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Research article

Exponential stabilization of quasi-one-sided Lipschitz systems with time delay

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Abstract: This paper aims to design Observer-Based (OB) controllers that ensure the exponential stability of a class of nonlinear time-delay systems. The nonlinear part of the system satisfies a weak Quasi-One-Sided Lipschitz (QOSL) condition characterized by the matrices $(\mathcal{L}_1, \mathcal{M}_1, \mathcal{N}_1)$, as well as a QOSL condition characterized by the matrices $(\mathcal{L}_2, \mathcal{M}_2, \mathcal{N}_2)$. First, we derive a sufficient condition formulated as a Linear Matrix Inequality (LMI) via a Lyapunov–Krasovskii (LK) functional. The main advantage of this design is that the controller and observer gains are computed in a single step. However, its main drawback is that the matrices $\mathcal{L}_1$, $\mathcal{M}_1$, and $\mathcal{N}_1$ are fixed rather than treated as decision variables. To overcome this limitation, we propose an improved design in which the matrices $\mathcal{L}_1$, $\mathcal{M}_1$, and $\mathcal{N}_1$ are treated as decision-variable matrices with a fixed structure. By using an appropriate decoupling technique, this approach provides greater flexibility in the selection of matrices and reduces conservatism. The efficacy of the developed OB controllers is demonstrated via a suitable numerical example.

Keywords: exponential stability; observer-based controller; time-delay systems; quasi-one-sided Lipschitz condition; decoupling technique
Mathematics Subject Classification: 93D23, 93C10, 93C43

1. Introduction

Due to the reliance on sensors for measurement, along with transmission and transport lags as well as processing latencies, time delays have become an unavoidable challenge in the control of dynamic systems. As a result, numerous control and analysis techniques originally developed for delay-free systems have been adapted for time-delay systems. The original theoretical extension began in the 20th century when Krasovskii [1] and Razumikhin [2] proposed fundamental theories to address time-delay equations by extending Lyapunov's theory. Historically, the literature shows that building on these extensions, a variety of results and related developments have emerged, evolving from the simplest time-delay systems to increasingly complex ones. For example, Razumikhin's methodology was initially applied in control to analyze uniform asymptotic stability for the simplest class of systems, namely those with a single time delay [3].

Research on time-delay systems has continuously grown with the advent of Linear Matrix Inequalities (LMIs) in control theory [4]. For the class of time-delay linear systems, numerous studies have been conducted to incorporate new techniques aimed at updating and improving existing results. For instance, asymptotic stability has been investigated in [5, 6], while exponential stability has been addressed in [7–9]. For time-delay nonlinear systems approximated by a Takagi–Sugeno fuzzy model, a vast number of stability results have been established, such as asymptotic [10, 11] and exponential [12, 13]. In the following sections, we focus on systems with Lipschitz-type properties, including standard Lipschitz, One-Sided Lipschitz (OSL), and Quasi-One-Sided Lipschitz (QOSL) conditions, considering both time-delay and delay-free cases.

The problems of observer design and Observer-Based (OB) control have been widely investigated for systems characterized by Lipschitz-type properties. These conditions offer convenient mathematical bounds on nonlinearities, which allow efficient observer and OB controller synthesis via LMI-based techniques. Initial research in this area started with nonlinear systems that adhere to the standard Lipschitz condition [14–16]. A primary limitation of the standard Lipschitz condition is that it becomes conservative with large Lipschitz constants. To overcome this, the OSL condition was introduced in [17]. Consequently, several notable works for such systems have been developed [18–20]. As is well known, incorporating more detailed information about the system into the analysis and design process allows the derivation of less conservative results. Following this principle, the authors in [21] introduced the QOSL condition that is less restrictive than the OSL condition, and consequently, than the standard one. Recently, some results have been published on observer design for QOSL systems [22].

As highlighted in the first two sections, considering the presence of time delays is necessary for analyzing and controlling dynamical systems. Initially, various classes of systems were studied without delays. Then, research has progressively evolved to address the challenges posed by time delays. More recently, the QOSL systems have been extended to handle delayed dynamics. Only a limited number of OB control strategies have been proposed for this class of systems. For instance, the design in [23] employs separation principle, whereas the approach in [24] is based on the reduced-order observer. Reference [25] presents an OB controller design that incorporates input saturation constraints.

