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Research article

Study on the exponential stability of stochastic functional differential equations with random impulses

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Abstract: This paper investigates the pth moment exponential stability of stochastic functional differential equations (SFDEs) with random impulses. By employing the Razumikhin-type method in combination with mathematical induction, we derive an exponential stability criterion for the considered system. In addition, we elucidate the intermediate mechanisms through which impulsive disturbances affect system stability, thereby extending and generalizing existing results in the literature. Finally, two numerical examples are presented to confirm the correctness and validity of the theoretical findings.

Keywords: Razumikhin-type condition; exponential stability; random impulses
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1. Introduction

Hybrid systems exhibit both continuous evolution and instantaneous jumps, integrating essential features of continuous-time and discrete-time dynamical systems [1–3]. They serve as powerful modeling tools for a wide range of real-world applications and complex physical phenomena, many of which cannot be adequately characterized by purely continuous or purely discrete frameworks. A prominent subclass of hybrid systems is impulsive systems, which are particularly well-suited to capturing abrupt state changes in practical scenarios, such as those arising in neural networks, networked control systems, multi-agent systems [4,5], and biological systems [6,7]. Owing to their extensive applicability, researchers have devoted substantial effort to investigating the fundamental theoretical foundations of impulsive differential systems, leading to a wealth of impactful results [8,9].

Time delays are ubiquitous in real-world scenarios, particularly in the transmission of coupling and communication signals [10]. For example, even information propagating at the speed of light requires a finite amount of time to traverse long distances. Likewise, the population dynamics of biological species are strongly influenced by the developmental stages of immature individuals [11]. Consequently, the evolution of a system is determined not only by its current state but also by its historical behavior. Existing studies have shown that time delays can adversely affect system performance, potentially inducing instability and impairing controllability. Conversely, they can also be harnessed as a regulatory tool to stabilize control systems [12,13]. Neglecting time delays may lead to defective control strategies or even entirely erroneous analytical conclusions. Importantly, time delays are rarely constant in practice; instead, they often exhibit time-varying characteristics, necessitating the use of diverse mathematical methodologies to characterize them with high precision.

Over the past few decades, impulsive stochastic functional differential equations (ISFDEs) have attracted significant attention, owing to their wide-ranging and impactful applications across fields such as electronics, economics, automatic control, and population dynamics [14–16]. Consequently, investigating the stability and exponential stability of ISFDEs is not merely valuable but also indispensable, which has led to the development of a rich body of stability theories. For instance, Chen [17] studied the pth moment exponential stability of mild solutions to ISFDEs by constructing an impulsive-integral inequality. Building on this work, Gao and Li [18] extended Chen's inequality and, by employing the M*ö*nch fixed point theorem, derived results on the existence and mean-square exponential stability of mild solutions for ISFDEs with time-varying delays. Separately, Xiao and Chen [19] applied the Banach fixed point theorem together with inequality techniques to examine the existence and exponential stability of ISFDEs. It is noteworthy that although the Razumikhin technique is a relatively simple yet effective tool for analyzing ISFDE stability, it was not utilized in the aforementioned studies by Chen [17], Gao and Li [18], and Xiao and Chen [19]. In contrast, several researchers have employed Razumikhin techniques to analyze ISFDE with impulses occurring at deterministic times. For example, Kao et al. [20] established multiple stability criteria by combining the Razumikhin technique with Gronwall's inequality; Huang and Li [21,22]. investigated the pth moment exponential stability of ISFDEs using the Razumikhin technique, the comparison principle, Lyapunov functions; and Guo et al. [23] explored both pth moment and almost sure exponential stability of ISFDEs with both bounded and unbounded time-varying delays, also relying on the Razumikhin technique. Additional contributions to the application of Razumikhin methods in this context can be found in Li et al. [24], Peng and Zhang [25], Cao and Zhu [26], and Yu et al. [27].

Most existing studies on impulsive dynamical systems have focused on deterministic impulses. However, neglecting the randomness of impulsive timings in real-world scenarios may severely limit the practical applicability of impulsive stochastic systems in engineering contexts [28,29]. When addressing random impulses, two core characteristics required attention: random impulsive density and random impulsive intensity. For example, Tang et al. [30] investigated the input-to-state stability of nonlinear ordinary differential systems that incorporate both random impulsive intensity and random impulsive density. In particular, Hu and Zhu [31,32] successfully examined the pth moment exponential stability for a class of impulsive stochastic functional differential equations with randomly occurring impulses have been shown to yield meaningful results. An interesting observation, however, is that the upper bound coefficient of the Lyapunov differential operator established in prior work is assumed to be a positive constant-a setting that may fail to accurately capture the dynamic characteristics of such systems. This raises an important question: is it feasible to develop stability criteria that do not impose this constant-coefficient restriction? Furthermore, the aforementioned results seldom account for scenarios involving indefinite continuous dynamics-a common feature in stochastic systems with time-varying coefficients. Motivated by this observation, this brief investigates the pth moment exponential stability of stochastic functional differential equations (SFDEs) with random impulses, where the continuous dynamics of the addressed systems are indefinite.

The main contributions of this article are summarized as follows:

(i) While previous studies [17–21] examined impulsive stochastic functional differential equations (ISFDEs) with deterministically timed impulses, the present work investigates the case where impulses occur at random instants.

(ii) We treat the upper-bound coefficient of the Lyapunov differential operator as a bounded time-varying function. This approach generalizes the findings of Hu and Zhu [31,32], where the coefficient was restricted to either a positive or negative constant. As elaborated in Remark 3.3, our results provide a more comprehensive characterization than Theorem 1 in Hu and Zhu [31] and Theorem III.1 in Hu and Zhu [32].

(iii) In contrast to Hu and Zhu [31], we explicitly clarify the quantitative relationship between the impulse intensity η, the upper bound of the Lyapunov differential operator β, and the distribution of impulse intervals λ.

