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Research article

A new score function of interval-valued intuitionistic fuzzy values based on prospect theory

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Abstract: As a fundamental tool for interval-valued intuitionistic fuzzy sets (IVIFSs), the score function (SF) plays a pivotal role in quantifying interval-valued intuitionistic fuzzy values (IVIFVs) and facilitating their comparative analysis. However, a notable limitation of existing SFs is their potential to assign identical scores to distinct IVIFVs, thereby compromising the discrimination capability. To address this challenge, this study introduces IVIFS-SF, a novel score function grounded in prospect theory, and proposes two innovative assessment methodologies. First, we employed prospect theory to develop an interval-valued evaluation method (IVEM), which converts the interval into a crisp number. Second, using IVEM, we developed the new score function IVIFS-SF and present its properties. Third, we put forward pass rate and variance as metrics to analyze and compare SFs. Rigorous comparative analysis demonstrated that IVIFS-SF achieves superior performance in both pass rate and variance metrics when benchmarked against existing state-of-the-art SFs. Furthermore, sensitivity analysis confirmed the robustness of IVIFS-SF across the parameter spectrum of prospect theory. Empirical case studies revealed that while IVIFS-SF identifies the same optimal alternative as competing SFs, it exhibits the highest variance among them, suggesting enhanced discriminative power.

Keywords: prospect theory; IVIFS; interval-valued evaluation; score function; IVIFV
Mathematics Subject Classification: 03E27, 90B50

1. Introduction

Interval-valued intuitionistic fuzzy sets (IVIFSs) [1], which extend both interval-valued fuzzy sets (IVFSs) [2] and intuitionistic fuzzy sets (IFSs) [3], provide a more flexible representation of uncertainty. They have been extensively applied in areas such as intelligent decision-making, target selection, and risk assessment [4,5]. When measuring the interval-valued intuitionistic fuzzy value (IVIFV) [1], commonly used methods include the score function (SF), distance, similarity, and entropy [6–8]. Unlike other measures, the SF can derive a crisp value directly from a single IVIFV. Consequently, researchers have placed considerable emphasis on studying the SF and its properties. However, the single, one-dimensional output of the SF makes it mathematically challenging to distinguish among all possible IVIFVs, which inherently possess a complex four-dimensional structure. Therefore, numerous studies have been conducted to enhance the discriminative ability of the SF in practical applications and to mitigate its mathematical limitations. The SF for the IVIFV was first proposed by Xu [9], can intuitively reflect the positive and negative attitudes of decision-makers, and has been widely used [10]. However, Xu also noted that some IVIFVs cannot be distinguished using this method [9]. To enhance the comparative ability of two IVIFVs, the accuracy function (AF) was then introduced. Despite their wide application in multi-attribute group decision-making (MADM) problems [4,11,12], it has been observed that some IVIFVs still cannot be effectively distinguished using the SF and AF [11,12]. For example, a_1 and a_2 cannot be distinguished when $a_1 = ([0.2, 0.6], [0.3, 0.4])$ and $a_2 = ([0.3, 0.5], [0.2, 0.5])$. In order to improve the comparative ability of IVIFVs by using the SF and AF, researchers have continued exploring some new SFs and AFs. However, they still show some degree of indistinguishability when $a_3 = ([0.45,0.45], [0.45,0.45])$ and $a_4 = ([0.35, 0.35], [0.35,0.35])$ [13], and when $a_5 = ([0.3,0.45], [0.27,0.51])$ and $a_6 = ([0.15, 0.55], [0.32,0.37])$ [14]. To enhance the distinguishability of IVIFVs and effectively utilize the SF in MADM, extensive investigations into score function techniques have been conducted in recent years [15–17]. For example, Kumar [17] developed an SF based on set pair analysis theory (SPA), and while building on Kumar's work, Wu [18] outlined one based on the beta function. Chen made improvements to this work a year later [19], but when $a_7 = ([0.3,0.45], [0.27,0.51])$, $a_8 = ([0.15, 0.55], [0.32,0.37])$, and the calculation results retain four decimal places, the SF cannot distinguish a_7 and a_8. Chen and Tsai [20] then presented improved SFs for the IVIFV that could easily distinguish IVIFVs rounded to two decimal places, and Chen [15,19] and Wu [18] studied SFs without the need of AFs to efficiently solve MADM problems. The details of these works are given in Section 4.2 and detailed explanations of the problems are given in Section 2.3. Cheng [16] studied the measurement method based on the Gold Rule and T-S theory. This successfully solved the measurement problem when $s = ([a, b], [c, d])$ meets the condition of $a = b = c = d$. However, it is necessary to determine the value of the parameter k based on the actual application environment. Our investigation indicates that existing SFs inadequately capture the uncertainties inherent in the membership and non-membership intervals, as well as in the dynamic nature of decision-making. Hence, addressing interval uncertainty remains a crucial challenge.

To overcome the limitations of existing score functions (SFs), researchers have incorporated entropy-based techniques into the design of SFs for IVIFVs [21–23]. Specifically, Ye [22] proposed an SF that combines entropy and the correlation coefficient. Wei [21] introduced SFs that integrate entropy and similarity, a combination that has proven not only superior to the SF developed by Bai

[24] but also widely applied in fuzzy decision-making applications. However, Guo [25] identified several shortcomings of these SFs. In efforts to enhance the processing capabilities of SFs, some researchers have introduced SFs based on knowledge measures and entropy. For example, Guo [25] proposed an SF based on membership, non-membership, and hesitation of the IVIFV. Kumar [26] studied the SF for transportation models. While Nguyen [27] investigated SFs based on knowledge measurement, in [28] he then investigated the SF by using a generalized p-Norm. However, when the norm value is $p = 1$, $a_9 = ([0.0,0.4],[0.0,0.5])$, and $a_{10} = ([0.1,0.3\,],[0.1,0.4])$, the Nguyen SF [28] fails to compare a_9 and a_{10}. Although entropy- and knowledge-based approaches have shown promise in constructing SFs for IVIFVs, the SFs of their products do not adequately reflect the psychological change process.

Despite these challenges, researchers have discovered that prospect theory effectively captures the psychological changes decision-makers experience [29]. Decision-makers tend to exhibit risk aversion when facing gains and show a preference for risk when facing losses. As the frequency of gains and losses increases for decision-makers, the marginal value of each subsequent gain or loss diminishes [30]. Consequently, the integration of fuzzy theory has received substantial attention [31,32], leading to significant methodological advances [33]. For instance, Fan [34] proposed a MEREC-MABAC approach for evaluating the performance of wearable health technology devices within a prospect-theory-based framework. Wang put forward a decision method based on prospect theory [12]. Gao [35] also developed an SF based on prospect theory, but could not distinguish IVIFVs with equal membership and non-membership. Wang et al. [36] proposed an SF based on the prospect value function, in which the reference point corresponds to the expected value of multiple IVIFVs and the resulting output is an interval. When there is only one IVIFV, this SF degenerates into a membership interval minus a non-membership interval value, which cannot be converted into a crisp number. Moreover, it fails to account for the dynamic psychological transition that decision-makers undergo when shifting from support to opposition.

In general, existing SFs for IVIFSs have demonstrated strong applicability in areas such as decision-making, evaluation, and intelligent analysis, but they also have several shortcomings. Some SFs can rapidly calculate and intuitively reflect the positive and negative attitudes of decision-makers, yet they fail to distinguish and compare a large number of different IVIFVs. Some SFs incorporate entropy-based measures, but they are not able to reflect the psychological change process of decision-makers. Therefore, the main research drivers behind this paper are summarized as follows.

(1) Although some functions can map the interval $[a,b]$ ($[a,b] \subseteq [0,1]$) to a crisp number, they fail to reflect the psychology of decision-makers. This paper will use prospect theory to study the interval mapping function and map $[a,b]$ to a crisp number.

(2) Many existing SFs of IVIFVs struggle to compare two IVIFVs with obvious differences. Despite the use of AFs to enhance the distinction of IVIFVs, a significant number of them remain incomparable (see Section 2.3). It is necessary to develop a new SF to improve the ability to compare IVIFVs without requiring an AF, thus enhancing the practical applicability of IVIFVs.

(3) Although researchers [36] have proposed SFs based on prospect theory before, their methods do not fully account for the prospect values of interior points in an interval, nor do their SFs result in a crisp number. Our proposal in this paper is to develop an SF that considers the prospect values of interior points and yields a crisp number.

(4) The selection and development of SFs need a set of standards and methods for comparison. To the best of our knowledge, no study has proposed an evaluation method or criteria for evaluating SFs, a gap which this paper aims to fill.

Inspired by prospect theory [29], this paper transforms continuous decision-making information into crisp numbers that can also reflect the decision-making psychology of decision-makers. An interval-valued evaluation method (IVEM) based on prospect theory is thus proposed to transform the IVIFV into an intuitionistic fuzzy value (IFV). The key contributions of this paper are:

(1) The development of an IVEM based on prospect theory, which can improve the discriminative ability in practical applications and mitigating mathematical limitations.

(2) The transformation of the IFN based on the IVEM.

(3) The design of an IVIFS-SF and the investigation of its properties based on the IVEM.

(4) The proposal of pass rate and variance as the SFs assessment methods, where a generated dataset is used for illustration.

(5) A comparative analysis and verification of the IVIFS-FS with the state-of-the-art SFs through a case study.

The remaining sections of this paper are organized as follows. Section 2 provides a background discussion of prospect theory, Section 3 describes the IVEM that has been developed as a result of that theory, Section 4 develops a new SF and SF assessment methods, Section 5 verifies the proposed SF with a case study, and concluding remarks are found in the final section.

2. Preliminary

2.1. IVIFS

Definition 1 [1]. Let X be the universe. The IVIFS A on X is defined in Eq (1):

$$A = \{(x, u_A(x), v_A(x)) | x \in X\}. \tag{1}$$

In Eq (1), the membership function is an interval mapping: $u_A(x) = [u_A^-(x), u_A^+(x)] \subseteq [0,1]$ and the non-membership function is a mapping of interval value: $v_A(x) = [v_A^-(x), v_A^+(x)] \subseteq [0,1]$. They satisfy $u_A^-(x) \geq 0, v_A^-(x) \geq 0,$ and $0 \leq u_A^+(x) + v_A^+(x) \leq 1$. The hesitation degree of A is

$$\rho_A(x) = [\pi_A^-(x), \pi_A^+(x)] = [1 - u_A^+(x) - v_A^+(x), 1 - u_A^-(x) - v_A^-(x)]. \tag{2}$$

In [12], $a = ([u^-, u^+], [v^-, v^+])$ is regarded as an interval-valued intuitionistic fuzzy value (IVIFV) where $[u^-, u^+] \subseteq [0,1], [v^-, v^+] \subseteq [0,1]$, and $u^+ + v^+ \leq 1$.

Definition 2 [9]. Let $a_i = ([u_{a_1}^-, u_{a_1}^+], [v_{a_1}^-, v_{a_1}^+])$ be a set of IVIFVs and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ be the weight vector satisfying $\sum_{i=1}^n \omega_i = 1,\ \omega_i \geq 0,\ i = 1,2, \cdots, n$. The IIFWA operator is defined as

$$\text{IIFWA}\ (a_1, a_2, \cdots, a_n) = ([1 - \prod_{i=1}^n (1 - u_i^-)^{\omega_i}, 1 - \prod_{i=1}^n (1 - u_i^+)^{\omega_i}], [\prod_{i=1}^n (v_i^-)^{\omega_i}, \prod_{i=1}^n (v_i^+)^{\omega_i}]). \tag{3}$$

2.2. Prospect theory

In prospect theory [29], the value of a prospect is evaluated relative to a reference point from decision-makers. The prospect value function is S-shaped, being concave in the gain domain and convex in the loss domain, as illustrated in Figure 1.

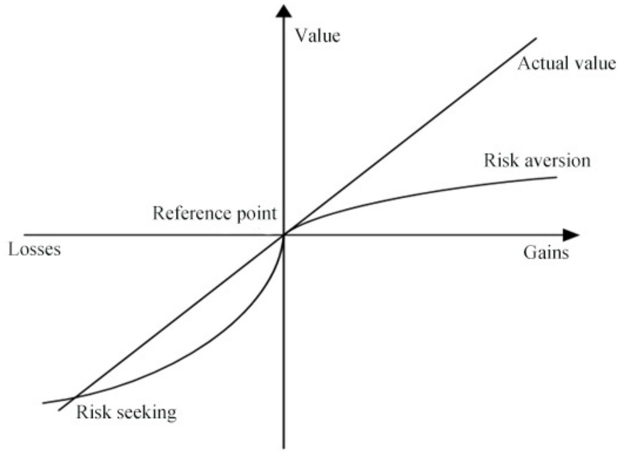

Figure 1. Value function of prospect theory.

In prospect theory, it has been observed that different decision-makers may have different choices for the same problem, as their subjective preferences influence the resulting decision outcomes. The prospect value is thus jointly determined by the value function and probabilistic weight function as shown in Eq (4).

$$V = v(\Delta x)w(p), \tag{4}$$

where V represents the prospect value, $v(\Delta x)$ represents the value function of a decision-maker's subjective feelings, and $w(p)$ represents the weight function of probability. If we define Δx as $x_i - x_*$, where x_* is the reference point and x_i represents the evaluation point, the calculation of $v(\Delta x)$ is shown in Eq (5).

$$V(\Delta x) = \begin{cases} (\Delta x)^{\alpha}, \Delta x \geq 0, \\ -\theta(-\Delta x)^{\beta}, \Delta x < 0. \end{cases} \tag{5}$$

In Eq (5), when $\Delta x \geq 0$, it is expressed as gain. Otherwise, it is regarded as loss. $\alpha(0 < \alpha < 1)$ and $\beta(0 < \beta < 1)$ are the risk attitude coefficients, which are shown in Figure 1 as the degree of concavity and convexity of the value function in the gain domain and the loss domain, respectively. They reflect the decreasing speed of the decision-maker's sensitivity. The larger their values are, the more likely the decision-makers are to take risks. θ is the loss avoidance coefficient, which is the steepness of the value function in the loss domain in Figure 1, that is, it reflects the degree of loss avoidance of a decision-maker. When $\theta > 1$, it indicates a decision-maker's loss aversion.

In Eq (4), the probability weight function $w(p)$ is determined by the probability of occurrence p of the event x_i, which is a monotonically increasing function as shown in Eq (6). In Eq (6), $\gamma > 0$ and $\delta > 0$ represent the risk attitude coefficients of the gain domain and the loss domain, respectively,

$$w(p) = \begin{cases} \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}}, \Delta x \geq 0, \\ \frac{p^{\delta}}{\left(p^{\delta}+(1-p)^{\delta}\right)^{1/\delta}}, \Delta x < 0. \end{cases} \tag{6}$$

2.3. Analysis of existing score functions

Since the first SF of IVIFSs was proposed in 2007 [9], over forty have been developed and investigated. However, many of these methods still have several shortcomings. To illustrate the performance of existing SFs, we scrutinize ten of them focusing on their indistinguishability.

The score function S_x of the IVIFVs proposed by Xu [9] has a simple calculation process and obtains results easily. Definition 3 shows that at present, it is one of the most widely used SFs, while Example 1 demonstrates its indistinguishability.