In this paper, we contribute to the advancement of OB controller design for QOSL systems with time delay. To the best of our knowledge, all existing OB controllers ensure only asymptotic stability. In contrast, this work focuses on designing an OB controller that guarantees exponential stability, leading to faster convergence and enhanced performance. Compared to [23–25], which ensure only asymptotic stability, the proposed method guarantees exponential stability. Asymptotic stability can be seen as a special case of exponential stability with zero decay rate, so the proposed result ensures a more rapid convergence of system trajectories. Furthermore, we observe that the primary limitation of the works in [23,24] lies in two aspects. First, the design of the observer and the controller is carried out separately, which increases the computational cost. In this work, we aim to simultaneously design both the observer and the controller within a unified framework. The second drawback, observed not only in [23,24] but more clearly highlighted in [25], lies in the use of the Lyapunov matrix both to verify the QOSL condition and to establish the stabilization conditions. As a result, the authors are forced to fix the Lyapunov matrix as a constant and reuse it in the LK functional, which inevitably leads to increased conservatism in the obtained results. Focusing on the primary reason that compels the authors to use a constant matrix instead of a matrix treated as a decision variable and flexible structure, we observe that this limitation arises from the pre- and post-multiplication by the Lyapunov matrix combined with the use of an inadequate decoupling technique. We propose two results to mitigate this second limitation. For the case of a constant Lyapunov matrix, we present a dedicated stability result. For the more challenging case of an unknown Lyapunov matrix with fixed structure, we develop an improved result by employing an adequate decoupling technique without the need for pre- and post-multiplying by the Lyapunov matrix.

The subsequent sections of this paper are arranged in the following manner. Section 2 begins by establishing key notations and defining the central problem being examined. The primary focus of this work is detailed in Section 3, which introduces the development of two control strategies based on observers. A numerical example in Section 4 demonstrates the effectiveness of the proposed methods. The article concludes with a summary and discussion in Section 5.

2. Notations and problem statement

2.1. Notations

Let $(k, m) \in \mathbb{N}^* \times \mathbb{N}^*$, we introduce the following notations used in the work:

- The standard notations $\mathbb{R}_{>0}$ (resp. $\mathbb{R}_{\geq 0}$), $\|.\|$, $\langle ., . \rangle$, $\lambda_{min}(.)$(resp. $\lambda_{max}(.)$) and I_k represent, in order, the set of strictly positive (resp. non-negative) real numbers, the Euclidean norm of a vector, the inner product of two vectors, the smallest (resp. largest) eigenvalue of a matrix and the $k \times k$ identity matrix.
- $\nabla_k := \{1, \dots, k\}$.
- $\mathbb{R}^{k\times m}$ and $\mathbb{S}^{k\times k}$ denote, in order, the sets of all $k \times m$ real and $k \times k$ symmetric; adding the subscript $_{>0}$ (resp. $_{\geq 0}$, $_{<0}$, $_{\leq 0}$) to the set $\mathbb{S}^{k\times k}$ indicates that the matrix is positive definite (resp. positive semi-definite, negative definite, negative semi-definite).
- Let $\bar{\varrho} \in \mathbb{R}_{>0}$. $C\Big(\begin{bmatrix} -\bar{\varrho} & 0 \end{bmatrix}, \mathbb{R}^k\Big)$ denotes the space of continuous functions from $\begin{bmatrix} -\bar{\varrho} & 0 \end{bmatrix}$ to $\mathbb{R}^k$; For $D(t) \in C\Big(\begin{bmatrix} -\bar{\varrho} & 0 \end{bmatrix}, \mathbb{R}^k\Big)$, $\|D\|_s = sup_{t\in\begin{bmatrix} -\bar{\varrho} & 0 \end{bmatrix}} \|D(t)\|$; $x_t \in C\Big(\begin{bmatrix} -\bar{\varrho} & 0 \end{bmatrix}, \mathbb{R}^k\Big)$ is defined as $x_t := \{x(t+\delta),\ \delta \in \begin{bmatrix} -\bar{\varrho} & 0 \end{bmatrix}\}$.
- Let $\mathcal{P} \in \mathbb{S}^{k\times k}$. $\mathcal{P} > 0$, (resp. $\mathcal{P} < 0$) means that $\mathcal{P}$ is positive definite (resp. negative definite).
- Let $\mathcal{P} \in \mathbb{R}^{k\times k}$. $\{\mathcal{P}\}_* = \mathcal{P} + \mathcal{P}^T$.
- The symbol $\bullet$ indicates the symmetric entries in a matrix expression.

