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Research article

A dynamic traffic assignment model for solving overlapping path issues and perfectly rational issues under stochastic time-varying conditions

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Abstract: To effectively handle the overlapping path issue in the multinomial logit (MNL) model and perfectly rational issue in the expected utility theory (EUT) while capturing the time-varying probabilistic distribution characteristics of origin—destination (OD) demand, this study develops a reliability-based dynamic traffic assignment (R-DTA) model, that is, a cumulative prospect value (CPV)-based generalized nested logit (GNL) stochastic user equilibrium (SUE) model under stochastic time-varying conditions. Specifically, the proposed R-DTA model is established by replacing the utility value with the CPV as the path performance within the GNL model framework. An equivalent variational inequality model is provided for the proposed R-DTA model, which is solved by the method of successive averages (MSA). Moreover, the existence and equivalence of the solution to the equivalent model are also proved. The proposed R-DTA model is tested on two networks to demonstrate its performance. The corresponding results demonstrate that the model can jointly deal with the perfectly rational issues and the overlapping path issues; also, the model can effectively capture the time-varying probabilistic distribution characteristics of OD demand.

Keywords: stochastic user equilibrium; cumulative prospect theory; generalized nested logit; method of successive averages; stochastic time-varying conditions
Mathematics Subject Classification: 00A06, 00A71

1. Introduction

Traffic assignment is a fundamental methodology in transportation science that has contributed significantly to mitigating urban traffic congestion. It is well known that urban traffic systems exhibit inherent stochasticity due to unpredictable factors, such as adverse weather [1], traffic accidents [2,3], transportation management and control [4], natural disasters [5], traffic information [6], and traveler characteristics [7]. These uncertainties induce stochastic fluctuations in origin—destination (OD) demand [8] and the degradation of link capacity, ultimately leading to travel time variability.

Currently, the existing research primarily handles travel time uncertainty through metrics like effective travel time (ETT) or travel time budget (TTB). In this regard, scholars have developed various traffic assignment models to handle uncertainty, for example, the probabilistic user equilibrium (PUE) model [9], the demand-driven travel time reliability-based user equilibrium (DRUE) model [10], the percentile travel time (PTT) model [2], the mean-excess traffic equilibrium (METE) model [6]. Nevertheless, most traffic assignment models are developed under deterministic conditions, and a limited number of studies incorporate the stochasticity of both OD demand and link capacity in their formulation. Among those that do consider OD demand stochasticity, a common approach is to rely on the probability distribution of demand observed in the same time period across multiple days as model input. However, this method fails to account for the time-varying characteristic of the distribution. Therefore, traffic assignment modeling that accounts for the stochastic time-varying characteristic of OD demand needs to be further investigated.

Beyond the challenges of uncertainty modeling, another critical issue concerns the route choice model. The route choice model is predominantly constructed within the multinomial logit (MNL) framework. The MNL model, introduced by Luce in 1959 [11], exhibits the independence of irrelevant alternatives (IIA) property. A direct consequence of this property is that the model inherently fails to account for the overlapping relationships among alternative routes [12,13]. To mitigate this limitation, researchers have developed various discrete choice models, which can be broadly divided into two categories. The first category is modified MNL models, such as C-logit [14], path size logit (PSL) [15], and path size correction logit (PSCL) [16]. These models typically calibrate route utilities by introducing adjustment or correction terms into the fixed utility term. The second category is generalized extreme value (GEV) models. These models assume that the stochastic utility terms follow a joint generalized extreme value distribution, thereby allowing correlations among alternatives to be captured through these terms. Furthermore, the GEV model allows for alternatives to belong to different nests. Researchers have derived various models depending on the specific generating function used in the GEV framework, such as nested logit (NL) [17,18], paired combinatorial logit (PCL) [19,20], cross-nested logit (CNL) [21,22], and generalized nested logit (GNL) [23].

In addition to structural route choice issues, the utility of route choice model also warrants attention. Traffic assignment models are generally developed based on expected utility theory, which assumes that decision-makers are perfectly rational. However, expected utility theory deviates from travelers' actual perception of path utility in complex decision-making environments. Consequently, most studies have introduced prospect theory (PT) [24] and cumulative prospect theory (CPT) [25] into traffic assignment modeling. For example, Connors and Sumalee [26] used cumulative prospect values as route choice decision criteria in user equilibrium model. Xu et al. [27] developed a CPT-based traffic assignment model by replacing fixed exogenous reference points with endogenous ones. Wang and Sun [28] proposed a user equilibrium model based on cumulative prospect theory (CPT). Yang et al. [29] further extended the CPT-based user equilibrium model to the CPT-based stochastic user equilibrium model. Despite these advances, it should be noted that these models typically handle either overlapping path issues or perfectly rational issues. Traffic assignment models that simultaneously address both issues need to be further researched.

To bridge these research gaps, this paper investigates dynamic traffic assignment models under stochastic time-varying conditions. First, a stochastic time-varying route choice model that simultaneously handles overlapping path issues and perfectly rational issues is established by replacing expected utility values with CPV as the path utility based on the GNL model framework.

Second, the reliability-based dynamic traffic assignment (R-DTA) model is formulated by incorporating the stochastic user equilibrium conditions based on the route choice model. Subsequently, an equivalent variational inequality model is derived, with the existence and equivalence of solutions for both models rigorously proven. Finally, the proposed R-DTA model is validated using both the Braess network and the Nguyen-Dupuis network.

2. Proposed methodology

In this section, we first analyze stochastic time-varying probability distribution characteristics of traffic parameters (i.e., OD demand, path flow, link flow, link travel time, path travel time). Then, the CPV-based GNL route choice model under stochastic time-varying conditions is provided by using the CPV instead of the utility value as path performance in the GNL model framework. Finally, the R-DTA model and its corresponding solution algorithm are presented.

2.1. The probabilistic characteristics of stochastic time-varying traffic parameters

This paper applies the mean and variance of OD demand to describe its stochastic probability distribution characteristics. In this study, OD demand is treated as a stochastic variable, with q_h^w denoting the mean of the OD demand Q_h^w, that is,

$$q_h^w = E[Q_h^w],\ \forall w \in W, h \in H, \tag{2.1}$$

where Q_h^w is stochastic travel demand between OD pair w during time period h, w is OD pair, and h is period, $h = 1,2 \cdots, \breve{m}$.

Let $\varepsilon_{Q_h^w}$ and $\sigma_{Q_h^w}$ represent the variance and standard deviation of the OD demand, respectively. Assuming that the standard deviation of the OD demand is a continuously non-decreasing function $W(\cdot)$ of the mean OD demand,

$$\sigma_{Q_h^w} = \sqrt{\varepsilon_{Q_h^w}} = W(q_h^w),\ \forall w \in W, h \in H. \tag{2.2}$$

The ratio between the standard deviation and the mean of OD demand is defined as

$$cv_h^w = \frac{\sigma_{Q_h^w}}{q_h^w},\ \forall w \in W, h \in H. \tag{2.3}$$

Given that OD demand is a stochastic variable, it follows that link flows and path flows are also stochastic variables. Based on the relationship between OD demand, path flow, and link flow, we have

$$\sigma_{F_{kh}^w} = \sqrt{\varepsilon_{F_{kh}^w}} = f_{kh}^w cv_h^w = f_{kh}^w \frac{\sigma_{Q_h^w}}{q_h^w},\ \forall w \in W, k \in K_w, h \in H, \tag{2.4}$$

$$\sigma_{X_{ah}} = \sqrt{\varepsilon_{X_{ah}}} = \sqrt{\sum_w \sum_k \delta_{ak}^w\, \varepsilon_{F_{kh}^w}} = \sqrt{\sum_w \sum_k \delta_{ak}^w (f_{kh}^w)^2 \left(\frac{\sigma_{Q_h^w}}{q_h^w}\right)^2},\ \forall a \in A, h \in H, \tag{2.5}$$

where $\sigma_{F_{kh}^w}$ is the standard deviation of the path flow, and $\sigma_{X_{ah}}$ is the standard deviation of the link flow.

Then, the link travel time is described by the Bureau of Public Roads (BPR) function [30], which can be expressed as follows:

$$T_{ah} = t_{ah}^0 \left[1 + \alpha \left(\frac{x_{ah}}{c_{ah}}\right)^{\ddot{n}}\right],\ \forall a \in A, \tag{2.6}$$

where t_{ah}^0 and c_{ah} represent the free-flow travel time and capacity of link a during time period h,

x_{ah} is mean link flow during time period h, the subscript a is a specific link, and α and $\ddot{n}$ are deterministic parameters.