The paper is structured as follows: Section 2 introduces the necessary notations, lemmas, and definitions that lay the foundation for subsequent analysis. Section 3 is devoted to analyzing the pth moment exponential stability of stochastic functional differential equations with randomly occurring impulsive effects. To verify the effectiveness of the proposed theoretical results, two numerical examples are presented in Section 4. Finally, Sections 5 summarizes the key findings of this work and offers concluding remarks.

2. Preliminaries

Notations: $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = (0, \infty)$, $\mathbb{N} = \{1, 2, \cdots\}$ and $\mathbb{R}^n$ stand for n-dimensional Euclidean space; $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ is a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions; $C([t_0, \infty), \mathbb{R})$ denotes the family of continuous function from $[t_0, \infty)$ to $\mathbb{R}$; $C^2(\mathbb{R}^n, \mathbb{R}^+)$ denotes the family of nonnegative, twice continuously differentiable functions; $PC([-\tau, 0], \mathbb{R}^n) = \{\xi : [-\tau, 0] \to \mathbb{R}^n\}$ denotes the family of piecewise right-continuous functions ξ, with norm defined by $\|\xi\| = \sup_{-\tau\leq\theta\leq 0} |\xi(\theta)|$;$PC^b_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^n)(PC^b_{\mathcal{F}_t}([-\tau, 0], \mathbb{R}^n))$ denote the families of all $\mathcal{F}_0$-measurable ($\mathcal{F}_t$-measurable) PC-valued random function ξ; $w(t)$ is an m-dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ satisfying $\mathbb{E}\{dw(t)\} = 0$ and $\mathbb{E}\{[dw(t)]^2\} = dt$.

Consider the impulsive stochastic functional differential equations as follows:

$$\begin{cases} \mathrm{d}x(t) = f(t, x_t)\mathrm{d}t + g(t, x_t)\mathrm{d}w(t), t_k < t < t_{k+1}, t \geq 0, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = I_k(x(t_k^-)), k = 1, 2, \cdots, \\ x_0(\theta) = \zeta(\theta), \theta \in [-\tau, 0], \end{cases} \tag{2.1}$$

where $f : [t_0, \infty) \times PC^b_{\mathcal{F}_t}([-\tau, 0], \mathbb{R}^n) \longrightarrow \mathbb{R}^n$, $g : [t_0, \infty) \times PC^b_{\mathcal{F}_t}([-\tau, 0], \mathbb{R}^n) \longrightarrow \mathbb{R}^{n\times m}$, x_t is regarded as a PC-valued stochastic process, with $x_t = x(t + \theta)$, where $\theta \in [-\tau, 0]$. $I_k(x(t_k^-))$ denote the impulsive perturbation of x at time t_k is $\mathcal{F}_{t_k^-}$-measurable. $x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits at t_k, respectively. Let $t_0 = 0$, $t_k = \sum_{i=1}^{k} r_i$, where $\{r_i\}_{i=1}^{\infty}$ is a sequence of independent exponentially distribution random variables with parameter $\lambda > 0$ and is independent of $w(t)$. For the initial condition

$\zeta \in PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$, we further assume that $f(t,\xi)$ and $g(t,\xi)$ are continuous and both satisfy the local Lipschitz condition and the linear growth condition in ξ, and $|I_k(\xi)| < \infty$. For any $\zeta \in PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$, system (2.1) has a unique solution $x(t,t_0,\zeta)$. Moreover, when $f(t,0)=0$, $g(t,0)=0$ and $I_k(0)=0$ for all $t \geq t_0$, system (2.1) has a trivial solution $x(t)=0$.

Based on system (2.1), the number of impulses up to time t follows a Poisson process, denoted by N_t. We also assume that the standard Brownian motion $w(t)$ and the Poisson process N_t are independent. The notation $x_{N_t}(t,\zeta)$ represents the state of the stochastic process $x(t,\zeta)$ with N_t impulses occurring up to time t. The notation $x_r^{(n)}(t;\zeta)$ refers to the state of the process $x(t,\zeta)$ with n impulses having occurred up to time t, where the exact positions of these impulses are unspecified. In contrast, $x^{(n)}(t,\zeta)$ denotes the state of $x(t,\zeta)$ with n impulses having occurred up to time t, and their positions are predetermined.

Definition 2.1. (see [31]) For a function $V : C^{1,2}([t_0-\tau,\infty)\times\mathbb{R}^n,\mathbb{R}^+)$, $\zeta \in PC^b_{\mathcal{F}_t}([-\tau,0],\mathbb{R}^n)$, the operator $\mathcal{L}$ associated with system (2.1), defined as

$$\mathcal{L}V(t,\xi) = \frac{\partial V(t,\xi(0))}{\partial t} + \frac{\partial V(t,\xi(0))}{\partial x} f(t,\xi) + \frac{1}{2}\text{trace}\left[g^T(t,\xi)\frac{\partial V^2(t,\xi(0))}{\partial x^2} g(t,\xi)\right].$$

For more details about the X-valued stochastic integral of an $L_2^0(Y,X)$-valued, $\mathfrak{F}_t$-adapted predictable process $h(t)$ with respect to the Q-Wiener process $\omega(t)$, see reference [1].

Definition 2.2. (see [32]) The trivial solution of system (2.1) is termed pth moment exponentially stable if there exist positive constants ε and M such that

$$\mathbb{E}|x_{N_t}(t,\zeta)|^p < M\mathbb{E}\|\zeta\|^p e^{-\varepsilon t}, \quad \zeta \in PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n), \quad t \geq 0.$$

Lemma 2.1. (see [31]) Suppose the number of impulses $N_t = n$, the joint density of $t_1, t_2, \cdots, t_n$ is $f(t_1,t_2,\cdots,t_n) = \frac{n!}{t^n}, 0 < t_1 < t_2 < \cdots < t_n < t$.