2.2. Time-delay QOSL system

Consider a time-delay nonlinear system, described as follows:

$$\begin{cases} \dot{\nu}(t) = A\nu(t) + \bar{A}\nu(t-\varrho(t)) + \phi(\nu(t), \nu(t-\varrho(t))) + B\mu(t), \\ y(t) = C\nu(t), \\ \nu(t) = d(t), t \in [-\bar{\varrho}, 0], \end{cases} \tag{2.1}$$

where:

- $\varrho(t)$ is a time-varying delay satisfying

$$0 \leq \varrho(t) \leq \bar{\varrho}, \ \dot{\varrho}(t) \leq \check{\varrho} < 1, \tag{2.2}$$

  where $\bar{\varrho} \in \mathbb{R}_{>0}$ and $\check{\varrho} \in \mathbb{R}_{\geq 0}$,
- $\nu(t) \in \mathbb{R}^p$, $\mu(t) \in \mathbb{R}^q$, $y(t) \in \mathbb{R}^r$ are, in order, the state vector, control input vector and measured output vector,
- A, $\bar{A}$, B and C are matrices whose dimensions are consistent with the number of states, inputs, and outputs,
- $d(t) \in \mathcal{C}\left(\begin{bmatrix} -\bar{\varrho} & 0 \end{bmatrix}, \mathbb{R}^p\right)$ defines the initial state vector,
- $\phi(\nu(t), \nu(t-\varrho(t)))$ is a nonlinear function of $\nu(t)$ and $\nu(t-\varrho(t))$ that represents the nonlinear time-delay dynamics of the system and fulfills the condition $\phi(0,0) = 0$.

Definition 2.1. *[23] The nonlinear function ϕ is defined as QOSL if $\{\forall (u_k, v_k) \in \mathbb{R}^{\mathrm{p}} \times \mathbb{R}^{\mathrm{p}},\ k \in \nabla_2\}$, we have:*

$$\langle \mathcal{L}(\phi(u_1,u_2) - \phi(v_1,v_2)), u_1 - v_2 \rangle \ \leq \ \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \end{bmatrix}^T \begin{bmatrix} \mathcal{M} & 0 \\ 0 & \mathcal{N} \end{bmatrix} \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \end{bmatrix}, \tag{2.3}$$

where $\mathcal{L} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{>0}$ to be computed; $\mathcal{M} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}$ and $\mathcal{N} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{\geq 0}$ which are determined by the choice of $\mathcal{L}$, are called QOSL matrices.

Since $\phi(0,0) = 0$, it follows that the QOSL condition (2.3) simplifies to the following weak QOSL condition:

$$\langle \mathcal{L}\phi(u_1,u_2), u_1 \rangle \ \leq \ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}^T \begin{bmatrix} \mathcal{M} & 0 \\ 0 & \mathcal{N} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{2.4}$$

Remark 1. *The QOSL condition presented in Definition 2.1 can be verified using the Mean-Value Theorem (MVT). As reported in the literature, two different cases can be identified:*

Case 1: Given a constant matrix $\mathcal{L} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{>0}$, we determine constant matrices $\mathcal{M} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}$ and $\mathcal{N} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{\geq 0}$. As an illustration, consider the function ϕ presented in the numerical example of [23]. The authors first set $\mathcal{L} = I_3$, after which the MVT is applied to obtain:

$$\mathcal{M} = \begin{bmatrix} -\frac{7}{4} & 0 & 0 \\ 0 & -\frac{7}{4} & 0 \\ 0 & 0 & \frac{9}{4} \end{bmatrix}, \ \mathcal{N} = \frac{1}{4} I_3.$$

