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Research article

The novel stochastic solutions for ion sound and Langmuir waves model in plasma physics

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Abstract: This paper analyzes and examines the ion sound and Langmuir waves (ISALWs) model with noise term utilizing the powerful extended tanh technique and the complete discrimination system for the polynomial method. Namely, we introduce some new stochastic solutions for the ISALWs model with multiplicative noise in the Itô sense. Ion sound waves and Langmuir waves are distinguished as fundamental plasma waves, characterized by varying frequencies, wavelengths, and modes of propagation. The combination of ion inertia and electron pressure results in the propagation of low-frequency electrostatic waves, referred to as ion sound waves, within a plasma. Langmuir waves refer to the high-frequency electrostatic oscillations of electrons within a plasma, while ions, due to their larger mass, are considered to be almost at rest. We also investigate how the noise term affects the structural characteristics of waves. These results have potential relevance to plasma physics applications, such as solar wind dynamics, Langmuir turbulence, and burst-like wave phenomena in cusp regions. Utilizing the capabilities of the MATLAB release, several profile pictures are created to demonstrate the behavior of the presented stochastic solutions. Ultimately, the suggested methodology has the potential to be adapted for various other models in the applied science.

Keywords: ISALWs model; extended tanh technique; complete discrimination system; stochastic solutions; wave propagation; plasma physics
Mathematics Subject Classification: 35C07, 35Q60, 60H15, 60H40, 82D10

1. Introduction

Deterministic or stochastic nonlinear wave equations can be used to represent a wide range of intricate nonlinear physical processes. Nonlinear wave equations represent a category of partial differential equations (PDEs) that describe wave phenomena where the amplitude of the wave influences its velocity, shape, stability, and chaotic dynamics. These kinds of equations are essential in many applied scientific domains, such as biology, plasma physics, superfluids, quantum mechanics, and many others [1, 2]. One of the most captivating topics for researchers today is nonlinear phenomena [3, 4]. The construction of diverse soliton solutions for nonlinear partial differential equations (NPDEs) not only aids in elucidating the physical properties of light signals traveling through nonlinear media, but also plays a role in the enhancement or creation of mathematical models. As a result, more and more researchers are working to create efficient analytical and numerical methods for producing different optical soliton solutions. These include the enhanced algebraic method [5], complete discrimination system method [6], trial equations method [7], planar dynamical properties [8], and more.

Stochastic partial differential equations (SPDEs), which are believed to be an extension of dynamical systems theory to incorporate noise in models, have been the subject of excessive attention recently [9–11]. This represents a considerable expansion, as existing systems are fundamentally incapable of being entirely isolated from their environment, indicating that this external random influence is always present. Specifically, the SPDEs offer a robust framework for representing intricate systems characterized by the interplay of randomness and nonlinearity [12]. A stochastic process is a mathematical framework employed to characterize a system that changes over time due to random influences. The Wiener process serves as a quintessential illustration of a stochastic process that demonstrates the characteristics of a Markov and a martingale process [13]. This process represents a crucial continuous-time stochastic model employed to represent random events across diverse disciplines, including biology, engineering, economy, physics, etc [14]. This process is fundamental to SPDEs, serving as the primary catalyst for random fluctuations.

The ion sound waves and Langmuir waves are two forms of plasma waves distinguished by their respective frequencies, wavelengths, and propagation processes. These waves are critical components in understanding various plasma phenomena. They have been intensively investigated in space plasmas, laboratory plasmas, and astrophysical process. Thejappa and MacDowall elucidated the nonlinear dynamics of field wave packets observed in the solar wind. Their findings indicated that the intensity peaks of these wave packets conform to the criteria for soliton formation and threshold conditions. Furthermore, the intense Langmuir turbulence facilitates the transformation of Langmuir structures into electromagnetic structures at higher-order harmonics of plasma frequency [15]. The model of ion sound and Langmuir waves reads as follows:

$$\begin{aligned} &Q_{tt} - Q_{xx} - 2\,(|\,E\,|^2)_{xx} = 0\,,\\ &iE_t + \frac{1}{2}E_{xx} - \; Q\,E = 0. \end{aligned} \tag{1.1}$$

$E\,e^{-iw_p t}$ denotes the normalized electric field of the Langmuir oscillation, and Q represents the normalized density perturbation. This model investigates the nonlinear phenomenon referred to as the Langmuir collapsing wave. It offers significant physical insights essential for comprehending intense Langmuir turbulence, encompassing the yet-to-be-defined limits of subsonic and supersonic behavior, along with the detection of Langmuir waves during solar radio bursts. Additionally, this model within the quantum plasma framework is examined for its potential to generate Langmuir waves in an isotropic gas plasma [16–19]. In this study, we investigate model (1.1) using the Wiener process, as follows:

$$\begin{aligned} &Q_{tt} - Q_{xx} - 2\,(|\,E\,|^2)_{xx} = 0\,,\\ &iE_t + \frac{1}{2}E_{xx} - \; Q\,E - i\sigma\,E\beta_t = 0. \end{aligned} \tag{1.2}$$

The parameter σ denotes the noise strength, where β_t is the time derivative of the Wiener process $\beta(t)$. This process is a stochastic process, which adheres to the following characteristics:

(a) $\beta(t)_{t\geq 0}$ is a continuous function of t,
(b) $\beta(t) - \beta(s)$ is independent of increments for $s < t$,
(c) $\beta(t) - \beta(s)$ follows a normal distribution with a mean of 0 and variance of $t - s$.