3. Main results

Theorem 3.1. *Suppose there exists a function* $V \in C^{1,2}([t_0-\tau,\infty)\times\mathbb{R}^n,\mathbb{R}^+)$ *along with positive constants* $p \geq 2, c_1, c_2, \eta, \beta$ *and* $\sigma > 0$*, such that*
(H_1) $c_1|x|^p \leq V(t,x) \leq c_2|x|^p$;
(H_2) $\forall k \in \mathbb{N}$ *and* $x \in \mathbb{R}^n$, $\mathbb{E}V(t,x+I_k(x)) \leq \eta\mathbb{E}V(t,x)$;
(H_3) For $t \geq t_0, t \neq t_k, k \in \mathbb{N}$ *and* $\xi \in PC_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$*, if* $\mathbb{E}V(t+\theta,\xi(\theta)) \leq q\mathbb{E}V(t,\xi(0))$*, then* $\mathbb{E}\mathcal{L}V(t+\theta,\xi) \leq \mu(t)\mathbb{E}V(t,\xi(0))$*, where* $q \geq \max\{\eta,\frac{1}{\eta}\}e^{\sigma\tau}$, $\mu : [t_0,\infty) \to (-\infty,\beta]$;
(H_4) $\frac{\lambda\min\{\eta,\frac{1}{\eta}\}}{\lambda-(\sigma+\beta)} < 1$ *and* $\sigma + (1-\eta)\lambda > 0$.
Then, system (2.1) *is pth moment exponentially stable.*

Proof. At the beginning, with the help of the Fubini theorem and Lemma 3.2 in reference [33], one has

$$d\mathbb{E}V(t,x^{(k)}(t)) = \mathbb{E}\mathcal{L}V(t,x_t)\mathrm{d}t,\ t \in [t_k, t_k + r_{k+1}),\ k \in \mathbb{N}\cup\{0\}. \tag{3.1}$$

Next, we divide the proof into the following two cases: $0 < \eta < 1$ and $\eta \geq 1$.

Case 1. $0 < \eta < 1$. Since $\eta > 0$, there exists $M_1 > 0$ such that $c_2 \le \eta M_1$. In order to obtain the desired result, we need to prove below that

$$\mathbb{E}V(t, x^{(k)}(t)) \le M_1 \mathbb{E}\|\zeta\|^p e^{-\sigma t}, \quad t \in [t_k, t_k + r_{k+1}),\ k \in \mathbb{N} \cup \{0\}. \tag{3.2}$$

According to the previous assumptions, there is no impulse occurs at $[t_0 - \tau, t_0]$, that is

$$\begin{aligned} & \mathbb{E}V(t, x^{(0)}(t)) \le \mathbb{E}V(t+\theta, x^{(0)}(\theta)) \le c_2 \mathbb{E}\|\zeta\|^p \\ \le\ & \eta M_1 \mathbb{E}\|\zeta\|^p \le M_1 \mathbb{E}\|\zeta\|^p,\ t \in [t_0 - \tau, t_0]. \end{aligned} \tag{3.3}$$

For $t \in [t_0, t_0 + r_1)$, there is still no impulse. If (3.2) is false on $[t_0, t_0 + r_1)$, there exist some $t \in [t_0, t_0 + r_1)$ such that

$$e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) > M_1 \mathbb{E}\|\zeta\|^p > \eta M_1 \mathbb{E}\|\zeta\|^p. \tag{3.4}$$

Without loss of generality, let $t^* = \inf\{t \in [t_0, t_0 + r_1) : e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) > M_1 \mathbb{E}\|\zeta\|^p\}$. Since $e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t))$ is continuous on any interval $[t_k, t_k + r_{k+1})$, $k \in \mathbb{N} \cup \{0\}$, one has

$$e^{\sigma t^*}\mathbb{E}V(t^*, x^{(0)}(t^*)) = M_1 \mathbb{E}\|\zeta\|^p;\ e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) < M_1 \mathbb{E}\|\zeta\|^p,\ t \in [t_0, t^*). \tag{3.5}$$

Define $t_* = \sup\{t \in [t_0 - \tau, t^*) : e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) \le \eta M_1 \mathbb{E}\|\zeta\|^p\}$. The same method can be easily adjusted to obtain

$$e^{\sigma t_*}\mathbb{E}V(t_*, x^{(0)}(t_*)) = \eta M_1 \mathbb{E}\|\zeta\|^p;\ e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) > \eta M_1 \mathbb{E}\|\zeta\|^p,\ t \in (t_*, t^*]. \tag{3.6}$$

In view of (3.5) and (3.6) for $\theta \in [-\tau, 0]$, one can see

$$e^{\sigma(t+\theta)}\mathbb{E}V(t+\theta, x^{(0)}(t+\theta)) \le \frac{1}{\eta}e^{\sigma t_*}\mathbb{E}V(t_*, x^{(0)}(t_*)) \le \frac{1}{\eta}e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)),\ t \in [t_*, t^*]. \tag{3.7}$$

That is,

$$\mathbb{E}V(t+\theta, x^{(0)}(t+\theta)) \le \frac{1}{\eta}e^{-\sigma\theta}\mathbb{E}V(t, x^{(0)}(t)) \le \frac{1}{\eta}e^{\sigma\tau}\mathbb{E}V(t, x^{(0)}(t)) \le q\mathbb{E}V(t, x^{(0)}(t)), \tag{3.8}$$

which lmplies that $\mathbb{E}\mathcal{L}V(t+\theta, x^{(0)}(t+\theta)) \le \mu(t)\mathbb{E}V(t, x^{(0)}(t))$ for all $t \in [t_*, t^*]$. From (3.1), (3.5) and (3.6), one has

$$\begin{aligned} \mathbb{E}V(t^*, x^{(0)}(t^*)) &= \mathbb{E}V(t_*, x^{(0)}(t_*))e^{\int_{t_*}^{t^*}\mu(s)\mathrm{d}s} = \eta M_1 \mathbb{E}\|\zeta\|^p e^{-\sigma t_*} e^{\int_{t_*}^{t^*}\mu(s)\mathrm{d}s} \\ &\le \eta M_1 \mathbb{E}\|\zeta\|^p e^{-\sigma t^*} e^{\int_{t_*}^{t^*}\sigma+\mu(s)\mathrm{d}s} \le \eta\mathbb{E}V(t^*, x^{(0)}(t^*))e^{(\sigma+\beta)r_1}, \end{aligned} \tag{3.9}$$

which means that $\eta e^{(\sigma+\beta)r_1} \ge 1$. Combining properties of the Poisson distribution, it can be concluded that

$$1 = \int_0^\infty \lambda e^{-\lambda r_1}\mathrm{d}(r_1) \le \int_0^\infty \lambda e^{-\lambda r_1}\eta e^{(\sigma+\beta)r_1}\mathrm{d}(r_1) = \frac{\lambda\eta}{\lambda - (\sigma+\beta)}. \tag{3.10}$$

