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Research article

Optimal refinancing strategy for a jump-diffusion mortgage rate model

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Abstract: Using a jump-diffusion process to characterize the mortgage rate, we consider the optimal refinancing strategy for interest-only mortgages. After transforming the two-dimensional refinancing problem into a one-dimensional optimization problem, we find that the optimal refinancing strategy is of a threshold type. The system of equations satisfied by the value function under the optimal refinancing strategy is also derived. Assuming that the jump sizes of the jump-diffusion process follow different distributions, we obtain the optimal refinancing threshold values and explicit expressions of the value function. Finally, some numerical results are provided to analyze the impact of some key parameters on the optimal refinancing strategy.

Keywords: mortgage; refinancing strategy; jump-diffusion process; threshold type
Mathematics Subject Classification: 91B70, 60G40

1. Introduction

The interest-only (IO) mortgage is one of the important products in real estate finance. It requires borrowers to pay only the interest during the loan term, with the principal repaid at the end. Due to their repayment structure that helps reduce borrowers' financial pressure, interest-only mortgages are favored by investors and home buyers with limited loan budgets. As noted by Barlevy and Fisher [1], interest-only mortgages are more prevalent in cities with hot housing markets, and their share exceeded 40% at the peak in some cities, such as Phoenix. Lenders, borrowers, and investors continue to pay close attention to this type of mortgage.

Historically, due to the important role of interest-only mortgages in the mortgage market, many studies have examined and discussed them from various perspectives. Aghili [2] provided a brief overview of the mechanics of adjustable-rate mortgages and considered the advantages and caveats of interest-only adjustable mortgages. Seay et al. [3] explored the relationship between financial literacy and the use of interest-only mortgages, and indicated that the individuals who incorrectly answered questions related to basic finance were more likely to be using an interest-only mortgage. Barlevy and Fisher [4] developed a model to investigate the reasons behind the popularity of interest-only mortgages during the U.S. housing boom. Results of Bäckman and Lutz [5] found that the introduction of IO loans led to an increase in housing turnover and transactions. Using Danish mortgage data, Larsen et al. [6] examined how interest-only mortgages affect consumption and savings over households' lifetimes. They found that young and old households are more likely to use interest-only mortgages compared with middle-aged households, and discussed the differences in consumption between households with IO mortgages and households with repayment mortgages. Similarly, Bäckman and Khorunzhina [7] explored the impact of interest-only mortgages on consumption.

If the borrower repays the loan according to the terms specified in the contract, the resulting cash flow is predictable. However, the borrower has the right to refinance when the market mortgage rate falls below the rate agreed upon in the contract in order to reduce future interest payments. This refinancing behavior not only reduces the lender's interest income but also complicates risk measurement and asset valuation. Therefore, it is essential for lenders to predict and mitigate refinancing risk before issuing a mortgage. Meanwhile, the issue of refinancing has attracted widespread attention from researchers. The review article by Krainer and Marquis [8] surveys research literature on factors influencing refinancing decisions and discusses developments in the mortgage refinancing market. For interest-only mortgages, Kimura and Makimoto [9] developed a model of rational mortgage refinancing where the drift and volatility of the interest rate process switch between two regimes and found that the optimal refinancing strategy is a threshold type. Agarwal et al. [10] assumed that the mortgage rate and inflation follow Brownian motion and derived a closed-form optimal refinancing strategy for housing mortgages. Xie et al. [11] used a Monte Carlo algorithm to study borrowers' refinancing behavior, where they modeled market mortgage rates with a two-dimensional Vasicek-type stochastic process. Wu et al. [12] examined optimal refinancing for fixed-rate mortgages based on the Vasicek interest rate model.

Due to the unique repayment structure of interest-only mortgages, the market mortgage rate significantly influences borrowers' refinancing behavior. To accurately capture the changing characteristics of the market mortgage rate and provide an appropriate refinancing strategy, we assume that the market mortgage rate follows a jump-diffusion process. This approach is chosen for two reasons: First, jump-diffusion processes are widely used in insurance and risk theory, which provides a solid theoretical foundation; second, the properties of the jump-diffusion process align well with the changes of market rate—particularly the jump component, which can capture large, sudden changes in rate over short periods.

Based on this foundation, our paper makes three contributions. First, the paper's main contribution lies in modeling mortgage rate dynamics using a jump-diffusion framework. Second, we transform the complex two-dimensional optimal stopping problem into a tractable one-dimensional problem, proving that the optimal strategy is of the threshold type. Finally, we derive explicit solutions for the optimal refinancing threshold under specific jump-size distributions.

Assuming that the state variable and production technology follow jump-diffusion processes, Ahn et al. [13] examined the term structure of interest rates. They revealed that bond prices are strictly higher under jump risks than in models without such risks. Using a double exponential jump-diffusion process to model the asset price, Kou [14] considered the option pricing problem. Using a stochastic impulse control approach, and considering the presence of fixed and proportional transaction costs, Zou et al. [15] discussed the dividend optimization problem for an insurer with a jump diffusion risk process. Under the condition that the dynamics of the risky underlying asset are driven by a Markov-modulated jump-diffusion model, Elliott et al. [16] considered the pricing of options. Assuming that the LIBOR interest rate follows a geometric Brownian motion with jump-diffusion terms, Mohamadinejad et al. [17] constructed a suitable model for pricing the spread options. By constructing a multi-dimensional jump-diffusion model, Melnikov and Nejad [18] calculated the upper and lower hedging prices. Other relevant studies include [19–22].

