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Research article

Input-to-state stability of the electro-hydraulic servo system with a backstepping-based impulsive correction control

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Abstract: In this paper, a backstepping-based impulsive correction control is proposed to solve the position control problem of the electro-hydraulic servo system (EHSS), where the input-to-state stability (ISS) of the error system is illustrated. A simplified mathematical model of the EHSS is developed, and a backstepping technique is adopted to develop the novel controller. A relationship between the impulsive control gain and the impulsive sequence is established. Compared with the existing results, the proposed controller can significantly reduce the control consumption. Finally, a numerical simulation is conducted to show the effectiveness of the theoretical results.

Keywords: electro-hydraulic servo system; input-to-state stability; impulsive correction control; backstepping-based control; Lyapunov method
Mathematics Subject Classification: 35R12, 49N25

1. Introduction

In recent years, the electro-hydraulic servo system (EHSS) has been rapidly developed in industrial applications due to the high load capability, high flexibility, fast response, and high power-to-weight ratio [1–4]. However, there are still many drawbacks in application, for example, the uncertain external disturbance [5], the nonlinearity of the mathematical model [6], time delay [7], and the massive control cost [8]. These mathematical properties bring difficulties to designing the controller. While the application of the EHSS is becoming complex, some modern controllers have been proposed.

To address these complex nonlinearities and uncertainties, advanced control strategies have been increasingly investigated for the EHSS. Intelligent control techniques, such as fuzzy logic control [9] and neural network-based control [10], have been applied due to their model-free nature and ability to approximate complex nonlinear functions.

In addition, backstepping is also a common method for designing controllers. In [11], authors provided a controller based on backstepping and an extended differentiator. In [12], a backstepping method was used to design a controller for electro-hydraulic servos on Romanian military IAR 99s to guarantee asymptotic stability of the position error. On the other hand, sliding-mode control can perfectly resist the matched external disturbance, which makes it appropriate to adopt for the EHSS. A sliding-mode controller was investigated in [13], where nonlinear unknown parameters was considered. In [14], an H_∞-based adaptive fractional-order sliding mode controller was developed for position tracking control of nonholonomic mobile robots. However, the controllers designed in these results are continuous, which depend on continuous feedback of the state and have much more control cost. They can neither analyze the relationship of the state nor the external disturbance. There are many ways for saving the control cost. Impulsive correction control is a kind of hybrid control, which exhibits the advantages of both impulsive control and continuous control [15–17]. It has been widely investigated in many areas [18, 19]. The nonlinear system with impulses exhibits continuous dynamics and discontinuous dynamics. However, the application of impulsive control in the EHSS still remains unexplored.

The disturbance cannot be avoided in the real world, especially the disturbance in stochastic phenomenon. It is critical to estimate the system state by the external disturbance. In [21], authors investigated the disturbance of observer-based fixed-time event-triggered control for the networked electro-hydraulic systems. A variable-bandwidth extended state observer was developed to further estimate the disturbance in [22]. Input-to-state stability (ISS) is a concept prompted by Sontag in [20] to characterize the effects of external inputs to the stability of control systems, which is used widely in many areas [23, 24]. If a system is ISS, then the state of the system is bounded. In terms of the dynamics system with impulsive control, the notion of the ISS was extended to the impulsive system in [25]. Readers are referred to [26, 27] for more results on the ISS of the impulsive system. As a useful tool, this notion is widely used in the EHSS. In [28], authors investigated the ISS property of the electro-hydraulic system with nonlinear control and extended state observer. The position control was achieved. The ISS was also obtained with a reference model-based disturbance observer (RMDO) control strategy in [29]. These results show that ISS is an important notion to characterize the effects of disturbance of the EHSS.

Inspired by the discussion above, we propose a backstepping-based impulsive correction control for position tracking of the EHSS. The main contribution of this paper can be summarized as follows: (1) A simplified mathematical model of the EHSS is proposed by taking full consideration of the physical characters of the EHSS. (2) Based on the mathematical model, a backstepping-based impulsive correction control is developed, where a relationship between impulsive control gain and the impulsive sequence is established. Compared with the results in [12, 28], the controller in this paper has the advantage of reducing the control cost. (3) The ISS of the EHSS with the backstepping-based impulsive correction control is investigated. The results show that the EHSS with the proposed controller is robust with respect to the external disturbance. A simulation of the result is provided to illustrate the effect of the theoretical results.

The rest of this paper is organized as follows: The notion of the ISS and the mathematical model of the EHSS are provided in Section 2. In Section 3, we explicitly introduce the design of backstepping-based impulsive correction control. A numerical simulation is conducted in Section 4, which shows the effectiveness of the theoretical results. Finally, conclusions are given in Section 5.