The link flow x_{ah} is replaced by the stochastic link flow X_{ah}, and the link capacity c_{ah} is replaced by the stochastic link capacity C_{ah}; we then have

$$T_{ah}(X_{ah}, C_{ah}) = t_{ah}^0\left[1+\alpha\left(\frac{X_{ah}}{C_{ah}}\right)^{\ddot{n}}\right],\ \forall a \in A, h \in H. \tag{2.7}$$

For period h, the mean and variance of the link travel time are obtained as follows, respectively:

$$t_{ah} = E[T_{ah}] = E\left[t_{ah}^0\left[1+\alpha\left(\frac{X_{ah}}{C_{ah}}\right)^{\ddot{n}}\right]\right] = t_{ah}^0 + \alpha t_{ah}^0 E\left[\left(\frac{X_{ah}}{C_{ah}}\right)^{\ddot{n}}\right],\ \forall a \in A, h \in H, \tag{2.8}$$

$$\varepsilon_{T_{ah}} = \mathrm{var}[T_{ah}] = E[(T_{ah})^2] - E^2[T_{ah}],\ \forall a \in A, h \in H. \tag{2.9}$$

Assuming X_{ah} and C_{ah} are independent, $E[T_{ah}]$ and $E[(T_{ah})^2]$ can be simplified as

$$E[T_{ah}] = t_{ah}^0 + \alpha t_{ah}^0 E\left[X_{ah}^{\ddot{n}}\right] E\left[\frac{1}{C_{ah}^{\ddot{n}}}\right],\ \forall a \in A, h \in H, \tag{2.10}$$

$$E[(T_{ah})^2] = (t_{ah}^0)^2 + \alpha^2 (t_{ah}^0)^2 E\left[X_{ah}^{2\ddot{n}}\right] E\left[\frac{1}{C_{ah}^{2\ddot{n}}}\right] + 2\alpha (t_{ah}^0)^2 E\left[X_{ah}^{\ddot{n}}\right] E\left[\frac{1}{C_{ah}^{\ddot{n}}}\right],\ \forall a \in A, h \in H. \tag{2.11}$$

To calculate the mean and variance of link travel time, it is necessary to calculate $E\left[\frac{1}{C_{ah}^{\ddot{n}}}\right]$, $E\left[\frac{1}{C_{ah}^{2\ddot{n}}}\right]$, $E\left[X_{ah}^{\ddot{n}}\right]$, and $E\left[X_{ah}^{2\ddot{n}}\right]$. The first two terms depend on the probability distribution of the stochastic link capacity C_{ah}, while the latter two depend on the probability distribution of the stochastic link flow X_{ah}. It is assumed that the OD demand follows a normal distribution, and the link capacity C_a follows a uniform distribution over interval $[\theta_a \bar{c}_a, \bar{c}_a]$. It should be explained that the upper bound $\bar{c}_a$ is the design capability, the lower bound $\theta_a \bar{c}_a$ is the worst-degraded capability, and θ_a is a fraction of the design capacity. We have

$$E\left[\frac{1}{C_{ah}^{\ddot{n}}}\right] = \int_{\theta_{ah}\bar{c}_{ah}}^{\bar{c}_{ah}} \frac{1}{C_{ah}^{\ddot{n}}} \frac{1}{\bar{c}_{ah} - \theta_{ah}\bar{c}_{ah}} \mathrm{d}c_{ah} = \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})},\ \forall a \in A, h \in H, \tag{2.12}$$

$$E\left[\frac{1}{C_{ah}^{2\ddot{n}}}\right] = \int_{\theta_{ah}\bar{c}_{ah}}^{\bar{c}_{ah}} \frac{1}{C_{ah}^{2\ddot{n}}} \frac{1}{\bar{c}_{ah} - \theta_{ah}\bar{c}_{ah}} \mathrm{d}c_{ah} = \frac{1-\theta_{ah}^{1-2\ddot{n}}}{\bar{c}_{ah}^{2\ddot{n}}(1-\theta_{ah})(1-2\ddot{n})},\ \forall a \in A, h \in H. \tag{2.13}$$

Let $Y_{ah} = \frac{X_{ah} - x_{ah}}{\sigma_{X_{ah}}}$, that is, $X_{ah} = x_{ah} + Y_{ah}\sigma_{X_{ah}}$. Then we can obtain

$$\begin{aligned} E\left[X_{ah}^{\ddot{n}}\right] &= E\left[\left(x_{ah} + Y_{ah}\sigma_{X_{ah}}\right)^{\ddot{n}}\right] \\ &= E\left[\sum_{b=0}^{\ddot{n}} C_n^b (x_{ah})^{\ddot{n}-b} \left(Y_{ah}\sigma_{X_{ah}}\right)^b\right] \\ &= \sum_{b=0}^{\ddot{n}} C_n^b (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b E[Y_{ah}]^b,\ \forall a \in A, h \in H. \end{aligned} \tag{2.14}$$

Substituting $E[Y_{ah}]^b = \begin{cases} (b-1)!! & \text{if } b \text{ is even} \\ 0 & \text{if } b \text{ is odd} \end{cases}$ into Eq (2.14), we can obtain:

$$E\left[X_{ah}^{\ddot{n}}\right] = \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!!,\ \forall a \in A, h \in H. \tag{2.15}$$

Similarly, we can obtain:

$$E\left[X_{ah}^{2\ddot{n}}\right] = \sum_{b=0}^{2\ddot{n}} \binom{2\ddot{n}}{b} (x_{ah})^{2\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!!,\ \forall a \in A, h \in H. \tag{2.16}$$

Substituting $E\left[\frac{1}{C_{ah}^{\ddot{n}}}\right]$, $E\left[\frac{1}{C_{ah}^{2\ddot{n}}}\right]$, $E\left[X_{ah}^{\ddot{n}}\right]$, and $E\left[X_{ah}^{2\ddot{n}}\right]$ into Eqs (2.10) and (2.11), the mean link travel time t_{ah} and $E[(T_{ah})^2]$ can be obtained as, respectively,

$$t_{ah} = t_{ah}^0 + \alpha t_{ah}^0 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{n}(1-\theta_{ah})(1-\ddot{n})} \forall a \in A, h \in H, \quad (2.17)$$

$$E[(T_{ah})^2] = (t_{ah}^0)^2 + \alpha^2 (t_{ah}^0)^2 \sum_{b=0}^{2\ddot{n}} \binom{2\ddot{n}}{b} (x_{ah})^{2\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-2\ddot{n}}}{\bar{c}_{ah}^{2\ddot{n}}(1-\theta_{ah})(1-2\ddot{n})}$$

$$+2\alpha (t_{ah}^0)^2 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})}, \ \forall a \in A, h \in H. \quad (2.18)$$

Taking the square of Eq (2.17), we can obtain

$$E^2[T_{ah}] = \left(t_{ah}^0 + \alpha t_{ah}^0 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})}\right)^2$$

$$= (t_{ah}^0)^2 + 2\alpha (t_{ah}^0)^2 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})}$$

$$+\left\{\alpha t_{ah}^0 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})}\right\}^2, \ \forall a \in A, h \in H. (2.19)$$

Furthermore, the variance of link travel time is obtained as follows:

$$\varepsilon_{T_{ah}} = \alpha^2 (t_{ah}^0)^2 \sum_{b=0}^{2\ddot{n}} \binom{2\ddot{n}}{b} (x_{ah})^{2\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-2\ddot{n}}}{\bar{c}_{ah}^{2\ddot{n}}(1-\theta_{ah})(1-2\ddot{n})}$$

$$-\left\{\alpha t_{ah}^0 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})}\right\}^2, \forall a \in A, h \in H. \ (2.20)$$

Given the relationship between link travel time and path travel time,

$$T_{kh}^w = \sum_a T_{ah} \delta_{ak}^w, \forall w \in W, k \in K_w, h \in H, \quad (2.21)$$

the mean and variance of path travel time can be obtained as shown below:

$$t_{kh}^w = \sum_a \left\{\delta_{ak}^w \left(t_{ah}^0 + \alpha t_{ah}^0 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})}\right)\right\},$$

$$\forall w \in W, k \in K_w, h \in H, \quad (2.22)$$

$$\varepsilon_{T_{kh}^w} = \sum_a \left\{\delta_{ak}^w \left(\alpha^2 (t_{ah}^0)^2 \sum_{b=0}^{2\ddot{n}} \binom{2\ddot{n}}{b} (x_{ah})^{2\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-2\ddot{n}}}{\bar{c}_{ah}^{2\ddot{n}}(1-\theta_{ah})(1-2\ddot{n})}\right.\right.$$

$$\left.\left.-\left\{\alpha t_{ah}^0 \sum_{b=0}^{\ddot{n}} \binom{\ddot{n}}{b} (x_{ah})^{\ddot{n}-b} \left(\sigma_{X_{ah}}\right)^b (b-1)!! \frac{1-\theta_{ah}^{1-\ddot{n}}}{\bar{c}_{ah}^{\ddot{n}}(1-\theta_{ah})(1-\ddot{n})}\right\}^2\right)\right\},$$

$$\forall w \in W, k \in K_w, h \in H, \quad (2.23)$$

where T_{kh}^w is stochastic travel time on path k between OD pair w during time period h, t_{kh}^w and $\sigma_{T_{kh}^w}$ are the mean and standard deviation of T_{kh}^w, $\delta_{a,k}^w$ is the relationship between path k and link a, and $\delta_{a,k}^w = 1$ if path k contains link a, and $\delta_{a,k}^w = 0$, otherwise.