Definition 3. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_X is defined in Eq (7):

$$S_X(a) = \frac{u^- + u^+ - v^- - v^+}{2}. \tag{7}$$

Example 1. Let $a_{11} = ([0.25, 0.3], [0.4, 0.6])$ and $a_{12} = ([0.15, 0.4], [0.5, 0.5])$. We can get $S_X(a_{11}) = S_X(a_{12}) = -0.45$, indicating that S_X cannot distinguish a_{11} and a_{12}. Although Xu [9] proposed the improved AF $H_X(\alpha) = \frac{1}{2}(u^- + u^+ + v^- + v^+)$, the calculation $H_X(a_{11}) = H_X(a_{12}) = 0.775$ fails to distinguish a_{11} and a_{12}. At the same time, for any two a_i and a_j, when $u_i^- + u_i^+ - v_i^- + v_i^+ = u_j^- + u_j^+ - v_j^- + v_j^+$, S_X cannot distinguish a_i and a_j.

Wang and Chen [13] proposed a score function S_{WC}, which not only integrates the support and opposition attitudes of decision-makers, but also considers the intersection of membership and non-membership as shown in Definition 4. Wei [7] pointed out that the parameters u^+ and v^- in S_{WC} do not satisfy monotonicity when $u^- = u^+ = v^- = v^+$. As shown in Example 2, IVIFVs cannot be distinguished.

Definition 4. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_{WC} is

$$S_{WC}(a) = \frac{(u^- + u^+)(u^- + v^-) - (v^- + v^+)(u^+ + v^+)}{2}. \tag{8}$$

Example 2. Let $a_{13} = ([0.45, 0.45], [0.45, 0.45])$ and $a_{14} = ([0.35, 0.35], [0.35, 0.35])$. This gives us $S_{WC}(a_{13}) = 0, S_{WC}(a_{14}) = 0$, and a S_{WC} that cannot distinguish a_{13} and a_{14}. At the same time, for any two a_i and a_j, when $u_i^- = u_i^+ = v_i^- = v_i^+, u_j^- = u_j^+ = v_j^- = v_j^+$, S_{WC} cannot distinguish a_i and a_j.

Wei and Li [7] proposed an information-based score function S_{WL} as shown in Definition 5. Their S_{WL} comprehensively reflects the multiple relationships between membership and non-membership of IVIFVs. Example 3 gives the indistinguishability in this case.

Definition 5. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_{WL} is

$$S_{WL}(a) = [1 + u^- + u^+ - v^- - v^+ + 0.5(|u^- - v^-| + |u^+ - v^+|)]\left[(1 + u^- + v^-)e^{u^- - v^- + u^- + u^+}/\right.$$

$$\left. e^3 + (1 + u^+ + v^+)e^{u^+ - v^+ - v^- - v^+}/e\right][(2 - v^- - v^+)/(4 - u^- - u^+ - v^- - v^+)]/16. \tag{9}$$

Example 3. Let $a_{15} = ([0.05, 0.07], [0.1, 0.55])$ and $a_{16} = ([0.23, 0.27], [0.45, 0.61])$. This gives us $S_{WL}(a_{15}) = S_{WL}(a_{16}) = 0.0049$, thus making S_{WL} unable to distinguish a_{15} and a_{16}. At the same time, when $a_1 = ([0.05, 0.07], [0.1, 0.55])$, $a_i = ([0.05 + k, 0.07 + 1.111k], [0.1 +$

$1.944k, 0.55 + 0.333k])(-0.05 \leq k \leq 0.263)$, and the score value is rounded to two decimal places, there are some a_j that S_{WL} cannot distinguish a_1 and a_j.

Bai [24] proposed a score function S_B based on the unknown degree as shown in Definition 6, but when $u^- = u^+ = 0$, S_B is always 0 as shown in Example 4.

Definition 6. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_B is shown in Eq (10):

$$S_B(a) = \frac{u^- + u^-(1 - u^- - v^-) + u^+ + u^+(1 - u^+ - v^+)}{2}. \tag{10}$$

Example 4. Let $a_{17} = ([0,0], [0.2,0.5])$ and $a_{18} = ([0,0], [0.1,0.2])$. $S_B(a_{17}) = S_B(a_{18}) = 0$ indicates that S_B fails to distinguish a_{17} and a_{18}. At the same time, for any two a_i and a_j,when $u_i^- = u_i^+ = 0$ and $u_j^- = u_j^+ = 0$, $S_B = 0$. So S_B cannot distinguish a_1 and a_j.

Garg [11] generalized score function S_G as shown in Definition 7. However, as shown in Example 5, when $u^- = u^+ = 0$, S_G is always 0 regardless of the values of k_1 or k_2.

Definition 7. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_G is

$$S_G(a) = \frac{\mu^- + \mu^+}{2} + k_1\mu^-(1 - \mu^- - v^-) + k_2\mu^+(1 - \mu^+ - v^+), \text{ and } k_1 + k_2 = 1, k_1, k_2 \geq 0. \tag{11}$$

Example 5. Let $a_{19} = ([0,0], [0.3,0.5])$ and $a_{20} = ([0,0], [0,0])$. This gives us $S_G(a_{19}) = S_G(a_{20}) = 0$ and an S_G that cannot distinguish a_{19} and a_{20}. At the same time, for any two a_i and a_j, when $u_i^- = u_i^+ = 0$ and $u_j^- = u_j^+ = 0$, $S_G = 0$. So S_G cannot distinguish a_1 and a_j.

Gao and Liu [35] proposed a score function S_{GL}, which reflects the degree of support by measuring the difference between the midpoint values of the membership and non-member degrees. By taking into account the effect of the degree of hesitation, it describes the integrated information of the IVIFV as indicated in Definition 8. When $u_a^- + u_a^+ = u_b^- + u_b^+$ and $v_a^- + v_a^+ = v_b^- + v_b^+$, S_{GL} cannot distinguish between the two IVIFVs as shown in Example 6.

Definition 8. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_{GL} is

$$S_{GL}(a) = \frac{\exp\{[u^- + u^+ - v^- - v^+]/2\}}{[1 - u^- - v^- + 1 - u^+ - v^+]/2 + 1}. \tag{12}$$

Example 6. Let $a_{21} = ([0,0.4], [0.4,0.6])$ and $a_{22} = ([0.2,0.2], [0.3,0.7])$. We have $S_{GL}(a_{20}) = S_G(a_{21}) = 0.5699$, which implies that S_{GL} cannot distinguish a_{20} and a_{21}. At the same time, there exist two a_i and a_j, $a_i \in A$ and $a_j \in A$, and $A = ([0.1 + k_1, 0.3 - k_1], [0.4 + k_2, 0.6 - k_2]), \{(k_1, k_2) \in \mathbb{R}^2 | -0.1 \leq k_1 \leq 0.1, -0.2 \leq k_2 \leq 0.1\}$, and S_{GL} cannot distinguish a_i and a_j when value is rounded to 1.

Chen and Tsai [20] proposed a score function S_{CT} as defined in Definition 9 to consider the hesitation, but the shortcoming of this approach is given in Example 7.

Definition 9. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_{CT} is

$$S_{CT}(a) = \frac{\sqrt{u^-} + \sqrt{u^+} + \sqrt{1 - v^-} + \sqrt{1 - v^+}}{2}. \tag{13}$$

Example 7. Let $a_{23} = ([0.1,0.2], [0.0,0.8])$ and $a_{24} = ([0.0,0.1], [0.0,0.2])$. This gives $S_{CT}(a_{23}) = S_{CT}(a_{24}) = 1.1053$, which means that S_{CT} cannot distinguish a_{23} and a_{24}. At the same time, there exist two a_i and a_j, $a_i \in A$ and $a_j \in A$ and $A = ([0.1 + k_1, 0.25 - k_1], [0.12 +$

$k_2, 0.43 - k_2]), \{(k_1, k_2) \in \mathbb{R}^2 | 0 \leq k_1 \leq 0.05, 0 \leq k_2 \leq 0.06\}$, and the score value is rounded to 1 decimal places. S_{CT} cannot distinguish a_i and a_j.

Chen and Deng [15] proposed a score function S_{CD} as presented in Definition 10, which extends S_X and S_{CT} by improving the comparison ability of IVIFVs. As shown in Example 8, however, S_{CD} is also insufficient to distinguish two IVIFVs under specific scenarios.

Definition 10. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_{CD} is

$$S_{CD}(a) = \frac{u^- - v^- + u^+ - v^+}{2} + \frac{\sin\left(u^- \times \frac{\pi}{2}\right) + \sin\left(u^+ \times \frac{\pi}{2}\right) + \sin\left((1 - v^-) \times \frac{\pi}{2}\right) + \sin\left((1 - v^+) \times \frac{\pi}{2}\right)}{2} + 2. \tag{14}$$

Example 8. Let $a_{25} = ([0.42, 0.71], [0.07, 0.08])$ and $a_{26} = ([0.61, 0.71], [0.2, 0.24])$. $S_{CD}(a_{25}) = S_{CD}(a_{26}) = 4.239$ indicates that S_{CD} cannot distinguish a_{25} and a_{26}. At the same time, for any two a_i and a_j, $a_i \in A$, and $a_j \in A$, and $A = ([0.5, 0.5], [0.1 + k, 0.5 - k]), -0.2 \leq k \leq 0.1$, and the score value is rounded to 3 decimal places. S_{CD} cannot distinguish a_i and a_j.

Kumar and Chen [17] proposed a score function S_{KC} as presented in Definition 11. It uses membership and non-member degrees and the degree of hesitation to construct the connection number *CN*. In Eq (15), *A* represents the same degree, *B* represents the degree of difference, and *C* represents the degree of opposites. Example 9 shows where S_{KC} is insufficient.

Definition 11. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_{KC} is

$$\begin{gathered} CN(a) = A + Bi + Cj, \\ S_{KC}(CN(a)) = \begin{cases} (A - C)(1 - B), & \text{if } A \neq C, \\ A(1 + B), & \text{if } A = C, \end{cases} \\ A = u^- + u^+ - u^- u^+ - \left(\frac{u^- v^+ + u^+ v^-}{2}\right), \\ C = v^- + v^+ - v^- v^+ - \left(\frac{u^- v^+ + u^+ v^-}{2}\right), \\ B = 1 - A - C. \end{gathered} \tag{15}$$

Example 9. Let $a_{27} = ([0.49, 0.52], [0.01, 0.48])$ and $a_{28} = ([0.08, 0.7], [0.22, 0.3])$. $S_{KC}(a_{27}) = S_{KC}(a_{28}) = 0.27$, implies that S_{KC} cannot distinguish a_{27} and a_{28}. At the same time, there exist two a_i and a_j, $a_i \in A$ and $a_j \in A$, and $A = ([0.40 + k_1, 0.60 - k_1], [0.16 + k_2, 0.34 - k_2 + 0.3k_1]), \{(k_1, k_2) \in \mathbb{R}^2 | -0.1 \leq k_1 \leq 0.1, -0.05 \leq k_2 \leq 0.05\}$, and the score value is rounded to 3 decimal places. S_{KC} cannot distinguish a_i and a_j.

Chen and Yu [19] proposed a score function S_{CY} based on the expected value of an interval. It not only considers the membership and non-membership relationships of the interval, but also considers their cross relationships as shown in Definition 12. Example 10 shows a case of indistinguishability.

Definition 12. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV. The S_{CY} is shown in Eq (16):

$$S_{CY} = \sqrt{u^- + u^+ + (1 - v^-) + (1 - v^+)} + \left(u^- \times \frac{3}{5} + u^+ \times \frac{2}{5}\right)\left[1 - \left(v^- \times \frac{3}{5} + v^+ \times \frac{2}{5}\right)\right] + \frac{\sqrt{u^-} + \sqrt{1 - v^-}}{2 - (u)^- \times (1 - v^-))} + \frac{\sqrt{u^+} + \sqrt{1 - v^+}}{2 - (u^+ \times (1 - v^+))} + 1. \tag{16}$$

Example 10. Let $a_{29} = ([0.27,0.37],[0.53,0.53])$ and $a_{30} = ([0.23,0.27],[0.18,0.69])$. This gives us $S_{CY}(a_{29}) = S_{CY}(a_{30}) = 3.7546$ and shows that S_{CY} cannot distinguish a_{29} and a_{30}. According to Definition 12, we can see that the S_{CY} lacks theoretical guidance and has unpredictable discrimination blind spots in high-dimensional spaces, leading to indistinguishable IVIFNs.

3. An interval-valued evaluation method (IVEM) based on prospect theory

In this section, prospect theory is introduced to evaluate $[a,b]$ that satisfies $[a,b] \subseteq [0,1]$ $(0 \le a \le b \le 1)$.

3.1. An interval-valued attitude analysis based on prospect theory

The interval $[a,b]$ is equally divided into $k(k \ge 1)$segments, with the length of each segment being $d = (b-a)/k$. The midpoint of each segment is selected to represent its judgment value; that is, we use x_i to represent the evaluation value of the i^{th} segment. If the decision-makers have the same attitude toward any points on $[a,b]$, the weights of each point on $[a,b]$ are equal, whereas any arbitrary method of division is equivalent.

In Figure 2, for example, the interval is equally divided into k segments, where $x_i = \frac{[a+i*d+a+(i-1)*d]}{2}, (i = 1,2,\dots k)$. The weight of x_i is equal, namely $w_i = 1/k$. The weighted average of $[a,b]$ can then be derived to be $\frac{a+b}{2}$. This calculation process is shown in Eq (17).

$$\sum_{i=1}^{k} x_i \times \frac{1}{k} = \sum_{i=1}^{k} \frac{a+i\times\frac{b-a}{k}+a+(i-1)\times\frac{b-a}{k}}{2k}.$$

$$= a - \frac{b-a}{2k} + \sum_{i=1}^{k} \frac{i\times(b-a)}{k^2} = a - \frac{b-a}{2k} + \frac{(b-a)}{k^2}\sum_{i=1}^{k} i = a - \frac{b-a}{2k} + \frac{(b-a)}{k^2}\frac{(k+1)k}{2} = \frac{a+b}{2}. \quad (17)$$

It can be seen from Eq (17) that, when the decision-makers have the same attitude toward each part of the interval, the midpoint of $[a,b]$ becomes the evaluation value, which is inconsistent with the uncertainty of the interval. Therefore, the weights of x_i should not be the same when $[a,b]$ is equally divided.

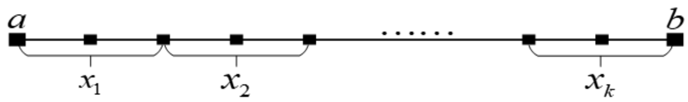

Figure 2. Interval segmentation process.

Prospect theory [29] is well able to reflect the change process of decision-makers' attitudes within $[a,b]$. As shown in Figure 1, the curve of the prospect value function takes the reference point as the base point to separate the loss domain and the gain domain. The membership and non-membership of IVIFVs are subsets of $[0,1]$ and meet $0 \le u_A^+(x) + v_A^+(x) \le 1$. When the endpoint of the membership (non-membership) is greater than the midpoint 0.5, the decision-makers usually think that the degree of support is greater than (less than) the degree of opposition. Thus, in this paper, 0.5 is taken as the reference point for prospect theory to separate the loss domain from the gain domain. The translated prospect theory curve is shown in Figure 3. By taking any point from

$[a, b]$ on the horizontal axis, a vertical line passing through this point intersects the prospect theoretical function curve L. The ordinate value corresponding to the intersection point is thus the prospect value.