Case 2: Given a decision-variable matrix $\mathcal{L} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}$ with a fixed structure, the matrices $\mathcal{M} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}$ and $\mathcal{N} \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}$ are determined such that they depend linearly on the entries of $\mathcal{L}$. For example, in the *numerical study of [24], $\mathcal{L}$ is defined as*

$$\mathcal{L} = \begin{bmatrix} \ell_{11} & 0 & 0 \\ 0 & \ell_{22} & \ell_{23} \\ 0 & \ell_{23} & \ell_{33} \end{bmatrix}, \tag{2.5}$$

for the proposed function ϕ. The MVT is then applied, resulting in

$$\begin{aligned} \mathcal{M} &= \begin{bmatrix} 2.25\ell_{11} & 0 & 0 \\ 0 & 0.25\ell_{23} - 1.75\ell_{22} & -2\ell_{23} \\ 0 & -2\ell_{23} & 0.25\ell_{23} - 1.75\ell_{33} \end{bmatrix}, \\ \mathcal{N} &= \begin{bmatrix} 0.25\ell_{11} & 0 & 0 \\ 0 & 0.25\ell_{23} + 0.25\ell_{22} & 0 \\ 0 & 0 & 0.25\ell_{23} + 0.25\ell_{33} \end{bmatrix}. \end{aligned} \tag{2.6}$$

The LMIs $\mathcal{L} > 0$ and $\mathcal{N} \geq 0$ are solved with additional LMIs formulated for the controller design.

Remark 2. *For Case 2 of Remark 1, $\mathcal{L}$ can initially take a general structure, and the structures of $\mathcal{M}$ and $\mathcal{N}$ are determined to ensure that inequality (2.4) is satisfied. The structure of $\mathcal{L}$ is then fixed so that $\mathcal{M}$ and $\mathcal{N}$ depend linearly on its entries. Also, we can fix the structure of $\mathcal{L}$ to simplify the design.*

2.3. OB controller

We consider the following OB controller:

$$\begin{cases} \dot{\hat{\nu}}(t) = A\hat{\nu}(t) + \bar{A}\hat{\nu}(t-\varrho(t)) + \phi(\hat{\nu}(t), \hat{\nu}(t-\varrho(t))) + B\mu(t) + \mathcal{G}(y(t) - \hat{y}(t)) \\ \qquad\quad + \mathcal{H}(y(t-\varrho(t)) - \hat{y}(t-\varrho(t))), \\ \hat{y}(t) = C\hat{\nu}(t), \\ \mu(t) = \mathcal{K}\hat{\nu}(t), \\ \hat{\nu}(t) = \bar{d}(t), t \in [-\bar{\varrho}, 0], \end{cases} \tag{2.7}$$

where $\bar{d}(t)$ is the initial condition. The notation $\hat{\nu}$ represents the estimated value of the state variable ν. This convention applies to all estimated variables throughout this paper.

In the rest, for brevity, the dependence on time t is omitted in each variable, and any variable depending on $t - \varrho(t)$ is denoted with a subscript ϱ.

The augmented system is:

$$\begin{cases} \dot{x} = \mathcal{A}x + \bar{\mathcal{A}}x_{\varrho} + \Phi(\tilde{\nu}), \\ x = D, t \in [-\varrho(t), 0], \end{cases} \tag{2.8}$$

where

$$\begin{aligned} x &= \begin{bmatrix} \nu \\ \nu - \hat{\nu} \end{bmatrix}, \mathcal{A} = \begin{bmatrix} A + B\mathcal{K} & -B\mathcal{K} \\ 0 & A - \mathcal{G}C \end{bmatrix}, \bar{\mathcal{A}} = \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{A} - \mathcal{H}C \end{bmatrix}, \\ \Phi(\tilde{\nu}) &= \begin{bmatrix} \phi(\nu, \nu_{\varrho}) \\ \phi(\nu, \nu_{\varrho}) - \phi(\hat{\nu}, \hat{\nu}_{\varrho}) \end{bmatrix}, D = \begin{bmatrix} d \\ d - \bar{d} \end{bmatrix}, \end{aligned}$$

in which $\tilde{\nu} = \begin{bmatrix} \nu & \nu_{\varrho} & \hat{\nu} & \hat{\nu}_{\varrho} \end{bmatrix}$.