Notation. In this paper, symbols $\mathbb{Z}_+, \mathbb{R}, \mathbb{R}_+, \mathbb{R}^n$ denote the set of all nonnegative integers, the set of all real numbers, the set of all positive real numbers, and the n dimensional Euclidean space equipped with the Euclidean norm $|\cdot|$, respectively. A function $\alpha : [0,\infty) \to [0,\infty)$ is of class $\mathcal{K}$ if α is continuous, strictly increasing, and $\alpha(0) = 0$. In addition, if α is unbounded, it is of class $\mathcal{K}_\infty$. A function $\beta : [0,\infty) \times [0,\infty) \to [0,\infty)$ is of class $\mathcal{KL}$ if $\beta(\cdot, t) \in \mathcal{K}$ for fixed $t \geq 0$ and $\beta(r,t)$ decreases to 0 as $t \to 0$ for fixed $r \geq 0$.

2. Preliminary

2.1. Input-to-state stability

In this section, some definitions and lemmas of the ISS are provided. Consider the following nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)), t \geq t_0, x(t_0) = x_0, \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a continuou function, and $u(t) \in \mathbb{R}^m$ is the external disturbance.

Definition 1 ([20]). *System* (2.1) *is* ISS *if there functions $\beta \in \mathcal{KL}, \gamma \in \mathcal{K}_\infty$ such that for each $t_0 \geq 0, x_0 \in \mathbb{R}^n$ and each input u, the solution satisfies*

$$|x(t)| \leq \beta(|x_0|, t - t_0) + \gamma(|u|_{[t_0,t]}),$$

for all $t \geq t_0$, where $|\cdot|_J$ denotes the supremum norm on the interval J.

Definition 2 ([20]). *A continuous positive definite function $V : \mathbb{R}^n \to \mathbb{R}_+$ is called an* ISS*-Lyapunov function, if there exist functions $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathcal{K}_\infty$ such that for all $x \in \mathbb{R}^n, u \in \mathbb{R}^m$, the following conditions hold:*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \tag{2.2}$$

$$\dot{V} \leq -\alpha_3(|x|) + \alpha_4(|u|). \tag{2.3}$$

Lemma 1 ([30]). *System* (2.1) *is* ISS *if it admits an* ISS*-Lyapunov function.*

Consider the following cascade system:

$$\dot{x}(t) = f_1(x, y), \tag{2.4}$$

$$\dot{y}(t) = f_2(y, u), \tag{2.5}$$

where the state $x, y \in \mathbb{R}^n$, the external disturbance $u \in \mathbb{R}^m$, and f_1, f_2 are continuous nonlinear functions. The initial values are defined as $x(t_0) = x_0, y(t_0) = y_0$. Then, the following Lemma provides a result on the ISS character of the cascade system.

Lemma 2 ([30]). *The cascade system of* (2.4) *and* (2.5) *is* ISS *if the subsystems* (2.4) *and* (2.5) *are* ISS*, respectively.*

Lemma 2 indicates that if two subsystems are ISS, then the cascade system is also ISS. The coupling effects have been discussed in [30]. The coupling effect of the impulsive control system is completely the same, which is the theoretical foundation of this paper.

2.2. Mathematical model of EHSS

In this part, we will establish the mathematical model of the EHSS, which is exhibited in Figure 1. The physical law used can be found in [2, 3]. The proportional valve is considered first, which can be described by a mass-spring-damper system.

$$\ddot{y}_v + 2\xi\omega\dot{y}_v + \omega^2 y_v = k\omega^2 u, \tag{2.6}$$

where y_v is the position of the spool valve, ξ is the damping ratio, k is the control gain, ω is the vibration frequency, and u is the control input voltage. For convenience, we assume that the spool valve position can be perfectly controlled. In other words, refer to [13, 31], where the position y_v is in proportion to the input u, i.e., there exists a constant $\lambda > 0$ such that $y_v = \lambda u$. Although this assumption simplifies the relationship between y_v and u, it is acceptable for the stability analysis. Then, the flow of the EHSS can be expressed by the control voltage.

