Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(12): 28514–28523.
DOI: 10.3934/math.20251255
Received: 13 June 2025
Revised: 13 November 2025
Accepted: 19 November 2025
Published: 03 December 2025

Research article

A class of small multiplicative functions in Piatetski-Shapiro sequences

Haihong Fan*

College of Science, Northwest A & F University, Yangling 712100, China

* **Correspondence:** Email: fanhaihong1@hotmail.com.

Abstract: For any real number x, $[x]$ denotes the integer part of x. $\mathcal{F}_1$ denotes a class of multiplicative functions that are small in a numerical sense. In this paper, we study the distribution of the functions from $\mathcal{F}_1$ in Piatetski-Shapiro sequences. In particular, we prove that for $1 < c < \frac{4}{3}$,

$$\sum_{\substack{n\le x\\ f\in\mathcal{F}_1}} f([n^c]) = \int_1^{x^c} \gamma u^{\gamma-1} d\left(\sum_{1\le n\le u} f(n)\right) + O(x^{1-\varepsilon}),$$

where $\gamma = \frac{1}{c}$ and $[n^c]$ is the Piatetski-Shapiro sequence.

Keywords: Piatetski-Shapiro sequences; multiplicative function; small value; exponential sum; distribution of primes
Mathematics Subject Classification: 11N05, 11N37

1. Introduction

In classical analytic number theory, the distribution of prime numbers has always been an important research topic. In addition to the qualitative result that there are infinitely many primes, people are also concerned about some quantitative results, such as the famous prime theorem. In addition, the representation of prime numbers by irreducible polynomials is one of the most important problems in the prime distribution module. At present, this problem has been completely solved in the case of linear polynomials, but it is still unknown whether $n(n \ge 2)$ degree polynomials represent prime numbers. In the middle of the two is the famous Piatetski -Shapiro prime number theorem.

The Piatetski-Shapiro sequences are sequences of the form

$$([n^c])_{n=1}^{\infty}, \quad c > 1, \quad c \notin \mathbb{N}.$$

In 1953, Piatetski-Shapiro [1] first proposed this sequence and proved that when $c \in (1, 12/11)$, the sequence contains infinite primes. More precisely, for such c, he showed that

$$\sum_{\substack{n\le x\\ [n^c]\in\mathbb{P}}} 1 \sim \frac{x}{c\log x} \quad as \quad x\to\infty. \tag{1.1}$$

Since then, the range of c has been continuously expanded by scholars; see [2–9]. At present, the best result is obtained by Rivat and Sargos [10], who proved that (1.1) is true for $c \in (1, 2817/2426)$. In addition to asymptotic results, Rivat and Wu [11] proved that there are infinitely many Piatetski-Shapiro primes for $c \in (1, 243/205)$.

In addition, scholars have also combined the Piatetski- Shapiro sequence with other famous number theory problems. Let $d(n)$ denote the number of positive divisors of natural numbers n. The study of sum $\sum_{n\le x} d(n)$, called the Dirichlet divisor problem, is a famous problem in number theory. In 1999, Arkhipov, Saliba, and Chubarikov [12] investigated the distribution of divisor functions in Piatetski-Shapiro sequences and proved that for $1 < c < 8/7$,

$$\sum_{n\le x} d([n^c]) = xP(\log x) + O\left(\frac{x}{\log x}\right),$$

where $P(x)$ is a polynomial of degree 1. Later, the range of c was improved by L*ü* and Zhai [13], and Wang and Zhang [14] to $1 < c < 495/433$ and $1 < c < 6/5$, respectively.

In 1975, Stux [15] was the first to study the distribution of square-free numbers of the form $[n^c]$. He proved that for any fixed $1 < c < \frac{4}{3}$ the asymptotic formula

$$\sum_{\substack{n\le x\\ [n^c]\ is\ square-free}} 1 = \frac{6}{\pi^2}x + O(x)$$

holds. In 1978, Rieger [16] improved the result of Stux, and showed that for $1 < c < 3/2$,

$$\sum_{\substack{n\le x\\ [n^c]\ is\ square-free}} 1 = \frac{6}{\pi^2}x + O(x^{\frac{2c+1}{4}+\varepsilon}).$$

Later, Cao and Zhai [17] improved Rieger's range by the method of exponential sums and proved that for $1 < c < 61/36$,

$$\sum_{\substack{n\le x\\ [n^c]\ is\ square-free}} 1 = \frac{6}{\pi^2}x + O(x^{\frac{36(c+1)}{97}+\varepsilon}).$$

Next, Cao and Zhai [18] further improved $1 < c < 61/36$ to $1 < c < 149/87$.

