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Research article

Sequential inspection sampling plan based on Burr-XII amputated life testing with numerical illustrations and industrial applications

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Abstract: It is often desired for practical reasons to cease a life test at a predetermined period t_0. In this study, we provide sequential inspection sampling plan (SISP) for amputated life tests underlying the Burr XII (BTXII) distribution. We considered the ρ^{th} percentile lifetime of a product as the quality parameter. The sequential sampling plan is a dynamic and efficient technique to quality control and statistical decision-making. Unlike fixed sample plans, which require a predetermined number of samples before deciding, sequential plan enables ongoing analysis as a batch is examined. This flexible plan enables decisions- such as rejecting, accepting or continuing the sampling process - to be made after each sample, which is cumulative data for the number of nonconforming items. Acceptance, rejection limit lines, and optimal sample size at levels of manufacturer and customer errors were analyzed. A technique is provided for calculating the operation characteristic (OC) function and average sample number (ASN) in the suggested SISP. The effectiveness of the SISP was compared to the single, double, and repeating sampling strategies. In comparison to the single, double, and repetitive acceptance sampling plans (RASPs) the proposed sequential sampling acceptance strategy requires fewer sample resources on average for amputated life testing. The suggested method's uses are demonstrated with illustrated instances. In industrial applications, two actual sets of data are employed to demonstrate the SISP's flexibility.

Keywords: multiple inspection sampling plan; sequential inspection sampling plan; average sample number; acceptable quality level; rejectable quality level; customer's error; manufacturer's error; sequential probability ratio test
Mathematics Subject Classification: 62N05; 62P30; 62L10; 62L10

1. Introduction

Acceptance sampling plans are a quality control technique to decide whether to accept or reject a batch of products or materials based on a sample. Rather than examining each item in a batch, which can be time-consuming and costly, acceptance sampling enables a smaller sample to be examined. The decision to accept or reject the batch is then based on the results from this sample. By statistically evaluating a smaller sample, acceptance sampling provides a balance between quality assurance and efficiency.

Acceptance sampling plans can be divided into variables sampling plans and attribute sampling plans. In variables sampling, only one quality characteristic of a unit is observed. In attribute sampling, a unit is classed as conforming or nonconforming based on whether it meets established specifications. Sequential sampling plan: This type, like multiple sampling, is an extension of double sampling, where a sample is drawn from a lot one at a time, and the decision to accept or reject the lot can be made after each sample is tested. If decision-making is difficult, the process of inspection continues until a conclusive decision is reached regarding this lot whether to accept or reject it. This plan can lead to faster decisions and lower inspection costs, especially when lots are consistently of good quality. Sequential plans are often used when sample size is crucial, requiring a small sample size must be drawn. They are more difficult to implement than multiple sampling plans because, in some cases, the amount of inspection work is not specified until the sample is obtained. The potential to take one sample at a time must exist; in some operations, this would be extremely difficult or impossible. Furthermore, the working procedure requires an astute and trustworthy examiner since it is somewhat more demanding than single, double, or multiple sampling. Sequential plans tend to be more flexible and efficient, whereas multiple sampling plans can provide a structured approach with fixed criteria for acceptance and rejection (see [1–3]).

The basic advantage of the sequential sampling plan is that it ensures the advantage of multiple sampling plans, which is represented in reducing the ASN. There are two kinds of sequential sampling plans:

i. Item-by-item sequential sampling means drawing one material is inspected at each sampling $(\text{i.e.}, n_i = 1), i = 1,2, \ldots$

ii. Group sequential sampling depends on drawing a group of items, the size of which is greater than one unit (two or more), is inspected at each sample. The group process is similar to multiple sampling plan except that the group sequential sampling process can continue indefinitely, whereas in multiple sampling, the sampling process is terminated after a certain number of units have been inspected.

Overall, sequential acceptance sampling plans are sophisticated tools that enable organizations to make informed decisions about product acceptance or rejection while optimizing resources and maintaining high standards of quality control. Their application continues to evolve with advancements in statistical methods and computational tools. In particular, our interest in this paper centers around item-by-item sequential acceptance sampling plans by attributes, as it is a statistical tool to accept or reject the submitted lots. The sequential inspection sampling plan based on lifetime distributions is adopted by some researchers. Many studies have also been conducted by [4–6].

The researchers in [7] proposed an innovative group acceptance sampling plans based on the percentiles of the Weibull-Fréchet probability model. Similarly, the researchers in [8] extended the quality control validation and testing using a new modified Lindley model and implemented some single, double, and multiple acceptance sampling plans, highlighting practical experimental applications and numerical examples.

Repetitive accepted sampling plan (RASP) has emerged as an efficient alternative to conventional sampling plans, particularly in scenarios requiring a balance between inspection effort and decision reliability. The concept originated with the researchers in [9], who proposed a cyclic attribute-based inspection approach to improve inspection economy. This was significantly extended to variable sampling by the researchers in [10], who introduced plans minimizing the ASN, followed by the researchers in [11], who refined the procedure for continuous measurements with ASN and OC curve analysis. Incorporating economic considerations, the researchers in [12] introduced variable RASPs accounting for process loss, while the researchers in [13] developed a plan based on the process capability index under unknown mean and variance conditions. The researchers in [14] further advanced RASP methodology by integrating quality loss and process yield, adding a dual economic-performance perspective.

The development of mixed and hybrid plans brought additional flexibility. The researchers in [15] proposed a mixed sampling plan which was further refined by the researchers in [16] through simulation-based optimization. The researchers in [17] contributed a new mixed RASP aimed at enhancing product acceptance decisions by combining process capability metrics with flexible sampling. Consumer-focused designs also emerged, notably by the researchers in [18], who emphasized protection against poor-quality lots. Multiple dependent state sampling was integrated by the researchers in [19] for Burr XII distributions, showcasing RASP adaptability to non-normal data.

Recent advancements also include in the work the researchers in [20], who embedded RASP into skip-lot frameworks to minimize inspection in high-quality processes. Collectively, these studies show the robust progression of RASP from simple attribute plans to sophisticated, variable, and mixed designs that accommodate process uncertainty, economic loss, product yield, and consumer protection, affirming RASP as a versatile tool in modern statistical quality control. The researchers in [21] analyzed a group inspection plan utilizing the amputated life-testing with the extended Dagum percentiles. Furthermore, the researchers in [22] explore Bayesian inference in the accelerated testing models under constant stress applications.

The following clarifies the design item-by-item sequential sampling plan. First, it is based on the sequential probability ratio test (SPRT) (see [3,23,24] for more details). The acceptance sampling is a test of the hypothesis:

$$H_0 : \pi = \pi_1 \qquad \forall \qquad H_1 : \pi = \pi_2,$$

where π_1 = AQL (acceptable quality level) and π_2 = RQL (Rejectable quality level). This hypothesis is tested using on the cumulative number of nonconforming units at a sampling. Sequential sampling operates in a rather unconventional manner; instead of a predetermined sample size, select one item (or a few) at a time and then test the hypothesis. There are three options:

i. Accept H_0 and stop,

ii. Reject H_0 in favor H_1 and stop,

iii. Failure to reach a decisive decision, continuing sampling.

The sampling continues, and the hypothesis is tested after each sampling process until a decisive decision is made regarding the sampling process (see [25]).

Our purpose of this study is to present a sequential inspection sampling plan (SISP) in which product life follows the BTXII distribution with a specified shape parameter based on the amputated life test. The remainder of the paper is structured as follows. Section 2 discusses the amputated life test for the BTXII distribution. Section 3 describes the design operational procedure for boundary lines of SISP. Section 4 presents the performance measures for SISP. Section 5 contains tables and instance descriptions. Section 6 covers industrial uses. In Section 7, a comparison study is offered. Finally, Section 8 contains a conclusion.

2. The amputated life test for BTXII distribution

A truncation life test refers to a type of experiment used in reliability engineering to assess the lifespan or durability of a product or component under specific conditions. The goal is to understand how long the item will last before failure. The CDF and PDF of the BTXII distribution are given by

$$F(\boldsymbol{t}) = 1 - \left[1 + \left(\frac{\boldsymbol{t}}{\alpha}\right)^{\beta}\right]^{-k}, \quad \boldsymbol{t} > 0\ , \alpha > 0, \beta > 0 \quad \text{and } k > 0, \tag{1}$$

$$f(\boldsymbol{t}) = \beta\alpha^{-1}k\left(\frac{\boldsymbol{t}}{\alpha}\right)^{\beta-1} 1 - \left[1 + \left(\frac{\boldsymbol{t}}{\alpha}\right)^{\beta}\right]^{-k-1}, \quad \boldsymbol{t} > 0, \alpha > 0, \beta > 0 \text{ and } k > 0,$$

where β and k are shape parameters and α is the scale parameter. Relations between the BTXII and other comparable distributions can be identified in [26]. The shape parameter β is crucial for the BTXII (see [14] for details). The BTXII is not symmetric, so the mean life may not be sufficient to explain the central tendency of this distribution. Moreover, no acceptance sampling plan has been created for BTXII, to the best of our knowledge. As a result, implementing a SISP for the percentile of BTXII is critical.

