AIMS Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(12): 29424–29453.
DOI: 10.3934/math.20251292
Received: 02 August 2025
Revised: 30 November 2025
Accepted: 08 December 2025
Published: 15 December 2025

Research article

On the construction of reversible DNA codes over $F_{4^{2m}}$ via T^n-set codes

Elif Segah Oztas[1], **Amal S. Alali**[2], **Shakir Ali**[3,4], **Mohd Azeem**[3,*], **Muhammad S. M. Asri**[4] and **Kok Bin Wong**[4]

[1] Department of Mathematics, Karamanoglu Mehmetbey University, Karaman-70100, Türkiye
[2] Department of Mathematical Sciences, College of Science, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh-11671, Saudi Arabia
[3] Department of Mathematics, Faculty of Science, Aligarh Muslim University, Aligarh-202002, India
[4] Institute of Mathematical Sciences, Faculty of Science, Universiti Malaya, Kuala Lumpur-50603, Malaysia

* **Correspondence:** Email: mohdazeem9211@gmail.com.

Abstract: The main objective of this paper is to introduce a new approach for constructing reversible and reversible DNA codes over the finite fields $F_{4^{2m}}, m \geq 1$ from any given polynomials by utilizing T^n-set codes. Notably, these polynomials are not required to be self-reciprocal divisors of $x^n - 1$. In addition to this method, we provide some results that demonstrate how to generate reversible and reversible DNA codes from any $[n, k, d]$-cyclic codes over $F_{4^{2m}}$ by applying T^n-set codes. Moreover, this approach allows us to determine a lower bound for the distance before completing the entire calculation process. We also construct a correspondence table between DNA sequences made up of 20-bases and all 256 elements of the linear code $\langle T^5_g \mid T^5_{(g^*)^{\circ 4}} \rangle$ over F_{16}, where $g = w^{13} + x + w^2x^2 + w^3x^3$ is a polynomial in x over F_{16} and $(g^*)^{\circ 4}$ represents the Hadamard 4^{th}-power of the reciprocal polynomial g^* to demonstrate the fact that non-reversible codes may correspond to reversible DNA codes. Additionally, we provide a detailed table presenting some outcomes related to l-MDS codes, self-orthogonal codes, and their associated parameters.

Keywords: reversible code; DNA code; reversible DNA code; T^n-set code; l-MDS code
Mathematics Subject Classification: 94B05, 94B15, 94B60, 94B65

1. Introduction

Bio-computing involves using biomolecules such as DNA (Deoxyribonucleic acid), RNA (Ribonucleic acid), and proteins to perform computational tasks, including encoding and processing data. DNA carries the genetic information of living organisms and is made up of four bases: adenine (A), guanine (G), thymine (T), and cytosine (C), which are called nucleotides. These bases form two strands that twist into a double helical structure and follow a pairing rule known as the Watson-Crick complement model. According to this model, adenine (A) pairs with thymine (T), and guanine (G) pairs with cytosine (C). This can be written as $A^c = T$, $T^c = A$, $G^c = C$, and $C^c = G$. In 1994, Adleman [3] introduced the field of DNA computing by solving a complex problem called the Hamiltonian path problem using the base pairing property of nucleotides and laboratory techniques. This experiment was based on the concept of DNA hybridization, which is crucial for DNA-based computations. However, DNA hybridization can lead to errors. To use the DNA for computing, specific sets of DNA sequences, known as DNA codes, were designed. These DNA codes meet certain constraints to avoid cross-hybridization, ensuring the sequences can perform the intended task correctly. Moreover, Boneh et al. [8] and Adleman et al. [4] demonstrated how DNA could be used to break the Data Encryption Standard (DES) algorithm. Many researchers have also explored DNA as a storage medium. The relationship between DNA and error-correcting codes has been studied by many researchers, including Liebovitch et al. [20], who looked for error-correcting codes in actual DNA sequences, and Brandao et al. [10], who further explored this area. DNA codes often include constraints like the Hamming distance constraint, reverse-complement constraint, reverse constraint, and fixed GC-weight constraint. These constraints are discussed in several studies. Generally, DNA codes are defined as codes over a ring R^n that meet at least one of these constraints. DNA corresponding to a code C keeps properties like being reversible or having a reversible complement.

Researchers have also constructed large sets of DNA codewords with specific minimum Hamming distances, as shown in previous studies (see [2, 18]). There have been further improvements and new methods for constructing DNA codes in [1, 11]. Some coding theorists have looked at four-element sets with algebraic structures because the DNA alphabet has four letters. Abualrub et al. [2] studied DNA codes using a finite field with four elements, and DNA codes over the finite ring $F_2[u]/(u^2-1)$ with four elements were explored in [17, 30]. Several researchers have used Z_4, F_4, and F_2 as base structures for ring extensions that correspond to DNA bases (viz.; [15, 29]). However, when the number of elements of the used algebraic structure is 4^{2m}, $m \geq 1$, generating DNA codes that satisfy the reverse-complement constraint is an open problem. This problem is called the DNA reversibility problem, which was introduced and studied in [26, 27]. The first solution for DNA codes over F_{16} was presented by Oztas and Siap [26], and later, this solution was generalized in [27] with the help of lifted polynomials. The main motivation for DNA codes originated from Adleman's experiment [3]. According to this experiment, the directed Hamiltonian path problem can be solved using reversible and reverse-complement DNA strings. Then, researchers focus on obtaining reversible and reversible DNA codes according to the code length. However, partitioning reversible DNA strings is unsuitable for the main motivation, as discussed in [14]. Building on these developments, researchers explored the properties of $[n, k, d]$-linear code C. The well-known Singleton bound $d \leq n-k+1$ leads to defining a non-negative integer $S(C) = n-k-d+1$, known as the Singleton defect of C (cf.; [7]). Moreover, when $d = n-k+1$, then C is a maximum distance separable (MDS) code [21]. When $S(C) = S(C^{\perp}) = l$, where $C^{\perp}$ is the dual code of C. Then, the code C is called l-maximum distance separable (l-MDS) code; these codes play a crucial role in various applications, including secret sharing schemes [24], binary index coding problems [32], and combinatorial designs [13]. In 2014, the concept of l-MDS code was introduced independently by Liao and Liao [19], and Tong et al. [33]. The class of l-MDS codes holds both theoretical and practical importance. Theoretically, many well-known linear codes are l-

MDS codes [19], such as binary and ternary extended Golay codes, quaternary (extended) quadratic-residual codes, q-ary Hamming codes, and twisted generalized Reed-Solomon codes (viz.; [16,21,31]). These codes are significant because l-MDS codes closely approach the maximum d (i.e., maximum of possible minimum distance) for a given n, k, and small l, especially when n is large relative to q, as the Singleton bound becomes less tight. For more examples and details, we refer the readers to [19, 33], where further references can be found. Reversible codes are important in DNA structure. The classical method of generating reversible codes over F_q (where q is a prime power) was first introduced by Massey in [23]. Later, Muttoo and Lal explored reversible codes over F_q (where q is a prime) in [25]. However, their codes were derived from a specially designed parity check matrix. In [12], Das and Tyagi proposed a generalized form of the parity check matrix to construct reversible codes for odd and even lengths. The method of generating reversible codes over F_q from any polynomials was introduced using restricted lifted polynomials (r-lifted polynomials) in [6]. However, in [6], r-lifted polynomials are generated by some factors of $x^n - 1$. Most of the existing algebraic constructions of reversible or reverse-complement DNA codes in the literature (e.g., [2, 5, 27]) fundamentally depend on conditions such as self-reciprocal divisors of $x^n - 1$, lifted or r–lifted polynomials, and often need additional selection among polynomials to meet DNA constraints. These requirements can increase the difficulty of finding suitable polynomials and reduce the set of eligible polynomials.

Motivated by this work, we explore a new method to construct reversible and reversible DNA codes from any polynomials. Moreover, the T^n-set approach developed here constructs reversible and reversible DNA codes from any polynomial $g(x)$ with $\deg g(x) < n$, without factoring $x^n - 1$ or restricting to self–reciprocal factors. Furthermore, the constructions $\langle T^n_g \mid T^n_{g^*} \rangle$ and $\langle T^n_g \mid T^n_{(g^*)^{\circ 4^m}} \rangle$ also offer a lower bound on the minimum Hamming distance before full calculation, which many earlier methods do not provide. Consequently, this methodology not only enlarges the design space for reversible and reversible-complement DNA codes but also guarantees inherent reversibility during construction, eliminating the need for post-processing or combinatorial verification. Finally, by utilizing the proposed methodology, we are able to derive l-MDS codes as well as self-orthogonal codes and their associated parameters.

The structure of this paper is as follows: In Section 2, we recall some basic definitions and notions. In Section 3, we prove some results to generate reversible codes using any polynomials and cyclic codes. In Section 4, we prove the theorems to generate reversible and reversible DNA codes by using any polynomials and cyclic codes. At the end of this paper, we present a correspondence table between DNA 20-bases and the elements of the linear code $\langle T^5_g \mid T^5_{(g^*)^{\circ 4}} \rangle$ over F_{16}. Moreover, we provide some results as a table that includes l-MDS codes, self-orthogonal codes, and their related parameters.

2. Preliminaries

We begin our discussions with the following notions: Let F_q be a finite field with q elements. Then, a linear code of length n over F_q is a subspace of F_q^n. A linear code of length n over F_q is said to be a cyclic code C of length n over F_q if it is invariant with respect to the right cyclic shift operator that maps a codeword $(c_0, c_1, \dots, c_{n-1}) \in C$ to another codeword $(c_{n-1}, c_0, \dots, c_{n-2})$ in C. For $n \geq 1$, let us consider a map $\gamma : F_q[x]/(x^n - 1) \longrightarrow C$ such that

$$\gamma(c_0 + c_1 x + \cdots + c_{n-1} x^{n-1}) = (c_0, c_1, \dots, c_{n-1}).$$

Now, from here onward, we can identify each codeword $(c_0, c_1, \ldots, c_{n-1}) \in C$ by the polynomial $c(x) = c_0 + c_1x + \cdots + c_{n-1}x^{n-1}$. For each codeword $c = (c_0, c_1, \ldots, c_{n-1})$, the reverse of c is defined as $c^r = (c_{n-1}, c_{n-2}, \ldots, c_0)$, where $c_i \in F_q$. To make the flow of ideas easier to follow, we summarize in Table 1 all the key notations and symbols that appear throughout the paper:

Table 1. Notations and symbols.

Symbol	Meaning
m, n, k, t	Positive integers; n is the code length, k is the dimension, and t is an integer used in Hadamard powers.
s, l	Non–negative integers; $s = \deg(g)$ denotes the degree of a polynomial $g(x)$, and l is the parameter in the l-MDS property.
p	A prime number (used in $q = p^m$).
q	A prime power, $q = p^m$.
F_q	Finite field with q elements.
F_{2^m}, $F_{4^{2m}}$	Finite fields of sizes 2^m and 4^{2m}, respectively; in particular $F_{4^{2m}}$ is used for DNA code constructions.
R	A commutative ring with identity.
w	A primitive element of F_{16}, or F_{256}.
F_q^n	The n-dimensional vector space of all n-tuples over F_q.
R^n	The R-module of all n-tuples.
$\mathbb{Z}_n$	The ring of integers modulo n.
C	A linear code of length n over F_q (or over R).
$[n, k, d]$	Standard parameters of a linear code: length n, dimension k, and minimum Hamming distance d.
$d(c, c')$	Hamming distance between two words c and c' of the same length.
$d(C)$	Minimum Hamming distance of a code C.
$w(c)$	Hamming weight of a codeword c.
$w(C)$	Minimum Hamming weight of a code C.
$S(C)$	Singleton defect of a code C, defined by $S(C) = n - k - d + 1$.
$C^\perp$	Dual code of C with respect to the standard Euclidean inner product.
MDS code	Maximum distance separable code.
AMDS code	Almost maximum distance separable code.
l-MDS code	A code C such that $S(C) = S(C^\perp) = l$.
c^r	The reverse of a codeword c.
A, G, C, T	DNA bases adenine (A), guanine (G), cytosine (C), and thymine (T).
x^c	Watson–Crick complement of a base $x \in \{A, G, C, T\}$; e.g., $A^c = T$, $T^c = A$, $G^c = C$, $C^c = G$.
Φ	DNA correspondence map from $F_{4^{2m}}$ (or F_{16}) to DNA 2-bases.
$\Phi(C)$	DNA code obtained as the coordinatewise image of a code C under Φ.
$\Phi(\alpha)^r$	Reverse of the DNA word $\Phi(\alpha)$.
$\Phi(\alpha)^c$	Watson–Crick complement of the DNA word $\Phi(\alpha)$.
γ	Coordinate map $\gamma : F_q[x]/(x^n - 1) \to F_q^n$ given by $\gamma(c_0 + c_1x + \cdots + c_{n-1}x^{n-1}) = (c_0, c_1, \ldots, c_{n-1})$.

Table 1. *(Continued.)* Notations and symbols.

Symbol	**Meaning**
$f(x)$, $g(x)$, $h(x)$	Polynomials over a field (or ring).
$f^*(x)$	Reciprocal polynomial of $f(x)$.
$P(x)$, $Q(x)$	Arbitrary polynomials over a ring R used to define the Hadamard product.
$P(x) \circ Q(x)$	Hadamard product of $P(x)$ and $Q(x)$.
$(P(x))^{\circ s}$	Hadamard s-power of a polynomial $P(x)$.
T^n_g	The T^n-generator set associated with a polynomial $g(x)$.
$T^n_{g^*}$	The T^n-generator set associated with the reciprocal polynomial $g^*(x)$ of a polynomial $g(x)$.
$T^n_{(g^*)^{\circ 2^t}}$	The T^n-generator set associated with the 2^t-th Hadamard power of the reciprocal polynomial $g^*(x)$ of polynomial $g(x)$.
$T^n_{(g^*)^{\circ 4^m}}$	The T^n-generator set associated with the 4^m-th Hadamard power of the reciprocal polynomial $g^*(x)$ of polynomial $g(x)$.
$G_{T^n_g}$	Generator matrix corresponding to the set T^n_g, obtained by writing the elements of T^n_g as rows in coefficient form or generator matrix of $\langle T^n_g \rangle$.
$G_{T^n_{g^*}}$	Generator matrix corresponding to the set $T^n_{g^*}$ or generator matrix $\langle T^n_{g^*} \rangle$.
$G_{T^n_{(g)^{\circ 2^t}}}$	Generator matrix corresponding to the set $T^n_{(g)^{\circ 2^t}}$ or generator matrix $\langle T^n_{(g)^{\circ 2^t}} \rangle$.
$G_{T^n_{(g)^{\circ 4^m}}}$	Generator matrix corresponding to the set $T^n_{(g)^{\circ 4^m}}$ or generator matrix of $\langle T^n_{(g)^{\circ 4^m}} \rangle$.
$\mathbf{r}_j$	The j-th row of a matrix of generator matrix $G_{T^n_g}$.
$\mathbf{r}'_j$	The j-th row of a matrix of generator matrix $G_{T^n_{g^*}}$.
$\mathbf{r}''_j$	The j-th row of a matrix of generator matrix $G_{T^n_{(g)^{\circ 4^m}}}$.
$\mathfrak{r}_j$	The j-th row of a matrix of generator matrix $[G_{T^n_g} \mid G_{T^n_{g^*}}]$; that is, $\mathfrak{r}_j = (\mathbf{r}_j \mid \mathbf{r}'_j)$.
$\mathfrak{r}^r_j$	The reverse of $\mathfrak{r}_j$, defined as $\mathfrak{r}^r_j = ((\mathbf{r}'_j)^r \mid (\mathbf{r}_j)^r)$.
$\mathfrak{r}'_j$	The j-th row of a matrix of generator matrix $[G_{T^n_g} \mid G_{T^n_{(g)^{\circ 4^m}}}]$; that is, $\mathfrak{r}'_j = (\mathbf{r}_j \mid \mathbf{r}''_j)$.
$\langle T^n_g \rangle$	The T^n-set code of length n over R generated by T^n_g, i.e., the R-module (or F_q-vector space) spanned by the rows of $G_{T^n_g}$.
$\langle g(x) \rangle$	Cyclic code of length n over F_q generated by $g(x)$, when $g(x)$ divides $x^n - 1$.
$\langle T^n_g \mid T^n_{g^*} \rangle$	Code constructed from a polynomial $g(x)$ and its reciprocal $g^*(x)$ using T^n-sets.
$\langle T^n_g \mid T^n_{(g^*)^{\circ 4^m}} \rangle$	Code constructed from $g(x)$ and the Hadamard 4^m-power of its reciprocal, used to obtain reversible DNA codes over $F_{4^{2m}}$.
$[G_{T^n_g} \mid G_{T^n_{g^*}}]$	Generator matrix of the code $\langle T^n_g \mid T^n_{g^*} \rangle$.
$[G_{T^n_g} \mid G_{T^n_{(g^*)^{\circ 4^m}}}]$	Generator matrix of the code $\langle T^n_g \mid T^n_{(g^*)^{\circ 4^m}} \rangle$.