$$\begin{aligned} Q_1 &= C_d \omega \lambda u \sqrt{\frac{2}{\rho} P_p}, \\ Q_2 &= C_d \omega \lambda u \sqrt{\frac{2}{\rho} P_r}, \end{aligned} \tag{2.7}$$

where C_d is the valve coefficient of discharge and ω is the valve orifice area gradient. The definition of P_p, P_r is provided as follows:

$$\begin{cases} P_p = |p_s - p_1|, \text{ if } u \geq 0, \\ P_p = |p_1 - p_a|, \text{ if } u < 0, \end{cases}$$

$$\begin{cases} P_r = |p_2 - p_a|, \text{ if } u \geq 0, \\ P_r = |p_s - p_2|, \text{ if } u < 0, \end{cases}$$

where p_1, p_2 are the pressures, p_s is the supply pressure, and p_a is the tank pressure, all of which are shown in Figure 1. In this paper, we neglect the internal and external leakages. Thus, the pressure of the compressible fluid volumes can be expressed as

$$\begin{aligned} \dot{p}_1 &= \frac{\beta}{V_{01} + A_1 x_1}(Q_1 - A_1 \dot{x}_1), \\ \dot{p}_2 &= \frac{\beta}{V_{02} - A_2 x_1}(-Q_2 + A_2 \dot{x}_1), \end{aligned} \tag{2.8}$$

where β is the fluid bulk modulus, A_1, A_2 are annulus areas of the piston and rod side of the cylinder, and V_{01}, V_{02} are the initial volumes of the cylinder. The part of the mechanical is expressed by the dynamic equation

$$\ddot{x} = \frac{1}{m}(p_1 A_1 - p_2 A_2 - b\dot{x} - cx - F_L), \tag{2.9}$$

where m is the mass of the piston and the load, b, c are parameters of the actuator and the external load and F_L is the uncertain interference. With the definition $x_1 = x, x_2 = \dot{x}, x_3 = \ddot{x}$, it follows from (2.7)–(2.9) that

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= x_3,\\ \dot{x}_3 &= \frac{1}{m}\left[\zeta_1 u - \zeta_2 x_2 - b x_3 - \dot{F}_L\right],\end{aligned} \tag{2.10}$$

where

$$\zeta_1 = \lambda\beta C_d\omega\left(\frac{A_1\sqrt{P_p}}{V_{01}+A_1x_1}+\frac{A_2\sqrt{P_r}}{V_{02}-A_2x_1}\right)\sqrt{\frac{2}{\rho}},$$
$$\zeta_2 = \beta\left(\frac{A_1^2}{V_{01}+A_1x_1}+\frac{A_2^2}{V_{02}-A_2x_2}\right)+c.$$

In this paper, we assume that the position, velocity, and acceleration can be monitored with feedback by hardware. The control object is to steer the piston position x_1 to the reference signal x_{1d}. We assume that all of the signals involved in this paper are right continuous and have left limits at all times. $(\cdot)^-$, $(\cdot)^+$ denote the left-limit and right-limit operators, respectively.

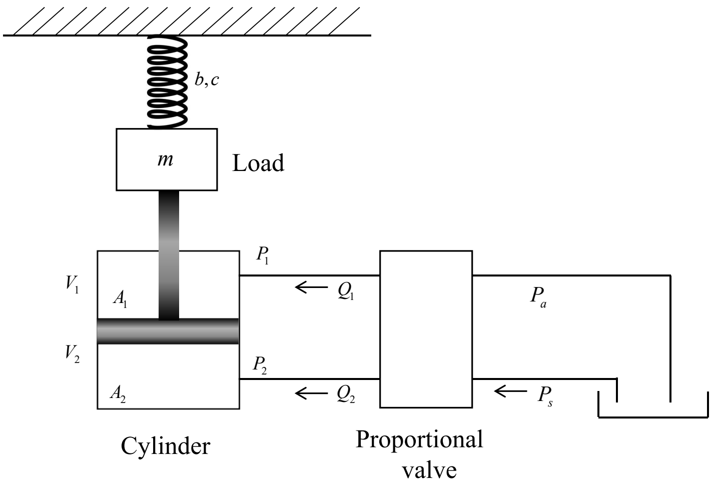

Figure 1. The diagram of the EHSS for mathematical model.

3. Main results

3.1. Design of backstepping-based control

In this section, we consider the x_1, x_2-system first, which is provided that

$$\dot{x}_1 = x_2, \tag{3.1}$$
$$\dot{x}_2 = x_3. \tag{3.2}$$

The object of the backstepping control is to design an appropriate virtual control x_{3d}. The real state x_3 can be expressed as $x_3 = x_{3d} + x_{3e}$, where x_{3e} is the error. Then, the x_1, x_2-system can be obtained that

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= x_{3d} + x_{3e}.\end{aligned} \tag{3.3}$$