2.2. The CPV-based route choice model

In this section, we apply CPV instead of the utility value as the path performance in the GNL model framework to construct the CPV-based route choice model under stochastic time-varying conditions to handle the overlapping path issue and the perfectly rational issue.

Step 1: the travel time budget.

The travel time budget is the sum of the mean path travel time and the buffer time. The expression for travel time budget is provided as follows:

$$\mathcal{B}_{kh}^{w} = t_{kh}^{w} + \xi\sigma_{T_{kh}^{w}},\ \forall w \in W, k \in K_w, h \in H, \tag{2.24}$$

where $\mathcal{B}_{kh}^{w}$ is travel time budget on path k between OD pair w during time period h, and ξ is a parameter related to on-time arrival probability.

After a series of derivations, the expression for the travel time budget of the path can be obtained as follows:

$$\mathcal{B}_{kh}^{w} = t_{kh}^{w} + \sigma_{T_{kh}^{w}}\Phi^{-1}(\rho_h),\ \forall w \in W, k \in K_w, h \in H, \tag{2.25}$$

where Φ^{-1} is the inverse function of the standard normal cumulative distribution function.

Step 2: CPV computation.

To calculate the cumulative prospect value of a path, it is necessary to first determine the reference point. This paper adopts an endogenous reference point proposed by Xu et al [27], expressed as follows:

$$u_{0h}^{w} = \min_{k}\{\mathcal{B}_{kh}^{w}\},\ \forall w \in W, h \in H, \tag{2.26}$$

where u_{0h}^{w} is reference point between OD pair w during time period h.

Based on the reference point, the cumulative prospect value of the path is calculated using the value function and probability weighting function. The expressions for the value function, probability weighting function, and cumulative prospect value are exhibited as follows:

$$g(T_{kh}^{w}) = \begin{cases} \left(u_{0h}^{w} - \check{T}_{kh}^{w}\right)^{\delta}, & \check{T}_{kh}^{w} \le u_{0h}^{w}, \\ -\lambda\left(\check{T}_{kh}^{w} - u_{0}^{w}\right)^{\eta}, & \check{T}_{kh}^{w} > u_{0h}^{w}, \end{cases}\ \forall w \in W, k \in K_w, h \in H, \tag{2.27}$$

$$w(p) = \exp[-(-\ln p)^{\gamma}], \tag{2.28}$$

$$v_{kh}^{w} = \int_{\underline{u}_{kh}^{w}}^{u_{0h}^{w}} \frac{dw^{+}[F(\check{T}_{kh}^{w})]}{d\check{T}_{kh}^{w}} g\left(\check{T}_{kh}^{w}\right) d\check{T}_{kh}^{w} + \int_{u_{0h}^{w}}^{\overline{u}_{kh}^{w}} -\frac{dw^{-}[1-F(\check{T}_{kh}^{w})]}{d\check{T}_{kh}^{w}} g\left(\check{T}_{kh}^{w}\right) d\check{T}_{kh}^{w},$$

$$\forall w \in W, k \in K_w, h \in H, \tag{2.29}$$

where δ and η are risk sensitivity parameters that reflect the degree of diminishing sensitivity in the value function. Larger values of these two parameters indicate that the decision-maker is more risk-sensitive, while smaller values suggest lower sensitivity to risk. The parameter λ denotes the loss aversion coefficient; a higher value implies that the decision-maker is more sensitive to losses. The parameter γ represents the curvature of the probability weighting function, a smaller value of γ corresponds to a more pronounced curvature. Typical values satisfy $0 < \delta, \eta < 1$, $\lambda \ge 1$, and $0 < \gamma < 1$. Additionally, $w(p)$ and p denote the perceived probability and the actual probability of an event, respectively. It is noteworthy that $w(p)$ increases monotonically with p and is independent of the outcome value γ [31]. In Eq (2.29), $\underline{u}_{kh}^{w}$ and $\overline{u}_{kh}^{w}$ represent the lower and upper bounds of the travel time for path k during time period h, respectively. The lower bound $\underline{u}_{kh}^{w}$ represents the free-flow travel time, while the upper bound is given by $\overline{u}_{kh}^{w} = \check{t}_{kh}^{w} + 3\sigma_{\check{T}_{kh}^{w}}$.

Step 3: Route choice probability.

The cumulative prospect value of a path as perceived by travelers is modeled as a stochastic variable. It consists of an actual observed value and a random error term and is expressed as follows:

$$\tilde{v}_{kh}^{w} = v_{kh}^{w} + \tilde{\varepsilon}_{kh}^{w},\ \forall w \in W, k \in K_w, h \in H, \tag{2.30}$$

where $\tilde{v}_{kh}^{w}$ is the perceived CPV of the path k between OD pair w during time period h, v_{kh}^{w} is the actual CPV of the path k between OD pair w during time period h, and $\tilde{\varepsilon}_{kh}^{w}$ is the random error.

In this study, we modify the traditional GNL model derived from GEV theory [32] by replacing the utility value with CPV. The probability of choosing path k between OD pair w during time period h is given as follows:

$$p_{kh}^{w} = \frac{\sum_m \left[\left(\alpha_{mk}^{w} \exp(\theta_h v_{kh}^{w}) \right)^{1/\mu_m^w} \left(\sum_{k \in K_m} \left(\alpha_{mk}^{w} \exp(\theta_h v_{kh}^{w}) \right)^{1/\mu_m^w} \right)^{\mu_m^w - 1} \right]}{\sum_m \left[\sum_{k \in K_m} \left(\alpha_{mk}^{w} \exp(\theta_h v_{kh}^{w}) \right)^{1/\mu_m^w} \right]^{\mu_m^w}}, \forall w \in W, k \in K_w, h \in H, \quad (2.31)$$

where v_{kh}^{w} is the cumulative prospect value of path k between OD pair w during time period h; θ_h is a dispersion parameter during time period h, which characterizes the familiarity of a user with the road network; K_m is set of all paths included in nest m; μ_m^w is nesting coefficients, which represent the degree of similarity between nests, $0 < \mu_m^w \le 1$; α_{mk}^{w} is inclusion coefficients, which represent the degree to which the path k between OD and w belongs to nest m; $\alpha_{mk}^{w} = \left(\frac{l_m}{l_k}\right)^{\tau} \delta_{mk}^{w}$, $\mu_m^w = 1 - \frac{1}{G_m^w} \sum_k \alpha_{mk}^{w}$, where l_m is the length of link m, l_k is the length of path k, and G_m^w is the number of paths containing link m between OD pair w; and τ is a parameter that characterizes the travelers' perception of the similarities between different routes, assumed as $\tau = 1$.

It is possible to decompose Eq (2.32) as marginal probability and conditional probability, that is,

$$P_{kh}^{w} = \sum_m P_h^w(m) P_h^w(k|m), \ \forall w \in W, k \in K_w, h \in H, \quad (2.32)$$

where $P_h^w(m)$ is the marginal probability that the travelers choose nest m between OD pair w during time period h, described as

$$P_h^w(m) = \frac{\left(\sum_k \left(\alpha_{mk}^{w} \exp(\theta_h v_{kh}^{w}) \right)^{1/\mu_m^w} \right)^{\mu_m^w}}{\sum_m \left(\sum_k \left(\alpha_{mk}^{w} \exp(\theta_h v_{kh}^{w}) \right)^{1/\mu_m^w} \right)^{\mu_m^w}}, \ \forall w \in W, m \in M, h \in H; \quad (2.33)$$

$P_h^w(k|m)$ is the conditional probability, namely, the probability of choosing path k when the nest m between OD pair w is selected, described as

$$P_h^w(k|m) = \frac{\left(\alpha_{mk}^{w} \exp(\theta_h v_{kh}^{w}) \right)^{1/\mu_m^w}}{\sum_k \left(\alpha_{mk}^{w} \exp(\theta_h v_{kh}^{w}) \right)^{1/\mu_m^w}} \quad \forall w \in W, m \in M, k \in K_w, h \in H. \quad (2.34)$$

2.3. *The reliability-based dynamic traffic assignment model*

Following the construction of the CPV-based route choice model, the CPV-based GNL SUE model is further developed by incorporating the SUE condition. According to Sheffi [33], the SUE conditions can be described as

$$\sum_m f_{mkh}^{w} = q_h^w P_{kh}^{w} \quad \forall w \in W, m \in M, k \in K_w, h \in H. \quad (2.35)$$

In addition, the regular network constraints must hold, that is,

$$q_h^w = \sum_m \sum_k f_{mkh}^{w} \quad \forall w \in W, h \in H, \quad (2.36)$$

$$x_{ah} = \sum_w \sum_k \sum_m f_{mkh}^{w} \delta_{akh}^{w}, \ \forall a \in A, h \in H, \quad (2.37)$$

$$f_{mkh}^{w} \ge 0 \quad \forall w \in W, m \in M, k \in K_w, h \in H, \quad (2.38)$$

where q_h^w is the travel demand between OD pair w during time period h, x_{ah} is the flow on link a during time period h, and f_{mkh}^w is the flow on path k of nest m between OD pair w during time period h.