Definition 1. *A code C of length n over a ring R is said to be a reversible code if, for any $c \in C$, $c^r \in C$.*

Let c and c' be two words of the same length. Then, the Hamming distance from c to c' is denoted by $d(c, c')$ and defined to be the number of places at which c and c' differ. Let C be a code containing at least two codewords. Then, the minimum distance (Hamming distance) of C is denoted by $d(C)$ and defined to be the minimum of all $d(c, c')$ with $c \neq c' \in C$. Additionally, let C be any code and $c \in C$. The weight (Hamming weight) of a codeword c is denoted by $w(c)$ and is defined as the number of non-zero coordinates in c. The minimum Hamming weight of the code C is the smallest of the weights of all non-zero codewords of C, and it is denoted by $w(C)$ (see, [21, 22] for details).

For an $[n, k, d]$-linear code C, the well-known Singleton bound states that $d \leq n - k + 1$, which leads to defining a non-negative integer $S(C) = n - k - d + 1$, called the Singleton defect of C (cf.; [7]). Moreover, when $d = n-k+1$, C is called a maximum distance separable (MDS) code [21]. If $d = n-k$, then C is called an almost maximum distance separable (AMDS) code [7].

Definition 2. *[19] A code C is called an l-MDS code, or said to satisfy the l-MDS property, if $S(C) = S(C^\perp) = l$, where $C^\perp$ is the dual code of C.*

The reciprocal of a polynomial $f(x) = c_0 + c_1 x + \cdots + c_s x^s$ with $c_s \neq 0$ is defined to be the polynomial $f^*(x) = x^s f(1/x) = c_s + c_{s-1} x + \cdots + c_0 x^s$. According to the reciprocal property, $\deg(f^*(x)) \leq \deg(f(x))$, and if $c_0 \neq 0$, then $\deg(f(x)) = \deg(f^*(x))$.

Definition 3. *[6] Let $s \geq 0$, $n \geq 1$ be fixed integers and $g(x) = z_0 + z_1 x + \cdots + z_s x^s$ be a polynomial of degree s over the ring R such that $\deg(g(x)) < n$. Then, the T_g^n generator set with respect to $g(x)$ is defined as: $T_g^n := \{g(x), xg(x), x^2 g(x), \ldots, x^{n-s-2} g(x), x^{n-s-1} g(x)\}$.*

$$
\begin{array}{rrcl}
\textit{Since,} & g(x) & = & z_0 + z_1 x + z_2 x^2 + \cdots + z_s x^s, \\
& xg(x) & = & z_0 x + z_1 x^2 + z_2 x^3 + \cdots + z_s x^{s+1}, \\
& x^2 g(x) & = & z_0 x^2 + z_1 x^3 + z_2 x^4 + \cdots + z_s x^{s+2}, \\
& \vdots & \vdots & \vdots \\
& x^{n-s-2} g(x) & = & z_0 x^{n-s-2} + z_1 x^{n-s-1} + z_2 x^{n-s} + \cdots + z_{s-1} x^{n-3} + z_s x^{n-2}, \\
& x^{n-s-1} g(x) & = & z_0 x^{n-s-1} + z_1 x^{n-s} + z_2 x^{n-s+1} + \cdots + z_{s-2} x^{n-3} + z_{s-1} x^{n-2} + z_s x^{n-1}.
\end{array}
$$

Therefore, the matrix form of T_g^n generator set as follows:

$$
G_{T_g^n} = \begin{bmatrix} g(x) \\ xg(x) \\ x^2 g(x) \\ \vdots \\ x^{n-s-2} g(x) \\ x^{n-s-1} g(x) \end{bmatrix} = \begin{bmatrix}
z_0 & z_1 & z_2 & \cdots & z_s & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & z_0 & z_1 & z_2 & \cdots & z_s & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & z_0 & z_1 & z_2 & \cdots & z_s & 0 & \cdots & 0 & 0 \\
\cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\
\cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\
0 & 0 & \cdots & z_0 & z_1 & z_2 & \cdots & & z_{s-1} & z_s & 0 \\
0 & 0 & 0 & \cdots & z_0 & z_1 & z_2 & \cdots & z_{s-2} & z_{s-1} & z_s
\end{bmatrix}. \qquad (2.1)
$$

Since, $\langle T_g^n \rangle$ is an R-module, so a code generated by T_g^n, i.e., $\langle T_g^n \rangle$ is called a T^n-set code of length n over R. Moreover, it is easy to observe that $\langle T_g^n \rangle$ generates a F_q-vector space if $R = F_q$.

Let $P(x) = \sum_{i=0}^{m} a_i x^i$ and $Q(x) = \sum_{i=0}^{n} b_i x^i$ be two polynomials over the ring R, then the Hadamard product $H(x)$ of $P(x)$ and $Q(x)$ is given by $H(x) = P(x) \circ Q(x)$, where $\circ$ denotes the Hadamard product *and the polynomial* $H(x)$ *is defined by its coefficients as follows:* $H(x) = \sum_{i=0}^{\min(m,n)} c_i x^i$, *where* $c_i = a_i b_i$ *for* $i = 0, 1, \ldots, \min(m, n)$. *The Hadamard s-power of* $P(x)$ *is denoted by* $(P(x))^{\circ s}$ *and defined as:* $(P(x))^{\circ s} = \sum_{i=0}^{m} (a_i)^s x^i$.

3. Reversible codes with T^n-set codes and cyclic codes

We now establish a foundational result, which shows that the T^n-set construction preserves the fundamental parameters of the corresponding code.

Theorem 1. *Let* $g(x)$ *be a polynomial over* F_q *and* $\deg(g(x)) < n$. *Then, the codes* $\langle T_g^n \rangle$ *and* $\langle T_{g^*}^n \rangle$ *have same* $[n, k, d]$ *parameters.*

Proof. Let $g(x) = z_0 + z_1 x + z_2 x^2 + \cdots + z_s x^s$ be a polynomial over F_q such that $\deg(g(x)) < n$ and $g^*(x) = z_s + z_{s-1}x + z_{s-2}x^2 + \cdots + z_0 x^s$ be the reciprocal polynomial of $g(x)$. Since $\deg(g^*(x)) \leq \deg(g(x))$ and $\deg(g(x)) < n$, so $\deg(g^*(x)) < n$. Then, the $T_{g^*}^n$ generator set with respect to $g^*(x)$ is $\{g^*(x), xg^*(x), x^2 g^*(x), \ldots, x^{n-s-2} g^*(x), x^{n-s-1} g^*(x)\}$. Moreover, the matrix form of $T_{g^*}^n$ generator set is

$$G_{T_{g^*}^n} = \begin{bmatrix} g^*(x) \\ xg^*(x) \\ x^2 g^*(x) \\ \vdots \\ x^{n-s-2} g^*(x) \\ x^{n-s-1} g^*(x) \end{bmatrix} = \begin{bmatrix} z_s & z_{s-1} & z_{s-2} & \cdots & z_0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & z_s & z_{s-1} & z_{s-2} & \cdots & z_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & z_s & z_{s-1} & z_{s-2} & \cdots & z_0 & 0 & \cdots & 0 & 0 \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ 0 & 0 & \cdots & z_s & z_{s-1} & z_{s-2} & \cdots & z_2 & z_1 & z_0 & 0 \\ 0 & 0 & \cdots & 0 & z_s & z_{s-1} & z_{s-2} & \cdots & z_2 & z_1 & z_0 \end{bmatrix}. \tag{3.1}$$

Since, $\langle T_g^n \rangle = \langle G_{T_g^n} \rangle$ and $\langle T_{g^*}^n \rangle = \langle G_{T_{g^*}^n} \rangle$, so from Eqs (2.1) and (3.1). It is easy to see that, all codewords of $\langle T_g^n \rangle$ are found in $\langle T_{g^*}^n \rangle$ in reverse order. This implies a direct relationship between these two codes, where the arrangement of elements in one generator set mirrors the reverse sequence of elements in the other. Additionally, if the code $\langle T_g^n \rangle$ has $[n, k, d]$ parameters, then $\langle T_{g^*}^n \rangle$ has the same $[n, k, d]$ parameters. This completes the proof. □

Remark 1. *Let* $g(x)$ *be a polynomial over* F_q *with* $g(x) \mid (x^n - 1)$. *Then, matrix of* T_g^n *is a generator matrix of the cyclic code* $C = \langle g(x) \rangle$.

Remark 2. *In the majority of our examples, we work with the finite field* F_{16}, *i.e.,* $F_{16} = \frac{F_2[w]}{\langle 1+w+w^4 \rangle} = \{\alpha_0 + \alpha_1 w + \alpha_2 w^2 + \alpha_3 w^3 \mid \alpha_0, \alpha_1, \alpha_2, \alpha_3 \in F_2\} = \{0, 1, w, w^2, w^3, w^4 = 1 + w, w^5 = w + w^2, w^6 = w^2 + w^3, w^7 = 1 + w + w^3, w^8 = 1 + w^2, w^9 = w + w^3, w^{10} = 1 + w + w^2, w^{11} = w + w^2 + w^3, w^{12} = 1 + w + w^2 + w^3, w^{13} = 1 + w^2 + w^3, w^{14} = 1 + w^3\}$.

The method of generating reversible codes over F_q from any polynomials was introduced using a restricted lifted polynomial (r-lifted polynomial) in [6]. However, in [6], r-lifted polynomials are generated by factors of $x^n - 1$. Here, however, we do not need to have any factor of any polynomial. Thus, the advantage of the following theorem is that we can produce reversible codes with any polynomial instead of self-reciprocal polynomials.

Theorem 2. *Let $g(x)$ be any polynomial over a commutative ring R with $\deg(g(x)) < n$ and $g^*(x)$ be the reciprocal polynomial of $g(x)$. Then, $\langle T_g^n \mid T_{g^*}^n\rangle$ is a reversible code over R, where the codes $\langle T_g^n\rangle$ and $\langle T_{g^*}^n\rangle$ have the same $[n, k, d]$ parameters.*

Proof. Let $G_{T_g^n}$ and $G_{T_{g^*}^n}$ be generator matrices of $\langle T_g^n\rangle$ and $\langle T_{g^*}^n\rangle$, respectively, as in Eqs (2.1) and (3.1). Their rows are obtained by successive right shifts of the coefficient vectors of $g(x)$ and $g^*(x)$, so the rows of $G_{T_g^n}$ and $G_{T_{g^*}^n}$ appear in reverse pairs. More precisely, if the rows of $G_{T_g^n}$ are $\mathbf{r}_1, \ldots, \mathbf{r}_k$ and the rows of $G_{T_{g^*}^n}$ are $\mathbf{r'}_1, \ldots, \mathbf{r'}_k$, then

$$\mathbf{r'}_j = \mathbf{r}_{k+1-j}^r \quad \text{and} \quad \mathbf{r}_j = \mathbf{r'}_{k+1-j}^r, \qquad 1 \le j \le k,$$

where $(\cdot)^r$ denotes coordinatewise reversal. The generator matrix of $C = \langle T_g^n \mid T_{g^*}^n\rangle$ is

$$G = [\, G_{T_g^n} \mid G_{T_{g^*}^n} \,],$$

and we denote its rows by $\mathfrak{r}_i = (\mathbf{r}_i \mid \mathbf{r'}_i)$, $1 \le i \le k$. The reversal of $\mathfrak{r}_j$ is

$$\mathfrak{r}_j^r = (\mathbf{r'}_j^r \mid \mathbf{r}_j^r) = (\mathbf{r}_{k+1-j} \mid \mathbf{r'}_{k+1-j}) = \mathfrak{r}_{k+1-j}, \qquad 1 \le j \le \left\lceil \frac{k}{2} \right\rceil.$$

Here, $\lceil\cdot\rceil$ denotes the greatest integer function. Let $\gamma : R[x]/(x^n - 1) \to R^n$ be the standard coordinate map. For any $\alpha \in R$ and any row polynomial $\mathfrak{r}_j(x)$ corresponding to the row $\mathfrak{r}_j$, linearity of γ and of reversal gives

$$\gamma(\alpha\mathfrak{r}_j(x))^r = (\alpha\,\gamma(\mathfrak{r}_j(x)))^r = \alpha\,\gamma(\mathfrak{r}_j(x))^r = \alpha\,\gamma(\mathfrak{r}_{k+1-j}(x)) = \gamma(\alpha\mathfrak{r}_{k+1-j}(x)). \tag{3.2}$$

Thus, the reversal of a scalar multiple of a generator row is again a scalar multiple of a generator row.

Now, let $c(x) \in C$, then $c(x) = \sum_{i=1}^k \alpha_i\mathfrak{r}_i(x)$ for some $\alpha_i \in R$. Using linearity of reversal and the Eq (3.2), we have

$$\gamma(c(x))^r = \left(\sum_{i=1}^k \alpha_i\gamma(\mathfrak{r}_i(x))\right)^r = \sum_{i=1}^k \alpha_i\,\gamma(\mathfrak{r}_i(x))^r = \sum_{i=1}^k \alpha_i\,\gamma(\mathfrak{r}_{k+1-i}(x)) \in C.$$

Hence, $C = \langle T_g^n \mid T_{g^*}^n\rangle$ is reversible. This completes the proof. □

The following theorem provides a method to determine a lower bound for the distance d_T of the reversible code $\langle T_g^n \mid T_{g^*}^n\rangle$ before completing the entire calculation process, i.e., $d_T \ge 2d$, where d is the distance of the given linear code $\langle T_g^n\rangle$ or $\langle T_{g^*}^n\rangle$.

Theorem 3. *Let $g(x)$ be a polynomial over $F_{4^{2m}}$ with $\deg(g(x)) < n$ and $g^*(x)$ be the reciprocal polynomial of $g(x)$. Then, the reversible code $\langle T_g^n \mid T_{g^*}^n\rangle$ has parameters $[2n, k, d_T]$, where $d_T \ge 2d$ and $\langle T_g^n\rangle$ is a $[n, k, d]$-code.*

Proof. Let $g(x) = z_0 + z_1x + \cdots + z_sx^s$ with $\deg(g) = s$. From Eq (2.1), the generator matrix $G_{T_g^n}$ of $\langle T_g^n\rangle$ has $k = n - s$ rows, each obtained by successive right shifts of the coefficient vector $(z_0, z_1, \ldots, z_s)$. In particular,

$$\mathbf{r}_1 = [z_0, z_1, \ldots, z_s, 0, \ldots, 0], \mathbf{r}_2 = [0, z_0, z_1, \ldots, z_s, 0, \ldots, 0], \ldots, \mathbf{r}_k = [0, \ldots, 0, z_0, z_1, \ldots, z_s].$$

For the reciprocal polynomial $g^*(x) = x^s g(1/x) = z_s + z_{s-1}x + z_{s-2}x^2 + \cdots + z_0 x^s$, the generator matrix $G_{T^n_{g^*}}$ in Eq (3.1) consists of successive shifts of the reversed coefficient vector $(z_s, z_{s-1}, \ldots, z_0)$:

$$\mathbf{r}'_1 = [z_s, z_{s-1}, \ldots, z_0, 0, \ldots, 0], \mathbf{r}'_2 = [0, z_s, z_{s-1}, \ldots, z_0, 0, \ldots, 0], \ldots, \mathbf{r}'_k = [0, \ldots, 0, z_s, z_{s-1}, \ldots, z_0].$$

Hence, each row of Eq (2.1) corresponds to the reverse of a row in Eq (3.1), that is, the first row of $G_{T^n_g}$ reverses to the last row of $G_{T^n_{g^*}}$, and so on. More generally, the j-th row of Eq (3.1) is the reverse of the $(k-j+1)$-th row of Eq (2.1), i.e., $\mathbf{r}'_j = (\mathbf{r}_{k-j+1})^r$. Re-indexing with $j = k - i + 1$ yields

$$(\mathbf{r}_i)^r = \mathbf{r}'_{k-i+1}. \tag{3.3}$$

It is easy to see that the length and dimension of the reversible code $\langle T^n_g \mid T^n_{g^*}\rangle$ depend on the structures of T^n_g and $T^n_{g^*}$. From Theorem 1, the codes $\langle T^n_g\rangle$ and $\langle T^n_{g^*}\rangle$ have same $[n, k, d]$ parameters. Hence, the length of $\langle T^n_g \mid T^n_{g^*}\rangle$ = length of $\langle T^n_g\rangle$ + length of $\langle T^n_{g^*}\rangle = n+n = 2n$ and its dimension is k. Thus, our goal is to determine the minimum distance of the reversible code $\langle T^n_g \mid T^n_{g^*}\rangle$. For any codeword $c \in \langle T^n_g\rangle$, there exist $a_1, a_2, \ldots, a_k \in F_{4^{2m}}$ such that

$$c = a_1\mathbf{r}_1 + a_2\mathbf{r}_2 + \cdots + a_k\mathbf{r}_k. \tag{3.4}$$

Since the minimum Hamming distance of linear code $\langle T^n_g\rangle$ is d, the minimum Hamming weight of $\langle T^n_g\rangle$ is also d. Thus, $w(c) \geq d$. Now, taking the reverse on both sides of Eq (3.4) and using the linearity of the reversal operator, we obtain

$$\begin{aligned} c^r &= (a_1\mathbf{r}_1 + a_2\mathbf{r}_2 + \cdots + a_k\mathbf{r}_k)^r \\ &= a_1(\mathbf{r}_1)^r + a_2(\mathbf{r}_2)^r + \cdots + a_k(\mathbf{r}_k)^r \\ &= a_1\mathbf{r}'_k + a_2\mathbf{r}'_{k-1} + \cdots + a_k\mathbf{r}'_1 \\ &= a_k\mathbf{r}'_1 + a_{k-1}\mathbf{r}'_2 + \cdots + a_1\mathbf{r}'_k. \end{aligned}$$

Since $\langle T^n_{g^*}\rangle$ is a linear code generated by the rows $\mathbf{r}'_1, \ldots, \mathbf{r}'_k$, so the vector c^r belongs to $\langle T^n_{g^*}\rangle$. Denote this vector by c', that is, $c' = a_k\mathbf{r}'_1 + a_{k-1}\mathbf{r}'_2 + \cdots + a_1\mathbf{r}'_k$. Therefore,

$$c^r = c'.$$

Hence, $w(c') \geq d$. Consequently, we conclude that $d(\langle T^n_g \mid T^n_{g^*}\rangle) \geq d + d = 2d$. This completes the proof. □

Theorem 2 shows how we generate a reversible code, while Theorem 3 demonstrates the minimum limit of the Hamming distance of generated reversible codes. The following example justifies these facts.