Our goal is to use the extended tanh approach [20] and the complete discrimination system for polynomial method [21,22] for the ISALWs model with multiplicative noise in the Itô sense. Namely, we introduce some new stochastic solutions through physical characteristics. The solutions offered pave the way for significant uses in solar-wind, magnetosphere, laboratory plasmas, nonlinear optics, and space plasmas. The suggested method is simple, effective, and precise. We also present the nonlinear dynamic behavior of certain chosen stochastic solutions through multiplicative noise. Demonstrating how the noise impact affects the suggested solutions is one of the most intriguing points. To our knowledge, there has been no prior research that employs the proposed technique to address the system of ISALWs influenced by multiplicative noise.

The structure of the article is organized in the following manner. Section 2 provides a condensed version of the extended tanh method and the complete discrimination system for the polynomial method. Section 3 outlines the mathematical model and translates it into the associated ordinary differential equation (ODE). Section 4 introduces novel stochastic solutions for the system of ISALWs influenced by multiplicative noise. Section 5 provides a physical interpretation for the obtained stochastic solutions. We also depict the influence of noise intensity on the dynamical structure of wave solutions. Ultimately, conclusions are presented in Section 6.

2. The description of the approaches

This section presents a simplified version of the extended tanh technique [20] and a brief description of the complete discrimination system for the polynomial method [21,22]. Assume the following form for the NPDEs for an unknown function $u(x, t)$:

$$\mathfrak{H}(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0. \tag{2.1}$$

Using the wave transformation

$$u(x, t) = u(\xi), \quad \xi = x + vt, \tag{2.2}$$

where v is the wave speed. Combining Eq (2.2) with Eq (2.1) results in the ODE

$$\mathfrak{R}(u, u', u'', u''', ...) = 0. \tag{2.3}$$

2.1. The extended tanh technique

The expanded tanh technique is briefly described as follows:

Step 1:

The solution to Eq (2.3) is provided as

$$u(\xi) = a_0 + \sum_{l=0}^{l=M} a_l \psi^l(\xi) + b_l \psi^{-l}(\xi). \tag{2.4}$$

The constants to be found are $a_i(l = 0, 1, ..., M)$. The function $u(\xi)$ satisfies the Riccati equation given by

$$\psi' = \varrho + \psi^2(\xi), \tag{2.5}$$

where ϱ is a constant. The verified solutions for Eq (2.5) are:

(1) When $\varrho < 0$, then

$$\psi(\xi) = -\sqrt{-\varrho}\, tanh(\sqrt{-\varrho}\,\xi),$$
$$\psi(\xi) = -\sqrt{-\varrho}\, coth(\sqrt{-\varrho}\,\xi). \tag{2.6}$$

(2) When $\varrho > 0$, then

$$\psi(\xi) = \sqrt{\varrho}\, tan(\sqrt{\varrho}\,\xi),$$
$$\psi(\xi) = -\sqrt{\varrho}\, cot(\sqrt{\varrho}\,\xi). \tag{2.7}$$

(3) When $\varrho = 0$, then

$$\psi(\xi) = -\frac{1}{\xi}. \tag{2.8}$$

Step 2:

Balancing the largest-power nonlinear term in (2.3) with the highest-order derivative term can result in M.

Step 3:

Substituting Eqs (2.4) and (2.5) into Eq (2.3) results in a set of nonlinear algebraic equations with zero coefficients for each power of $\psi(\xi)$. Solving these equations gives the expression for Eq (2.4). Using Eqs (2.6), (2.7), and (2.8) yields the needed explicit answers for Eq (2.1).

2.2. The complete discrimination system for the polynomial technique

The complete discrimination system for the polynomial method is briefly described as follows:

Equation (2.3) can be transformed into the following equation after a series of transformations

$$(u')^2 = \mu_4 u^4 + \mu_2 u^2 + \mu_0. \tag{2.9}$$

Based on [21–23], we make the following assumptions:

$$u = \pm\sqrt{(4\mu_4)^{-\frac{1}{3}}\omega},\, c_1 = 4\mu_2(4\mu_4)^{-\frac{2}{3}},\, c_0 = 4\mu_0(4\mu_4)^{-\frac{1}{3}},\, \xi_1 = (4\mu_4)^{\frac{1}{3}}\xi. \tag{2.10}$$

Then Eq (2.3) can be converted to

$$(\omega'_{\xi_1})^2 = \omega(\omega^2 + c_1\omega + c_0). \tag{2.11}$$

Integrating Eq (2.11), then we obtain

$$\pm(\xi_1 - \xi_0) = \int \frac{d\omega}{\sqrt{\omega F(\omega)}}, \tag{2.12}$$

where ξ_0 is an integral constant and $F(\omega) = \omega^2 + c_1\omega + c_0$.

The following is the complete discriminant system for the quadratic polynomial $F(\omega)$:

$$\Delta = c_1^2 - 4c_0. \tag{2.13}$$

The categorization of the solution to the equation can be derived, and the classification of the traveling wave solutions of the system of ISALWs influenced by multiplicative noise will be presented in the following section.