Since $\eta = \min\{\eta, 1/\eta\}$ for $\eta \in (0, 1)$, which implies that (3.10) contradiction to the given condition $\lambda\min\{\eta, 1/\eta\}/(\lambda - (\sigma+\beta)) < 1$. That is, the inequality (3.2) holds on $[t_0, t_0 + r_1)$. Indeed, further assume that (3.2) holds on $[t_0, t_{n-1} + r_n), n \in \mathbb{N}$. Next, we prove that (3.2) holds on $[t_n, t_n + r_{n+1})$, where $t_n = t_{n-1} + r_n, n \in \mathbb{N}$. If this does not hold, then there exists some $t \in [t_n, t_n + r_{n+1})$ such that

$$\begin{aligned} & e^{\sigma t}\mathbb{E}V(t, x^{(n)}(t)) > M_1\mathbb{E}\|\zeta\|^p > \eta M_1\mathbb{E}\|\zeta\|^p \geq \eta\mathbb{E}V(t_n^-, x^{(n-1)}(t_n^-))e^{\sigma t_n^-} \\ \geq \ & \mathbb{E}V(t_n^-, x^{(n-1)}(t_n^-) + I_n(x^{(n-1)}(t_n^-)))e^{\sigma t_n^-} = e^{\sigma t_n}\mathbb{E}V(t_n, x^{(n)}(t_n)). \end{aligned} \tag{3.11}$$

Similarly, select the stopping time $\bar{t}^* = \inf\{t \in [t_n, t_n + r_{n+1}) : e^{\sigma t}\mathbb{E}V(t, x^{(n)}(t)) \geq M_1\mathbb{E}\|\zeta\|^p\}$ and $\bar{t}_* = \sup\{t \in [t_n, \bar{t}^*) : e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) \leq \eta M_1\mathbb{E}\|\zeta\|^p\}$ such that

$$e^{\sigma\bar{t}^*}\mathbb{E}V(\bar{t}^*, x^{(n)}(\bar{t}^*)) = M_1\mathbb{E}\|\zeta\|^p,\ e^{\sigma t}\mathbb{E}V(t, x^{(n)}(t)) < M_1\mathbb{E}\|\zeta\|^p,\ \forall t \in [t_n, \bar{t}^*); \tag{3.12}$$

$$e^{\sigma\bar{t}_*}\mathbb{E}V(\bar{t}_*, x^{(n)}(\bar{t}_*)) = \eta M_1\mathbb{E}\|\zeta\|^p,\ e^{\sigma t}\mathbb{E}V(t, x^{(n)}(t)) > \eta M_1\mathbb{E}\|\zeta\|^p,\ \forall t \in (\bar{t}_*, \bar{t}^*]. \tag{3.13}$$

By the definitions of $\bar{t}_*$ and $\bar{t}^*$ for any $t \in [\bar{t}_*, \bar{t}^*]$, it is easy to show that $e^{\sigma\bar{t}_*}\mathbb{E}V(\bar{t}_*, x^{(n)}(\bar{t}_*)) \leq e^{\sigma t}\mathbb{E}V(t, x^{(n)}(t)) \leq e^{\sigma\bar{t}^*}\mathbb{E}V(\bar{t}^*, x^{(n)}(\bar{t}^*))$. For $t + \theta \geq t_n$ and $\theta \in [-\tau, 0]$, we have

$$e^{\sigma(t+\theta)}\mathbb{E}V(t+\theta, x^{(n)}(t+\theta)) \leq M_1\mathbb{E}\|\xi\|^p = \frac{1}{\eta}e^{\sigma\bar{t}_*}\mathbb{E}V(\bar{t}_*, x^{(n)}(\bar{t}_*)) \leq \frac{1}{\eta}e^{\sigma t}\mathbb{E}V(t, x^{(n)}(t)),\ t \in [\bar{t}_*, \bar{t}^*]. \tag{3.14}$$

Combining (3.14), $\theta \in [-\tau, 0]$ and $q \geq e^{\sigma\tau}/\eta$ gives

$$\mathbb{E}V(t+\theta, x^{(n)}(t+\theta)) \leq \frac{1}{\eta}e^{-\sigma\theta}\mathbb{E}V(t, x^{(n)}(t)) \leq \frac{1}{\eta}e^{\sigma\tau}\mathbb{E}V(t, x^{(n)}(t)) \leq q\mathbb{E}V(t, x^{(n)}(t)). \tag{3.15}$$

Since (3.2) holds on $[t_0, t_{n-1} + r_n), n \in \mathbb{N}$, $t_n = t_{n-1} + r_n$ and $t + \theta < t_n$, which yields

$$\begin{aligned} & \mathbb{E}V(t+\theta, x^{(n)}(t+\theta)) \leq M_1\mathbb{E}\|\xi\|^p e^{-\sigma(t+\theta)} = \frac{1}{\eta}e^{-\sigma\theta}\eta M_1\mathbb{E}\|\xi\|^p e^{-\sigma\bar{t}_*}e^{\sigma(\bar{t}_*-t)} \\ \leq \ & \frac{1}{\eta}e^{-\sigma\theta}\mathbb{E}V(\bar{t}_*, x^{(n)}(\bar{t}_*)) \leq \frac{1}{\eta}e^{\sigma\tau}\mathbb{E}V(t, x^{(n)}(t)) \leq \mathbb{E}V(t, x^{(n)}(t)). \end{aligned} \tag{3.16}$$

