned} \tag{3.19}$$

Combining (3.18) and (3.19), we obtain

$$\|x(t_{k+1}^+)\|\| \leq \mu\|x(t_k^+)\|, \tag{3.20}$$

where $\mu = \max\{\lambda, lc_\alpha\}$. It follows by i) and (3.20) that

$$\begin{aligned} V(t_{k+1}^+, x(t_{k+1}^+)) &\leq \frac{c_1}{c_2}\mu^2 V(t_k^+, x(t_k^+)) \\ &= \mu^2 V(t_k^+, x(t_k^+)). \end{aligned} \tag{3.21}$$

Choosing $l < \frac{1}{c_\alpha}$ guarantees that $\mu \in (0, 1)$, and condition iii) of Theorem 3.1 is then verified. The exponential convergence of systems (2.10)–(2.12) under control triggered by event $\mathcal{E}_1$ is obtained. □

Remark 3.3. *The sequence construction method is a flexible approach which allows solutions to exceed the limits set after the moment* (t_k)*, provided they return below them before the next moment* (t_{k+1})*. This postpones intervention and allows the system to self-adjust, limiting the number of interventions and associated costs.*

3.3. ETIC under actuation delay

The result in subsection 3.2 is based on the assumption that there are no delays in the execution of control measures. In reality, however, this is a hypothetical scenario, since obstacles may arise from various factors. Logistical, human, technical, and managerial reasons, may postpone the moment of intervention. This postponement could have an impact on the rate of decay or the type of convergence of the system. In this section, we assume that the control commands are executed after the event time t_k with a time delay of $\tau > 0$. As a result, the time sequence of the impulse is shifted from t_k to $(t_k + \tau)$, leading to a transformation of the event-triggered mechanism into the following:

$$E_{kd} = \{\delta_i, 1 \le i \le p, \|x(t_k + \delta_i + \tau)\| \ge \lambda \|x(t_k^+\|\}, \tag{3.22}$$

and the next $(t_{k+1}, u(x(t_{k+1}))$ is given by

$$\mathcal{E}_2 : \begin{cases} t_{k+1} = t_k + \underline{\delta}_{kd}, x((t_{k+1} + \tau)^+) = x(t_{k+1} + \tau) + u(x(t_{k+1})), \text{ if } E_{kd} \neq \emptyset, \\ t_{k+1} = t_k + \Delta \text{ and } x((t_{k+1} + \tau)^+) = x(t_{k+1} + \tau), \text{ if } E_{kd} = \emptyset, \end{cases}$$

where $\underline{\delta}_{kd} = \min E_{kd}$. Then, according to the control-triggering mechanism ($\mathcal{E}_2$), our system takes the following form:

$$^c D_{t_k}^{\alpha} x(t) = f(t, x(t)), \ t \in (t_k + \tau, t_{k+1}] \tag{3.23}$$

$$x((t_k + \tau)^+) = x(t_k + \tau) + u(x(t_k)), \ k = 1, 2, .. \tag{3.24}$$

$$x(t_0^+) = x_0. \tag{3.25}$$

The following theorem gives certain adequate conditions guaranteeing the exponential convergence of the systems (3.23)–(3.25) when it is under ETM ($\mathcal{E}_2$).

Theorem 3.3. *Consider systems (3.23)–(3.25), and suppose there exist* $r > 0$ *and* $l \in (0, 1)$ *such that, for all* $x \in B_r$,

$$\|x + u(x)\| \le l \|x\|, \tag{3.26}$$

and the sequence $(t_k, u(x(t_k))$ *is defined by mechanism* $\mathcal{E}_2$*. Then, the origin* $x = 0$ *is exponentially stable.*