Influenced by the above problems, scholars have studied cube-free and square-full numbers in the Piatetski-Shapiro sequence and its analogues. In 2017, Zhang and Li [21] proved that there are infinite cube-free numbers of the form $[n^c]$ for any fixed real number $c \in (1, 11/6)$. Subsequently, the range of c was extended to $c \in (1, 2)$ by Deshouillers [22]. In 2020, using the simple one-dimension exponent pair, Srichan and Tangsupphathawat [23] considered square-full numbers in the Piatetski-Shapiro sequence. In 2023, Dimitrov [19] investigated square-free values of $[n^c \tan^\theta(\log n)]$. In 2025, Dimitrov [20] investigated cube-free values of $[n^c \tan^\theta(\log n)]$.

In this paper, we will propose a special function class $\mathcal{F}_1$, which contains the multiplicative arithmetic functions $f(n)$ satisfying the following properties:

$(A.1)$ $f(p) = 1$ for any prime p;

$(A.2)$ $f(n) \ll_\varepsilon n^\varepsilon$ for $n \geq 1$ and any $\varepsilon > 0$.

Inspired by the above problems, we study the distribution of the functions from $\mathcal{F}_1$ in Piatetski-Shapiro sequences. Through the combination of classical Vaaler transformation and the two-dimensional exponential sum method, we obtain the following result.

Theorem 1.1. *Let $1 < c < \frac{4}{3}$. Then we have*

$$\sum_{\substack{n\leq x\\ f\in\mathcal{F}_1}} f([n^c]) = \int_1^{x^c} \gamma u^{\gamma-1} d\left(\sum_{1\leq n\leq u} f(n)\right) + O(x^{1-\varepsilon}), \tag{1.2}$$

where $\gamma = \frac{1}{c}$.

In fact, $\mathcal{F}_1$ includes many number-theoretic functions, and we may select a few important and common examples: $d^{(e)}(n)$, the number of exponential divisors of n; $\beta(n)$, the number of square-full divisors of n; $a(n)$, the number of non-isomorphic Abelian groups of order n; $\mu_2(n)$, the characteristic function of the square-free integers. Thus, from Theorem 1.1, we immediately obtain Theorem 1.2 as follows:

Theorem 1.2. *For $f(n) \in \{d^{(e)}(n), \beta(n), a(n), \mu_2(n)\}$, we have the asymptotic formula*

$$\sum_{n\leq x} f([n^c]) = \int_1^{x^c} \gamma u^{\gamma-1} d\left(\sum_{1\leq n\leq u} f(n)\right) + O(x^{1-\varepsilon}),$$

where $\gamma = \frac{1}{c}$ and $1 < c < \frac{4}{3}$.

2. Notations and preliminary lemmas

Throughout this paper, ε always denotes a small enough positive constant, which may alter the value in different situations. $e(x) = \exp(2\pi i x) = e^{2\pi i x}$. $m \sim M$ means $c_1 M \leq m \leq c_2 M$ for absolute constants c_1 and c_2. $\psi(t) = t - [t] - \frac{1}{2}$.

Next, we quote some lemmas used in this paper.

Lemma 2.1. *For any $H \geq 1$, there exist numbers a_h and b_h such that*

$$\left|\psi(t) - \sum_{0<|h|\leq H} a_h e(th)\right| \leq \sum_{|h|\leq H} b_h e(th), \quad a_h \ll \frac{1}{|h|}, \quad b_h \ll \frac{1}{H}.$$

Proof. See pages 206–210 of Vaaler [24]. □

We also need the following lemma about the two-dimensional exponential sum.