Equation (2.36) is the flow conservation constraints. Equation (2.37) denotes the incidence relationship between link flow and path flow; Eq (2.38) is the flow nonnegativity constraint. It should be pointed out that the value of P_{kh}^w in Eq (2.35) is calculated based on the CPV-based GNL route choice model under stochastic time-varying conditions.

(1) Equivalent variational inequality model.

Establishing a mathematical programming model is challenging due to the complexity of the cumulative prospect function of path utilities and the asymmetry inherent in dynamic traffic assignment. This paper proposes a variational inequality model based on path flows to describe the dynamic traffic assignment problem under stochastic supply-demand conditions. Therefore, the solution of the reliability-based dynamic traffic assignment (R-DTA) model is equivalent to finding a feasible mean path flow f_{mkh}^{w*}, which satisfies the following variational inequality (VI).

$$\sum_w \sum_m \sum_k \left[\frac{1-\mu_m^w}{\theta_h}\ln(\textstyle\sum_k f_{mkh}^{w*}) + \frac{\mu_m^w}{\theta_h} ln f_{mkh}^{w*} - \frac{1}{\theta_h} ln(\alpha_{mk}^w) - v_{kh}^w\right] \times (f_{mkh}^w - f_{mkh}^{w*}) \geq 0. \quad (2.39)$$

(2) Proof of equivalence.

The objective of the equivalence proof is to ensure that the variational inequality model satisfies the equilibrium condition of the R-DTA model. According to the variational inequality theorem, the variational inequality model is equivalent to the following complementary relaxation conditions:

$$\left(\frac{1-\mu_m^w}{\theta_h}\ln(\textstyle\sum_k f_{mkh}^{w*}) + \frac{\mu_m^w}{\theta_h} ln f_{mkh}^{w*} - \frac{1}{\theta_h} ln(\alpha_{mk}^w) - v_{kh}^w\right) \times f_{mkh}^{w*} = 0, \quad (2.40)$$

$$\left(\frac{1-\mu_m^w}{\theta_h}\ln(\textstyle\sum_k f_{mkh}^{w*}) + \frac{\mu_m^w}{\theta_h} ln f_{mkh}^{w*} - \frac{1}{\theta_h} ln(\alpha_{mk}^w) - v_{kh}^w\right) \geq 0. \quad (2.41)$$

When $f_{mkh}^{w*} > 0$, we obtain

$$\frac{1-\mu_m^w}{\theta_h}\ln(\textstyle\sum_k f_{mkh}^{w*}) + \frac{\mu_m^w}{\theta_h} ln f_{mkh}^{w*} - \frac{1}{\theta_h} ln(\alpha_{mk}^w) - v_{kh}^w = 0. \quad (2.42)$$

Multiply both sides of Eq (2.42) by θ_h. We can obtain

$$(1-\mu_m^w)\ln(\textstyle\sum_k f_{mkh}^{w*}) + \mu_m^w ln f_{mkh}^{w*} - ln\alpha_{mk}^w - \theta_h v_k^w = 0. \quad (2.43)$$

Then, divide both sides of Eq (2.43) by μ_m^w. We can obtain

$$\frac{(1-\mu_m^w)}{\mu_m^w}\ln(\textstyle\sum_k f_{mkh}^{w*}) + ln f_{mkh}^{w*} = \frac{ln\alpha_{mk}^w}{\mu_m^w} + \frac{\theta_h v_k^w}{\mu_m^w}. \quad (2.44)$$

Take the logarithm of exp as the base, Eq (2.44), can be rewritten as

$$(f_{mkh}^{w*})(\textstyle\sum_k f_{mkh}^{w*})^{\frac{1-\mu_m^w}{\mu_m^w}} = (\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]. \quad (2.45)$$

Summing Eq (2.45) by route k, we have

$$(\textstyle\sum_k f_{mkh}^{w*})^{\frac{1}{\mu_m^w}} = \sum_k (\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \times \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]. \quad (2.46)$$

Elevating Eq (2.46) both sides to μ_m^w, we have

$$\textstyle\sum_k f_{mkh}^{w*} = \left\{\sum_k (\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]\right\}^{\mu_m^w}. \quad (2.47)$$

Summing Eq (2.47) by link (nest) m, we have

$$\sum_m \sum_k f_{mkh}^{w*} = \sum_m \left\{\sum_k (\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]\right\}^{\mu_m^w}. \tag{2.48}$$

After Eq (2.47) is divided by Eq (2.48), the marginal probability is obtained as

$$P_h^w(m) = \frac{\sum_k f_{mkh}^{w*}}{\sum_m \sum_k f_{mkh}^{w*}} = \frac{\left\{\sum_k (\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]\right\}^{\mu_m^w}}{\sum_m \left\{\sum_k (\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]\right\}^{\mu_m^w}}. \tag{2.49}$$

The conditional probability is obtained by dividing Eq (2.46) by Eq (2.47) as

$$P_h^w(k|m) = \frac{f_{mkh}^{w*}}{\sum_k f_{mkh}^{w*}} = \frac{(\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]}{\sum_k (\alpha_{mk}^w)^{\frac{1}{\mu_m^w}} \exp\left[(\theta_h v_{kh}^w)/\mu_m^w\right]}. \tag{2.50}$$

By simultaneously substituting Eqs (2.49) and (2.50) into Eq (2.32), we can obtain:

$$P_{kh}^w = \sum_m P_h^w(m) P_h^w(k|m) = \frac{f_{mkh}^{w*}}{\sum_m \sum_k f_{mkh}^{w*}} = \frac{\sum_m f_{mkh}^{w*}}{q_h^w}. \tag{2.51}$$

We can further obtain

$$\sum_m f_{mkh}^{w*} = q_h^w P_{kh}^w. \tag{2.52}$$

At the optimal solution of the model, the equilibrium conditions of the R-DTA model are satisfied, thus proving the statement.

(3) Proof of existence.

Let $Z(f_{mkh}^{w*}) = \frac{1-\mu_m^w}{\theta} \ln(\sum_k f_{mkh}^{w*}) + \frac{\mu_m^w}{\theta} ln f_{mkh}^{w*} - \frac{1}{\theta} ln(\alpha_{mk}^w) - v_{kh}^w$. It is assumed that the distribution function of the link travel time is a continuous function of the link flow. According to the reference point Eq (2.26), the reference point is a continuous function of path flow; therefore, the CPV is also a continuous function of path flow. In addition, according to constraints (2.36)–(2.38), the feasible path flow set is a compact convex set. According to the theory of variational inequalities, the solution to the variational inequality Eq (2.39) exists.

2.4. Solution algorithm

Generally, the method of successive averages (MSA) introduced by Powell and Sheffi [34] offers a straightforward and practically feasible approach for engineering applications. Accordingly, this study adapts the conventional MSA algorithm to solve the proposed R-DTA model. The detailed procedural steps are outlined as follows.

Step 0: Initialization. Perform traffic assignment based on the initial cumulative prospect values $\{v_{kh}^{w(0)}\}$of paths to obtain the set of mean path flows $\{f_{mkh}^{w(n)}\}$. Set the iteration counter $n = 1$.

Step 1: Update cumulative prospect values of the path. Set $v_{kh}^{w(n)} = v_{kh}^w(f_{mkh}^{w(n)})$.

Step 2: Determine search direction. Perform traffic assignment using the current set of mean path travel times $\{f_{mkh}^{w(n)}\}$ to obtain the auxiliary mean path flow pattern $\{y_{mkh}^{w(n)}\}$.