The quantile function of BTXII, say $Q_\rho = F^{-1}(\boldsymbol{t})$, is given by

$$\boldsymbol{\mathcal{U}}_\rho = \alpha\left[\left(\frac{1}{1-\rho}\right)^{\frac{1}{k}} - 1\right]^{\frac{1}{\beta}}, 0 < \rho < 1. \tag{2}$$

By putting $\rho = 0.5$, in Eq (2) gives the median of BTXII distribution is shown as

$$\boldsymbol{\mathcal{U}}_{0.5} = \alpha\left[(2)^{\frac{1}{k}} - 1\right]^{\frac{1}{\beta}}.$$

The quantile $\boldsymbol{\mathcal{U}}_\rho$ is a function of the scale parameter α and at a pre-fixed value of $\boldsymbol{\mathcal{U}}_\rho$ say $\boldsymbol{\mathcal{U}}_\rho^0$, we may obtain the value of α, say α_0, as

$$\alpha_0 = \boldsymbol{\mathcal{U}}_\rho^0 \left[\left(\frac{1}{1-\rho}\right)^{\frac{1}{k}} - 1\right]^{-\frac{1}{\beta}}. \tag{3}$$

Clearly for a given $\boldsymbol{\mathcal{U}}_\rho, \boldsymbol{\mathcal{U}}_\rho^0, \alpha$ and α_0, we have

$$\boldsymbol{\mathcal{U}}_\rho \geq \boldsymbol{\mathcal{U}}_\rho^0 \Leftrightarrow \alpha \geq \alpha_0,$$

and we may also note α_0 depend on β_0 and k_0. To implement sequential sampling plans for BTXII distribution it is an ascertained that $\boldsymbol{\mathcal{U}}_\rho$ exceeds $\boldsymbol{\mathcal{U}}_\rho^0$ and α exceeds α_0.

We seek to demonstrate the use of a sequential inspection sampling plan based on the amputated life test. As a result, we suggest a sequential inspection sampling strategy in which the item's lifetime follows a BTXII distribution with known shape parameters. Furthermore, the suggested sequential sampling plan is compared to other plans, such as the double sampling plan for BTXII introduced by [27] and the repeating sampling plans for BTXII introduced by [15].

It is coveted to design a SISP to guarantee that the product's ρ^{th} percentile lifetime is at least $\boldsymbol{\mathcal{U}}_\rho$. The batch is accepted if there is sufficient evidence that $\boldsymbol{\mathcal{U}}_\rho \geq \boldsymbol{\mathcal{U}}_\rho^0$ at a specific degree of customer's and manufacturer's errors. To construct a SISP for the ρ^{th} percentile lifetime under amputated life test, the experimental time can be determined as $\boldsymbol{t}_0 = \mathcal{L}_\rho \boldsymbol{\mathcal{U}}_\rho^0$, where $\mathcal{L}_\rho$ is called coefficient factor. To get the genuine ρ^{th} percentile, $\boldsymbol{\mathcal{U}}_\rho$, of the BTXII distribution, the formula $\Pi = F(\boldsymbol{t}_0) = F\left(\mathcal{L}_\rho \boldsymbol{\mathcal{U}}_\rho^0\right)$ is used as

$$\Pi = 1 - \left[1 + \left(\frac{\mathcal{L}_\rho . \xi}{\varpi}\right)^\beta\right]^{-k}, \tag{4}$$

where

$$\xi = \left[\left(\frac{1}{1-\rho}\right)^{\frac{1}{k}} - 1\right]^{\frac{1}{\beta}}, \text{ and } \varpi = \frac{1}{\boldsymbol{\mathcal{U}}_\rho^0} \boldsymbol{\mathcal{U}}_\rho.$$

Here, the ratio ϖ is the quality level of each item. The proportion Π depends on the ratio ϖ. A submitted batch gets accepted or declined based on the outcomes of a life test carried out on selected items from the lot. As a result, inspection sampling plans are typically designed using a two-point approach that takes into account the manufacturer's error at the AQL and the customer's error at the RQL see [28,29]. The maximum allowable level of the percent of defects is called AQL, whereas the lowest impermissible level of the percent of defects is called RQL. Here, we express the AQL and RQL as the ratio of ϖ. Let the manufacturer's error be $\mathcal{P}^*$, the customer's error be $\mathcal{C}^*$, genuine ρ^{th} percentile of product lifetime be $\boldsymbol{\mathcal{U}}_\rho$ and the fixed ρ^{th} percentile be $\boldsymbol{\mathcal{U}}_\rho^0$. A product is regarded agreeable if $\boldsymbol{\mathcal{U}}_\rho \geq \boldsymbol{\mathcal{U}}_\rho^0$, and inadmissible otherwise.

A manufacturer wishes to employ an inspection sampling strategy that ensures a high acceptance probability for a submitted batch when the nonconforming level is less than or equal to AQL, expressed by π_1, to become greater than $(1 - \mathcal{P}^*)$. A customer may desire to utilize an inspection sampling strategy that provides a low acceptance probability for a submitted batch when the faulty level is greater than or equal to RQL, indicated by π_2, to become smaller than $\mathcal{C}^*$. The ratio of $\frac{\boldsymbol{\mathcal{U}}_\rho}{\boldsymbol{\mathcal{U}}_\rho^0}$ at AQL and RQL are ϖ_1 and ϖ_2, respectively, $\varpi_1 > \varpi_2$. For the specified values of the test termination coefficient $\mathcal{L}_\rho$, quality level ϖ and the shape parameters of the BTXII distribution, Eq (4) calculates the probability of product failing prior to $\boldsymbol{t}_0$. Hence, the values of π_1 and π_2 that equivalent to ϖ_1 and ϖ_2 can be obtained using Eq (4).

3. Design of sequential inspection sampling plan

The sequential inspection sampling plan starts by randomly selecting a single item from the batch and examining it. Depending on the result of an examination, the batch is either accepted or denied or a new item is selected, noticing that the decision taken depends on the number of non-conforming units which are observed from drawing this item and the relation of this number on the acceptance and rejection lines. If its:

i. $\sum_{n=1}^{x} d_{(n)} \leq x_A$, the lot is accepted, and the examination stops.

ii. $\sum_{n=1}^{x} d_{(n)} \geq x_R$, the lot is rejected, and the examination stops.

iii. $R_x < \sum_{n=1}^{x} d_{(n)} < A_x$, drawing and examing another new item until a decision is made.

Here, x_A and x_R are acceptance and rejection limit lines. The process provided is termed unit sequential inspection sampling since products are drawn item by item. In practice, SISP is easy to implement applying the graph provided in Figure 1, which plots the total number of nonconforming units discovered versus the total number of samples drawn.

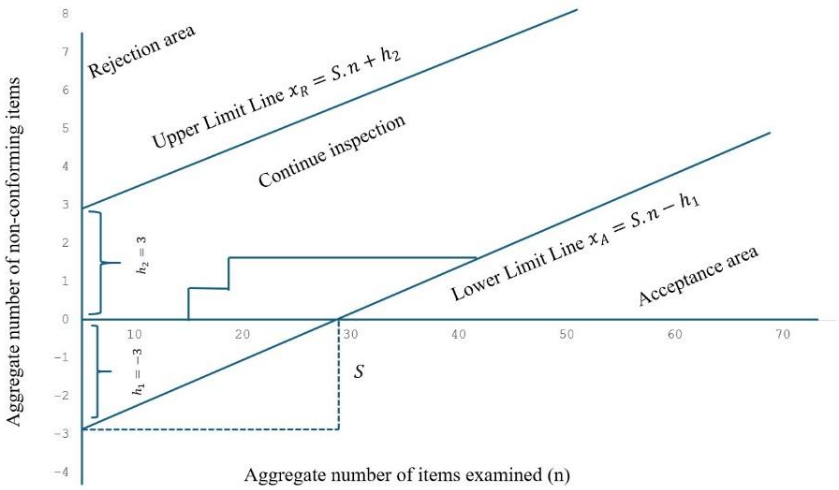

Figure 1. Sequential sampling plot.

According to Figure 1, the abscissa represents the total number of non-conforming units recorded. The diagram is split into three areas. If the total number of nonconforming items lies below the lower range, the lot is accepted. If the total number of non-conforming units lies within the upper range, the lot is declined. If the total number of nonconforming items is in the intermediate range, sampling proceeds.

Designing a sequential inspection plan means finding the values of x_A and x_R that achieve producer's risk $\mathcal{P}^*$ at an acceptable quality level (AQL = π_1) and consumer's risk $\mathcal{C}^*$ at a Rejectable quality level (RQL = π_2). Four values need to be specified before a sequential-sampling plan can be constructed and have been derived by [23,29]. They are π_1, π_2, $\mathcal{P}^*$, and $\mathcal{C}^*$. With these values, the two lines that separate the sequential-sampling chart into the three regions can be constructed using the following equations:

$$\text{Lower: } x_A = \mathcal{S}n - \hbar_1, \tag{5}$$

$$\text{Upper Line: } x_R = \mathcal{S}n + \hbar_2. \tag{6}$$

The continuation region for SPRT of strength $(\pi_1, \mathcal{P}^*, \pi_2, \mathcal{C}^*)$ is given by the inequality

$$x_A < d_{(n)} < x_R,$$

where

$d_{(n)}$: The number of defects observed during inspection of the first n unit.

x_A: Acceptance limit line.

x_R: Rejection limit line.

n: Sequential unit taken from the lot of sample items taken.