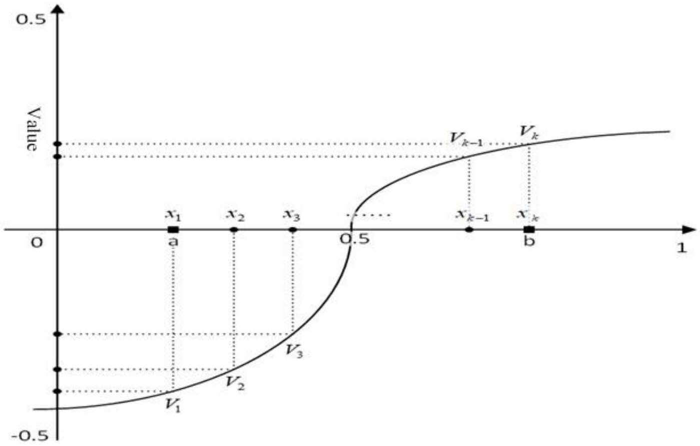

Figure 3. Mapping between the interval and prospect curve.

Figure 3 has $x_1 = a$ and $x_k = b$. The prospect values corresponding to $x_1, x_2, x_3, \cdots, x_k$ are $V_1, V_2, V_3, \cdots, V_k$. It can be seen that for any point x_i, the weight of x_i is set to be $w_i = \frac{V_i}{\sum_{i=1}^{k} V_i}, (i = 1,2, \cdots, k)$. The evaluation value of $[a, b]$ is calculated by the weighted average, defined as $E = \sum_{i=1}^{k} x_i w_i$. It is easy to verify that E is not necessarily equal to $\frac{a+b}{2}$. Therefore, the prospect value of x_i is used to calculate the weight of decision-maker that can overcome the shortcomings of equal weight on $[a, b]$. In addition, the prospect value of x_i can effectively capture the changes in the decision-maker's attitude.

3.2. IVEM based on the prospect value function

Since the interval $[a, b]$ is continuous, Eq (6) cannot be directly used to calculate the prospect value. Drawing inspiration from the concept of a hesitation fuzzy set [37], we fuse the finite number of evaluation values that can be used to express the decision-makers' attitudes. These fused data, derived from interval discretization, are then used to obtain the evaluation value of the interval. There are two common methods to discretize the interval: equal division and random division. The equal division method not only accurately reflects the decision-makers' attitudes but is also simpler to calculate. Since the weights of each point are different, the equal division method will be adopted in this study.

Suppose $[a, b](0 \le a \le b \le 1)$ to be an arbitrary interval. We use Eq (18) to discretize the interval $[a, b]$ into $k(k \ge 2)$ points. The set of discrete points is $X = \{x_1, x_2, x_3, \cdots, x_{k-1}, x_k\}$. When $k = 2$, X contains only the endpoints a and b.

$$x_i = a + (i - 1)\frac{b-a}{k-1}. \tag{18}$$

For any point x_i, as defined in Eq (6), both the reference point and the parameter k need to be considered when calculating its prospect value. When the decision-makers select different reference points, their subjective perception values are different. Thus, the prospect values are also different.

For all discrete points, however, the prospect value ratio of x_i does not change, a claim that will be proved in Section 3.3 For the convenience of calculation, the reference point in this paper is set to 0.5.

Each real number x_i in X is independent of the others. The probability of drawing x_i is equal, that is, $p_i = 1/k$. By substituting p_i, x_i, and $x_* = 0.5$ into Eqs (4)–(6), we can calculate the prospect value of x_i in Eq (19), where parameters $\alpha, \gamma, \beta, \delta,$ and θ have the same meaning as those in Eqs (5) and (6).

$$V(x_i) = \begin{cases} (x_i - x_*)^\alpha \dfrac{\left(\frac{1}{k}\right)^\gamma}{\left(\left(\frac{1}{k}\right)^\gamma + \left(1-\left(\frac{1}{k}\right)\right)^\gamma\right)^{1/\gamma}}, (x_i - x_*) \geq 0, \\ -\theta (x_* - x_i)^\beta \dfrac{\left(\frac{1}{k}\right)^\delta}{\left(\left(\frac{1}{k}\right)^\delta + \left(1-\left(\frac{1}{k}\right)\right)^\delta\right)^{\frac{1}{\delta}}}, (x_i - x_*) < 0. \end{cases} \tag{19}$$

When $a = b$, the interval degenerates into a real number a or b denoted as $[a, a]$ or $[b, b]$. In this case, the equal division of the interval is still the point itself and the probability of drawing x_i is $p_i = 1$. Substituting $x_i = a$ and $p_i = 1$ into Eqs (4)–(6), the prospect value of point a or b can be obtained using Eq (20):

$$v(a) = \begin{cases} (x_i - x_*)^\alpha, (x_i - x_*) \geq 0, \\ -\theta (x_* - x_i)^\beta, (x_i - x_*) < 0, \end{cases} \tag{20}$$

where x_* is the reference point. The meanings of α and β are explained in Eq (5). Combining Eqs (19) and (20), Eq (21) gives the prospect value of any discrete point on $[a, b]$.

$$V(x_i) = \begin{cases} (x_i - x_*)^\alpha \dfrac{\left(\frac{1}{k}\right)^\gamma}{\left(\left(\frac{1}{k}\right)^\gamma + \left(1-\left(\frac{1}{k}\right)\right)^\gamma\right)^{1/\gamma}}, (x_i - x_*) \geq 0, \\ -\theta (x_* - x_i)^\beta \dfrac{\left(\frac{1}{k}\right)^\delta}{\left(\left(\frac{1}{k}\right)^\delta + \left(1-\left(\frac{1}{k}\right)\right)^\delta\right)^{\frac{1}{\delta}}}, (x_i - x_*) < 0, \\ (x_i - x_*)^\alpha, (x_i - x_*) \geq 0 \text{ and } a = b = x_i, \\ -\theta (x_* - x_i)^\beta, (x_i - x_*) < 0 \text{ and } a = b = x_i. \end{cases} \tag{21}$$

As shown in Eq (21), the parameter k directly influences the prospect value of x_i. From the intuitive perspective of the decision-makers, the smaller the discrete number is, the smaller the prospect value discrimination of x_i becomes, while the value of k depends on the actual situation. In Section 4.3.2, we discuss whether or not k can take the value greater than or equal to 20.

As illustrated in Figure 3, the interval $[a, b]$ lies within $[0,1]$. The prospect value calculated by Eq (21) increases monotonously, which means that it cannot directly reflect the attitude of the decision-makers. Instead, by taking the *x*-axis as the symmetric axis, the gain part of curve L is mirrored below $y = 0$ as shown in Figure 4. The corresponding formula is given in Eq (22). The curve S only indirectly reflects the attitude of the decision-makers because part of S is the symmetry of curve L.

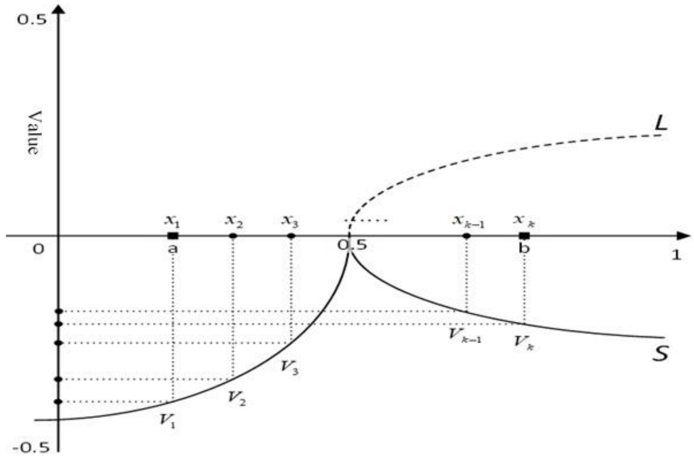

Figure 4. Curve *S* of transforming prospect theory.

$$V(x_i) = \begin{cases} 0-(x_i-x_*)^{\alpha}\dfrac{\left(\frac{1}{k}\right)^{\gamma}}{\left(\left(\frac{1}{k}\right)^{\gamma}+\left(1-\left(\frac{1}{k}\right)\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, (x_i-x_*)\geq 0 \text{ and } a<x_i\leq b, \\ -\theta(x_*-x_i)^{\beta}\dfrac{\left(\frac{1}{k}\right)^{\delta}}{\left(\left(\frac{1}{k}\right)^{\delta}+\left(1-\left(\frac{1}{k}\right)\right)^{\delta}\right)^{\frac{1}{\delta}}}, (x_i-x_*)<0 \text{ and } a\leq x_i<b, \\ 0-(x_i-x_*)^{\alpha}, (x_i-x_*)\geq 0 \text{ and } a=b=x_i, \\ -\theta(x_*-x_i)^{\beta}, (x_i-x_*)<0 \text{ and } a=b=x_i, \end{cases} \tag{22}$$

Similarly, in Eq (22), x_* represents the reference point and $1/k$ represents the probability of each point. The prospect value $V(x_\mathrm{i})$ of x_i is calculated by Eq (22). It can be seen from Figure 4 that all prospect values are negative. We introduce $Weight(x_\mathrm{i})$ as the weight of x_i in Definition 13 to normalize the prospect values.

Definition 13. The weight of $x_i (i=1,2,\cdots,k)$ on interval $[a,b]$ is denoted by $Weight(x_\mathrm{i})$:

$$Weight(x_i) = \begin{cases} \dfrac{V(x_i)-\min\limits_j\left(V(x_j)\right)}{\sum_{i=1}^{k}\left[V(x_i)-\min\limits_j\left(V(x_j)\right)\right]}, \sum_{i=1}^{k}\left[V(x_i)-\min\limits_j\left(V(x_j)\right)\right]\neq 0, \\ \dfrac{1}{k}, \sum_{i=1}^{k}\left[V(x_i)-\min\limits_j\left(V(x_j)\right)\right]=0, \end{cases} \tag{23}$$

where $\min\limits_j\left(V(x_j)\right)$ represents the minimum prospect value of all discrete points and $V(x_j)$ represents the prospect value of x_j. It can be seen that $Weight(x_i)\geq 0$ and $\sum_{i=1}^{k}Weight(x_i)=1$.

According to Eq (23), the weight of each point can be calculated by using the transformed prospect value function and the evaluation value of $[a,b]$ can be obtained by taking the weighted average given by Eq (24). In this paper, the weighted average of the discrete points on $[a,b]$ is regarded as the evaluation value $InValue$ of $[a,b]$:

$$InValue = \sum_{i=1}^{k} x_i * Weight(x_i). \tag{24}$$

When $\frac{a+b}{2}=x_*=0.5$ for any $[a,b]$, after the discretization by Eq (18), the weight of each discrete point can be obtained by Eq (23). It is found that the closer x_i locates to the reference point

x_*, the greater weight the point obtains. The change in the decision-maker's psychological state is consistent with prospect theory. When $\frac{a+b}{2} \neq 0.5$, however, the weight of the endpoint a or b of $[a, b]$ is the largest, making it difficult to map the change process of the decision-maker's attitude. Therefore, it is necessary to translate $[a, b]$ to make the midpoint of the interval equal 0.5. In this sense, the decision-maker has a greater weight near the midpoint of the interval. The length of translation, $MoveLength$, is defined in Eq (25).

$$MoveLength = \frac{1}{2} - \frac{a+b}{2}. \tag{25}$$

If the $MoveLength$ is less than (greater than) 0, $[a, b]$ is shifted to the left (right) to get a new interval $[\tilde{a}, \tilde{b}]$. With Eq (24), the evaluation value $InValue$ of $[\tilde{a}, \tilde{b}]$ can be obtained and with Eq (26), the evaluation value $EvValue$ of $[a, b]$ can be obtained:

$$EvValue = IVEM([a, b]) = InValue - MoveLength. \tag{26}$$

In summary, the calculation of the $EvValue$ of $[a, b]$ includes six steps: (1) $[a, b]$ is converted to $[\tilde{a}, \tilde{b}]$ by using Eq (25), where $\tilde{a}$ and $\tilde{b}$ satisfy $\frac{\tilde{a}+\tilde{b}}{2} = 0.5$; (2) $[\tilde{a}, \tilde{b}]$ is discretized into set $X = \{x_\mathrm{i} | i = 1,2, \ldots, k\}$ by Eq (18); (3) the prospect value $V(x_\mathrm{i})$ of x_i is calculated by Eq (22); (4) the weight $Weight(x_i)$ of x_i is derived by Eq (23); (5) the $InValue$ is calculated by Eq (24); and (6) the $EvValue$ of $[a, b]$ is obtained by Eq (26).

3.3. IVIFS-SF and its properties

For an arbitrary IVIFV $a = ([u^-, u^+], [v^-, v^+])$, $0 \leq u^- \leq u^+$, $0 \leq v^- \leq v^+$ and $u^+ + v^+ \leq 1$ are satisfied. The membership, non-membership, and hesitation intervals of a are $u_a = [u^-, u^+]$, $v_a = [v^-, v^+]$, and $\rho_a = [1 - u^+ - v^+, 1 - u^- - v^-]$, respectively. According to the IVEM, u_a and v_a can be transformed into two crisp numbers: u and v. The evaluation value u of u_a is

$$u = \sum_{i=1}^{k} x_i * Weight(x_i) + \frac{u^- + u^+ - 1}{2}. \tag{27}$$

Equation (27) first shifts u_a by $\frac{(u^- + u^+) - 1}{2}$ to get the interval $\left[u^- - \frac{(u^- + u^+) - 1}{2}, u^+ - \frac{(u^- + u^+) - 1}{2})\right]$. Equation (18) is then used to get the k discrete points $x_\mathrm{i} (i = 1,2, \cdots, k)$. The weight $Weight(x_\mathrm{i})$ of x_i is obtained according to Eq (23).

Similarly, the evaluation value v of v_a can be derived. v_a is translated by $\frac{(v^- + v^+) - 1}{2}$ to the interval $\left[v^- - \frac{(v^- + v^+) - 1}{2}, v^+ - \frac{(v^- + v^+) - 1}{2})\right]$ before applying Eq (18). The calculation of v is

$$v = \sum_{i=1}^{k} x_i * Weight(x_i) + \frac{v^- + v^+ - 1}{2}. \tag{28}$$

Then, u and v can be obtained by calculating the membership interval and non-membership interval of the IVIFV through Eqs (27) and (28). As shown in Theorem 1, we have $0 \leq u + v \leq 1$.

Theorem 1. For an arbitrary $a = ([u^-, u^+], [v^-, v^+])$ of the IVIFV, the evaluation values of its membership and non-membership intervals are u and v that satisfy $0 \leq u + v \leq 1$.

Proof. According to the interval value evaluation Eq (26), we have $u \geq 0$ and $v \geq 0$. If both $0 \leq u \leq u^+$ and $0 \leq v \leq v^+$ hold, Theorem 1 can be proved. Below, we will first prove $u \leq u^+$.

