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Research article

Analytical solution and asymptotic limit to one-dimensional compressible magnetohydrodynamic equations without magnetic diffusion

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Abstract: This paper gives the analytical solution to the one-dimensional compressible MHD equations without magnetic diffusion in half space. The interesting result is that the solution to one-dimensional compressible isentropic MHD equations without magnetic diffusion on $(x,t) \in [0,+\infty)\times R_+$ can be expressed as the initial density and boundary function of velocity strictly if and only if the solution to the Riccati differential equation of the boundary condition function exists under some assumptions. When the magnetic field is zero, the compressible MHD equations become compressible Navier–Stokes equations. The analytical solution was studied by Dou & Zhao 2021 on the 1D Navier–Stokes equations, which is a special case in this paper. The magnetic field and the coupling term of the magnetic field and velocity result in more complicated computations and estimates. And, we can prove that the analytical solution to compressible MHD equations without magnetic diffusion depends continuously on the initial value. The existence of a steady solution to compressible MHD equations is given, and the large-time behavior and asymptotic limit theorem of the analytical solution are also shown when the time tends to infinity and the initial data of magnetic field goes to zero. Finally, we give several examples to show that the set of this kind of solution is not empty.

Keywords: compressible MHD equations; Riccati differential equation; initial boundary value problem; analytical solution; asymptotic limit
Mathematics Subject Classification: 76N99, 35M33, 35Q30

1. Introduction

We consider the one-dimensional compressible MHD equations in half space in the following:

$$\begin{aligned}
&\rho_t + (\rho v)_x = 0 \quad (x,t) \in [0,+\infty) \times R_+,\\
&\rho(v_t + v v_x) + p_x = \mu v_{xx} + H_x H, \quad (x,t) \in [0,+\infty) \times R_+,
\end{aligned}$$

$$H_t + (vH)_x = \eta H_{xx}, \quad (x,t) \in [0,+\infty) \times R_+,$$

where $\rho(x,t)$, $v(x,t)$, and $H(x,t)$ stand for the density, velocity, and magnetic field of compressible flow, which are the functions of temporal variable t and spatial variable x. The constant μ is the viscosity coefficient, and η is the constant magnetic diffusion coefficient. $p = p(\rho)$ represents the pressure of the flow, which depends on temporal variable t and spatial variable x. We assume the initial data:

$$\rho(x,t=0) = \rho_0(x), v(x,t=0) = v_0(x), H(x,t=0) = H_0(x),$$

and the boundary condition:

$$v(0,t) = f(t), H(0,t) = g(t).$$

If we let magnetic diffusion viscosity $\eta = 0$, then we have

$$\rho_t + (\rho v)_x = 0 \;\; (x,t) \in [0,+\infty) \times R_+, \tag{1.1}$$

$$\rho(v_t + vv_x) + p_x = \mu v_{xx} + H_x H, \;\; (x,t) \in [0,+\infty) \times R_+, \tag{1.2}$$

$$H_t + (vH)_x = 0, \;\; (x,t) \in [0,+\infty) \times R_+. \tag{1.3}$$

And the initial data can be reduced to:

$$\rho(x,t=0) = \rho_0(x), H(x,t=0) = H_0, \tag{1.4}$$

and the boundary condition:

$$v(0,t) = f(t). \tag{1.5}$$

Here, we do not need the boundary condition of H. And let

$$\rho_0(x) > 0 \text{ and } \rho_0(x) \in C^2(0,+\infty), \;\; f(t) \in C^1(0,+\infty), \;\; p(\cdot) \in C^1(0,+\infty).$$

If the time goes to infinity, the compressible MHD equations become steady compressible MHD equations formally, as follows:

$$(\bar{\rho}\bar{v})_x = 0 \;\; x \in [0,+\infty), \tag{1.6}$$

$$\rho\bar{v}\bar{v}_x + \bar{p}_x = \mu\bar{v}_{xx} + \bar{H}_x\bar{H}, \;\; x \in [0,+\infty), \tag{1.7}$$

$$(\bar{v}\bar{H})_x = 0, \;\; x \in [0,+\infty). \tag{1.8}$$

The physical background, the well-posedness and the vanishing viscosity limit of compressible MHD equations have been studied by many applied mathematicians, due to its physical importance, complexity, rich phenomena, and mathematical challenges in [2, 30, 31]. Jiang et al. investigated the low Mach number limit of local smooth solutions to the full compressible MHD equations with heat conductivity in [14, 15] for the whole space or a torus. The existence of global weak solutions to the compressible MHD equations was established by [11, 33], and the low Mach number limit was studied in [12, 13]. The low Mach number limit was established in [12–15] for the compressible MHD equations in the whole space or a torus. Dou et al. considered the low Mach number limit of compressible MHD equations with the boundary conditions in [6, 7]. Liang et al. obtained the low Mach number limit of the three-dimensional non-isentropic MHD equations with or without the magnetic diffusion in a bounded domain if the temperature variation is large but finite in [24]. As for the analytical solution, Linshiz et al. showed the global well-posedness and regularity of solutions of a certain MHD-α model in [25]. Zhang et al. obtained an analytical solution to the famous Falkner–Skan equation of the MHD flow, that is, the sink flow with a velocity power index of -1 in [35]. Liu et al. proved the global existence and the large-time decay estimate of solutions to the two-dimensional MHD boundary layer equations with small initial data, which is analytical in the tangential variable in [26]. Recently, Hastir et al. gave an explicit solution to the operator Riccati equation, which solved the Linear–Quadratic optimal control problem for a class of boundary controlled hyperbolic partial differential equations defined on a one-dimensional spatial domain in [9]. David et al. [4] derived a set of simplified equations for numerical studies of reduced magnetohydrodynamic turbulence within a small patch of the radially expanding solar wind. Cho et al. showed Liouville-type theorems for the stationary compressible ideal MHD equations in [3].

