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Research article

Solitary wave solutions for the conformable time-fractional coupled Konno-Oono model via applications of three mathematical methods

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Abstract: This research extensively investigates the solitary wave solutions of the fractional coupled Konno–Oono model, a prevalent framework in diverse scientific and engineering disciplines. Three mathematical methods called the simple equation method, the modified extended auxiliary equation mapping method, and the exponential-expansion method are gradually employed to derive the analytical solutions. Moreover, Mathematica 13.0 software is used to perform the analytical computations and graphical simulations. The explored outcomes have significant applications in the realm of magnetic fields. After the careful selection of parametric values under constrained conditions, some solutions are plotted in 2-dimensional and 3-dimensional spaces to understand the physical phenomena of the concerned model. Importantly, our findings affirm that the employed methods not only yield complete and uniform responses but also showcase simplicity, effectiveness, and remarkable computational efficiency. Hence, our research contributes valuable insights into the behavior of the fractional coupled Konno–Oono model, paving the way for enhanced comprehension and potential applications in magnetic field studies.

Keywords: fractional coupled Konno–Oono model (FCKOM); mathematical methods; soliton solutions; fractional derivatives
Mathematics Subject Classification: 35C05, 35C07, 35C08, 47J35

1. Introduction

Nonlinear partial differential equations play an essential role in numerous applied science disciplines, like plasma physics, economics, propagation of shallow water waves, optics, solid-state physics, fluid dynamics, biology, electromagnetic theory, and many more. Recent studies have established the efficiency of analytical solutions in providing exact and decoupled solutions to the majority of nonlinear evolution equations encountered within physics and engineering. Zayed and Arnous [1] examined DNA dynamics via the homogeneous balance method, whose application of non-linear processes in biological processes was brought to the fore. Altalbe et al. [2] applied different analysis techniques to study the space–time fractional van der Waals equation with particular emphasis on fractional calculus in fluid dynamics. Ryabov et al. [3] applied Kudryashov's method for high-order evolution equations and derived new exact solutions. Similarly, Cevikel et al. [4] studied the FitzHugh–Nagumo model using conformable derivatives in order to investigate novel solitonic behaviors. Wang et al.'s classical (G'/G)-expansion approach [5] has been a device to solve nonlinear evolution equations. Darvishi et al. [6] have extended the BLMP model to (2+1)-dimension and uncovered diverse soliton structures. Nestor et al.'s [7] subsequent work uncovered solitary waves of the Klein–Gordon–Zakharov system and optical solitons of perturbed nonlinear Schrödinger equations with double-power law nonlinearity. Finally, Cevikel [8] also presented precise optical solutions to the (3+1)-dimensional YTSF equation, again indicating the generalizability of these approaches to higher-dimensional wave processes. All of these efforts combined are reflective of the growing significance of the exact analytical techniques in model construction and description of increasingly complex nonlinear systems [9].

Analytical solutions provide us with a meticulous understanding of how a system performs, grounded on definite conditions and characteristics. These solutions help reconnoiter a system's dynamics, revealing critical points, singularities, and stability. This inclination for analytical solutions is particularly evident in studying fractional partial differential equations, where it helps investigators make theoretical predictions and gain shrewd knowledge about the system's behavior, managing future research in this field. Many researchers created variable analytical methodologies to find the exact and soliton solutions of nonlinear partial differential equations, like bilinear transformation [10], Bäcklund transformation [11], Painlevé analysis [12], Hirota bilinear method [13,14], trilinear analysis method [15], modified extended direct algebraic method [16], Lie point symmetries analysis [17], improved tanh-function algorithm [18], extended generalized Riccati mapping method [19], F-expansion method [20,21], and many more.

Recently, certain analytical and semi-analytical methods have been employed to obtain the solution of nonlinear differential equations in applied mathematics and mathematical physics. Nadeem and He [22] successfully employed the He–Laplace variational iteration technique for the solution of nonlinear chemical kinetics and population dynamics problems, establishing the potency of variational principles in physical systems. In [23], the authors validated the Adomian Decomposition Method (ADM) for Volterra integral equations with a discontinuous kernel using the CESTAC method to ensure high numerical accuracy. The q-homotopy analysis technique was applied in [24] to the Cahn–Hilliard equation, which plays an important role in advection and reaction dynamics of phase separation processes. Modanli et al. [25] and Qazza et al. [26] employed the residual power series and ARA-residual power series methods, respectively, to obtain solutions of complex pseudo-hyperbolic and fractional systems with evidence of their effectiveness in problems of nonlocal and memory-dependent nature.

The authors of [27] presented new coupling doubly dispersive equation solutions using the extended Bernoulli sub-equation method, while Akbar et al. [28] explored soliton dynamics for perturbed Schrödinger equations and microtubules by using the generalized Kudryashov approach. Similarly, Hashemi and Mirzazadeh [29] used Lie symmetry analysis for the optical solitons of the same class of equations. In [30], they solved the variable coefficient Davey–Stewartson system to derive novel soliton and periodic solutions. Rezazadeh et al. [31] also used variable-coefficient analysis to study traveling wave structures in nonlinear Schrödinger systems. Ali et al. [32], Seadawy et al. [33], and Lu et al. [34] solved dispersive wave models and generalized equal width equations using extended simple equation and Riemann–Liouville methods.

Ahmad and Younas [36] also extended the above works by considering the conformable time-fractional Konno–Oono model, magnetic field stability outcomes, etc. Liang et al. [37] found different wave phenomena of the fractional Boiti–Leon–Manna–Pempinelli equation, i.e., kinks and breathers, and Li et al. [38] found a variational principle for the fractal derivative nonlinear Schrödinger equation. Collectively, these advances illustrate the significance of advanced analytical methods for characterizing complex soliton dynamics and for enriching our understanding of fractional and nonlinear evolution equations in a wide variety of physical settings.

The main motivation of this work is to investigate the solitary wave solutions of the nonlinear fractional coupled Konno-Oono model, a prevalent framework in diverse scientific and engineering disciplines. For this purpose, three analytical mathematical methods [32–34] are employed to the FCKOM with the assistance of the conformable derivative [35]. The solutions obtained using the employed methods are expressed in the form of exponential, trigonometric, and hyperbolic functions. Our obtained results are novel and have a variety of solutions as compared to results found in previous research articles [36, 39, 40]. Moreover, explored outcomes have significant applications in the realm of magnetic fields and have diverse applications in physics.

In Section 2, the definition of conformable fractional derivative and its properties are discussed. In Section 3, details of the proposed mathematical techniques are arranged. Section 4 represents the mathematical analysis of FCKOM. In Section 5, the proposed methods are applied to construct solitary wave solutions of FCKOM. In Section 6, the conclusion is discussed.

2. Conformable fractional derivative

Conformal fractional derivatives have been introduced to discourse the limitations of conventional fractional derivatives, which adhere to standard derivative rules, making them a subject of interest for several investigators.

2.1. Definition (conformable fractional derivative of order γ):

Let $U : [0, \infty] \to \mathbb{R}$ represent a function; the conformable fractional derivative of order γ is defined as;

$$T_\gamma(U)(t) = \operatorname*{Lim}_{\delta \to 0} \left(\frac{U\left(\delta t^{1-\gamma} + t\right) - U(t)}{\delta} \right), \text{ for all } t > 0, \quad 0 < \gamma \leq 1.$$

2.2. Some properties of conformable fractional derivative:

Let U_1 and U_2 be two γ-conformable differential functions with the inclusions α_1 and α_2 are constants then followings properties hold;

- $T_\gamma(\alpha_1 U_1 + \alpha_2 U_2) = \alpha_1 T_\gamma(U_1) + \alpha_2 T_\gamma(U_2)$.
- $T_\gamma(t^\omega) = \rho t^{\omega-\gamma}$ for all $\omega \in \mathbb{R}$.
- $T_\gamma(U_1 U_2) = U_1 T_\gamma(U_2) + U_2 T_\gamma(U_1)$.
- $T_\gamma\left(\frac{U_1}{U_2}\right) = \left(\frac{U_2 T_\gamma(U_1) - U_1 T_\gamma(U_2)}{U_2^2}\right)$.
- $T_\gamma(U)(t) = t^{1-\gamma}\frac{\mathrm{dU}}{\mathrm{dt}}(t)$.