3. Mathematical analysis

Utilizing the wave transformation

$$E(x,t) = u(\xi)e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}; Q(x,t) = \ q(\xi) \tag{3.1}$$

$$\xi = v\, x + \rho\, t,$$

where ρ, k, λ, and v are constants, yields

$$-\frac{1}{2}k^2u(\xi) + \frac{1}{2}v^2u''(\xi) - \lambda u(\xi) - u(\xi)q(\xi) = 0. \tag{3.2}$$

$$(\rho - v)(\rho + v)q''(\xi) - 4v^2e^{2\sigma(\beta(t)-\sigma t)}\left(u(\xi)u''(\xi) + u'(\xi)^2\right) = 0. \tag{3.3}$$

Taking the expectation on both sides of Eq (3.3), gives

$$(\rho - v)(\rho + v)q''(\xi) - 4v^2\left(u(\xi)u''(\xi) + u'(\xi)^2\right)e^{-2\sigma^2 t}\ E(e^{2\sigma\beta(t)}) = 0, \tag{3.4}$$

and since $E(e^{2\sigma\beta(t)}) = e^{2\sigma^2 t}$, Eq (3.4) is reduced to

$$(\rho - v)(\rho + v)q''(\xi) - 4v^2\left(u(\xi)u''(\xi) + u'(\xi)^2\right) = 0. \tag{3.5}$$

On the other hand, we have $\rho = -kv$. Solving Eq (3.5) gives

$$q(\xi) = \frac{2v^2}{\rho^2 - v^2}u^2(\xi) = \frac{2}{k^2-1}u^2(\xi),$$

Eq (3.2) becomes

$$\frac{1}{2}v^2u''(\xi) - \frac{2}{k^2-1}u^3(\xi) - (\frac{1}{2}k^2 + \lambda)u(\xi) = 0. \tag{3.6}$$

4. The stochastic solutions of ISALWs

We use the extended tanh approach and the complete discrimination system for the polynomial method to solve the following equation:

$$(k^2-1)v^2u''(\xi) - 4u^3(\xi) - (k^2-1)(k^2+2\lambda)u(\xi) = 0. \tag{4.1}$$

4.1. Solutions employing extended tanh technique

Balancing the highest-order derivatives with nonlinear terms yields $M = 1$.

$$u(\xi) = a_0 + a_1\psi + \frac{b_1}{\psi}. \tag{4.2}$$

$$u'(\xi) = a_1\varrho + a_1\psi^2 - \frac{b_1\varrho}{\psi^2} - b_1. \tag{4.3}$$

Substituting Eq (4.2) and its derivative into Eq (4.1) and collecting all terms with the same power of ψ^3, ψ^2, ψ, ψ^0, ψ^{-1}, ψ^{-2}, and ψ^{-3}, gives

$$(k^2-1)v^2a_1 - 2a_1^3 = 0, \tag{4.4}$$

$$-12a_0a_1^2 = 0, \tag{4.5}$$

$$-12a_0^2a_1 - 12a_1^2b_1 - (k^2-1)(k^2+2\lambda)a_1 + 2(k^2-1)v^2a_1\varrho = 0, \tag{4.6}$$

$$-4a_0^3 - 24a_0a_1b_1 - (k^2-1)(k^2+2\lambda)a_0 = 0, \tag{4.7}$$

$$-12a_0^2b_1 - 12a_1b_1^2 - (k^2-1)(k^2+2\lambda)b_1 + 2(k^2-1)v^2b_1\varrho = 0, \tag{4.8}$$

$$-12a_0b_1^2 = 0, \tag{4.9}$$

$$-2b_1^3 + (k^2-1)v^2b_1\varrho^2 = 0. \tag{4.10}$$

The following families result from solving these equations:

Case I:

$$a_0 = 0, \quad a_1 = \pm\sqrt{\frac{(k^2-1)v^2}{2}}, \; b_1 = 0, \varrho = \frac{k^2+2\lambda}{2v^2}. \tag{4.11}$$

The following solutions are obtained by combining Eq (4.11) with Eq (4.2):

For $k^2 + 2\lambda < 0$, the solutions of (4.1) are

$$\begin{aligned} u_{1,2}(\xi) &= \pm\sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{4}}\, tanh(\sqrt{\frac{-(k^2+2\lambda)}{2v^2}}\,\xi), \\ u_{3,4}(\xi) &= \pm\sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{4}}\, coth(\sqrt{\frac{-(k^2+2\lambda)}{2v^2}}\,\xi). \end{aligned} \tag{4.12}$$

Consequently, the solutions to (1.2) are

$$\begin{aligned} E_{1,2}(x,t) &= \pm\sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{4}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\, tanh(\sqrt{\frac{-(k^2+2\lambda)}{2v^2}}\,(v\,x+\rho\,t)), \\ E_{3,4}(x,t) &= \pm\sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{4}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\, coth(\sqrt{\frac{-(k^2+2\lambda)}{2v^2}}\,(v\,x+\rho\,t)). \end{aligned} \tag{4.13}$$

For $k^2 + 2\lambda > 0$, the solutions of (4.1) are

$$u_{5,6}(\xi) = \pm\sqrt{\frac{(k^2-1)(k^2+2\lambda)}{4}}\, tan(\sqrt{\frac{k^2+2\lambda}{2v^2}}\,\xi),$$
$$u_{7,8}(\xi) = \pm\sqrt{\frac{(k^2-1)(k^2+2\lambda)}{4}}\, cot(\sqrt{\frac{k^2+2\lambda}{2v^2}}\,\xi). \tag{4.14}$$