As for the related compressible Navier–Stokes system (the system (1.1)–(1.3) with $H = 0$), there exists much literature on the global existence and large-time behavior of solutions. Kanel [18] addressed the problems for sufficiently smooth data when the viscosity is a positive constant and the initial density is greater than zero. Serre [27] and Hoff [10] considered the compressible Navier–Stokes equations when the initial data is discontinuous. As viscosity $\mu = \mu(\rho)$ has a positive constant lower bound, global well-posedness and large-time behavior of solutions to the system were given in [1, 19, 29] as the initial data is away from vacuum. When we get the well-posedness of solutions to the compressible Navier–Stokes equations, the existence of vacuum is a major difficulty. Ding et al. obtained the global existence of classical solutions to 1D compressible Navier–Stokes equations in bounded domains in [5], provided that $\mu \in C^2[0;\infty)$ satisfies $0 < \bar{\mu} \le \mu(\rho) \le C(1 + P(\rho))$. Ye got the global classical large solutions to the Cauchy problem (1.1)-(1.2) with $\mu(\rho) = 1+\rho^\beta, 0 \le \beta < \gamma$ in [32]. Zhang et al. arrived at the global existence of classical solution to the 1D Navier–Stokes equations for viscous compressible and heat-conducting fluids when temperature satisfies the Robin boundary condition in [36]. Li et al. got the uniform upper bound of density and the global well-posedness of strong (classical) solutions with the external force in [22]. For the 2D case, if the viscous coefficients μ is constant and $\lambda = \lambda(\rho)$, global well-posedness of classical solutions to compressible Navier–Stokes equations with vacuum was obtained in [16, 17, 21]. Li et al. reached the global well-posedness and large-time asymptotic behavior of strong (classical) solutions to the Cauchy problem of the Navier–Stokes equations for viscous compressible barotropic flows in 2D or 3D space with vacuum as far-field density in [23], when the initial energy is small and the viscosity coefficients are constants. Recently, Dou et al. obtained the analytical solution to one-dimensional compressible Navier–Stokes equations on the half space and gave the expression of solutions by the initial data and boundary condition in [8], which is a special case when the magnetic field $H = 0$ in our paper. Shi et al. got the determination of the 3D Navier–Stokes equations with damping in [28].

In this paper, we obtain an interesting result on the solution to one-dimensional compressible MHD equations (1.1)-(1.2) without magnetic diffusion; that is, the solutions to the initial boundary value problem of 1D compressible MHD equations (1.1)-(1.2) in half space can be turned into the solution to the Riccati differential equation under some suitable assumptions. And we give the existence of a steady solution to compressible MHD equations which is not trivial solution, and the large-time behavior and asymptotic limit theorem of the analytical solution when the initial data of the magnetic field goes to zero. Furthermore, several examples can show the set of this kind of solution is not empty.

First, we denote

$$A(x,t) = \mu\left(\frac{\rho_0''(t+x)}{\rho_0^2(t+x)} - \frac{2(\rho_0'(t+x))^2}{\rho_0^3(t+x)}\right), \tag{1.9}$$

$$B(x,t) = \frac{\rho_0'(t+x)}{\rho_0^2(t+x)}, \tag{1.10}$$

$$C(x,t) = p'(\rho_0(t+x))\rho_0'(t+x). \tag{1.11}$$

And we give the following assumption:
Assumption 1. $f(t)$ satisfies the Riccati differential equation:

$$f'(t) + Q(t)(1+f(t)) + P(t)(1+f(t))^2 = R(t), \tag{1.12}$$

when

$$\rho_0(x) \in C^2(0,+\infty), \quad f(t) \in C^1(0,+\infty), \quad p(\cdot) \in C^1(0,+\infty), \tag{1.13}$$

and there exist three functions $Q(t)$, $P(t)$, $R(t)$, which only depend on t such that

$$Q(t) = \frac{\rho_0'(t)}{\rho_0(t)} + A(x,t),\ P(t) = -\rho_0(t)B(x,t),\ R(t) = -\frac{C(x,t)}{\rho_0(t)} \tag{1.14}$$

for $(x,t) \in [0,+\infty) \times [0,+\infty)$.