3. Description of mathematical methods

Let

$$F_1(P, P_x, P_t, P_{xx}, P_{tt} \ldots) = 0, \tag{3.1}$$

consider,

$$P = P(\xi), \qquad \xi = x - \omega t. \tag{3.2}$$

Put Eq (3.1) into Eq (3.2),

$$F_2\left(P, P', \omega P', P'', \omega^2 P'' \ldots\right) = 0. \tag{3.3}$$

3.1. Extended simple equation method:

Let Eq (3.3) have a solution,

$$P(\xi) = \sum_{i=-N}^{N} A_i \Psi^i(\xi). \tag{3.4}$$

Let Ψ'

$$\Psi' = L_0 + L_1\Psi + L_2\Psi^2 + L_3\Psi^3. \tag{3.5}$$

The general solutions to Eq (3.5) are as follows;

If $L_3 = 0$, then the solution of Eq (3.5) is;

$$\Psi = -\left(\frac{L_1 - \sqrt{4L_0L_2 - L_1^2}\tan\left(\frac{1}{2}\sqrt{4L_0L_2 - L_1^2}(\xi + \delta_0)\right)}{2L_2}\right),\ 4L_0L_2 > L_1^2. \tag{3.6}$$

If $L_0 = L_3 = 0$, then the solution of Eq (3.5) is;

$$\Psi(\xi) = \left(\frac{L_1 \exp(L_1(\xi + \delta_0))}{1 - L_2 \exp(L_1(\xi + \delta_0))}\right),\ L_1 > 0, \tag{3.7}$$

$$\Psi(\xi) = -\left(\frac{L_1 \exp(L_1(\xi + \delta_0))}{L_2 \exp(L_1(\xi + \delta_0)) + 1}\right),\ L_1 < 0. \tag{3.8}$$

If $L_1 = L_3 = 0$, then the solution of Eq (3.5) is;

$$\Psi(\xi) = \left(\frac{\sqrt{L_0L_2}\tan\left(\sqrt{L_0L_2}(\xi + \delta_0)\right)}{L_2}\right),\ \ L_0L_2 > 0, \tag{3.9}$$

$$\Psi(\xi) = \left(\frac{\sqrt{-L_0 L_2} \tanh\left(\sqrt{-L_0 L_2} \, (\xi + \delta_0) \right)}{L_2} \right), \quad L_0 L_2 < 0. \tag{3.10}$$

Put (3.4) with (3.5) in (3.3) and solved for the solutions of Eq (3.1) with Eqs (3.6)–(3.10).

3.2. Modified extended auxiliary equation mapping method:

Let Eq (3.3) have solution

$$P(\xi) = \sum_{i=0}^{N} A_i \Psi^i + \sum_{i=-1}^{-N} B_{-i} \Psi^i + \sum_{i=2}^{N} C_i \Psi^{i-2} \Psi' + \sum_{i=1}^{N} D_i \left(\frac{\Psi'}{\Psi} \right)^i. \tag{3.11}$$

Let Ψ'

$$\Psi' = \sqrt{l_1 \Psi^2 + l_2 \Psi^3 + l_3 \Psi^4}. \tag{3.12}$$

The solutions of (3.12) are

$$\Psi = -\left(\frac{l_1 \left(\epsilon \coth\left(\frac{1}{2} \sqrt{l_1} \, (\xi + \delta_0) \right) + 1 \right)}{l_2} \right), \; l_1 > 0, \; \epsilon = 1 \; or - 1, \; l_2^2 - 4 l_1 l_3 = 0. \tag{3.13}$$

$$\Psi = - \sqrt{\frac{l_1}{4 l_3}} \left(\frac{\epsilon \sinh\left(\sqrt{l_1} \, (\xi + \delta_0) \right)}{\cosh\left(\sqrt{l_1} \, (\xi + \delta_0) \right) + \eta} + 1 \right), \; l_1 > 0, \; l_3 > 0, \; l_2 = \sqrt{4 l_1 l_3},$$
$$(\epsilon, \eta) = (1,1), (1,-1), \; (-1,1), \; (-1,-1). \tag{3.14}$$

$$\Psi = -\left(\frac{l_1 \left(\frac{\epsilon \left(\sinh\left(\sqrt{l_1} (\xi + \delta_0) \right) + R \right)}{\cosh\left(\sqrt{l_1} (\xi + \delta_0) \right) + \eta \sqrt{R^2 + 1}} + 1 \right)}{l_2} \right), \; l_1 > 0,$$
$$(\epsilon, \eta) = (1,1), \; (1,-1), \; (-1,1), \; (-1,-1). \tag{3.15}$$

Put (3.11) with (3.12) in (3.3), solved for Eq (3.1) with Eqs (3.13)–(3.15).

3.3. The $Exp(-\Psi(\xi))$-expansion method:

Let Eq (3.3) have solution

$$P(\xi) = A_N \, (Exp(-\Psi(\xi)^N + ..., A_N \neq 0. \tag{3.16}$$

Let

$$\Psi' = Exp(-\Psi(\xi)) + \mu_1 Exp(\Psi(\xi)) + \lambda_1. \tag{3.17}$$

For $\lambda_1^2 - 4\mu_1 > 0, \;\; \mu_1 \neq 0$ then the solution of Eq (3.17) is as follows:

$$\Psi = \log \left(\frac{-\sqrt{\lambda_1^2 - 4\mu_1} \tanh\left(\frac{1}{2} \sqrt{\lambda_1^2 - 4\mu_1} (\xi + \delta_0) \right) - \lambda_1}{2\mu_1} \right). \tag{3.18}$$

For $\lambda_1^2 - 4\mu_1 > 0,\ \ \mu_1 = 0$, then the solution of Eq (3.17) is

$$\Psi = -\log\left(\frac{\lambda_1}{e^{\lambda_1(\xi+\delta_0)} - 1}\right). \tag{3.19}$$

For $\lambda_1^2 - 4\mu_1 = 0,\ \ \lambda_1 \neq 0$, then the solution of Eq (3.17) is

$$\Psi = \log\left(\frac{2 - 2\lambda_1(\xi+\delta_0)}{\lambda_1^2(\xi+\delta_o)}\right). \tag{3.20}$$

For $\lambda_1^2 - 4\mu_1 = 0,\ \ \mu_1 = \lambda_1 = 0$, then the solution of Eq (3.17) is

$$\Psi = \log(\xi+\delta_0). \tag{3.21}$$

For $\lambda_1^2 - 4\mu_1 < 0$, then the solution of Eq (3.17) is

$$\Psi = \log\left(\frac{-\sqrt{4\mu_1-\lambda_1^2}\tan\left(\frac{1}{2}(\xi+\delta_0)\sqrt{4\mu_1-\lambda_1^2}\right) - \lambda_1}{2\mu_1}\right). \tag{3.22}$$

Put (3.16) with (3.17) in (3.3) and solved for the solution of Eq (3.1) with the help of Eqs (3.18)–(3.22).