The content of this chapter is arranged as follows. In Section 2, we introduce a refinancing model for interest-only mortgages, in which a jump-diffusion process is used to model the mortgage rate. In Section 3, the optimal refinancing strategy and the system of equations that the value function satisfies are presented. In Section 4, we derive the optimal refinancing threshold and the mortgage valuation under the optimal refinancing strategy by considering different distributions for the jump sizes. Finally, numerical results are provided to examine the impact of some parameters on the optimal refinancing threshold and mortgage valuation.

2. The model

In this section, we construct a repayment model to determine the borrower's optimal refinancing strategy. We assume that the borrower is risk-neutral and that the risk-free interest rate is a constant ρ. The borrower repays the full principal M at the termination time Θ, which follows an exponential distribution with parameter η. The contractual borrowing rate is defined as the process $m(t)$. Until time Θ, the borrower only pays interest at the rate $m(t)M$. The borrower may choose to refinance at any time; however, the mortgage borrowing rate is reset to the current market mortgage rate, and a transaction cost δ is incurred at refinancing time.

If the mortgage holder does not have a refinance option, the expected present value of total payments with the initial borrowing rate m is given by

$$E\left[mM\int_0^{\Theta} e^{-\rho t}dt + e^{-\rho\Theta}M\right] = (\eta + m)\omega, \tag{1}$$

where $\omega = \frac{M}{\rho+\eta}$.

We assume that the market mortgage rate process is defined as

$$r(t) = r(0) + \mu t + \sigma B_t + \sum_{i=1}^{N(t)} Y_i, \tag{2}$$

where B_t is a standard Brownian motion with $B_0 = 0$, $\{N(t)\}_{t\geq 0}$ is a Poisson process with rate λ, constant μ and $\sigma > 0$ are the drift and volatility of the diffusion part, respectively, and the jump sizes $\{Y_1, Y_2, ...\}$ are independent and identically distributed (i.i.d.) random variables. The distribution function and density function are respectively defined as $F_Y(y)$ and $f_Y(y)$. Assuming that $\{B_t\}_{t\geq 0}$, $\{N(t)\}_{t\geq 0}$, and random variables $\{Y_1, Y_2, ...\}$ are independent. The linear drift and the Brownian motion in Eq(2) represent the continuous and normal changes in the market mortgage rate, while the compound Poisson process characterizes abnormal jumps.

Note that $r(t)$ is a continuous-time Markov process. When the initial market mortgage rate $r(0) = r_1 > 0$, the market mortgage rate $r(t)$ is denoted as $r^{(r_1)}(t)$. If $r^{(r_1)}(t)$ and $r^{(r_2)}(t)$ are constructed via the same paths of $\{B_t\}_{t\geq 0}$ and $\{N(t)\}_{t\geq 0}$, random variables $Y_1, Y_2, ...$, we have

$$r^{(r_1)}(t) - r^{(r_2)}(t) = r_1 - r_2, \quad \forall t \geq 0. \tag{3}$$

Denote $\tau = (\tau_1, \tau_2, ..., \tau_N)$ as the refinancing time sequence. Specifically, $\tau_0 = 0$, and N is the number of refinances before time Θ, and τ_j, $j = 1, ..., N$, is the j-th refinancing time. Note that $N = 0$ means the mortgage holder does not refinance before Θ. Borrowing rate and mortgage rate process $\{r_t\}_{t\geq 0}$ at initial time $t = 0$ are denoted by (m, r) if $m(0) = m$ and $r(0) = r$. Thus, the present value of total payments starting with initial state (m, r) under refinancing time sequence $\tau = (\tau_1, ..., \tau_N)$ is given by

$$\begin{aligned} U^\tau(m, r) =& I_{\{N\neq 0\}} mM \int_0^{\tau_1} e^{-\rho t} dt + \sum_{k=1}^{N-1} r(\tau_k) M \int_{\tau_k}^{\tau_{k+1}} e^{-\rho t} dt + I_{\{N\neq 0\}} r(\tau_N) M \int_{\tau_N}^{\Theta} e^{-\rho t} dt \\ &+ I_{\{N=0\}} mM \int_0^{\Theta} e^{-\rho t} dt + \sum_{k=1}^{N} \delta e^{-\rho\tau_k} + e^{-\rho\Theta} M. \end{aligned} \tag{4}$$

Equation (4) shows the present value of different parts of the mortgage. The first four terms are the present value of the interest paid by the borrower, the fifth term is the present value of the refinancing penalty, and the last term is the present value of the mortgage principal M. We use the indicator function $I_{\{\cdot\}}$ so that Eq (4) holds when $N = 0$. The objective is to find a refinancing time sequence τ that minimizes $E\left[U^\tau(m, r)\right]$. We denote the expected present value under the optimal refinancing strategy by

$$V(m, r) = \min_{\tau} E\left[U^\tau(m, r)\right], \tag{5}$$

which is also called the value function.