Theorem 1.1. (Analytical solution) The functions $(\rho, v, H)(t,x) = \Big(\rho_0(t+x), (1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1, H_0 \exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)} ds\}\Big)$ is the solution to compressible MHD equations (1.1)–(1.3) with the initial data (1.4) and boundary condition (1.5), if and only if Assumption 1 holds. Furthermore, if the following condition holds

$$PP'' + PQP' - \frac{3}{2}(P')^2 - PQ' - \frac{1}{2}P^2Q^2 - 2P^2R < 0, \tag{1.15}$$

then we reach the global existence and uniqueness of the analytical solution to (1.12).
(Continuous dependence) If the initial data has a small perturbation as

$$\rho_\epsilon(x, t=0) = \rho_0(x) + \epsilon\rho_0(x), H_\epsilon(x, t=0) = H_0, \tag{1.16}$$

where ϵ is small. Then, the functions $(\rho_\epsilon, v_\epsilon, H_\epsilon)(t,x) = \Big(\rho_0(t+x) + \epsilon\rho_0(t+x), (1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1, H_0 \exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)} ds\}\Big)$ is the solution to compressible MHD equations (1.1)–(1.3) with the initial data (1.16) and boundary condition (1.5), if and only if Assumption 1 holds.

And, we have the following large-time behavior and asymptotic limit theorem of the analytical solution to compressible MHD equations when the time tends to infinity and the initial data of the magnetic field goes to zero.

Theorem 1.2. (Existence of steady solution) Assume $|\bar{v}| \geq \delta > 0$; we obtain the existence of steady solution $(\bar{\rho}, \bar{v}, \bar{H}) = (\bar{\rho}(x), \bar{v}(x), \bar{H}(x))$ to the system (1.6)–(1.8) with the boundary condition

$$(\bar{\rho}(x=0), \bar{v}(x=0), \bar{H}(x=0), \bar{v}_x(x=0)) = (\bar{\rho}(0), \bar{v}(0), \bar{H}(0), \bar{v}_x(0)).$$

(Large-time behavior) If

$$f(t) \to \bar{f}, \quad \text{as } t \to \infty,$$

the analytical solution $(\rho(t,x), v(t,x),\ H(t,x))$ to compressible MHD equations (1.1)–(1.3) with the initial data (1.4) and boundary condition (1.5) will go to the steady solution, i.e.,

$$(\rho(t,x), v(t,x), H(t,x)) \to (\bar{\rho}, \bar{v}, \bar{H}) = (\bar{\rho}(x), \bar{f}\bar{\rho}(0)/\bar{\rho}(x), \bar{\rho}(x)\bar{H}(0)/\bar{\rho}(0)), \quad \text{as } t \to \infty.$$

(Asymptotic limit) When $H_0 \to 0$, the analytical solution $(\rho(t,x),\ v(t,x),\ H(t,x))$ to compressible MHD equations tends to $(\tilde{\rho}(t,x), \tilde{v}(t,x), 0)$, which satisfy the following compressible Navier–Stokes equations

$$\tilde{\rho}_t + (\tilde{\rho}\tilde{v})_x = 0, \quad (x,t) \in [0,+\infty) \times R_+, \tag{1.17}$$

$$\tilde{\rho}(\tilde{v}_t + \tilde{v}\tilde{v}_x) + \tilde{p}_x = \mu \tilde{v}_{xx}, \quad (x,t) \in [0,+\infty) \times R_+, \tag{1.18}$$

with the initial data

$$\tilde{\rho}(x, t=0) = \tilde{\rho}_0(x) \tag{1.19}$$

and the boundary condition

$$\tilde{v}(0,t) = f(t). \tag{1.20}$$

Here, the pressure $\tilde{p} = p(\tilde{\rho})$.

Remark 1.3. As we all know, the existence of a general solution to the Riccati equation is still an open problem. However, if $P(t)$, $Q(t)$, and $R(t)$ satisfy (1.13)–(1.15), we can get the global existence of an analytical solution to (1.12), which can be seen in Theorem 1.1.

Remark 1.4. The analytical solution $(\rho, v, H)(t,x)$ obtained by Theorem 1.1 satisfies the divergence-free condition of the magnetic field, i.e., $H_x = 0$. And this means that the magnetic field is spatially constant. Because the divergence of H is free in MHD equations, this condition degenerates to $H = H(t)$, which is spatially constant in the one-dimensional case.

Remark 1.5. In Theorem 1.1, the initial value $\rho_0(x), v_0(x), H_0(x)$ can be bounded or not in Sobolev space, which is determined by the given condition of the initial value.

1) Bounded case. If the initial data satisfy

$$\|\rho_0(x)\|_{L^1([0,+\infty))} \leq C,\ \rho_0(0) = 0,\ \|H_0(x)\|_{L^1([0,+\infty))} \leq C$$

and $f(t) = 0$, then we have the initial energy $\rho_0(x)v_0^2(x) \in L^1([0,+\infty))$. In fact,

$$\begin{aligned}
&\int_0^\infty \rho_0(x)v_0^2(x)dx \\
=\ &\int_0^\infty \rho_0(x)\left(\frac{(1+f(0))\rho_0(0)}{\rho_0(x)} - 1\right)^2 (x)dx
\end{aligned}$$

$$= \int_0^\infty \rho_0(x)dx \le C.$$

As a result of basic energy estimate, we can get

$$\int_0^\infty \rho(x,t)v(x,t)^2 dx \le C.$$

2) Unbounded case. It is obvious to get that the energy estimate is unbounded when the boundary condition of velocity $f(t) \neq 0$.