$\mathcal{S}$: Equilibrium quality or slope of the acceptance and rejection boundaries, where

$$\mathcal{S} = \frac{1}{\varrho} \ln \left[\frac{1-\pi_1}{1-\pi_2}\right]. \tag{7}$$

h_1: Intercept of the acceptance line, where

$$h_1 = \frac{1}{\varrho} \ln \left[\frac{1-\mathcal{P}^*}{\mathcal{C}^*}\right]. \tag{8}$$

h_2: Intercept of the rejection lines, where

$$h_2 = \frac{1}{\varrho} \ln \left[\frac{1-\mathcal{C}^*}{\mathcal{P}^*}\right], \tag{9}$$

and

$$\varrho = \ln \left[\frac{1}{\pi_1(1-\pi_2)} \pi_2(1-\pi_1)\right]. \tag{10}$$

Employing these metrics, SISP for attribute examination may be easily established and described.

4. Performance measures of the sequential plan

The indicators of performance aid in monitoring the implementation of each inspection sampling plan. The following equations can be used to calculate the probability of acceptance lot (L_a), the probability of non-conforming proportion (Π), and the average sample number ASN $(\bar{n})$

$$L_a = \frac{\left[\frac{(1-\mathcal{C}^*)}{\mathcal{P}^*}\right]^{\delta} - 1}{\left[\frac{(1-\mathcal{C}^*)}{\mathcal{P}^*}\right]^{\delta} - \left[\frac{\mathcal{C}^*}{(1-\mathcal{P}^*)}\right]^{\delta}}, \tag{11}$$

$$\Pi = \frac{1 - \left[\frac{(1-\pi_2)}{(1-\pi_1)}\right]^{\delta}}{\left(\frac{\pi_2}{\pi_1}\right)^{\delta} - \left[\frac{(1-\pi_2)}{(1-\pi_1)}\right]^{\delta}}, -\infty < \delta < \infty, \delta \neq 0, \tag{12}$$

Table 1 gives the sequential inspection sampling plan for proportion defective points on the OC, ASN and AOQ curves. Exceptionally, Table 1 shows the formulas found when $\delta = -\infty$, -1, 0, 1, ∞. By combining L_a and Π, we may obtain the generic formula for ASN.

$$\bar{n} = \frac{L_a \log\left[\frac{\mathcal{C}^*}{(1-\mathcal{P}^*)}\right] + (1-L_a) \log\left[\frac{(1-\mathcal{C}^*)}{\mathcal{P}^*}\right]}{\Pi \log\left(\frac{\pi_2}{\pi_1}\right) + (1-\Pi) \log\left[\frac{(1-\pi_2)}{1-\pi_1}\right]}. \tag{13}$$

It is worth noting that the ASN in a sequential sampling plan is benchmark to optimizing inspection efficiency, controlling costs, and ensuring reliable quality control. Generally, the following optimization problem is applied for deriving the parameters of SISP.

$$\text{Minimum ASN} = \bar{n} = \frac{L_a \log\left[\frac{\mathcal{C}^*}{(1-\mathcal{P}^*)}\right] + (1-L_a)\log\left[\frac{(1-\mathcal{C}^*)}{\mathcal{P}^*}\right]}{\Pi \log\left(\frac{\pi_2}{\pi_1}\right) + (1-\Pi)\log\left[\frac{(1-\pi_2)}{1-\pi_1}\right]}.$$

Subject to

$$L_a\left(\text{AQL}\left|\frac{1}{\mathcal{U}_\rho^0}\mathcal{U}_\rho = \varpi_1\right.\right) \geq 1 - \mathcal{P}^*,$$

$$L_a\left(\text{LTQL}\left|\frac{1}{\mathcal{U}_\rho^0}\mathcal{U}_\rho = \varpi_2\right.\right) \leq \mathcal{C}^*,$$

$$\varpi_1 > \varpi_2 \text{ and } \mathcal{U}_\rho \geq \mathcal{U}_\rho^0.$$

The parameters $\mathcal{P}^*$ and $\mathcal{C}^*$ are balanced to achieve the desired confidence level in the decision-making process. Also, the parameters ϖ_1 and ϖ_2 are the life ratios at the producer's and consumer's errors respectively. The product's lifetime is taken as a quality attribute and is stated using the ratio ϖ. Hence, to calculate the probability of lot acceptance in the OC function, the ratio ϖ is used. Increasing ϖ increases the probability of lot acceptance. Additionally, the ASN is another measure for evaluating the sampling plan's performance. In this section, we present a mechanism for computing the proposed SISP's OC and ASN. The proposed mechanism for computing $\bar{n}$ and L_a is as follows.

Mechanism 1.

i. Employing Eq (4), the value of Π for the given ϖ is calculated.

ii. Incorporate the value of Π into Eq (12), to get the corresponding value of δ.

iii. The value of L_a is Calculated utilizing the value of δ in Eq (11).

iv. In Eq (13), the values of L_a and Π are entered to calculate the value of ASN ($\bar{n}$).

In this investigation, the ratio ϖ_2 was fixed to one, hence the LTQL or (RQL) is $\mathcal{U}_\rho^0$. Performing mechanism 1 for ϖ_1 and ϖ_2 yields the outcomes shown in Table 1.

Table 1. Sequential inspection sampling plan for proportion defective points on the OC, ASN and AOQ curves.

δ	Π	L_a	$\bar{n}$	AOQ
$-\infty$	0	1	$\frac{\hbar_1}{\mathcal{S}}$	0
-1	π_1	$1-\mathcal{P}^*$	$\frac{(1-\mathcal{P}^*)\hbar_1 - \mathcal{P}^*\hbar_2}{\mathcal{S}-\pi_1}$	$(1-\mathcal{P}^*)\pi_1$
0	$\mathcal{S}$	$\left[\frac{\hbar_2}{(\hbar_1+\hbar_2)}\right]$	$\left[\frac{1}{\mathcal{S}(1-\mathcal{S})}\hbar_1\hbar_2\right]$	$\left[\frac{1}{(\hbar_1+\hbar_2)}\mathcal{S}\hbar_2\right]$
1	π_2	$\mathcal{C}^*$	$\frac{(1-\mathcal{C}^*)\hbar_2 - \mathcal{C}^*\hbar_1}{\pi_2-\mathcal{S}}$	$\mathcal{C}^*\pi_2$
∞	1	0	$\frac{1}{(1-\mathcal{S})}\hbar_2$	0

5. Illustrative examples

The numerical outcomes for T ~ BTXII $(\boldsymbol{t};\beta,k)$ when β = 0.75, 2 and k = 3, 2 are given in Tables 2–12. Tables 2 and 5 display the results of acceptance and rejection limit lines for SISP with parameter β= 0.75 and k = 3 under percentiles ρ = 0.10 and 0.50, respectively. Tables 3, 4, 6 and 7 display the results of OC and ASN for SISP with parameter β= 0.75 and k = 3 under percentiles ρ = 0.10 and 0.50 when ϖ_1= 2 and 4, respectively. Table 8 displays the results of acceptance and rejection limit lines for SISP under 10th percentile with parameters β = 2 and k = 2. Tables 9 and 11 display the results of acceptance and rejection limit lines for SISP under 10th percentile when $\hat{\beta}$ = 0.85, 5.47 and $\hat{k}$ = 5.49, 0.08. Table 10 displays the results of OC and ASN for SISP under 10th percentile when parameters $\hat{\beta}$ = 0.85, 5.47 and 2, $\hat{k}$ = 5.49, 0.08, and 2. Table 12 displays the comparison of SISP, SASP, double acceptance sampling plan (DASP), and RASP. In quality control and decision-making processes, the probability of accepting a lot refers to the probability that a batch of products or items will be accepted during inspection, even if it contains some defective items. A good strategy is one that brings the probability of acceptance for the batch close to one. These tables exhibit that the probability of acceptance (L_a) rises to 1 as the quality ratio (ϖ) increases. Moreover, decreasing the values of $(\mathcal{C}^*)$ leads to decreases the measure (L_a) with fixed producer's risk. Decreasing the value of $\mathcal{P}^*$ leads to increases in the value of measure (L_a) with fixed consumer's risk. The measure (L_a) decreases when the value of AQL (ϖ_1) increases from 2 to 4. Decreasing $\mathcal{P}^*$and $\mathcal{C}^*$ values increase the value of the ASN ($\bar{n}$). Increasing the quality attribute level (ϖ) increases L_a while decreasing ASN ($\bar{n}$). Increasing the coefficient factor ($\mathcal{L}_\rho$) reduces the ASN ($\bar{n}$) but raises the value of measure (L_a) while maintaining the customer and manufacturer risks. Increasing the value of ρ^{th} percentile leads to decreases the value of measure (L_a) and ASN ($\bar{n}$) with fixed the value of AQL(ϖ_1). Additionally, reducing the experiment's duration results in a rise in the ASN. Also, raising AQL levels results in a drop in ASN. Now, two illustrative experiments are presented.