To calculate the evaluation value u of $[u^-, u^+]$, the translation is first calculated. From Eq (18), the interval after translation of $[u^-, u^+]$ becomes $\left[u^- + \frac{1-(u^-+u^+)}{2}, u^+ + \frac{1-(u^-+u^+)}{2}\right]$. The k discrete points are $(x_1, x_2, x_3, \cdots, x_{k-1}, x_k)$ and meet $u^- + \frac{1-(u^-+u^+)}{2} = x_1 \leq x_2 \leq \cdots, x_{k-1} \leq x_k = u^+ + \frac{1-(u^-+u^+)}{2}$. Therefore, we get

$$\sum_{i=1}^{k} x_i * Weight(x_i) \leq \sum_{i=1}^{k} x_k * Weight(x_i).$$

Given the weight of each point $Weight(x_i) > 0$ and $\sum_{i=1}^{k} Weight(x_i) = 1$, we have

$$\sum_{i=1}^{k} x_i * Weight(x_i) \leq \sum_{i=1}^{k} x_k * Weight(x_i) \leq x_k * \sum_{i=1}^{k} Weight(x_i) = x_k = u^+ - \frac{u^- + u^+ - 1}{2}.$$

Substituting $x_k = u^+ - \frac{u^-+u^+-1}{2}$ into Eq (27) leads to

$$u = \sum_{i=1}^{k} x_i * Weight(x_i) + \frac{u^- + u^+ - 1}{2} \leq x_k + \frac{u^- + u^+ - 1}{2}.$$

By rearranging the above equation, we have

$$u \leq \left(u^+ - \frac{u^- + u^+ - 1}{2}\right) + \frac{u^- + u^+ - 1}{2} = u^+.$$

That is to say, $u \leq u^+$ is established and $0 \leq u \leq u^+$ holds.

Using the same method, we can prove $0 \leq v \leq v^+$. Thus, it is easy to see $0 \leq u + v \leq u^+ + v^+$. Since $0 \leq u^+ + v^+ \leq 1$ and $0 \leq u + v \leq 1$ are now established, Theorem 1 is proved.

□

Remark 1. According to Theorem 1, using the IVEM, the $EvValue$ of the membership and non-membership intervals of the IVIFV can be obtained, which satisfy $0 \leq u + v \leq 1$. By defining the hesitation degree as $\rho = 1 - u - v$, the corresponding IFV can be constructed. Therefore, the IVEM based on prospect theory can not only reflect the psychological change process of the decision-makers, but also transform the IVIFV into an IFV.

Corollary 1. For an arbitrary $a = ([u^-, u^+], [v^-, v^+])$ of the IVIFV, if we have $u_a = [u^-, u^+]$ and $v_a = [v^-, v^+]$, $IVEM(u_a) = u$ and $IVEM(v_a) = v$ can be obtained by Eq (26). $a_i = (u, v)$ is an IFV, where u and v are the membership and non-membership of a_i, respectively, and the hesitation degree is $\rho = 1 - u - v$.

Remark 2. For an arbitrary $a = ([u^-, u^+], [v^-, v^+])$ of the IVIFV, the evaluation values of its membership, non-membership, and hesitation are u, v, and ρ_{IV}, respectively. As can be seen in Example 11, $u + v + \rho_{IV}$ may be greater than 1.

Example 11. For $a_{31} = ([0.2,0.3], [0.4,0.5])$, it is possible to get $u_{a_{31}} = [0.2,0.3]$, $v_{a_{31}} = [0.4,0.5]$, and $\rho_{a_{31}} = [0.2,0.4]$. If the results are rounded to 4 decimal places, $IVEM(u_{a_{31}}) = u =$

0.2578, $IVEM(v_{a_{31}}) = v = 0.4578$, and $IVEM(\rho_{a_{31}}) = \rho_{IV} = 0.3155$ are obtained. We then have $u + v + \rho_{IV} = 1.0311 > 1$ and $\rho = 1 - u - v = 0.1844 < \rho_{IV}$.

Example 11 demonstrates that the IFV can be constructed by using the evaluation values u and v. As Wang suggested [36], this method adjusts the degree of hesitation and enables the construction of a new IFV.

According to Corollary 1, any IVIFV can be transformed into a new IFV score function. Consequently, the evaluation value of the IVIFV can be used to develop a new SF. In other words, a new SF can be developed by using the membership and non-membership of the IFV. In the practical decision-making process, the decision-makers typically prefer options that have the highest positive part, and the lowest negative and hesitation parts. If the deviation between the membership and non-membership becomes greater and the hesitation value becomes smaller, the corresponding alternative is better. According to the above analysis, an IVIFS-SF, S_{NEW}, based on the sine function and cosine function is designed and defined in Definition 14, which is affected by the hesitation.

Definition 14. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFV, and u and v are the evaluation values of the interval membership and non-membership obtained by Eqs (27) and (28). The IVIFS-SF of a is defined in Eq (29):

$$S_{NEW} = \left[\sin\left((u - v)\frac{\pi}{2}\right) + 1\right]\left[\cos\left((1 - u - v)\frac{\pi}{2}\right) + 3\right] - 4, \tag{29}$$

where $u - v$ is the deviation between the evaluation values of interval membership and non-membership. $1 - u - v$ represents the degree of hesitation after evaluation. When $u - v$ equals 0, as the degree of hesitation decreases, the score increases, indicating a better alternative. For convenience, IVIFS-SF is regarded as S_{NEW} in this paper. The calculation process is illustrated in Example 12.

Example 12. Suppose $a_{32} = ([0.25, 0.3], [0.4, 0.6])$. First, the membership degree $[0.25, 0.3]$ of a_5 is translated to $[0.475, 0.525]$ by Eq (25). It is then discretized by Eq (18). Equation (23) is used to calculate the weight of each discrete point and Eq (24) is used to calculate the weighted average value of the discrete points. Finally, we use Eq (27) to compute the evaluation value, which is $u = 0.2789$ for $[0.25, 0.3]$. Similarly, the evaluation value of the non-membership of a_{25} is $v = 0.5155$. $S_{New} = -1.4858$ of a_{32} can be obtained by Eq (29).

For an arbitrary $a = ([u^-, u^+], [v^-, v^+])$ of the IVIFV, it can be seen from Eq (29) that the minimum value of S_{NEW} is -4 and the maximum value is 4. More specifically, four properties of S_{NEW} can be given.

Property 1. $-4 \le S_{NEW}(a) \le 4$.

Property 2. If $a = ([0,0], [1,1])$, then $u = 0, v = 1$, and $S_{NEW}(a) = -4$.

Property 3. If $a = ([1,1], [0,0])$, then $u = 1, v = 0$, and $S_{NEW}(a) = 4$.

Property 4. If $a = ([0.5,0.5], [0.5,0.5])$, then $u = 0.5, v = 0.5$, and $S_{NEW}(a) = 0$.

Using the IVEM, the evaluations of $[0,0]$, $[1,1]$, and $[0.5,0.5]$ are 0, 1, and 0.5, respectively. For Properties 2–4, the corresponding score values can be obtained by substituting the evaluation of u and v into Eq (29). These results are easy to deduce and prove, so their proofs have been omitted. The proof of Property 1 is as follows.

Proof. The partial derivative of S_{NEW} with respect to u is calculated to prove the monotonicity of S_{NEW} for the evaluation value u of interval membership.

$$\frac{\partial S_{NEW}(a)}{\partial u}=\frac{\partial\left[\sin\left(\frac{\pi(u-v)}{2}\right)+1\right]\left[\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\right]}{\partial u}$$
$$=\frac{\pi}{2}\left[\cos\left(\frac{\pi(u-v)}{2}\right)\left(\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\right)+\left(\sin\left(\frac{\pi(u-v)}{2}\right)+1\right)\sin\left(\frac{\pi(1-u-v)}{2}\right)\right].$$

According to Theorem 1, we have $0\le u\le 1$, $0\le v\le 1$, and $0\le u+v\le 1$, which indicates $\cos\left(\frac{\pi(u-v)}{2}\right)\ge 0,\left(\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\right)\ge 0$, $\left(\sin\left(\frac{\pi(u-v)}{2}\right)+1\right)\ge 0$, and $\sin\left(\frac{\pi(1-u-v)}{2}\right)\ge 0$.

Hence, we have

$$\frac{\partial S_{NEW}(a)}{\partial u}=\frac{\pi}{2}\left[\cos\left(\frac{\pi(u-v)}{2}\right)\left(\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\right)+\left(\sin\left(\frac{\pi(u-v)}{2}\right)+1\right)\sin\left(\frac{\pi(1-u-v)}{2}\right)\right]\ge 0.$$

Next, we calculate the partial derivative of $S_{NEW}(a)$ with respect to the non-membership degree evaluation value v. The following results can be obtained:

$$\frac{\partial S_{NEW}(a)}{\partial v}=\frac{\partial\left[\sin\left(\frac{\pi(u-v)}{2}\right)+1\right]\left[\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\right]}{\partial v}$$
$$=-\frac{\pi}{2}\left[\cos\left(\frac{\pi(u-v)}{2}\right)\left(\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\right)-\left(1+\sin\left(\frac{\pi(u-v)}{2}\right)\right)\cos\left(\frac{\pi(u+v)}{2}\right)\right]$$
$$=-\frac{\pi}{2}[\cos\left(\frac{\pi(u-v)}{2}\right)\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\cos\left(\frac{\pi(u-v)}{2}\right)-\cos\left(\frac{\pi(u+v)}{2}\right)$$
$$-\sin\left(\frac{\pi(u-v)}{2}\right)\cos\left(\frac{\pi(u+v)}{2}\right)]$$
$$=-\frac{\pi}{2}\left[\sin(\pi v)+3\cos\left(\frac{\pi(u-v)}{2}\right)-\cos\left(\frac{\pi(u+v)}{2}\right)\right]$$
$$=-\frac{\pi}{2}\left[\sin(\pi v)+2\cos\left(\frac{\pi(u-v)}{2}\right)+\cos\left(\frac{\pi(u-v)}{2}\right)-\cos\left(\frac{\pi(u+v)}{2}\right)\right]$$
$$=-\frac{\pi}{2}\left[\sin(\pi v)+2\cos\left(\frac{\pi(u-v)}{2}\right)+2\sin\frac{\pi v}{2}\sin\frac{\pi u}{2}\right].$$

According to Theorem 1, we have $0\le u\le 1$, $0\le v\le 1$, and $0\le u+v\le 1$, which indicates $\sin(\pi v)\ge 0$, $2\sin\frac{\pi v}{2}\sin\frac{\pi u}{2}\ge 0$, and $-1\le u-v\le 1$. From that it is possible to get $2\cos\left(\frac{\pi(u-v)}{2}\right)\ge 0$.

By applying Theorem 1 again, each sub-item in the above equation is greater than 0. Thus, the partial derivative of $S_{NEW}(a)$ with respect to v is less than 0, that is,

$$\frac{\partial S_{NEW}(a)}{\partial v}$$
$$=-\frac{\pi}{2}\left[\cos\left(\frac{\pi(u-v)}{2}\right)\left(\cos\left(\frac{\pi(1-u-v)}{2}\right)+3\right)-\left(\sin\left(\frac{\pi(v-u)}{2}\right)+1\right)\sin\left(\frac{\pi(1-u-v)}{2}\right)\right]\le 0.$$

Given the monotonicity of $S_{NEW}(a)$ with respect to the evaluation values u and v, Definition 1 and Theorem 1, when $a=([0,0],[1,1])$, $u=0$, and $v=1$, $S_{NEW}(a)$ reaches the minimum value of -4. When $a=([1,1],[0,0])$, $u=0$, and $v=1$, $S_{NEW}(a)$ reaches the maximum value of 4.

Thus, for an arbitrary a, the property $-4\le S_{NEW}(a)\le 4$ holds.

□

4. Comparison analysis and sensitivity analysis of the IVIFS-SF

Assessment methods of SFs are introduced in Section 4.1. The IVISF-SF is compared with other SFs in Section 4.2. Sensitivity analysis of the IVIFS-SF is presented in Section 4.3. For the prospect theory that we will use in this paper, Kahneman and Tversky [30] have obtained the values of the empirical parameters ($\alpha = 0.88, \gamma = 0.61,\ \beta = 0.88, \theta = 2.25,$ and $\delta = 0.69$).

4.1. Assessment methods for SFs

To assist researchers in selecting or developing more suitable SFs, we will construct a method for evaluating them. This approach includes introducing pass rate and variance as criteria for assessing SFs. Two synthetic datasets are generated for comparative analysis.

4.1.1. SF pass rate

Each of the SFs was used to calculate the scores of the IVIFVs in the dataset. The ratio between the number of IVIFVs that can be distinguished from each other and the total number of IVIFVs in the dataset is defined as the pass rate P, as shown in Eq (30).

$$P = \frac{diffNumber(IVIFVs)}{Total(IVIFVs)}, \tag{30}$$

where $diffNumber(IVIFVs)$ is the number of IVIFVs with different scores in the dataset and $Total(IVIFVs)$ is the total number of IVIFVs in the dataset. For some cases, IVIFVs cannot be counted into $diffNumber(IVIFVs)$ and the scores cannot be calculated by the SF because the denominator is 0 [38].

4.1.2. SF variance

For different score functions, when the pass rate is the same, the greater the difference of scores in the actual application, the easier it is for decision-makers to distinguish. Therefore, on the basis of the pass rate, we introduce the variance of the score function to further measure the difference between score functions. Considering the different ranges of the SFs, the score values were normalized in Eq (31), where $S(a_i)$ is the score of any IVIFV a_i in the dataset, and $Score_{Max}$ and $Score_{Min}$ are the maximum and minimum scores in the dataset:

$$Score_{Norm}^{i} = \frac{S(a_i)\text{-}Score_{Min}}{Score_{Max} - Score_{Min}}. \tag{31}$$

Based on $Score_{Norm}^{i}$, the expected values and the variances of all the IVIFVs' scores in the dataset were calculated by Eqs (32) and (33).

$$\overline{Score_{Norm}} = \frac{\sum_{i=1}^{n} Score_{Norm}^{i}}{n}, \tag{32}$$

$$Variance = \frac{\sum_{i=1}^{n} \left(Score_{Norm}^{i} - \overline{Score_{Norm}}\right)^2}{n}, \tag{33}$$

where n is the number of IVIFVs in the dataset. Some IVIFVs were excluded from the variance calculation, as their scores cannot be computed by the SF due to a denominator of 0.

4.1.3. Synthetic dataset

IVIFVs rounded to two decimal places are widely used in the actual decision-making process. Therefore, before calculating the pass rate and variance of the SF, we constructed an IVIFV dataset with two decimal places denoted as IVIFDataSet1, such as $a_{33} = ([0.35, 0.45], [0.15, 0.55])$. This gives us a dataset with 4,598,126 IVIFVs. The construction process adopted the exhaustive attack method and the algorithm was implemented in the Python programming language. The pseudo-code is shown in Algorithm 1, where step = 0.01 ranges from 0 to 1. This process ensures that both the membership and non-membership are exhausted and no duplication occurs.

Algorithm 1. Generate IVIFDataSet1.

Input: step=0.01
Output: IVIFDataSet1
Create DataFrame IVIFDataSet1
for $u^- = 0 \to u^- = 1$ **do**
    $u^- += step$
    for $u^+ = u^- \to u^+ = 1$ **do**
        $u^+ += step$
        for $v^- = 0 \to v^- = 1$ **do**
            $v^- += step$
            for $v^+ = v^- \to v^+ = 1$ **do**
                $v^+ += step$
                if $u^+ + v^+ \le 1$ **then**
                    IVIFDataSet1 append (u^-, u^+, v^-, v^+)
                end if
            end for
        end for
    end for
end for
return IVIFDataSet1

To systematically evaluate the pass rate and variance of different score functions, we constructed a random dataset named IVIFDataSet2. This dataset comprises 10 million randomly generated IVIFVs, generated using Algorithm 2. The algorithm operates through the following core steps: (1) Generate a batch data (BatchData) containing 1.5 million quadruples (a, b, c, d) with each element satisfying $0 \le a, b, c, d \le 1$; (2) filter the BatchData to extract quadruples that meet the IVIFVs definition, forming a valid batch data (VBatchData); and (3) select IVIFVs from VBatchData that are not already present in IVIFDataSet2 and add them to the dataset. This process iterates until IVIFDataSet2 reaches its target size of 10 million IVIFVs. Algorithm 2 is shown as follows.