2. Proof of main theorems

First of all, we will prove Theorem 1.1.

Proof. **Analytical solution** Firstly, we show that the analytical solution to the initial boundary value problem of the one-dimensional compressible MHD equations can be turned to looking for the solution to the Riccati equation.

($\Rightarrow$) If we have the analytical function:

$$\rho(t,x) = \rho_0(t+x),$$

we arrive at

$$\begin{aligned} v(x,t) &= \exp\{-\int_0^x \frac{\rho_0'(y+t)}{\rho_0(y+t)}dy\}(1+v(0,t)) - 1 \\ &= \frac{\rho_0(t)}{\rho_0(t+x)}(1+f(t)) - 1, \end{aligned} \tag{2.1}$$

though the equation

$$\rho_0'(t+x) + \rho_0(t+x)v_x + v\rho_0'(t+x) = 0$$

and the boundary condition (1.5).

And we reach

$$H(x,t) = H_0 \exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\},$$

due to the magnetic equation (1.3), the expression (2.1) of $v(x,t)$, and the condition (1.14).

So, the derivatives of $v(x,t)$ and $H(x,t)$ are

$$v_t(x,t) = f'(t)\frac{\rho_0(t)}{\rho_0(t+x)} + (1+f(t))\frac{\rho_0'(t)\rho_0(t+x) - \rho_0(t)\rho_0'(t+x)}{\rho_0^2(t+x)}, \tag{2.2}$$

$$v_x(x,t) = -(1+f(t))\frac{\rho_0(t)\rho_0'(t+x)}{\rho_0^2(t+x)}, \tag{2.3}$$

$$v_{xx}(x,t) = -(1+f(t))\frac{\rho_0(t)\rho_0^2(t+x)\rho_0''(t+x) - 2\rho_0(t)\rho_0(t+x)(\rho_0'(t+x))^2}{\rho_0^3(t+x)}, \tag{2.4}$$

and

$$H_t(x,t) = H_0(1+f(t))\frac{\rho_0(t)\rho_0'(t+x)}{\rho_0^2(t+x)}\exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}, \tag{2.5}$$

$$\begin{aligned} H_x(x,t) = H_0 \exp\{\textstyle\int_0^t \quad & (1+f(s))\rho_0(s) \\ & \cdot\frac{\rho_0''(x+s)\rho^2(x+s) - 2\rho_0(x+s)(\rho_0'(x+s))^2}{\rho_0^4(x+s)}ds\}. \end{aligned} \tag{2.6}$$

Substituting (2.2)–(2.6) into the moment equation (1.2) and the magnetic equation (1.3), we obtain

$$\begin{aligned} & f'(t) + (1+f(t))\left[\frac{\rho_0'(t)}{\rho_0(t)} + \mu\left(\frac{\rho_0''(t+x)}{\rho_0^2(t+x)} - \frac{2(\rho_0'(t+x))^2}{\rho_0^3(t+x)}\right)\right] \\ & -(1+f(t))^2\frac{\rho_0(t)}{\rho_0(t+x)}\frac{\rho_0'(t+x)}{\rho_0(t+x)} \\ = \quad & -\frac{p'(\rho_0(t+x))}{\rho_0(t)}\rho_0'(t+x). \end{aligned}$$

Due to (1.14), we have $\frac{\rho_0''(t+x)}{\rho_0^2(t+x)} - \frac{2(\rho_0'(t+x))^2}{\rho_0^3(t+x)}$, $\frac{\rho_0'(t+x)}{\rho_0^2(t+x)}$ and $p'(\rho_0(t+x))\rho_0'(t+x)$ is not dependent on x. Therefore, $f(t)$ is the solution to the Riccati equation (1.12) when the conditions (1.13)-(1.14) hold. ($\Leftarrow$) If $f(t)$ satisfies (1.12) and the conditions (1.13)-(1.14) hold, it is obvious to obtain

$$\begin{aligned} & [\rho(t+x)]_t + \left(\rho(t+x)[(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1]\right)_x = 0, \\ & \rho(t+x)\Big([(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1]_t \\ & \qquad +[(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1][(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1]_x\Big) \\ & \qquad +p'(\rho(t+x))\rho'((t+x)) = \mu\left[(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)}\right]_{xx}, \\ & \Big[H_0\exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}\Big]_t + \Big([(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1] \\ & \qquad \cdot H_0\exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}\Big)_x = 0. \end{aligned}$$

Then, $(\rho, v, H) = \Big(\rho_0(t+x), (1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1, H_0\exp\{\int_0^t(1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}\Big)$ is the solution to compressible MHD equations (1.1)–(1.3) with the initial data (1.4) and boundary condition (1.5).