The following definition and lemma will be useful for the subsequent developments.

Definition 2.2. *[8] For given $\sigma \in \mathbb{R}_{>0}$, system (2.8) is σ-exponential (σ−E) stable if $\exists\alpha \in \mathbb{R}_{>0}$ satisfying*

$$\|x(t,D)\| \leq \alpha e^{-\sigma t}\|D\|_s, \ \forall t \in \mathbb{R}_{\geq 0}. \tag{2.9}$$

Lemma 2.1. *[26] Let Λ, Υ, $\mathcal{R}$ and Θ be dimensionally conformable matrices, and $\kappa \in \mathbb{R}$. Provided that*

$$\begin{bmatrix} \Lambda & \kappa\Upsilon + \mathcal{R}^T\Theta^T \\ \bullet & -\kappa(\{\Theta\}_*) \end{bmatrix} < 0, \tag{2.10}$$

then,

$$\Lambda + \{\Upsilon\mathcal{R}\}_* < 0. \tag{2.11}$$

The main objective of this article is to design the OB controller gains $\mathcal{G}$, $\mathcal{H}$ and $\mathcal{K}$ such that system (2.8) achieves σ-E stability.

3. Main results

The design of the OB controller that ensure σ-E stability for system (2.8) follows the algorithm below:

Algorithm 1.

Step 1: *For $\mathcal{L}_1 \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{>0}$, find $\mathcal{M}_1 \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}$ and $\mathcal{N}_1 \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{\geq 0}$ satisfy the following weak QOSL condition:*

$$\langle \mathcal{L}_1\phi(\nu,\nu_\varrho),\nu\rangle \leq \begin{bmatrix} \nu \\ \nu_\varrho \end{bmatrix}^T \begin{bmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{N}_1 \end{bmatrix} \begin{bmatrix} \nu \\ \nu_\varrho \end{bmatrix}, \tag{3.1}$$

and for $\mathcal{L}_2 \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{>0}$, find $\mathcal{M}_2 \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}$ and $\mathcal{N}_2 \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}}_{\geq 0}$ satisfy the following QOSL condition:

$$\langle \mathcal{L}_2(\phi(\nu,\nu_\varrho)-\phi(\hat{\nu},\hat{\nu}_\varrho)),\nu-\hat{\nu}\rangle \leq \begin{bmatrix} \nu-\hat{\nu} \\ \nu_\varrho-\hat{\nu}_\varrho \end{bmatrix}^T \begin{bmatrix} \mathcal{M}_2 & 0 \\ 0 & \mathcal{N}_2 \end{bmatrix} \begin{bmatrix} \nu-\hat{\nu} \\ \nu_\varrho-\hat{\nu}_\varrho \end{bmatrix}. \tag{3.2}$$

Step 2: *Let $\{\mathcal{L}_k,\ \mathcal{M}_k,\mathcal{N}_k,\ k \in \nabla_2\}$ be the obtained matrices. For given $\sigma \in \mathbb{R}_{>0}$, establish the LMI conditions that guarantee the σ-E stability for system (2.8).*

Next, we focus on Step 2 of Algorithm 1, where the LMI conditions are established. We examine two categories of results derived from Step 1 of Algorithm 1.

Case 1: $\{\mathcal{L}_1,\ \mathcal{M}_1,\mathcal{N}_1\}$ are constant matrices such that $\mathcal{L}_1 > 0,\ \mathcal{N}_1 \geq 0$, while $\{\mathcal{L}_2,\ \mathcal{M}_2,\mathcal{N}_2\}$ are decision-variable matrices with a fixed structure.