Lemma 2.2. *Suppose M and N are large positive numbers, $A > 0, \alpha, \beta$ are rational numbers (not non-negative numbers). Suppose $m \sim M, n \sim N, F = AM^{\alpha}N^{\beta} \gg N, |a(m)| \leq 1, |b(n)| \leq 1$. Then*

$$\begin{aligned} S &= \sum_{M<m\leq 2M} \sum_{N<n\leq 2N} a(m)b(n)e(Am^{\alpha}n^{\beta}) \\ &\ll (MN^{1/2} + F^{4/20}M^{13/20}N^{15/20} + F^{4/23}M^{15/23}N^{18/23} \\ &\quad + F^{1/6}M^{2/3}N^{7/9} + F^{1/5}M^{3/5}N^{4/5} + F^{1/10}M^{4/5}N^{7/10}) \log^4 F. \end{aligned}$$

Proof. See Proposition 1 of Zhai and Cao [25]. □

Lemma 2.3. *Let $q \geq 0$ be an integer. Suppose that f has $q + 2$ continuous derivatives on I, and that $I \subseteq (N, 2N]$. Assume also that there is some constant F such that*

$$|f^{(r)}(x)| \approx FN^{-r},$$

for $r = 1, \cdots, q + 2$. Then

$$\sum_{n\in I} e(f(n)) \ll F^{1/(4Q-2)}N^{1-(q+2)/(4Q-2)} + F^{-1}N,$$

where $Q = 2^q$.

Proof. See Theorem 2.9 in Graham and Kolesnik [26]. □

3. Proof of Theorem 1.1

In this section, we prove Theorem 1.1.

For $f \in \mathcal{F}_{\infty}$, $f(n)$ is a multiplicative function, and for any prime p, $f(p) = 1$. Therefore, using the Euler product formula, we obtain

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{f(n)}{n^s} &= \prod_p \left(1 + \frac{1}{p^s} + \frac{f(p^2)}{p^{2s}} + \frac{f(p^3)}{p^{3s}} + \cdots\right) \\ &= \zeta(s) \prod_p \left(1 + \frac{f(p^2) - 1}{p^{2s}} + \frac{f(p^3) - f(p^2)}{p^{3s}} + \cdots\right) \\ &= \zeta(s)G(s). \end{aligned} \tag{3.1}$$

When $Res > 1/2$, $G(s) = \Sigma_{n=1}^{\infty} \frac{g(d)}{d^s}$ converges absolutely. Through simple analysis, it can be concluded that

$$\sum_{d\leq z} |g(d)| \ll_{\varepsilon} z^{\frac{1}{2}+\varepsilon}. \tag{3.2}$$

Comparing the left and right sides of Eq (3.1), we found that

$$f(n) = \sum_{n=ld} g(d). \tag{3.3}$$

Let $\gamma = 1/c$. $[n^c] = m$ means that

$$-(m + 1)^{\gamma} < -n \leq -m^{\gamma}.$$

Hence, we get

$$\begin{aligned}\sum_{n\le x} f([n^c]) &= \sum_{m\le x^c}([-m^\gamma]-[-(m+1)^\gamma])f(m)+O(x^\varepsilon)\\ &= \sum_{m\le x^c}((m+1)^\gamma-m^\gamma)f(m)+\sum_{m\le x^c}(\psi(-(m+1)^\gamma)-\psi(-m^\gamma))f(m)+O(x^\varepsilon)\\ &:= S_1+S_2+O(x^\varepsilon). \end{aligned}\tag{3.4}$$

First, we estimate S_1. Taylor expansion

$$(x+1)^\gamma - x^\gamma = \gamma x^{\gamma-1}+O(x^{\gamma-2})$$

and partial summation provides that

$$\begin{aligned} S_1 &= \gamma\sum_{m\le x^c} f(m)m^{\gamma-1}+O\left(\sum_{m\le x^c} f(m)m^{\gamma-2}\right)\\ &= \int_1^{x^c}\gamma u^{\gamma-1}d\left(\sum_{1\le n\le u} f(n)\right)+O(x^{1-c}). \end{aligned}\tag{3.5}$$

Now we consider the sum S_2. In view of Lemma 2.1, we get

$$S_2 = S_{21}+O(S_{22}),\tag{3.6}$$

where

$$\begin{aligned} S_{21} &= \sum_{m\le x^c} f(m)\sum_{0<|h|\le H} a(h)[e(-h(m+1)^\gamma)-e(-hm^\gamma)],\\ S_{22} &= \sum_{m\le x^c} f(m)\sum_{|h|\le H} b(h)[e(-h(m+1)^\gamma)+e(-hm^\gamma)]. \end{aligned}$$

There is an additional term, $h=0$, in the sum S_{22}, and the processing of S_{22} is similar to that of S_{21}, which we will explain later.