Secondly, motivated by the results in [8,20,34], the global existence for Riccati equation (1.12) will be given under the assumption (1.15) of $P(t)$, $Q(t)$, and $R(t)$. By taking

$$V(t) = P(t)(1 + f(t)),$$

the Eq (1.12) becomes

$$V'(t) = -V^2(t) - f(t)V(t) + R(t),$$

where $f(t) = Q(t) - \frac{P'(t)}{P(t)}$. Because of $\rho_0(x) \in C^2(0, +\infty)$ and (1.15), we get

$$f(t) \in C^1(0, +\infty),\ R(t) \in C^1(0, +\infty) \tag{2.7}$$

$$\frac{1}{2}f'(t) + \frac{1}{4}f^2(t) + R(t) > 0. \tag{2.8}$$

With (2.7)-(2.8), we arrive at the global existence of (1.12).

Continuous dependence ($\Rightarrow$) When the initial data (1.16) is given, the analytical function $(\rho_\epsilon, v_\epsilon, H_\epsilon)(t, x)$ of initial boundary value problem of compressible MHD equations (1.1)–(1.3) becomes

$$\rho_\epsilon(t, x) = \rho_0(t + x) + \epsilon\rho_0(t + x),$$

we arrive at

$$\begin{aligned} v_\epsilon(x, t) &= \exp\{-\int_0^x \frac{\rho_0'(y + t) + \epsilon\rho_0'(y + t)}{\rho_0(y + t) + \epsilon\rho_0(y + t)}dy\}(1 + v(0, t)) - 1 \\ &= \frac{\rho_0(t)}{\rho_0(t + x)}(1 + f(t)) - 1, \end{aligned} \tag{2.9}$$

due to the equation

$$\begin{aligned} &\rho_0'(t + x) + \epsilon\rho_0'(t + x) + (\rho_0(t + x) + \epsilon\rho_0(t + x))v_x \\ &\quad + v(\rho_0'(t + x) + \epsilon\rho_0'(t + x)) = 0 \end{aligned}$$

and the boundary condition (1.5).

And we reach

$$H_\epsilon(x, t) = H_0 \exp\{\int_0^t (1 + f(s))\frac{\rho_0(s)\rho_0'(x + s)}{\rho_0^2(x + s)}ds\},$$

for the magnetic equations (1.3), (2.9), and (1.14).

So, the derivatives of $v_\epsilon(x, t)$ and $H_\epsilon(x, t)$ are

$$v_{\epsilon t}(x, t) = f'(t)\frac{\rho_0(t)}{\rho_0(t + x)} + (1 + f(t))\frac{\rho_0'(t)\rho_0(t + x) - \rho_0(t)\rho_0'(t + x)}{\rho_0^2(t + x)}, \tag{2.10}$$

$$v_{\epsilon x}(x, t) = -(1 + f(t))\frac{\rho_0(t)\rho_0'(t + x)}{\rho_0^2(t + x)}, \tag{2.11}$$

$$v_{\epsilon xx}(x,t) = -(1+f(t))\frac{\rho_0(t)\rho_0^2(t+x)\rho_0''(t+x) - 2\rho_0(t)\rho_0(t+x)(\rho_0'(t+x))^2}{\rho_0^3(t+x)} \tag{2.12}$$

and

$$H_{\epsilon t}(x,t) = H_0(1+f(t))\frac{\rho_0(t)\rho_0'(t+x)}{\rho_0^2(t+x)}\exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}, \tag{2.13}$$

$$\begin{aligned} H_{\epsilon x}(x,t) = H_0 \exp\{\int_0^t & (1+f(s))\rho_0(s) \\ & \cdot\frac{\rho_0''(x+s)\rho^2(x+s) - 2\rho_0(x+s)(\rho_0'(x+s))^2}{\rho_0^4(x+s)}ds\}. \end{aligned} \tag{2.14}$$

With (2.10)–(2.14), the moment equation (1.2) and the magnetic equation (1.3) have

$$\begin{aligned} & f'(t) + (1+f(t))\left[\frac{\rho_0'(t)}{\rho_0(t)} + \mu\left(\frac{\rho_0''(t+x)}{\rho_0^2(t+x)} - \frac{2(\rho_0'(t+x))^2}{\rho_0^3(t+x)}\right)\right] \\ & -(1+f(t))^2\frac{\rho_0(t)}{\rho_0(t+x)}\frac{\rho_0'(t+x)}{\rho_0(t+x)} \\ = \; & -\frac{p'(\rho_0(t+x))}{\rho_0(t)}\rho_0'(t+x). \end{aligned}$$

For (1.14), $\frac{\rho_0''(t+x)}{\rho_0^2(t+x)} - \frac{2(\rho_0'(t+x))^2}{\rho_0^3(t+x)}$, $\frac{\rho_0'(t+x)}{\rho_0^2(t+x)}$ and $p'(\rho_0(t+x))\rho_0'(t+x)$ only depend on the temporal variable t. So, $f(t)$ satisfies the Riccati differential equation (1.12) under the assumption of (1.13)-(1.14). ($\Leftarrow$) If $f(t)$ is solution to the Riccati differential equation (1.12) with (1.13)-(1.14), we have

$$\begin{aligned} & [\rho(t+x) + \epsilon\rho(t+x)]_t + (\rho(t+x) + \epsilon\rho(t+x))\left([(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1]\right)_x \\ & \quad +(\rho(t+x) + \epsilon\rho(t+x))_x[(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1] = 0, \\ & [\rho(t+x) + \epsilon\rho(t+x)]\Big([(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1]_t \\ & \quad +[(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1][(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1]_x\Big) \\ & \quad +p'(\rho(t+x))(1+\epsilon)\rho'((t+x)) = \mu\left[(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)}\right]_{xx}, \\ & \Big[H_0\exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}\Big]_t + \Big([(1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1] \\ & \quad \cdot H_0\exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}\Big)_x = 0. \end{aligned}$$