Example 1. *Let $g(x) = w + w^2x + w^3x^2 + w^4x^3$ be a polynomial over F_{16} and $n = 7$. Then, the matrix of the T^7_g generator set is*

$$\begin{bmatrix} w & w^2 & w^3 & w^4 & 0 & 0 & 0 \\ 0 & w & w^2 & w^3 & w^4 & 0 & 0 \\ 0 & 0 & w & w^2 & w^3 & w^4 & 0 \\ 0 & 0 & 0 & w & w^2 & w^3 & w^4 \end{bmatrix}.$$

Thus, $\langle T_g^7 \rangle$ *is a* $[7,4,2]$*-linear code over* F_{16}*. We obtain the reciprocal polynomial* $g^*(x) := w^4 + w^3x + w^2x^2 + wx^3$*. Further, the matrix of* $T_{g^*}^7$ *generator set is*

$$\begin{bmatrix} w^4 & w^3 & w^2 & w & 0 & 0 & 0 \\ 0 & w^4 & w^3 & w^2 & w & 0 & 0 \\ 0 & 0 & w^4 & w^3 & w^2 & w & 0 \\ 0 & 0 & 0 & w^4 & w^3 & w^2 & w \end{bmatrix}.$$

Therefore, $\langle T_{g^*}^7 \rangle$ *is a* $[7,4,2]$*-linear code over* F_{16}*. In view of Theorems 2 and 3, the generator matrix of* $C = \langle T_g^7 \mid T_{g^*}^7 \rangle$ *is*

$$\begin{bmatrix} w & w^2 & w^3 & w^4 & 0 & 0 & 0 & w^4 & w^3 & w^2 & w & 0 & 0 & 0 \\ 0 & w & w^2 & w^3 & w^4 & 0 & 0 & 0 & w^4 & w^3 & w^2 & w & 0 & 0 \\ 0 & 0 & w & w^2 & w^3 & w^4 & 0 & 0 & 0 & w^4 & w^3 & w^2 & w & 0 \\ 0 & 0 & 0 & w & w^2 & w^3 & w^4 & 0 & 0 & 0 & w^4 & w^3 & w^2 & w \end{bmatrix}.$$

Hence, the code C *is a reversible* $[14,4,7]$*-linear code over* F_{16}*, and the minimum distance* $d(C) = 7 > 2 \cdot d(\langle T_g^7 \rangle) = 2 \cdot 2 = 4$*.*

Theorem 4. *Let* $C = \langle g(x) \rangle$ *be a* $[n,k,d]$*-cyclic code over* $F_{4^{2m}}$*,* $g(x) \mid (x^n - 1)$ *and* $g^*(x)$ *be the reciprocal polynomial of* $g(x)$*. Then, the code* $C' = \langle \langle g(x) \rangle \mid T_{g^*}^n \rangle$ *is reversible over* $F_{4^{2m}}$*. Moreover,* C' *has parameters* $[2n, k, d_T]$*, where* $d_T \geq 2d$*.*

Proof. The proof is similar to that of Theorem 2. □

Example 2. *Let* $g(x) = w^9 + w^{13}x + w^{10}x^2 + w^2x^3 + x^4 + w^{12}x^5 + w^7x^6 + w^{12}x^7 + w^9x^8 + w^2x^9 + w^4x^{10} + wx^{11} + x^{12}$ *be a polynomial over* F_{16}*,* $n = 15$ *and* $g(x) \mid (x^{15} - 1)$ *over* F_{16}*. Then, the generator matrix of* $\langle g(x) \rangle$ *is*

$$\begin{bmatrix} w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 & 0 \\ 0 & w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 \\ 0 & 0 & w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 \end{bmatrix}.$$

Thus, $C = \langle g(x) \rangle$ *is a* $[15,3,10]$*-cyclic code over* F_{16}*. Further, we have the reciprocal polynomial* $g^*(x) := 1 + wx + w^4x^2 + w^2x^3 + w^9x^4 + w^{12}x^5 + w^7x^6 + w^{12}x^7 + x^8 + w^2x^9 + w^{10}x^{10} + w^{13}x^{11} + w^9x^{12}$*. Then, the matrix of the* $T_{g^*}^{15}$ *generator set is*

$$\begin{bmatrix} 1 & w & w^4 & w^2 & w^9 & w^{12} & w^7 & w^{12} & 1 & w^2 & w^{10} & w^{13} & w^9 & 0 & 0 \\ 0 & 1 & w & w^4 & w^2 & w^9 & w^{12} & w^7 & w^{12} & 1 & w^2 & w^{10} & w^{13} & w^9 & 0 \\ 0 & 0 & 1 & w & w^4 & w^2 & w^9 & w^{12} & w^7 & w^{12} & 1 & w^2 & w^{10} & w^{13} & w^9 \end{bmatrix}.$$

Therefore, $\langle T_{g^*}^{15} \rangle$ *is a* $[15,3,10]$*-cyclic code over* F_{16}*. Then, by Theorem* 4*, the generator matrix of* $C' = \langle \langle g(x) \rangle \mid T_{g^*}^{15} \rangle$ *is*

$$\begin{bmatrix} w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 & 0 & 1 & w & w^4 & w^2 & w^9 & w^{12} & w^7 & w^{12} & 1 & w^2 & w^{10} & w^{13} & w^9 & 0 & 0 \\ 0 & w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 & 0 & 1 & w & w^4 & w^2 & w^9 & w^{12} & w^7 & w^{12} & 1 & w^2 & w^{10} & w^{13} & w^9 & 0 \\ 0 & 0 & w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 & 0 & 1 & w & w^4 & w^2 & w^9 & w^{12} & w^7 & w^{12} & 1 & w^2 & w^{10} & w^{13} & w^9 \end{bmatrix}.$$

Hence, the code C' *is a reversible* $[30,3,24]$*-linear code over* F_{16}*, and the minimum distance* $d(C') = 24 > 2 \cdot d(C) = 2 \cdot 10 = 20$*.*

Example 3. *Let* $g(x) = 1 + w^{10}x + w^6x^2 + w^5x^3 + w^{11}x^4 + w^9x^5 + w^3x^6$ *be a polynomial over the field* F_{256} *and* $n = 8$*. Then, the matrix of the* T_g^8 *generator set is*

$$\begin{bmatrix} 1 & w^{10} & w^6 & w^5 & w^{11} & w^9 & w^3 & 0 \\ 0 & 1 & w^{10} & w^6 & w^5 & w^{11} & w^9 & w^3 \end{bmatrix}.$$

Hence, $\langle T_g^8 \rangle$ *is a* $[8, 2, 7]$*-linear code over* F_{256}*. The reciprocal polynomial of* $g(x)$ *is* $g^*(x) := w^3 + w^9x + w^{11}x^2 + w^5x^3 + w^6x^4 + w^{10}x^5 + x^6$*. Further, the matrix of the* $T_{g^*}^8$ *generator set is*

$$\begin{bmatrix} w^3 & w^9 & w^{11} & w^5 & w^6 & w^{10} & 1 & 0 \\ 0 & w^3 & w^9 & w^{11} & w^5 & w^6 & w^{10} & 1 \end{bmatrix}.$$

Therefore, the code $\langle T_{g^*}^8 \rangle$ *is a* $[8, 2, 7]$*-linear code over* F_{256}*. In view of Theorems 2 and 3, the generator matrix of* $C = \langle T_g^8 \mid T_{g^*}^8 \rangle$ *is*

$$\left[\begin{array}{cccccccccccccccc} 1 & w^{10} & w^6 & w^5 & w^{11} & w^9 & w^3 & 0 & w^3 & w^9 & w^{11} & w^5 & w^6 & w^{10} & 1 & 0 \\ 0 & 1 & w^{10} & w^6 & w^5 & w^{11} & w^9 & w^3 & 0 & w^3 & w^9 & w^{11} & w^5 & w^6 & w^{10} & 1 \end{array} \right].$$

Hence, the code C is a reversible $[16, 2, 14]$*-linear code over* F_{256}*. Also, C is a self-orthogonal and 1-MDS (AMDS) code. Moreover, the dual of* $\langle T_g^8 \mid T_{g^*}^8 \rangle$ *is a* $[16, 14, 2]$*-linear code over* F_{256}*.*

Example 4. *Let* $g(x) = w^{13} + x + w^2x^2 + w^3x^3 + w^6x^4 + x^5 + w^5x^6 + w^6x^7$ *be a polynomial over* F_{16} *and* $n = 10$*. Then, the matrix of the* T_g^{10} *generator set is*

$$\begin{bmatrix} w^{13} & 1 & w^2 & w^3 & w^6 & 1 & w^5 & w^6 & 0 & 0 \\ 0 & w^{13} & 1 & w^2 & w^3 & w^6 & 1 & w^5 & w^6 & 0 \\ 0 & 0 & w^{13} & 1 & w^2 & w^3 & w^6 & 1 & w^5 & w^6 \end{bmatrix}.$$

Thus, $\langle T_g^{10} \rangle$ *is a* $[10, 3, 7]$*-linear code over* F_{16}*. Moreover, the reciprocal polynomial* $g^*(x) := w^6 + w^5x + x^2 + w^6x^3 + w^3x^4 + w^2x^5 + x^6 + w^{13}x^7$*. We obtain the matrix of the* $T_{g^*}^{10}$ *generator set as*

$$\begin{bmatrix} w^6 & w^5 & 1 & w^6 & w^3 & w^2 & 1 & w^{13} & 0 & 0 \\ 0 & w^6 & w^5 & 1 & w^6 & w^3 & w^2 & 1 & w^{13} & 0 \\ 0 & 0 & w^6 & w^5 & 1 & w^6 & w^3 & w^2 & 1 & w^{13} \end{bmatrix}.$$

Hence, $\langle T_{g^*}^{10} \rangle$ *is a* $[10, 3, 7]$*-linear code over* F_{16}*.*
According to Theorems 2 and 3, the generator matrix of $\langle T_g^{10} \mid T_{g^*}^{10} \rangle$ *is*

$$\left[\begin{array}{cccccccccccccccccccc} w^{13} & 1 & w^2 & w^3 & w^6 & 1 & w^5 & w^6 & 0 & 0 & w^6 & w^5 & 1 & w^6 & w^3 & w^2 & 1 & w^{13} & 0 & 0 \\ 0 & w^{13} & 1 & w^2 & w^3 & w^6 & 1 & w^5 & w^6 & 0 & 0 & w^6 & w^5 & 1 & w^6 & w^3 & w^2 & 1 & w^{13} & 0 \\ 0 & 0 & w^{13} & 1 & w^2 & w^3 & w^6 & 1 & w^5 & w^6 & 0 & 0 & w^6 & w^5 & 1 & w^6 & w^3 & w^2 & 1 & w^{13} \end{array} \right].$$

Hence, the code $\langle T_g^{10} \mid T_{g^*}^{10} \rangle$ *is reversible* $[20, 3, 16]$*- linear code over* F_{16}*, which is self-orthogonal as well. Moreover, the dual of* $\langle T_g^{10} \mid T_{g^*}^{10} \rangle$ *is a* $[16, 14, 2]$*-linear, 2-MDS code over* F_{16}*.*

Lemma 1. *[28, Lemma 1] For any* $\alpha \in F_{4^{2m}}$*,* $\Phi(\alpha^{4^m}) = \Phi(\alpha)^r$ *and* $\Phi(\alpha + 1) = \Phi(\alpha)^c$*, where* $\Phi(\alpha)^r$ *is the reverse of* $\Phi(\alpha)$ *and* $\Phi(\alpha)^c$ *is the Watson-Crick complement of* $\Phi(\alpha)$*.*

4. Reversible DNA codes with Hadamard power and T^n-set codes

In this section, we focus on DNA reversible codes with Hadamard power and discuss the results related to T^n-set codes. However, there is an issue that arises concerning the reversibility of DNA codes over $F_{4^{2m}}$. The reversibility problem can be illustrated as follows for F_{16}: Consider the codeword (w^3, w^{11}, w), which corresponds to the DNA sequence AGCTAT, where w^3 maps to AG, w^{11} maps to CT, and w maps to AT, with w^3, w^{11}, and w are elements of F_{16}. The reverse of the codeword (w^3, w^{11}, w) is (w, w^{11}, w^3), which is corresponding to the DNA sequence ATCTAG. However, ATCTAG is not the reverse of AGCTAT because the reverse of AGCTAT is TATCGA. This DNA reversibility problem was introduced and solved by Oztas and Siap [26]; later, this solution was generalized in [27] with the help of lifted polynomials.

Let Φ represent the bijective mapping that defines DNA correspondence for the elements of the field. For example, $\Phi(w^7) = GT$ over F_{16}. If we add 1 to any element of $F_{4^{2m}}$, then its image under Φ produces a complement to the DNA correspondence. For example, over F_{16}, $\Phi(w^7 + 1) = \Phi(w^9) = CA$. Additionally, taking the 4^m-th power of the elements produces the reverse of the DNA correspondence (see Lemma 1). For example, over F_{16}, $\Phi((w^7)^4) = \Phi(w^{13}) = TG$. Thus, Φ can be extended to codewords and codes such as $\Phi(c_0, c_1, \ldots, c_{n-1}) = (\Phi(c_0), \Phi(c_1), \ldots, \Phi(c_{n-1}))$ and $\Phi(C)$ correspond to DNA codes.

In this paper, we use the following DNA correspondence Table 2, which was developed by Oztas and Siap [26] for the finite field F_{16}.

Table 2. DNA correspondence table for F_{16}.