3. Optimal refinancing strategy and equations value function satisfied

In this section, we will show that optimal refinancing strategy and the system of equations $H(x) = V(0, x)$ satisfied.

Proposition 3.1. *Value function $V(m + z, r + z)$ satisfies the following equation:*

$$V(m + z, r + z) = V(m, r) + \omega z. \tag{6}$$

Proof. From Eqs (2) and (4), for the same refinancing time sequence τ, terminate time Θ, $\{N(t)\}_{t\geq 0}$, $\{r(t)\}_{t\geq 0}$, and $Y_1, ..., Y_{N(t)}$, we have

$$U^\tau(m + z, r + z) = U^\tau(m, r) + zM \int_0^{\Theta} e^{-\rho t} dt. \tag{7}$$

Taking expectation on both sides of Eq (7), we obtain

$$E\left[U^\tau(m + z, r + z)\right] = E\left[U^\tau(m, r)\right] + E\left[zM \int_0^{\Theta} e^{-\rho t} dt\right] = E\left[U^\tau(m, r)\right] + \omega z. \tag{8}$$

If τ is the optimal refinancing strategy of $U^{\tau}(m, r)$, it follows from Eq (8) that

$$V(m+z, r+z) \leq E\left[U^{\tau}(m+z, r+z)\right] \leq V(m, r) + \omega z. \tag{9}$$

Similarly, let τ be the optimal refinancing strategy of $U^{\tau}(m+z, r+z)$, we can obtain that

$$V(m+z, r+z) \geq V(m, r) + \omega z. \tag{10}$$

From Eqs (9) and (10), we note that Eq (6). □

From Proposition 3.1, we can rewrite $V(m, r)$ as

$$V(m, r) = V(m, m+x) = H(x) + m\omega, \tag{11}$$

where $x = r - m$ is the difference between the initial market interest rate and the initial borrowing rate, and $H(x) := V(0, x)$. Then, we reduce the two-dimensional optimal refinancing problem to a one-dimensional problem, i.e., the optimization problem for $V(m, r)$ is transformed into the optimization problem for $H(x)$. We assume that $H(x)$ is a continuous and integrable function. The expressions $H'(x)$, $H''(x)$, $H'''(x)$, and $H^{(4)}(x)$ denote the first, second, third, and fourth derivatives of the function $H(x)$, respectively.

By following the approach of Kimura and Makimoto [9], we obtain the following proposition, which presents the optimal refinancing strategy.

Proposition 3.2. *The optimal refinancing strategy is of threshold type, i.e., refinance when the difference between the market mortgage rate and the borrowing rate falls below a negative threshold θ, where θ is a constant to be determined.*

Proof. We prove this proposition by contradiction. For $x_1 > x_2$, suppose that an immediate refinance is optimal for the initial state $(m, m+x_1)$ and it is not optimal for $(m, m+x_2)$. Thus, from the definition of value function $V(m, r)$, we have

$$V(m, m+x_1) = V(m+x_1, m+x_1) + \delta = V(0,0) + \omega(m+x_1) + \delta, \tag{12}$$

$$V(m, m+x_2) < V(m+x_2, m+x_2) + \delta = V(0,0) + \omega(m+x_2) + \delta. \tag{13}$$

We consider the first refinancing time τ_1 in the optimal strategy when the initial state is $(m, m+x_2)$. It is obvious that this strategy is not an optimal refinancing time for the initial state $(m, m+x_1)$. Then we have

$$V(m, m+x_1) < E_{m+x_1}\left[V_{\tau_1} + I_{\{\tau_1<\Theta\}} e^{-\rho\tau_1} \left(V(r(\tau_1), r(\tau_1)) + \delta\right)\right], \tag{14}$$

$$V(m, m+x_2) = E_{m+x_2}\left[V_{\tau_1} + I_{\{\tau_1<\Theta\}} e^{-\rho\tau_1} \left(V(r(\tau_1), r(\tau_1)) + \delta\right)\right], \tag{15}$$

where

$$E_x[\cdot] = E[\cdot | r(0) = x]$$

and

$$V_{\tau_1} = \int_0^{\tau_1 \wedge \Theta} mMe^{-\rho t} dt + I_{\{\tau_1>\Theta\}} e^{-\rho\Theta} M.$$

By Eqs (12)–(15), we have

$$\begin{aligned}&E_{m+x_1}\left[V_{\tau_1}+I_{\{\tau_1<\Theta\}}e^{-\rho\tau_1}\left(V(r(\tau_1),r(\tau_1))+\delta\right)\right]-E_{m+x_2}\left[V_{\tau_1}+I_{\{\tau_1<\Theta\}}e^{-\rho\tau_1}\left(V(r(\tau_1),r(\tau_1))+\delta\right)\right]\\&>V(m,m+x_1)-V(m,m+x_2)>\omega(x_1-x_2).\end{aligned}\tag{16}$$