Consequently, the solutions to (1.2) are

$$E_{5,6}(x,t) = \pm\sqrt{\frac{(k^2-1)(k^2+2\lambda)}{4}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\, tan(\sqrt{\frac{k^2+2\lambda}{2v^2}}\,(v\,x+\rho\,t)),$$
$$E_{7,8}(x,t) = \pm\sqrt{\frac{(k^2-1)(k^2+2\lambda)}{4}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\, cot(\sqrt{\frac{k^2+2\lambda}{2v^2}}\,(v\,x+\rho\,t)). \tag{4.15}$$

For $k^2 + 2\lambda = 0$, the solutions of (4.1) are

$$u_{9,10}(\xi) = \pm\sqrt{\frac{(k^2-1)v^2}{2}}\,\frac{-1}{\xi}. \tag{4.16}$$

Consequently, the solutions to (1.2) are

$$E_{9,10}(x,t) = \pm\sqrt{\frac{(k^2-1)v^2}{2}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\,\frac{-1}{v\,x+\rho\,t}. \tag{4.17}$$

Case II:

$$a_0 = 0,\quad a_1 = \pm\sqrt{\frac{(k^2-1)v^2}{2}},\; b_1 = \mp\frac{-(k^2-1)(k^2+2\lambda)}{2\sqrt{2}\sqrt{4(k^2-1)v^2}}, \varrho = \frac{-(k^2+2\lambda)}{4v^2}. \tag{4.18}$$

For $k^2 + 2\lambda > 0$, the solutions of (4.1) are

$$u_{11,12}(\xi) = \pm\sqrt{\frac{(k^2-1)(k^2+2\lambda)}{2}}\, csch\left(\sqrt{\frac{k^2+2\lambda}{v^2}}\,\xi\right). \tag{4.19}$$

Consequently, the solutions to (1.2) are

$$E_{11,12}(x,t) = \pm\sqrt{\frac{(k^2-1)(k^2+2\lambda)}{2}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\, csch\left(\sqrt{\frac{k^2+2\lambda}{v^2}}\,(vx+\rho t)\right). \tag{4.20}$$

For $k^2 + 2\lambda < 0$, the solutions of (4.1) are

$$u_{13,14}(\xi) = \pm\sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{2}}\, csc\left(\sqrt{\frac{-(k^2+2\lambda)}{v^2}}\,\xi\right). \tag{4.21}$$

Consequently, the solutions to (1.2) are

$$E_{13,14}(x,t) = \pm\sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{2}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\, csc\left(\sqrt{\frac{-(k^2+2\lambda)}{v^2}}\,(vx+\rho t)\right). \tag{4.22}$$

For $k^2 + 2\lambda = 0$, the solutions of (1.2) are the same as the solutions given in (4.17).

Case III:

$$a_0 = 0, \quad a_1 = \pm\sqrt{\frac{(k^2-1)v^2}{2}},\ b_1 = \mp\frac{-(k^2-1)(k^2+2\lambda)}{16\sqrt{(k^2-1)v^2}}, \varrho = \frac{(k^2+2\lambda)}{8v^2}. \tag{4.23}$$

For $k^2 + 2\lambda > 0$, the solutions of (4.1) are

$$u_{15,16}(\xi) = \pm\sqrt{\frac{(k^2-1)(k^2+2\lambda)}{16}}\left(tan\left(\sqrt{\frac{k^2+2\lambda}{8v^2}}\,\xi\right) - cot\left(\sqrt{\frac{k^2+2\lambda}{8v^2}}\,\xi\right)\right). \tag{4.24}$$

Thus, the solution of (1.2) is

$$\begin{aligned} E_{15,16}(x,t) = \pm & \sqrt{\frac{(k^2-1)(k^2+2\lambda)}{16}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t} \times \\ & \left(tan\left(\sqrt{\frac{k^2+2\lambda}{8v^2}}\,(vx+\rho t)\right) - cot\left(\sqrt{\frac{k^2+2\lambda}{8v^2}}\,(v\,x+\rho\,t)\right)\right). \end{aligned} \tag{4.25}$$

For $k^2 + 2\lambda < 0$, the solutions of (4.1) are

$$u_{17,18}(\xi) = \pm\sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{16}}\left(tahn\left(\sqrt{\frac{-(k^2+2\lambda)}{8v^2}}\,\xi\right) + coth\left(\sqrt{\frac{-(k^2+2\lambda)}{8v^2}}\,\xi\right)\right). \tag{4.26}$$

Thus, the solution of (1.2) is

$$\begin{aligned} E_{17,18}(x,t) = \pm & \sqrt{\frac{-(k^2-1)(k^2+2\lambda)}{16}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t} \times \\ & \left(tahn\left(\sqrt{\frac{-(k^2+2\lambda)}{8v^2}}\,(v\,x+\rho\,t)\right) + coth\left(\sqrt{\frac{-(k^2+2\lambda)}{8v^2}}\,(v\,x+\rho\,t)\right)\right). \end{aligned} \tag{4.27}$$

For $k^2 + 2\lambda = 0$, the solutions of (1.2) are the same as the solutions given in (4.17).

4.2. Solutions employing complete discrimination system for polynomial method

Here, we give more traveling wave solutions for Eq (1.2). Namely, we mention only the types of families, which were not given in the previous section. Multiplying u' on both sides of Eq (4.1), and then integrating once, we get

$$(u')^2 = \mu_4 u^4 + \mu_2 u^2 + \mu_0, \tag{4.28}$$

where $\mu_4 = \frac{2}{(k^2-1)v^2}$, $\mu_2 = \frac{(k^2+2\lambda)}{v^2}$, and μ_0 is an arbitrary constant.