S. No.	DNA 2-bases	F_{16} (Multiplicative)	Additive
1	AA	0	0
2	TT	w^0	1
3	AT	w^1	w
4	GC	w^2	w^2
5	AG	w^3	w^3
6	TA	w^4	$1 + w$
7	CC	w^5	$w + w^2$
8	AC	w^6	$w^2 + w^3$
9	GT	w^7	$1 + w + w^3$
10	CG	w^8	$1 + w^2$
11	CA	w^9	$w + w^3$
12	GG	w^{10}	$1 + w + w^2$
13	CT	w^{11}	$w + w^2 + w^3$
14	GA	w^{12}	$1 + w + w^2 + w^3$
15	TG	w^{13}	$1 + w^2 + w^3$
16	TC	w^{14}	$1 + w^3$

Theorem 5. *Let $g(x)$ be a polynomial over F_{2^m} with $\deg(g(x)) < n$. Then, the codes $\langle T^n_g \rangle$ and $\langle T^n_{(g^*)^{\circ 2^t}} \rangle$ ($2^t < 2^m - 1$) have the same $[n, k, d]$ parameters.*

Proof. Let $g(x) = z_0 + z_1x + z_2x^2 + \cdots + z_sx^s$ be a polynomial over F_{2^m}, such that $\deg(g(x)) = s < n$. Then, following similar steps as in Eq (2.1) over F_{2^m}, we obtain

$$G_{T_g^n} = \begin{bmatrix} g(x) \\ xg(x) \\ x^2g(x) \\ \vdots \\ x^{n-s-2}g(x) \\ x^{n-s-1}g(x) \end{bmatrix} = \begin{bmatrix} z_0 & z_1 & z_2 & \cdots & z_s & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & z_0 & z_1 & z_2 & \cdots & z_s & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & z_0 & z_1 & z_2 & \cdots & z_s & 0 & \cdots & 0 & 0 \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ 0 & 0 & \cdots & z_0 & z_1 & z_2 & \cdots & & z_{s-1} & z_s & 0 \\ 0 & 0 & 0 & \cdots & z_0 & z_1 & z_2 & \cdots & z_{s-2} & z_{s-1} & z_s \end{bmatrix}, \tag{4.1}$$

and $g^*(x) = z_s + z_{s-1}x + z_{s-2}x^2 + \cdots + z_0x^s$ be the reciprocal polynomial of $g(x)$, since $\deg(g^*(x)) \le \deg(g(x))$ and $\deg(g(x)) < n$, so $\deg(g^*(x)) < n$, and also, the Hadamard $(2^t)^{th}$-power of $g^*(x)$ is $(g^*(x))^{\circ 2^t} = (z_s)^{2^t} + (z_{s-1})^{2^t}x + (z_{s-2})^{2^t}x^2 + \cdots + (z_0)^{2^t}x^s$, obviously, $\deg(g^*(x))^{\circ 2^t} = \deg(g^*(x))$ as coefficients of $(g^*(x))$ and $(g^*(x))^{\circ 2^t}$ are from the field F_{2^m}, therefore, $\deg(g^*(x))^{\circ 2^t} < n$. Then, the $T^n_{(g^*)^{\circ 2^t}}$ generator set with respect to $(g^*(x))^{\circ 2^t}$ is $\left\{(g^*(x))^{\circ 2^t}, x\,(g^*(x))^{\circ 2^t}, x^2\,(g^*(x))^{\circ 2^t}, \ldots, x^{n-s-2}\,(g^*(x))^{\circ 2^t}, x^{n-s-1}\,(g^*(x))^{\circ 2^t}\right\}$. Moreover, the matrix form of the $T^n_{(g^*)^{\circ 2^t}}$ generator set is

$$G_{T^n_{(g^*)^{\circ 2^t}}} = \begin{bmatrix} (g^*(x))^{\circ 2^t} \\ x\,(g^*(x))^{\circ 2^t} \\ x^2\,(g^*(x))^{\circ 2^t} \\ \vdots \\ x^{n-s-2}\,(g^*(x))^{\circ 2^t} \\ x^{n-s-1}\,(g^*(x))^{\circ 2^t} \end{bmatrix}$$

$$= \begin{bmatrix} (z_s)^{2^t} & (z_{s-1})^{2^t} & (z_{s-2})^{2^t} & \cdots & (z_0)^{2^t} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & (z_s)^{2^t} & (z_{s-1})^{2^t} & (z_{s-2})^{2^t} & \cdots & (z_0)^{2^t} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & (z_s)^{2^t} & (z_{s-1})^{2^t} & (z_{s-2})^{2^t} & \cdots & (z_0)^{2^t} & 0 & \cdots & 0 & 0 \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ 0 & 0 & \cdots & (z_s)^{2^t} & (z_{s-1})^{2^t} & (z_{s-2})^{2^t} & \cdots & (z_2)^{2^t} & (z_1)^{2^t} & (z_0)^{2^t} & 0 \\ 0 & 0 & \cdots & 0 & (z_s)^{2^t} & (z_{s-1})^{2^t} & (z_{s-2})^{2^t} & \cdots & (z_2)^{2^t} & (z_1)^{2^t} & (z_0)^{2^t} \end{bmatrix}. \tag{4.2}$$

From Eqs (4.1) and (4.2), each row of Eq (4.2) can be obtained from the reverse of a row in Eq (4.1) by replacing every entry with its 2^t-th power, that is, the first row of Eq (4.2) is reverse of the last row of Eq (4.1) with each entry raise to the power 2^t and so on. Since $\langle T_g^n \rangle = \langle G_{T_g^n} \rangle$ and $\langle T^n_{(g^*)^{\circ 2^t}} \rangle = \langle G_{T^n_{(g^*)^{\circ 2^t}}} \rangle$, then all codewords of $\langle T_g^n \rangle$ are found in $\langle T^n_{(g^*)^{\circ 2^t}} \rangle$ in reverse order by replacing every entry with its 2^t-th power. Hence, the length and dimension of codes $\langle T_g^n \rangle$ and $\langle T^n_{(g^*)^{\circ 2^t}} \rangle$ are preserved. Now we need to proceed with the Hamming distance. Let x', y' be any nonzero entries of generator matrix $G_{T_g^n}$ in the same column but different rows with $x' \neq y'$ and let x, y be any two nonzero elements of F_{2^t}, such that $xx' + yy' \neq 0$. Then, $x^{2^t}(x')^{2^t} + y^{2^t}(y')^{2^t} \neq 0$ because $x^{2^t}(x')^{2^t} + y^{2^t}(y')^{2^t} = (xx' + yy')^{2^t}$ over F_{2^t}. Since the finite field has characteristic 2, the 2^t-th power preserves zero positions in a codeword. Therefore, the Hamming distance is preserved. This completes the proof. □

As an immediate consequence of Theorem 5, we have the following result.

Corollary 6. *Let $g(x)$ be a polynomial over $F_{4^{2m}}$ and $\deg(g(x)) < n$. Then, the codes $\langle T^n_g \rangle$ and $\langle T^n_{(g^*)^{\circ 4^m}} \rangle$ have the same $[n, k, d]$ parameters.*

The method of generating reversible DNA codes from any polynomials using r-lifted polynomials was introduced in [6]. However, in that work, the r-lifted polynomials were generated by factors of $x^n - 1$. In this study, we do not require any factors of any polynomial. Thus, the advantage of the following theorem is that we can produce reversible DNA codes with any polynomials, not just self-reciprocal polynomials.

Theorem 7. *Let $g(x)$ be a polynomial over $F_{4^{2m}}$ with $\deg(g(x)) < n$ and $g^*(x)$ be the reciprocal polynomial of $g(x)$. Then, the code $C' = \langle T^n_g \mid T^n_{(g^*)^{\circ 4^m}} \rangle$ has $[2n, k, d_T]$ parameters, and $\Phi(C')$ is a reversible DNA code, where $d_T \geq 2d$, $\langle T^n_g \rangle$ is a $[n, k, d]$-code. Moreover, if $C'' = \langle (T^n_g \mid T^n_{(g^*)^{\circ 4^m}}), (b(x) \mid b(x)) \rangle$, where $b(x) = 1 + x + x^2 + \cdots + x^{n-1}$, then $\Phi(C'')$ is a reversible complement DNA code.*

Proof. By Corollary 6 and Theorem 3, the code $C' = \langle T^n_g \mid T^n_{(g^*)^{\circ 4^m}} \rangle$ has parameters $[2n, k, d_T]$ with $d_T \geq 2d$, where $\langle T^g_n \rangle$ is an $[n, k, d]$-code. It remains to show that $\Phi(C')$ is a reversible DNA code. Let $g(x) = z_0 + z_1 x + z_2 x^2 + \cdots + z_s x^s$ be a polynomial over $F_{4^{2m}}$ such that $\deg(g(x)) = s < n$. Then, following similar steps as in Eq (2.1) over $F_{4^{2m}}$, we obtain

$$G_{T^n_g} = \begin{bmatrix} g(x) \\ xg(x) \\ x^2 g(x) \\ \vdots \\ x^{n-s-2} g(x) \\ x^{n-s-1} g(x) \end{bmatrix} = \begin{bmatrix} z_0 & z_1 & z_2 & \cdots & z_s & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & z_0 & z_1 & z_2 & \cdots & z_s & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & z_0 & z_1 & z_2 & \cdots & z_s & 0 & \cdots & 0 & 0 \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ 0 & 0 & \cdots & z_0 & z_1 & z_2 & \cdots & & z_{s-1} & z_s & 0 \\ 0 & 0 & 0 & \cdots & z_0 & z_1 & z_2 & \cdots & z_{s-2} & z_{s-1} & z_s \end{bmatrix}, \tag{4.3}$$

and $g^*(x) = z_s + z_{s-1} x + z_{s-2} x^2 + \cdots + z_0 x^s$ be the reciprocal polynomial of $g(x)$, by applying the same steps as in the proof of Theorem 5, we obtain the $T^n_{(g^*)^{\circ 4^m}}$ generator set with respect to $(g^*(x))^{\circ 4^m}$ is $\left\{ (g^*(x))^{\circ 4^m}, x\,(g^*(x))^{\circ 4^m}, x^2\,(g^*(x))^{\circ 4^m}, \ldots, x^{n-s-2}\,(g^*(x))^{\circ 4^m}, x^{n-s-1}\,(g^*(x))^{\circ 4^m} \right\}$. Moreover, the matrix form of $T^n_{(g^*)^{\circ 4^m}}$ generator set is

$$G_{T^n_{(g^*)^{\circ 4^m}}} = \begin{bmatrix} (g^*(x))^{\circ 4^m} \\ x\,(g^*(x))^{\circ 4^m} \\ x^2\,(g^*(x))^{\circ 4^m} \\ \vdots \\ x^{n-s-2}\,(g^*(x))^{\circ 4^m} \\ x^{n-s-1}\,(g^*(x))^{\circ 4^m} \end{bmatrix}$$

$$= \begin{bmatrix} (z_s)^{4^m} & (z_{s-1})^{4^m} & (z_{s-2})^{4^m} & \cdots & (z_0)^{4^m} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & (z_s)^{4^m} & (z_{s-1})^{4^m} & (z_{s-2})^{4^m} & \cdots & (z_0)^{4^m} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & (z_s)^{4^m} & (z_{s-1})^{4^m} & (z_{s-2})^{4^m} & \cdots & (z_0)^{4^m} & 0 & \cdots & 0 & 0 \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ 0 & 0 & \cdots & (z_s)^{4^m} & (z_{s-1})^{4^m} & (z_{s-2})^{4^m} & \cdots & (z_2)^{4^m} & (z_1)^{4^m} & (z_0)^{4^m} & 0 \\ 0 & 0 & \cdots & 0 & (z_s)^{4^m} & (z_{s-1})^{4^m} & (z_{s-2})^{4^m} & \cdots & (z_2)^{4^m} & (z_1)^{4^m} & (z_0)^{4^m} \end{bmatrix}. \tag{4.4}$$

From Eqs (4.3) and (4.4), it follows that each row in Eq (4.4) is derived from the reverse of a row in Eq (4.3) by raising every entry to its 4^m-th power. Let $\mathbf{r}_j$ denote the j-th row of the matrix $G_{T_g^n}$ in Eq (4.3) and $\mathbf{r}''_j$ denote the j-th row of the matrix $G_{T^n_{(g^*)^{\circ 4^m}}}$ in Eq (4.4). Then, for each $1 \le j \le k$, we have

$$\mathbf{r}''_j \;=\; (\mathbf{r}^r_{k+1-j})^{\circ 4^m} \quad \text{and} \quad \mathbf{r}_j = \left((\mathbf{r}''_{k+1-j})^r\right)^{\circ 4^m}, \qquad 1 \le j \le k.$$

The generator matrix of $C \;=\; \langle T_g^n \mid T^n_{(g^*)^{\circ 4^m}} \rangle$ is

$$G = \left[G_{T_g^n} \mid G_{T^n_{(g^*)^{\circ 4^m}}} \right],$$

and we denote its rows by $\mathrm{r}'_i = (\mathbf{r}_i \mid \mathbf{r}''_i)$, $1 \le i \le k$. The reverse of r'_j is

$$(\mathrm{r}'_j)^r = (\mathbf{r}''^r_j \mid \mathbf{r}^r_j) = ((\mathbf{r}_{k+1-j})^{\circ 4^m} \mid (\mathbf{r}''_{k+1-j})^{\circ 4^m}) = (\mathbf{r}_{k+1-j} \mid \mathbf{r}''_{k+1-j})^{\circ 4^m} = (\mathrm{r}'_{k+1-j})^{\circ 4^m}, \quad 1 \le j \le \left\lceil \frac{k}{2} \right\rceil. \tag{4.5}$$

Here, $\lceil \cdot \rceil$ denotes the greatest integer function. Let $\gamma : F_{4^{2m}}[x]/(x^n - 1) \to F^n_{4^{2m}}$ be the standard coordinate map and Φ be the DNA correspondence map. For any $\alpha \in F_{4^{2m}}$ and any row polynomial $\mathrm{r}'_j(x)$ corresponding to the row r'_j, using Lemma 1, Eq (4.5), DNA correspondence map Φ and linearity of γ gives

$$\Phi\Big(\gamma(\alpha \mathrm{r}'_j(x))\Big)^r = \Phi\Big((\gamma(\alpha \mathrm{r}'_j(x)))^{\circ 4^m}\Big) = \Phi\Big((\alpha\gamma(\mathrm{r}'_j(x)))^{\circ 4^m}\Big) = \Phi\Big(\alpha^{4^m}(\gamma(\mathrm{r}'_j(x))^{\circ 4^m}\Big) = \Phi\Big(\gamma\Big(\alpha^{4^m}\mathrm{r}'_{k+1-j}(x)\Big)\Big). \tag{4.6}$$

Now, let $c \in C'$, then the corresponding polynomial $c(x) = \sum_{i=1}^{k} \alpha_i \mathrm{r}'_i(x)$ for some $\alpha_i \in F_{4^{2m}}$. Using the linearity of γ and Eq (4.6), we have

$$\Phi\Big(\gamma(c(x))\Big)^r = \Phi\Big(\gamma\Big(\sum_{i=1}^{k} \alpha_i \mathrm{r}'_i(x)\Big)\Big)^r = \Phi\Big(\gamma\Big(\sum_{i=1}^{k} \alpha_i^{4^m} \mathrm{r}'_i(x)\Big)\Big) \in \Phi(C').$$

Since $\gamma\Big(\sum_{i=1}^{k} \alpha_i^{4^m} \mathrm{r}'_i(x)\Big) \in C'$. Hence, the code $C' = \langle T_g^n \mid T^n_{(g^*)^{\circ 4^m}} \rangle$ is reversible. Moreover, for the given polynomial $b(x) = 1 + x + x^2 + \cdots + x^{n-1}$, $\gamma\Big(b(x) \mid b(x)\Big)$ shows the codeword $(1, 1, 1, \ldots, 1, 1)$ as the vector of length $2n$ in the generator matrix G. Following the same steps as used above for establishing the reversibility of C', and by applying Lemma 1, it can be concluded that the image of C'' under Φ, that is, $\Phi(C'')$, also satisfies the complement property. Therefore, $\Phi(C'')$ is a reversible complement DNA code. This completes the proof. □

Example 5. *Let $g(x) = w + w^2x + w^6x^2 + w^4x^3$ be a polynomial over F_{16} and $n = 7$. Then, the matrix of the T_g^7 generator set is*

$$\begin{bmatrix} w & w^2 & w^6 & w^4 & 0 & 0 & 0 \\ 0 & w & w^2 & w^6 & w^4 & 0 & 0 \\ 0 & 0 & w & w^2 & w^6 & w^4 & 0 \\ 0 & 0 & 0 & w & w^2 & w^6 & w^4 \end{bmatrix}.$$

Hence, the code $\langle T_g^7 \rangle$ is a $[7, 4, 3]$-linear code over F_{16}. Further, we obtain $g^*(x) := w^4 + w^6x + w^2x^2 + wx^3$. Then, using the Hadamard 4^{th}-power of $g^*(x)$, the matrix of $T^7_{(g^*)^{\circ 4}}$ generator set is

$$\begin{bmatrix} w & w^9 & w^8 & w^4 & 0 & 0 & 0 \\ 0 & w & w^9 & w^8 & w^4 & 0 & 0 \\ 0 & 0 & w & w^9 & w^8 & w^4 & 0 \\ 0 & 0 & 0 & w & w^9 & w^8 & w^4 \end{bmatrix}.$$

Therefore, $\langle T^7_{(g^*)^{\circ 4}} \rangle$ is a $[7, 4, 3]$-linear code over F_{16}. Then by Theorem 7, the generator matrix of $C' = \langle T_g^7 \mid T^7_{(g^*)^{\circ 4}} \rangle$ is

$$\begin{bmatrix} w & w^2 & w^6 & w^4 & 0 & 0 & 0 & w & w^9 & w^8 & w^4 & 0 & 0 & 0 \\ 0 & w & w^2 & w^6 & w^4 & 0 & 0 & 0 & w & w^9 & w^8 & w^4 & 0 & 0 \\ 0 & 0 & w & w^2 & w^6 & w^4 & 0 & 0 & 0 & w & w^9 & w^8 & w^4 & 0 \\ 0 & 0 & 0 & w & w^2 & w^6 & w^4 & 0 & 0 & 0 & w & w^9 & w^8 & w^4 \end{bmatrix}.$$

Hence, the linear code $C' = \langle T_g^7 \mid T^7_{(g^*)^{\circ 4}} \rangle$ over F_{16} having parameters $[14, 4, 8]$ that corresponds to a reversible DNA code, and the minimum distance $d(C') = 8 > 2 \cdot d(\langle T_g^7 \rangle) = 2 \cdot 3 = 6$.