However, Eqs (14) and (15) have the same τ_1, Θ and V_{τ_1} which are independent of x_1, we can obtain that

$$\begin{aligned}&E_{m+x_1}\left[V_{\tau_1}+I_{\{\tau_1<\Theta\}}e^{-\rho\tau_1}\left(V(r(\tau_1),r(\tau_1))+\delta\right)\right]\\&\quad-E_{m+x_2}\left[V_{\tau_1}+I_{\{\tau_1<\Theta\}}e^{-\rho\tau_1}\left(V(r(\tau_1),r(\tau_1))+\delta\right)\right]\\&=E\left[I_{\{\tau_1<\Theta\}}e^{-\rho\tau_1}\omega\left(r^{(x_1)}(\tau_1)-r^{(x_2)}(\tau_1)\right)\right]\\&=E\left[I_{\{\tau_1<\Theta\}}e^{-\rho\tau_1}\omega(x_1-x_2)\right]<\omega(x_1-x_2).\end{aligned}\tag{17}$$

Since (16) contradicts (17), the proof has been completed. □

Remark 3.1. *The optimal refinancing threshold θ has a straightforward economic interpretation as the minimum required interest rate spread to justify modifying a contract under uncertainty. If the spread is above θ, the interest savings from a new contract are insufficient to cover the transaction cost, making it optimal to wait for market rates to decrease. If the spread falls below θ, the reduced interest is enough to compensate for the transaction cost, triggering the optimal action to refinance immediately. A lower value of θ indicates a more conservative refinancing strategy, while a higher value suggests a more aggressive one.*

Proposition 3.3. *Value function $H(x)$ satisfies the following system of equations:*

$$\begin{cases}H(x)=H(0)+\delta+\omega x, \quad x\le\theta,\\ \dfrac{\sigma^2}{2}H''(x)+\mu H'(x)-(\rho+\lambda+\eta)H(x)+\lambda\displaystyle\int_{-\infty}^{+\infty}H(x+y)dF_Y(y)+\eta M=0,\quad x>\theta,\end{cases}\tag{18}$$

where $H(x)$ is defined in Eq (11).

Proof. Under the optimal strategy, the mortgage holder should refinance immediately for $x\le\theta$, then

$$V(m,m+x)=V(m+x,m+x)+\delta.\tag{19}$$

By Eq (6), we have

$$V(m+x,m+x)=V(0,0)+\omega(m+x).\tag{20}$$

Substituting Eqs (11) and (20) into Eq (19), we can obtain that

$$H(x)=H(0)+\delta+\omega x,\qquad x\le\theta.$$

For $x>\theta$, the mortgage holder should not refinance immediately under the optimal strategy. We consider an infinitesimal time interval $[0,dt]$ and separate the four possible cases as follows:

(1) $\Theta\le dt$ (the probability is $1-e^{-\eta dt}$);

(2) $\Theta > dt$ and no jump of the process $N(t)$ occurs in $[0, dt]$ (the probability is $e^{-(\lambda+\eta)dt}$);
(3) $\Theta > dt$ and one jump of the process $N(t)$ occurs in $[0, dt]$ (the probability is $\lambda dte^{-\lambda dt}e^{-\eta dt}$);
(4) $\Theta > dt$ and more than one jump of the process $N(t)$ occurs in $[0, \varepsilon]$ (the probability is $o(dt)$).

Then we get

$$\begin{aligned} H(x) =& e^{-\rho dt}\left\{(\eta dt + o(dt))\, M + (1 - (\lambda + \eta)dt + o(dt))E\left[H(x + \mu dt + \sigma dB_t)\right]\right. \\ &\left. +(\lambda dt + o(dt))E\left[H(x + \mu dt + \sigma dB_t + Y_1)\right] + o(dt)\right\}, \end{aligned}$$

where, as usual, $o(dt)$ means that $o(dt)/dt \to 0$ as $dt \to 0$. Thus, we have

$$\frac{\sigma^2}{2}H''(x) + \mu H'(x) - (\rho + \lambda + \eta)H(x) + \lambda \int_{-\infty}^{+\infty} H(x + y)dF(y) + \eta M = 0. \tag{21}$$

□

Remark 3.2. *After obtaining the solution to the system of equations in Eq* (18)*, we apply the relevant boundary conditions to determine the unknown constants in the solution. From the value matching and the smooth pasting conditions, which are widely known as optimality conditions, we have*

$$\lim_{x\uparrow\theta} H(x) = \lim_{x\downarrow\theta} H(x), \quad \lim_{x\uparrow\theta} H'(x) = \lim_{x\downarrow\theta} H'(x). \tag{22}$$

4. Optimal refinancing threshold value and valuation of mortgage

In this section, we derive the explicit expression of the value function $H(x)$ and the value of the optimal refinancing threshold θ for the distribution $F_Y(y)$ of some common forms.