4. Mathematical analysis of fractional coupled Konno-Oono model

Consider the mathematical form of the fractional coupled Konno–Oono model [36, 39, 40],

$$\begin{aligned} &D_t^\gamma P_x(x,t) - 2P(x,t)Q(x,t) = 0,\\ &D_t^\gamma Q(x,t) + 2P(x,t)P_x(x,t) = 0. \end{aligned} \tag{4.1}$$

FCKOM has fruitful applications where the conservation of quantities and symmetries is essential. When γ tends towards 1, the system (4.1) is simplified to the integer-order derivative with the following new form:

$$\begin{aligned} &P_{xt}(x,t) - 2P(x,t)Q(x,t) = 0,\\ &Q_t(x,t) + 2P(x,t)P_x(x,t) = 0. \end{aligned} \tag{4.2}$$

Let

$$\begin{aligned} &P(x,t) = P(\xi), \qquad \xi = k\left(x - \frac{\omega t^\gamma}{\gamma}\right),\\ &Q(x,t) = Q(\xi), \qquad \xi = k\left(x - \frac{\omega t^\gamma}{\gamma}\right). \end{aligned} \tag{4.3}$$

Put Eq (4.3) in Eq (4.2), we have

$$\begin{aligned} &-k\omega P'' - 2PQ = 0,\\ &2kPP' - \omega Q' = 0. \end{aligned} \tag{4.4}$$

Integrating the second equation from the system Eq (4.4), we obtained,

$$Q(\xi) = \frac{P^2 + \chi}{k}. \tag{4.5}$$

Here χ is a constant of integration; substitute (4.5) into the first equation in (4.4),

$$2\chi \mathrm{P} + k^2\omega P'' + 2P^3 = 0. \tag{4.6}$$

5. Applications

5.1. Application of ESE method

Let Eq (4.6) have a solution

$$P = A_1\Psi + \frac{A_{-1}}{\Psi} + A_0. \tag{5.1}$$

Put Eq (5.1) with Eq (3.5) into Eq (4.6).
Case 1. $L_3 = 0$,
F-I

$$A_{-1} = 0,\ A_1 = -\frac{2iL_2\sqrt{\chi}}{\sqrt{L_1^2 - 4L_0L_2}},\ A_0 = -\frac{iL_1\sqrt{\chi}}{\sqrt{L_1^2 - 4 = L_0L_2}},\ \omega = \frac{4\chi}{k^2\left(L_1^2 - 4L_0L_2\right)}. \tag{5.2}$$

Put (5.2) in (5.1),

$$P_1(x,t) = -\left(\frac{i\sqrt{\chi}\left(\sqrt{4L_0L_2 - L_1^2}\tan\left(\frac{1}{2}\sqrt{4L_0L_2 - L_1^2}(\delta_0 + \xi)\right) + L_1\right)}{\sqrt{L_1^2 - 4L_0L_2}}\right) - \left(\frac{iL_1\sqrt{\chi}}{\sqrt{L_1^2 - 4 = L_0L_2}}\right).\ \ 4L_0L_2 > L_1^2. \tag{5.3}$$

$$Q_1(x,t) = \frac{1}{k}\left(\left(\frac{i\sqrt{\chi}\left(\sqrt{4L_0L_2 - L_1^2}\tan\left(\frac{1}{2}\sqrt{4L_0L_2 - L_1^2}(\delta_0 + \xi)\right) + L_1\right)}{\sqrt{L_1^2 - 4L_0L_2}}\right) - \left(\frac{iL_1\sqrt{\chi}}{\sqrt{L_1^2 - 4 = L_0L_2}}\right)\right)^2 + \left(\frac{\chi}{k}\right). \tag{5.4}$$

The solution's representation of Eqs (5.3) and (5.4) is shown in Figure 1.

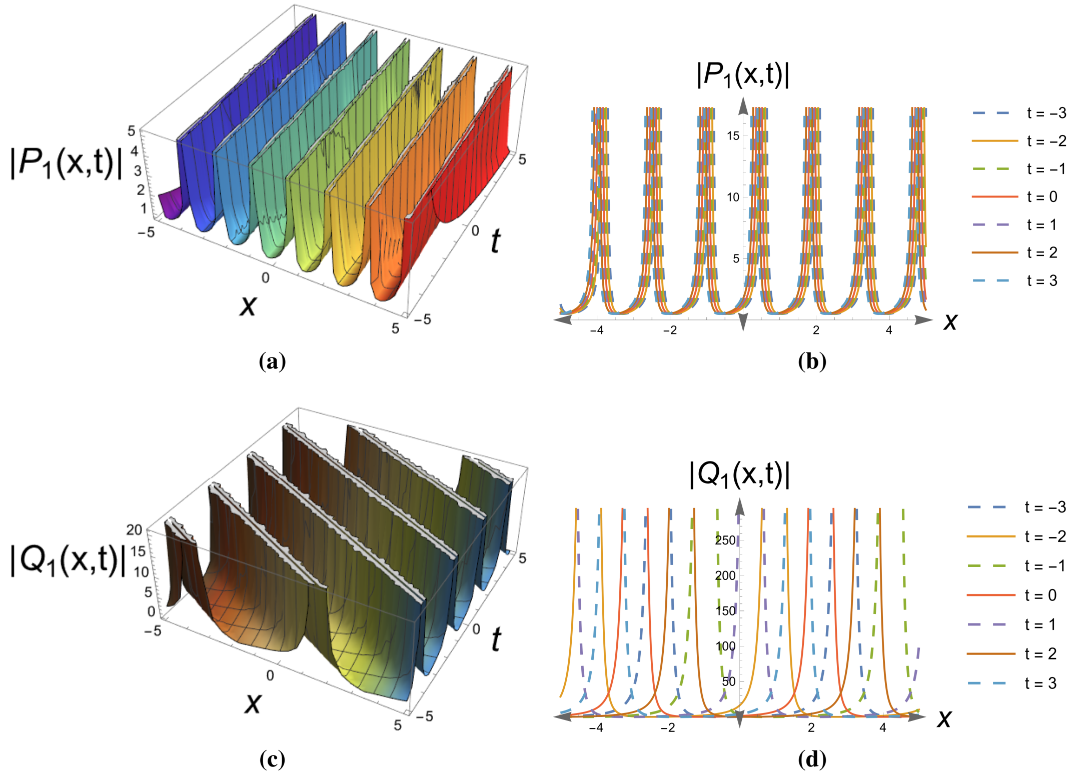

Figure 1. The 3-dimensional and 2-dimensional plots of periodic solitary wave solutions of P_1**(a,b)** and Q_1**(c,d)** with $\gamma = 1,\ \delta_0 = 0.7,\ k = 2.5,\ L_0 = L_1 = L_2 = 1,\ \chi = 0.5$ and $\gamma = 1,\ \delta_0 = 0.2,\ k = 0.7,\ L_0 = L_1 = L_2 = 1,\ \chi = 0.5$.