Theorem 8. *Let $C = \langle g(x) \rangle$ be a $[n, k, d]$-cyclic code over $F_{4^{2m}}$ and $g^*(x)$ be the reciprocal polynomial of $g(x)$. Then, the code $C' = \langle \langle g(x) \rangle \mid T^n_{(g^*)^{\circ 4^m}} \rangle$ has $[2n, k, d_T]$ parameters, where $d_T \geq 2d$ and $\Phi(C')$ is a reversible DNA code over $F_{4^{2m}}$. Moreover, if $C'' = \langle (\langle g(x) \rangle \mid T^n_{(g^*)^{\circ 4^m}}), (b(x) \mid b(x)) \rangle$, where $b(x) = 1 + x + x^2 + \cdots + x^{n-1}$, then $\Phi(C'')$ is a reversible complement DNA code.*

Proof. The proof is similar to that of Theorem 7. □

Example 6. *Let $g(x) = w^9 + w^{13}x + w^{10}x^2 + w^2x^3 + x^4 + w^{12}x^5 + w^7x^6 + w^{12}x^7 + w^9x^8 + w^2x^9 + w^4x^{10} + wx^{11} + x^{12}$ be a polynomial over F_{16} and $n = 15$, we already find the generator matrix of $\langle g(x) \rangle$ in Example 2 and $d(\langle g(x) \rangle) = 10$, also we obtained $g^*(x) := 1 + wx + w^4x^2 + w^2x^3 + w^9x^4 + w^{12}x^5 + w^7x^6 + w^{12}x^7 + x^8 + w^2x^9 + w^{10}x^{10} + w^{13}x^{11} + w^9x^{12}$. Then, using the Hadamard 4^{th}-power of $g^*(x)$, the matrix of $T^{15}_{(g^*)^{\circ 4}}$ generator set is*

$$\begin{bmatrix} 1 & w^4 & w & w^8 & w^6 & w^3 & w^{13} & w^3 & 1 & w^8 & w^{10} & w^7 & w^6 & 0 & 0 \\ 0 & 1 & w^4 & w & w^8 & w^6 & w^3 & w^{13} & w^3 & 1 & w^8 & w^{10} & w^7 & w^6 & 0 \\ 0 & 0 & 1 & w^4 & w & w^8 & w^6 & w^3 & w^{13} & w^3 & 1 & w^8 & w^{10} & w^7 & w^6 \end{bmatrix}.$$

Thus, the code $\langle T^{15}_{(g^*)^{\circ 4}} \rangle$ is a $[15, 3, 10]$-cyclic code over F_{16}.

By Theorem 8, the generator matrix of $C' = \langle \langle g(x) \rangle \mid T^{15}_{(g^*)^{\circ 4}} \rangle$ is

$$\begin{bmatrix} w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 & 0 & 1 & w^4 & w & w^8 & w^6 & w^3 & w^{13} & w^3 & 1 & w^8 & w^{10} & w^7 & w^6 & 0 & 0 \\ 0 & w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 & 0 & 1 & w^4 & w & w^8 & w^6 & w^3 & w^{13} & w^3 & 1 & w^8 & w^{10} & w^7 & w^6 & 0 \\ 0 & 0 & w^9 & w^{13} & w^{10} & w^2 & 1 & w^{12} & w^7 & w^{12} & w^9 & w^2 & w^4 & w & 1 & 0 & 0 & 1 & w^4 & w & w^8 & w^6 & w^3 & w^{13} & w^3 & 1 & w^8 & w^{10} & w^7 & w^6 \end{bmatrix}.$$

Hence, the linear code $C' = \langle\langle g(x)\rangle \mid T^{15}_{(g^*)^{\circ 4}}\rangle$ *over* F_{16} *having parameters* $[30, 3, 20]$*, that corresponds to a reversible DNA code, and the minimum distance* $d(C') = 20 = 2 \cdot d(\langle g(x)\rangle) = 2 \cdot 10 = 20$.

Example 7. *From the Example 3, applying Hadamard* 4^{th}*-power on the reciprocal polynomial* $g^*(x)$*, then the matrix of* $T^{8}_{(g^*)^{\circ 4}}$ *generator set is*

$$\begin{bmatrix} w^{48} & w^{144} & w^{176} & w^{80} & w^{96} & w^{160} & 1 & 0 \\ 0 & w^{48} & w^{144} & w^{176} & w^{80} & w^{96} & w^{160} & 1 \end{bmatrix}.$$

Thus, $\langle T^{8}_{(g^*)^{\circ 4}}\rangle$ *is a* $[8, 2, 7]$*-linear code over* F_{256}*. In view of Theorem 7, the generator matrix of* $\langle T^{8}_{g} \mid T^{8}_{(g^*)^{\circ 4}}\rangle$ *is*

$$\begin{bmatrix} 1 & w^{10} & w^{6} & w^{5} & w^{11} & w^{9} & w^{3} & 0 & w^{48} & w^{144} & w^{176} & w^{80} & w^{96} & w^{160} & 1 & 0 \\ 0 & 1 & w^{10} & w^{6} & w^{5} & w^{11} & w^{9} & w^{3} & 0 & w^{48} & w^{144} & w^{176} & w^{80} & w^{96} & w^{160} & 1 \end{bmatrix}.$$

Hence, the code $\langle T^{8}_{g} \mid T^{8}_{(g^*)^{\circ 4}}\rangle$ *is an 1-MDS (AMDS),* $[16, 2, 14]$*-linear code over the field* F_{256}*, which corresponds to a reversible DNA code.*

Example 8. *According to the Example 4, the matrix of* $T^{10}_{(g^*)^{\circ 4}}$ *generator set is*

$$\begin{bmatrix} w^{9} & w^{5} & 1 & w^{9} & w^{12} & w^{8} & 1 & w^{7} & 0 & 0 \\ 0 & w^{9} & w^{5} & 1 & w^{9} & w^{12} & w^{8} & 1 & w^{7} & 0 \\ 0 & 0 & w^{9} & w^{5} & 1 & w^{9} & w^{12} & w^{8} & 1 & w^{7} \end{bmatrix}.$$

Thus, $\langle T^{10}_{(g^*)^{\circ 4}}\rangle$ *is a* $[10, 3, 7]$*-linear code over* F_{16}*. Then, by Theorem 7, the generator matrix of* $\langle T^{10}_{g} \mid T^{10}_{(g^*)^{\circ 4}}\rangle$ *is*

$$\begin{bmatrix} w^{13} & 1 & w^{2} & w^{3} & w^{6} & 1 & w^{5} & w^{6} & 0 & 0 & w^{9} & w^{5} & 1 & w^{9} & w^{12} & w^{8} & 1 & w^{7} & 0 & 0 \\ 0 & w^{13} & 1 & w^{2} & w^{3} & w^{6} & 1 & w^{5} & w^{6} & 0 & 0 & w^{9} & w^{5} & 1 & w^{9} & w^{12} & w^{8} & 1 & w^{7} & 0 \\ 0 & 0 & w^{13} & 1 & w^{2} & w^{3} & w^{6} & 1 & w^{5} & w^{6} & 0 & 0 & w^{9} & w^{5} & 1 & w^{9} & w^{12} & w^{8} & 1 & w^{7} \end{bmatrix}.$$

Hence, the code $\langle T^{10}_{g} \mid T^{10}_{(g^*)^{\circ 4}}\rangle$ *is a self-orthogonal,* 2*-MDS* $[20, 3, 16]$*-linear code over* F_{16}*, which corresponds to a reversible DNA code.*

Let us consider codewords of $\langle T^{10}_{g} \mid T^{10}_{(g^*)^{\circ 4}}\rangle$ with the rows r_1, r_2, and r_3 being the first, second, and third rows of its generator matrix, respectively. Then, $c_1 = w \cdot r_1 + w^3 \cdot r_2 = w \cdot (w^{13}, 1, w^2, w^3, w^6, 1, w^5, w^6, 0, 0, w^9, w^5, 1, w^9, w^{12}, w^8, 1, w^7, 0, 0) + w^3 \cdot (0, w^{13}, 1, w^2, w^3, w^6, 1, w^5, w^6, 0, 0, w^9, w^5, 1, w^9, w^{12}, w^8, 1, w^7, 0) = (w^{14}, 0, 0, w^8, w^{10}, w^3, w^2, w^{11}, w^9, 0, w^{10}, w^4, w^{10}, w^{12}, w, w^7, w^6, w^{13}, w^{10}, 0)$, that is, $c_1 = w^{14}00w^8w^{10}w^3w^2w^{11}w^9 0w^{10}w^4w^{10}w^{12}ww^7w^6w^{13}w^{10}0$. From the Table 2, c_1 corresponds to $\Phi(c_1) = TCAAAACGGGAGGCCTCAAAGGTAGGGAATGTACTGGGAA$.

According to the proof of Theorem 7, we write $c_2 = (w)^4 \cdot r_3 + (w^3)^4 \cdot r_2 = (w)^4 \cdot (0, 0, w^{13}, 1, w^2, w^3, w^6, 1, w^5, w^6, 0, 0, w^9, w^5, 1, w^9, w^{12}, w^8, 1, w^7) + w^{12} \cdot (0, w^{13}, 1, w^2, w^3, w^6, 1, w^5, w^6, 0, 0, w^9, w^5, 1, w^9, w^{12}, w^8, 1, w^7, 0) = (0, w^{10}, w^7, w^9, w^{13}, w^4, w^3, w^{10}, w, w^{10}, 0, w^6, w^{14}, w^8, w^{12}, w^{10}, w^2, 0, 0, w^{11})$, i.e., $c_2 = 0w^{10}w^7w^9w^{13}w^4w^3w^{10}ww^{10}0w^6w^{14}w^8w^{12}w^{10}w^2 00w^{11}$. Table 2 shows that c_2 is corresponds to $\Phi(c_2) = AAGGGTCATGTAAGGGATGGAAACTCCGGAGGGCAAAACT$. Thus, c_1 and c_2 are not reverse to each other. However, $\Phi(c_1)$ and $\Phi(c_2)$ are reverses of each other.

The following example presents a correspondence table between DNA sequences composed of 20 bases and all 256 elements of the linear code $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ over F_{16} and justify the fact that non-reversible codes may correspond to reversible DNA codes.

Example 9. *Let $g(x) = w^{13} + x + w^2x^2 + w^3x^3$ be a polynomial over F_{16} and $n = 5$. Then, the matrix of T_g^5 generator set is*

$$\begin{bmatrix} w^{13} & 1 & w^2 & w^3 & 0 \\ 0 & w^{13} & 1 & w^2 & w^3 \end{bmatrix}.$$

Thus, $\langle T_g^5 \rangle$ is a $[5, 2, 3]$-linear code over F_{16}. Further, we obtain the reciprocal polynomial $g^*(x) = w^3 + w^2x + x^2 + w^{13}x^3$. Now, applying the Hadamard 4^{th}-power on $g^*(x)$. Then, the matrix of $T_{(g^*)^{\circ 4}}^5$ generator set is

$$\begin{bmatrix} w^{12} & w^8 & 1 & w^7 & 0 \\ 0 & w^{12} & w^8 & 1 & w^7 \end{bmatrix}.$$

Therefore, the code $\langle T_{(g^*)^{\circ 4}}^5 \rangle$ is also a $[5, 2, 3]$-linear code over F_{16}. In view of Theorem 7, the generator matrix of $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ is

$$\begin{bmatrix} w^{13} & 1 & w^2 & w^3 & 0 & w^{12} & w^8 & 1 & w^7 & 0 \\ 0 & w^{13} & 1 & w^2 & w^3 & 0 & w^{12} & w^8 & 1 & w^7 \end{bmatrix}.$$

Hence, the code $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ is a 1-MDS (AMDS) $[10, 2, 8]$-linear code over F_{16}, which is reversible DNA code but not reversible.

We know that, every codeword x of $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ can be expressed as, $x = \alpha \cdot (w^{13}, 1, w^2, w^3, 0, w^{12}, w^8, 1, w^7, 0) + \beta \cdot (0, w^{13}, 1, w^2, w^3, 0, w^{12}, w^8, 1, w^7)$ for all $\alpha, \beta \in F_{16}$. In particular, if $\alpha = w, \beta = 0$, then we obtain, $x = w \cdot (w^{13}, 1, w^2, w^3, 0, w^{12}, w^8, 1, w^7, 0) + 0 \cdot (0, w^{13}, 1, w^2, w^3, 0, w^{12}, w^8, 1, w^7) = (w^{14}, w, w^3, w^4, 0, w^{13}, w^9, w, w^8, 0)$, i.e., $x = w^{14}ww^3w^40w^{13}w^9ww^80$. Now, from the Table 2, x corresponds to $\Phi(x) = (TC, AT, AG, TA, AA, TG, CA, AT, CG, AA)$, that is, $\Phi(x) = TCATAGTAAATGCAATCGAA$. Clearly, $x \in \langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$. However, $x^r = 0w^8ww^9w^{13}0w^4w^3ww^{14} \notin \langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$. Therefore, $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ is not reversible code. Moreover, for any codeword $y \in \Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$, its reverse $y^r \in \Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$. Hence, $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$ is reversible code, *i.e.*, $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ is reversible DNA code. In a similar way, we can find all the elements of $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ for different choices of $\alpha, \beta \in F_{16}$. However, the Magma computational algebra system [9] is used to complete all of the computations in Tables 3 and 4. In Table 3 (refer to Appendix A), we present a DNA correspondence for the linear code $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ over F_{16} and the sequences of DNA 20-bases, i.e., correspondence between all the elements of length 10 of the linear code $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ over F_{16} and all the elements of $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$.

In Appendix B, Table 4 provides a detailed analysis of linear codes derived from polynomials $g(x)$ over the finite field F_{16}. It highlights essential parameters, including length, dimension, and minimum Hamming distance of the codes $\langle T_g^n \rangle$, $\langle T_{g^*}^n \rangle$, $\langle T_g^n \mid T_{g^*}^n \rangle$, and $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle$, as well as their duals. The table also examines whether these codes and their transformations show self-orthogonality or satisfy the criteria for l-MDS codes, which are optimal in terms of distance and dimension. Self-orthogonality is indicated (T for True, F for False), while the dual properties provide additional insights into the relationship between code structure and performance.