We will first consider estimating the upper bound of S_{21}. For convenience, we set

$$\phi_h(t) = e(h(t^\gamma-(t+1)^\gamma))-1,$$

then S_{21} can be expressed as

$$S_{21} = \sum_{m\le x^c} f(m)\sum_{0<|h|\le H} a(h)\phi_h(m)e(-hm^\gamma).$$

By a splitting argument, we only need to estimate

$$S_{21} = \sum_{m\sim M} f(m)\sum_{0<|h|\le H} a(h)\phi_h(m)e(-hm^\gamma),$$

where $M\le x^c$. Using partial summation and noting $a_h \ll \frac{1}{|h|}$, we obtain

$$S_{21} \ll \sum_{0<|h|\le H}\frac{1}{h}\left|\sum_{m\sim M} f(m)\phi_h(m)e(-hm^\gamma)\right|$$

$$
\begin{aligned}
&\ll \sum_{0<|h|\le H}\frac{1}{h}\left|\int_M^{2M}\phi_h(t)d\left(\sum_{M\le m\le t}f(m)e(-hm^\gamma)\right)\right| \\
&\ll \sum_{0<|h|\le H}\frac{1}{h}\left|\phi_h(2M)\sum_{M\le m\le 2M}f(m)e(-hm^\gamma)\right| \\
&\quad+\int_M^{2M}\sum_{0<|h|\le H}\frac{1}{h}\left|\frac{d\phi_h(t)}{dt}\right|\left|\sum_{M\le m\le t}f(m)e(-hm^\gamma)\right|dt \\
&\ll M^{\gamma-1}\max_{M\le t\le 2M}\sum_{0<|h|\le H}\left|\sum_{M\le m\le t}f(m)e(-hm^\gamma)\right| \\
&\ll x^{1-c}\max_{M\le t\le 2M}\sum_{0<|h|\le H}\left|\sum_{M\le m\le t}f(m)e(-hm^\gamma)\right|, \qquad (3.7)
\end{aligned}
$$

where we have used the bounds

$$
\phi_h(t)\ll ht^{\gamma-1},\ \frac{d\phi_h(t)}{dt}\ll ht^{\gamma-2}.
$$

From (3.7) and by a splitting argument, we need to estimate

$$
\Sigma := \sum_{h\sim H'}\left|\sum_{m\sim M}f(m)e(hm^\gamma)\right|, \qquad (3.8)
$$

where $1\le H^{'}\le \frac{H}{2}$. Noting that

$$
|e(f(n))| = 1,
$$

when n is real, we use (3.3) and triangle inequality to get

$$
\begin{aligned}
\Sigma &= \sum_{h\sim H'}\left|\sum_{ld\sim M}g(d)e(hl^\gamma d^\gamma)\right| \\
&= \sum_{h\sim H'}\left|\sum_{1\le d\le 2M}g(d)\sum_{l\sim L}e(hl^\gamma d^\gamma)\right| \\
&\le \sum_{h\sim H'}\sum_{1\le d\le 2M}|g(d)|\left|\sum_{l\sim L}e(hl^\gamma d^\gamma)\right| \\
&:= \Sigma^*, \qquad (3.9)
\end{aligned}
$$

where $L=\frac{M}{d}$.

Now, we split Σ^* into two parts:

$$
\begin{aligned}
\Sigma^* &= \sum_{1\le d\le\sqrt{M}}|g(d)|\sum_{h\sim H'}\left|\sum_{l\sim L}e(hl^\gamma d^\gamma)\right| + \sum_{\sqrt{M}\le d\le 2M}|g(d)|\sum_{h\sim H'}\left|\sum_{l\sim L}e(hl^\gamma d^\gamma)\right|, \\
&:= \Sigma_1+\Sigma_2. \qquad (3.10)
\end{aligned}
$$

For Σ_1, we apply Lemma 2.3 ($q = 0$) to deal with the innermost sum about l, then from (3.2) we get

$$\begin{aligned}\Sigma_1 &\ll \sum_{1\le d\le\sqrt{M}} |g(d)| \sum_{h\sim H'} \left|(hd^\gamma L^\gamma)^{\frac{1}{2}}\right| \\ &\ll_\varepsilon H^{\frac{3}{2}} M^{\frac{\gamma}{2}+\frac{1}{4}+\varepsilon}. \end{aligned} \tag{3.11}$$