In view of (3.16) and (H_3), we get

$$\begin{aligned} & \mathbb{E}V(\bar{t}^*, x^{(n)}(\bar{t}^*)) = \mathbb{E}V(\bar{t}_*, x^{(n)}(\bar{t}_*))e^{\int_{\bar{t}_*}^{\bar{t}^*}\mu(s)\mathrm{d}s} = \eta M_1\mathbb{E}\|\zeta\|^p e^{-\sigma\bar{t}_*}e^{\int_{\bar{t}_*}^{\bar{t}^*}\mu(s)\mathrm{d}s} \\ \leq \ & \eta M_1\mathbb{E}\|\zeta\|^p e^{-\sigma\bar{t}^*}e^{\int_{\bar{t}_*}^{\bar{t}^*}\sigma+\mu(s)\mathrm{d}s} \leq \eta\mathbb{E}V(\bar{t}^*, x^{(n)}(\bar{t}^*))e^{(\sigma+\beta)r_n},\ t \in [\bar{t}_*, \bar{t}^*], \end{aligned} \tag{3.17}$$

which implies that $\eta e^{(\sigma+\beta)r_n} \geq 1$. Notice that the random variable r_n follows an exponential distribution, which leads to the following estimate

$$1 = \int_0^\infty \lambda e^{-\lambda r_n}\mathrm{d}(r_n) \leq \int_0^\infty \lambda e^{-\lambda r_n}\eta e^{(\sigma+\beta)r_n}\mathrm{d}(r_n) = \frac{\lambda\eta}{\lambda - (\sigma+\beta)}, \tag{3.18}$$

which is contradicts the condition $\lambda\eta/(\lambda - (\sigma+\beta)) < 1$ given in (H_4). Therefore, (3.2) holds on $[t_n, t_n + r_{n+1})$, we further assert that (3.2) is true via the mathematical induction.

Finally, the Lemma 2.1 and (3.2) gives $\mathbb{E}V(t, x_r^{(k)}(t)) \leq M_1\mathbb{E}\|\zeta\|^p e^{-\sigma t}, t \geq t_0$. Meanwhile, From the (2) of reference [11] and (H_1), it is obtained that

$$\mathbb{E}|x_{N_t}(t)|^p \leq \frac{M_1}{c_1}\mathbb{E}\|\zeta\|^p e^{-\sigma t},\ x_{N_t}(t) = x_{N_t}(t, \zeta). \tag{3.19}$$

Case 2. $\eta \geq 1$. There exist a constant M_2 such that $c_2 \leq \frac{M_2}{\eta}$. Next, we verify that

$$\mathbb{E}V(t, x^{(k)}(t)) \leq M_2\eta^k \mathbb{E}\|\zeta\|^p e^{-\sigma t}, \ t \in [t_k, t_k + r_{k+1}). \tag{3.20}$$

For $t \in [t_0, t_0 + r_1)$, there is no impulse. If (3.20) is not true, there exist some $t \in [t_0, t_0 + r_1)$ such that

$$e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) > M_2\mathbb{E}\|\zeta\|^p > \frac{M_2}{\eta}\mathbb{E}\|\zeta\|^p. \tag{3.21}$$

Thus, let $\tilde{t}^* = \inf\{t \in [t_0, t_0 + r_1) : e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) > M_2\mathbb{E}\|\zeta\|^p\}$. From the continuity property of the function $e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t))$ on the interval $[t_0, t_0 + r_1)$, which yields

$$e^{\sigma \tilde{t}^*}\mathbb{E}V(\tilde{t}^*, x^{(0)}(\tilde{t}^*)) = M_2\mathbb{E}\|\zeta\|^p; \ e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) < M_2\mathbb{E}\|\zeta\|^p, \ t \in [t_0, \tilde{t}^*). \tag{3.22}$$

Let $\tilde{t}_* = \sup\{t \in [t_0 - \tau, \tilde{t}^*) : e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) \leq \frac{M_2}{\eta}\mathbb{E}\|\zeta\|^p\}$. The same procedure may be easily adapted to obtain that

$$\begin{cases} e^{\sigma \tilde{t}_*}\mathbb{E}V(\tilde{t}_*, x^{(0)}(\tilde{t}_*)) = \frac{M_2}{\eta}\mathbb{E}\|\zeta\|^p, \\ e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)) > \frac{M_2}{\eta}\mathbb{E}\|\zeta\|^p, \quad \forall t \in (\tilde{t}_*, \tilde{t}^*], \end{cases} \tag{3.23}$$

Noting that (3.22) and (3.23) for $\theta \in [-\tau, 0]$, it immediately follows that

$$e^{\sigma(t+\theta)}\mathbb{E}V(t+\theta, x^{(0)}(t+\theta)) \leq M_2\mathbb{E}\|\zeta\|^p = \eta e^{\sigma t_*}\mathbb{E}V(t_*, x^{(0)}(t_*)) \leq \eta e^{\sigma t}\mathbb{E}V(t, x^{(0)}(t)), \ t \in [t_*, t^*]. \tag{3.24}$$

In view of (3.24), $\eta = \max\{\eta, 1/\eta\}$, $\eta e^{\sigma\tau} \leq q$ for $\theta \in [-\tau, 0]$, which yields

$$\mathbb{E}V(t+\theta, x^{(0)}(t+\theta)) \leq \eta e^{-\sigma\theta}\mathbb{E}V(t, x^{(0)}(t)) \leq \eta e^{\sigma\tau}\mathbb{E}V(t, x^{(0)}(t)) \leq q\mathbb{E}V(t, x^{(0)}(t)). \tag{3.25}$$