5. Conclusions

In this paper, we introduced a new approach for constructing reversible and reversible DNA codes using any polynomials or cyclic codes over the finite field $F_{4^{2m}}$. A key contribution of our work is demonstrating that the codes $\langle T_g^n \rangle$ and $\langle T_{g^*}^n \rangle$ preserve their fundamental parameters $[n, k, d]$, as established in Theorem 1, where $g(x)$ is a polynomial over F_q with $\deg(g(x)) < n$. This result plays a crucial role in confirming that the transformation of polynomials to their reciprocal polynomials does not degrade the essential properties of the code, thereby maintaining its error correcting capability. Furthermore, we have proven that the extended code $\langle T_g^n \mid T_{g^*}^n \rangle$ maintains reversibility over the ring R, leading to more significant structural constraints and potential applications (Theorem 2). Notably, Theorem 3 established an essential result that the reversible code $\langle T_g^n \mid T_{g^*}^n \rangle$ having parameters $[2n, k, d_T]$ maintains the improved distance bound $d_T \geq 2d$, which enhances error stability, making it suitable for real-world error correction. Extending our study to DNA codes over $F_{4^{2m}}$, we examined the Hadamard s-power to demonstrated that the codes $\langle T_g^n \rangle$ and $\langle T_{(g^*)^{\circ 2^t}}^n \rangle$ $(2^t < 2^m - 1)$ exhibit equivalent parameters, as shown in Theorem 5. This highlights the structural consistency of these codes under reciprocality, which is essential for DNA-based storage and bioinformatics applications. Furthermore, in Theorem 7, we established that the extended codes $C' = \langle T_g^n \mid T_{(g^*)^{\circ 4^m}}^n \rangle$ and $C'' = \langle (T_g^n \mid T_{(g^*)^{\circ 4^m}}^n), (b(x) \mid b(x)) \rangle$, where $b(x) = 1 + x + x^2 + \cdots + x^{n-1}$ correspond to a reversible DNA code and a reversible-complement DNA code, respectively, under the given map Φ. These results highlight the robustness of our approach in constructing reliable DNA codes that can be utilized in various biological computing systems. Additionally, we established a correspondence table that links DNA sequences of 20 bases with all 256 elements of the linear code $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ over F_{16} with respect to the polynomial $g = w^{13} + x + w^2x^2 + w^3x^3$ in x over F_{16}. This linear code demonstrates the fact that non-reversible codes can correspond to reversible DNA codes. Moreover, our method also produces l-MDS and self-orthogonal codes, highlighting the capability of our method. These results emphasize the broader implications of our methodology for DNA computing and error-correcting codes. Furthermore, our tabulated analysis (Table 4) based on different polynomials over F_{16} provides further insights into their characteristics, including their status such as MDS or AMDS codes and their self-orthogonality properties, which are crucial in optimizing code performance. However, determining whether these MDS or AMDS codes possess quasi-cyclic properties remains an open problem for future research.

Author contributions

All authors contributed equally to the conception, execution, and writing of this manuscript. All authors read and approved the final version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

The authors thank to the anonymous reviewers for their valuable suggestions and comments which significantly improved both the quality and the presentation of this paper.

Moreover, the authors extend their appreciation to Princess Nourah bint Abdulrahman University (PNU), Riyadh, Saudi Arabia, for funding this research under the Researchers Supporting Project Number (PNURSP2025R231). Furthermore, the research of the first and third authors is supported by TUBITAK–the Scientific and Technological Research Council of Türkiye, under the program 2221-Fellowship for Visiting Professor/Scientists at Karamanoglu Mehmetbey University (KMU), Türkiye. Also, the research of the fourth author is supported by the University Grants Commission (UGC), Government of India (Ref. No.: 211610001225).

Conflict of interest

The authors declare that they have no conflicts of interest.

References

1. N. Aboluion, D. H. Smith, S. Perkins, Linear and nonlinear constructions of DNA codes with Hamming distance d, constant GC-content and a reverse-complement constraint, *Discrete Math.*, **312** (2012), 1062–1075. https://doi.org/10.1016/j.disc.2011.11.021
2. T. Abualrub, A. Ghrayeb, X. N. Zeng, Construction of cyclic codes over $GF(4)$ for DNA computing, *J. Franklin I.*, **343** (2006), 448–457. https://doi.org/10.1016/j.jfranklin.2006.02.009
3. L. Adleman, Molecular computation of solutions to combinatorial problems, *Science*, **266** (1994), 1021–1024. https://doi.org/10.1126/science.7973651
4. L. Adleman, P. W. K. Rothemund, S. Roweis, E. Winfree, On applying molecular computation to the data encryption standard, *J. Comput. Biol.*, **6** (1999), 53–63. https://doi.org/10.1089/cmb.1999.6.53
5. S. Ali, A. S. Alali, M. Azeem, A. A. Khan, K. B. Wong, On double cyclic codes over finite chain rings for DNA computing, *Entropy*, **27** (2025), 1187. https://doi.org/10.3390/e27121187
6. T. Alsuraiheed, E. S. Oztas, S. Ali, M. B. Yilgor, Reversible codes and applications to DNA codes over $F_{4^{2t}}[u]/(u^2-1)$, *AIMS Math.*, **8** (2023), 27762–27774. https://doi.org/10.3934/math.2024358
7. M. A. D. Boer, Almost MDS codes, *Design Code. Cryptogr.*, **9** (1996), 143–155. https://doi.org/10.1007/BF00124590
8. D. Boneh, C. Dunworth, R. J. Lipton, Breaking DES using a molecular computer, *DNA Based Comput.*, **27** (1995), 37–66. https://doi.org/10.1090/dimacs/027/03
9. W. Bosma, J. Cannon, *Handbook of Magma functions*, University of Sydney, 1995.
10. M. M. Brandao, L. Spoladore, L. C. B. Faria, A. S. L. Rocha, M. C. S. Filho, R. Palazzo, Ancient DNA sequence revealed by error-correcting codes, *Sci. Rep.*, **5** (2015), 1–9. https://doi.org/10.1038/srep12051

11. Y. M. Chee, S. Ling, Improved lower bounds for constant GC-content DNA codes, *IEEE T. Inform. Theory*, **54** (2008), 391–394. https://doi.org/10.1109/TIT.2007.911167

12. P. K. Das, V. Tyagi, Reversible and (n_1, n_2)-blockwise reversible code, *Int. J. Algebra*, **6** (2012), 185–191.

13. C. Ding, C. Tang, *Designs from linear codes*, 2 Eds, Singapore: World Scientific, 2022. https://doi.org/10.1142/12697

14. H. Q. Dinh, S. Pathak, A. K. Upadhyay, W. Yamaka, New DNA codes from cyclic codes over mixed alphabets, *Mathematics*, **8** (2020), 1–24. https://doi.org/10.3390/math8111977

15. H. Q. Dinh, A. K. Singh, S. Pattanayak, Cyclic DNA codes over the ring $F_2 + uF_2 + vF_2 + uvF_2 + v^2F_2 + uv^2F_2$, *Design Code. Cryptogr.*, **86** (2018), 1451–1467. https://doi.org/10.1007/s10623-017-0405-x

16. H. Gu, J. Zhang, On twisted generalized Reed-Solomon codes with ℓ twists, *IEEE T. Inform. Theory*, **70** (2024), 145–153. https://doi.org/10.1109/TIT.2023.3314059

17. K. Guenda, T. A. Gulliver, Construction of cyclic codes over $F_2 + uF_2$ for DNA computing, *Appl. Algebr. Eng. Commun.*, **24** (2013), 445–459. https://doi.org/10.1007/s00200-013-0188-x

18. O. D. King, Bounds for DNA codes with constant GC-content, *Electron. J. Comb.*, **10** (2003), 1–13. https://doi.org/10.37236/1726

19. Q. Liao, H. Liao, On m-MDS codes over finite fields, *Int. J. Comput. Math.*, **91** (2014), 863–871. https://doi.org/10.1080/00207160.2013.818669

20. L. S. Liebovitch, Y. Tao, A. T. Todorov, L. Levine, Is there an error correcting code in the base sequence in DNA? *Biophys. J.*, **71** (1996), 1539–1544. https://doi.org/10.1016/S0006-3495(96)79356-6

21. S. Ling, C. Xing, *Coding theory: A first course*, Cambridge University Press, 2004.

22. F. J. MacWilliams, N. J. A. Sloane, *The theory of error-correcting codes*, Elsevier, **16** (1977).

23. J. L. Massey, Reversible codes, *Inf. Control*, **7** (1964), 369–380. https://doi.org/10.1016/S0019-9958(64)90438-3

24. S. Mehta, V. Saraswat, S. Sen, *Secret sharing using near-MDS codes*, In: Cryptology and Information Security, Springer, Cham, **11445** (2019), 195–214. https://doi.org/10.1007/978-3-030-16458-4_12

25. S. K. Muttoo, S. Lal, A reversible code over $GF(q)$, *Kybernetika*, **22** (1986), 85–91.

26. E. S. Oztas, I. Siap, Lifted polynomials over F_{16} and their applications to DNA codes, *Filomat*, **27** (2013), 461–468. https://doi.org/10.2298/FIL1303459O

27. E. S. Oztas, I. Siap, On a generalization of lifted polynomials over finite fields and their applications to DNA codes, *Int. J. Comput. Math.*, **92** (2015), 1976–1988. https://doi.org/10.1080/00207160.2014.930449

28. E. S. Oztas, A. S. Alali, S. Ali, M. Azeem, M. S. M. Asri, K. B. Wong, A matrix-theoretic approach to the construction of reversible DNA codes over $F_{4^{2t}}$, *IEEE Access*, 2025. https://doi.org/10.1109/ACCESS.2025.3638664

29. O. Prakash, A. Singh, R. K. Verma, P. Sole, W. Cheng, DNA code from cyclic and skew cyclic codes over $F_4[v]/ < v^3 >$, *Entropy*, **25** (2023), 239. https://doi.org/10.3390/e25020239
30. I. Siap, T. Abualrub, A. Ghrayeb, Cyclic DNA codes over the ring $F_2[u]/(u^2 - 1)$ based on the deletion distance, *J. Franklin I.*, **346** (2009), 731–740. https://doi.org/10.1016/j.jfranklin.2009.07.002
31. J. Sui, Q. Yue, X. Li, D. Huang, MDS, near-MDS or 2-MDS self-dual codes via twisted generalized Reed-Solomon codes, *IEEE T. Inform. Theory*, **68** (2022), 7832–7841. https://doi.org/10.1109/TIT.2022.3190676
32. A. Thomas, B. Rajan, *Binary index codes using l-th NMDS codes*, In: IEEE International Conference on Communications (ICC), 2017, 1–6. https://doi.org/10.1109/ICC.2017.7997268
33. H. Tong, W. Chen, F. Yu, On ℓ-th NMDS codes, *Chinese J. Electron.*, **23** (2014), 454–457. https://doi.org/10.23919/CJE.2014.10851235

Appendix A

Table 3. DNA correspondence table of $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ according to the Example 9.

S. No.	**Elements of** $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$	**DNA 20-bases of** $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$
1	0000000000	AAAAAAAAAAAAAAAAAAAA
2	$0w^{13}1w^2w^30w^{12}w^81w^7$	AATGTTGCAGAAGACGTTGT
3	$0w^{14}ww^3w^40w^{13}w^9ww^8$	AATCATAGTAAATGCAATCG
4	$01w^2w^4w^50w^{14}w^10w^2w^9$	AATTGCTACCAATCGGGCCA
5	$0ww^3w^5w^601w^{11}w^3w^{10}$	AAATAGCCACAATTCTAGGG
6	$0w^2w^4w^6w^70ww^{12}w^4w^{11}$	AAGCTAACGTAAATGATACT
7	$0w^3w^5w^7w^80w^2w^{13}w^5w^{12}$	AAAGCCGTCGAAGCTGCCGA
8	$0w^4w^6w^8w^90w^3w^{14}w^6w^{13}$	AATAACCGCAAAAGTCACTG
9	$0w^5w^7w^9w^{10}0w^41w^7w^{14}$	AACCGTCAGGAATATTGTTC
10	$0w^6w^8w^{10}w^{11}0w^5ww^81$	AAACCGGGCTAACCATCGTT
11	$0w^7w^9w^{11}w^{12}0w^6w^2w^9w$	AAGTCACTGAAAACGCCAAT
12	$0w^8w^{10}w^{12}w^{13}0w^7w^3w^{10}w^2$	AACGGGGATGAAGTAGGGGC
13	$0w^9w^{11}w^{13}w^{14}0w^8w^4w^{11}w^3$	AACACTTGTCAACGTACTAG
14	$0w^{10}w^{12}w^{14}10w^9w^5w^{12}w^4$	AAGGGATCTTAACACCGATA
15	$0w^{11}w^{13}1w0w^{10}w^6w^{13}w^5$	AACTTGTTATAAGGACTGCC
16	$0w^{12}w^{14}ww^20w^{11}w^7w^{14}w^6$	AAGATCATGCAACTGTTCAC
17	$w^{13}1w^2w^30w^{12}w^81w^70$	TGTTGCAGAAGACGTTGTAA
18	$w^{13}w^6w^8w^6w^3w^{12}w^9w^2w^9w^7$	TGACCGACAGGACAGCCAGT
19	$w^{13}w^3w^50w^4w^{12}w^3w^7w^{14}w^8$	TGAGCCAATAGAAGGTTCCG
20	$w^{13}00w^7w^5w^{12}w^6w^5w^{12}w^9$	TGAAAAGTCCGAACCCGACA
21	$w^{13}w^4w^6w^{11}w^6w^{12}w^2w^{12}w^4w^{10}$	TGTAACCTACGAGCGATAGG
22	$w^{13}w^8w^{10}w^2w^7w^{12}w^{10}w^{11}w^3w^{11}$	TGCGGGGCGTGAGGCTAGCT
23	$w^{13}w^{14}ww^4w^8w^{12}1w^6w^{13}w^{12}$	TGTCATTACGGATTACTGGA
24	$w^{13}ww^3w^{13}w^9w^{12}w^{13}w^3w^{10}w^{13}$	TGATAGTGCAGATGAGGGTG
25	$w^{13}w^{10}w^{12}ww^{10}w^{12}w^500w^{14}$	TGGGGAATGGGACCAAAATC
26	$w^{13}w^{13}1w^{12}w^{11}w^{12}w^4w^4w^{11}1$	TGTGTTGACTGATATACTTT
27	$w^{13}w^9w^{11}w^5w^{12}w^{12}w^{14}w^81w$	TGCACTCCGAGATCCGTTAT
28	$w^{13}w^2w^4w^{10}w^{13}w^{12}w^{11}w^{14}w^6w^2$	TGGCTAGGTGGACTTCACGC
29	$w^{13}w^7w^9w^8w^{14}w^{12}0ww^8w^3$	TGGTCACGTCGAAAATCGAG
30	$w^{13}w^5w^711w^{12}w^{12}w^{10}w^2w^4$	TGCCGTTTTTGAGAGGGCTA
31	$w^{13}w^{12}w^{14}w^{14}ww^{12}ww^{13}w^5w^5$	TGGATCTCATGAATTGCCCC
32	$w^{13}w^{11}w^{13}w^9w^2w^{12}w^7w^9ww^6$	TGCTTGCAGCGAGTCAATAC
33	$w^{14}ww^3w^40w^{13}w^9ww^80$	TCATAGTAAATGCAATCGAA
34	$w^{14}w^{12}w^{14}w^{10}w^3w^{13}w^8w^{10}w^2w^7$	TCGATCGGAGTGCGGGGCGT
35	$w^{14}w^7w^9w^7w^4w^{13}w^{10}w^3w^{10}w^8$	TCGTCAGTTATGGGAGGGCG
36	$w^{14}w^4w^60w^5w^{13}w^4w^81w^9$	TCTAACAACCTGTACGTTCA
37	$w^{14}00w^8w^6w^{13}w^7w^6w^{13}w^{10}$	TCAAAACGACTGGTACTGGG

Table 3. *(Continued.) DNA correspondence table of* $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ *according to the Example 9.*