4.1. Exponential distribution

Suppose that the jump sizes Y_i are exponential distribution with density function

$$f_Y(y) = \begin{cases} \alpha e^{-\alpha y}, & y > 0, \\ 0, & y \le 0, \end{cases}$$

where $\alpha > 0$. Equation (21) can be rewritten as

$$\frac{\sigma^2}{2}H''(x) + \mu H'(x) - (\rho + \lambda + \eta)H(x) + \lambda\alpha \int_{0}^{+\infty} H(x + y)e^{-\alpha y}dy + \eta M = 0, \ x > \theta. \tag{23}$$

Let

$$U(x) = \int_0^{+\infty} H(x + y)e^{-\alpha y}dy = e^{\alpha x} \int_x^{+\infty} H(y)e^{-\alpha y}dy.$$

Equation (23) becomes

$$\frac{\sigma^2}{2}H''(x) + \mu H'(x) - (\rho + \lambda + \eta)H(x) + \lambda\alpha U(x) + \eta M = 0. \tag{24}$$

After differentiate Eq (24) with respect to x, we get

$$\frac{\sigma^2}{2}H'''(x) + \mu H''(x) - (\rho + \lambda + \eta)H'(x) + \lambda\alpha U'(x) = 0.$$

From $U'(x) = \alpha U(x) - H(x)$, we obtain

$$\frac{\sigma^2}{2}H'''(x) + \mu H''(x) - (\rho + \lambda + \eta)H'(x) + \lambda\alpha\left(\alpha U(x) - H(x)\right) = 0. \tag{25}$$

We multiply Eq (24) by $-\alpha$ and add Eq (25) and obtain

$$\frac{\sigma^2}{2}H'''(x) + (\mu - \frac{1}{2}\alpha\sigma^2)H''(x) - (\rho + \lambda + \eta + \mu\alpha)H'(x) + \alpha(\rho + \eta)H(x) - \alpha\eta M = 0. \tag{26}$$

Function $g_1(u)$ is given by

$$g_1(u) = \frac{\sigma^2}{2}u^3 + (\mu - \frac{1}{2}\alpha\sigma^2)u^2 - (\rho + \lambda + \eta + \mu\alpha)u + (\rho + \eta)\alpha.$$

From the fact that

$$g_1(0) = (\rho + \eta)\alpha > 0,\ g_1(\alpha) = -\lambda\alpha < 0,\ \lim_{u\to+\infty} g_1(u) > 0,\ \lim_{u\to-\infty} g_1(u) < 0,$$

we know that $g_1(u) = 0$ has three real roots, u_1, u_2, and u_3 with $u_1 < 0 < u_2 < \alpha < u_3$.

Then for $x > \theta$, the solution of Eq (26) is given by

$$H(x) = C_1e^{u_1x} + C_2e^{u_2x} + C_3e^{u_3x} + \omega\eta,$$

where C_1, C_2, and C_3 are constants. Together with condition that $\lim_{x\to+\infty} H(x) = \omega\eta$, it obvious that $C_2 = C_3 = 0$, i.e.,

$$H(x) = C_1e^{u_1x} + \omega\eta.$$

By Eq (22), we have

$$\begin{cases} C_1 + \delta + \omega\theta = C_1e^{u_1\theta}, \\ \omega = C_1u_1e^{u_1\theta}. \end{cases} \tag{27}$$

By substituting $C_1 = \omega/(u_1e^{u_1\theta})$ into the first equation of Eq (27) and performing a simplification, we have

$$e^{-u_1\theta} = 1 - \frac{u_1\delta}{\omega} - u_1\theta = \nu - u_1\theta, \tag{28}$$

where $\nu = 1 - u_1\delta/\omega$. Equation (28) can be rewritten as $(u_1\theta - \nu)\,e^{u_1\theta-\nu} = -e^{-\nu}$, i.e.,

$$u_1\theta - \nu = W(-e^{-\nu}), \tag{29}$$

where $W(\cdot)$ is the Lambert W function and satisfies $W(x)e^{W(x)} = x$. Therefore, we can obtain that

$$\theta = \frac{1}{u_1}\left(\nu + W(-e^{-\nu})\right).$$

From Eq (29) and the equation $W(-e^{-\nu})e^{W(-e^{-\nu})} = -e^{-\nu}$, we have

$$e^{u_1\theta} = e^{\nu}e^{W(-e^{-\nu})} = -\frac{1}{W(-e^{-\nu})}.$$

Then, we obtain that

$$C_1 = \frac{\omega}{u_1 e^{u_1\theta}} = -\frac{\omega}{u_1}W\left(-e^{-\nu}\right).$$

4.2. Double exponential distribution

Suppose that Y_i follows the double exponential jump diffusion process, i.e., the density function is

$$f_Y(y) = p\alpha_1 e^{-\alpha_1 y}I_{\{y\geq 0\}} + q\alpha_2 e^{\alpha_2 y}I_{\{y<0\}}, \tag{30}$$

here $p, q \geq 0$ are constants, $p + q = 1$, and $\alpha_1, \alpha_2 > 0$. Therefore, for $x > \theta$, we have

$$\begin{aligned}\frac{\sigma^2}{2}H''(x) + \mu H'(x) - (\rho + \lambda + \eta)H(x) + \lambda p\alpha_1 \int_0^{+\infty} H(x+y)e^{-\alpha_1 y}dy \\ +\lambda q\alpha_2 \int_{-\infty}^{0} H(x+y)e^{\alpha_2 y}dy + \eta M = 0, \quad x > \theta.\end{aligned} \tag{31}$$