By (3.25) and (H_3), we have

$$\begin{aligned} \mathbb{E}V(\tilde{t}^*, x^{(0)}(\tilde{t}^*)) &= \mathbb{E}V(\tilde{t}_*, x^{(0)}(\tilde{t}_*))e^{\int_{\tilde{t}_*}^{\tilde{t}^*}\mu(s)\mathrm{d}s} = \frac{M_2}{\eta}\mathbb{E}\|\zeta\|^p e^{-\sigma\tilde{t}_*}e^{\int_{\tilde{t}_*}^{\tilde{t}^*}\mu(s)\mathrm{d}s} \\ \leq \ & \frac{M_2}{\eta}\mathbb{E}\|\zeta\|^p e^{-\sigma\tilde{t}^*}e^{\int_{\tilde{t}_*}^{\tilde{t}^*}\sigma+\mu(s)\mathrm{d}s} \leq \frac{1}{\eta}\mathbb{E}V(\tilde{t}^*, x^{(0)}(\tilde{t}^*))e^{(\sigma+\beta)r_1}, \end{aligned} \tag{3.26}$$

which implies that $\frac{1}{\eta}e^{(\sigma+\beta)r_1} \geq 1$. Similarly to (3.10), we obtain $1 \leq \lambda/(\eta[\lambda-(\sigma+\beta)])$, which contradicts condition $\lambda/(\eta[\lambda - (\sigma+\beta)]) < 1$. Thus, inequality (3.20) holds on the interval $[t_0, t_0 + r_1)$. In general, assume that (3.20) holds on the interval $[t_0, t_{n-1} + r_n)$. By repeating the above analytical process, we obtain that inequality (3.20) valid for $k = n$. Then,

$$\mathbb{E}V(t, x_{N_t}) = \sum_{k=0}^{\infty}\mathbb{E}V(t, x_r^{(k)})\frac{e^{-\lambda t}(\lambda t)^k}{k!} \leq M_2\mathbb{E}\|\zeta\|^p e^{-(\sigma+\lambda)t}\sum_{k=0}^{\infty}\frac{(\lambda t)^k\eta^k}{k!} \leq M_2\mathbb{E}\|\zeta\|^p e^{-(\sigma+(1-\eta)\lambda)t}. \tag{3.27}$$

Combining (3.27) and (H_1), we get

$$\mathbb{E}|x_{N_t}|^p \leq \frac{M_2}{c_1}\mathbb{E}\|\zeta\|^p e^{-(\sigma+(1-\eta)\lambda)t}. \tag{3.28}$$

In view of (3.19), (3.28), and Definition 2.2, we conclude that system (2.1) is pth moment exponentially stable. This completes the proof.

Corollary 3.1. Assume that there exists a function $V \in C^{1,2}([t_0 - \tau, \infty) \times \mathbb{R}^n, \mathbb{R}^+)$ together with positive constants $p, c_1, c_2, \sigma > 0, \beta, \eta_k > -1, k \in \mathbb{N}$ such that the following conditions are satisfied:
($\mathbf{H_5}$): $c_1|x|^p \le V(t, x) \le c_2|x|^p$;
($\mathbf{H_6}$): For $k \in \mathbb{N}$ and $x \in \mathbb{R}^n$, $\mathbb{E}V(t, x + I_k(x)) \le (1 + \eta_k)\mathbb{E}V(t, x)$;
($\mathbf{H_7}$): For $t \ge t_0, t \ne t_k, k \in \mathbb{N}$ and $\xi \in PC_{\mathcal{F}_t}([-\tau, 0], \mathbb{R}^n)$, if $\mathbb{E}V(t + \theta, \xi(\theta)) \le q\mathbb{E}V(t, \xi(0))$, then $\mathbb{E}\mathcal{L}V(t + \theta, \xi) \le \mu(t)\mathbb{E}V(t, \xi(0))$, where $q \ge \max\{1 + \eta_k, \frac{1}{1+\eta_k}\}e^{\sigma\tau}$, $\mu : [t_0, \infty) \to \mathbb{R}$ and $\mu(t) \le \beta$;
($\mathbf{H_8}$): $\frac{\lambda \min\{1+\eta_k, \frac{1}{1+\eta_k}\}}{\lambda-(\sigma+\beta)} < 1$ and $\sigma - \eta_k\lambda > 0$ for all $k \in \mathbb{N}$.
Then, system (2.1) is pth moment exponentially stable.

Remark 3.1. According to Theorem 3.1, the system can be stabilized by impulsive control for any $\eta \in (0, 1)$, regardless of whether the underlying continuous-time dynamics are stable ($\beta \le 0$) or potentially unstable ($\beta > 0$). For the case $\eta \in [1, \infty)$, Condition (H_4) specifies the extent to which the system can tolerate impulsive effects without compromising stability. In particular, stability is guaranteed provided that $1 \le \eta < \frac{\sigma}{\lambda} + 1$.

Remark 3.2. It is evident that exponential stability of system (2.1) cannot be achieved under fixed impulse times when $\beta > 0$. However, our results not only reinforce the conclusions reported in [31, 32], which demonstrate that randomness in the impulse instants plays a crucial role in stabilizing the system, but also reveal that the impulse intensity η, the upper bound β of the Lyapunov differential operator, and the distribution parameter λ of the impulsive intervals must satisfy the condition $\lambda \min\{\eta, 1/\eta\}/[\lambda - (\sigma + \beta)] < 1$.

Remark 3.3. In Hu and Zhu [31], the coefficient of the upper bound of the Lyapunov differential operator is taken as a positive constant, namely, $\mathbb{E}\mathcal{L}V(t, \xi) \le v\mathbb{E}V(\xi(0))$, $v > 0$. In Hu and Zhu [32], $\mathbb{E}\mathcal{L}V(t, \xi) \le -\gamma\mathbb{E}V(\xi(0))$, with $\gamma > 0$. For systems with time-varying coefficients, as considered in this paper and in Hu and Zhu [31], identifying a suitable constant that satisfies the required condition is challenging. Even if such a constant were found, the condition would fail to accurately capture the properties of impulsive systems in which the continuous dynamics alternate between stable and unstable states. On the one hand, Theorem 3.1 provides a broader result that encompasses Hu and Zhu [31, 32], namely, $\mathbb{E}\mathcal{L}V(t, \xi) \le \mu(t)\mathbb{E}V(\xi(0))$, where $\mu(\cdot) : [t_0, \infty) \to (-\infty, \beta]$. On the other hand, the constants $M > 1$ and c_1 in Hu and Zhu [31] are required to simultaneously satisfy the constraints $c_1Me^{-\gamma r} > 1$ and $c_1e^{-\gamma r} \le 1$. However, our findings show that the restriction on c_1 in Hu and Zhu [31] is unnecessary, and this observation becomes particularly evident when $M = \max\{\eta, 1/\eta\}$.