Step 3: Update mean path flows. Let $f_{mkh}^{w(n+1)} = f_{mkh}^{w(n)} + (1/n)\left(y_{mkh}^{w(n)} - f_{mkh}^{w(n)}\right)$, and calculate the new mean path flows.

Step 4: Convergence check. If $\| f_{mkh}^{w(n+1)} - f_{mkh}^{w(n)} \| / \| f_{mkh}^{w(n)} \| \leq \varepsilon$, then stop. If not, set $n = n + 1$ and go to Step 1.

3. Numerical examples

To verify the performance of the proposed R-DTA model in this paper, this section will analyze three aspects, that is, the overlapping path issue, the perfectly rational issue, and the time-varying traffic parameters.

3.1. Analysis of the overlapping path issue

The Braess network includes 1 OD pair, 5 links, and 3 paths, as shown in Figure 1. Path 1 includes nodes 1, 2, and 4; path 2 includes nodes 1, 2, 3, and 4; and path 3 includes nodes 1-3-4. It should be noted that link 1 and link 5 are overlapping sections, while link 2, link 3, and link 4 are non-overlapping sections. The parameters in the BPR function are $\alpha = 0.15$ and $\beta = 4$. The dispersion parameter θ is set as 0.5. The on-time arrival probability ρ is assumed to be 0.8. According to Tversky and Kahneman [25], the parameters of the value function in Eq (2.27) are assumed to be $\delta = \eta = 0.88$ and $\lambda = 2.25$. According to Prelec [35], the probability weighting function in Eq (2.28) is considered as $\gamma = 0.74$. The iteration accuracy ε of the MSA algorithm is set as 0.0001.

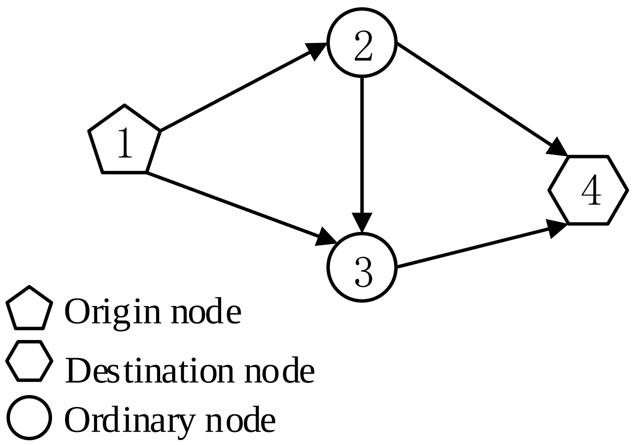

Figure 1. Braess network.

The free-flow travel time and capacity of each link are shown in Table 1, whose values are adopted from Li and Lang [36].

Table 1. Link characteristics.

No.	Link	t_a^0	$\bar{c}_a$	θ_{ah}
1	1-2	5	600	0.6
2	1-3	6	500	0.7
3	2-3	7	600	0.8
4	2-4	8	500	0.7
5	3-4	3	700	0.6

To obtain the mean and variance of OD demand, with each 15-minute interval as one time period, there are a total of 96 time periods in a day (00:00 to 24:00). First, for each 15-minute time period t ($t = t_1, t_2, t_3 \cdots t_h$), the maximum value of OD demand for each OD pair is determined based on the characteristics of morning peak, evening peak, and off-peak OD demand data. Second, thirty stochastic numbers between 0.9 and 1 are generated, and the maximum OD demand value is multiplied by each of the thirty random numbers to obtain 30 sets of OD demand data for that 15-minute time period. Finally, according to the calculation equation of mean and variance, the mean and variance of OD demand for that 15-minute time period are calculated. This process is repeated to obtain the mean and variance of OD demand data for one or multiple days. The process of obtaining the mean and variance of OD demand is illustrated in Figure 2.

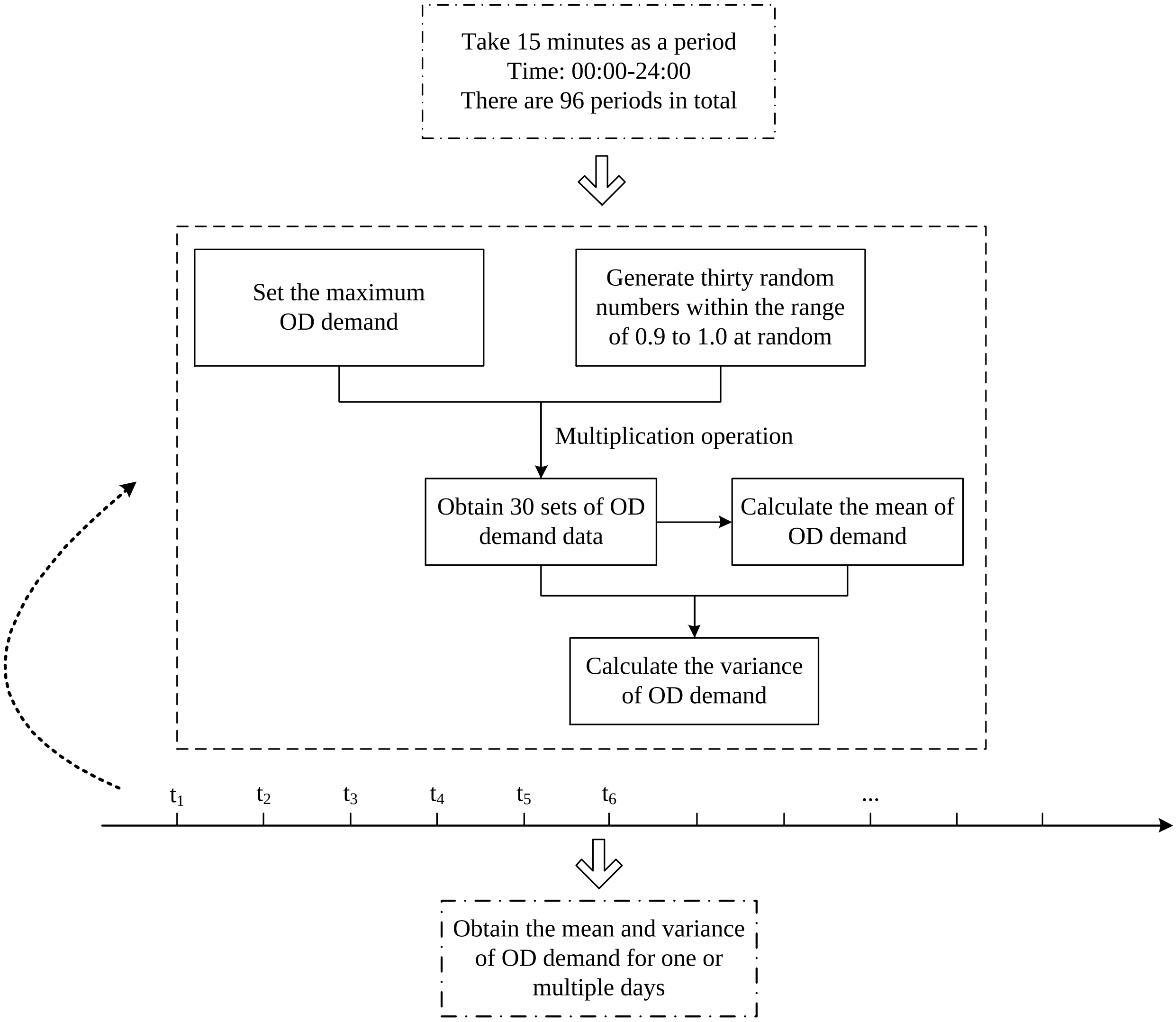

Figure 2. The process of obtaining the mean and variance of OD demand.

For OD (1, 4), the mean and variance data of OD demand for a specific day are selected to describe the time-series of OD demand mean and variance, as shown in Figures 3 and 4.

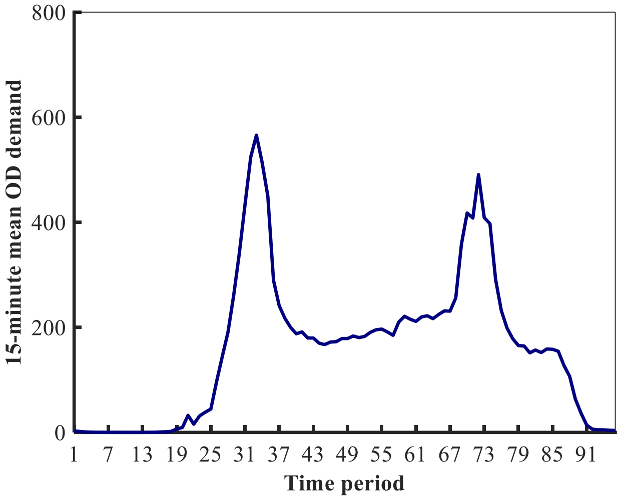

Figure 3. 15-minute mean OD demand time series diagram.