Table 2. The results of acceptance and rejection limit lines for SISP under 10th percentile BTXII model with parameter β = 0.75 and k = 3.

n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R
1	#	3	11	#	3	21	0	4	31	1	5	41	1	5	51	2	6
2	#	3	12	#	3	22	0	4	32	1	5	42	1	5	52	2	6
3	#	3	13	#	3	23	0	4	33	1	5	43	1	5	53	2	6
4	#	3	14	#	3	24	0	4	34	1	5	44	2	5	54	2	6
5	#	3	15	#	4	25	0	4	35	1	5	45	2	5	55	2	6
6	#	3	16	#	4	26	0	4	36	1	5	46	2	5	56	2	6
7	#	3	17	#	4	27	0	4	37	1	5	47	2	6	57	2	6
8	#	3	18	#	4	28	1	4	38	1	5	48	2	6	58	2	6
9	#	3	19	0	4	29	1	4	39	1	5	49	2	6	59	3	6
10	#	3	20	0	4	30	1	4	40	1	5	50	2	6	60	3	6

Table 3. OC value and ASN for SISP under 10th percentile BTXII model at β = 0.75, k = 3 and ϖ_1 = 2.0.

$\mathcal{C}^*$	ϖ	$\mathcal{L}_{0.10}$ = 0.5						$\mathcal{L}_{0.10}$ = 1.0					
		$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01		$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01	
		L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$
0.25	2	0.91	162	0.953	193	0.992	224.4	0.92	99.1	0.955	118	0.994	137
	4	0.997	92.7	0.999	97.4	1	100.6	0.998	56.3	0.999	59.2	1	61.1
	6	0.999	76.5	1	79.8	1	82.3	0.999	46.3	1	48.4	1	49.9
	8	1	69.6	1	72.6	1	74.8	1	42.1	1	43.9	1	45.3
	10	1	65.8	1	68.6	1	70.7	1	39.8	1	41.4	1	42.7
0.10	2	0.91	299	0.951	339	0.991	378.4	0.92	183	0.952	208	0.992	232
	4	0.998	160	1	164	1	167.6	0.998	97.1	1	99.9	1	102
	6	1	131	1	135	1	137.1	1	79.6	1	81.6	1	83.1
	8	1	119	1	122	1	124.6	1	72.3	1	74.0	1	75.4
	10	1	113	1	116	1	117.7	1	68.2	1	69.9	1	71.2
0.05	2	0.902	404	0.95	451	0.991	495.0	0.903	247	0.951	276	0.992	303
	4	0.998	211	1	215	1	218.2	0.998	128	1	131	1	133
	6	1	173	1	176	1	178.5	1	105	1	107	1	108
	8	1	157	1	160	1	162.3	1	95.1	1	96.8	1	98.2
	10	1	148	1	151	1	153.3	1	89.7	1	91.4	1	92.7
0.01	2	0.901	650	0.95	710	0.991	765.9	0.902	398	0.951	435	0.991	469
	4	0.998	328	1	333	1	335.8	0.998	199	1	202	1	204
	6	1	269	1	272	1	274.7	1	163	1	165	1	167
	8	1	245	1	248	1	249.8	1	148	1	150	1	151
	10	1	231	1	234	1	236	1	140	1	141	1	143

Table 4. OC value and ASN for SISP under 10th percentile BTXII model at β = 0.75, k = 3 and ϖ_1= 4.0.

$\mathcal{C}^*$	ϖ	$\mathcal{L}_{0.10}$ = 0.5						$\mathcal{L}_{0.10}$ = 1.0					
		$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01		$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01	
		L_a	n̄	L_a	n̄	L_a	n̄	L_a	n̄	L_a	n̄	L_a	n̄
0.25	2	0.643	62.8	0.706	87.2	0.8	139.9	0.643	38.4	0.706	53.3	0.799	85.5
	4	0.9	55.2	0.95	65.7	0.99	76.5	0.9	33.5	0.95	39.9	0.99	46.5
	6	0.957	48.9	0.985	54.6	0.999	58.7	0.957	29.6	0.985	33.1	0.999	35.5
	8	0.977	45.2	0.993	49.1	1	51.6	0.977	27.3	0.993	29.7	1	31.2
	10	0.985	42.8	0.996	45.9	1	47.8	0.985	25.9	0.996	27.7	1	28.9
0.10	2	0.545	119	0.614	158	0.723	244.9	0.545	72.4	0.613	96.7	0.722	150
	4	0.9	102	0.95	116	0.99	129.0	0.9	61.9	0.95	70.2	0.99	78.3
	6	0.963	87.3	0.987	93.7	0.999	98.0	0.963	52.9	0.987	56.7	0.999	59.3
	8	0.982	79.5	0.995	83.5	1	86.0	0.982	48.0	0.995	50.5	1	52.0
	10	0.989	74.7	0.997	77.8	1	79.6	0.989	45.1	0.997	46.9	1	48.1
0.05	2	0.491	161	0.561	213	0.677	325.1	0.49	98.4	0.56	130	0.675	199
	4	0.9	138	0.95	154	0.99	168.8	0.9	83.7	0.95	93.3	0.99	103
	6	0.965	116	0.988	123	0.999	127.7	0.965	70.4	0.988	74.6	0.999	77.3
	8	0.983	105	0.995	110	1	112.0	0.983	63.6	0.995	66.2	1	67.7
	10	0.99	98.7	0.998	102	1	103.7	0.99	59.6	0.998	61.5	1	62.6
0.01	2	0.402	260	0.472	340	0.594	513.9	0.401	159	0.471	207	0.593	314
	4	0.9	222	0.95	242.	0.99	261.2	0.9	135	0.95	147	0.99	159
	6	0.967	183	0.988	192	0.999	196.7	0.967	111	0.988	116	0.999	119
	8	0.984	165	0.996	170	1	172.4	0.984	99.7	0.996	103	1	104
	10	0.991	154	0.998	158	1	159.6	0.991	93.1	0.998	95.2	1	96.4

Table 5. The results of acceptance and rejection limit lines for SISP under 50th percentile BTXII model with parameter β = 0.75 and k = 3.

n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R
1	#	3	11	3	6	21	6	10	31	10	13	41	14	17	51	17	21
2	#	3	12	3	7	22	7	10	32	10	14	42	14	17	52	18	21
3	#	3	13	4	7	23	7	11	33	11	14	43	14	18	53	18	21
4	#	4	14	4	7	24	7	11	34	11	14	44	15	18	54	18	22
5	0	4	15	4	8	25	8	11	35	11	15	45	15	18	55	19	22
6	1	4	16	5	8	26	8	12	36	12	15	46	15	19	56	19	22
7	1	5	17	5	8	27	9	12	37	12	16	47	16	19	57	19	23
8	2	5	18	5	9	28	9	12	38	13	16	48	16	19	58	20	23
9	2	5	19	6	9	29	9	13	39	13	16	49	16	20	59	20	23
10	2	6	20	6	9	30	10	13	40	13	17	50	17	20	60	20	24

Table 6. OC value and ASN for SISP under median BTXII model at β = 0.75, k = 3 and ϖ_1= 2.0

$\mathcal{C}^*$	ϖ	$\mathcal{L}_{0.50}$ = 0.5						$\mathcal{L}_{0.50}$ = 1.0					
		$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01		$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01	
		L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$	L_a	$\bar{n}$
0.25	2	0.9	28.9	0.95	34.4	0.99	40.1	0.9	20.9	0.95	24.9	0.99	29.0
	4	0.996	15.5	0.999	16.3	1	16.8	0.997	10.7	1	11.2	1	11.6
	6	0.999	12.5	1	13.0	1	13.4	1	8.4	1	8.8	1	9.1
	8	1	11.2	1	11.7	1	12.0	1	7.5	1	7.8	1	8.1
	10	1	10.5	1	10.9	1	11.3	1	7.0	1	7.3	1	7.5
0.10	2	0.9	53.4	0.95	60.6	0.99	67.5	0.9	38.6	0.95	43.8	0.99	48.8
	4	0.998	26.7	1	27.5	1	28.0	0.998	18.4	1	18.9	1	19.3
	6	1	21.4	1	22.0	1	22.4	1	14.5	1	14.8	1	15.1
	8	1	19.2	1	19.7	1	20.1	1	12.9	1	13.2	1	13.4
	10	1	18.0	1	18.4	1	18.8	1	12.0	1	12.3	1	12.5
0.05	2	0.9	72.2	0.95	80.5	0.99	88.4	0.9	52.2	0.95	58.2	0.99	63.9
	4	0.998	35.2	1	36.0	1	36.5	0.998	24.3	1	24.8	1	25.1
	6	1	28.2	1	28.7	1	29.1	1	19.1	1	19.4	1	19.7
	8	1	25.3	1	25.8	1	26.1	1	16.9	1	17.2	1	17.5
	10	1	23.7	1	24.1	1	24.5	1	15.7	1	16.0	1	16.3
0.01	2	0.9	116	0.95	127	0.99	136.7	0.9	83.9	0.95	91.7	0.99	98.8
	4	0.998	54.8	1	55.6	1	56.2	0.998	37.8	1	38.3	1	38.7
	6	1	43.9	1	44.4	1	44.8	1	29.7	1	30.0	1	30.3
	8	1	39.4	1	39.8	1	40.2	1	26.3	1	26.7	1	26.9
	10	1	36.9	1	37.3	1	37.6	1	24.5	1	24.8	1	25.0

Table 7. OC value and ASN for SISP median BTXII model at β = 0.75, k = 3 and ϖ_1 = 4.0.