So, $(\rho_\epsilon, v_\epsilon, H_\epsilon) = \Big(\rho_0(t+x)+\epsilon\rho_0(t+x), (1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)}-1, H_0\exp\{\int_0^t(1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}\Big)$ is the solution to compressible MHD equations (1.1)–(1.3) with the initial boundary condition (1.16) and (1.5). □

Next, we will prove Theorem 1.2.

Proof. **Existence of steady solution** From (1.6) and (1.8), we have

$$\bar{\rho}(x)\bar{v}(x) = C_1 = \bar{\rho}(0)\bar{v}(0), \quad \bar{v}(x)\bar{H}(x) = C_3 = \bar{v}(0)\bar{H}(0).$$

Then,

$$\bar{H}(x) = \frac{C_3}{\bar{v}(x)}, \text{ and} \Big(\frac{H^2}{2}\Big)_x = \Big(\frac{C_3^2}{2\bar{v}^2}\Big)_x.$$

And by (1.7), we obtain

$$\bar{v}_x = \frac{C_1}{\mu}\bar{v} + \frac{aC_1^\gamma}{\mu}\frac{1}{\bar{v}^\gamma} - \frac{C_3^2}{2\mu}\frac{1}{\bar{v}^2} + \frac{C_2}{\mu},$$

where $C_2 = \mu\bar{v}_x(0) - \bar{\rho}(0)\bar{v}^2(0) + a\bar{\rho}^\gamma(0)$. Let

$$a = \frac{C_1}{\mu}, \ \ b = \frac{aC_1^\gamma}{\mu}, \ \ c = -\frac{C_3^2}{2\mu}, \ \ d = \frac{C_2}{\mu},$$

one get

$$\bar{v}_x = a\bar{v} + b\frac{1}{\bar{v}^\gamma} + c\frac{1}{\bar{v}^2} + d. \tag{2.15}$$

Let

$$F(\bar{v}) = a\bar{v} + b\frac{1}{\bar{v}^\gamma} + c\frac{1}{\bar{v}^2} + d,$$

we can see $F(\bar{v})$ is continuous with respect to $\bar{v}$ in the interval $|\bar{v}| \geq \delta > 0$ and satisfies the Lipschitz condition. According to the Picard–Lindel*ö*f theorem, we have, for any initial value in that interval, the Eq (2.15) has a unique solution $\bar{v}(x)$. Therefore, we have the solution $(\bar{\rho}, \bar{v}, \bar{H}) = (\bar{\rho}(0)\bar{v}(0)/\bar{v}(x), \bar{v}(x), \bar{H}(0)\bar{v}(0)/\bar{v}(x))$ satisfying the Eqs (1.6)–(1.8).

Large-time behavior When $\rho(t,x) \to \bar{\rho}(0)$, we have

$$\rho(t,x) = \rho_0(t,x) \to \bar{\rho}(0), \ \text{ and } \rho_0^{'}(t,x) \to 0 \text{ as } t \to \infty.$$

Therefore,

$$\begin{aligned}
& (\rho, v, H)(t,x) \\
= \ & \Big(\rho_0(t+x), (1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1, H_0\exp\{\int_0^t (1+f(s))\frac{\rho_0(s)\rho_0'(x+s)}{\rho_0^2(x+s)}ds\}\Big) \\
\to \ & (\bar{\rho}(0), \bar{f}, \bar{H}(0)).
\end{aligned}$$

Asymptotic limit When the initial data $H_0 \to 0$, it is easy to get that

$$\begin{aligned}
& (\rho(t,x), v(t,x), H(t,x)) \\
\to \ & (\tilde{\rho}(t,x), \tilde{v}(t,x), \tilde{H}(t,x)) = (\rho_0(t+x), (1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1, 0).
\end{aligned}$$

By direct calculation, we can obtain that $(\tilde{\rho}(t,x), \tilde{v}(t,x)) = (\rho_0(t+x), (1+f(t))\frac{\rho_0(t)}{\rho_0(t+x)} - 1)$ satisfy the mass conversation equation

$$\tilde{\rho}_t + (\tilde{\rho}\tilde{v})_x = 0$$

and the moment conversation law

$$\tilde{\rho}(\tilde{v}_t + \tilde{v}\tilde{v}_x) + \tilde{p}_x = \mu\tilde{v}_{xx}$$

with the pressure $\tilde{p} = p(\tilde{\rho})$. And the initial data and boundary condition are also fit for the analytical solution $(\tilde{\rho}(t,x), \tilde{v}(t,x))$. Therefore, $(\tilde{\rho}(t,x), \tilde{v}(t,x))$ is the analytical solution of the system (1.17)–(1.20). □

3. Examples

We give some examples to show that the set of solutions in the above Theorem 1.1 is not empty in this section. First, we can check Example 3.1 easily.