Algorithm 2. Randomly generate IVIFDataSet2.

Input: TargetNumber = 10,000,000.

Output: IVIFDataSet2.

```
GenerateNumber = 0, BatchData=1,500,000, IVIFDataSet2=Null;
While (GenerateNumber < TargetNumber) do
    Randomly generate BatchData of [a,b,c,d] with 7 decimal places retained;
    Get VBatchData that satisfy the constraints of IVIFVs from BatchData;
    for (i=1; i <= size(VBatchData); i++) do
        if (VBatchData[i] not in IVIFDataSet2 && GenerateNumber < TargetNumber) then
            Add VBatchData[i] to IVIFDataSet2;
            GenerateNumber++;
            end if
        end if
    end for
end while
return IVIFDataSet2;
```

4.2. Comparison analysis of the IVIFS-SF

To validate the effectiveness of the proposed score function in this paper, this section presents a comparative analysis using two synthetic datasets and examples present in Section 2.3. The synthetic datasets are used to evaluate the general performance and the examples are used to test the extreme robustness.

4.2.1. Comparison analysis on the synthetic dataset

The pass rates and variances of each score function under three synthetic datasets are shown in Figure 5. The greater the variances are, the higher the dispersion of the score values becomes and the better the discrimination of the SFs perform. The results demonstrate that the proposed score function performed best compared with $S_X, S_{WC}, S_{WL}, S_B, S_G, S_{GL}, S_{CT}, S_{CD}, S_{KC}$, and S_{CY}.

As can be seen from Figure 5, the score function proposed in this paper is reliable and effective. The subgraphs (a) and (b) in Figure 5 indicate that S_{NEW} can distinguish more IVIFVs than $S_X, S_{WC}, S_{WL}, S_B, S_G, S_{GL}, S_{CT}, S_{CD}, S_{KC}$, and S_{CY}, reducing the number of indistinguishable IVIFVs from 68,972 to 32,186 on IVIFDataSet1 and from 370,000 to 160,000 on IVIFDataSet2. From the subgraphs (c) and (d) in Figure 5, S_{NEW} achieves enhanced discriminative capability than other SFs with higher dispersion of the score values, indicating superior separation performance. From the comparative experimental results on the synthetic dataset, the proposed score function is reliable and has strong discrimination ability.

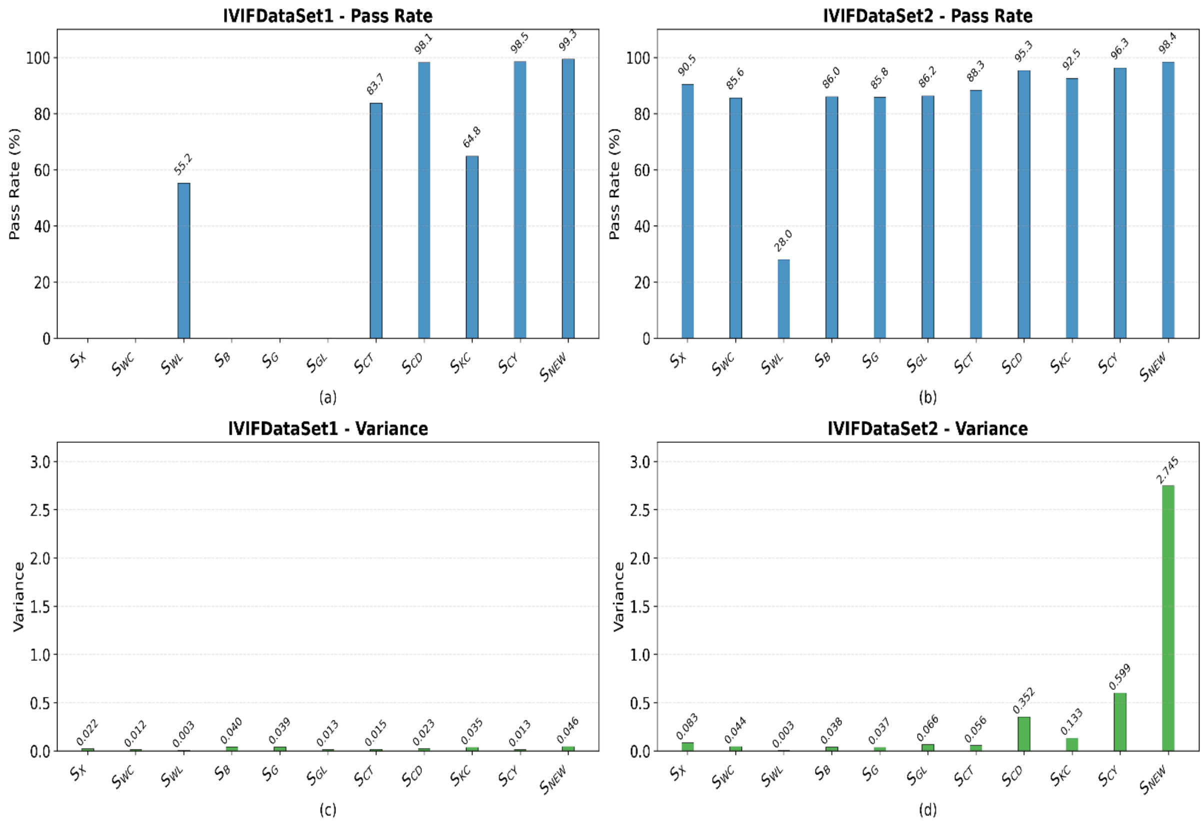

Figure 5. Comparison results on synthetic datasets.

4.2.2. Comparison analysis on examples

It can be seen in Examples 1 to 10 that there are scenarios that cannot be distinguished by the SFs given in Section 2.3. This is primarily due to the loss of fuzzy information in the process of converting IVIFVs, which can lead to the failure of distinguishing two obviously different IVIFVs. At this point in our test, $S_X, S_{WC}, S_{WL}, S_B, S_G, S_{GL}, S_{CT}, S_{CD}, S_{KC}, S_{CY,}$ and S_{NEW} were used for computing all of the 10 groups of IVIFVs given in Section 2.3. The results are shown in Table 1.

As show in Table 1, S_{NEW} is able to distinguish all 10 groups of IVIFVs. S_X fails to distinguish two groups ($a_{11}, a_{12},$ and a_{21}, a_{22}), while S_{WC}, S_B, S_G, and S_{GL} fail to distinguish two groups. The remaining SFs present the indistinguishability of one group. Thus, in terms of distinguishability, S_{NEW} outperforms the other compared SFs.

Table 1. Comparison of evaluation methods and related score functions.

$IVIFVs$	S_X	S_{WC}	S_{WL}	S_B	S_G	S_{GL}	S_{CT}	S_{CD}	S_{KC}	S_{CY}	S_{NEW}
a_{11} a_{12}	$a_{11}=a_{12}$	$a_{11}=a_{12}$	$a_{11}<a_{12}$	$a_{11}>a_{12}$	$a_{11}>a_{12}$	$a_{11}=a_{12}$	$a_{11}>a_{12}$	$a_{11}<a_{12}$	$a_{11}<a_{12}$	$a_{11}>a_{12}$	$a_{11}<a_{12}$
a_{13} a_{14}	$a_{13}>a_{14}$	$a_{13}=a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$	$a_{13}>a_{14}$
a_{15} a_{16}	$a_{15}>a_{16}$	$a_{15}>a_{16}$	$a_{15}=a_{16}$	$a_{15}<a_{16}$	$a_{15}<a_{16}$	$a_{15}<a_{16}$	$a_{15}<a_{16}$	$a_{15}<a_{16}$	$a_{15}>a_{16}$	$a_{15}<a_{16}$	$a_{15}<a_{16}$
a_{17} a_{18}	$a_{17}<a_{18}$	$a_{17}<a_{18}$	$a_{17}<a_{18}$	$a_{17}=a_{18}$	$a_{17}=a_{18}$	$a_{17}<a_{18}$	$a_{17}<a_{18}$	$a_{17}<a_{18}$	$a_{17}<a_{18}$	$a_{17}<a_{18}$	$a_{17}<a_{18}$
a_{19} a_{20}	$a_{19}<a_{20}$	$a_{19}<a_{20}$	$a_{19}<a_{20}$	$a_{19}=a_{20}$	$a_{19}=a_{20}$	$a_{19}<a_{20}$	$a_{19}<a_{20}$	$a_{19}<a_{20}$	$a_{19}<a_{20}$	$a_{19}<a_{20}$	$a_{19}<a_{20}$
a_{21} a_{22}	$a_{21}=a_{22}$	$a_{21}<a_{22}$	$a_{21}>a_{22}$	$a_{21}<a_{22}$	$a_{21}<a_{22}$	$a_{21}=a_{22}$	$a_{21}<a_{22}$	$a_{21}>a_{22}$	$a_{21}>a_{22}$	$a_{21}<a_{22}$	$a_{21}>a_{22}$
a_{23} a_{24}	$a_{23}<a_{24}$	$a_{23}<a_{24}$	$a_{23}<a_{24}$	$a_{23}>a_{24}$	$a_{23}>a_{24}$	$a_{23}>a_{24}$	$a_{23}=a_{24}$	$a_{23}<a_{24}$	$a_{23}<a_{24}$	$a_{23}<a_{24}$	$a_{23}<a_{24}$
a_{25} a_{26}	$a_{25}>a_{26}$	$a_{25}<a_{26}$	$a_{25}>a_{26}$	$a_{25}>a_{26}$	$a_{25}>a_{26}$	$a_{25}<a_{26}$	$a_{25}>a_{26}$	$a_{25}=a_{26}$	$a_{25}>a_{26}$	$a_{25}>a_{26}$	$a_{25}>a_{26}$
a_{27} a_{28}	$a_{27}>a_{28}$	$a_{27}>a_{28}$	$a_{27}>a_{28}$	$a_{27}>a_{28}$	$a_{27}>a_{28}$	$a_{27}>a_{28}$	$a_{27}>a_{28}$	$a_{27}>a_{28}$	$a_{27}=a_{28}$	$a_{27}>a_{28}$	$a_{27}>a_{28}$
a_{29} a_{30}	$a_{29}<a_{30}$	$a_{29}>a_{30}$	$a_{29}<a_{30}$	$a_{29}>a_{30}$	$a_{29}>a_{30}$	$a_{29}>a_{30}$	$a_{29}>a_{30}$	$a_{29}>a_{30}$	$a_{29}>a_{30}$	$a_{29}=a_{30}$	$a_{29}>a_{30}$

4.3. Sensitivity analysis of the IVIFS-SF

4.3.1. Sensitivity analysis of the IVEM

For the prospect theory that we will use in this paper, Kahneman and Tversky [30] have obtained the values of the empirical parameters ($\alpha = 0.88, \gamma = 0.61,\ \beta = 0.88, \theta = 2.25,$ and $\delta = 0.69$). We will then discuss the reference point and the discrete point numbers of the IVEM.

(1) Impact on the reference point

When the reference point shifts, the impact of prospect value of the interval, weight of the discretized points, and $EvValue$ of the interval will be analyzed, respectively.

We compare how the selection of different reference points influences the change in prospect value of the discretized points. Figure 6 presents one of the experimental results, where the randomly generated data was $c_1 = [0.2, 0.6]$ and was discretized into 21 points.

In Figure 6, the value lines for all of the reference points are overlapped, which means that when the reference point ranges from 0 to 0.9, the prospect value of each discrete point is unchanged. Therefore, the changes in the reference point do not affect the prospect value.

Using the same setting, we compare the change in weight of the discretized points when different reference points are selected. The corresponding weight change of each discrete point in the interval $[0.2, 0.6]$ is shown in Figure 7.

Similarly, in Figure 7, the weight change lines of the reference points are overlapped, which means that when the reference point ranges from 0 to 0.9, the weight of each discrete point is unchanged. Therefore, the change of reference point does not affect the weight.

We consider an arbitrary interval $[a, b]$ and compare the changing status of the $EvValue$ of each interval when selecting different reference points. An experiment was selected with intervals $C_1 = [0.2, 0.6]$, $C_2 = [0.3, 0.5]$, $C_3 = [0.15, 0.8]$, $C_4 = [0.3, 0.7]$, and $C_5 = [0.1, 0.85]$, which were discretized into 21 points. The results are shown in Figure 8.

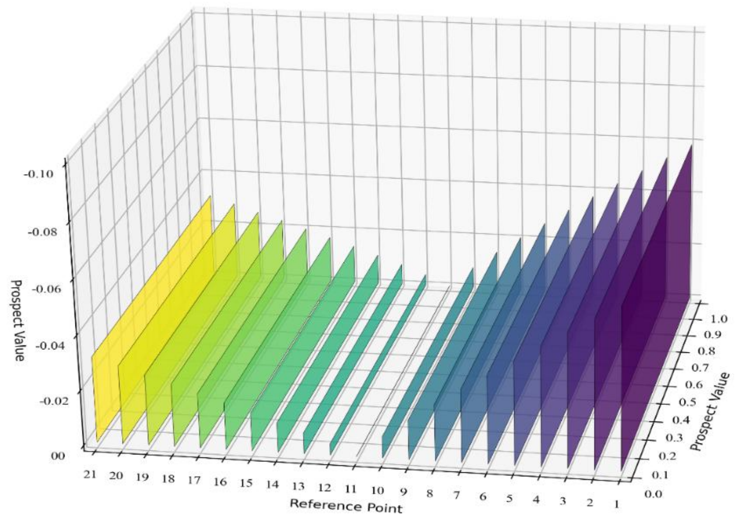

Figure 6. Change in prospect value for different reference points.

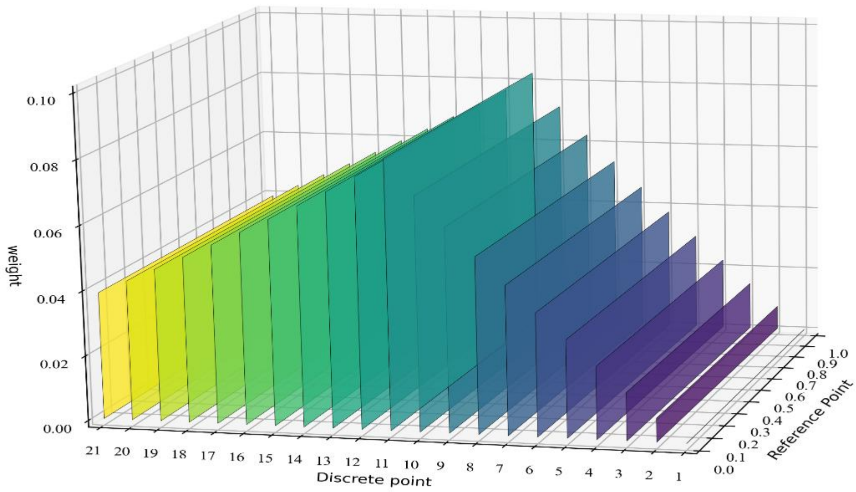

Figure 7. Change in weight for different reference points.