		$\mathcal{L}_{0.50}$ = 0.5						$\mathcal{L}_{0.50}$ = 1.0					
$\mathcal{C}^*$	ϖ	$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01		$\mathcal{P}^*$ = 0.10		$\mathcal{P}^*$ = 0.05		$\mathcal{P}^*$ = 0.01	
		L_a	n̄	L_a	n̄	L_a	n̄	L_a	n̄	L_a	n̄	L_a	n̄
0.25	2	0.649	11.0	0.71	15.2	0.805	24.6	0.649	7.8	0.708	10.8	0. 80	17.5
	4	0.9	9.2	0.95	10.9	0.99	12.7	0.9	6.3	0.95	7.5	0.99	8.7
	6	0.958	7.9	0.985	8.8	0.999	9.5	0.959	5.3	0.985	5.9	0.999	6.4
	8	0.978	7.2	0.994	7.8	1	8.2	0.979	4.8	0.994	5.2	1	5.4
	10	0.986	6.8	0.997	7.2	1	7.5	0.987	4.5	0.997	4.8	1	5.0
0.10	2	0.55	20.7	0.619	27.6	0.734	42.9	0.547	14.7	0.619	19.6	0.728	30.6
	4	0.9	16.9	0.95	19.2	0.99	21.4	0.9	11.6	0.95	13.1	0.99	14.7
	6	0.964	14.1	0.987	15.1	0.999	15.8	0.965	9.5	0.988	10.2	0.999	10.6
	8	0.983	12.7	0.995	13.3	1	13.7	0.984	8.4	0.995	8.8	1	9.1
	10	0.99	11.8	0.998	12.3	1	12.5	0.991	7.8	0.998	8.1	1	8.3
0.05	2	0.50	28.0	0.563	37.1	0.679	56.9	0.501	19.9	0.562	26.3	0.676	40.5
	4	0.9	22.9	0.95	25.5	0.99	28.0	0.9	15.7	0.95	17.5	0.99	19.2
	6	0.966	18.8	0.988	19.9	0.999	20.6	0.967	12.6	0.988	13.4	0.999	13.8
	8	0.984	16.8	0.996	17.4	1	17.8	0.985	11.1	0.996	11.6	1	11.8
	10	0.991	15.6	0.998	16.0	1	16.3	0.992	10.3	0.998	10.6	1	10.8
0.01	2	0.405	45.2	0.473	59.1	0.60	89.8	0.401	32.0	0.473	41.8	0.595	63.8
	4	0.9	36.8	0.95	40.2	0.99	43.3	0.9	25.2	0.95	27.5	0.99	29.7
	6	0.968	29.6	0.989	30.9	0.999	31.7	0.969	19.9	0.989	20.8	0.999	21.3
	8	0.985	26.2	0.996	26.9	1	27.4	0.986	17.4	0.996	17.9	1	18.2
	10	0.992	24.3	0.998	24.8	1	25.1	0.992	16.0	0.998	16.4	1	16.6

Table 8. The results of acceptance and rejection limit lines for SISP under 10th percentile BTXII model with shape parameters $\beta = 2$ and $k = 2$.

n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R
1	#	2	11	#	3	21	0	3	31	1	4	41	1	4	51	2	5
2	#	2	12	#	3	22	0	3	32	1	4	42	1	4	52	2	5
3	#	2	13	#	3	23	0	3	33	1	4	43	1	4	53	2	5
4	#	2	14	#	3	24	0	3	34	1	4	44	2	4	54	2	5
5	#	2	15	#	3	25	0	3	35	1	4	45	2	4	55	2	5
6	#	2	16	#	3	26	1	3	36	1	4	46	2	4	56	2	5
7	#	2	17	#	3	27	1	3	37	1	4	47	2	5	57	2	5
8	#	2	18	0	3	28	1	3	38	1	4	48	2	5	58	2	5
9	#	2	19	0	3	29	1	4	39	1	4	49	2	5	59	2	5
10	#	2	20	0	3	30	1	4	40	1	4	50	2	5	60	2	5

Table 9. The results of acceptance and rejection limit lines for SISP under 10th percentile BTXII model with parameter $\hat{\beta} = 0.85$ and $\hat{k} = 5.49$.

n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R
1	#	5	11	#	5	21	#	6	31	1	8	41	1	8	51	2	8
2	#	5	12	#	5	22	#	6	32	1	8	42	1	8	52	2	8
3	#	5	13	#	5	23	#	6	33	1	8	43	1	8	53	2	9
4	#	5	14	#	6	24	#	6	34	1	8	44	1	8	54	2	9
5	#	5	15	#	6	25	#	6	35	1	8	45	1	8	55	2	9
6	#	5	16	#	6	26	#	6	36	1	8	46	1	8	56	2	9
7	#	5	17	#	6	27	#	7	37	1	8	47	1	8	57	2	9
8	#	5	18	#	6	28	#	7	38	1	8	48	1	8	58	2	9
9	#	5	19	#	6	29	0	7	39	2	8	49	2	8	59	2	9
10	#	5	20	#	6	30	0	7	40	2	8	50	2	8	60	2	9

Table 10. OC value and ASN for SISP 10th percentile BTXII model at ϖ_1 = 2.0 and $\mathcal{P}^*$ = 0.05.

$\mathcal{C}^*$	ϖ	$\beta =$ 0.85 and k = 5.49				$\beta =$ 5.47 and k = 0.08				$\beta =$ 2 and k = 2			
		$\mathcal{L}_{0.10}$ = 0.5		$\mathcal{L}_{0.10}$ = 1.0		$\mathcal{L}_{0.10}$ = 0.5		$\mathcal{L}_{0.10}$ = 1.0		$\mathcal{L}_{0.10}$ = 0.5		$\mathcal{L}_{0.10}$ = 1.0	
		OC	ASN	OC	ASN	OC	ASN	OC	ASN	OC	ASN	OC	ASN
0.25	2	0.95	166.9	0.95	94.7	0.95	266.2	0.95	13.2	0.95	105.3	0.95	27.5
	4	0.999	87.2	0.999	49.1	0.998	284.1	0.999	13.3	0.999	74.9	0.999	19.3
	6	1	72.6	1	40.8	1	285.3	1	13.3	1	70.0	1	18.0
	8	1	66.6	1	37.4	1	285.5	1	13.3	1	68.4	1	17.5
	10	1	63.3	1	35.5	1	285.6	1	13.3	1	67.6	1	17.3
0.10	2	0.95	293.7	0.95	166.7	0.95	468.6	0.95	23.3	0.95	185.4	0.95	48.4
	4	1	147.2	1	83.0	0.998	480.8	0.999	22.4	0.999	126.5	0.999	32.6
	6	1	122.4	1	68.8	1	481.5	1	22.4	1	118.0	1	30.3
	8	1	112.3	1	63.0	1	481.6	1	22.4	1	115.3	1	29.6
	10	1	106.8	1	59.9	1	481.6	1	22.4	1	114.1	1	29.2
0.05	2	0.95	390.3	0.95	221.5	0.95	622.7	0.95	31.0	0.95	246.4	0.95	64.3
	4	1	192.6	1	108.5	0.999	629.3	0.999	29.3	0.999	165.5	0.999	42.62
	6	1	160.1	1	89.9	1	629.8	1	29.3	1	154.4	1	39.6
	8	1	146.9	1	82.4	1	629.9	1	29.3	1	150.8	1	38.7
	10	1	139.7	1	78.3	1	629.9	1	29.3	1	149.2	1	38.3
0.01	2	0.95	615.2	0.95	349.2	0.95	981.5	0.95	46.9	0.95	388.4	0.95	101.3
	4	1	297.9	1	167.9	0.999	974.0	0.999	45.4	0.999	256.1	0.999	65.9
	6	1	247.5	1	139.1	1	974.2	1	45.3	1	238.8	1	61.3
	8	1	227.2	1	127.5	1	974.2	1	45.3	1	233.2	1	59.8
	10	1	216.0	1	121.1	1	974.2	1	45.3	1	230.7	1	59.2

Table 11. The results of acceptance and rejection limit lines for SISP under 10th percentile BTXII model with parameter $\hat{\beta} = 5.47$ and $\hat{k} = 0.08$.

n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R	n	x_A	x_R
1	#	1	11	#	1	21	#	2	31	0	2	41	0	2	51	1	3
2	#	1	12	#	1	22	#	2	32	0	2	42	0	2	52	1	3
3	#	1	13	#	1	23	#	2	33	0	2	43	0	2	53	1	3
4	#	1	14	#	1	24	#	2	34	0	2	44	0	2	54	1	3
5	#	1	15	#	1	25	#	2	35	0	2	45	1	2	55	1	3
6	#	1	16	#	1	26	#	2	36	0	2	46	1	2	56	1	3
7	#	1	17	#	1	27	#	2	37	0	2	47	1	2	57	1	3
8	#	1	18	#	2	28	#	2	38	0	2	48	1	2	58	1	3
9	#	1	19	#	2	29	#	2	39	0	2	49	1	3	59	1	3
10	#	1	20	#	2	30	0	2	40	0	2	50	1	3	60	1	3

Table 12. Comparison of SISP, SASP, DASP, and RASP for ϖ_1 = 2.0.