4. Examples

In this section, we substantiate our main results by presenting effective illustrative examples as follows:

Example 4.1. Consider the following two-dimensional ISDEs:

$$\begin{cases} dx_1(t) = \frac{1}{8}x_1(t)e^{-2t}dt + \frac{1}{5}x_1(t - 0.5)\sin t dw_1(t) \\ dx_2(t) = \frac{1}{10}x_2(t)e^{-2t}dt + \frac{1}{5}x_2(t - 0.5)\sin t dw_2(t), t_k < t < t_{k+1}, \\ \triangle x_1(t_k) = -0.5x_1(t_k^-),\ \triangle x_2(t_k) = -0.5x_2(t_k^-), t = t_k, k = 1, 2, \cdots, \\ x_1(0) = \xi_1(\theta),\ x_2(0) = \xi_2(\theta),\ \theta \in [-0.5, 0]. \end{cases} \tag{4.1}$$

where $t_k = \sum_{i=1}^{k} r_i$, $\{r_i\}_{i=1}^{\infty}$ is a sequence and follows an independent exponential distribution of parameter $\lambda = 0.9$,

$$A = \begin{bmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{10} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}.$$

Let $V(t,x) = |x|^2$, which yields $V(t_k, x(t_k)) = |x(t_k)|^2 \leq 0.25|x(t_k^-)|^2$ and

$$\begin{aligned} &\mathbb{E}\mathcal{L}V(t,\xi) \leq \mathbb{E}\{(2\xi^T(0)Ae^{-2t}\xi(0)) + [(\xi(-0.5)\sin t)^T B^T B(\xi(-0.5)\sin t)]\} \\ \leq\ & 2\|A\|e^{-2t}\mathbb{E}|\xi(0)|^2 + \|B\|^2\mathbb{E}(|\xi(-0.5)|^2 \sin^2 t) \leq \frac{1}{4}\mathbb{E}|\xi(0)|^2 + \frac{1}{25}\mathbb{E}|\xi(-0.5)|^2. \end{aligned} \tag{4.2}$$

Then, let $\sigma = 0.1$, $\eta = 0.25$, $q = 4e^{\sigma\tau}$, $\beta = 0.5$ in Theorem 3.1. Based on this, we have

$$\frac{\lambda \min\{\eta, \frac{1}{\eta}\}}{\lambda - (\sigma + \beta)} = \frac{0.25 \times 0.9}{0.9 - (0.5 + 0.1)} < 1, \quad \sigma + (1-\eta)\lambda = 0.775 > 0. \tag{4.3}$$

From (4.2) and (4.3), it follows that system (4.1) satisfies the conditions of Theorem 3.1. According to Theorem 3.1, system (4.1) is exponentially stable, and the corresponding simulation is shown in Figure 1. In the left subfigure of Figure 1, three sample paths of the impulsive stochastic delay system are depicted for both state components $x_1(t)$ and $x_2(t)$. The trajectories illustrate the influence of stochastic perturbations and exponentially distributed impulse times (with parameter $\lambda = 1$), highlighting the qualitative behavior generated by the drift, diffusion, and impulsive dynamics. In the right subfigure of Figure 1, the curve of $\log(\mathbb{E}|x(t)|^2) - t$ computed from 100 simulated paths. The approximately linear decreasing trend provides numerical evidence of exponential stability, consistent with the theoretical results established in this manuscript.

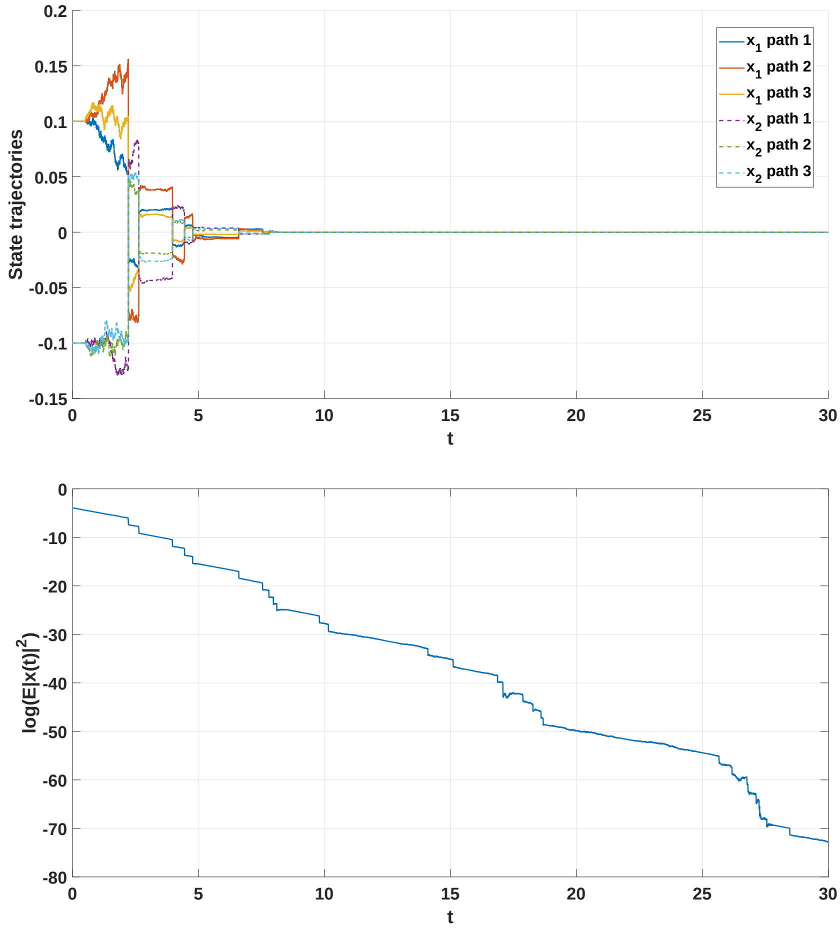

Figure 1. Sample paths of system (4.1) under different initial conditions. Evolution of $\mathbb{E}|x(t)|^2$ for system (4.1) across multiple sample paths.