For Σ_2, we first estimate the innermost double sum. Regarding the two-dimensional exponential sum of h and l in (3.10), we can use Lemma 2.2 with $m = l, n = h, F = d^\gamma H L^\gamma$ to obtain

$$\begin{aligned}\sum_{h\sim H'}\sum_{l\sim L} e(hl^\gamma d^\gamma) &\ll H^{\frac{1}{2}} M d^{-1} + H^{\frac{19}{20}} M^{\frac{4}{20}\gamma+\frac{13}{20}} d^{-\frac{13}{20}} + H^{\frac{22}{23}} M^{\frac{4}{23}\gamma+\frac{15}{23}} d^{-\frac{15}{23}} \\ &\quad + H^{\frac{1}{6}+\frac{7}{9}} M^{\frac{\gamma}{6}+\frac{2}{3}} d^{-\frac{2}{3}} + H M^{\frac{\gamma}{5}+\frac{3}{5}} d^{-\frac{3}{5}} + H^{\frac{8}{10}} M^{\frac{\gamma}{10}+\frac{4}{5}} d^{-\frac{4}{5}}. \end{aligned} \tag{3.12}$$

Substituting (3.12) into (3.10), noting (3.2), and using the partial summation formula yields

$$\begin{aligned}\Sigma_2 &\ll H^{\frac{1}{2}} M^{\frac{3}{4}+\varepsilon} + H^{\frac{19}{20}} M^{\frac{4}{20}\gamma+\frac{23}{40}+\varepsilon} + H^{\frac{22}{23}} M^{\frac{4}{23}\gamma+\frac{53}{92}+\varepsilon} + H^{\frac{17}{18}} M^{\frac{\gamma}{6}+\frac{7}{12}+\varepsilon} \\ &\quad + H M^{\frac{\gamma}{5}+\frac{11}{20}+\varepsilon} + H^{\frac{8}{10}} M^{\frac{\gamma}{10}+\frac{13}{20}+\varepsilon}. \end{aligned} \tag{3.13}$$

Combining the estimates of (3.7), (3.9), (3.10), (3.11) , and (3.13) immediately provides the following upper bound result for S_{21}:

$$\begin{aligned}S_{21} &\ll H^{\frac{3}{2}} x^{\frac{3}{2}-\frac{3}{4}c+\varepsilon} + H^{\frac{1}{2}} x^{1-\frac{c}{4}+\varepsilon} + H^{\frac{19}{20}} x^{\frac{24}{20}-\frac{17}{40}c+\varepsilon} + H^{\frac{22}{23}} x^{\frac{27}{23}-\frac{39}{92}c+\varepsilon} \\ &\quad + H^{\frac{17}{18}} x^{\frac{7}{6}-\frac{5}{12}c+\varepsilon} + H x^{\frac{6}{5}-\frac{9}{20}c+\varepsilon} + H^{\frac{8}{10}} x^{\frac{11}{10}-\frac{7}{20}c+\varepsilon}. \end{aligned} \tag{3.14}$$

We now consider the upper bound of S_{22}. The contribution of $h = 0$ is

$$\begin{aligned}&\ll b_0 \sum_{m\le x^c} f(m) \\ &\ll_\varepsilon H^{-1} x^{c+\varepsilon}. \end{aligned} \tag{3.15}$$

By noting $b_h \ll \frac{1}{H}$, the contribution of $h \neq 0$ is

$$S_{22} \ll \frac{1}{H} \sum_{1\le|h|\le H} \left| \sum_{m\le x^c} f(m)(e(h(m+1)^\gamma) + e(hm^\gamma)) \right|.$$

By substituting variables, we only need to estimate

$$\frac{1}{H} \sum_{1\le|h|\le H} \left| \sum_{m\le x^c} f(m) e(hm^\gamma) \right|. \tag{3.16}$$