Assume the reference position signal for the EHSS is x_{1d}. Then, the candidate Lyapunov function can be defined as

$$V_1 = \frac{1}{2}(x_1 - x_{1d})^2.$$

Differentiating it gives

$$\dot{V}_1 = (x_1 - x_{1d})(x_2 - \dot{x}_{1d}).$$

Then, we can choose

$$x_{2d} = -\frac{1}{2}(x_1 - \dot{x}_{1d}) + \dot{x}_{1d}. \tag{3.4}$$

Consider a new candidate Lyapunov function

$$V_2 = V_1 + \frac{1}{2}(x_2 - x_{2d})^2.$$

By differentiating it, it can be obtained that

$$\begin{aligned}\dot{V}_2 &= (x_1 - x_{1d})(x_2 - \dot{x}_{1d}) + (x_2 - x_{2d})(x_3 - \dot{x}_{2d})\\ &= (x_1 - x_{1d})(x_2 - \dot{x}_{1d}) + (x_2 - x_{2d})(x_3 + \frac{1}{2}(\dot{x}_1 - \dot{x}_{1d}) - \ddot{x}_{1d}).\end{aligned}$$

Thus, we can deduce the virtual control that

$$x_{3d} = -\frac{1}{2}(x_2 - \dot{x}_{1d}) + \ddot{x}_{1d} - (x_2 - x_{2d}) - (x_1 - x_{1d}). \tag{3.5}$$

Now, we can analyze the x_1, x_2-system (3.3) by choosing the following Lyapunov function:

$$V_3 = \frac{1}{2}(x_1 - x_{1d})^2 + \frac{1}{2}(x_2 - x_{2d})^2.$$

The differentiate can deduce that

$$\begin{aligned}\dot{V}_3 &= (x_1 - x_{1d})(x_2 - \dot{x}_{1d}) + (x_2 - x_{2d})(x_{3e} + x_{3d} - \dot{x}_{2d})\\ &= (x_1 - x_{1d})(x_2 - \dot{x}_{1d}) + (x_2 - x_{2d})\left[(x_{3e} + x_{3d}) + \frac{1}{2}(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d})\right]\\ &= (x_1 - x_{1d})(x_2 - \dot{x}_{1d}) + (x_2 - x_{2d})x_{3e} + (x_2 - x_{2d})\left[x_{3d} + \frac{1}{2}(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d}\right].\end{aligned}$$

By substituting (3.5) into it, we have

$$\begin{aligned}\dot{V}_3 &= (x_1 - x_{1d})(x_2 - \dot{x}_{1d}) + (x_2 - x_{2d})x_{3e} + (x_2 - x_{2d})\left[-(x_2 - x_{2d}) - (x_1 - x_{1d})\right]\\ &\leq (x_1 - x_{1d})(x_2 - \dot{x}_{1d}) + \frac{1}{2}(x_2 - x_{2d})^2 + \frac{1}{2}x_{3e}^2 - (x_2 - x_{2d})^2 - (x_2 - x_{2d})(x_1 - x_{1d})\\ &\leq (x_1 - x_{1d})\left[(x_2 - \dot{x}_{1d}) - (x_2 - x_{2d})\right] - \frac{1}{2}(x_2 - x_{2d})^2 + \frac{1}{2}x_{3e}^2\\ &\leq (x_1 - x_{1d})\left[(x_2 - \dot{x}_{1d}) - x_2 - \frac{1}{2}(x_1 - x_{1d}) + \dot{x}_{1d}\right] - \frac{1}{2}(x_2 - x_{2d})^2 + \frac{1}{2}x_{3e}^2\\ &\leq -\frac{1}{2}(x_1 - x_{1d})^2 - \frac{1}{2}(x_2 - x_{2d})^2 + \frac{1}{2}x_{3e}^2\\ &\leq -V_3 + \frac{1}{2}x_{3e}^2.\end{aligned}$$

According to Lemma 1, the ISS of the error system with respect to x_{1d}, x_{2d} can be guaranteed.

3.2. Design of impulsive correction control

In this section, let x_{3e} denote the error between the dynamics of x_3 and virtual control signal x_{3d}, i.e., $x_{3e} = x_3 - x_{3d}$. The behaviors can be expressed as $\dot{x}_{3e} = \dot{x}_3 - \dot{x}_{3d}$. With impulsive correction, the error system can be written as

$$\dot{x}_{3e} = \frac{1}{m}(\xi_1 u - \xi_2 x_2 - bx_3 - \dot{F}_L) - \dot{x}_{3d}. \tag{3.6}$$