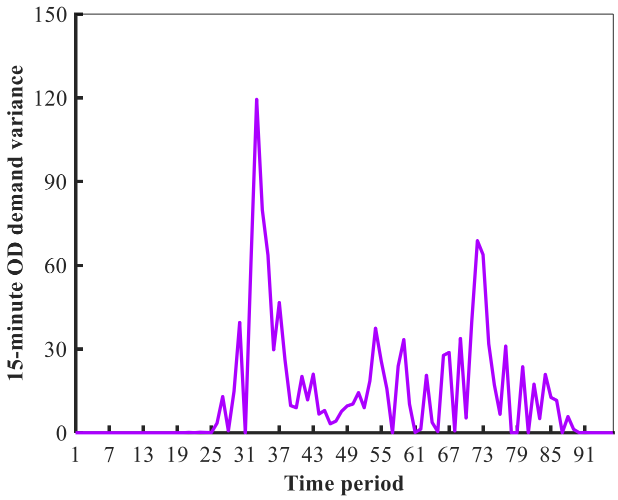

Figure 4. 15-minute OD demand variance time series diagram.

It can be seen from Figures 3 and 4 that the proposed method in this study can generate OD demand data with time-varying characteristics. Specifically, travel demand between OD (1, 4) is significantly higher during morning peak hours (07:00-09:00) and evening peak hours (17:00-19:00), while demand decreases during off-peak periods. In addition, the OD demand exhibits obvious heteroscedasticity, that is, the variance of demand varies across each 15-minute interval.

To verify that the proposed R-DTA model (i.e., the CPV-based GNL-SUE model under stochastic time-varying conditions) can effectively handle overlapping path issues, the mean and variance of OD demands observed during the morning peak from 8:00-8:15 were used as model inputs. Both the R-DTA model and the CPV-based MNL-SUE model under stochastic time-varying conditions were run on the Braess network, yielding the equilibrium mean link flow shown in Figure 5.

It can be seen from Figure 5 that the proposed R-DTA model assigns less traffic flow on overlapping links (link 1 and link 5) and greater traffic flow on non-overlapping links (link 2 and link 4) than the CPV-based MNL-SUE model under stochastic time-varying conditions. This is because the CPV-based MNL-SUE model under stochastic time-varying conditions exhibits the independence of irrelevant alternatives (IIA) property, which fails to account for overlapping paths. Consequently, it tends to overestimate traffic flow on overlapping links while underestimating flow on non-overlapping links. In contrast, the proposed R-DTA model assigns less traffic flow on overlapping links than the CPV-based MNL-SUE model under stochastic time-varying conditions. This demonstrates its capability to partially overcome the IIA property inherent in the CPV-based MNL-SUE model under stochastic time-varying conditions, as clearly evidenced by our mean traffic flow distribution results. The proposed R-DTA model assigns less traffic to heavily overlapped segments compared to the CPV-based MNL-SUE model under stochastic time-varying conditions. A comparative analysis of mean link flow assignment confirms that the R-DTA model successfully handles the inherent IIA limitation associated with the CPV-based MNL-SUE model under stochastic time-varying conditions.

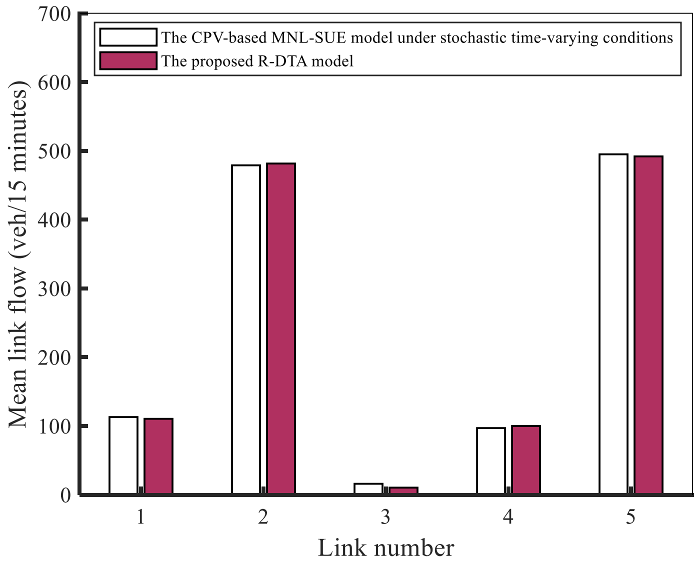

Figure 5. Comparison of mean link flows.

3.2. Analysis of perfectly rational issue

The Nguyen-Dupuis network consists of 4 OD pairs, 13 nodes, and 19 links, as shown in Figure 6. It should be noted that the other model parameters are the same as those in the Braess network.

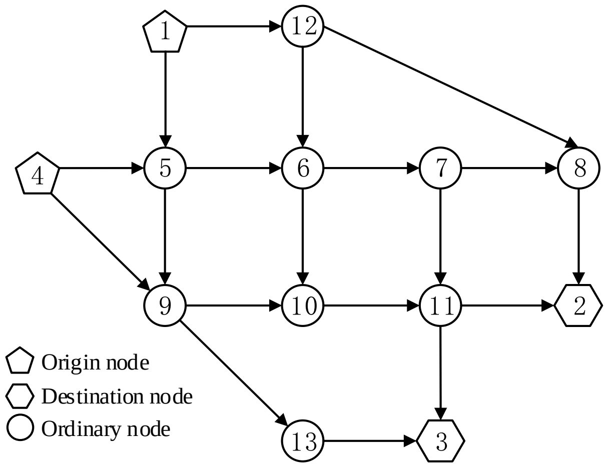

Figure 6. Nguyen-Dupuis network.

The free-flow travel time t_{ah}^0, link capacity $\bar{c}_{ah}$, and the worst-degraded coefficient θ_{ah} are shown in Table 2, the values of which come from the research of Xu et al. [27]. The values of the path node sequence and path length can be found in [37].

Table 2. Link characteristics of Nguyen-Dupuis network.

No.	link	t_{ah}^{0}	$\bar{c}_{ah}$	θ_{ah}	No.	link	t_{ah}^{0}	$\bar{c}_{ah}$	θ_{ah}
1	1-5	7	600	0.8	11	8-2	9	600	0.7
2	1-12	9	600	0.8	12	9-10	10	600	0.6
3	4-5	9	600	0.7	13	9-13	9	600	0.8
4	4-9	12	600	0.8	14	10-11	6	600	0.7
5	5-6	3	600	0.6	15	11-2	9	600	0.7
6	5-9	9	600	0.6	16	11-3	8	600	0.6
7	6-7	5	600	0.7	17	12-6	7	600	0.8
8	6-10	13	600	0.8	18	12-8	14	600	0.7
9	7-8	5	600	0.7	19	13-3	11	600	0.7
10	7-11	9	600	0.7					

The mean and variance data for OD demands on a specific day is selected, and these values are plotted over time in Figures 7 and 8.