Using (2.10), Eq (4.28) can be expressed as follows:

$$(\omega'_{\xi_1})^2 = \omega^3 + c_1\omega^2 + c_0\omega. \tag{4.29}$$

The integral expression of Eq (4.29) may thus be obtained as

$$\pm(\xi_1 - \xi_0) = \int \frac{d\omega}{\sqrt{\omega(\omega^2 + c_1\omega + c_0)}}. \tag{4.30}$$

Here, we get $F(\omega) = \omega^2 + c_1\omega + c_0, \Delta = c_1^2 - 4c_0$. We examine the third-order polynomial discrimination system in four scenarios in order to solve Eq (4.29).

Case 1: $\Delta = 0$, $\omega > 0$.

When $c_1 < 0$, Eq (4.29) admits the hyperbolic tanh-coth solution as given in Case I in Section 4.1.

When $c_1 > 0$, Eq (4.29) admits the trigonometric tan solution as given in Case I in Section 4.1.

When $c_1 = 0$, Eq (4.29) has the following solution:

$$\omega_1 = \frac{4}{(\xi_1 - \xi_0)^2}. \tag{4.31}$$

Thus, the solution of (1.2) is

$$E_{19,20}(x,t) = \pm\sqrt{\left(\frac{8}{(k^2-1)v^2}\right)^{-\frac{1}{3}}}\, e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}\, \frac{2}{\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}} (v\, x + \rho\, t) - \xi_0}. \tag{4.32}$$

Case 2: $\Delta = 0$, and $c_0 = 0$.

When $\omega > -c_1$ and $c_1 > 0$, Eq (4.29) has the following solution:

$$\omega_2 = \frac{c_1}{2}\tanh^2\left(\frac{1}{2}\sqrt{\frac{c_1}{2}}(\xi_1 - \xi_0)\right) - c_1. \tag{4.33}$$

$$\omega_3 = \frac{c_1}{2}\coth^2\left(\frac{1}{2}\sqrt{\frac{c_1}{2}}(\xi_1 - \xi_0)\right) - c_1. \tag{4.34}$$

Thus, the solutions of (1.2) are

$$E_{21}(x,t) = \sqrt{\left(\frac{8}{(k^2-1)v^2}\right)^{-\frac{1}{3}}\left(\frac{c_1}{2}\tanh^2\left(\frac{1}{2}\sqrt{\frac{c_1}{2}}(\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}}(vx+\rho t)-\xi_0)\right)-c_1\right)}\times e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \tag{4.35}$$

$$E_{22}(x,t) = \sqrt{\left(\frac{8}{(k^2-1)v^2}\right)^{-\frac{1}{3}}\left(\frac{c_1}{2}\coth^2\left(\frac{1}{2}\sqrt{\frac{c_1}{2}}(\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}}(v\,x+\rho\,t)-\xi_0)\right)-c_1\right)}\times$$

$$e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t},\tag{4.36}$$

where $c_1 = \frac{4(k^2+2\lambda)}{v^2}(\frac{8}{(k^2-1)v^2})^{-\frac{2}{3}}$.

When $\omega > -c_1$ and $c_1 < 0$, Eq (4.29) has the following solution:

$$\omega_4 = -\frac{c_1}{2}\tan^2\left(\frac{1}{2}\sqrt{-\frac{c_1}{2}}(\xi_1-\xi_0)\right) - c_1.\tag{4.37}$$

Thus, the solution of (1.2) is

$$E_{23}(x,t) = \sqrt{\left(\frac{8}{(k^2-1)v^2}\right)^{-\frac{1}{3}}\left(-\frac{c_1}{2}\tan^2\left(\frac{1}{2}\sqrt{-\frac{c_1}{2}}(\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}}(v\,x+\rho\,t)-\xi_0)\right)-c_1\right)}\times$$

$$e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t},\tag{4.38}$$

where $c_1 = \frac{4(k^2+2\lambda)}{v^2}(\frac{8}{(k^2-1)v^2})^{-\frac{2}{3}}$.

Case 3: $\Delta > 0$, and $c_0 \neq 0$.

Let $\alpha < \delta < \gamma$, there α, δ and γ, where one of them is zero and the two others are the roots of $F(\omega) = 0$.

When $\alpha < \omega < \delta$, Eq (4.29) has the following solution:

$$\omega_5 = \alpha + (\delta-\alpha)sn^2(\frac{\sqrt{\gamma-\alpha}}{2}(\xi_1-\xi_0), m),\tag{4.39}$$

where $m^2 = \frac{\delta-\alpha}{\gamma-\alpha}$. Thus, the solution of (1.2) is

$$E_{24}(x,t) = \sqrt{\left(\frac{8}{(k^2-1)v^2}\right)^{-\frac{1}{3}}\left(\alpha+(\delta-\alpha)sn^2(\frac{\sqrt{\gamma-\alpha}}{2}(\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}}(vx+\rho t)-\xi_0), m)\right)}\times$$

$$e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}.\tag{4.40}$$

When $\omega > \gamma$, Eq (4.29) has the following solution:

$$\omega_6 = \frac{\gamma - \delta.sn^2(\frac{\sqrt{\gamma-\alpha}}{2}(\xi_1-\xi_0), m)}{cn^2(\frac{\sqrt{\gamma-\alpha}}{2}(\xi_1-\xi_0), m)}.\tag{4.41}$$

Thus, the solution of (1.2) is

$$E_{25}(x,t) = \sqrt{\left(\frac{8}{(k^2-1)v^2}\right)^{-\frac{1}{3}}\left(\frac{\gamma-\delta.sn^2(\frac{\sqrt{\gamma-\alpha}}{2}(\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}}(vx+\rho t)-\xi_0), m)}{cn^2(\frac{\sqrt{\gamma-\alpha}}{2}(\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}}(vx+\rho t)-\xi_0), m)}\right)}e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}.\tag{4.42}$$

Case 4: $\Delta < 0$.