So using the same processing method as (3.8), the contribution of $h \neq 0$ is

$$\begin{aligned}&\ll H^{\frac{1}{2}} x^{\frac{1}{2}+\frac{c}{4}+\varepsilon} + H^{-\frac{1}{2}} x^{\frac{3}{4}c+\varepsilon} + H^{-\frac{1}{20}} x^{\frac{4}{20}+\frac{23}{40}c+\varepsilon} + H^{-\frac{1}{23}} x^{\frac{4}{23}+\frac{53}{92}c+\varepsilon} \\ &\quad + H^{-\frac{1}{18}} x^{\frac{1}{6}+\frac{7}{12}c+\varepsilon} + x^{\frac{1}{5}+\frac{11}{20}c+\varepsilon} + H^{-\frac{1}{5}} x^{\frac{1}{10}+\frac{13}{20}c+\varepsilon}. \end{aligned} \tag{3.17}$$

Combining (3.6), (3.14), (3.15) , and (3.17), we obtain

$$\begin{aligned} S_2 \ll\ & H^{\frac{3}{2}} x^{\frac{3}{2}-\frac{3}{4}c+\varepsilon} + H^{\frac{1}{2}} x^{1-\frac{c}{4}+\varepsilon} + H^{\frac{19}{20}} x^{\frac{24}{20}-\frac{17}{40}c+\varepsilon} \\ & + H^{\frac{22}{23}} x^{\frac{27}{23}-\frac{39}{92}c+\varepsilon} + H^{\frac{17}{18}} x^{\frac{7}{6}-\frac{5}{12}c+\varepsilon} + H x^{\frac{6}{5}-\frac{9}{20}c+\varepsilon} \\ & + H^{\frac{8}{10}} x^{\frac{11}{10}-\frac{7}{20}c+\varepsilon} + H^{-1} x^{c+\varepsilon} + H^{\frac{1}{2}} x^{\frac{1}{2}+\frac{c}{4}+\varepsilon} \\ & + H^{-\frac{1}{2}} x^{\frac{3}{4}c+\varepsilon} + H^{-\frac{1}{20}} x^{\frac{4}{20}+\frac{23}{40}c+\varepsilon} + H^{-\frac{1}{23}} x^{\frac{4}{23}+\frac{53}{92}c+\varepsilon} \\ & + H^{-\frac{1}{18}} x^{\frac{1}{6}+\frac{7}{12}c+\varepsilon} + x^{\frac{1}{5}+\frac{11}{20}c+\varepsilon} + H^{-\frac{1}{5}} x^{\frac{1}{10}+\frac{13}{20}c+\varepsilon}. \end{aligned} \tag{3.18}$$

Choosing $H = x^{\frac{7c-6}{10}}$, we obtain

$$S_2 \ll x^{1-\varepsilon} \tag{3.19}$$

as long as $c < \frac{4}{3}$, where ε will be chosen with respect to c.

Finally, combining (3.4), (3.5) , and (3.19), we proved Theorem 1.1.

4. Conclusions

In this article, we studied the distribution of a special class of small- value multiplicative functions in Piatetski-Shapiro sequences and obtained a good range of applicability for the constant c. This result will have certain significance for enriching the relevant theories of Piatetski-Shapiro sequences and provide some enlightenment for the related research of other function types.

Use of Generative-AI tools declaration

The author declares she has not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

The author would like to express the most sincere gratitude to the referee for his/her patience in refereeing this paper. This work is supported by the Fundamental Research Funds for the Central Universities (Grant No. 2452024154).

Conflict of interest

The author declares that no conflicts of interest exist in this manuscript.

References

1. I. I. Piatetski-Shapiro, On the distribution of prime numbers in sequences of the form $[f(n)]$, *Mat. Sb. (N.S.)*, **75** (1953), 559–566.
2. G. A. Kolesnik, The distribution of primes in sequences of the form $[n^c]$, *Mat. Zametki.*, **2** (1967), 117–128.
3. D. R. Heath-Brown, The Pjateckiĭ -Šapiro prime number theorem, *J. Number Theory*, **16** (1983), 242–266.