The impulsive correction controller is expressed as follows:

$$u = \frac{m}{\zeta_1}\left(\frac{1}{m}\zeta_2 x_2 + \frac{1}{m}bx_3 + \dot{x}_{3d} + \sum_{n\in\mathbb{Z}_+}(q-1)x_{3e}\delta(t-t_n)\right), \tag{3.7}$$

where δ is the Dirac function and $0 < q < 1$ is the impulsive control gain. Denote a class of impulsive sequence $\mathcal{F}$. A sequence of discrete time instants $\{t_n\} \in \mathcal{F}$ if it satisfies $0 = t_0 < t_1 < t_2 < ...$, $\lim_{n\to\infty} t_n = \infty$, and $0 < t_{n+1} - t_n \le \tau$ for all $n \in \mathbb{Z}_+$ and a positive constant τ. In this paper, we ignore internal and external leakage and Coulomb friction, keeping only viscous damping b and stiffness c. These two constants are used in the controller for feedback. Hence, the stability and the steady-state error bounds will not be affected by them.

3.3. Stability analysis

Theorem 1. *Error system* (3.6) *with the impulsive correction controller* (3.7) *is* ISS *over the class* $\mathcal{F}$ *if there exist constants* $0 < q < 1$, $\tau > 0$ *such that*

$$\frac{\ln q}{\tau} + 1 < 0. \tag{3.8}$$

Proof. Denote x_{3e} as the solution of system (3.6) with initial value $x_{3e0} = x_{3e}(t_0)$. Consider the candidate Lyapunov function

$$V_4 = \frac{1}{2}x_{3e}^2.$$

Obviously, the Lyapunov function is positive, and there exist functions $\alpha_1(x), \alpha_2(x) \in \mathcal{K}_\infty$ that

$$\alpha_1(|x_{3e}|) \le V_4 \le \alpha_2(|x_{3e}|).$$

If $t = t_n$, one can deduce that

$$V_4(t_n) = \frac{1}{2}q^2 x_{3e}(t_n^-) = q^2 V_4(t_n^-). \tag{3.9}$$

If $t \ne t_n$, it holds that

$$\begin{aligned} D^+V_4 &= x_{3e}\left[\frac{1}{m}(\xi_1 u - \xi_2 x_2 - bx_3 - \dot{F}_L) - \dot{x}_{3d}\right] \\ &= x_{3e}\left[\frac{1}{m}\left(\xi_2 x_2 + bx_3 + m\dot{x}_{3d} - \xi_2 x_2 - bx_3 - \dot{F}_L\right) - \dot{x}_{3d}\right] \\ &\le |x_{3e}|\left|\frac{1}{m}\dot{F}_L\right| \\ &\le V_4 + \frac{1}{2}\left|\frac{1}{m}\dot{F}_L\right|^2. \end{aligned}$$

Define $\epsilon = \frac{1}{m}\dot{F}_L$. For $|x_{3e}| \geq |\epsilon|$, we have

$$D^+V_4 \leq x_{3e}^2 = 2V_4, t \neq t_n, n \in \mathbb{Z}_+. \tag{3.10}$$

If $|x_{3e}| \geq |\epsilon|$ holds on a time interval $[t', t'']$, then suppose that there exists an impulsive sequence such that $t' = t_0 < t_1 < t_2 < t_3 < ... < t_n = t''$. For $[t_0, t_1)$, it follows from (3.10) that

$$V_4(t) \leq e^{2(t-t_0)}V_4(t_0). \tag{3.11}$$

For $[t_1, t_2)$, it follows from (3.9)–(3.11) that

$$\begin{aligned} V_4(t) &\leq e^{2(t-t_1)}V_4(t_1) \\ &\leq q^2 e^{2(t-t_0)}V_4(t_0). \end{aligned}$$

By induction and condition (3.8), we have

$$V_4(t) \leq \Xi V_4(t_0) \exp\left[-\eta(t - t_0)\right],$$

where $t \in [t', t'')$, the constant $\Xi = e^{-2\ln q}$, and $\eta = -(\frac{2\ln q}{\tau} + 2)$. According to (3.3), it holds for $t' \leq t \leq t''$ that

$$|x_{3e}(t)| \leq \alpha_1^{-1}\left(\Xi e^{-\eta(t-t_0)}\alpha_2\left(|x_{3e0}|\right)\right) = \beta(|x_{3e0}|, t - t_0), \tag{3.12}$$

where β is a $\mathcal{KL}$-class function. Define the ball around the origin $\mathcal{B} = \{x_{3e} : |x_{3e}| \leq |\epsilon|\}$. Define $\check{t}_1 = \inf\{t \geq t_0 : |x_{3e}| \leq |\epsilon|\}$. If $\check{t}_1 = \infty$, then system (3.6) is ISS. If $\check{t}_1 < \infty$, denote $\hat{t}_1 = \inf\{t > \check{t}_1, |x_{3e}| \geq |\epsilon|\}$. If $\hat{t}_1 = \infty$, then the system satisfies