Proof. As in the proof of Theorem 3.2, Zeno's behavior is avoided. Consider the Lyapunov function defined for all $t \ge t_0$ by $V(t, x(t)) = \|x(t)\|^2$ and simply denoted by $v(t)$. Conditions i) and ii) are trivially satisfied. It remains to prove point iii). To achieve this, we proceed as follows:

$$\begin{aligned} x((t_k + \tau)^+) &= x(t_k + \tau) + u(x(t_k)) \\ &= x(t_k) + \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t_k+\tau} (t_k + \tau - s)^{\alpha - 1} f(s, x(s)) ds + u(x(t_k)). \end{aligned} \tag{3.27}$$

It follows from assumption (3.26) that

$$\begin{aligned} \|x((t_k+\tau)^+)\| &\le l\|x(t_k)\| + \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t_k+\tau}(t_k+\tau-s)^{\alpha-1}\|f(s,x(s))\|ds \\ &\le l\|x(t_k)\| + \frac{L}{\Gamma(\alpha)}\int_{t_k}^{t_k+\tau}(t_k+\tau-s)^{\alpha-1}\|x(s)\|ds \\ &\le l\|x(t_k)\| + \frac{L}{\Gamma(\alpha)}\int_{0}^{\tau}(\tau-u)^{\alpha-1}\|x(t_k)\|E_\alpha(Lu^\alpha)du \\ &\le (l + L\tau^\alpha E_{\alpha,\alpha+1}(L\tau^\alpha))\|x(t_k)\| \\ &\le \rho(\tau)\|x(t_k)\|, \end{aligned} \tag{3.28}$$

where $\rho(\tau) = l + L\tau^\alpha E_{\alpha,\alpha+1}(L\tau^\alpha)$. As

$$\|x(t_k)\| \le \|x((t_{k-1}+\tau)^+)\|\; E_\alpha(L(\Delta-\tau)^\alpha) \tag{3.29}$$

we obtain

$$v((t_k+\tau)^+)) \le \frac{c_2}{c_1}\rho^2(\tau)E_\alpha^2(L(\Delta-\tau)^\alpha)v((t_{k-1}+\tau)^+)). \tag{3.30}$$

Let $\phi(\tau) = \rho(\tau)E_\alpha(L(\Delta-\tau)^\alpha)$. Then, ϕ is a continuous function on $(0,\Delta]$, $\phi(\Delta) = \rho(\Delta) > 1$, and $\lim_{\tau\to 0^+}\phi(\tau) = lE_\alpha(L\Delta^\alpha)$. We can select $l > 0$ such that $l^2E_\alpha^2(L\Delta^\alpha) < 1$ and then pick τ with $\frac{c_2}{c_1}\phi^2(\tau) < 1$. It turns out that condition iii) of Theorem 3.1 is fulfilled; thus, $x = 0$ is exponentially stable. □

Remark 3.4. *Note that the choice of τ that satisfies $\frac{c_2}{c_1}\phi^2(\tau) < 1$ is strongly correlated with the choice of the control u. Since the Mittag-Leffler function is exponentially increasing, a sufficiently small τ must be allowed to ensure the convergence of the system.*

Remark 3.5. *The exponential stability established in Theorems 3.2 and 3.3 depends on the scalar*

$$\mu = \max\{\lambda,\ l\,c_\alpha\}, \qquad c_\alpha = E_\alpha(L\theta^\alpha)\,.$$

Hence, the choice of the fractional order α and the threshold λ is crucial:

i) A larger α (closer to 1) generally decreases c_α and accelerates convergence, whereas a smaller α slows down decay, yet stability is preserved as long as $\mu < 1$.
ii) A smaller λ tightens the triggering rule, lowering μ and enlarging the stability margin, at the cost of more frequent interventions (see Section 4).

For actuation delays, Theorem 3.3 shows that stability holds provided

$$\rho(\tau)\,E_\alpha(L(\Delta-\tau)^\alpha) < \sqrt{c_1/c_2},$$

where $\rho(\tau)$ and $\phi(\tau)$ (cf. Remark 3.4) quantify the impact of τ. Although increasing τ enlarges $\rho(\tau)$ and reduces robustness, exponential stability persists for a practical range of small-to-moderate delays, as confirmed numerically in Section 4.