F-II

$$A_{-1} = \frac{2iL_0\sqrt{\chi}}{\sqrt{L_1^2 - 4L_0L_2}},\ A_1 = 0,\ A_0 = \frac{iL_1\sqrt{\chi}}{\sqrt{L_1^2 - 4L_0L_2}},\ \omega = \frac{4\chi}{k^2\left(L_1^2 - 4L_0L_2\right)}. \tag{5.5}$$

Substitute (5.5) in (5.1),

$$P_2(x,t) = \left(\frac{i\sqrt{\chi}\left(L_1 - \frac{4L_0L_2}{\sqrt{4L_0L_2 - L_1^2}\tan\left(\frac{1}{2}\sqrt{4L_0L_2 - L_1^2}(\delta_0+\xi)\right)+L_1}\right)}{\sqrt{L_1^2 - 4L_0L_2}}\right),\ 4L_0L_2 > L_1^2. \tag{5.6}$$

$$Q_2(x,t) = \frac{1}{k}\left(\frac{i\sqrt{\chi}\left(L_1 - \frac{4L_0L_2}{\sqrt{4L_0L_2 - L_1^2}\tan\left(\frac{1}{2}\sqrt{4L_0L_2 - L_1^2}(\delta_0+\xi)\right)+L_1}\right)}{\sqrt{L_1^2 - 4L_0L_2}}\right)^2 + \left(\frac{\chi}{k}\right). \tag{5.7}$$

Case 2. $L_0 = L_3 = 0$,

$$A_{-1} = 0,\ A_1 = \frac{2iL_2\sqrt{\chi}}{L_1},\ A_0 = i\sqrt{\delta},\ \omega = \frac{4\chi}{k^2L_1^2}. \tag{5.8}$$

Put (5.8) in (5.1),

$$P_3(x,t) = \left(i\sqrt{\chi} + \frac{\left(2iL_2\sqrt{\chi}\right)\left(L_1 \exp\left(L_1\left(\delta_0+\xi\right)\right)\right)}{L_1\left(1 - L_2\exp\left(L_1\left(\delta_0+\xi\right)\right)\right)} \right),\ L_1 > 0. \tag{5.9}$$

$$Q_3(x,t) = \frac{1}{k}\left(i\sqrt{\chi} + \frac{\left(2iL_2\sqrt{\chi}\right)\left(L_1 \exp\left(L_1\left(\delta_0+\xi\right)\right)\right)}{L_1\left(1 - L_2\exp\left(L_1\left(\delta_0+\xi\right)\right)\right)} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.10}$$

$$P_4(x,t) = \left(i\sqrt{\chi} - \frac{\left(2iL_2\sqrt{\chi}\right)\left(L_1 \exp\left(L_1\left(\delta_0+\xi\right)\right)\right)}{L_1\left(L_2\exp\left(L_1\left(\delta_0+\xi\right)\right)+1\right)} \right),\ L_1 < 0. \tag{5.11}$$

$$Q_4(x,t) = \frac{1}{k}\left(i\sqrt{\chi} - \frac{\left(2iL_2\sqrt{\chi}\right)\left(L_1 \exp\left(L_1\left(\delta_0+\xi\right)\right)\right)}{L_1\left(L_2\exp\left(L_1\left(\delta_0+\xi\right)\right)+1\right)} \right)^2 + \left(\frac{\chi}{k}\right),\ L_1 < 0. \tag{5.12}$$

The solution's representation of Eqs (5.9) and (5.10) is shown in Figure 2.

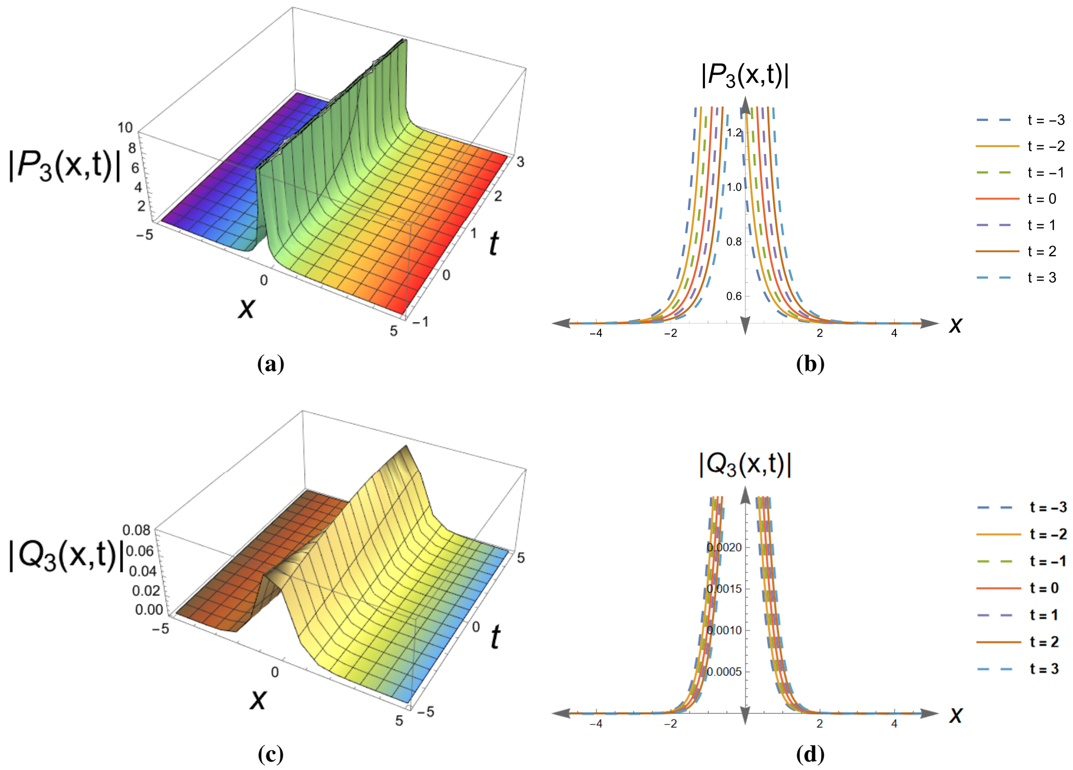

Figure 2. The 3-dimensional and 2-dimensional plots of bright solitary wave solutions of P_3**(a,b)** and Q_3**(c,d)** with $\gamma = 1$, $\delta_0 = 0.7$, $k = 2.4$, $L_1 = 1$, $L_2 = 1$, $\chi = 0.5$, and $\gamma = 1$, $\delta_0 = -1.7$, $k = -2.4$, $L_1 = 1$, $L_2 = -4$, $\chi = -0.2$.

Case 3. $L_1 = L_3 = 0$,

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$$A_{-1} = 0,\ A_1 = \frac{\sqrt{\chi \mathrm{L}_2}}{\sqrt{L_0}},\ A_0 = 0,\ \omega = -\frac{\chi}{k^2 L_0 L_2}. \tag{5.13}$$

Put (5.13) in (5.1),

$$P_5(x,t) = \left(\frac{\sqrt{\chi \mathrm{L}_2}\left(\sqrt{L_0 L_2}\tan\left(\sqrt{L_0 L_2}\,(\delta_0 + \xi)\right)\right)}{\sqrt{L_0} L_2} \right),\ L_0 L_2 > 0. \tag{5.14}$$

$$Q_5(x,t) = \frac{1}{k}\left(\frac{\sqrt{\chi \mathrm{L}_2}\left(\sqrt{L_0 L_2}\tan\left(\sqrt{L_0 L_2}\,(\delta_0 + \xi)\right)\right)}{\sqrt{L_0} L_2} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.15}$$

$$P_6(x,t) = \left(\frac{\sqrt{\chi \mathrm{L}_2}\left(\sqrt{-L_0 L_2}\tanh\left(\sqrt{-L_0 L_2}\,(\delta_0 + \xi)\right)\right)}{\sqrt{L_0} L_2} \right),\ L_0 L_2 < 0. \tag{5.16}$$

$$Q_6(x,t) = \frac{1}{k}\left(\frac{\sqrt{\chi \mathrm{L}_2}\left(\sqrt{-L_0 L_2}\tanh\left(\sqrt{-L_0 L_2}\,(\delta_0 + \xi)\right)\right)}{\sqrt{L_0} L_2} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.17}$$