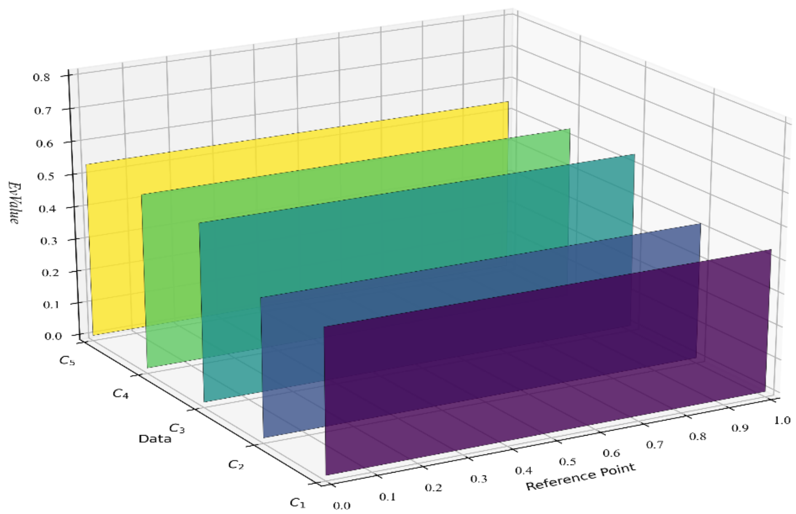

Figure 8. Change in $EvValue$ for different reference points.

From Figure 8, the change in the reference point has no impact on the interval $EvValue$. Thus, the change in the reference point does not affect the prospect value, the interval $EvValue$, or the weight of the discrete point.

(2) Impact on the prospect value of discrete points

We took a random interval [0.4,0.8], discretized it into k points, and analyzed the change in the prospect value when varying the number of discrete points. We used the interval [0.4,0.8]. The change in prospect value is shown in Figure 9.

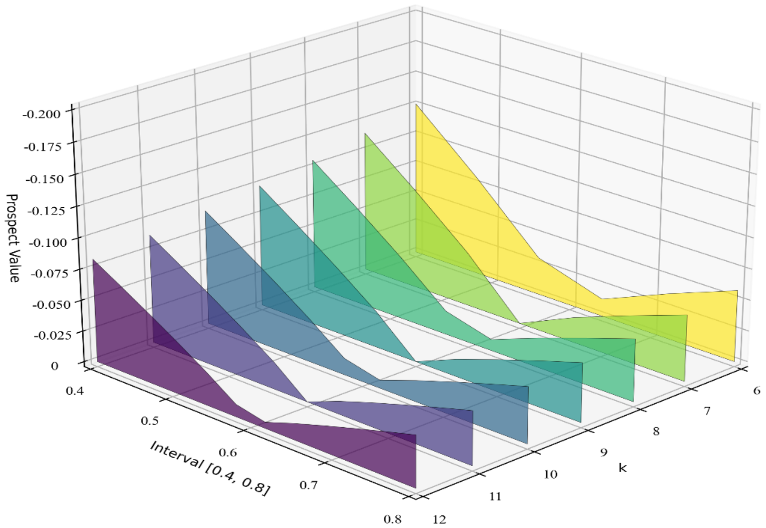

Figure 9. Change in prospect value for different discrete point numbers.

It can be seen from Figure 9 that with the increase in the number of discrete points k, the function curve gradually becomes smooth. The description of each discrete point on the interval $[0.4,0.8]$ becomes more detailed, which mimics the true attitude of the decision-makers.

The experiment conducted 100 independent trials, with each trial randomly generating an interval. We then took an arbitrary interval and examined the change in the $EvValue$ for different k. For the selected interval $[0.45,0.8]$, the change is shown in Figure 10.

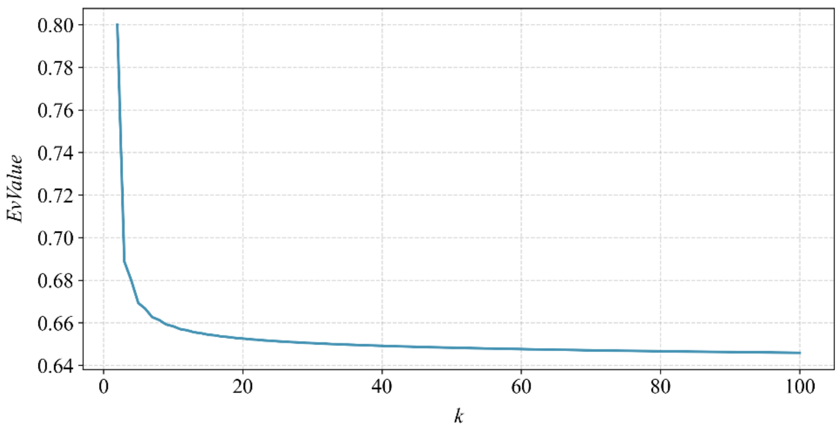

Figure 10. Change in interval $EvValue$ for different discrete point numbers.

As can be seen from Figure 10, the $EvValue$ decreases with the increase of k. When $k < 20$, the $EvValue$ decreases, but it changes gently and gradually converges around 0.645 when $k \geq$

20. The $EvValue$ is always greater than the midpoint 0.625 of the interval [0.45,0.8], which further explains that the midpoint of the interval cannot reflect the fuzziness of the interval.

4.3.2. Sensitivity analysis of the IVIFS-SF

In this section, the reference point x_* and the number of discrete points k are investigated for S_{NEW} by using the settings $\alpha = 0.88, \gamma = 0.61$, $\beta = 0.88, \theta = 2.25$, and $\delta = 0.69$.

(1) Influence of x_* on S_{NEW}

This experiment was repeated 100 times. This led to the random generation of 15 IVIFVs for each experiment, noted as $b_1, b_2, \dots, b_{15}$. The membership and non-membership intervals of $b_1, b_2, \dots, b_{15}$ were discretized to $k = 21$ points. When x_* (reference point) ranged from 0 to 1, the scores of S_{NEW} were observed to be stable in each experiment. This means that the change of the reference point does not affect the score of each b_i (i = 1, 2, …, 15). Figure 11 presents the results from one of the experiment iterations, where the randomly generated data is

$$b_1 = ([0.04, 0.16], [0.09, 0.40]),\ \ b_2 = ([0.02, 0.02], [0.56, 0.71]),$$
$$b_3 = ([0.12, 0.17], [0.15, 0.39]),\ \ b_4 = ([0.12, 0.25], [0.41, 0.49]),$$
$$b_5 = ([0.22, 0.31], [0.20, 0.36]),\ \ b_6 = ([0.01, 0.62], [0.33, 0.37]),$$
$$b_7 = ([0.00, 0.56], [0.19, 0.23]),\ \ b_8 = ([0.21, 0.26], [0.02, 0.02]),$$
$$b_9 = ([0.17, 0.24], [0.11, 0.67]),\ \ b_{10} = ([0.16, 0.23], [0.01, 0.39]),$$
$$b_{11} = ([0.28, 0.42], [0.19, 0.24]),\ \ b_{12} = ([0.26, 0.66], [0.03, 0.19]),$$
$$b_{13} = ([0.09, 0.20], [0.57, 0.60]),\ \ b_{14} = ([0.32, 0.51], [0.41, 0.41]),$$
$$b_{15} = ([0.21, 0.55], [0.09, 0.23]).$$

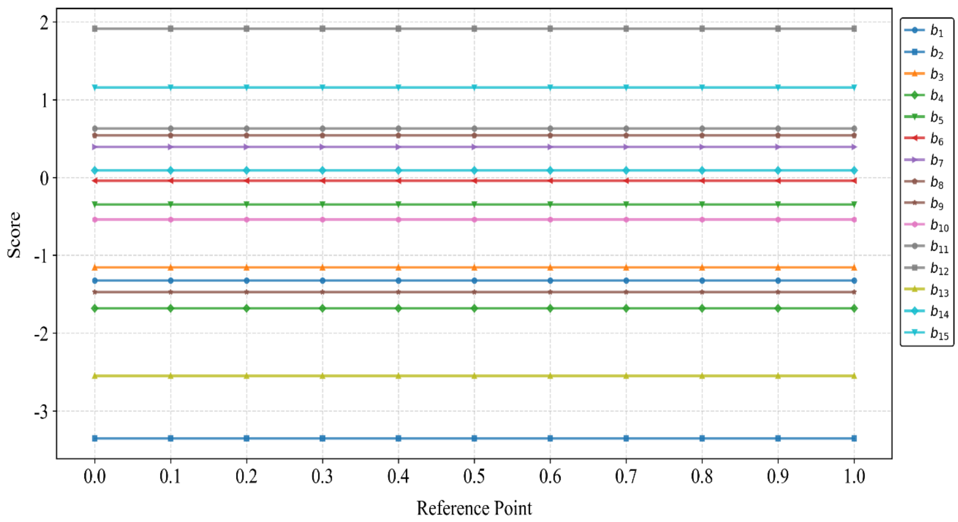

Figure 11. Scores for different reference points.

(2) Influence of parameter k on S_{NEW}

Similarly, we ran another 100 experiments with 15 IVIFVs ($d_1, d_2, \dots, d_{15}$) in each experiment by letting $x_* = 0.5$ and varying k from 3 to 30. In one of the experiments, when $3 \leq k \leq 10$, the scores of d_i ($i = 1, 2, \dots, 15$) fluctuated greatly and seemed to cross over. When $k > 15$, there were almost no fluctuations in the scores and the order of the scores did not change. In this sense, it seemed better to take $k > 15$. We in fact used $k = 21$ for all the experiments in this paper. Figure 12 presents the results from another of the experiment iterations:

$$d_1 = ([0.38, 0.69], [0.07, 0.08]),\ \ d_2 = ([0.05, 0.09], [0.59, 0.62]),$$
$$d_3 = ([0.57, 0.79], [0.02, 0.11]),\ \ d_4 = ([0.35, 0.38], [0.34, 0.34]),$$
$$d_5 = ([0.25, 0.32], [0.35, 0.42]),\ \ d_6 = ([0.19, 0.37], [0.39, 0.60]),$$
$$d_7 = ([0.04, 0.58], [0.24, 0.41]),\ \ d_8 = ([0.03, 0.31], [0.63, 0.66]),$$
$$d_9 = ([0.18, 0.49], [0.19, 0.27]),\ \ d_{10} = ([0.01, 0.02], [0.52, 0.94]),$$
$$d_{11} = ([0.18, 0.20], [0.32, 0.32]),\ \ d_{12} = ([0.14, 0.19], [0.65, 0.65])$$
$$d_{13} = ([0.02, 0.23], [0.19, 0.52]),\ \ d_{14} = ([0.25, 0.85], [0.01, 0.08]),$$
$$d_{15} = ([0.06, 0.06], [0.24, 0.47]).$$

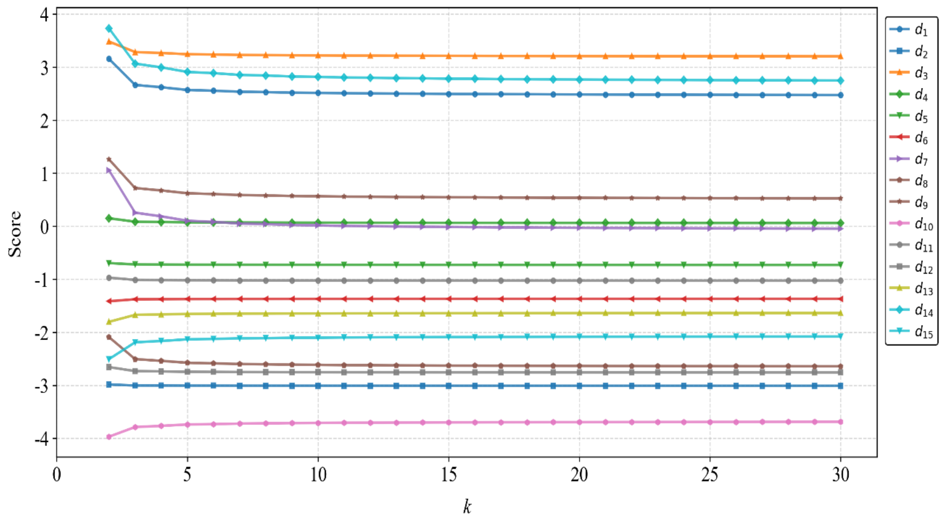

Figure 12. Scores for different numbers of discrete points.

From Figure 12, it is possible to see that $S_{NEW}(d_7) > S_{NEW}(d_4)$ and $S_{NEW}(d_{14}) > S_{NEW}(d_3)$ when $k \leq 7$, but $S_{NEW}(d_7) < S_{NEW}(d_4)$ and $S_{NEW}(d_{14}) < S_{NEW}(d_3)$ when $k > 7$.

5. Cases

Two cases are presented for comparison and analysis of the proposed IVIFS-SF in this section. An alternative sensitivity analysis of the IVIFS-SF is also provided in Case 1. For the prospect theory that we will use in this paper, Kahneman and Tversky [30] have obtained the values of the empirical parameters ($\alpha = 0.88, \gamma = 0.61,\ \beta = 0.88, \theta = 2.25,$ and $\delta = 0.69$).

5.1. Case 1

The problem of cadre selection [9] is used to verify the proposed SF in Case 1. In the problem, there are five candidates $A_i\,(i = 1, 2, \cdots, 5)$ and six assessment indices $G_j\,(j = 1, 2, \cdots, 6)$ including ideology and morality (G_1), work attitude (G_2), work style (G_3), cultural level and knowledge structure (G_4), leadership ability (G_5), and pioneering ability (G_6). The index weight vector is $\omega = (0.20, 0.10, 0.25, 0.10, 0.15, 0.20)^T$. As a result of the consultation with the masses and their subsequent recommendations, the five candidates are evaluated according to the above-mentioned six indices. This is then followed by statistical processing. Specifically, there are four steps necessary to rank the candidates.

Step 1. After the normalization [9], the decision matrix D of the IVIFVs is obtained.

$$D=\begin{pmatrix} ([0.2.0.3],[0.4,0.5]),([0.6,0.7],[0.2.0.3]),([0.4,0.5],[0.2.0.4]),([0.7,0.8],[0.1,0.2]),([0.1,0.3],[0.5,0.6]),([0.5,0.7],[0.2,0.3]) \\ ([0.6.0.7],[0.2.0.3]),([0.5,0.6],[0.1.0.3]),([0.6.0.7],[0.2.0.3]),([0.6.0.7],[0.1,0.2]),([0.3,0.4],[0.5,0.6]),([0.4,0.7],[0.1,0.2]) \\ ([0.4,0.5],[0.3.0.4]),([0.7,0.8],[0.1,0.2]),([0.5,0.6],[0.3,0.4]),([0.6,0.7],[0.1,0.3]),([0.4,0.5],[0.3,0.4]),([0.3,0.5],[0.1,0.3]) \\ ([0.6.0.7],[0.2,0.3]),([0.5,0.7],[0.1,0.3]),([0.7,0.8],[0.1,0.2]),([0.3,0.4],[0.1,0.2]),([0.5,0.6],[0.1,0.3]),([0.7,0.8],[0.1,0.2]) \\ ([0.5,0.6],[0.3,0.4]),([0.3,0.4],[0.3,0.5]),([0.6,0.7],[0.1,0.3]),([0.6,0.8],[0.1,0.2]),([0.6,0.7],[0.2,0.3]),([0.5,0.6],[0.2,0.4]) \end{pmatrix}.$$

Step 2. The IIFWA [9] operator is used to aggregate the information. The evaluation value of the corresponding candidate A_i, r_i $(i = 1, 2, \cdots, 5)$, can be then obtained:

$$r_1 = ([0.4165,0.5597],[0.2459,0.3804]),\ r_2 = ([0.5176,0.6574],[0.1739,0.2947]),$$
$$r_3 = ([0.4703,0.5900],[0.1933,0.3424]),\ r_4 = ([0.6070,0.7203],[0.1149,0.2400]),$$

and

$$r_5 = ([0.5375,0.6536],[0.1772,0.3402]).$$

Step 3. r_i is used to calculate S_X, S_{WC}, S_{WL}, S_B, S_G, S_{GL}, S_{CT}, S_{CD}, S_{KC}, S_{CY} and S_{New} is denoted as $(i = 1, 2, \cdots, 5)$ as shown in Table 2.