Remark 3.6. *It is pertinent to point out that the field of stability of fractional-order differential systems with impulses and delays, as addressed in this study via the Caputo derivative, is also the subject of complementary research using other definitions of fractional operators. For example, recent work has explored the stability, particularly in finite time, of conformable fractional-order delay differential systems with impulses [18,47]. These parallel studies, although based on a different formulation of the fractional derivative, share fundamental concerns about the influence of impulsive events and delays on the convergence behavior of systems.*

4. Numerical validation

In this part, we show by computer simulation the effect of ETM $\mathcal{E}_1$ and $\mathcal{E}_2$ introduced respectively in Sections 3.1 and 3.2 on a system of fractional differential equations with chaotic behavior. For this, we consider a system of fractional differential equations used in finance and economics given in [6,7,23], modeling the interaction between three variables $x_i(t)$, $i = 1, \ldots, 3$ which respectively represent the interest rate, the demand for investment, and the price index. This model is governed by the following system:

$$\begin{array}{rcl} {}^cD_0^{\alpha_1}x_1 & = & x_3 + (x_2 - a)x_1, \\ {}^cD_0^{\alpha_2}x_2 & = & 1 - bx_2 - sx_1^2, \\ {}^cD_0^{\alpha_3}x_3 & = & -x_1 - cx_3, \end{array} \tag{4.1}$$

where a, b, and c are three parameters representing the amount of savings, the cost per investment, and the elasticity of demand in the commercial market, respectively. Note that, as is mentioned in [33], when the condition $b - c - abc \geq 0$ is satisfied, then system (4.1) admits three equilibrium points; otherwise, it admits only one point $x_e^* = (0, 1/b, 0)$.

In what follows, we successively apply the $\mathcal{E}_1$ and $\mathcal{E}_2$ stabilizing strategies to system (4.1) around the x_e^* equilibrium point, and we replace x by $y = x - x_e^*$ to satisfy the condition $x_e = 0$ mentioned in Theorem 3.1. Thus, under these assumptions, system (4.1) is rewritten in the compact form

$$\begin{array}{rcl} {}^cD_{t_0}^{\alpha}y(t) & = & f(t, y(t)); \quad t \geq t_0 \\ y(t_0^+) & = & y_0, \end{array} \tag{4.2}$$

where $f(t, y(t)) = (y_3 + y_1y_2 + (\frac{1}{b} - a)y_1, -by_2 - sy_1^2, -cy_3 - y_1)$.

To carry out this experimental study, we consider the following values for the parameters of system (4.1): $a = 0.9$, $b = 0.2$, $c = 0.5$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = 0.95$, and $x_0 = (1, 2, 0.9)$, where x_0 corresponds to the initial state of system (4.1). In this case, $b - c - abc = -1.57$. Therefore, system (4.1) admits a single equilibrium point $x_e^* = (0, 5, 0)$.

The control times $\delta_1, \delta_2, \delta_3$, the delay time τ, and the threshold λ cited in ETM $\mathcal{E}_1$ and $\mathcal{E}_2$ are given by: $\delta_1 = 0.3$, $\delta_2 = 0.7$, $\delta_3 = 0.9, \lambda = 0.6$, and $\tau = 0.01$. The control matrix $C(t)$ is given by

$$C(t) = \begin{pmatrix} -1 & 0 & -\frac{2}{3}\cos(\pi t) \\ 0 & -1 - \frac{2}{3}\sin(\pi t) & 0 \\ -\frac{2}{3}\sin(2t) & 0 & -1 \end{pmatrix} \tag{4.3}$$