C^*	ϖ	$\beta = 0.85$ and $k = 5.49$							
		$\mathcal{L}_{0.10}$ = 0.5				$\mathcal{L}_{0.10}$ = 1.0			
		(ASN) SISP	(ASN) SASP	(ASN) DASP	(ASN) RASP	(ASN) SISP	(ASN) SASP	(ASN) DASP	(ASN) RASP
0.25	2	166.9	361	347.01	674.9	94.7	205	197.17	213.4
	4	87.2	109	99.01	90.91	49.1	62	67.82	58.71
	6	72.6	89	77.06	68.33	40.8	51	43.80	38.84
	8	66.6	68	57.53	43.59	37.4	39	33.19	24.82
0.10	2	293.7	*	*	653.5	166.7	*	*	386.07
	4	147.2	183	173.77	129.8	83.0	116	106.94	74.82
	6	122.4	139	112.04	82.92	68.8	78	69.81	47.22
	8	112.3	116	100.18	82.92	63.0	65	56.53	46.64
0.05	2	390.3	*	*	1610.9	221.5	*	*	995.94
	4	192.6	250	231.23	144.70	108.5	142	131.44	82.17
	6	160.1	182	162.40	112.20	89.9	103	92.45	63.46
	8	146.9	134	117.60	84.94	82.4	76	66.77	48.16
0.01	2	615.2	*	*	1401.4	349.2	*	*	1345.9
	4	297.9	252	*	210.09	167.9	20	200.98	122.35
	6	247.5	226	229.52	144.47	139.1	142	143.28	81.50
	8	227.2	220	201.86	116.78	127.5	127	115.10	66.06

Continued on next page

		$\beta = 5.47$ and $k = 0.08$							
		$\mathcal{L}_{0.10} = 0.5$				$\mathcal{L}_{0.10} = 1.0$			
$\mathcal{C}^*$	ϖ	(ASN) SISP	(ASN) SASP	(ASN) DASP	(ASN) RASP	(ASN) SISP	(ASN) SASP	(ASN) DASP	(ASN) RASP
0.25	2	266.2	−	456.33	526.3	13.2	−	27.98	24.82
	4	284.1	−	457.56	↑	13.3	−	34.90	↑
	6	285.3	−	454.55	↑	13.3	−	29.35	↑
	8	285.5	−	457.82	↑	13.3	−	38.07	↑
0.10	2	468.6	−	625.44	662.7	23.3	−	45.58	31.23
	4	480.8	−	626.82	↑	22.4	−	32.54	↑
	6	481.5	−	633.58	↑	22.4	−	39.78	↑
	8	481.6	−	625.91	↑	22.4	−	32.31	↑
0.05	2	622.7	−	745.98	760.3	31.0	−	46.76	34.94
	4	629.3	−	742.55	↑	29.3	−	50.26	↑
	6	629.8	−	742.24	↑	29.3	−	40.65	↑
	8	629.9	−	746.79	↑	29.3	−	46.68	↑
0.01	2	981.5	−	1016.6	1016.4	46.9	−	66.03	47.05
	4	974.0	−	1015.2	↑	45.4	−	58.98	↑
	6	974.2	−	1018.2	↑	45.3	−	62.16	↑
	8	974.2	−	1015.8	↑	↑	−	53.10	↑
		$\beta = 2$ and $k = 2$							
		$\mathcal{L}_{0.10} = 0.5$				$\mathcal{L}_{0.10} = 1.0$			
$\mathcal{C}^*$	ϖ	(ASN) SISP	(ASN) SASP	(ASN) DASP	(ASN) RASP	(ASN) SISP	(ASN) SASP	(ASN) DASP	(ASN) RASP
0.25	2	105.3	×	×	146.78	27.5	×	×	45.55
	4	74.9	×	×	94.99	19.3	×	×	24.82
	6	70.0	×	×	↑	18.0	×	×	↑
	8	68.4	×	×	↑	17.5	×	×	↑
0.10	2	185.4	×	×	233.55	48.4	×	×	61.39
	4	126.5	×	×	118.98	32.6	×	×	31.23
	6	118.0	×	×	↑	30.3	×	×	↑
	8	115.3	×	×	↑	29.6	×	×	↑
0.05	2	246.4	×	×	264.24	64.3	×	×	70.04
	4	165.5	×	×	135.64	42.6	×	×	34.94
	6	154.4	×	×	↑	39.6	×	×	↑
	8	150.8	×	×	↑	38.7	×	×	↑
0.01	2	388.4	×	×	329.14	101.3	×	×	87.11
	4	256.1	×	×	181.18	65.9	×	×	47.05
	6	238.8	×	×	↑	61.3	×	×	↑
	8	233.2	×	×	↑	59.8	×	×	↑

5.1. First experiment

A company manufactures electronic components and needs to test whether the components meet the reliability requirement of the 10th percentile life of 4000 h. The company wants to design a sequential sampling plan based on a time-truncated life test, where the test is stopped after a fixed time if a decision can be made early. Assume the lifetime of the components follows BTXII distribution with the shape parameters β = 0.75 and k = 3. The test will be stopped at 1000 hours (i.e., all items will be tested for 1000h or until failure, whichever comes first). The manufacturers need to inspect a batch to ensure the 10th percentile life is 4000 h (the minimum AQL), while the client wants to take the risk of accepting the batch with the 10th percentile value of 1000h. The risk of rejecting a batch that meets the required 10th percentile life, typically set to 0.05 while the risk of accepting a batch that does not meet the required 10th percentile life, typically set to 0.75. Considering this data, it found that $\mathcal{L}_{0.10}$ = 1.0, ϖ_1 = 4.0, ϖ_2 = 1.0, $\boldsymbol{\mathcal{U}}^0_{0.10}$= 4000, $\mathcal{P}^*$ = 0.05 and $\mathcal{C}^*$ = 0.25. To set the acceptance and rejection limit lines, sequential sampling utilizes statistical criteria based on the stipulated errors ($\mathcal{P}^*$ and $\mathcal{C}^*$). The decision criteria are listed below:

i. The lot is accepted if the cumulative non-conforming units indicate that the 10th percentile life is equal to or greater than 4000h.
ii. The lot is rejected if the cumulative non-conforming units suggest that the 10th percentile life is less than 1000 hours.
iii. Continue the sampling process if the cumulative data does not lead to a clear decision.

The initial operation of designing a SISP in this test is to compute calculate π_1, π_2. Equation (4) shows that π_1 and π_2 have values of 0.037 and 0.1, respectively. From the Eqs (7)–(10), the values of ϱ, $\hbar_1$, $\hbar_2$ and $\mathcal{S}$ are 1.063, 1.256, 2.548, and 0.064, respectively. The acceptance limit line (x_A) and rejection limit line (x_R) are as follows:

$$x_A = 0.064\,n - 1.256, \tag{14}$$

$$x_R = 0.064\,n + 2.548, \tag{15}$$

Table 2 gives the results of acceptance and rejection limit lines for SISP under 10th percentile BTXII model with parameter β = 0.75 and k = 3. Table 3 below presents the OC value and ASN for SISP under 10th percentile BTXII model at β = 0.75, k = 3 and ϖ_1 = 2.0. Table 4 shows the OC value and ASN for SISP under 10th percentile BTXII model at β = 0.75, k = 3 and ϖ_1 = 4.0. The results of x_A and x_R for the SISP are illustrated in Table 2. For example, take the situation of obtaining the acceptance and rejection limit line for n = 30. Substituting n = 30 into Eqs (14) and (15) gets the following:

$$x_A = 0.064\,(20) - 1.256 = 0.024,$$

$$x_R = 0.064\,(20) + 2.548 = 3.828.$$

It is noted that from Table 2, the cells with (#) indicate that the acceptance is not allowed at stage i. Also, both the acceptance and the rejection limits must be integers. Therefore, the values of x_A and x_R are rounded to the nearest whole. Hence, the acceptance and rejection limit for n = 20 are 0 and 4, respectively. In light of this outcome, for n = 20, if the aggregate number of non-conforming item until this phase is zero, accept the batch; if the aggregate number of recorded non-conforming item up to this point is 1, 2, or 3, the sampling procedure ought to be continued; otherwise, if the total number of non-conforming items until this phase is four or more, reject the batch.