Example 3.1. If $\rho_0(x), v_0(x), H_0(x)$ are constants and suppose the pressure $p(\rho) = \rho^\gamma$ (for any $\gamma > 0$), the solution to compressible MHD equations (1.1)–(1.3) with (1.4) and (1.5) exists and satisfies Theorem 1.1.

Proof. For the initial data $\rho_0(x), v_0(x), H_0(x)$ are constants, we suppose

$$(\rho_0(x), v_0(x), H_0(x)) = (C_\rho, C_v, C_H).$$

So,

$$(\rho_0)'(t+x) = (\rho_0)''(t+x) = p'(\rho_0(t+x)) = 0.$$

Then,

$$A(t,x) = B(t,x) = C(t,x) = 0,$$

and

$$Q(t) = P(t) = R(t) = 0.$$

Due to the above, (1.12) becomes $f'(t) = 0$, which implies $f(t) = f(0) = v_0(x) = C_v$. Therefore, the boundary value problem of compressible MHD equations has the analytical solution $(\rho, v, H) = (C_\rho, C_v, C_H)$. □

Then, we show the following interesting example.

Example 3.2. Let

$$\begin{aligned} \rho_0(x) &= \frac{1}{x+1}, \\ v_0(x) &= \frac{(2c_0+1)x+2}{2c_0-1}, \\ H_0(x) &= \left|\frac{2c_0}{1-2c_0}\right| \quad (c_0 \neq \frac{1}{2}). \end{aligned} \tag{3.1}$$

Then, the analytical solution to the compressible MHD equations (1.1)–(1.3) with the pressure $p(\rho) = -\frac{1}{\rho}$ can be obtained as

$$\rho(x,t) = \frac{1}{1+t+x},$$

$$v(x,t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(1+t+x)-1, \tag{3.2}$$
$$H(x,t) = \left|\frac{2c_0e^{t}}{1-2c_0e^{2t}}\right|.$$

Moreover, we can get the particle path of compressible flow

$$x(t) = \frac{x(0)+1}{2c_0-1}\cdot\frac{2c_0e^{2t}-1}{e^{t}}-(t+1),$$

where $x(0)$ means the initial position of the particle.

Proof. For (3.1), we get

$$\rho(x,0) = \rho_0(x) = \frac{1}{x+1},$$
$$v(x,0) = v_0(x) = \frac{2c_0+1}{2c_0-1}x+\frac{2}{2c_0-1},$$

and compatibility condition

$$f(0) = v(0,t)|_{t=0} = v(0,0) = v_0(0) = \frac{2}{2c_0-1}.$$

By the initial data, we have

$$A(x,t) = \mu\left(\frac{\rho_0''(t+x)}{\rho_0^2(t+x)}-\frac{2(\rho_0'(t+x))^2}{\rho_0^3(t+x)}\right) = 0,$$
$$B(x,t) = \frac{\rho_0'(t+x)}{\rho_0^2(t+x)} = -1,$$
$$C(x,t) = p'(\rho_0(t+x))\rho_0'(t+x) = -1.$$

Consequently,

$$Q(t) = -\frac{1}{1+t},\ P(t) = \frac{1}{1+t},\ R(t) = 1+t.$$

So, we have

$$(1+f(t))' - \frac{1}{1+t}(1+f(t)) + \frac{1}{1+t}(1+f(t))^2 = 1+t.$$

Let

$$U(t) = f(t)-t, \tag{3.3}$$

then $U(t)$ satisfies

$$U'(t) = \left(\frac{1}{1+t}-2\right)U(t)-\frac{1}{1+t}U^2(t). \tag{3.4}$$

The above Eq (3.4) divided by $-U^2$, we get

$$\left(\frac{1}{U(t)}\right)' = \left(2 - \frac{1}{1+t}\right)\frac{1}{U(t)} + \frac{1}{1+t}\left(\frac{1}{U(t)}\right)^2.$$

Due to the method of constant variation, we arrive at

$$U(t) = \frac{2(1+t)}{2c_0e^{2t} - 1}.$$

Here, we have used the compatibility condition $f(0) = v_0(0) = \frac{2}{2c_0-1}$.

By (3.3), we have

$$f(t) = \frac{2(1+t)}{2c_0e^{2t} - 1} + t. \tag{3.5}$$

From the result of Theorem 1.1 and (3.5), we get

$$\rho(x,t) = \frac{1}{1+t+x},$$
$$v(x,t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(1+t+x) - 1,$$

and

$$lnH(x,t) = lnH_0 + \int_0^t v_x ds$$
$$= -\int_0^t s + \frac{2}{2c_0e^{2s}-1}ds$$
$$= -t - \int_0^t \frac{2e^{-2s}}{2c_0 - e^{-2s}}ds$$
$$= -t \cdot ln\left|\frac{2c_0}{e^{-2t} - 2c_0}\right|.$$

i.e.,

$$H(x,t) = \left|\frac{2c_0e^{t}}{1-2c_0e^{2t}}\right|.$$

So, we obtain the solution (3.2).