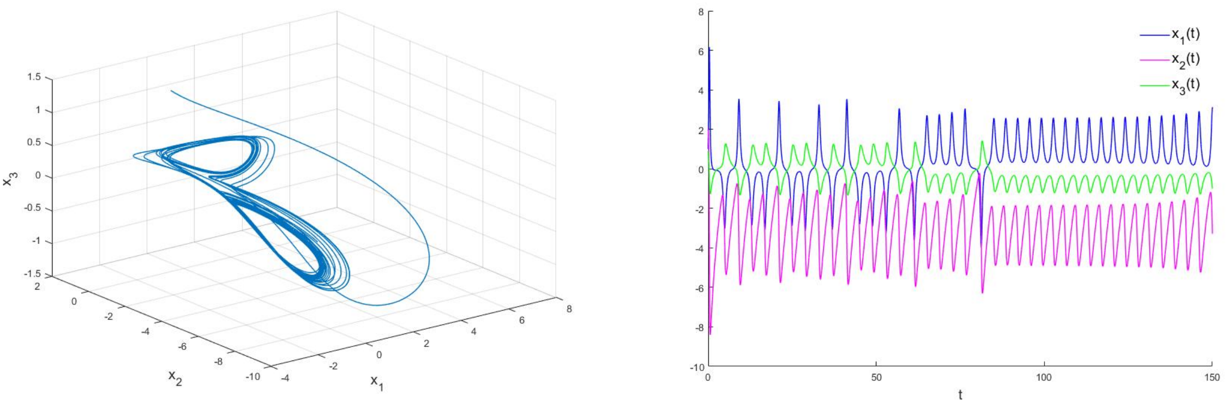
The fractional differential systems (4.1) and (4.2) are simulated with MATLAB R2017a. The numerical resolution of these systems is carried out using the `fde12` solver of fractional differential equations proposed in [5], and implemented in [16].

Remark 4.1. *In the implementation of the event-triggered impulsive control (ETIC) strategy, the design procedure is guided by the theoretical stability conditions derived in Theorems 3.2 and 3.3. Specifically, the control u is selected such that the inequality* $\mu = \max\{\lambda, lc_\alpha\} < 1$ *is satisfied. The event-triggering threshold* λ *is tuned to ensure a trade-off between stability and control efficiency. The sampling instants* $\{\delta_i\}$ *are chosen according to two principles: they should not be too small, in order to highlight the control effect when the system state becomes large; and they should not be too large, as excessively large* δ_i *are ineffective and often never activated in practice. The actuation delay* τ *is constrained to satisfy the admissibility condition involving the delay-dependent function* $\rho(\tau)$ *and the Mittag-Leffler function. This design ensures that all parameters used in the simulations comply with the analytical results and illustrate meaningful control behavior.*

Results and discussion

Figure 1(a) shows the phase portraits of system (4.1), while Figure 1(b) presents the temporal evolution of the components of x for $t \in [0, 150]$.

Based on Figure 1, we can see the chaotic behavior of the solutions of system (4.1) in the absence of control procedures. This result is not new; it has already been proven mathematically by Diouf and Sene in [6], and we have just validated it by simulation.

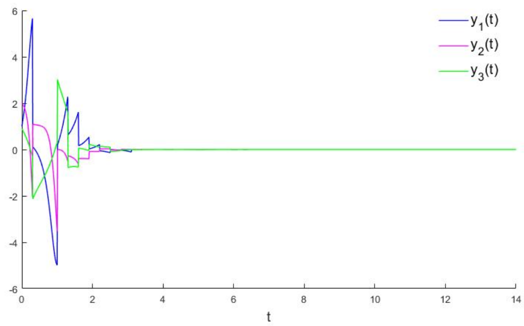


(a) Phase portraits of (4.1)