When $\omega > 0$, Eq (4.29) has the following solution:

$$\omega_7 = \frac{2\sqrt{c_0}}{1 + cn(c_0^{\frac{1}{4}}(\xi_1 - \xi_0), m)} - \sqrt{c_0}. \tag{4.43}$$

Thus, the solution of (1.2) is

$$E_{26}(x,t) = \sqrt{\left(\frac{8}{(k^2-1)v^2}\right)^{-\frac{1}{3}} \left(\frac{2\sqrt{c_0}}{1 + cn(c_0^{\frac{1}{4}}(\left(\frac{8}{(k^2-1)v^2}\right)^{\frac{1}{3}}(vx+\rho t) - \xi_0), m)} - \sqrt{c_0}\right)} e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}, \tag{4.44}$$

where $m^2 = \frac{1}{2}\left(1 - \frac{c_1}{2\sqrt{c_0}}\right)$, $c_0 = 4\mu_0(\frac{8}{(k^2-1)v^2})^{-\frac{1}{3}}$, and $c_1 = \frac{4(k^2+2\lambda)}{v^2}(\frac{8}{(k^2-1)v^2})^{-\frac{2}{3}}$.

5. Influences of noise

Theoretical frameworks that characterize ion sound waves and Langmuir waves are crucial for the advancement of innovative analytical and numerical techniques. The ISALW model with multiplicative noise is transformed into a standard nonlinear ordinary differential form utilizing a time-dependent Wiener process $\beta(t)$. By using $E(e^{2\sigma\beta(t)}) = e^{2\sigma^2 t}$, the ISALW model (1.2) transforms into Eq (4.1). Then, we applied the extended tanh approach and the complete discrimination system for the polynomial method to provide novel stochastic solutions for the ISALW model (1.2) with multiplicative noise in the Itô sense. This model emerges in plasma and is roughly equivalent to conventional sound waves in a neutral gas, and longitudinal waves composing rarefaction and compressions proceeding in the medium. As a result, the suggested stochastic solutions allow for some intriguing plasma physics applications, such as space plasmas, laboratory plasmas, and fusion devices.

We employ the symbolic program MATLAB to generate visual representations for specific stochastic solutions in the Itô sense. Additionally, we examine the impact of multiplicative noise on the solitary wave solutions of the stochastic ISALW model. For particular solutions of the proposed model, we provide both 2D and 3D graphs based on appropriate parameter selections.

The solution (4.13) displays 2D and 3D periodic waves profiles as seen in Figure 1 in the absence of noise term (i.e., $\sigma = 0$). Figure 2 displays the graphs of 2D and 3D periodic pulse wave solutions (4.15). Conversely, the investigation of how external effective impulse noises influence the properties and components of solitary waves, which could lead to collapse or significant dissipation, is conducted. These effects are analyzed and detailed based on the solutions derived from electrostatic wave fields. Figures 3 and 4 illustrate how the stochastic electrostatic solution of $E(x,t)$ (4.13) changes with space x and noise intensity σ. These figures show that the wave behaved like a dissipative wave as σ rose, and Figure 3 shows that the shock wave amplitude may be inferred from this. It was indicated that σ influenced both the amplitude and width of the periodic pulse wave solution (4.15) depicted in Figure 5, as well as the explosive pulse wave shown in Figure 6.

In summary, the features of stochastic solutions for the model of ISALWs imposed by multiplicative noise in the Itô sense were examined via the extended tanh technique to improve the wave properties under stochastic influences. These solutions hold considerable relevance in various fields, including space plasmas, solar wind, optical analogy, and astrophysical plasmas. The results presented demonstrate the effectiveness and dependability of the suggested method for identifying stochastic solutions in complex models within the realms of new physics and applied science.