F-II

$$A_{-1} = 0,\ A_1 = -\frac{\sqrt{L_2}\sqrt{\chi}}{\sqrt{L_0}},\ A_0 = 0,\ \omega = -\frac{\chi}{k^2 L_0 L_2}. \tag{5.18}$$

Put (5.18) in (5.1),

$$P_7(x,t) = \left(\frac{1}{\frac{\sqrt{L_0 L_2}\tan\left(\sqrt{L_0 L_2}(\delta_0+\xi)\right)}{L_2}} - \frac{\sqrt{L_2}\sqrt{\chi}}{\sqrt{L_0}} \right),\ L_0 L_2 > 0. \tag{5.19}$$

$$Q_7(x,t) = \frac{1}{k}\left(\frac{1}{\frac{\sqrt{L_0 L_2}\tan\left(\sqrt{L_0 L_2}(\delta_0+\xi)\right)}{L_2}} - \frac{\sqrt{L_2}\sqrt{\chi}}{\sqrt{L_0}} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.20}$$

$$P_8(x,t) = \left(\frac{1}{\frac{\sqrt{-c_0 c_2}\tanh\left(\sqrt{-L_0 L_2}(\delta_0+\xi)\right)}{c_2}} - \frac{\sqrt{L_2}\sqrt{\chi}}{\sqrt{L_0}} \right),\ L_0 L_2 < 0. \tag{5.21}$$

$$Q_8(x,t) = \frac{1}{k}\left(\frac{1}{\frac{\sqrt{-c_0 c_2}\tanh\left(\sqrt{-L_0 L_2}(\delta_0+\xi)\right)}{c_2}} - \frac{\sqrt{L_2}\sqrt{\chi}}{\sqrt{L_0}} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.22}$$

F-III

$$A_{-1} = -\frac{\sqrt{L_0}\sqrt{\chi}}{2\sqrt{L_2}},\ A_1 = \frac{\sqrt{\delta}\sqrt{L_2}}{2\sqrt{L_0}},\ A_0 = 0,\ \omega = -\frac{\chi}{4k^2 L_0 L_2}. \tag{5.23}$$

Put (5.23) in (5.1),

$$P_9(x,t) = \left(\frac{1}{2}\tan\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right)\left(\sqrt{\delta} - \sqrt{\chi}\cot^2\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right)\right)\right),\ L_0L_2 > 0. \tag{5.24}$$

$$Q_9(x,t) = \frac{1}{k}\left(\frac{1}{2}\tan\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right)\left(\sqrt{\delta} - \sqrt{\chi}\cot^2\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right)\right)\right)^2 + \left(\frac{\chi}{k}\right). \tag{5.25}$$

$$P_{10}(x,t) = \left(\frac{1}{2}\sqrt{\chi}\cot\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right) - \frac{1}{2}\sqrt{\delta}\tan\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right)\right),\ L_0L_2 < 0. \tag{5.26}$$

$$Q_{10}(x,t) = \frac{1}{k}\left(\frac{1}{2}\sqrt{\chi}\cot\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right) - \frac{1}{2}\sqrt{\delta}\tan\left(\sqrt{L_0}\sqrt{L_2}(\delta_0+\xi)\right)\right)^2 + \left(\frac{\chi}{k}\right). \tag{5.27}$$

5.2. Application of MEAEM method

Let solution of Eq (4.6) as;

$$P(\xi) = A_1\Psi + A_0 + \frac{B_1}{\Psi} + D_1\left(\frac{\Psi'}{\Psi}\right). \tag{5.28}$$

Put (5.28) with (3.12) in (4.6),

$$A_0 = 0,\ A_1 = \frac{i\sqrt{l_3}\sqrt{\chi}}{\sqrt{l_1}},\ D_1 = \frac{i\sqrt{\chi}}{\sqrt{l_1}},\ B_1 = 0,\ \omega = \frac{4\chi}{k^2 l_1}. \tag{5.29}$$

Put (5.29) in (5.28).

Case I.

$$P_{11}(x,t) = \left(-\frac{i\sqrt{\chi}\left(2\sqrt{l_1}\sqrt{l_3}\left(\epsilon\coth\left(\frac{1}{2}\sqrt{l_1}(\delta_0+\xi)\right)+1\right)^2 + l_2\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{l_1}(\delta_0+\xi)\right)\right)}{2l_2\left(\epsilon\coth\left(\frac{1}{2}\sqrt{l_1}(\delta_0+\xi)\right)+1\right)}\right),$$
$$l_1 > 0,\ l_2^2 - 4l_1ł_3 = 0. \tag{5.30}$$

$$Q_{11}(x,t) = \frac{1}{k}\left(-\frac{i\sqrt{\chi}\left(2\sqrt{l_1}\sqrt{l_3}\left(\epsilon\coth\left(\frac{1}{2}\sqrt{l_1}(\delta_0+\xi)\right)+1\right)^2 + l_2\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{l_1}(\delta_0+\xi)\right)\right)}{2l_2\left(\epsilon\coth\left(\frac{1}{2}\sqrt{l_1}(\delta_0+\xi)\right)+1\right)}\right)^2$$
$$+\left(\frac{\chi}{k}\right). \tag{5.31}$$

Case II.

$$P_{12}=\left(-\frac{i\sqrt{\chi}\left(2\sqrt{\frac{l_1}{l_3}}\sqrt{l_3}\left(\eta+\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\epsilon\sinh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)\right)^2-4\sqrt{l_1}\epsilon\left(\eta\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+1\right)\right)}{4\sqrt{l_1}\left(\eta+\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)\right)\left(\eta+\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\epsilon\sinh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)\right)}\right),$$
$$l_1>0,\ l_3>0,\ l_2=(4l_1l_3)^{1/2}. \tag{5.32}$$

$$Q_{12}=\frac{1}{k}\left(-\frac{i\sqrt{\chi}\left(2\sqrt{\frac{l_1}{l_3}}\sqrt{l_3}\left(\eta+\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\epsilon\sinh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)\right)^2-4\sqrt{l_1}\epsilon\left(\eta\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+1\right)\right)}{4\sqrt{l_1}\left(\eta+\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)\right)\left(\eta+\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\epsilon\sinh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)\right)}\right)^2$$
$$+\left(\frac{\chi}{k}\right). \tag{5.33}$$

Figure 3 is plotted to represent the solutions of Eqs (5.32) and (5.33) and display their characteristics.