Case 2: $\{\mathcal{L}_k,\ \mathcal{M}_k,\mathcal{N}_k,\ k \in \nabla_2\}$ are decision-variable matrices with a fixed structure.

Remark 3. *It is important to note that when the matrices are decision-variable matrices with a fixed structure, $\{\mathcal{M}_k,\mathcal{N}_k\}$ depend linearly on $\mathcal{L}_k$, $\forall k \in \nabla_2$. In this case, $\mathcal{L}_k$ and $\mathcal{N}_k \geq 0$ are obtained by solving additional LMIs formulated in Step 2 of Algorithm 1.*

3.1. OB controller design (Case 1)

Theorem 3.1. *For given $\sigma \in \mathbb{R}_{>0}$, system (2.8) is σ-E stable if there are $\{\mathcal{Q}_k \in \mathbb{S}^{p\times p},\ k \in \nabla_2\}$, $\mathcal{K} \in \mathbb{R}^{q\times p}$ and $(\tilde{\mathcal{G}}, \tilde{\mathcal{H}}) \in \mathbb{R}^{p\times r} \times \mathbb{R}^{p\times r}$ such that the following LMIs are satisfied:*

$$\mathcal{L}_2 > 0,\ \mathcal{N}_2 \geq 0, \tag{3.3}$$

$$\mathcal{Q}_k > 0,\ \textit{for } k \in \nabla_2, \tag{3.4}$$

$$\mathbf{V} < 0, \tag{3.5}$$

where

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}(1,1) & -\mathcal{L}_1 B\mathcal{K} & \mathcal{L}_1\bar{A} & 0 \\ \bullet & \mathbf{V}(2,2) & 0 & \mathcal{L}_2\bar{A} - \tilde{\mathcal{H}}C \\ \bullet & \bullet & -(1-\breve{\varrho})e^{-2\sigma\bar{\varrho}}\mathcal{Q}_1 + 2\mathcal{N}_1 & 0 \\ \bullet & \bullet & \bullet & -(1-\breve{\varrho})e^{-2\sigma\bar{\varrho}}\mathcal{Q}_2 + 2\mathcal{N}_2 \end{bmatrix},$$

in which

$$\begin{aligned} \mathbf{V}(1,1) &= \{\mathcal{L}_1 A + \mathcal{L}_1 B\mathcal{K} + \sigma\mathcal{L}_1\}_* + 2\mathcal{M}_1 + \mathcal{Q}_1, \\ \mathbf{V}(2,2) &= \{\mathcal{L}_2 A - \tilde{\mathcal{G}}C + \sigma\mathcal{L}_2\}_* + 2\mathcal{M}_2 + \mathcal{Q}_2. \end{aligned}$$

The OB controller gains are

$$\mathcal{K},\ \mathcal{G} = \mathcal{L}_2^{-1}\tilde{\mathcal{G}},\ \mathcal{H} = \mathcal{L}_2^{-1}\tilde{\mathcal{H}}. \tag{3.6}$$

Furthermore, $x(t, D)$ satisfies

$$\|x(t,D)\| \leq \sqrt{\frac{\overline{\tau}}{\underline{\tau}}} e^{-\sigma t}\|D\|_s, \tag{3.7}$$

where

$$\underline{\tau} = \lambda_{min}(\mathcal{L}),\ \overline{\tau} = \lambda_{max}(\mathcal{L}) + \bar{\varrho}\lambda_{max}(\mathcal{Q}),$$

in which

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_2 \end{bmatrix},\ \mathcal{Q} = \begin{bmatrix} \mathcal{Q}_1 & 0 \\ 0 & \mathcal{Q}_2 \end{bmatrix}. \tag{3.8}$$

Proof. Define the following LK functional:

$$\vartheta(x_t) = \vartheta_1 + \vartheta_2, \tag{3.9}$$

where

$$\vartheta_1 = x^T\mathcal{L}x,\ \vartheta_2 = \int_t^{t+\varrho(t)} e^{2\sigma(k-t-\varrho(t))}x^T(k-\varrho(t))\mathcal{Q}x(k-\varrho(t))dk. \tag{3.10}$$