From

$$\begin{aligned}\int_{-\infty}^{0} H(x+y)e^{\alpha_2 y}dy &= \int_{\theta-x}^{0} H(x+y)e^{\alpha_2 y}dy + \int_{-\infty}^{\theta-x} H(x+y)e^{\alpha_2 y}dy \\ &= \int_{\theta-x}^{0} H(x+y)e^{\alpha_2 y}dy + \int_{-\infty}^{\theta-x} \left(H(0) + \delta + \omega(x+y)\right)e^{\alpha_2 y}dy \\ &= \int_{\theta-x}^{0} H(x+y)e^{\alpha_2 y}dy + \left[\frac{H(0)+\delta}{\alpha_2} + \frac{\omega}{\alpha_2}\left(\theta - \frac{1}{\alpha_2}\right)\right]e^{\alpha_2(\theta-x)} \\ &:= \int_{\theta-x}^{0} H(x+y)e^{\alpha_2 y}dy + Je^{-\alpha_2 x},\end{aligned}$$

where

$$J = \left[\frac{H(0)+\delta}{\alpha_2} + \frac{\omega}{\alpha_2}\left(\theta - \frac{1}{\alpha_2}\right)\right]e^{\alpha_2\theta},$$

Eq (31) can be rewritten as

$$\frac{\sigma^2}{2}H''(x) + \mu H'(x) - (\rho + \lambda + \eta)H(x) + U_1(x) + U_2(x) + \lambda q\alpha_2 Je^{-\alpha_2 x} + \eta M = 0, \tag{32}$$

where

$$U_1(x) = \lambda p\alpha_1 \int_0^{+\infty} H(x+y)e^{-\alpha_1 y}dy = \lambda p\alpha_1 e^{\alpha_1 x}\int_x^{+\infty} H(y)e^{-\alpha_1 y}dy,$$

$$U_2(x) = \lambda q\alpha_2 \int_{\theta-x}^{0} H(x+y)e^{\alpha_2 y}dy = \lambda q\alpha_2 e^{-\alpha_2 x}\int_{\theta}^{x} H(y)e^{\alpha_2 y}dy.$$

It can be found that

$$U_1'(x) = \alpha_1 U_1(x) - \lambda p\alpha_1 H(x),$$
$$U_2'(x) = -\alpha_2 U_2(x) + \lambda q\alpha_2 H(x).$$

After different Eq (32) with respect to x, we get

$$\frac{\sigma^2}{2}H'''(x) + \mu H''(x) - (\rho+\lambda+\eta)H'(x) + (\lambda q\alpha_2 - \lambda p\alpha_1)H(x) + \alpha_1 U_1(x) - \alpha_2 U_2(x) - \lambda q\alpha_2^2 J e^{-\alpha_2 x} = 0. \quad (33)$$

After multiply Eq (32) by α_2 and add Eq (33), we have

$$\frac{\sigma^2}{2}H'''(x) + (\mu + \frac{1}{2}\alpha_2\sigma^2)H''(x) - (\rho+\lambda+\eta-\mu\alpha_2)H'(x) + (\alpha_1+\alpha_2)U_1(x) + (\lambda q\alpha_2 - \lambda p\alpha_1 - (\rho+\lambda+\eta)\alpha_2)H(x) + \eta\alpha_2 M = 0. \quad (34)$$

Different Eq (34) again with respect to x, we obtain

$$\frac{\sigma^2}{2}H^{(4)}(x) + (\mu + \frac{1}{2}\alpha_2\sigma^2)H'''(x) - (\rho+\lambda+\eta-\mu\alpha_2)H''(x) + \alpha_1(\alpha_1+\alpha_2)U_1(x) - \lambda p\alpha_1(\alpha_1+\alpha_2)H(x) + (\lambda q\alpha_2 - \lambda p\alpha_1 - (\rho+\lambda+\eta)\alpha_2)H'(x) = 0. \quad (35)$$

Similarly, we multiply Eq (34) by $-\alpha_1$ and add Eq (35) and have

$$\begin{aligned}\frac{\sigma^2}{2}H^{(4)}(x) + (\mu + \frac{1}{2}\alpha_2\sigma^2 - \frac{1}{2}\alpha_1\sigma^2)H'''(x)\\ -(\rho+\lambda+\eta-\mu\alpha_2+\mu\alpha_1+\frac{1}{2}\alpha_1\alpha_2\sigma^2)H''(x)\\ +(\lambda q\alpha_2 - \lambda p\alpha_1 - (\rho+\lambda+\eta)\alpha_2 + (\rho+\lambda+\eta-\mu\alpha_2)\alpha_1)H'(x)\\ + (\rho+\eta)\,\alpha_1\alpha_2 H(x) - \eta\alpha_1\alpha_2 M = 0. \quad (36)\end{aligned}$$

Let

$$g_2(u) = a_4u^4 + a_3u^3 + a_2u^2 + a_1u + a_0,$$

where

$$a_4 = \frac{\sigma^2}{2},\ \ a_3 = \mu + \frac{1}{2}\alpha_2\sigma^2 - \frac{1}{2}\alpha_1\sigma^2,\ \ a_2 = -\rho-\lambda-\eta+\mu\alpha_2-\mu\alpha_1-\frac{1}{2}\alpha_1\alpha_2\sigma^2,$$
$$a_1 = \lambda q\alpha_2 - \lambda p\alpha_1 - (\rho+\lambda+\eta)\alpha_2 + (\rho+\lambda+\eta-\mu\alpha_2)\alpha_1,\ \ a_0 = (\rho+\eta)\,\alpha_1\alpha_2.$$