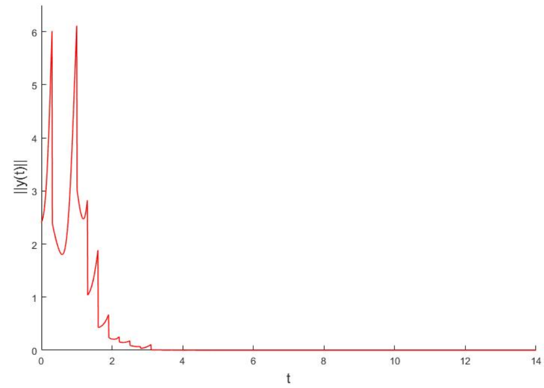
(b) Trajectories of components x_1, x_2, x_3

Figure 1. Illustration of the chaotic behavior of the system (4.1) for $t \in [0, 150]$.

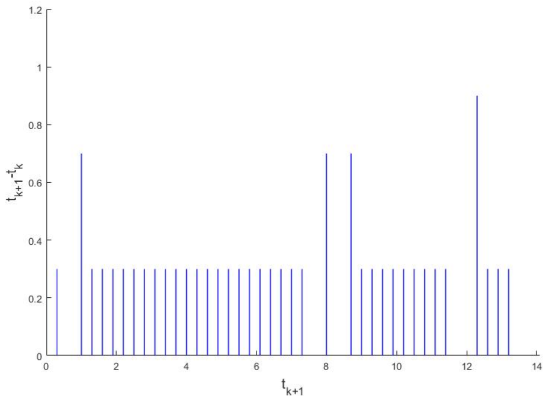
Figure 2 summarizes the effect of control procedure $\mathcal{E}_1$ on system (4.2). Thus, Figure 2(a) illustrates the exponential convergence of y towards the equilibrium point $(0, 0, 0)$. Furthermore, Figure 2(b) shows the exponential convergence of $\|y\|$ towards 0. Finally, Figure 2(c) presents both the intervention instants $t_k,\ k \geq 0$, and the duration $t_{k+1} - t_k,\ k \geq 0$ between two instants of successive interventions.



(a) Evolution of $y_i(t)$, $t \in [0, 14]$



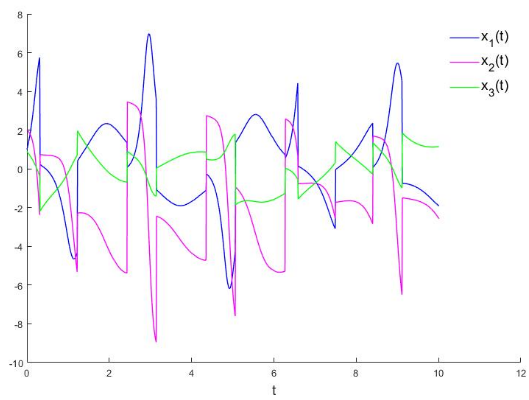
(b) Evolution $\|y(t)\|$, $t \in [0, 14]$



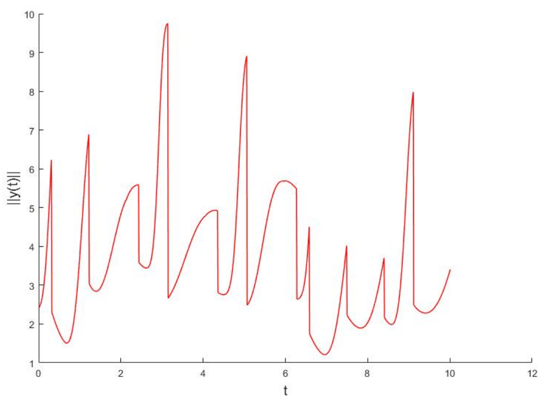
(c) $t_{k+1} - t_k$ vs t_{k+1}

Figure 2. Resolution of system (4.2) under ETM ($\mathcal{E}_1$).

Figure 3 summarizes the effect of the control procedure $\mathcal{E}_2$ on system (4.2). Figure 3(a),(b) illustrates the temporal evolution of y and $\|y\|$, respectively.