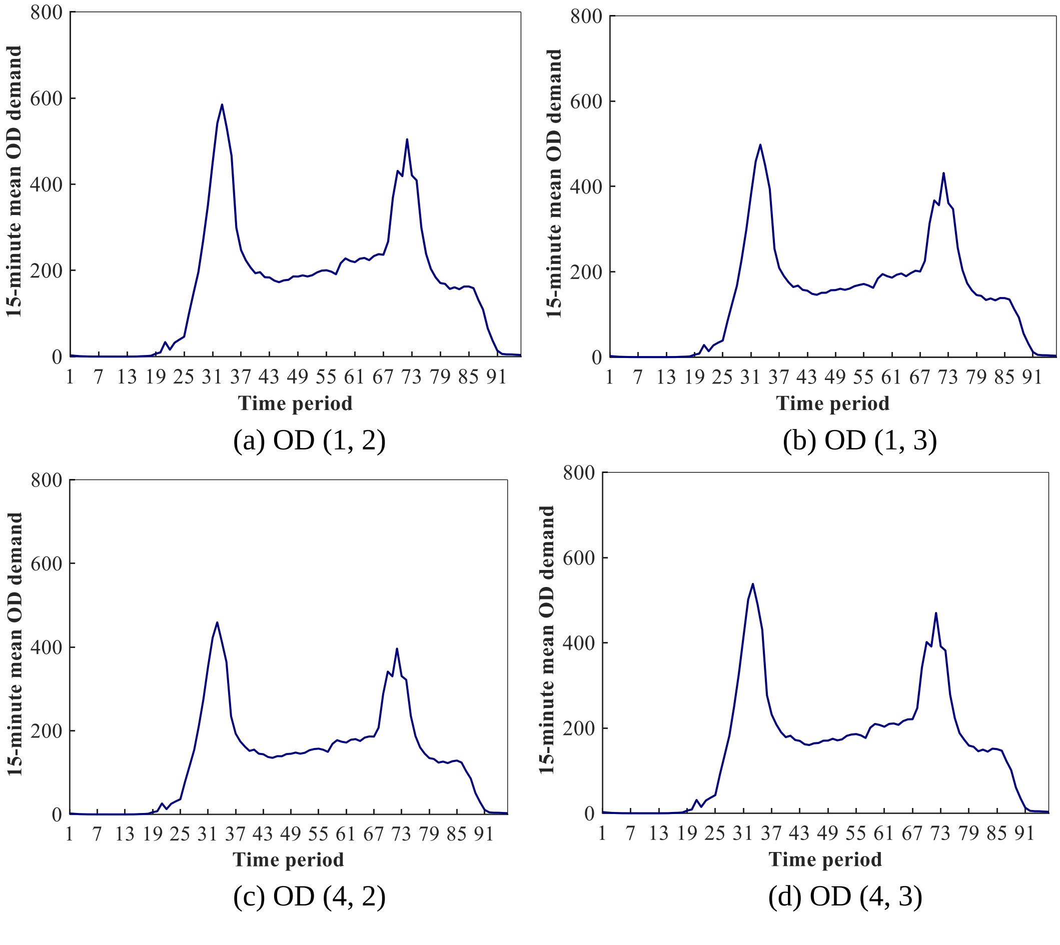

Figure 7. 15-minute mean OD demand time series diagram.

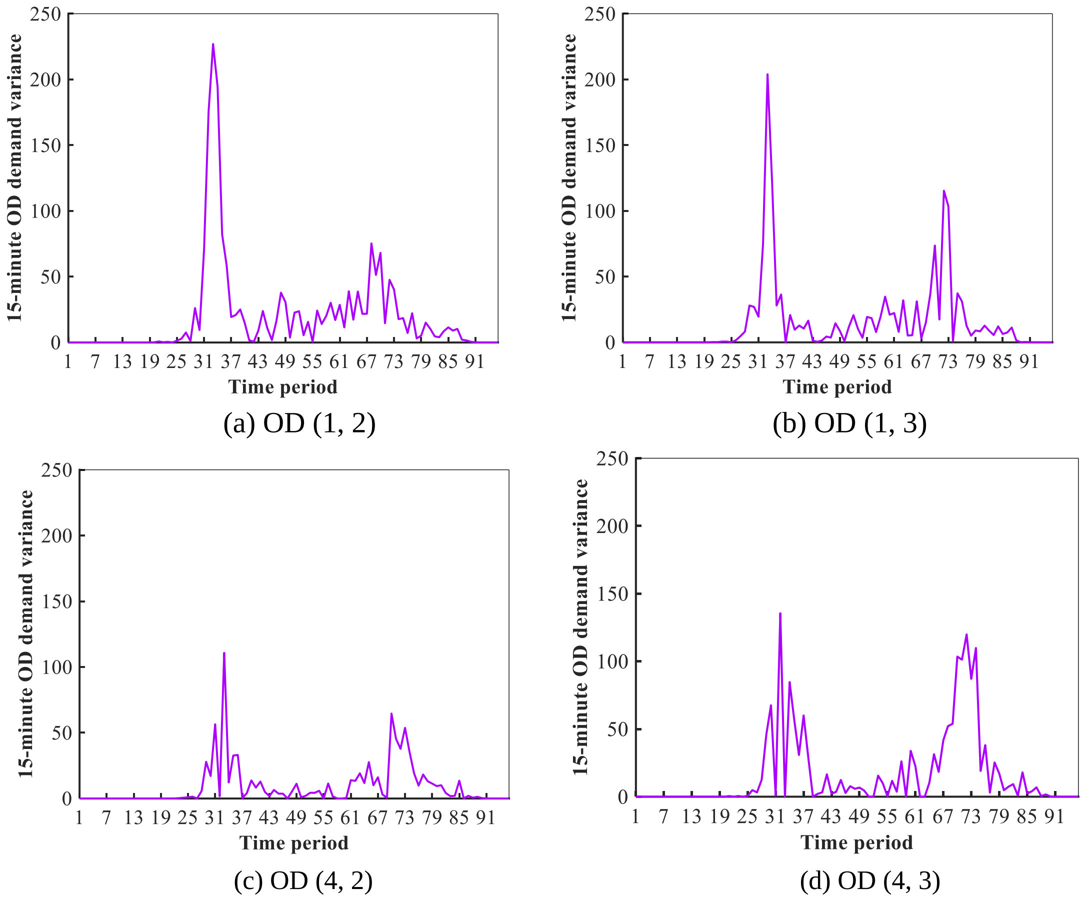

Figure 8. 15-minute OD demand variance time series diagram.

It can be seen from Figures 7 and 8 that the approach proposed in this paper can generate OD demand data with time-varying characteristics. Specifically, all OD demand data has morning and evening peak characteristics, and OD demand has obvious heteroscedasticity.

To verify that the proposed R-DTA model can handle the perfectly rational issue, the mean and variance of OD demand during the morning peak period (8:00-8:15) were used as input data for both the R-DTA model (i.e., the CPV-based GNL SUE model under stochastic time-varying conditions) and the expected utility value (EUV)-based GNL SUE model under stochastic time-varying conditions. The resulting equilibrium traffic flow distribution patterns in the road network are shown in Tables 3 and 4.

It can be seen from Tables 3 and 4 that there exist significant differences in path flow assigned by the proposed R-DTA model and the EUV-based GNL SUE model under stochastic time-varying conditions. This is because the two models have different assumptions about travel behavior. Specifically, the EUV-based GNL SUE model under stochastic time-varying conditions assumes that all travelers are perfectly rational in their route choice decisions. In other words, these travelers only consider the mean path time as the criterion for route choice. In contrast, the proposed R-DTA model assumes that travelers exhibit bounded rationality in route choice, taking into account both the mean travel time and the standard deviation of travel time when choosing paths.

Table 3. The assignment results of the proposed R-DTA model.

OD	Path	f_{kh}^{w}	v_{kh}^{w}	t_{kh}^{w}	$\sigma_{T_{kh}^{w}}$
(1, 2)	1	426.35	0.95	38.72	1.94
	2	80.97	-1.98	40.96	2.81
	3	58.21	-2.93	41.80	2.15
	4	10.04	-5.98	43.71	1.80
	5	4.28	-7.38	44.36	0.61
	6	10.51	-5.20	43.27	2.18
	7	4.71	-6.79	44.11	1.23
	8	0.61	-10.10	46.01	0.39
(1, 3)	9	178.12	0.74	42.64	1.36
	10	183.28	0.90	42.07	2.37
	11	55.23	-1.22	43.98	2.07
	12	37.76	-1.82	44.63	1.18
	13	43.36	-1.55	44.38	1.59
	14	8.89	-4.68	46.29	1.09
(4, 2)	15	180.52	0.55	41.87	0.98
	16	142.99	0.12	41.52	2.77
	17	88.31	-0.57	42.35	2.10
	18	19.51	-3.16	44.26	1.75
	19	8.09	-4.39	44.92	0.41
(4, 3)	20	358.25	0.84	40.14	1.56
	21	85.09	-1.62	42.14	1.41
	22	27.42	-3.29	43.19	1.29
	23	64.12	-2.60	42.62	2.33
	24	11.06	-5.53	44.53	2.02
	25	3.61	-6.80	45.19	1.10

More specifically, in Table 3, the mean travel time of path 15 is greater than that of path 16, while its standard deviation is smaller. The proposed R-DTA model assigns more traffic flow to path 15 than to path 16. This demonstrates that the proposed R-DTA model takes into account both the mean and standard deviation of path travel time when assigning traffic flows. In Table 4, travelers make route choices exclusively based on mean path travel time. In other words, paths with smaller mean travel times are assigned greater traffic flows, while those with larger mean travel times are assigned lower traffic flows. Therefore, it can be concluded that the proposed R-DTA model effectively handles the perfectly rational issue in travelers' route choice decision-making.

Table 4. The assignment results of the EUV-based GNL SUE model under stochastic time-varying conditions.