Table 2. Comprehensive index scores of candidates.

Score	$S(r_1)$	$S(r_2)$	$S(r_3)$	$S(r_4)$	$S(r_5)$	Ranking result
S_X	0.1749	0.3532	0.2623	0.4862	0.3369	$A_4 > A_2 > A_5 > A_3 > A_1$
S_{WC}	0.0289	0.1831	0.1021	0.3087	0.1686	$A_4 > A_2 > A_5 > A_3 > A_1$
S_{WL}	0.0392	0.0862	0.0577	0.1468	0.0814	$A_4 > A_2 > A_5 > A_3 > A_1$
S_B	0.5752	0.6831	0.6292	0.7624	0.6743	$A_4 > A_2 > A_5 > A_3 > A_1$
S_G	0.5645	0.6702	0.6174	0.7483	0.6593	$A_4 > A_2 > A_5 > A_3 > A_1$
S_{GL}	0.9937	1.2083	1.0815	1.4032	1.2223	$A_4 > A_5 > A_2 > A_3 > A_1$
S_{CT}	1.5245	1.6395	1.5815	1.7202	1.6305	$A_4 > A_2 > A_5 > A_3 > A_1$
S_{CD}	3.7408	4.0745	3.9054	4.3032	4.0494	$A_4 > A_2 > A_5 > A_3 > A_1$
S_{KC}	0.2060	0.4112	0.3062	0.5565	0.3820	$A_4 > A_2 > A_5 > A_3 > A_1$
S_{CY}	4.6913	5.2011	4.9330	5.6290	5.1636	$A_4 > A_2 > A_5 > A_3 > A_1$
$\boldsymbol{S_{NEW}}$	**1.0405**	**2.0680**	**1.5327**	**2.7213**	**1.9710**	$\mathbf{A_4 > A_2 > A_5 > A_3 > A_1}$

Step 4. Candidates are ranked according to the $S(r_i)$.

As can be seen from Table 2, only the ranking of S_{GL} is slightly different from the others, while the remaining 10 SFs give a consistent ranking. The results verify that S_{New} is accurate.

The 11 groups of scores were normalized by Eq (31) to calculate the variances indicated by Eq (33). The results are shown in Table 3 and Figure 13.

As can be seen from Table 3, the normalized r_4 of S_{NEW} has the second highest score and the largest variance, which makes it easier for S_{NEW} to distinguish the differences among alternatives. S_{NEW} thus has better applicability than the other SFs.

Table 3. Normalized scores and variances.

Score	$S(r_1)$	$S(r_2)$	$S(r_3)$	$S(r_4)$	$S(r_5)$	Variance
S_X	0.5875	0.6766	0.6312	0.7431	0.6684	0.0027
S_{WC}	0.5145	0.5916	0.5510	0.6543	0.5843	0.0022
S_{WL}	0.0392	0.0862	0.0577	0.1468	0.0814	0.0013
S_B	0.5752	0.6831	0.6292	0.7624	0.6743	0.0038
S_G	0.5645	0.6702	0.6174	0.7483	0.6593	0.0037
S_{GL}	0.2663	0.3576	0.3036	0.4405	0.3635	0.0035
S_{CT}	0.7623	0.8197	0.7908	0.8601	0.8152	0.0011
S_{CD}	0.6852	0.7686	0.7264	0.8258	0.7624	0.0022
S_{KC}	0.6030	0.7056	0.6531	0.7783	0.6910	0.0034
S_{CY}	0.5273	0.6002	0.5619	0.6613	0.5948	0.0020
$\boldsymbol{S_{NEW}}$	**0.6301**	**0.7585**	**0.6916**	**0.8402**	**0.7464**	**0.0049**

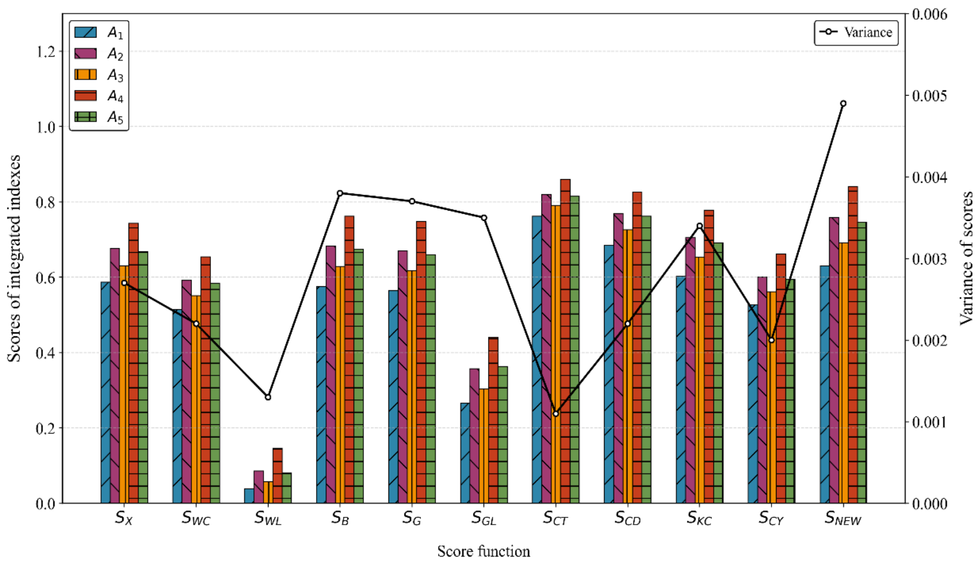

Figure 13. Comparison of normalized scores and variances.

As can be seen from Figure 13, the variance of S_{New} is the largest, meaning that the dispersion degree of the candidates (or alternatives) is also the largest. The variances of S_B, S_G, and S_{GL} are also relatively high. The score difference between r_2 and r_5 is relatively small, which may explain the ranking difference between S_{GL} and the other SFs.

In order to compare the deviation of each SF still further, each of the 11 groups of scores based on the normalized data was sorted from the best (No. 1) to the worst (No. 5). The deviation of the scores between the two neighboring rankings and the averages between them were calculated and given in Table 4.

As can be seen from Table 4, both the average deviation and the individual deviation of S_{NEW} are the largest amongst all the SFs. S_{NEW} makes it easier to distinguish each alternative and thus facilitates the work of the decision-makers, a conclusion also confirmed by Figure 14.

Table 4. Normalized deviation sequence of comprehensive indices.

Score	S(No. 4) − S(No. 5)	S(No. 3) − S(No. 4)	S(No. 2) − S(No. 3)	S(No. 1) − S(No. 2)	Average value of the deviation
S_X	0.0437	0.0373	0.0082	0.0665	0.0389
S_{WC}	0.0366	0.0332	0.0073	0.0628	0.0350
S_{WL}	0.0186	0.0237	0.0048	0.0606	0.0269
S_B	0.0540	0.0450	0.0088	0.0793	0.0468
S_G	0.0529	0.0420	0.0109	0.0781	0.0460
S_{GL}	0.0373	0.0540	0.0060	0.0769	0.0436
S_{CT}	0.0285	0.0245	0.0045	0.0404	0.0245
S_{CD}	0.0411	0.0360	0.0063	0.0572	0.0352
S_{KC}	0.0501	0.0379	0.0146	0.0727	0.0438
S_{CY}	0.0345	0.0329	0.0053	0.0611	0.0334
$\boldsymbol{S_{NEW}}$	**0.0615**	**0.0548**	**0.0121**	**0.0817**	**0.0525**

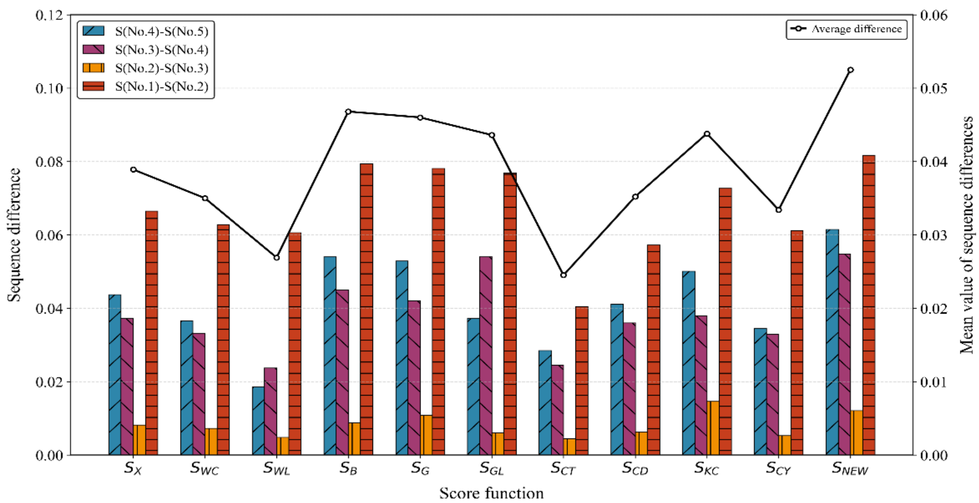

Figure 14. Sequence deviation of normalized comprehensive indices.

5.2. Case 2

To further validate the effectiveness of the score function proposed in this paper, we will compare and analyze it with Chen's method [19]. Chen [19] outlined an IVIFV decision-making matrix $A = \left([u_{ij}^-, u_{ij}^+], [v_{ij}^-, v_{ij}^+]\right)_{3\times 3}$ (see Example 5.4 in [19]) with three alternatives r_1, r_2, and r_3. For each alternative, there are three attributes, g_1, g_2, and g_3, and their weights are W_1, W_2, and W_3. The specifications are

$$A = \begin{pmatrix} ([0.30,0.30],[0.10,0.10]),([0.60,0.60],[0.25,0.25]),([0.80,0.80],[0.20,0.20]) \\ ([0.20,0.20],[0.15,0.15]),([0.68,0.68],[0.20,0.20]),([0.45,0.45],[0.50,0.50]) \\ ([0.20,0.20],[0.45,0.45]),([0.70,0.70],[0.05,0.05]),([0.60,0.60],[0.30,0.30]) \end{pmatrix},$$

$$W_1 = ([0.25,0.25],[0.25,0.25]),\ W_2 = ([0.35,0.35],[0.40,0.40]),$$

and

$$W_3 = ([0.30,0.30],[0.65,0.65]).$$

In [7], the scores of the alternatives are $S_{CY}(r_1) = 19.909$, $S_{CY}(r_2) = 18.586$, and $S_{CY}(r_3) = 19.176$, and the ranking result is $r_1 \succ r_3 \succ r_2$.

We replaced S_{CY} with S_{NEW} without changing the calculation steps, which are indicated below.

Step 1. W_i is calculated by S_{NEW} and $\widetilde{W_i}$ $(i = 1,2,3)$ is normalized by Eq (34).

By calculating W_i, we can get the real number weights $\widetilde{W_i}$:

$$\widetilde{W_1} = -0.293,\ \widetilde{W_2} = -0.384,\ \widetilde{W_3} = -2.091.$$

After the normalization of $\widetilde{W_i}$, we have $\overline{W_1} = 0.106, \overline{W_2} = 0.139, \overline{W_3} = 0.755$, and

$$\overline{W_J} = \frac{\widetilde{W_J}}{\sum_{l=1}^{3}\widetilde{W_J}}. \tag{34}$$

Step 2. The weighted decision-making matrix $D = (\widetilde{d_{ij}})_{3\times 3}$ is calculated by the power operator given by Chen [19]:

$$\widetilde{d_{ij}} = \left(\left[u_{ij}^{-\overline{W_J}}, u_{ij}^{+\overline{W_J}}\right], \left[1-(1-v_{ij}^{-})^{\overline{W_J}}, 1-(1-v_{ij}^{+})^{\overline{W_J}}\right]\right). \tag{35}$$

The result is

$$D = \begin{pmatrix} ([0.880,0.880],[0.011,0.011]), ([0.932,0.932],[0.039,0.039]), ([0.845,0.845],[0.155,0.155]) \\ ([0.843,0.843],[0.017,0.017]), ([0.948,0.948],[0.030,0.030]), ([0.547,0.547],[0.408,0.408]) \\ ([0.843,0.843].[0.061,0.061]), ([0.952,0.952],[0.007,0.007]), ([0.680,0.680],[0.236,0.236]) \end{pmatrix}.$$

Step 3. The score of each element in D is calculated and cost is turned into benefit in matrix D:

$$\overline{d_{ij}} = \begin{cases} S_{NEW}(\widetilde{d_{ij}}), if\ G_j\ is\ a\ benefit-type\ attribute, \\ 9 - S_{NEW}(\widetilde{d_{ij}}), if\ G_j\ is\ a\ cost-type\ attribute. \end{cases} \tag{36}$$

The score of each element in D given by S_{NEW} is

$$S_{NEW}(\widetilde{d_{11}}) = 3.887,\ S_{NEW}(\widetilde{d_{12}}) = 3.941,\ S_{NEW}(\widetilde{d_{13}}) = 3.534,$$
$$S_{NEW}(\widetilde{d_{21}}) = 3.805,\ S_{NEW}(\widetilde{d_{22}}) = 3.965,\ S_{NEW}(\widetilde{d_{23}}) = 0.866,$$
$$S_{NEW}(\widetilde{d_{31}}) = 3.746,\ S_{NEW}(\widetilde{d_{32}}) = 3.981,\ S_{NEW}(\widetilde{d_{33}}) = 2.553.$$

The normalized matrix is

$$M = \left(\overline{d_{ij}}\right)_{3\times 3} = \begin{pmatrix} 3.887, 3.941, 3.534 \\ 3.805, 3.965, 0.866 \\ 3.746, 3.981, 2.553 \end{pmatrix}.$$

Step 4. The score of alternatives is calculated in Eq (37):

$$SC(r_i) = \sum_{j=1}^{3} \overline{d_{ij}}. \tag{37}$$

The results are

$$S_{NEW}(r_1) = 11.363,\ S_{NEW}(r_2) = 8.636,\ \text{and}\ S_{NEW}(r_3) = 10.28.$$

It is easy to see that $S_{NEW}(r_1) > S_{NEW}(r_3) > S_{NEW}(r_2)$.

Step 5. The ranking result of alternatives is obtained.

According to the score of alternatives, the ranking result is $r_1 \succ r_3 \succ r_2$, which is the same as the ranking in [19]. Eq (33) is used to calculate the variances of S_{CY} and S_{NEW}, which are 0.0060 for S_{CY} and 0.0196 for S_{NEW}. We can also get $S_{CY}(r_1) - S_{CY}(r_2)$ = 0.733, $S_{CY}(r_3) - S_{CY}(r_2)$ = 0.590, $S_{NEW}(r_1) - S_{NEW}(r_3)$ = 1.083, and $S_{NEW}(r_3) - S_{NEW}(r_2)$ =1.644. It is evident that 1.083 is greater than 0.733 and 1.644 is greater than 0.590, making S_{NEW} exhibit better ability of discrimination than S_{CY}.