(a) Evolution of $y_i(t)$

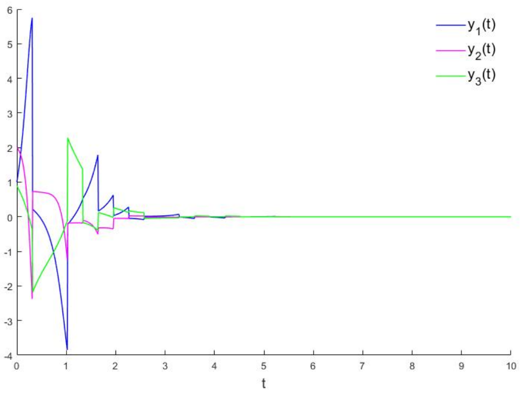


(b) Evolution $\|y(t)\|$

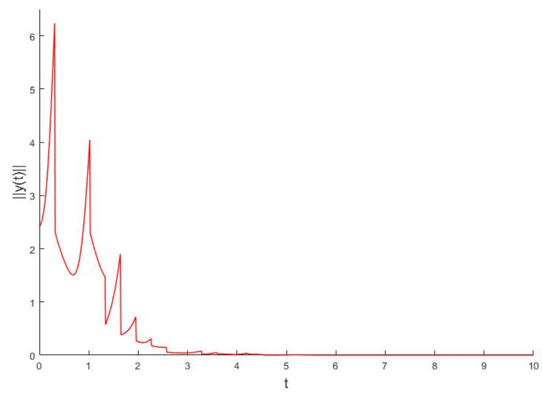
Figure 3. Resolution of system (4.2) under ETM ($\mathcal{E}_2$), $t \in [0, 10]$, $\tau = 0.01$, $\lambda = 0.6$.

According to Figure 3(a),(b), the solution y of system (4.2) is no longer convergent. We have thus shown that when faced with a very small delay ($\tau = 0.01$), the solution of system (4.2) becomes divergent. This reinforces the chaotic nature of system (4.1). To overcome this drawback, we repeated the same experiment using a smaller λ value equal to 0.1 instead of 0.6.

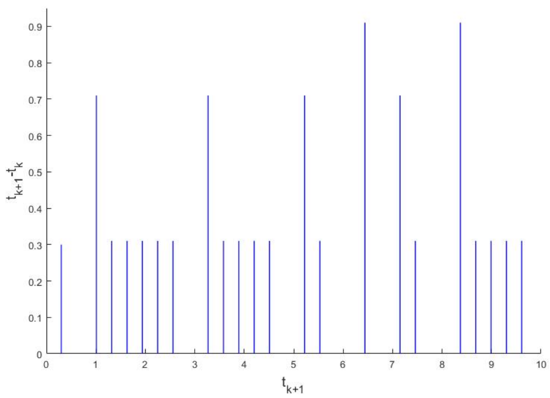
Figure 4 summarizes the effect of control procedure $\mathcal{E}_2$ on system (4.2) with $\lambda = 0.1$. Figure 4(a),(b) illustrates the temporal evolution of y and $\|y\|$, respectively. Figure 4(c) presents both the intervention instants $t_k,\ k \geq 0$, and the duration $t_{k+1} - t_k,\ k \geq 0$ between two instants of successive interventions.