$$|x_{3e}(t)| \leq \beta(|x_{3e}(t_0)|, t - t_0) + |\epsilon|_{[t_0,t]}, t \geq t_0, \tag{3.13}$$

which indicates the system is also ISS. If $\hat{t}_1 < \infty$, then one can obtain that $|x_{3e}(\hat{t}_1)| \leq |\epsilon(\hat{t}_1)|$. Assume the second time $x_{3e} \in \mathcal{B}$ is $\check{t}_2 = \inf\{t > \hat{t}_1, |x_{3e}| \leq |\epsilon|\}$. For $\hat{t}_1 \leq t \leq \check{t}_2$, we have

$$\begin{aligned} |x_{3e}(t)| &\leq \alpha_1^{-1}\left(\Xi e^{-\eta(t-\hat{t}_1)}\alpha_2\left(|x_{3e}(\hat{t}_1)|\right)\right) \\ &\leq \alpha_1^{-1}\left(\Xi e^{-\eta(t-\hat{t}_1)}\alpha_2\left(|\epsilon(\hat{t}_1)|\right)\right) \\ &\leq \alpha_1^{-1}\left(\Xi\alpha_2\left(|\epsilon|_{[0,t]}\right)\right). \end{aligned} \tag{3.14}$$

Define $\gamma(|\epsilon|_{[t_0,t]}) = \max\left\{|\epsilon|_{[0,t]}, \alpha_1^{-1}\left(\Xi\alpha_2\left(|\epsilon|_{[0,t]}\right)\right)\right\}$. It follows from (3.12)–(3.14) that

$$|x_{3e}(t)| \leq \beta\left(|x_{3e0}|, t - t_0\right) + \gamma\left(|\epsilon|_{[t_0,t]}\right),\ t \geq t_0.$$

Thus, system (3.6) is ISS. The proof is completed. □

Theorem 2. *The errors between the reference signals* (x_{1d}, x_{2d}, x_{3d}) *and the full* EHSS *system* (2.10) *with the impulsive correction controller* (3.7) *are* ISS *if condition* (3.8) *holds.*

Proof. The full EHSS system is composed of two subsystems, x_1, x_2-system (3.3) and x_3-system (3.6). With the impulsive correction controller (3.7), the error system between x_{3d} and the state of the x_3-system is ISS by Theorem 2. On the other hand, the errors between desired signals (x_{1d}, x_{2d}) and the state of the x_1, x_2-system (3.3) are also ISS. Then, according to Lemma 2, the errors of the full cascade system are ISS. Thus, the proof is completed. □

Remark 1. *In view of Theorem 2, the position tracking problem is solved by controller* (3.7)*. In controller* (3.7)*, the continuous part plays a role that resists the dynamics of the EHSS, and the impulsive correction part is only triggered in discrete time instants to reduces the control consumption. Condition* (3.8) *establishes a relationship between the impulsive control gain and the impulsive sequence. Compared with the existing results [12, 32] of position control for the EHSS, the controller proposed by* (3.7) *only feedbacks the state* x_{3e} *at the given time sequences by impulsive jump, which reduce the control consumption and the burden of communication. Furthermore, the result of the ISS reveals that the errors between the states and the reference signals are bounded by* $\dot{F}_L$*. If* $\dot{F}_L \to 0$ *as* $t \to \infty$*, then we can derive that* $(x_1, x_2, x_3) \to (x_{1d}, x_{2d}, x_{3d})$*. It should be noticed that the impulsive control is an ideal control, which has to be approximated by continuous control when deployed in the real world. Readers are referred to [19,33,34]. There are many ways to implement this ideal controller in the real world. One can adopt a high-gain controller to approximate the performance of the Dirac function. Besides, a class of smooth function can also be used to approximate the Dirac function* $\delta(\cdot)$*. These methods validate the feasibility of implementing the impulsive control strategy.*

4. Simulation

In this section, a numerical simulation is conducted to verify the effectiveness of the proposed backstepping-based impulsive correction control. The simulation time is set at 30 s. The control frequency in the experiment is selected at 100 Hz. The total length of the hydraulic cylinder is $l = 1$. Assume the EHSS remains stationary at the initial values of the system states $x_1 = x_2 = x_3 = 0$, and the hydraulic cylinder without piston and with piston are equal in length at the beginning. Then, we can deduce that the volume $V_{01} = \frac{1}{2}A_1$ and $V_{02} = \frac{1}{2}A_2$. The rest of the parameters involved in the simulation is provided in Table 1.