5.2. Second experiment

A company produces car batteries and claims that their batteries have a mean life of 4000 h (the target median life). To verify this claim, the company will conduct a time-truncated life test using a sequential sampling plan. The test will stop when either a decision is reached or will stop at 1000 h. The battery lifetimes follow a BTXII distribution with the shape parameters β = 0.75 and k = 3. The batch is acceptable if the median life is 4000 h or more (AQL), the manufacturer wants the risk of rejecting a good batch be 5%. The batch is rejected if the mean life is 1000h or less (LTPD), the customer wants the risk of rejecting a good batch be 25%. The data indicates that $\mathcal{L}_{0.5}$ = 1.0, ϖ_1 = 4.0, ϖ_2 = 1.0, $\mathcal{P}^*$ = 0.05 and $\mathcal{C}^*$ = 0.25. The initial process in creating a SISP in the above experiment is to calculate π_1, π_2. According to Eq (4), the values of $\pi_1$and π_2 are 0.232 and 0.5. From the Eqs (7–10), the values of ϱ, $\hbar_1$, $\hbar_2$ and $\mathcal{S}$ are 1.198, 1.114, 2.260 and 0.358, respectively. The acceptance limit line (x_A) and the rejection limit line (x_R) are as follows:

$$x_A = 0.358\ n - 1.114, \tag{16}$$

$$x_R = 0.358\ n + 2.260. \tag{17}$$

Table 5 gives the results of acceptance and rejection limit lines for SISP under 50th percentile BTXII model with parameter β = 0.75 and k = 3. Table 6 presents the OC value and ASN for SISP under median BTXII model at β = 0.75, k = 3 and ϖ_1 = 2.0. Table 7 presents the OC value and ASN for SISP median BTXII model at β = 0.75, k = 3 and ϖ_1 = 4.0. The outputs of x_A and x_R for the SISP are illustrated in Table 5. Take the example of calculating the acceptance and rejection limit line for n = 20. Substituting n = 20 in (16) and (17) gets the following:

$$x_A = 0.358\ (20) - 1.114 = 6.046,$$

$$x_R = 0.358\ (20) + 2.260 = 9.42.$$

This example shows how a sequential sampling strategy based on a time- amputated life test works. By testing 20 batteries, computing the aggregate number of observed nonconforming items, and comparing it to pre-determined decision boundaries, the corporation may swiftly and efficiently decide whether to accept or reject a batch of batteries. Based on Table 5, the acceptance and rejection line limits for n = 20 are 6 and 9, respectively. The batch is accepted if the aggregate number of nonconforming items until this phase is 6 items. If the aggregate number of recorded flaws up to this phase is 7 or 8, the sampling process must continue. The batch is rejected if the aggregate number of nonconforming items until this phase is more than or equal to 9 items.

5.3. Third experiment

The researchers in [23] had provided the repetitive acceptance sampling plans for BTXII distribution with shape parameters β = 2 and k = 2. Suppose that the quality controller intends to establish the genuine unknown 10th lifetime $\boldsymbol{\mathcal{U}}_p^0$ = 1000 hours. Consider the customer needs the error of accepting the batch with the 10th percentile of 1000 h to be less than 25%, whereas the manufacturer needs the error of rejecting the batch with the 10th percentile of 2000 to be less than 5%. This data leads to $\mathcal{L}_{0.10}$ = 1.0, $\mathcal{C}^*$ = 0.25, $\mathcal{P}^*$ = 0.05 ϖ_1 = 2.0, ϖ_2 = 1.0. For SISP, according to mechanism 1 for the above data the boundary lines are:

$$x_A = 0.056\ n - 0.949,$$

$$x_R = 0.358\ n + 1.926.$$

Based on mechanism 1, the ASN for SISP is 27.5 represented in Table 10. The results of x_A and x_R for the SISP under BTXII distribution as $\beta = 2$ and $k = 2$ are presented in Table 8. If the batch is not decided until item 17, item 18 is examined for $\mathcal{U}^0_{0.10}$ = 2000h. The batch is rejected if the aggregate number of nonconforming items exceeds three items, accepted if the aggregate number of nonconforming items up to this point is zero.

This illustrative experiment shows how a sequential sampling plan based on a time-amputated life test is designed, focusing on decision-making to assess the reliability of products. The key idea is to make a decision based on cumulative failure data, comparing it with predetermined acceptance and rejection criteria, while maximizing efficiency in testing. According to Table 8, the acceptance and rejection limit lines for n = 20 are 0 and 3, respectively. For n = 20, the batch is approved if the aggregate number of nonconforming items at this phase eqal to zero. The sampling process continues if the aggregate number of nonconforming up to this phase is one or two items. The batch is rejected if the aggregate number of nonconforming until this phase is three or more.

6. Industrial applications

The proposed sequential inspection sampling plan using two real-world data sets from [30] are discussed in this section. The first dataset includes the 19 times in minutes oil breakdown of an insulating fluid under high test voltage (34 kV). [31] first contributed this dataset, which is provided below (see Tables 13 and 14).

Table 13. Time (in minutes) for an insulating fluid to break down at a voltage of 34 kV.

0.19	0.31	0.78	0.96	2.78	3.16	4.15	4.67	4.85	6.50
7.35	8.01	8.27	12.06	31.75	32.52	33.91	36.71	72.89	

Table 14. The second dataset pertains to the first failure times of small electric carts employed for internal delivery and movement in a major industrial facility. And it is given as follows: lifetime (in months) to first failure of 20 electric carts.

0.9	1.5	2.3	3.2	3.9	5.0	6.2	7.5	8.3	10.4
11.1	12.6	15.0	16.3	19.3	22.6	24.8	31.5	38.1	53.0

The researchers in [32] discussed both datasets in their BTXII analysis of reliability. [30] found that the BTXII distribution fitted well for both of the above data sets applying the AIC measure and K-S statistics, and they generated maximum likelihood estimates (MLEs) of BTXII parameters. Many articles have utilized both data sets to investigate various elements of reliability. Readers can learn more about these data sets by reviewing the papers listed earlier.

6.1. Industrial example 1

The researchers in [32] introduced the MLEs of shape parameters of the BTXII model as $\hat{\beta}$ = 0.85 and $\hat{k}$ = 5.49, to estimate the lifespan of oil breakdown in an insulating fluid under high test voltage (34 kV). Assume the testers are interested in applying the proposed SISP to check if the genuine unidentified 10th percentile lifespan is more than the predefined life, $\mathcal{U}_{0.10}$ = 3.0 min. The customer's error is 25% when the genuine 10th percentile lifespan is 3.0 minutes, and the manufacturer's error is 5% when the genuine 10th percentile lifespan is 6.0 minutes. The experiment would be stopped at 3.0

minutes, yielding the ratio $\mathcal{L}_{0.10} = 1.0$. This directs to ϖ_1 = 2.0, ϖ_2= 1.0, $\mathcal{P}^*$ = 0.05 and $\mathcal{C}^*$ = 0.25. For SISP, according to method 1 for above data the boundary lines are:

$$x_A = 0.077\, n - 2.194,$$

$$x_R = 0.077\, n + 4.450.$$

Table 9 presents the results of acceptance and rejection limit lines for SISP under 10th percentile BTXII model with parameter $\hat{\beta} = 0.85$ and $\hat{k} = 5.49$. Table 10 shows the OC value and ASN for SISP 10th percentile BTXII model at ϖ_1= 2.0 and $\mathcal{P}^*$= 0.05. Table 11 gives the results of acceptance and rejection limit lines for SISP under 10th percentile BTXII model with parameter $\hat{\beta}$ =5.47 and k ̂=0.08.

The results of x_A and x_R for the SISP under BTXII distribution as $\hat{\beta} = 0.85$ and $\hat{k} = 5.49$ are presented in Table 9. If the batch is not judged till item 28, item 29 is tested for $\mathcal{U}^0_{0.10}$ = 3.0 minutes. The batch is then approved if the aggregate number of nonconforming up to this time is zero, refused if the aggregate number of nonconforming exceeds seven items. Using mechanism 1, the ASN for SISP is 94.7, as shown in Table 10.

6.2. Industrial example 2

The researchers in [30] derived the MLEs of shape parameters of the BTXII distribution as $\hat{\beta} = 5.47$ and $\hat{k} = 0.08$, to estimate the lifespan of 20 miniature electric carts. Presume quality controller is interested in applying the proposed SISP to ensure the genuine unknown 10th percentile lifetime of a tiny electric carts for at least two months ($\mathcal{U}_{0.10}$ = 2 months). It is considered that the customer's error is 5% when the genuine 10th percentile lifespan is two months and 5% when the genuine 10th percentile lifespan is four months. Suppose that the experimenter wants to run the test within two months, which returns the ratio of $\mathcal{L}_{0.10} = 1.0$. This leads to $\mathcal{P}^*$ = 0.05, $\mathcal{C}^*$ = 0.05, ϖ_1 = 2.0, and ϖ_2 = 1.0. The boundary lines for SISP, according to mechanism 1 for the above data, are:

$$x_A = 0.032\, n - 0.937,$$

$$x_R = 0.032\, n + 0.937.$$

Based on mechanism 1, the ASN for SISP is 31.0, represented in Table 10. The results of x_A and x_R for the SISP under BTXII distribution as $\hat{\beta} = 5.47$ and $\hat{k} = 0.08$ are presented in Table 11. As an instance, if the batch is not judged up until item 29, item 30 is examined for $\mathcal{U}^0_{0.10}$ = 2 months. The lot is approved if the total number of defective items up to this phase is equal to zero; it is rejected if the total number of defective items exceeds two.