From the fact that the function $g_2(u)$ satisfies

$$\lim_{u\to-\infty} g_2(u) > 0, \quad \lim_{u\to+\infty} g_2(u) > 0, \quad g_2(0) = (\rho+\eta)\,\alpha_1\alpha_2 > 0,$$

$$g_2(\alpha_1) = -\lambda p\alpha_1(\alpha_1 + \alpha_2) < 0, \quad g_2(-\alpha_2) = -\lambda q\alpha_2(\alpha_1 + \alpha_2) < 0,$$

we know that $g_2(u) = 0$ has four real roots, u_1, u_2, u_3, and u_4 with $u_1 < -\alpha_2 < u_2 < 0 < u_3 < \alpha_1 < u_4$. Therefore, the solution to Eq (36) is given by

$$H(x) = D_1 e^{u_1 x} + D_2 e^{u_2 x} + D_3 e^{u_3 x} + D_4 e^{u_4 x} + \omega\eta,$$

where D_1, D_2, D_3, and D_4 are constants to be determined. Together with condition that $\lim_{x\to+\infty} H(x) = \frac{\eta}{\rho+\eta}M$, it obvious that $D_3 = D_4 = 0$, i.e.,

$$H(x) = D_1 e^{u_1 x} + D_2 e^{u_2 x} + \omega\eta. \tag{37}$$

By the conditions Eqs (18) and (22), we have

$$\begin{cases} D_1 + D_2 + \delta + \omega\theta = D_1 e^{u_1\theta} + D_2 e^{u_2\theta}, \\ \omega = D_1 u_1 e^{u_1\theta} + D_2 u_2 e^{u_2\theta}. \end{cases} \tag{38}$$

After substituting Eq (37) into Eq (32), from the coefficient of $e^{-\alpha_2 x}$, we obtain

$$\left(1 - \frac{\alpha_2}{u_1 + \alpha_2} e^{u_1\theta}\right) D_1 + \left(1 - \frac{\alpha_2}{u_2 + \alpha_2} e^{u_2\theta}\right) D_2 + \omega\left(\theta - \frac{1}{\alpha_2}\right) + \delta = 0. \tag{39}$$

From Eqs (38) and (39), we obtain that θ is the negative root of equation

$$\frac{(u_1 + \alpha_2)u_2}{(u_2 - u_1)u_1\alpha_2} e^{-u_1\theta} + \frac{(u_2 + \alpha_2)u_1}{(u_1 - u_2)u_2\alpha_2} e^{-u_2\theta} + \theta + \frac{\delta}{\omega} - \frac{1}{u_1} - \frac{1}{u_2} - \frac{1}{\alpha_2} = 0,$$

and

$$D_1 = \frac{(u_1 + \alpha_2)u_2\omega}{(u_2 - u_1)u_1\alpha_2} e^{-u_1\theta}, \quad D_2 = \frac{(u_2 + \alpha_2)u_1\omega}{(u_1 - u_2)u_2\alpha_2} e^{-u_2\theta}.$$

5. Numerical illustration

Since mortgage rates can experience both upward and downward jumps in reality, we present some numerical results for the case where the jump size follows a double exponential distribution introduced in Section 4.2. The parameters in Eq (30) are set to $p = 0.45$, $q = 0.55$, $\alpha_1 = 400$, and $\alpha_2 = 400$. Without loss of generality, we set $M = 1$. Other parameters are set to $\rho = 0.03$, $\mu = 0.003$, $\sigma = 0.012$, $\eta = 0.05$, $\delta = 0.02$, $\lambda = 2$*.

The parameter δ represents the transaction costs that the borrower must pay when refinancing. When refinancing costs rise, borrowers need to wait longer for a lower mortgage rate to offset these costs. Figure 1 also shows that $-\theta$ increases with higher transaction costs. This indicates that transaction costs have a significant impact on the optimal refinancing threshold†.

*The parameter values employed in this section are chosen for illustrative purposes to conduct a sensitivity analysis. They are not based on empirical estimation.

†Since refinancing is triggered when the interest rate spread falls below θ, a higher value of $-\theta$ implies a more conservative strategy and a longer expected waiting time. Thus, plotting the change in $-\theta$ directly illustrates the trend in the expected waiting time until refinancing.

By setting the parameter η, we can obtain the expected time until future full repayment (i.e., $1/\eta$). A larger η implies that the borrower is more likely to repay the mortgage early. Figure 2 shows the relationship between η and $-\theta$. Compared to long-term mortgages (with smaller η), borrowers with short-term loans (larger η) tend to wait longer before refinancing. The reason is that the economic benefit of a lower post-refinancing rate requires time to build up, whereas a short-term loan might be repaid soon after refinancing. As a result, borrowers with short-term loans require a lower mortgage rate to achieve a larger interest rate spread.