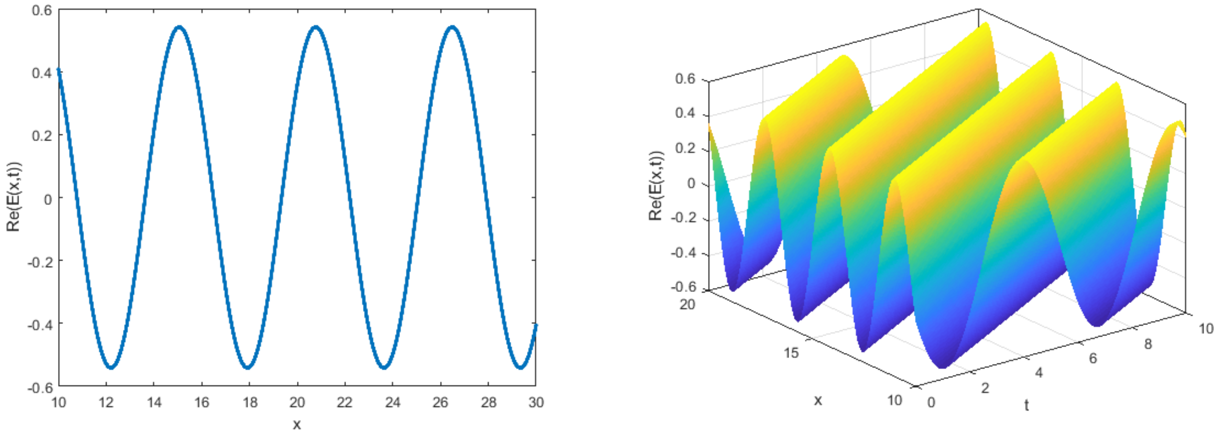

Figure 1. 2D and 3D periodic wave solution $E_1(x, t)$ in (4.13) for $k = 1.1, v = 0.5, \lambda = -2$, and $t = 2$.

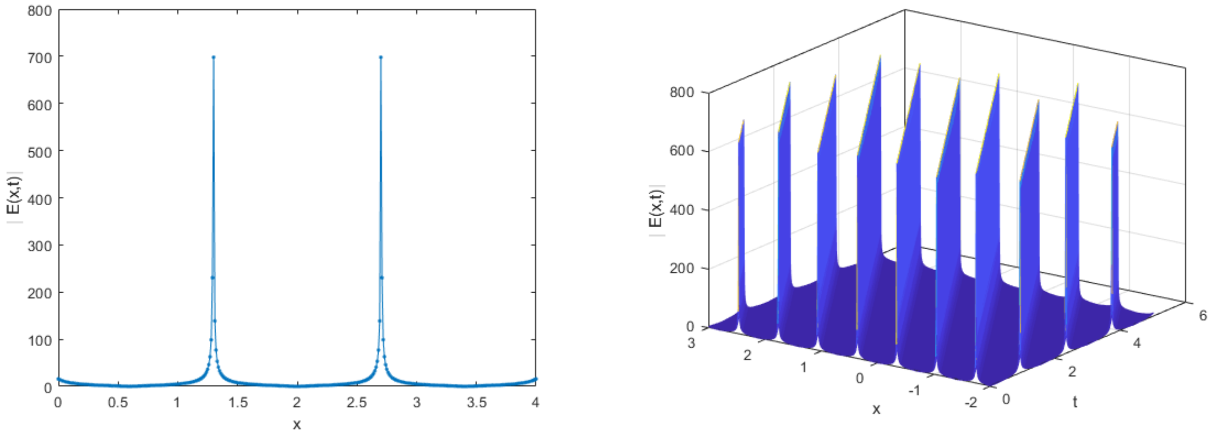

Figure 2. 2D and 3D periodic pulse wave solution $E_5(x, t)$ in (4.15) for$k = 2, v = 0.5, \lambda = 3$, and $t = 1$.

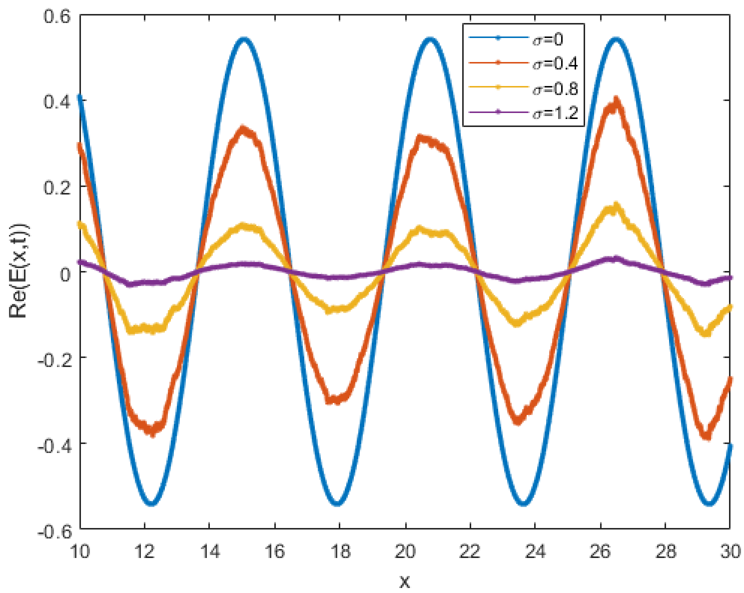

Figure 3. 2D periodic wave $E_1(x,t)$ in (4.13) for $k = 1.1, v = 0.5, \lambda = -2$, and $t = 2$ with different values of σ.