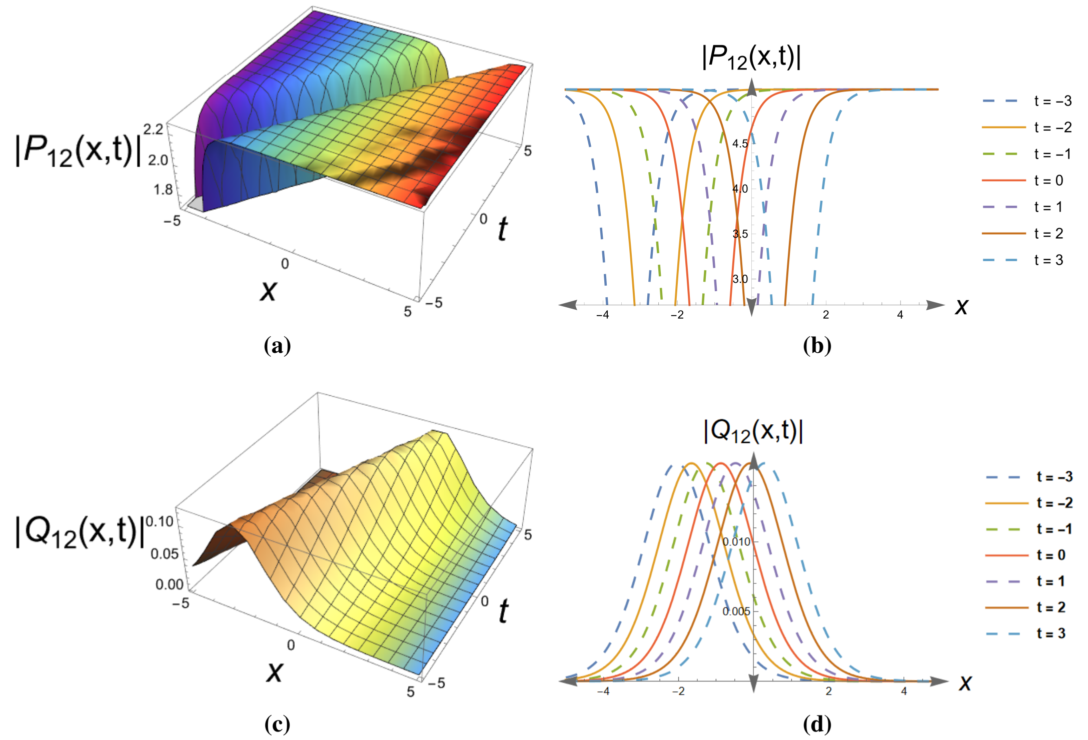

Figure 3. The 3-dimensional and 2-dimensional plots of singular solitary wave solutions of P_{12}**(a,b)** and Q_{12}**(c,d)** with $\gamma=1,\ \delta_0=-2.7,\ \eta=1,\ k=-2.4,\ l_1=2,\ l_2=4,\ l_3=2,\ \chi=-5.1,\ \epsilon=1$, and $\gamma=1,\ \delta_0=0.7,\ \eta=1,\ k=0.8,\ l_1=2,\ l_2=4,\ l_3=2,\ \chi=0.1,\ \epsilon=1$.

Case III.

$$P_{13} = \left(\frac{i\sqrt{\chi}\left(\epsilon\left(\eta\sqrt{R^2+1}\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right) - R\sinh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+1\right)\right)}{\left(\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\eta\sqrt{R^2+1}\right)\left(\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\eta\sqrt{R^2+1}\right) - \left(\frac{\frac{\epsilon\left(\sinh\left(\sqrt{l_1}(\xi+\xi_0)\right)+R\right)}{\cosh\left(\sqrt{l_1}(\delta_0+\xi)\right)+\eta\sqrt{R^2+1}}+1}{l_2}\right)} \right),$$

$$l_1 > 0. \tag{5.34}$$

$$Q_{13} = \frac{1}{k}\left(\frac{i\sqrt{\chi}\left(\epsilon\left(\eta\sqrt{R^2+1}\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right) - R\sinh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+1\right)\right)}{\left(\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\eta\sqrt{R^2+1}\right)\left(\cosh\left(\sqrt{l_1}\,(\delta_0+\xi)\right)+\eta\sqrt{R^2+1}\right) - \left(\frac{\frac{\epsilon\left(\sinh\left(\sqrt{l_1}(\xi+\xi_0)\right)+R\right)}{\cosh\left(\sqrt{l_1}(\delta_0+\xi)\right)+\eta\sqrt{R^2+1}}+1}{l_2}\right)} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.35}$$

5.3. Application of $Exp(-\Psi(\xi))$-expansion method

Let the solution of Eq (4.6) be

$$P(\xi) = A_1 \exp(-\Psi(\xi)) + A_2 \exp(-2\Psi(\xi)) + A_0. \tag{5.36}$$

Put (5.36) with (3.17) in (4.6),

$$A_0 = \frac{\lambda_1\sqrt{\chi}}{\sqrt{4\mu_1-\lambda_1^2}},\ A_1 = \frac{2\sqrt{\chi}}{\sqrt{4\mu_1-\lambda_1^2}},\ \omega = \frac{4\chi}{k^2\left(\lambda_1^2-4\mu_1\right)}. \tag{5.37}$$

Putting (5.37) with (5.36), the following cases exist.
Case I. $\lambda_1^2 - 4\mu_1 > 0,\ \ \mu_1 \neq 0$,

$$P_{14}(x,t) = \left(\frac{\sqrt{\chi}\left(2\log\left(-\frac{\sqrt{\lambda_1^2-4\mu_1}\tanh\left(\frac{1}{2}(\delta_0+\xi)\sqrt{\lambda_1^2-4\mu_1}\right)+\lambda_1}{2\mu_1}\right)+\lambda_1\right)}{\sqrt{4\mu_1-\lambda_1^2}} \right). \tag{5.38}$$

$$Q_{14}(x,t) = \frac{1}{k}\left(\frac{\sqrt{\chi}\left(2\log\left(-\frac{\sqrt{\lambda_1^2-4\mu_1}\tanh\left(\frac{1}{2}(\delta_0+\xi)\sqrt{\lambda_1^2-4\mu_1}\right)+\lambda_1}{2\mu_1}\right)+\lambda_1\right)}{\sqrt{4\mu_1-\lambda_1^2}} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.39}$$

The solution's representation of Eqs (5.38) and (5.39) is shown in Figure 4.

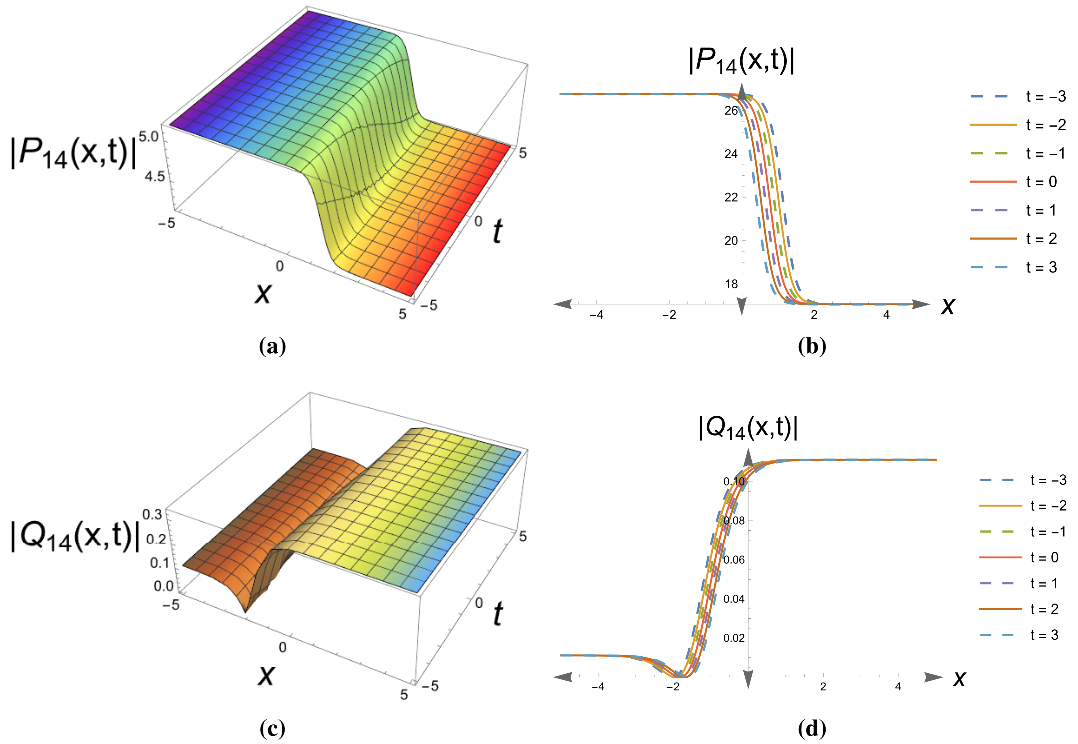

Figure 4. The 3-dimensional and 2-dimensional plots of kinked solitary wave solutions of P_{14}**(a,b)** and Q_{14}**(c,d)** with $\gamma = 1,\ \delta_0 = 1.7,\ \lambda_1 = 3,\ k = -2.4,\ \mu_1 = 1,\ \chi = 2.1$, and $\gamma = 1,\ \delta_0 = 1.7,\ \lambda_1 = -4,\ k = 0.8,\ \mu_1 = 1,\ \chi = 0.1$.