In the industrial examples in this Section, we adopted MLEs of β and k. Using these estimates, the authors design a plan (e.g., for acceptance of sampling). Since the plan is based on the assumption that the estimated parameters are exact, it introduces a layer of conditionality. That is, the resulting plan is conditional on the estimated parameters and might not perform exactly as intended if the estimates deviate from the true parameters. It is worth mentioning that this assumption can lead to slight discrepancies between the theoretical and practical performance of the plan according to the inherent uncertainty in parameter estimation. Such deviations are typically small, but their acknowledgment is important for rigorous analysis.

7. Comparison study

In this section, the comparative study among SASP, DASP, and RASP sampling plan and proposed SISP for BTXII are discussed in Table 12. Also, ASN helps balance the risks of accepting a defective batch or rejecting a good batch. It provides insights into how robust and reliable the sampling plan is in terms of minimizing these risks. In SASP, since the decision is made after inspecting exactly n units, the ASN is simply equal to n. This is because the sample size is fixed and there is no further inspection once the initial sample has been evaluated. Hence, no matter what the outcome is (accept or reject), you always inspect exactly n units, making the $ASN = n$. From Table 12, it is concluded that the proposed SISP has the best performance in comparison with the DASP and RASP based on ASN with parameter $\beta = 5.47$ and $k = 0.08$ for all quality level (ϖ). Also, the proposed SISP has smaller ASN than the RASP with parameter $\beta = 2$ and $k = 2$ for all quality level (ϖ) when $\mathcal{C}^* = 0.25$ but the RASP has the best performance in comparison with the SISP based on ASN when $\mathcal{C}^*$ = 0.10, 0.05 and 0.01 at $\varpi = 4$ for $\mathcal{L}_{0.10} = 0.5$ and 1.0. When the tester uses the parameters $\beta = 0.85$ and $k = 5.49$ the RASP has smaller ASN than the SISP, SASP, and DASP when $\varpi \geq 4$ for $\mathcal{C}^*$ = 0.10, 0.05 and 0.01 at $\mathcal{L}_{0.10} = 0.5$ and 1.0. The proposed SISP has smaller ASN than the SASP, DASP and RASP when $\varpi \leq 4$ for $\mathcal{C}^* = 0.25$. The SASP has the best performance in comparison with the SISP, DASP and RASP based on ASN when $\varpi = 4$ for $\mathcal{C}^* = 0.01$. According to this the proposed SISP provides smaller ASN than the single, double, and repetitive acceptance sampling plans. Hence, the proposed SISP is more efficient and economical than the single, double and repetitive acceptance sampling plans in terms ASN. In summary, the ASN is a key metric in evaluating acceptance sampling that helps in understanding and optimizing the efficiency of the sampling process. By focusing on ASN, organizations can better manage their quality control processes and make more informed decisions regarding their acceptance sampling plans.

Notes:

i. "*" indicates value in this cell is not available in research paper introduced by [30].

ii. "↑" indicates that the same value of the above cell applies.

iii. "−" indicates that the research paper introduced by [30] did not include a study of these parameters.

iv. "×" indicates that the research paper introduced by [23] did not include a study of these plans of at these parament

Sequential sampling and repetitive sampling plans differ significantly not only in their decision-making processes but also in terms of cost efficiency. As highlighted in the paper, sequential sampling plans (SISP) offer a more dynamic approach by enabling decisions to be made after each item is inspected, which often results in a lower ASN. This reduction in ASN directly translates to lower inspection costs, reduced time for testing, and minimized resource utilization, making SISP particularly suitable for scenarios with time constraints or expensive testing procedures. On the other hand, repetitive sampling plans (RSP), while providing structured acceptance and rejection criteria at fixed stages, may require multiple samples to reach a conclusive decision. Although RSP can offer higher accuracy and confidence in quality assessments, they may incur higher overall costs due to the need for repeated inspections. The trade-off between cost and precision must be carefully considered when selecting an appropriate sampling strategy. The paper demonstrates through numerical examples and industrial applications that SISP generally outperforms repetitive sampling in terms of cost-effectiveness, especially when early decisions can be made without compromising the risks associated with accepting defective batches or rejecting good ones. However, in cases where stringent quality standards demand repeated verification, such as in pharmaceuticals or high-precision manufacturing, the additional costs incurred by RSP may be justified.

8. Conclusions

In this study, the lifespan of product is considered a random variable following the BTXII distribution. Because the lifespan of products usually is high, the amputation test is utilized for assessing and the real percentile lifetime. Amputation life test refers to the stopping rule for the life test, which can be based on a fixed time or the number of failures observed. The goal is to understand how long the item will last before failure. In such tests, truncation implies that the test is ended after a predetermined time or after a set number of failures, even if some units are still functioning. Consequently, the sequential inspection sampling plans have been introduced based on the percentiles of BTXII when the life test is amputated at a pre-determined time. A sequential sampling plan enables continuous evaluation of items one by one rather than committing to a fixed sample size upfront. Testing continues until a clear decision can be made, either to accept or reject the lot based on the performance of the tested items. The optimum parameters are calculated for the proposed plan by satisfying both consumers and producer's risks. A procedure is presented for computing characteristic functions and ASN in the SISP plan. The proposed sequential sampling plan has been compared with the corresponding single, double and repetitive sampling plan. At the parameters $\hat{\beta}$ = 5.47 and $\hat{k}$ = 0.08, the Comparisons have indicated that the SISP is more efficient than the DASP and RASP in terms ASN. Moreover, the proposed SISP are more efficient than SASP, and DASP when the parameters $\hat{\beta}$ = 0.85 and $\hat{k}$ = 5.49 except for one point only, which is that the SASP is more efficient than the SISP when consumer's risk ($\mathcal{C}^*$) = 0.01 and quality ratio (ϖ) = 4. At parameters $\hat{\beta}$ = 0.85 and $\hat{k}$ = 5.49, the SISP is more efficient than the RASP in terms ASN when quality ratio (ϖ) < 4. The proposed SISP is more efficient than RASP in terms ASN when the parameters β = 2 and k = 2, except the RASP is more efficient than the SISP when quality ratio (ϖ) = 4 for consumer's risk ($\mathcal{C}^*$) < 0.25. This comparison study shows that the suggested SISP can greatly lower the ASN and has a good performance compared to the other plans. Also, the sequential sampling plan gives significant results than repetitive acceptance sampling plan in terms of reducing the ASN. Some useful tables are provided and applied to establish proposed sampling plans for many examples.

Sequential sampling plans are highly efficient in terms of resource utilization. Unlike fixed-sample plans, SISP evaluates data as samples are examined, enabling dynamic decisions on accepting, rejecting, or continuing inspection based on cumulative results. This feature reduces the ASN, minimizing time, cost, and material waste during amputated life tests.

As a significant potential future research direction, the effect of cost on sequential sampling and repetitive sampling should be explored in greater depth, particularly under emerging applications. This includes analyzing how varying inspection costs, equipment usage, labor expenses, and time constraints influence the efficiency and feasibility of each sampling method. In the future, researchers could develop comprehensive cost models that integrate both fixed and variable costs associated with sampling plans. These models would help identify the most economical approach under different industrial conditions. Additionally, comparing cost implications across industries, such as healthcare, automotive, or electronics, can provide tailored guidelines for quality control practitioners.

Author contributions

The authors contributed equally and significantly to all stages of this article. All authors of this article contributed equally.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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Appendix

Table A. Notations.

Notation	Description
AIC	Akaike Information Criterion
AQL	Acceptable Quality Level
ASN	Average Sample Number
CDF	Cumulative Distribution Function
DASP	Double Acceptance Sampling Plan
K-S statistics	Kolmogorov-Smirnov statistics
LTPD	Lot Tolerance Percent Defective
MLES	Maximum likelihood estimates
OC	Operation Characteristic
PDF	Probability Distribution Function
RASP	Repetitive Acceptance Sampling Plan
RQL	Rejectable Quality Level
SASP	Single Acceptance Sampling Plan
SISP	Sequential Inspection Sampling Plan
SPRT	Sequential Probability Ratio Test

Table B. Symbols.

Symbol	Definition
α	Scale parameter of the Burr XII distribution
β and k	Shape parameters of the Burr XII distribution
t_0	Truncation time, the predetermined cutoff for testing
$\mathcal{L}_\rho$	Coefficient factor. Used in setting the test termination time $t_0 = \mathcal{L}_\rho \mu_\rho^0$.
ϖ	Quality ratio of each item, used to adjust decision criteria in sequential plans.
Π	Failure probability at time t_0
$\mathcal{P}^*$	Manufacturer's error, the probability of rejecting a lot with no defects
$\mathcal{C}^*$	Customer's error, the probability of accepting a lot with defects
$d_{(n)}$	The number of defects observed during inspection of the first n unit
x_A	Acceptance limit line
x_R	Rejection limit line
n	Sequential unit taken from the lot of sample items taken
S	Equilibrium quality or slope of the acceptance and rejection boundaries
δ	Decision criterion. A parameter used to define the acceptance or rejection boundaries
L_a	Probability of acceptance lot based on the nonconforming proportion
μ_ρ^0	The ρ^{th} percentile lifetime,