OD	Path	f_{kh}^{w}	t_{kh}^{w}	$\sigma_{T_{kh}^{w}}$
(1, 2)	1	394.94	38.16	1.81
	2	84.02	40.95	2.80
	3	51.36	41.96	2.21
	4	16.38	44.00	1.87
	5	12.44	44.19	0.65
	6	18.28	43.18	2.13
	7	11.03	44.61	1.27
	8	7.22	46.24	0.44
(1, 3)	9	170.61	42.45	1.31
	10	180.46	42.28	2.46
	11	55.00	44.32	2.15
	12	37.24	44.93	1.25
	13	47.32	44.51	1.66
	14	16.01	46.56	1.16
(4, 2)	15	166.22	41.54	0.87
	16	154.12	41.64	2.76
	17	79.29	42.65	2.17
	18	24.35	44.70	1.82
	19	15.44	45.31	0.49
(4, 3)	20	351.94	39.38	1.44
	21	80.20	41.86	1.38
	22	34.39	43.15	1.24
	23	58.42	42.97	2.42
	24	15.93	45.02	2.11
	25	8.67	45.63	1.18

3.3. Analysis of time-varying characteristics of traffic parameters

To investigate the time-varying characteristics of the R-DTA model, this study uses the Nguyen-Dupuis network as the test network. The mean and variance of OD demand for 96 time periods throughout the day (i.e., each period representing 15 minutes) are used as inputs for the R-DTA model. The study analyzes the variations of mean path flow, standard deviation of path flow, mean path time, standard deviation of path time, and cumulative prospect values over time in the network equilibrium state are shown in Figure 9.

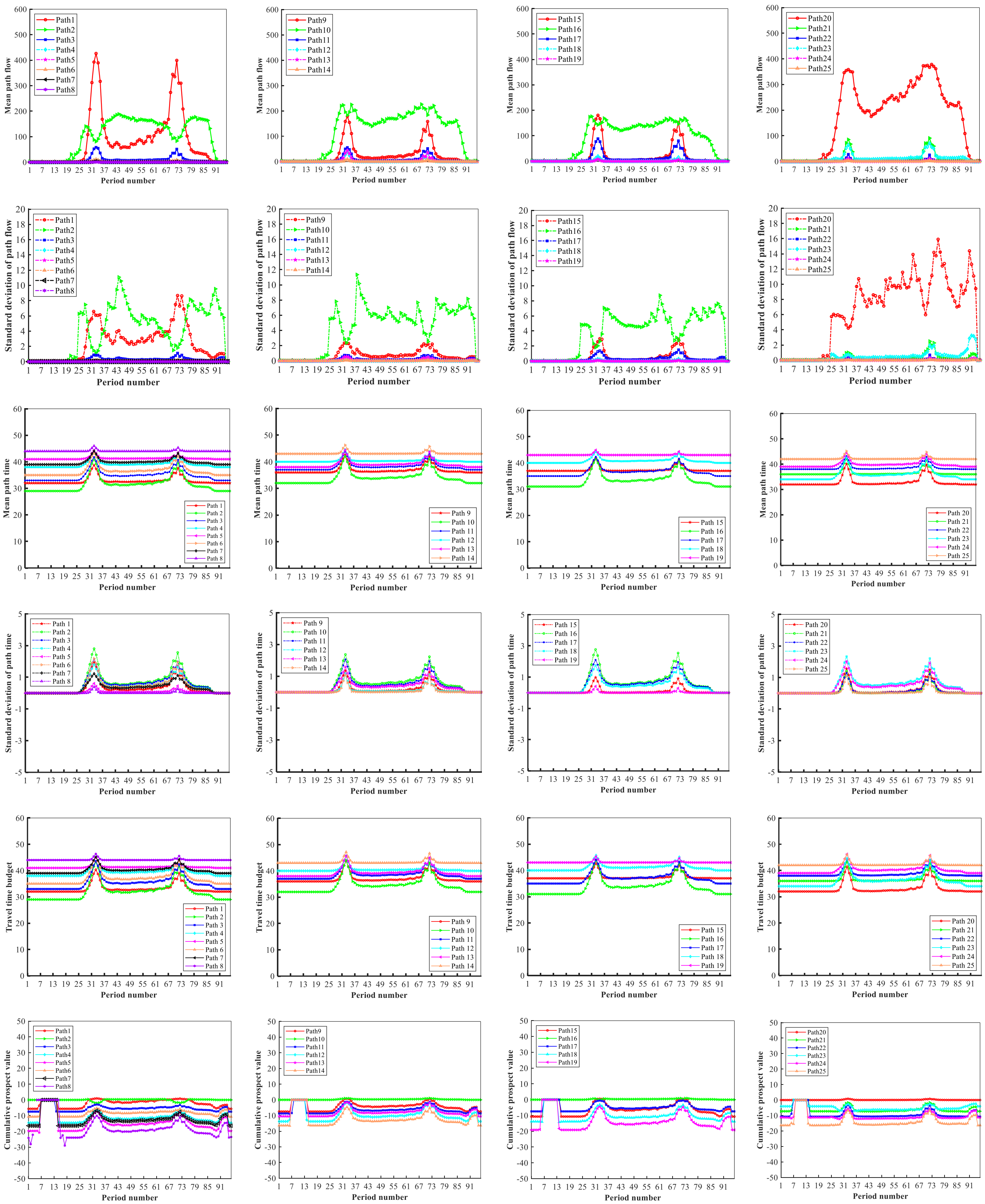

Figure 9. Variation of traffic parameters over time.

The proposed R-DTA model effectively captures time-varying patterns in path flow, path time, and cumulative prospect values, demonstrating clear peak-hour characteristics. During the morning peak hours from 07:00 to 09:00 and the evening peak hours from 17:00 to 19:00, the traffic flows on path 1, path 9, path 15, and path 20 are relatively high, while the traffic on path 2, path 10, and path 16 shows a decreasing trend. This phenomenon may be attributed to the flow conservation constraint required for achieving equilibrium in the road network. During the period from 06:00 to 23:00, the traffic flow on path 1, path 2, path 10, path 16, and path 20 fluctuates significantly, whereas from 00:00 to 06:00, the traffic flow on all paths shows minimal variation. This is likely because traverlers, travel activities are primarily concentrated between 06:00 and 23:00, while they typically rest or sleep from 00:00 to 06:00.

During the morning peak hours (07:00-09:00) and evening peak hours (17:00-19:00), both the mean travel time and standard deviation on all paths are relatively high, whereas during off-peak hours, both the mean travel time and standard deviation on all paths are relatively low. The trend of the travel time budget aligns with that of the mean and standard deviation of path time, as it is calculated based on these two metrics. During the period from 00:00 to 06:00, the path travel time budget shows almost no variation, while from 10:00 to 17:00 or 20:00 to 23:00, it exhibits relatively minor fluctuations. In addition, during the morning peak hours (07:00-09:00) and evening peak hours (17:00-19:00), the CPVs for all paths are relatively high, while during the off-peak period (09:00-17:00), the CPVs for all paths are relatively low.

It can be concluded that the path flow assignment results are consistent with the flow distribution principles of the R-DTA model proposed in this study, thereby validating the effectiveness of the R-DTA model.

4. Conclusions

Traffic assignment is vital for policy making in transportation management and operations. Existing studies often fail to consider the stochastic time-varying characteristic of OD demand and the inherent variability of link capacity. Moreover, the joint solution to the path overlapping issue and the perfectly rational issue is still lack of adequate investigations in the context of traffic assignment. Therefore, this paper proposes a R-DTA model, that is, a cumulative prospect value (CPV)-based generalized nested logit (GNL) stochastic user equilibrium (SUE) model under stochastic time-varying conditions.

The R-DTA model effectively handles the path overlapping issue by employing a flexible nesting structure and correlation adjustment parameters. This allows overlapping paths to share correlations within nests, thereby correcting the overestimation of choice probabilities caused by the independence of irrelevant alternatives (IIA) property of the traditional MNL model. Furthermore, the R-DTA model overcomes the limitations of the perfect rationality assumption in expected utility theory (EUT) by integrating a probability weighting function and a value function. These functions replace objective probabilities and actual outcomes with psychologically perceived evaluations, thereby capturing bounded-rational decision-making behavior that better reflects real-world risk choice patterns.

The proposed R-DTA model is tested on both the Braess network and the Nguyen-Dupuis network to validate its performance. The results indicate that the proposed R-DTA model can deal with the overlapping path issue and the perfectly rational issue. Moreover, the R-DTA model captures the time-varying probability distribution of traffic parameters, which reflects peak-hour characteristics.

In future, multiclass users will be incorporated into the R-DTA modeling process. In addition, more efficient model-solving methods will be developed and applied to larger-scale transportation networks.

Author contributions

Dongmei Yan: Conceptualization, methodology, writing—original draft preparation; Jianmei Cheng: Conceptualization, methodology, writing-review & editing; Jing Gan: Writing—original draft, writing—review & editing; Yue Wang: writing—review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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