Case II. $\lambda_1^2 - 4\mu_1 > 0,\ \mu_1 = 0,$

$$P_{15}(x,t) = \left(\frac{\lambda_1 \sqrt{\chi}}{\sqrt{-\lambda_1^2}} - \frac{\left(2\sqrt{\chi}\right) \log\left(\frac{\lambda_1}{\exp(\lambda_1(\delta_0+\xi))-1}\right)}{\sqrt{-\lambda_1^2}} \right). \tag{5.40}$$

$$Q_{15}(x,t) = \frac{1}{k} \left(\frac{\lambda_1 \sqrt{\chi}}{\sqrt{-\lambda_1^2}} - \frac{\left(2\sqrt{\chi}\right) \log\left(\frac{\lambda_1}{\exp(\lambda_1(\delta_0+\xi))-1}\right)}{\sqrt{-\lambda_1^2}} \right)^2 + \left(\frac{\chi}{k}\right). \tag{5.41}$$

Case III. $\lambda_1^2 - 4\mu_1 < 0,$

$$P_{16}(x,t) = \left(\frac{\left(2\sqrt{\chi}\right) \log\left(\frac{-\sqrt{4\mu_1-\lambda_1^2}\tan\left(\frac{1}{2}(\delta_0+\xi)\sqrt{4\mu_1-\lambda_1^2}\right)-\lambda_1}{2\mu_1} \right)}{\sqrt{4\mu_1-\lambda_1^2}} + \frac{\lambda_1\sqrt{\chi}}{\sqrt{4\mu_1-\lambda_1^2}} \right). \tag{5.42}$$

$$Q_{16}(x,t) = \frac{1}{k}\left(\frac{\left(2\sqrt{\chi}\right)\log\left(\frac{-\sqrt{4\mu_1-\lambda_1^2}\tan\left(\frac{1}{2}(\delta_0+\xi)\sqrt{4\mu_1-\lambda_1^2}\right)-\lambda_1}{2\mu_1}\right)}{\sqrt{4\mu_1-\lambda_1^2}} + \frac{\lambda_1\sqrt{\chi}}{\sqrt{4\mu_1-\lambda_1^2}}\right)^2 + \left(\frac{\chi}{k}\right). \tag{5.43}$$

6. Conclusions

This research extensively investigates the solitary wave solutions of the fractional coupled Konno–Oono model, a prevalent framework in diverse scientific and engineering disciplines. The innovation of this work lies in its comprehensive investigation of FCKOM by employing the three analytical methods, which provides a perspective platform to indulge the dynamic behaviors of the model, leading to the detection of various soliton solutions. The explored outcomes have significant applications in the realm of magnetic fields. To visually interpret and dissect our results, some solutions are plotted in 2-dimensional and 3-dimensional forms by assigning specific values to the parameters. Our research recommends that the suggested methods not only hold promise for application in nonlinear models but also pave the way for wider solicitations across numerous scientific disciplines.

Author contributions

Aly R. Seadawy: Formal analysis, software, methodology; Asghar Ali: Validation, methodology; Taha Radwan: Investigation, writing-review & editing, supervision; Wael W. Mohammed: Investigation, writing-review & editing, supervision; Karim K. Ahmed: Methodology, resources, writing-review & editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

References

1. E. M. E. Zayed, A. H. Arnous, DNA dynamics studied using the homogeneous balance method, *Chinese Phys. Lett.*, **29** (2012), 080203. Available from: `https://iopscience.iop.org/article/10.1088/0256-307X/29/8/080203`.

2. A. Altalbe, A. Zaagan, A. Bekir, A. Cevikel, The nonlinear wave dynamic of the space-time fractional van der Waals equation via three analytical methods, *Phys. Fluids*, **36** (2024), 027140. https://doi.org/10.1016/j.ijleo.2019.164133
3. P. N. Ryabov, D. I. Sinelshchikov, M. B. Kochanov, Application of the Kudryashov method for finding exact solutions of the high order nonlinear evolution equations, *Appl. Math. Comput.*, **218** (2011), 3965. https://doi.org/10.1016/j.amc.2011.09.027
4. A. Cevikel, A. Bekir, O. Guner, Exploration of new solitons solutions for the Fitzhugh-Nagumo-type equations with conformable derivatives, *Int. J. Mod. Phys. B*, **37** (2023), 2350224. https://doi.org/10.1142/S0217979223502247
5. M. Wang, X. Li, J. Zhang, The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, *Phys. Lett. A*, **372** (2008), 417–423. https://doi.org/10.1016/j.physleta.2007.07.051
6. M. T. Darvishi, M. Naja, S. B. Arbabi, H. Rezazadeh, A. Bekir, A. Cevikel, New extensions of (2+1)-dimensional BLMP models with soliton solutions, *Opt. Quant. Electron.*, **55** (2023), 568. https://doi.org/10.1007/s11082-023-04862-1
7. S. Nestor, A. Houwe, H. Rezazadeh, A. Bekir, G. Betchewe, S. Y. Doka, New solitary waves for the Klein–Gordon-Zakharov equations, *Mod. Phys. Lett. B*, **34** (2020), 2050246. https://doi.org/10.1142/S0217984920502462
8. A. Cevikel, Optical solutions for the (3+1)-dimensional YTSF equation, *Opt. Quant. Electron.*, **55** (2023), 510. https://doi.org/10.1007/s11082-023-04787-9
9. S. Nestor, G. Betchewe, H. Rezazadeh, A. Bekir, S. Y. Doka, Exact optical solitons to the perturbed nonlinear Schrödinger equation with dual-power law of nonlinearity, *Opt. Quant. Electron.*, **52** (2020), 1–16. https://doi.org/10.1007/s11082-020-02412-7
10. C. R. Jisha, R. K. Dubey, Wave interactions and structures of (4+1)-dimensional Boiti-Leon-Manna-Pempinelli equation, *Nonlinear Dynam.*, **110** (2022), 3685–3696. https://doi.org/10.1007/s11071-022-07816-6
11. X. Zhao, F. Pang, H. Gegen, Interactions among two-dimensional nonlinear localized waves and periodic wave solution for a novel integrable (2+1)-dimensional KdV equation, *Nonlinear Dynam.*, **110** (2022), 3629–3654. https://doi.org/10.1007/s11071-022-07785-w
12. A. M. Wazwaz, N. S. Alatawi, W. Albalawi, S. A. E. Tantawy, Painlevé analysis for a new (3+1)-dimensional KP equation: Multiple-soliton and lump solutions, *Eur. Phys. Lett.*, **140** (2022), 52002. Available from: `https://iopscience.iop.org/article/10.1209/0295-5075/aca49f/meta`.
13. H. F. Ismae, A. N. Akkilic, M. A. S. Murad, H. Bulut, W. Mahmoud, M. S. Osman, Boiti-Leon-Manna-Pempinelli equation including time-dependent coefficient (vcBLMPE): A variety of nonautonomous geometrical structures of wave solutions, *Nonlinear Dynam.*, **110** (2022), 3699–3712. https://doi.org/10.1007/s11071-022-07817-5
14. S. Kumar, B. Mohan, A novel and efficient method for obtaining Hirota's bilinear form for the nonlinear evolution equation in $(n+1)$-dimensions, *Part. Differ. Equ. Appl. Math.*, **5** (2022), 100274. https://doi.org/10.1016/j.padiff.2022.100274
15. O. A. Ilhan, J. Manafian, H. M. Baskonus, M. Lakestani, Solitary wave solitons to one model in the shallow water waves, *Eur. Phys. J. Plus.*, **136** (2021), 337. https://doi.org/10.1140/epjp/s13360-021-01327-w