From (3.2),

$$x'(t) = v(x,t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(1+t+x) - 1. \tag{3.6}$$

Let

$$P_1(t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1},$$

and

$$Q_1(t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(1+t) - 1,$$

(3.6) becomes

$$x'(t) = P_1(t)x + Q_1(t).$$

Therefore, the particle path is

$$x(t) = \frac{x(0)+1}{2c_0-1} \cdot \frac{2c_0e^{2t}-1}{e^t} - (t+1),$$

due to the method of constant variation. □

And we have the continuous dependence of the analytical solution, which lies in Example 3.2 in the following Remark 3.3. This proof is similar to that in Example 3.2. Here we omit it.

Remark 3.3. Let

$$\begin{aligned} \rho_{\epsilon 0}(x) &= \frac{1}{x+1} + \frac{\epsilon}{x+1}, \\ v_{\epsilon 0}(x) &= \frac{(2c_0+1)x+2}{2c_0-1}, \\ H_{\epsilon 0}(x) &= \Big|\frac{2c_0}{1-2c_0}\Big| \ (c_0 \neq \frac{1}{2}). \end{aligned} \tag{3.7}$$

Then, we get the analytical solution $(\rho_\epsilon, v_\epsilon, H_\epsilon)(t,x)$ to the compressible MHD equations (1.1)–(1.3) with the pressure $p(\rho) = -\frac{1}{\rho}$ and the initial data (3.7) as follows:

$$\begin{aligned} \rho_\epsilon(x,t) &= \frac{1}{1+t+x} + \frac{\epsilon}{1+t+x}, \\ v_\epsilon(x,t) &= \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(1+t+x) - 1, \\ H_\epsilon(x,t) &= \Big|\frac{2c_0e^t}{1-2c_0e^{2t}}\Big|. \end{aligned}$$

When $\epsilon \to 0$, the analytical solution $(\rho_\epsilon, v_\epsilon, H_\epsilon)(t,x)$ to the compressible MHD equations (1.1)–(1.3) with the pressure $p(\rho) = -\frac{1}{\rho}$ and the initial data (3.7) tends to the analytical solution $(\rho_\epsilon, v_\epsilon, H_\epsilon)(t,x)$ to the compressible MHD equations (1.1)–(1.3) with the pressure $p(\rho) = -\frac{1}{\rho}$ and the initial data (3.1). And the particle path of compressible MHD flow is

$$x(t) = \frac{x(0)+1}{2c_0-1} \cdot \frac{2c_0e^{2t}-1}{e^t} - (t+1),$$

which is the same as the particle path in Example 3.2.

Remark 3.4. In Example 3.2, we can obtain that, when $\forall p > 1$,

$$\begin{aligned} \|\rho_0(x)\|_{L^p([0,+\infty))} &= \int_0^\infty \frac{1}{(x+1)^p} dx \leq 1, \\ \|\rho(x,t)\|_{L^p([0,+\infty))} &= \int_0^\infty \frac{1}{(1+t+x)^p} dx \leq \frac{1}{(1+t)^{\frac{p-1}{p}}}. \end{aligned}$$

Meanwhile, when $p = 1$, we can see that $\|\rho_0(x)\|_{L^1([0,+\infty))}$ is unbounded, and it is difficult to get the boundedness of $\|\rho_0 v_0^2\|_{L^1([0,+\infty))}$ and $\|\rho v^2\|_{L^1([0,+\infty))}$ for the solution to compressible MHD equations with the given initial data (3.1).

If $c_0 = 1$, we have $v_0(x) = 3x + 2$. So, the solution of compressible MHD equations (1.1)–(1.3) with the pressure $p(\rho) = -\rho^{-1}$ can be written as

$$\rho(x,t) = \frac{1}{1+t+x},$$

$$v(x,t) = \frac{2e^{2t}+1}{2e^{2t}-1}(1+t+x) - 1,$$

$$H(x,t) = \frac{2e^{t}}{2e^{2t}-1}.$$

Moreover, the particle path of compressible flow is expressed as

$$x(t) = (x(0)+1)(2e^{t} + e^{-t}) - t - 1.$$

4. Conclusions

The analytical solution to the one-dimensional compressible isentropic MHD equations without magnetic diffusion in half space is found to be expressed as the initial density and boundary function of velocity strictly if and only if the solution to the Riccati differential equation of the boundary condition function exists under some assumptions in this study. And the compressible MHD equations become compressible Navier–Stokes equations when the magnetic field is zero. The analytical solution to compressible MHD equations without magnetic diffusion depends continuously on the initial value. The existence of a steady solution to compressible MHD equations is given, and the large-time behavior and asymptotic limit theorem of the analytical solution are also shown when the time tends to infinity and the initial data of magnetic field goes to zero. Several examples are given to ensure that the set of this kind of the analytical solution is not empty.

Author contributions

Changsheng Dou: Conceptualization, Methodology, Writing–original draft, Writing–review and editing; Hongtian Zhang: Methodology, Writing–original draft, Writing–review and editing; Tengzhe Zhao: Methodology, Writing–original draft, Writing–review and editing. All authors have been working together on the mathematical development of the manuscript, and have read and approved the final version of the manuscript for publication.

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The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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