We then used S_X, S_{WC}, S_{WL}, S_B, S_G, S_{GL}, S_{CT}, S_{CD}, S_{KC} to replace S_{CY} in decision-making matrix A. The alternative scores and ranking results are shown in Table 5.

Table 5. Alternative scores and ranking results of the SFs.

Score	$S(r_1)$	$S(r_2)$	$S(r_3)$	Ranking result
S_X	2.5481	1.9679	2.3215	$r_1 \succ r_3 \succ r_2$
S_{WC}	2.5374	1.9474	2.2543	$r_1 \succ r_3 \succ r_2$
S_{WL}	1.3264	1.0874	1.2918	$r_1 \succ r_3 \succ r_2$
S_B	2.6886	2.5258	2.5888	$r_1 \succ r_3 \succ r_2$
S_G	2.6886	2.5258	2.5888	$r_1 \succ r_3 \succ r_2$
S_{GL}	5.8012	5.1126	5.4046	$r_1 \succ r_3 \succ r_2$
S_{CT}	5.5899	5.4123	5.4592	$r_1 \succ r_3 \succ r_2$
S_{CD}	14.0204	13.5674	13.6728	$r_1 \succ r_3 \succ r_2$
S_{CY}	19.9151	18.5883	19.1778	$r_1 \succ r_3 \succ r_2$
$\boldsymbol{S_{NEW}}$	**11.3626**	**8.6361**	**10.2799**	$\boldsymbol{r_1 \succ r_3 \succ r_2}$

It can be seen from Table 5 that S_X, S_{WC}, S_{WL}, S_B, S_G, S_{GL}, S_{CT}, S_{CD}, $S_{CY,}$ and S_{NEW} have the same ranking. It further confirms the feasibility and effectiveness of S_{NEW}. However, we find that $S_{KC}(W_1) = 0.4688, S_{KC}(W_2) = -0.0586$, and $S_{KC}(W_3) = -0.3666$ cannot be normalized by Eq (34). Thus, S_{KC} is not suitable for the settings of decision-making matrix A.

S_X, S_{WC}, S_{WL}, S_B, S_G, S_{GL}, S_{CT}, S_{CD}, S_{CY}, and S_{NEW} were normalized by Eq (31) to calculate the variances indicated in Eq (33). The results are shown in Table 6 and Figure 15.

Table 6. Normalized scores and variances.

Score	$S(r_1)$	$S(r_2)$	$S(r_3)$	Variance
S_X	1.7741	1.4840	1.6608	0.0143
S_{WC}	1.7687	1.4737	1.6272	0.0145
S_{WL}	1.3264	1.0874	1.2918	0.0111
S_B	2.6886	2.5258	2.5888	0.0045
S_G	2.6886	2.5258	2.5888	0.0045
S_{GL}	2.3117	2.0187	2.1429	0.0144
S_{CT}	2.7950	2.7062	2.7296	0.0014
S_{CD}	3.2551	3.1419	3.1682	0.0023
S_{CY}	2.7022	2.5126	2.5968	0.0060
$\boldsymbol{S_{NEW}}$	**1.9203**	**1.5795**	**1.7850**	**0.0196**

As can be seen from Table 6, the highest score and the largest variance are found at the normalized S_{NEW}, which implies that it is more efficient for S_{NEW} to distinguish the differences between alternatives and S_{NEW} has greater applicability. The largest variance at S_{NEW} can also be seen from Figure 15 meaning that the dispersion degree of the alternatives is also the largest.

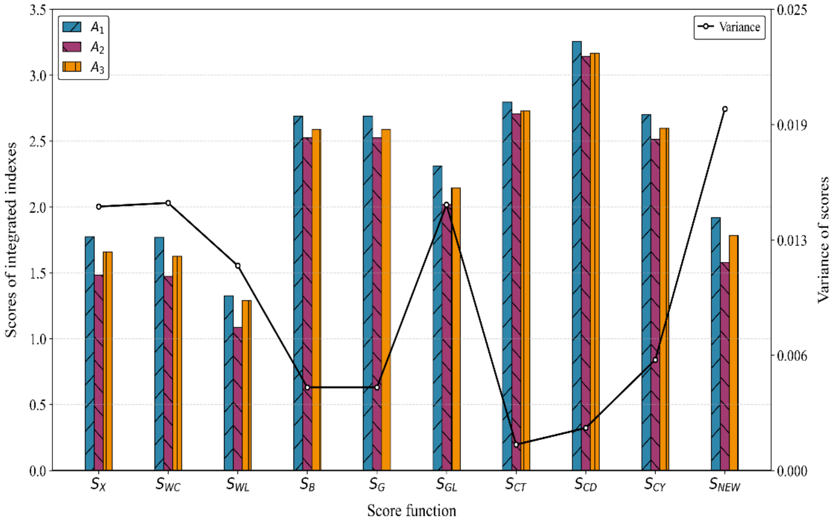

Figure 15. Comparison of normalized scores and variances.

The deviation of the scores between two neighboring rankings and the averages of them were calculated and given Table 7.

Table 7. Normalized deviation sequence of alternative score.

Score	S(No. 2) − S(No. 1)	S(No. 3) − S(No. 2)	Average value of the deviation
S_X	0.1768	0.1133	0.1451
S_{WC}	0.1535	0.1415	0.1475
S_{WL}	0.2044	0.0346	0.1195
S_B	0.0630	0.0998	0.0814
S_G	0.0630	0.0998	0.0814
S_{GL}	0.1242	0.1687	0.1465
S_{CT}	0.0234	0.0654	0.0444
S_{CD}	0.0264	0.0869	0.0567
S_{CY}	0.0842	0.1053	0.0948
$\boldsymbol{S_{NEW}}$	**0.2055**	**0.1353**	**0.1704**

As can be seen from Table 7 and Figure 16, both the average deviation and individual deviation of S_{NEW} are the largest among the SFs. It makes S_{NEW} more outstanding in distinguishing each alternative and facilitating decision-makers to select an alternative.

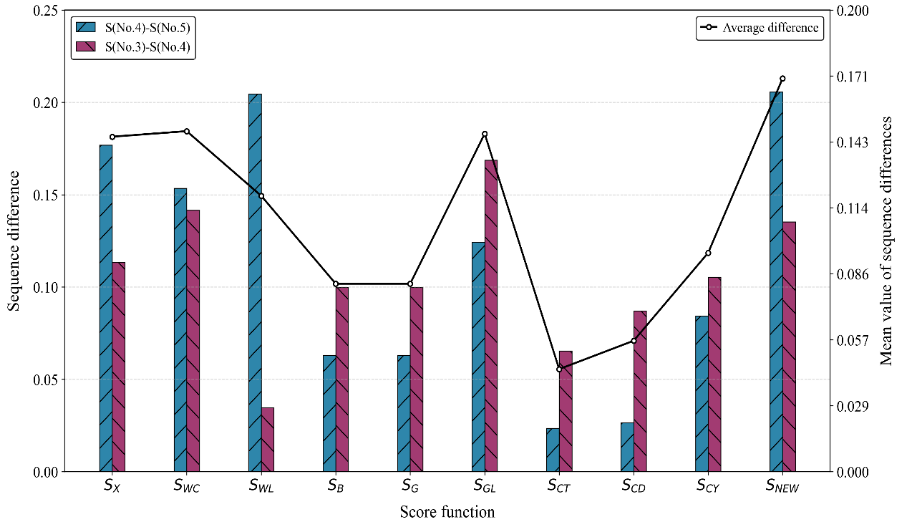

Figure 16. Sequence deviation of the normalized alternative score.

5.3. Sensitivity analysis of the IVIFS-SF in Case 1

In order to further investigate this new SF, we use Case 1 to analyze the effects of the parameters in S_{NEW}. When $\alpha, \gamma, \beta,$ and δ change from 0.2 to 1.0 in Eq (22), the scores of the five alternatives described in Case 1 are shown in Figure 17.

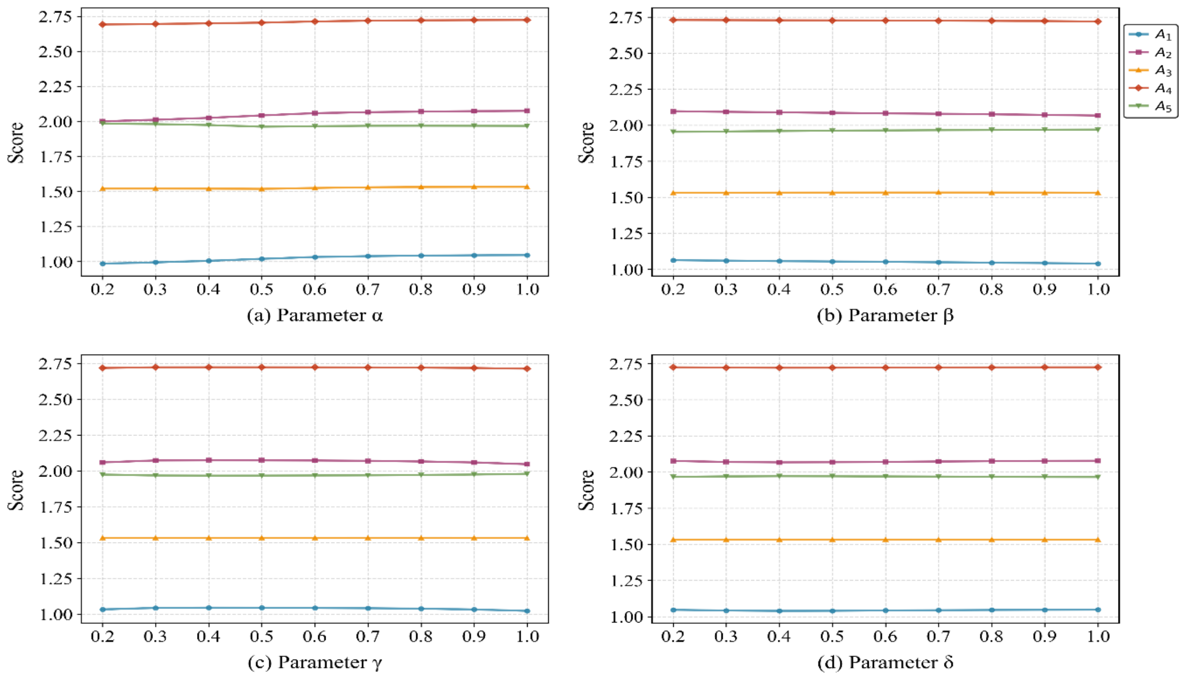

Figure 17. Sorting results for the alternatives with S_{New} parameter change.

As can be seen from Figure 17, when parameters of $\alpha, \gamma, \beta,$ and δ are changed in Eq (22), S_{NEW} does not change drastically and the ranking of alternatives remains unaffected. The reason for this is that when the parameters change, the $EvValue$ of the alternative changes very little, leading to only a slight weight change. Similarly, when θ changes, the change in the weight of the discrete points is also small and the ranking of the alternatives is not affected here either.

Another element of our sensitivity analysis was our investigation of the influence of the number of discrete points on the alternative ranking. Figure 18 shows the scores for S_{NEW} when k varies from 2 to 20.

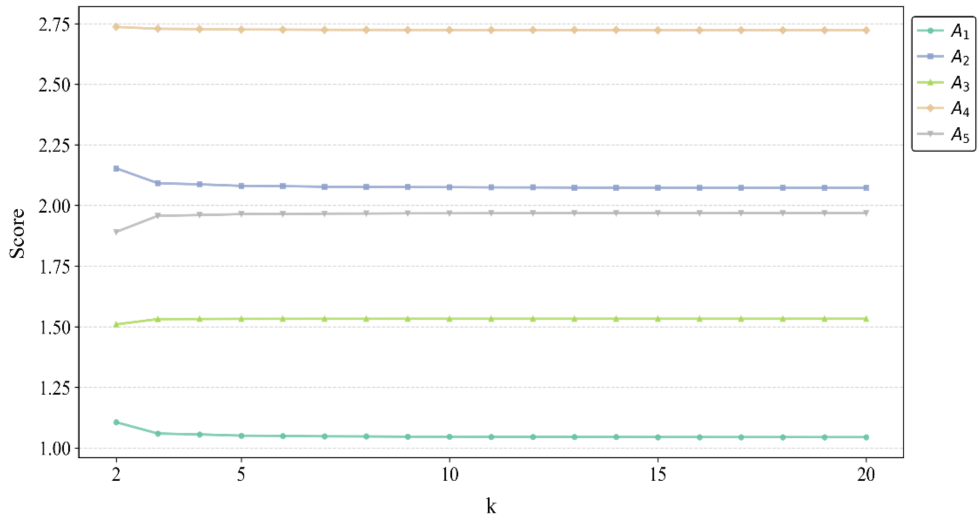

Figure 18. The scores of the alternatives when parameter k of S_{NEW} changes.

When k changes from 2 to 3, the ranking order of the alternatives does not change despite the change of the scores. When k is greater than 3, the alternatives in this case have invisible fluctuations and the ranking order remains unchanged.

From Case 1 and comparation analysis, S_{NEW} obtained the same optimal alternative as the other SFs and the same ranking results. At the same time, of all of the SFs, S_{NEW} achieved the highest dispersion degree between the alternatives and the greatest deviation in neighboring score values. Therefore, these results have clearly shown that S_{NEW} outperforms the other compared SFs.

6 Conclusions

Building upon the interval-valued evaluation method (IVEM) derived from prospect theory, this study proposes a novel score function for interval-valued intuitionistic fuzzy sets (IVIFS-SF). The proposed IVEM framework transforms an interval $[a, b]$ into a crisp numerical value by integrating the value function and probability weight function from prospect theory. This methodology not only establishes a quantitative relationship between interval values and crisp numbers but also effectively models the psychological decision-making process of human evaluators. To facilitate the systematic evaluation and selection of score functions for IVIFVs, we introduce two comprehensive assessment metrics: pass rate and variance. Extensive comparative analyses demonstrate that the IVIFS-SF achieves superior performance, attaining the highest pass rate and largest variance among existing score function (SF) and accuracy function (AF) methods—all without requiring additional accuracy functions. Empirical case studies confirm that while the IVIFS-SF identifies the same optimal alternative as competing methods, it maintains its advantage in terms of variance. Furthermore, sensitivity analysis validates the robustness of the IVIFS-SF across the parameter space of prospect theory.

While the IVIFS-SF has demonstrated promising performance in terms of pass rate optimization and variance reduction within synthetic datasets, thereby addressing some limitations of conventional

SF and AF methodologies, it still faces significant challenges in managing IVIFV-related uncertainty. For instance, the proposed score function remains unable to distinguish certain IVIFVs, which is an inherent limitation of this scoring approach. Developing new interpretability measurement methods will be a valuable research direction. As a fundamental aspect of IVIFS research, the measurement method of the IVIFS provides the basis for further studies. In this paper, the interval $[a, b]$ is divided into k $(k \geq 1)$ equally spaced segments, and the case of randomly partitioned k-segment intervals will also be investigated.

Author contributions

Benting Wan: Supervision, Conceptualization, Data curation, Investigation, Methodology, Validation, Visualization, Writing–original draft; Jun Wan: Data curation, Methodology, Project administration, Resources, Supervision, Writing–review & editing; Juan Zhang: Data curation, Investigation, Writing–review & editing; Jin Xie: Supervision, Formal analysis, Software, Writing–review & editing; Youyu Cheng: Supervision, Data curation, Investigation, Validation, Writing–original draft; Wenzhong Peng: Data curation, Formal analysis, review & editing.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no competing interests among them.

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