S. No.	**Elements of** $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$	**DNA 20-bases of** $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$
38	$w^{14}w^5w^7w^{12}w^7w^{13}w^3w^{13}w^5w^{11}$	TCCCGTGAGTTGAGTGCCCT
39	$w^{14}w^9w^{11}w^3w^8w^{13}w^{11}w^{12}w^4w^{12}$	TCCACTAGCGTGCTGATAGA
40	$w^{14}1w^2w^5w^9w^{13}ww^7w^{14}w^{13}$	TCTTGCCCCATGATGTTCTG
41	$w^{14}w^2w^4w^{14}w^{10}w^{13}w^{14}w^4w^{11}w^{14}$	TCGCTATCGGTGTCTACTTC
42	$w^{14}w^{11}w^{13}w^2w^{11}w^{13}w^6001$	TCCTTGGCCTTGACAAAATT
43	$w^{14}w^{14}ww^{13}w^{12}w^{13}w^5w^5w^{12}w$	TCTCATTGGATGCCCCGAAT
44	$w^{14}w^{10}w^{12}w^6w^{13}w^{13}1w^9ww^2$	TCGGGAACTGTGTTCAATGC
45	$w^{14}w^3w^5w^{11}w^14w^{13}w^{12}1w^7w^3$	TCAGCCCTTCTGGATTGTAG
46	$w^{14}w^8w^{10}w^91w^{13}0w^2w^9w^4$	TCCGGGCATTTGAAGCCATA
47	$w^{14}w^6w^8www^{13}w^{13}w^{11}w^3w^5$	TCACCGATATTGTGCTAGCC
48	$w^{14}w^{13}11w^2w^{13}w^2w^{14}w^6w^6$	TCTGTTTTGCTGGCTCACAC
49	$1w^2w^4w^50w^{14}w^{10}w^2w^90$	TTGCTACCAATCGGGCCAAA
50	$1w^{14}www^3w^{14}w^31w^7w^7$	TTTCATATAGTCAGTTGTGT
51	$1w^{13}1w^{11}w^4w^{14}w^9w^{11}w^3w^8$	TTTGTTCTTATCCACTAGCG
52	$1w^8w^{10}w^8w^5w^{14}w^{11}w^4w^{11}w^9$	TTCGGGCGCCTCCTTACTCA
53	$1w^5w^70w^6w^{14}w^5w^9ww^{10}$	TTCCGTAAACTCCCCAATGG
54	$100w^9w^7w^{14}w^8w^7w^{14}w^{11}$	TTAAAACAGTTCCGGTTCCT
55	$1w^6w^8w^{13}w^8w^{14}w^4w^{14}w^6w^{12}$	TTACCGTGCGTCTATCACGA
56	$1w^{10}w^{12}w^4w^9w^{14}w^{12}w^{13}w^5w^{13}$	TTGGGATACATCGATGCCTG
57	$1ww^3w^6w^{10}w^{14}w^2w^81w^{14}$	TTATAGACGGTCGCCGTTTC
58	$1w^3w^51w^{11}w^{14}1w^5w^{12}1$	TTAGCCTTCTTCTTCCGATT
59	$1w^{12}w^{14}w^3w^{12}w^{14}w^700w$	TTGATCAGGATCGTAAAAAT
60	$11w^2w^{14}w^{13}w^{14}w^6w^6w^{13}w^2$	TTTTGCTCTGTCACACTGGC
61	$1w^{11}w^{13}w^7w^{14}w^{14}ww^{10}w^2w^3$	TTCTTGGTTCTCATGGGCAG
62	$1w^4w^6w^{12}1w^{14}w^{13}ww^8w^4$	TTTAACGATTTCTGATCGTA
63	$1w^9w^{11}w^{10}ww^{14}0w^3w^{10}w^5$	TTCACTGGATTCAAAGGGCC
64	$1w^7w^9w^2w^2w^{14}w^{14}w^{12}w^4w^6$	TTGTCAGCGCTCTCGATAAC
65	$ww^3w^5w^601w^{11}w^3w^{10}0$	ATAGCCACAATTCTAGGGAA
66	$ww^8w^{10}w^3w^311w^{13}w^5w^7$	ATCGGGAGAGTTTTTGCCGT
67	$w1w^2w^2w^41w^4ww^8w^8$	ATTTGCGCTATTTAATCGCG
68	$ww^{14}ww^{12}w^51w^{10}w^{12}w^4w^9$	ATTCATGACCTTGGGATACA
69	$ww^9w^{11}w^9w^61w^{12}w^5w^{12}w^{10}$	ATCACTCAACTTGACCGAGG
70	$ww^6w^80w^71w^6w^{10}w^2w^{11}$	ATACCGAAGTTTACGGGCCT
71	$w00w^{10}w^81w^9w^81w^{12}$	ATAAAAGGCGTTCACGTTGA
72	$ww^7w^9w^{14}w^91w^51w^7w^{13}$	ATGTCATCCATTCCTTGTTG
73	$ww^{11}w^{13}w^5w^{10}1w^{13}w^{14}w^6w^{14}$	ATCTTGCCGGTTTGTCACTC
74	$ww^2w^4w^7w^{11}1w^3w^9w1$	ATGCTAGTCTTTAGCAATTT
75	$ww^4w^6ww^{12}1ww^6w^{13}w$	ATTAACATGATTATACTGAT
76	$ww^{13}1w^4w^{13}1w^800w^2$	ATTGTTTATGTTCGAAAAGC

Table 3. *(Continued.) DNA correspondence table of* $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ *according to the Example 9.*

S. No.	Elements of $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$	DNA 20-bases of $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$
77	$www^3 1w^{14} 1w^7 w^7 w^{14} w^3$	ATATAGTTTCTTGTGTTCAG
78	$ww^{12} w^{14} w^8 11w^2 w^{11} w^3 w^4$	ATGATCCGTTTTGCCTAGTA
79	$ww^5 w^7 w^{13} w1w^{14} w^2 w^9 w^5$	ATCCGTTGATTTTCGCCACC
80	$ww^{10} w^{12} w^{11} w^2 10w^4 w^{11} w^6$	ATGGGACTGCTTAATACTAC
81	$w^2 w^4 w^6 w^7 0ww^{12} w^4 w^{11} 0$	GCTAACGTAAATGATACTAA
82	$w^2 w^{11} w^{13} w^{12} w^3 w0w^5 w^{12} w^7$	GCCTTGGAAGATAACCGAGT
83	$w^2 w^9 w^{11} w^4 w^4 www^{14} w^6 w^8$	GCCACTTATAATATTCACCG
84	$w^2 ww^3 w^3 w^5 ww^5 w^2 w^9 w^9$	GCATAGAGCCATCCGCCACA
85	$w^2 1w^2 w^{13} w^6 ww^{11} w^{13} w^5 w^{10}$	GCTTGCTGACATCTTGCCGG
86	$w^2 w^{10} w^{12} w^{10} w^7 ww^{13} w^6 w^{13} w^{11}$	GCGGGAGGGTATTGACTGCT
87	$w^2 w^7 w^9 0w^8 ww^7 w^{11} w^3 w^{12}$	GCGTCAAACGATGTCTAGGA
88	$w^2 00w^{11} w^9 ww^{10} w^9 ww^{13}$	GCAAAACTCAATGGCAATTG
89	$w^2 w^8 w^{10} 1w^{10} ww^6 ww^8 w^{14}$	GCCGGGTTGGATACATCGTC
90	$w^2 w^{12} w^{14} w^6 w^{11} ww^{14} 1w^7 1$	GCGATCACCTATTCTTGTTT
91	$w^2 w^3 w^5 w^8 w^{12} ww^4 w^{10} w^2 w$	GCAGCCCGGAATTAGGGCAT
92	$w^2 w^5 w^7 w^2 w^{13} ww^2 w^7 w^{14} w^2$	GCCCGTGCTGATGCGTTCGC
93	$w^2 w^{14} ww^5 w^{14} ww^9 00w^3$	GCTCATCCTCATCAAAAAAG
94	$w^2 w^2 w^4 w1ww^8 w^8 1w^4$	GCGCTAATTTATCGCGTTTA
95	$w^2 w^{13} 1w^9 www^3 w^{12} w^4 w^5$	GCTGTTCAATATAGGATACC
96	$w^2 w^6 w^8 w^{14} w^2 w1w^3 w^{10} w^6$	GCACCGTCGCATTTAGGGAC
97	$w^3 w^5 w^7 w^8 0w^2 w^{13} w^5 w^{12} 0$	AGCCGTCGAAGCTGCCGAAA
98	$w^3 w^7 w^9 1w^3 w^2 ww^4 w^{11} w^7$	AGGTCATTAGGCATTACTGT
99	$w^3 w^{12} w^{14} w^{13} w^4 w^2 0w^6 w^{13} w^8$	AGGATCTGTAGCAAACTGCG
100	$w^3 w^{10} w^{12} w^5 w^5 w^2 w^2 1w^7 w^9$	AGGGGACCCCGCGCTTGTCA
101	$w^3 w^2 w^4 w^4 w^6 w^2 w^6 w^3 w^{10} w^{10}$	AGGCTATAACGCACAGGGGG
102	$w^3 ww^3 w^{14} w^7 w^2 w^{12} w^{14} w^6 w^{11}$	AGATAGTCGTGCGATCACCT
103	$w^3 w^{11} w^{13} w^{11} w^8 w^2 w^{14} w^7 w^{14} w^{12}$	AGCTTGCTCGGCTCGTTCGA
104	$w^3 w^8 w^{10} 0w^9 w^2 w^8 w^{12} w^4 w^{13}$	AGCGGGAACAGCCGGATATG
105	$w^3 00w^{12} w^{10} w^2 w^{11} w^{10} w^2 w^{14}$	AGAAAAGAGGGCCTGGGCTC
106	$w^3 w^9 w^{11} ww^{11} w^2 w^7 w^2 w^9 1$	AGCACTATCTGCGTGCCATT
107	$w^3 w^{13} 1w^7 w^{12} w^2 1ww^8 w$	AGTGTTGTGAGCTTATCGAT
108	$w^3 w^4 w^6 w^9 w^{13} w^2 w^5 w^{11} w^3 w^2$	AGTAACCATGGCCCCTAGGC
109	$w^3 w^6 w^8 w^3 w^{14} w^2 w^3 w^8 1w^3$	AGACCGAGTCGCAGCGTTAG
110	$w^3 1w^2 w^6 1w^2 w^{10} 00w^4$	AGTTGCACTTGCGGAAAATA
111	$w^3 w^3 w^5 w^2 ww^2 w^9 w^9 ww^5$	AGAGCCGCATGCCACAATCC
112	$w^3 w^{14} ww^{10} w^2 w^2 w^4 w^{13} w^5 w^6$	AGTCATGGGCGCTATGCCAC
113	$w^4 w^6 w^8 w^9 0w^3 w^{14} w^6 w^{13} 0$	TAACCGCAAAAGTCACTGAA
114	$w^4 1w^2 w^{11} w^3 w^3 w^5 w^{14} w^6 w^7$	TATTGCCTAGAGCCTCACGT

Table 3. *(Continued.) DNA correspondence table of* $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ *according to the Example 9.*

S. No.	Elements of $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$	DNA 20-bases of $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$
115	$w^4w^8w^{10}ww^4w^3w^2w^5w^{12}w^8$	TACGGGATTAAGGCCCGACG
116	$w^4w^{13}1w^{14}w^5w^30w^7w^{14}w^9$	TATGTTTCCCAGAAGTTCCA
117	$w^4w^{11}w^{13}w^6w^6w^3w^3ww^8w^{10}$	TACTTGACACAGAGATCGGG
118	$w^4w^3w^5w^5w^7w^3w^7w^4w^{11}w^{11}$	TAAGCCCCGTAGGTTACTCT
119	$w^4w^2w^41w^8w^3w^{13}1w^7w^{12}$	TAGCTATTCGAGTGTTGTGA
120	$w^4w^{12}w^{14}w^{12}w^9w^31w^81w^{13}$	TAGATCGACAAGTTCGTTTG
121	$w^4w^9w^{11}0w^{10}w^3w^9w^{13}w^5w^{14}$	TACACTAAGGAGCATGCCTC
122	$w^400w^{13}w^{11}w^3w^{12}w^{11}w^31$	TAAAAATGCTAGGACTAGTT
123	$w^4w^{10}w^{12}w^2w^{12}w^3w^8w^3w^{10}w$	TAGGGAGCGAAGCGAGGGAT
124	$w^4w^{14}ww^8w^{13}w^3ww^2w^9w^2$	TATCATCGTGAGATGCCAGC
125	$w^4w^5w^7w^{10}w^{14}w^3w^6w^{12}w^4w^3$	TACCGTGGTCAGACGATAAG
126	$w^4w^7w^9w^41w^3w^4w^9ww^4$	TAGTCATATTAGTACAATTA
127	$w^4ww^3w^7ww^3w^{11}00w^5$	TAATAGGTATAGCTAAAACC
128	$w^4w^4w^6w^3w^2w^3w^{10}w^{10}w^2w^6$	TATAACAGGCAGGGGGGCAC
129	$w^5w^7w^9w^{10}0w^41w^7w^{14}0$	CCGTCAGGAATATTGTTCAA
130	$w^5w^5w^7w^4w^3w^4w^{11}w^{11}w^3w^7$	CCCCGTTAAGTACTCTAGGT
131	$w^5ww^3w^{12}w^4w^4w^61w^7w^8$	CCATAGGATATAACTTGTCG
132	$w^5w^9w^{11}w^2w^5w^4w^3w^6w^{13}w^9$	CCCACTGCCCTAAGACTGCA
133	$w^5w^{14}w1w^6w^40w^81w^{10}$	CCTCATTTACTAAACGTTGG
134	$w^5w^{12}w^{14}w^7w^7w^4w^4w^2w^9w^{11}$	CCGATCGTGTTATAGCCACT
135	$w^5w^4w^6w^6w^8w^4w^8w^5w^{12}w^{12}$	CCTAACACCGTACGCCGAGA
136	$w^5w^3w^5ww^9w^4w^{14}ww^8w^{13}$	CCAGCCATCATATCATCGTG
137	$w^5w^{13}1w^{13}w^{10}w^4ww^9ww^{14}$	CCTGTTTGGGTAATCAATTC
138	$w^5w^{10}w^{12}0w^{11}w^4w^{10}w^{14}w^61$	CCGGGAAACTTAGGTCACTT
139	$w^500w^{14}w^{12}w^4w^{13}w^{12}w^4w$	CCAAAATCGATATGGATAAT
140	$w^5w^{11}w^{13}w^3w^{13}w^4w^9w^4w^{11}w^2$	CCCTTGAGTGTACATACTGC
141	$w^51w^2w^9w^{14}w^4w^2w^3w^{10}w^3$	CCTTGCCATCTAGCAGGGAG
142	$w^5w^6w^8w^{11}1w^4w^7w^{13}w^5w^4$	CCACCGCTTTTAGTTGCCTA
143	$w^5w^8w^{10}w^5ww^4w^5w^{10}w^2w^5$	CCCGGGCCATTACCGGGCCC
144	$w^5w^2w^4w^8w^2w^4w^{12}00w^6$	CCGCTACGGCTAGAAAAAAC
145	$w^6w^8w^{10}w^{11}0w^5ww^810$	ACCGGGCTAACCATCGTTAA
146	$w^6w^3w^5w^9w^3w^5w^{13}00w^7$	ACAGCCCAAGCCTGAAAAGT
147	$w^6w^6w^8w^5w^4w^5w^{12}w^{12}w^4w^8$	ACACCGCCTACCGAGATACG
148	$w^6w^2w^4w^{13}w^5w^5w^7ww^8w^9$	ACGCTATGCCCCGTATCGCA
149	$w^6w^{10}w^{12}w^3w^6w^5w^4w^7w^{14}w^{10}$	ACGGGAAGACCCTAGTTCGG
150	$w^61w^2ww^7w^50w^9ww^{11}$	ACTTGCATGTCCAACAATCT
151	$w^6w^{13}1w^8w^8w^5w^5w^3w^{10}w^{12}$	ACTGTTCGCGCCCCAGGGGA
152	$w^6w^5w^7w^7w^9w^5w^9w^6w^{13}w^{13}$	ACCCGTGTCACCCAACTGTG
153	$w^6w^4w^6w^2w^{10}w^51w^2w^9w^{14}$	ACTAACGCGGCCTTGCCATC

Table 3. *(Continued.) DNA correspondence table of* $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ *according to the Example 9.*