16. A. S. Khalifa, H. M. Ahmed, N. M. Badra, W. B. Rabie, Exploring solitons in optical twin-core couplers with Kerr law of nonlinear refractive index using the modified extended direct algebraic method, *Opt. Quant. Electron.*, **56** (2024), 1060. https://doi.org/10.1007/s11082-024-06882-x
17. O. D. Adeyemo, L. Zhang C. M. Khalique, Bifurcation theory, lie group-invariant solutions of subalgebras and conservation laws of a generalized (2+1)-dimensional BK equation type II in plasma physics and fluid mechanics, *Mathematics*, **10** (2022), 2391. https://doi.org/10.3390/math10142391
18. M. E. Ramadan, H. M. Ahmed, A. S. Khalifa, K. K. Ahmed, Invariant solitons and travelling-wave solutions to a higher-order nonlinear Schrödinger equation in an optical fiber with an improved tanh-function algorithm, *J. Appl. Anal. Comput.*, **15** (2025), 3270–3289. https://doi.org/10.11948/20250042
19. Z. Islam, A. Abdeljabbar, M. A. N. A. Sheikh, M. A. Taher, Stability and spin solitonic dynamics of the HFSC model: Effects of neighboring interactions and crystal field anisotropy parameters, *Opt. Quant. Electron.*, **56** (2024), 190. https://doi.org/10.1007/s11082-023-05739-z
20. C. Li, L. Chen, G. Li, Optical solitons of space-time fractional Sasa-Satsuma equation by F-expansion method, *Optik*, **224** (2020), 165527. https://doi.org/10.1016/j.ijleo.2020.165527
21. A. Akbulut, R. Islam, Y. Arafat, F. Tascan, A novel scheme for SMCH equation with two different approaches, *Comput. Methods Differ.*, **11** (2023), 263–280. https://doi.org/10.22034/cmde.2022.50363.2093
22. M. Nadeem, J. H. He, He–Laplace variational iteration method for solving the nonlinear equations arising in chemical kinetics and population dynamics, *J. Math. Chem.*, **59** (2021), 1234–1245. https://doi.org/10.1007/s10910-021-01236-4
23. S. Noeiaghdam, D. Sidorov, A. M. Wazwaz, N. Sidorov, V. Sizikov, The numerical validation of the adomian decomposition method for solving Volterra integral equation with discontinuous kernels using the CESTAC method, *Mathematics*, **9** (2021), 260. https://doi.org/10.3390/math9030260
24. S. Hussain, A. Shah, A. Ullah, F. Haq, The q-homotopy analysis method for a solution of the Cahn-Hilliard equation in the presence of advection and reaction terms, *J. Taibah Univ. Sci.*, **40** (2022), 813–819. https://doi.org/10.1080/16583655.2022.2119746
25. M. Modanli, S. T. Abdulazeez, A. M. Husien, A residual power series method for solving pseudo hyperbolic partial differential equations with nonlocal conditions, *Numer. Meth. Part. D. E.*, **40** (2021), 2235–2243. https://doi.org/10.1002/num.22683
26. A. Qazza, A. Burqan, R. Saadeh, Application of ARA-Residual power series method in solving systems of fractional differential equations, *Math. Probl. Eng.*, **2022** (2022), 6939045. https://doi.org/10.1155/2022/6939045
27. F. Dusunceli, E. Celik, M. Askin, H. Bulut, New exact solutions for the doubly dispersive equation using the improved Bernoulli sub-equation function method, *Indian. J. Phys.*, **95** (2021) 309–314. https://doi.org/10.1007/s12648-020-01707-5
28. A. Akbar, A. M. Wazwaz, F. Mahmud, D. Baleanu, R. Roy, H. K. Barman, et al., Dynamical behavior of solitons of the perturbed nonlinear Schrödinger equation and microtubules through the generalized Kudryashov scheme, *Res. Phys.*, **95** (2022), 106079. https://doi.org/10.1016/j.rinp.2022.106079

29. M. S. Hashemi, M. Mirzazadeh, Optical solitons of the perturbed nonlinear Schrödinger equation using Lie symmetry method, *Optik*, **281** (2023), 170816. https://doi.org/10.1016/j.ijleo.2023.170816
30. R. M. E. Shiekh, M. Gaballah, Novel solitons and periodic wave solutions for Davey–Stewartson system with variable coefficients, *J. Taibah Univ. Sci.*, **14** (2020), 783–789. https://doi.org/10.1080/16583655.2020.1774975
31. H. Rezazadeh, K. U. Tariq, J. Sabiu, A. Bekir, Abundant traveling wave solutions to the resonant nonlinear Schrödinger's equation with variable coefficients, *Mod. Phys. Lett. B*, **34** (2020). https://doi.org/10.1142/S0217984920501183
32. A. Ali, A. R. Seadawy, D. Lu, Dispersive solitary wave soliton solutions of (2+1)-dimensional Boussineq dynamical equation via extended simple equation method, *J. King Saud. Univ. Sci.*, **13** (2019), 653–658. https://doi.org/10.1016/j.jksus.2017.12.015
33. A. R. Seadawy, A. Ali, S. Althobaiti, K. E. Rashidy, Construction of abundant novel analytical solutions of the space-time fractional nonlinear generalized equal width model via Riemann–Liouville derivative with application of mathematical methods, *Open Phys.*, **19** (2021), 657–668. https://doi.org/10.1515/phys-2021-0076
34. D. Lu, A. R. Seadawy, A. Ali, Dispersive traveling wave solutions of the equal-width and modified equal-width equations via mathematical methods and its applications, *Res. Phys.*, **13** (2018), 313–320. https://doi.org/10.1016/j.rinp.2018.02.036
35. R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, *J. Comput. Appl. Math.*, **34** (2014), 65–70. https://doi.org/10.1016/j.cam.2014.01.002
36. J. Ahmad, T. Younas, A comprehensive study of the conformable time-fractional coupled Konno-Oono equation: New methodologies and stability analysis in magnetic field, *Opt. Quant. Electron.*, **56** (2024), 911. https://doi.org/10.1007/s11082-024-06752-6
37. H. Y. Liang, J. K. Wang, Z. X. Hou, Multiple kink-soliton, breather wave, interaction wave and the travelling wave solutions to the fractional (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation, *Fractals*, 2025. https://doi.org/10.1142/S0218348X25500823
38. W. L. Li, S. H. Chen, K. J. Wang, A variational principle of the nonlinear Schrodinger equation with fractal derivatives, *Fractals*, 2025. https://doi.org/10.1142/S0218348X25500690
39. T. S. Shaikh, M. Z. Baber, N. Ahmed, N. Shahid, A. Akgül, M. D. L. Sen, On the soliton solutions for the stochastic Konno-Oono system in magnet field with the presence of noise, *Mathematics*, **11** (2023), 1472. https://doi.org/10.3390/math11061472
40. E. M. Elbrolosy, A. A. Elmandouh, Dynamical behaviour of conformable time-fractional coupled Konno-Oono equation in magnetic field, *Math. Probl. Eng.*, **1** (2022), 3157217. https://doi.org/10.1155/2022/3157217