It follows that

$$\underline{\tau}\|x(t)\|^2 \leq \vartheta(x_t) \leq \overline{\tau}\|x_t\|_s^2. \tag{3.11}$$

$\dot{\vartheta}_1$ and $\dot{\vartheta}_2$ are given as follows:

$$\dot{\vartheta}_1 = 2x^T\mathcal{L}\dot{x} = 2x^T\mathcal{L}\Big(\mathcal{A}x + \bar{\mathcal{A}}x_\varrho + \Phi(\tilde{\nu})\Big), \tag{3.12}$$

$$\dot{\vartheta}_2 = x^TQx - (1-\dot{\varrho}(t))e^{-2\sigma\varrho(t)}x_\varrho^TQx_\varrho - 2\sigma\vartheta_2. \tag{3.13}$$

From (2.2), it follows that

$$\dot{\vartheta}_2 \leq x^TQx - (1-\breve{\varrho})e^{-2\sigma\bar{\varrho}}x_\varrho^TQx_\varrho - 2\sigma\vartheta_2. \tag{3.14}$$

We have

$$2x^T\mathcal{L}\Phi(\tilde{\nu}) = 2\langle\mathcal{L}_1\phi(\nu,\nu_\varrho),\nu\rangle + 2\langle\mathcal{L}_2(\phi(\nu,\nu_\varrho)-\phi(\hat{\nu},\hat{\nu}_\varrho)),\nu-\hat{\nu}\rangle. \tag{3.15}$$

Therefore, from Step 1 of Algorithm 1, it follows that:

$$2x^T\mathcal{L}\Phi(\tilde{\nu}) \leq 2x^T\mathcal{M}x + 2x_\varrho^T\mathcal{N}x_\varrho, \tag{3.16}$$

where

$$\mathcal{M} = \begin{bmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{bmatrix},\ \mathcal{N} = \begin{bmatrix} \mathcal{N}_1 & 0 \\ 0 & \mathcal{N}_2 \end{bmatrix}.$$

The combination of (3.12)–(3.14) and (3.16) implies

$$\dot{\vartheta}(x_t) + 2\sigma\vartheta(x_t) \leq x^T\mathbf{V}x. \tag{3.17}$$

Solving LMI (3.5) leads to

$$\dot{\vartheta}(x_t) \leq -2\sigma\vartheta(x_t), \tag{3.18}$$

which gives

$$\vartheta(x_t) \leq \vartheta(D)e^{-2\sigma t}. \tag{3.19}$$

The validity of (3.7) follows from (3.11) and (3.19). □

Remark 4. *Compared to [23, 24], in addition to guaranteeing exponential stability instead of asymptotic stability, the proposed design in Theorem 3.1 integrates the observer and controller synthesis into a single-step procedure rather than treating them separately, thereby reducing computational complexity. Moreover, a time-varying delay is considered instead of a constant time delay.*

Remark 5. *While condition (3.5) forms an LMI for the first case, it becomes a BMI for the second case due to the presence of the bilinear term* $\mathcal{L}_1\mathrm{B}\mathcal{K}$*. This BMI cannot be eliminated by using the decoupling technique of [25], which is based on pre- and post-multiplying by* $\tilde{\mathcal{L}}_1 = \mathcal{L}_1^{-1}$*. In fact, while posing* $\tilde{Q}_1 = \tilde{\mathcal{L}}_1Q_1\tilde{\mathcal{L}}_1$ *with* Q_1 *independent of* $\mathcal{L}_1$ *does not cause any issues, we cannot similarly set* $\tilde{\mathcal{M}}_1 = \tilde{\mathcal{L}}_1\mathcal{M}_1\tilde{\mathcal{L}}_1$ *when* $\mathcal{M}_1$ *has a fixed structure and depends linearly on the variables of* $\mathcal{L}_1$*.*

3.2. OB controller design (Case 2)

The following Theorem is proposed for the general case where $\{\mathcal{L}_k,\ \mathcal{M}_k, \mathcal{N}_k,\ k \in \nabla_2\}$ are decision-variable matrices with a fixed structure.