The parameter λ represents the average number of mortgage rate jumps per unit of time. Therefore, a larger λ implies more jumps occurring during the mortgage period. As shown in Figure 3, $-\theta$ is monotonically increasing with λ. Under our parameter settings, more frequent jumps in interest rate will lead to a longer waiting period for the borrower before refinancing. Figure 4 shows how the jump affects the mortgage valuation, where $m(0) = r(0) = 0.06$. It can be observed that the valuation V decreases as the parameter λ increases, which means the mortgage valuation decreases when jumps are more frequent.

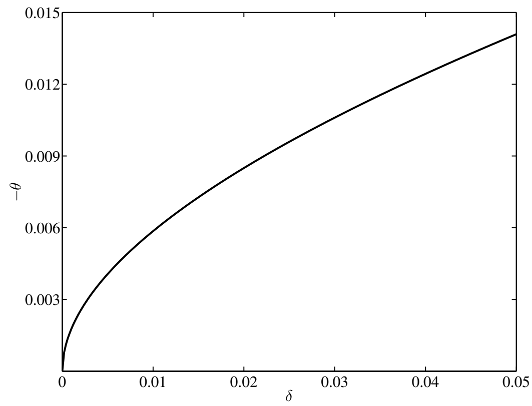

Figure 1. The relationship between δ and $-\theta$.

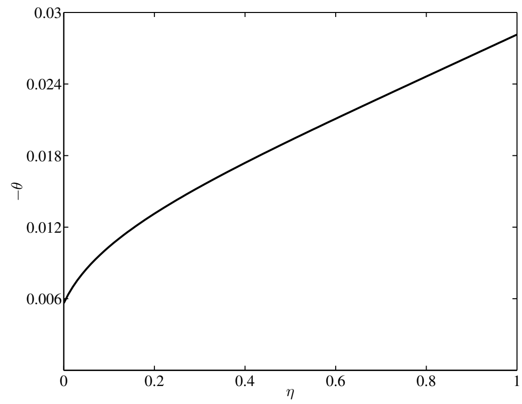

Figure 2. The relationship between η and $-\theta$.

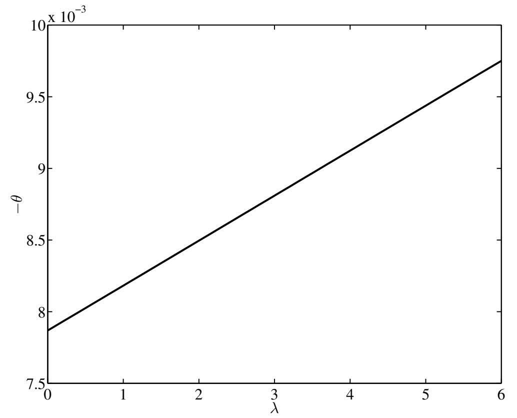

Figure 3. The relationship between λ and $-\theta$.

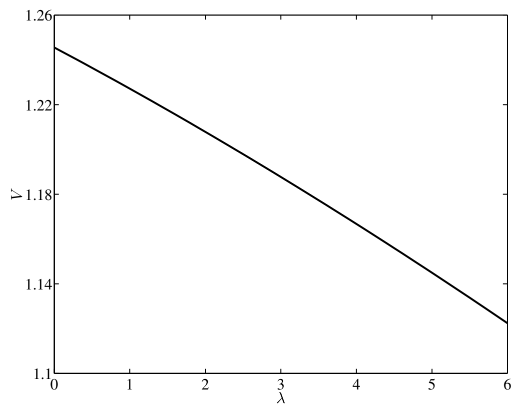

Figure 4. The relationship between the valuation V and λ.

6. Conclusions

This paper examines the optimal refinancing strategy for interest-only mortgages under a jump-diffusion mortgage rate model. Unlike traditional short-rate models such as Vasicek or CIR processes that primarily describe the continuous evolution of the mortgage rate, the jump-diffusion process incorporates a jump component. This addition is critical for capturing the sudden, sharp movements in the mortgage rate that often occur in response to macroeconomic shocks or policy announcements. By transforming the two-dimensional optimization problem into a one-dimensional problem, it is shown that the optimal refinancing strategy is of a threshold type. Furthermore, a system of equations concerning the value function and the optimal refinancing threshold is derived. When the jump size follows an exponential distribution, we obtain an explicit formula for the optimal refinancing threshold; when it follows a double exponential distribution, we derive the equation that the threshold must satisfy. Finally, numerical examples are used to explore the effects of transaction costs, mortgage term, and jump frequency on the borrower's optimal refinancing strategy and mortgage valuation.

Under some simplifying assumptions (such as a constant risk-free rate), the paper derives several results related to mortgage refinancing. However, given the complex market environment and real-world economic conditions, these simplified assumptions have certain limitations. By these assumptions, the existing model can be extended to yield refinancing strategies that are more aligned with market fluctuations. For instance, the evolution of the risk-free rate could be modeled using a stochastic process, or we could assume that $N(t)$ is a Cox process to capture the changes in the frequency of shocks.

Author contributions

Congjin Zhou contributed to the conception of the study and wrote the main manuscript text; Yinghui Dong and Wanrong Mu provided constructive suggestions in the derivation of propositions. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no potential conflicts of interest.

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