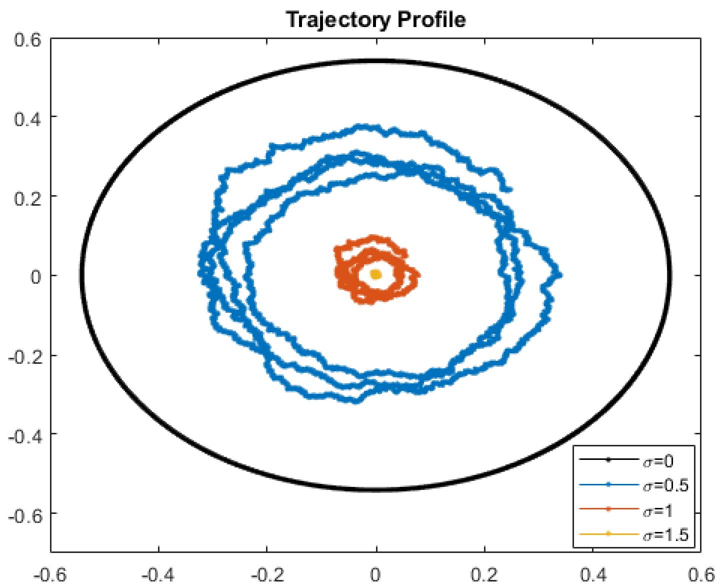

Figure 4. Trajectory of solitary wave $E_1(x,t)$ in (4.13) for $k = 1.1, v = 0.5, \lambda = -2$, and $t = 2$ with different values of σ.

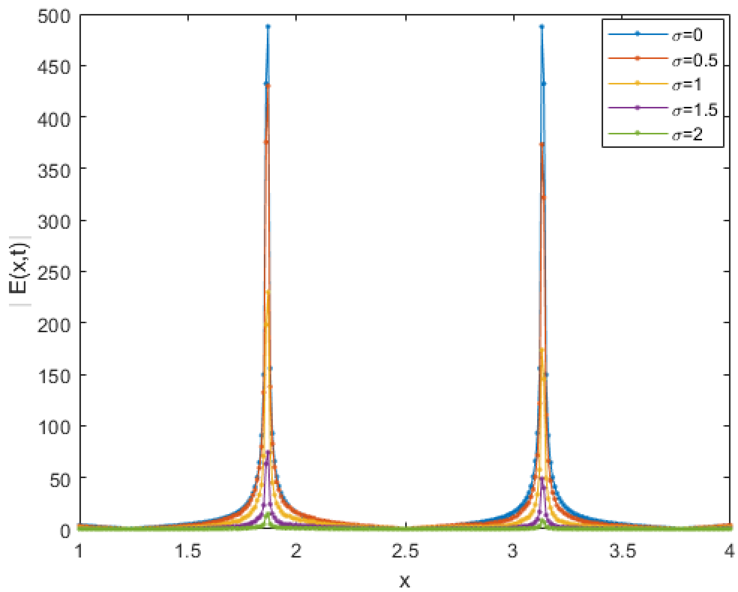

Figure 5. 2D periodic pulse wave solution $E_5(x, t)$ in (4.15) for $k = 2.5, v = 0.5, \lambda = 3$, and $t = 1$.

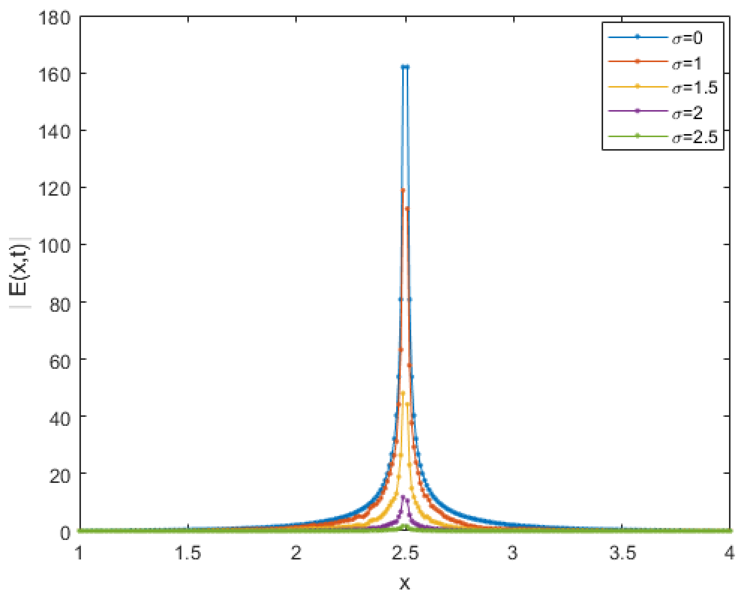

Figure 6. 2D explosive pulse wave solution $E_{13}(x, t)$ in (4.22) for $k = 2.5, v = 0.5, \lambda = -5$, and $t = 1$.

6. Conclusions

This work successfully developed novel stochastic solitary wave solutions for the ion sound and Langmuir waves system using an extended tanh approach and the complete discrimination system for the polynomial method. This model illustrates the bands of the plasma electrostatic field associated with Langmuir waves. We have incorporated new behaviors into the stochastic ISALWs model with multiplicative noise intensity, including periodic envelopes, rational solitonic waves, periodic pulse wave, and explosive pulse wave, etc. These solutions have important uses in astrophysical plasmas, solar wind, plasma heating, nonlinear optics, tokamak plasmas, bursty waves in cusps regions, space plasmas, and magnetospheric waves. The impact of noise factors on the structural behaviors of waves has been examined. Some modulations in wave structures, including rapid amplitude changes, falling envelope amplitudes, frequency fluctuations, and collapsing soliton tails, have been found to be produced by the stochastic parameter σ. In future studies, we will utilize different analytical approaches to derive additional stochastic traveling waves solutions for ion sound and Langmuir waves model.

Use of Generative-AI tools declaration

The author declares he has not used artificial intelligence (AI) tools in the creation of this article.

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Conflict of interest

The author declares no conflicts of interest.

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