(a) Evolution of $y_i(t)$



(b) Evolution $\|y(t)\|$



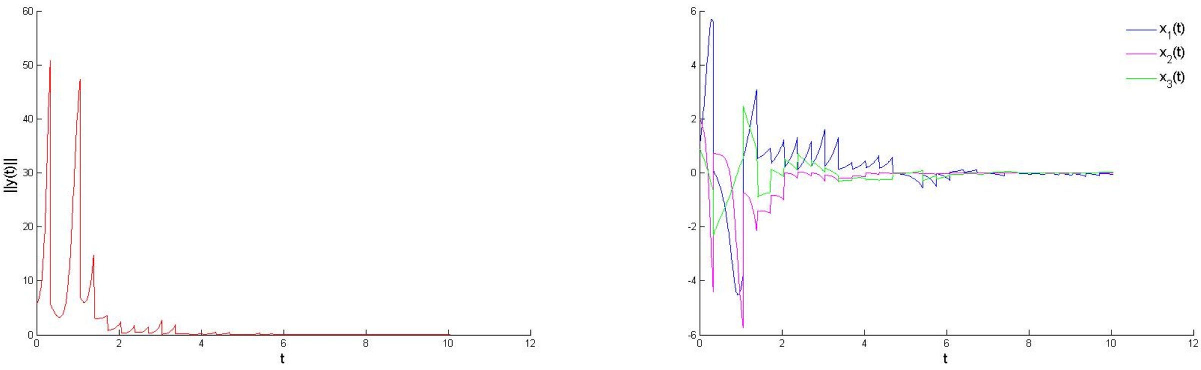
(c) $t_{k+1} - t_k$ vs t_{k+1}

Figure 4. Resolution of system (4.2) under ETM ($\mathcal{E}_2$), $t \in [0, 10]$, $\tau = 0.01$, $\lambda = 0.1$.

According to Figure 4(a),(b), the solution y of the system (4.2) is now convergent, and the procedure $\mathcal{E}_2$ has successfully controlled the chaos of the system.

Remark 4.2. *The numerical simulations in Figures 3 and 4 clearly illustrate the practical trade-off between the triggering threshold λ and system performance under actuation delay. When $\lambda = 0.6$, the event-triggered control strategy fails to stabilize the system (Figure 3), as the delay combined with a relaxed triggering condition results in insufficient intervention. However, reducing the threshold to $\lambda = 0.1$ (Figure 4) enables successful exponential stabilization, despite the presence of delay. This highlights that while a smaller λ leads to more frequent triggering and higher control cost, it significantly improves robustness and stability margin. The optimal choice of λ must therefore balance* *performance and resource constraints based on application needs.*

Remark 4.3. *Figure 5 confirms that the proposed ETIC strategy remains exponentially stable for fractional order* $\alpha \geq 0.90$ *and actuation delays* $\tau < 0.04$*, provided that the triggering threshold* λ *is chosen appropriately. This supports the theoretical robustness of the method with respect to variations in* α *and* τ*.*



(a) Evolution of $y_i(t)$ **(b)** Evolution $\|y(t)\|$

Figure 5. Resolution of (4.2) under ETM ($\mathcal{E}_2$), $t \in [0, 10]$, $\alpha = 0.9$, $\tau = 0.03$, $\lambda = 0.1$.

5. Conclusions

In this paper, exponential stabilization via event-triggered control is studied for fractional-order systems with $\alpha \in (0, 1)$. Sufficient conditions for exponential stability of impulsive systems are derived, leading to two event-triggered impulsive control (ETIC) strategies.

The first strategy triggers control after a limited set of non-periodic measurements, providing greater flexibility and reducing both control cost and data storage. The second strategy examines system convergence when there is an actuation delay in the control law.

While the present study addresses constant delays, the robustness of ETIC schemes with respect to *time-varying and uncertain delays*, along with data-loss effects, remains an open problem and will be pursued in future work. Finally, the effectiveness of the proposed methods is illustrated through a simulation of a chaotic financial model governed by fractional-order dynamics.

Author contributions

Mohsen DLALA: Conceptualization, supervision, investigation, drafting of theoretical proofs, formal analysis, writing, review, editing, acquisition of funding; Abdelhamid ZAIDI: Investigation, validation, programming, writing the numerical validation, review, and editing; Farida ALHARBI: Investigation, validation, writing, preparation of the original project, methodology, formal analysis, revision and editing of the writing, acquisition of funding. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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