Example 4.2. Consider the following two-dimensional ISDEs:

$$\begin{cases} dx_1(t) = -x_1(t)dt + x_1(t-\tau)dw_1(t) \\ dx_2(t) = -x_2(t)dt + x_2(t-\tau)dw_2(t), t_k < t < t_{k+1}, \\ \triangle x_1(t_k) = 0.1x_1(t_k^-),\ \triangle x_2(t_k) = 0.1x_2(t_k^-), t = t_k, k = 1, 2, \cdots, \\ x_1(0) = \xi_1(\theta),\ x_2(0) = \xi_2(\theta), \theta \in [-0.5, 0], \end{cases} \tag{4.4}$$

where $t_k = \sum_{i=1}^{k} r_i$ is a sequence and follows a Poisson distribution of parameter $\lambda = 1$,

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Let $V(t, x) = |x|^2$, which yields $V(t_k, x(t_k)) = |x(t_k)|^2 \leq 1.21|x(t_k^-)|^2$ and

$$\begin{aligned} &\mathbb{E}\mathcal{L}V(t,\xi) \leq \mathbb{E}\{(2\xi^T(0)A\xi(0)) + (\xi(-0.5)B^T B(\xi(-0.5))\} \\ \leq\ & 2\|A\|e^{-2t}\mathbb{E}|\xi(0)|^2 + \|B\|^2\mathbb{E}|\xi(-0.5)|^2 \leq -2\mathbb{E}|\xi(0)|^2 + \mathbb{E}|\xi(-0.5)|^2. \end{aligned} \tag{4.5}$$

Let $\sigma = 0.3, \eta = 1.21, q = 1.21e^{\sigma\tau}, \beta = -0.5$ in Theorem 3.1. Based on this, we get

$$\frac{\lambda \min\{\eta, \frac{1}{\eta}\}}{\lambda - (\sigma + \beta)} = \frac{1}{1.21[1 - (0.3 - 0.5)]} < 1, \qquad \sigma + (1 - \eta)\lambda = 0.3 + (1 - 1.21)1 = 0.09 > 0. \tag{4.6}$$

From (4.5) and (4.6), it follows that system (4.4) satisfies the conditions of Theorem 3.1. According to Theorem 3.1, system (4.4) is exponentially stable, and the corresponding simulation is shown in Figure 2.

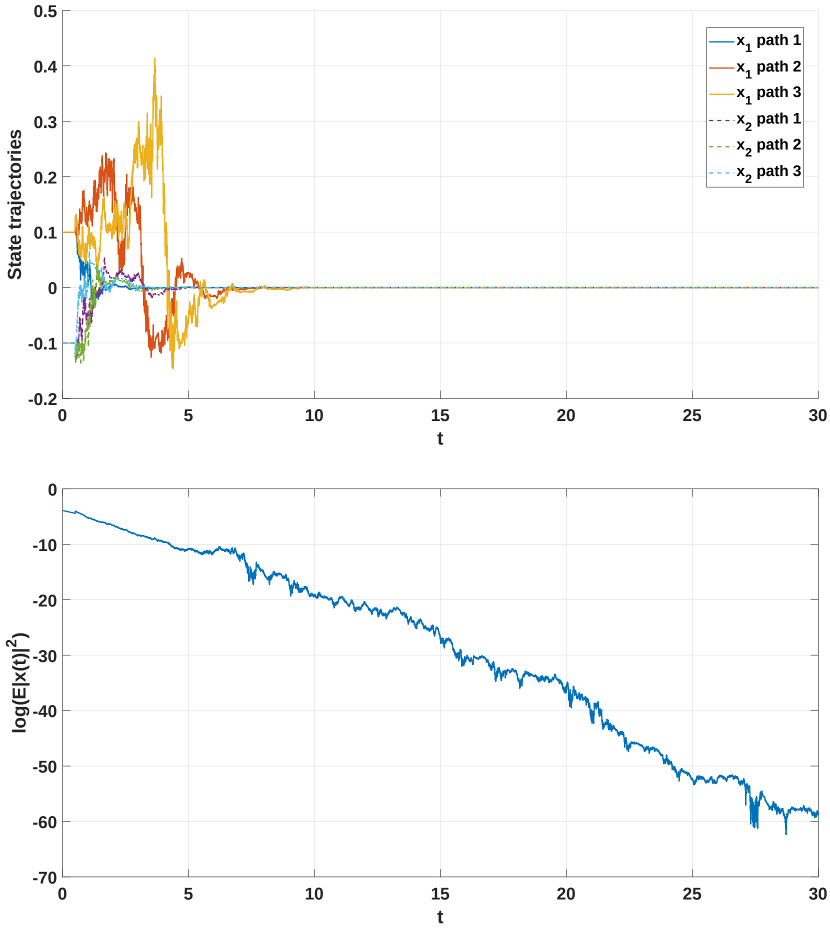

Figure 2. Sample paths of system (4.4) under different initial conditions. Evolution of $\mathbb{E}|x(t)|^2$ for system (4.4) across multiple sample paths.

5. Conclusions

In this paper, we establish novel criteria for the pth-moment exponential stability of stochastic functional differential equations (SFDEs) with randomly occurring impulses, employing the Razumikhin-type condition, Ito's formula, and stochastic analysis theory. To demonstrate the effectiveness and practicality of the proposed criteria, two illustrative examples are presented. For future research, we aim to extend this work by investigating the stability of such equations under both impulsive intensity and impulsive density, with particular emphasis on the application of vector Lyapunov functions.

Author contributions

Dongdong Gao: Writing-original draft, writing-review and editing, validation, supervision; Maosheng Ye: Conceptualization, writing-review and editing, resources. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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