S. No.	Elements of $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$	DNA 20-bases of $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$
154	$w^6w^{14}ww^{14}w^{11}w^5w^2w^{10}w^2 1$	ACTCATTCCTCCGCGGGCTT
155	$w^6w^{11}w^{13}0w^{12}w^5w^{11}1w^7w$	ACCTTGAAGACCCTTTGTAT
156	$w^6 001w^{13}w^5w^{14}w^{13}w^5w^2$	ACAAAATTTGCCTCTGCCGC
157	$w^6w^{12}w^{14}w^4w^{14}w^5w^{10}w^5w^{12}w^3$	ACGATCTATCCCGGCCGAAG
158	$w^6ww^3w^{10}1w^5w^3w^4w^{11}w^4$	ACATAGGGTTCCAGTACTTA
159	$w^6w^7w^9w^{12}ww^5w^8w^{14}w^6w^5$	ACGTCAGAATCCCGTCACCC
160	$w^6w^9w^{11}w^6w^2w^5w^6w^{11}w^3w^6$	ACCACTACGCCCACCTAGAC
161	$w^7w^9w^{11}w^{12}0w^6w^2w^9w0$	GTCACTGAAAACGCCAATAA
162	$w^7w^{10}w^{12}w^7w^3w^6w^7w^{12}w^4w^7$	GTGGGAGTAGACGTGATAGT
163	$w^7w^4w^6w^{10}w^4w^6w^{14}00w^8$	GTTAACGGTAACTCAAAACG
164	$w^7w^7w^9w^6w^5w^6w^{13}w^{13}w^5w^9$	GTGTCAACCCACTGTGCCCA
165	$w^7w^3w^5w^{14}w^6w^6w^8w^2w^9w^{10}$	GTAGCCTCACACCGGCCAGG
166	$w^7w^{11}w^{13}w^4w^7w^6w^5w^8 1w^{11}$	GTCTTGTAGTACCCCGTTCT
167	$w^7ww^3w^2w^8w^60w^{10}w^2w^{12}$	GTATAGGCCGACAAGGGCGA
168	$w^7w^{14}ww^9w^9w^6w^6w^4w^{11}w^{13}$	GTTCATCACAACACTACTTG
169	$w^7w^6w^8w^8w^{10}w^6w^{10}w^7w^{14}w^{14}$	GTACCGCGGGACGGGTTCTC
170	$w^7w^5w^7w^3w^{11}w^6ww^3w^{10}1$	GTCCGTAGCTACATAGGGTT
171	$w^7 1w^2 1w^{12}w^6w^3w^{11}w^3w$	GTTTGCTTGAACAGCTAGAT
172	$w^7w^{12}w^{14}0w^{13}w^6w^{12}ww^8w^2$	GTGATCAATGACGAATCGGC
173	$w^7 00ww^{14}w^6 1w^{14}w^6w^3$	GTAAAAATTCACTTTCACAG
174	$w^7w^{13}1w^5 1w^6w^{11}w^6w^{13}w^4$	GTTGTTCCTTACCTACTGTA
175	$w^7w^2w^4w^{11}ww^6w^4w^5w^{12}w^5$	GTGCTACTATACTACCGACC
176	$w^7w^8w^{10}w^{13}w^2w^6w^9 1w^7w^6$	GTCGGGTGGCACCATTGTAC
177	$w^8w^{10}w^{12}w^{13}0w^7w^3w^{10}w^2 0$	CGGGGATGAAGTAGGGGCAA
178	$w^8w^9w^{11}w^{14}w^3w^7w^{10}ww^8w^7$	CGCACTTCAGGTGGATCGGT
179	$w^8w^{11}w^{13}w^8w^4w^7w^8w^{13}w^5w^8$	CGCTTGCGTAGTCGTGCCCG
180	$w^8w^5w^7w^{11}w^5w^7 100w^9$	CGCCGTCTCCGTTTAAAACA
181	$w^8w^8w^{10}w^7w^6w^7w^{14}w^{14}w^6w^{10}$	CGCGGGGTACGTTCTCACGG
182	$w^8w^4w^6 1w^7w^7w^9w^3w^{10}w^{11}$	CGTAACTTGTGTCAAGGGCT
183	$w^8w^{12}w^{14}w^5w^8w^7w^6w^9ww^{12}$	CGGATCCCCGGTACCAATGA
184	$w^8w^2w^4w^3w^9w^70w^{11}w^3w^{13}$	CGGCTAAGCAGTAACTAGTG
185	$w^8 1w^2w^{10}w^{10}w^7w^7w^5w^{12}w^{14}$	CGTTGCGGGGGTGTCCGATC
186	$w^8w^7w^9w^9w^{11}w^7w^{11}w^8 11$	CGGTCACACTGTCTCGTTTT
187	$w^8w^6w^8w^4w^{12}w^7w^2w^4w^1 1w$	CGACCGTAGAGTGCTACTAT
188	$w^8ww^3ww^{13}w^7w^4w^{12}w^4w^2$	CGATAGATTGGTTAGATAGC
189	$w^8w^{13}10w^{14}w^7w^{13}w^2w^9w^3$	CGTGTTAATCGTTGGCCAAG
190	$w^8 00w^2 1w^7w 1w^7w^4$	CGAAAAGCTTGTATTTGTTA
191	$w^8w^{14}ww^6ww^7w^{12}w^7w^{14}w^5$	CGTCATACATGTGAGTTCCC

Table 3. *(Continued.) DNA correspondence table of* $\langle T_g^5 \mid T^5_{(g^*)^{\circ 4}} \rangle$ *according to the Example 9.*

S. No.	Elements of $\langle T_g^5 \mid T^5_{(g^*)^{\circ 4}} \rangle$	DNA 20-bases of $\Phi(\langle T_g^5 \mid T^5_{(g^*)^{\circ 4}} \rangle)$
192	$w^8w^3w^5w^{12}w^2w^7w^5w^6w^{13}w^6$	CGAGCCGAGCGTCCACTGAC
193	$w^9w^{11}w^{13}w^{14}0w^8w^4w^{11}w^30$	CACTTGTCAACGTACTAGAA
194	$w^9w^4w^6w^{13}w^3w^8w^6w^7w^{14}w^7$	CATAACTGAGCGACGTTCGT
195	$w^9w^{10}w^{12}1w^4w^8w^{11}w^2w^9w^8$	CAGGGATTTACGCTGCCACG
196	$w^9w^{12}w^{14}w^9w^5w^8w^9w^{14}w^6w^9$	CAGATCCACCCGCATCACCA
197	$w^9w^6w^8w^{12}w^6w^8w00w^{10}$	CAACCGGAACCGATAAAAGG
198	$w^9w^9w^{11}w^8w^7w^811w^7w^{11}$	CACACTCGGTCGTTTTGTCT
199	$w^9w^5w^7ww^8w^8w^{10}w^4w^{11}w^{12}$	CACCGTATCGCGGGTACTGA
200	$w^9w^{13}1w^6w^9w^8w^7w^{10}w^2w^{13}$	CATGTTACCACGGTGGGCTG
201	$w^9w^3w^5w^4w^{10}w^80w^{12}w^4w^{14}$	CAAGCCTAGGCGAAGATATC
202	$w^9ww^3w^{11}w^{11}w^8w^8w^6w^{13}1$	CAATAGCTCTCGCGACTGTT
203	$w^9w^8w^{10}w^{10}w^{12}w^8w^{12}w^9ww$	CACGGGGGGACGGACAATAT
204	$w^9w^7w^9w^5w^{13}w^8w^3w^5w^{12}w^2$	CAGTCACCTGCGAGCCGAGC
205	$w^9w^2w^4w^2w^{14}w^8w^5w^{13}w^5w^3$	CAGCTAGCTCCGCCTGCCAG
206	$w^9w^{14}w01w^8w^{14}w^3w^{10}w^4$	CATCATAATTCGTCAGGGTA
207	$w^900w^3ww^8w^2ww^8w^5$	CAAAAAAGATCGGCATCGCC
208	$w^91w^2w^7w^2w^8w^{13}w^81w^6$	CATTGCGTGCCGTGCGTTAC
209	$w^{10}w^{12}w^{14}10w^9w^5w^{12}w^40$	GGGATCTTAACACCGATAAA
210	$w^{10}ww^3w^8w^3w^9w^{14}w^9ww^7$	GGATAGCGAGCATCCAATGT
211	$w^{10}w^5w^7w^{14}w^4w^9w^7w^81w^8$	GGCCGTTCTACAGTCGTTCG
212	$w^{10}w^{11}w^{13}ww^5w^9w^{12}w^3w^{10}w^9$	GGCTTGATCCCAGAAGGGCA
213	$w^{10}w^{13}1w^{10}w^6w^9w^{10}1w^7w^{10}$	GGTGTTGGACCAGGTTGTGG
214	$w^{10}w^7w^9w^{13}w^7w^9w^200w^{11}$	GGGTCATGGTCAGCAAAACT
215	$w^{10}w^{10}w^{12}w^9w^8w^9www^8w^{12}$	GGGGGACACGCAATATCGGA
216	$w^{10}w^6w^8w^2w^9w^9w^{11}w^5w^{12}w^{13}$	GGACCGGCCACACTCCGATG
217	$w^{10}w^{14}ww^7w^{10}w^9w^8w^{11}w^3w^{14}$	GGTCATGTGGCACGCTAGTC
218	$w^{10}w^4w^6w^5w^{11}w^90w^{13}w^51$	GGTAACCCCTCAAATGCCTT
219	$w^{10}w^2w^4w^{12}w^{12}w^9w^9w^7w^{14}w$	GGGCTAGAGACACAGTTCAT
220	$w^{10}w^9w^{11}w^{11}w^{13}w^9w^{13}w^{10}w^2w^2$	GGCACTCTTGCATGGGGCGC
221	$w^{10}w^8w^{10}w^6w^{14}w^9w^4w^6w^{13}w^3$	GGCGGGACTCCATAACTGAG
222	$w^{10}w^3w^5w^31w^9w^6w^{14}w^6w^4$	GGAGCCAGTTCAACTCACTA
223	$w^{10}1w^20ww^91w^4w^{11}w^5$	GGTTGCAAATCATTTACTCC
224	$w^{10}00w^4w^2w^9w^3w^2w^9w^6$	GGAAAATAGCCAAGGCCAAC
225	$w^{11}w^{13}1w0w^{10}w^6w^{13}w^50$	CTTGTTATAAGGACTGCCAA
226	$w^{11}00w^5w^3w^{10}w^4w^3w^{10}w^7$	CTAAAACCAGGGTAAGGGGT
227	$w^{11}w^2w^4w^9w^4w^{10}1w^{10}w^2w^8$	CTGCTACATAGGTTGGGCCG
228	$w^{11}w^6w^81w^5w^{10}w^8w^9ww^9$	CTACCGTTCCGGCGCAATCA
229	$w^{11}w^{12}w^{14}w^2w^6w^{10}w^{13}w^4w^{11}w^{10}$	CTGATCGCACGGTGTACTGG
230	$w^{11}w^{14}ww^{11}w^7w^{10}w^{11}ww^8w^{11}$	CTTCATCTGTGGCTATCGCT

Table 3. *(Continued.) DNA correspondence table of* $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$ *according to the Example 9.*

S. No.	**Elements of** $\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle$	**DNA 20-bases of** $\Phi(\langle T_g^5 \mid T_{(g^*)^{\circ 4}}^5 \rangle)$
231	$w^{11}w^8w^{10}w^{14}w^8w^{10}w^3 00w^{12}$	CTCGGGTCCGGGAGAAAAGA
232	$w^{11}w^{11}w^{13}w^{10}w^9w^{10}w^2w^2w^9w^{13}$	CTCTTGGGCAGGGCGCCATG
233	$w^{11}w^7w^9w^3w^{10}w^{10}w^{12}w^6w^{13}w^{14}$	CTGTCAAGGGGGGAACTGTC
234	$w^{11}1w^2w^8w^{11}w^{10}w^9w^{12}w^4 1$	CTTTGCCGCTGGCAGATATT
235	$w^{11}w^5w^7w^6w^{12}w^{10}0w^{14}w^6w$	CTCCGTACGAGGAATCACAT
236	$w^{11}w^3w^5w^{13}w^{13}w^{10}w^{10}w^8 1w^2$	CTAGCCTGTGGGGGCGTTGC
237	$w^{11}w^{10}w^{12}w^{12}w^{14}w^{10}w^{14}w^{11}w^3w^3$	CTGGGAGATCGGTCCTAGAG
238	$w^{11}w^9w^{11}w^7 1w^{10}w^5w^7w^{14}w^4$	CTCACTGTTTGGCCGTTCTA
239	$w^{11}w^4w^6w^4ww^{10}w^7 1w^7w^5$	CTTAACTAATGGGTTTGTCC
240	$w^{11}ww^3 0w^2w^{10}ww^5w^{12}w^6$	CTATAGAAGCGGATCCGAAC
241	$w^{12}w^{14}ww^2 0w^{11}w^7w^{14}w^6 0$	GATCATGCAACTGTTCACAA
242	$w^{12}w^2w^4 0w^3w^{11}w^2w^6w^{13}w^7$	GAGCTAAAAGCTGCACTGGT
243	$w^{12}00w^6w^4w^{11}w^5w^4w^{11}w^8$	GAAAAAACTACTCCTACTCG
244	$w^{12}w^3w^5w^{10}w^5w^{11}ww^{11}w^3w^9$	GAAGCCGGCCCTATCTAGCA
245	$w^{12}w^7w^9ww^6w^{11}w^9w^{10}w^2w^{10}$	GAGTCAATACCTCAGGGCGG
246	$w^{12}w^{13}1w^3w^7w^{11}w^{14}w^5w^{12}w^{11}$	GATGTTAGGTCTTCCCGACT
247	$w^{12}1w^2w^{12}w^8w^{11}w^{12}w^2w^9w^{12}$	GATTGCGACGCTGAGCCAGA
248	$w^{12}w^9w^{11}1w^9w^{11}w^4 00w^{13}$	GACACTTTCACTTAAAAATG
249	$w^{12}w^{12}w^{14}w^{11}w^{10}w^{11}w^3w^3w^{10}w^{14}$	GAGATCCTGGCTAGAGGGTC
250	$w^{12}w^8w^{10}w^4w^{11}w^{11}w^{13}w^7w^{14}1$	GACGGGTACTCTTGGTTCTT
251	$w^{12}ww^3w^9w^{12}w^{11}w^{10}w^{13}w^5w$	GAATAGCAGACTGGTGCCAT
252	$w^{12}w^6w^8w^7w^{13}w^{11}01w^7w^2$	GAACCGGTTGCTAATTGTGC
253	$w^{12}w^4w^6w^{14}w^{14}w^{11}w^{11}w^9ww^3$	GATAACTCTCCTCTCAATAG
254	$w^{12}w^{11}w^{13}w^{13}1w^{11}1w^{12}w^4w^4$	GACTTGTGTTCTTTGATATA
255	$w^{12}w^{10}w^{12}w^8ww^{11}w^6w^8 1w^5$	GAGGGACGATCTACCGTTCC
256	$w^{12}w^5w^7w^5w^2w^{11}w^8ww^8w^6$	GACCGTCCGCCTCGATCGAC

Appendix B

Table 4. Code parameters and MDS/AMDS/self-orthogonality over F_{16} derived from polynomial $g(x)$, where T: True, F: False.

Polynomial $g(x)$ over F_{16}	**Length of $\langle T_g^n \rangle$**	**Dimension of $\langle T_g^n \rangle$**	**Distance of $\langle T_g^n \rangle$**	**$\langle T_g^n \rangle$ is self-orthogonal**	**$\langle T_{g*}^n \rangle$ is self-orthogonal**	**$\langle T_{(g*)^{\circ 4}}^n \rangle$ is self-orthogonal**	**Length of $\langle T_g^n \mid T_{g^*}^n \rangle$**	**Dimension of $\langle T_g^n \mid T_{g^*}^n \rangle$**	**Distance of $\langle T_g^n \mid T_{g^*}^n \rangle$**	**l of l-MDS code of $\langle T_g^n \mid T_{g^*}^n \rangle$**	**Length of $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle$**	**l of l-MDS code of $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle$**	**Distance of $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle$**	**Length of $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle^\perp$**	**Dimension of $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle^\perp$**	**Distance of $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle^\perp$**	**l of l-MDS code of $\langle T_g^n \mid T_{(g^*)^{\circ 4}}^n \rangle^\perp$**
$w^{13}x^5 + w^9x^4 + wx^3 + w^2x^2 + x + 1$	7	2	6	F	F	F	14	2	12	1	14	2	12	14	12	2	1
$w^3x^6 + w^9x^5 + w^{11}x^4 + w^5x^3 + w^6x^2 + w^{10}x + 1$	8	2	7	T	T	T	16	2	14	1	16	2	14	16	14	2	1
$w^8x^7 + w^9x^6 + w^2x^5 + wx^4 + w^5x^3 + w^{12}x^2 + w^2x + w^2$	9	2	8	T	T	T	18	2	16	1	18	2	16	18	16	2	1
$w^{12}x^8 + w^3x^7 + w^{11}x^6 + w^{12}x^5 + w^{11}x^4 + wx^3 + w^{12}x^2 + w^7x + w^{11}$	10	2	9	F	F	F	20	2	18	1	20	2	18	20	18	2	1
$w^6x^9 + x^8 + x^7 + w^3x^6 + x^5 + w^8x^4 + x^3 + w^{13}x^2 + w^9x + 1$	11	2	10	F	F	F	22	2	20	1	22	2	20	22	20	2	1
$w^{11}x^4 + w^{11}x^3 + w^4x^2 + w^8x + 1$	7	3	5	F	F	F	14	3	10	2	14	3	10	14	11	2	2
$x^6 + w^9x^5 + w^{11}x^4 + w^{14}x^3 + w^{13}x^2 + w^{10}x + 1$	9	3	6	F	F	F	18	3	14	2	18	3	12	18	15	2	NO
$w^6x^7 + w^5x^6 + x^5 + w^6x^4 + w^3x^3 + w^2x^2 + x + w^{13}$	10	3	7	F	F	F	20	3	16	2	20	3	16	20	17	2	2
$w^6x^8 + w^{13}x^7 + w^3x^6 + w^5x^5 + w^5x^4 + w^{12}x^3 + wx^2 + w^4x + w^5$	11	3	8	F	F	F	22	3	18	2	22	3	16	22	19	2	NO
$w^{11}x^8 + w^5x^7 + w^7x^6 + w^{14}x^5 + w^{10}x^4 + w^3x^3 + x^2 + w^4x + w^5$	11	3	8	F	F	F	22	3	18	2	22	3	16	22	19	2	NO

AIMS Press