Table 1. Parameters of the EHSS in simulation.

Parameter	Meaning	Value	Unit
λ	Proportionality factor	20	-
β	Fluid bulk modulus	700	MPa
C_d	Valve coefficient of discharge	1	-
ω	Valve orifice area gradient	1	m
ρ	Density of hydraulic oil	900	kg/m^3
A_1	Annulus areas of the piston	1.3	m^2
A_2	Rod side of the cylinder	1.2	m^2
c	Viscous damping coefficient of the load stiffness	0.1	N·s/m
b	Viscous damping coefficient of the actuator	100	N·s/m
P_s	Supply pressure	35000	kPa
P_a	Tank pressure	30000	kPa
q	Impulsive control gain	0.5	-

Specifically, the parameters b, c are viscous damping and stiffness, both of which are assumed to be known in advance and may not affect the stability and the steady-state error bounds. Consider the desired position $x_{1d} = \sin(t)$, which gives $\dot{x}_{1d} = \cos(t), \ddot{x}_{1d} = -\sin(t)$, and $\dddot{x}_{1d} = -\cos(t)$. In the simulation, the multiple disturbances are considered as $F_{L1} = 60t + 100$ and $F_{L2} = \sin(t)$. In the impulsive control law, we set the impulsive gain $q = 0.5$ and the impulsive sequence $\{t_n : t_n = 0.3n, n \in \mathbb{Z}_+\}$. Then, condition (3.8) holds. By applying Theorem 2, the errors of the EHSS and the reference signals are ISS. Figures 2 and 3 give the results under the disturbance F_{L1}. In Figure 2, the states x_2, x_3 are driven to the desired signals x_{2d}, x_{3d}, respectively. In particular, the state x_3 behaves as an impulsive jump due to impulsive correction control. On the other hand, the errors are both bounded, which means the ISS character is also exhibited. The position of the EHSS x_1, the reference signal x_{1d}, and the error x_{1e} are shown in Figure 3, respectively.

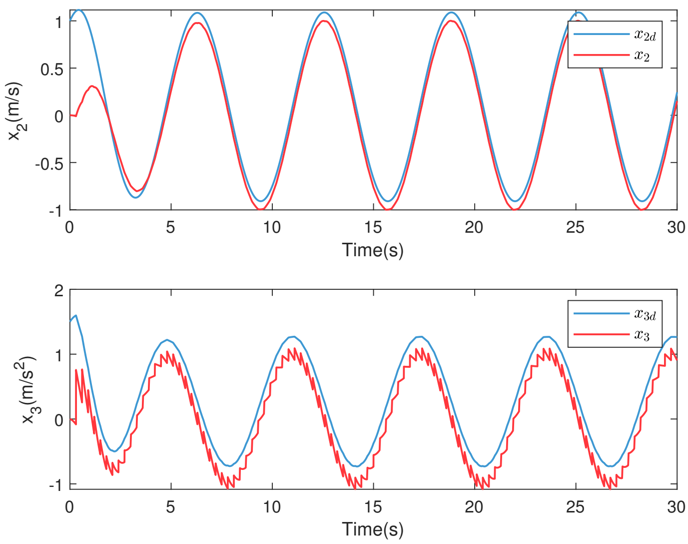

Figure 2. The velocity and the acceleration of the EHSS.

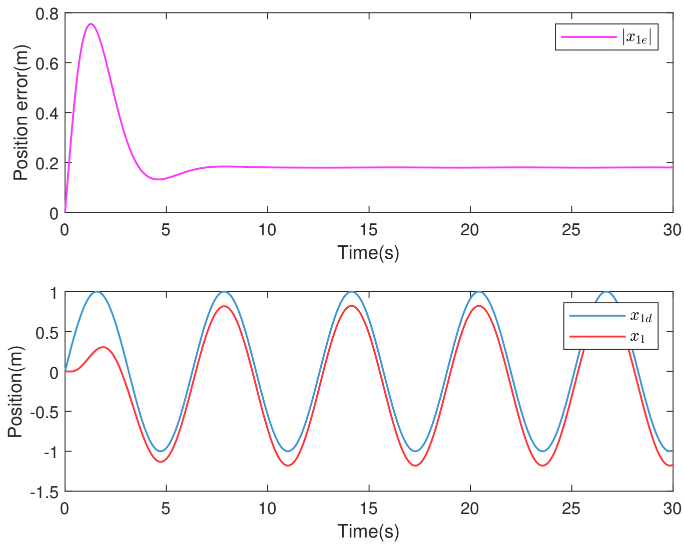

Figure 3. The position and the position error of the EHSS.