4. G. A. Kolesnik, Primes of the form $[n^c]$, *Pac. J. Math.*, **118** (1985), 437–447. http://doi.org/10.2140/pjm.1985.118.437

5. H. Q. Liu, J. Rivat, On the PjateckiĬ-Šapiro prime number theorem, *Bull. Lond. Math. Soc.*, **24** (1992), 143–147. https://doi.org/10.1112/blms/24.2.143

6. C. H. Jia, On PjateckiĬ-Šapiro prime number theorem II, *Sci. China Ser. A.*, **36** (1993), 913–926.

7. C. H. Jia, On PjateckiĬ-Šapiro prime number theorem, *Chin. Ann. Math. Ser. B.*, **15** (1994), 9–22.

8. R. C. Baker, G. Harman, J. Rivat, Primes of the form $[n^c]$, *J. Number Theory*, **50** (1995), 261–277. http://doi.org/10.1006/jnth.1995.1020

9. A. V. Kumchev, On the distribution of prime numbers of the form $[n^c]$, *Glasg. Math. J.*, **41** (1999), 85–102. http://doi.org/10.1017/s0017089599970477

10. J. Rivat, P. Sargos, Nombres premiers de la forme $\lfloor n^c \rfloor$, *Canad. J. Math.*, **53** (2001), 414–433. https://doi.org/10.4153/CJM-2001-017-0

11. J. Rivat, J. Wu, Prime numbers of the form $[n^c]$, *Glasg. Math. J.*, **43** (2001), 237–254. https://doi.org/10.1017/S0017089501020080

12. G. I. Arkhipov, Kh. M. Saliba, V. N. Chubarikov, On the sum of values of a multidimensional function of divisors on a sequence of type $[n^c]$, *Vestnik Moskov. Univ. Ser. 1. Mat. Mekh.*, 1999, 28–35.

13. G. S. Lü, W. G. Zhai, The sum of multidimensional divisor functions on a special sequence, *Adv. Math.*, **32** (2003), 660–664.

14. H. Wang, Y. Zhang, On the divisor function over Piatetski-Shapiro sequences, *Czechoslovak Math. J.*, **73** (2023), 613–620. http://doi.org/10.21136/CMJ.2023.0205-22

15. I. E. Stux, Distribution of squarefree integers in non-linear sequences, *Pac. J. Math.*, **59** (1975), 577–584.

16. G. J. Rieger, Remark on a paper of Stux concerning squarefree numbers in non-linear sequences, *Pac. J. Math.*, **78** (1978), 241–242. http://doi.org/10.2140/pjm.1978.78.241

17. X. D. Cao, W. G. Zhai, The distribution of square-free numbers of the form $[n^c]$, *J. Theor. Nombr. Bordx.*, **10** (1998), 287–299.

18. X. D. Cao, W. G. Zhai, On the distribution of square-free numbers of the form $[n^c]$ (II) , *Acta Math. Sin. (Chin. Ser.)*, **51** (2008), 1187–1194. https://doi.org/10.12386/A2008sxxb0138

19. S. I. Dimitrov, Square-free values of $[n^c \tan^\theta(\log n)]$, In: *Exploring mathematical analysis, approximation theory, and optimization*, Cham: Springer, **207** (2023), 33–43. https://doi.org/10.1007/978-3-031-46487-4_3

20. S. I. Dimitrov, Cube-free values of $[n^c \tan^\theta(\log n)]$, In: *Mathematical analysis, optimization, approximation and applications*, 2025, 217–226. https://doi.org/10.1142/9789811293009_0010

21. M. Zhang, J. J. Li, Distribution of cube-free numbers with form $[n^c]$, *Front. Math. China*, **12** (2017), 1515–1525. https://doi.org/10.1007/s11464-017-0652-1

22. J. M. Deshouillers, A remark on cube-free numbers in Segal-Piatestki-Shapiro sequences, *Hardy-Ramanujan J.*, 2019. https://doi.org/10.46298/hrj.2019.5114

23. T. Srichan, P. Tangsupphathawat, Square-full numbers in Piatetski-Shapiro sequences, *Ann. Math. Québec,* **44** (2020), 385–391. https://doi.org/10.1007/s40316-020-00132-8

24. J. D. Vaaler, Some extremal functions in Fourier analysis, *Bull. Amer. Math. Soc. (N.S.)*, **12** (1985), 183–216.

25. W. G. Zhai, X. D. Cao, On the number of coprime integer pairs within a circle, *Acta Arith.*, **90** (1999), 1–16. http://doi.org/10.4064/aa-90-1-1-16

26. S. W. Graham, G. Kolesnik, *Van Der Corput's method of exponential sums*, Cambridge University Press, 2010. https://doi.org/10.1017/CBO9780511661976