One can find that the error is regulated to be bounded, i.e., the position error is ISS. We can also conclude that the position of the EHSS can basically track the desired position x_{1d} with a small error. If the disturbance $\dot{F}_L \to 0$ as $t \to \infty$, then the asymptotic stability of the error can be derived. The results under the disturbance F_{L2} are shown in Figures 4 and 5, both of which lead to the same conclusion. Define the control consumption metric as $J(t) = \int_{t_0}^{t} |u(s)|ds$, which can be calculated to verify the reduction of the control consumption. Finally, the theoretical results are effective.

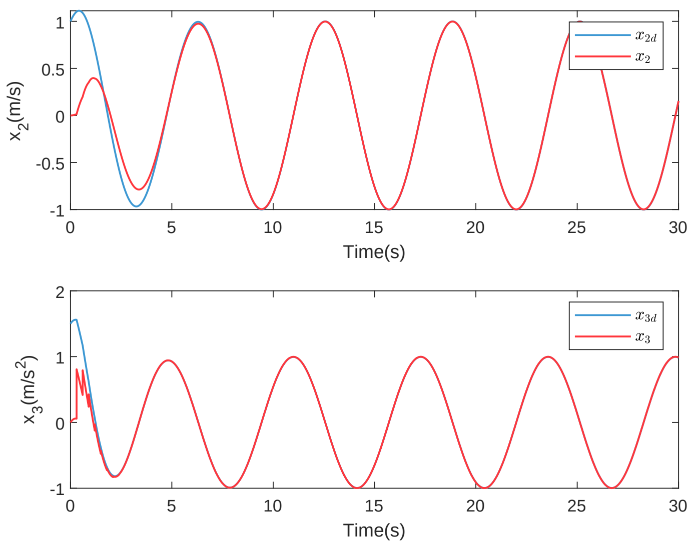

Figure 4. The position and the position error of the EHSS.

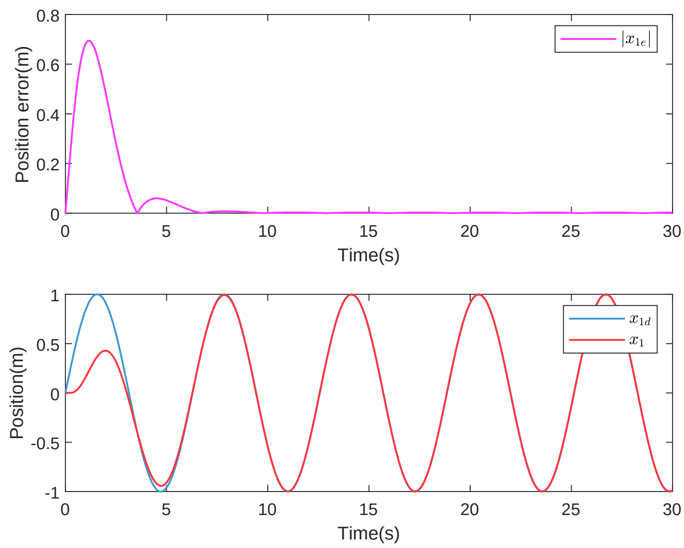

Figure 5. The position and the position error of the EHSS.

5. Conclusions

This paper investigates the ISS character of the EHSS with backstepping-based impulsive correction control. We provide a simplified mathematical model of the EHSS, which can basically describe the mathematical property of the EHSS. Based on the model, a backstepping-based impulsive correction control is proposed, which can significantly reduce the control cost. A relationship between the impulsive control gain and the impulsive sequence is established. The ISS character of the errors between desired signals and the EHSS is also demonstrated by the Lyapunov method.

Based on the controller proposed in this paper, one can also compensate the external disturbances by introducing disturbance observers according to [21], and it will be an interesting work. In the future, we can also consider adopting the dynamic surface control to address the derivative explosion problem caused by the backstepping method. We will further consider analyzing the impact of sampling and actuation delays in practice. Another interesting topic is developing a fuzzy-based method for the position control of the EHSS [35] and considering the uncertainties of parameters.

Author contributions

Wei Li: original draft, investigation; Mingzhong Li: review, editing; Jian Liu: data curation, investigation, methodology; Guofa Wang: validation, software; Yuhao Qi: data analysis, visualization; Wei Wang: deduction, methodology; Yongming Li: validation, supervision; Shuai Liu: methodology, supervision. All authors have read and agreed to the published version of the article.

Use of Generative-AI tools declaration

The authors declare that they have not use any artificial intelligence (AI) tools in the writing of this article.

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Conflict of interest

The authors declare there are no conflicts of interest.

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