Theorem 3.2. *For given $\sigma \in \mathbb{R}_{>0}$ and $\kappa \in \mathbb{R}$, system (2.8) is σ-E stable if there are $\{\mathcal{Q}_k \in \mathbb{S}^{\mathrm{p}\times\mathrm{p}},\ k \in \nabla_2\}$, $\tilde{\mathcal{K}} \in \mathbb{R}^{\mathrm{q}\times\mathrm{p}}$ $(\tilde{\mathcal{G}}, \tilde{\mathcal{H}}) \in \mathbb{R}^{\mathrm{p}\times\mathrm{r}} \times \mathbb{R}^{\mathrm{p}\times\mathrm{r}}$ and $\tilde{\Theta} \in \mathbb{R}^{\mathrm{q}\times\mathrm{q}}$ such that the following LMIs are satisfied:*

$$\mathcal{L}_k \succ 0,\ \mathcal{N}_k \geq 0,\ \textit{for } k \in \nabla_2, \tag{3.20}$$

$$\mathcal{Q}_k \succ 0,\ \textit{for } k \in \nabla_2, \tag{3.21}$$

$$\mathbf{W} \prec 0, \tag{3.22}$$

with

$$\mathbf{W} = \begin{bmatrix} \Lambda & \tilde{\Upsilon} \\ \bullet & -\{\tilde{\Theta}\}_* \end{bmatrix}, \tag{3.23}$$

where

$$\Lambda = \begin{bmatrix} \Lambda(1,1) & -\mathrm{B}\tilde{\mathcal{K}} & \mathcal{L}_1\bar{\mathrm{A}} & 0 \\ \bullet & \Lambda(2,2) & 0 & \Lambda(2,4) \\ \bullet & \bullet & -(1-\breve{\varrho})e^{-2\sigma\bar{\varrho}}\mathcal{Q}_1 + 2\mathcal{N}_1 & 0 \\ \bullet & \bullet & \bullet & -(1-\breve{\varrho})e^{-2\sigma\bar{\varrho}}\mathcal{Q}_2 + 2\mathcal{N}_2 \end{bmatrix},\ \tilde{\Upsilon} = \begin{bmatrix} \tilde{\Upsilon}(1) \\ -\tilde{\mathcal{K}}^T \\ 0 \\ 0 \end{bmatrix},$$

in which

$$\begin{aligned} \Lambda(1,1) &= \{\mathcal{L}_1\mathrm{A} + \mathrm{B}\tilde{\mathcal{K}} + \sigma\mathcal{L}_1\}_* + 2\mathcal{M}_1 + \mathcal{Q}_1, \\ \Lambda(2,2) &= \{\mathcal{L}_2\mathrm{A} - \tilde{\mathcal{G}}\mathrm{C} + \sigma\mathcal{L}_2\}_* + 2\mathcal{M}_2 + \mathcal{Q}_2, \\ \Lambda(2,4) &= \mathcal{L}_2\bar{A} - \tilde{\mathcal{H}}\mathrm{C}, \\ \tilde{\Upsilon}(1) &= \kappa\mathcal{L}_1\mathrm{B} - \mathrm{B}\tilde{\Theta} + \tilde{\mathcal{K}}^T. \end{aligned}$$

The gains are obtained as

$$\mathcal{K} = \Theta^{-1}\tilde{\mathcal{K}},\ \mathcal{G} = \mathcal{L}_2^{-1}\tilde{\mathcal{G}},\ \mathcal{H} = \mathcal{L}_2^{-1}\tilde{\mathcal{H}}, \tag{3.24}$$

with

$$\Theta = \frac{1}{\kappa}\tilde{\Theta}. \tag{3.25}$$

Furthermore, (3.7) is satisfied.

Proof. Condition (3.22) implies $\{\tilde{\Theta}\}_* \succ 0$, so Θ is nonsingular.

In addition, $\tilde{\Upsilon}$ can be written in the form:

$$\tilde{\Upsilon} = \kappa\Upsilon + \mathcal{R}^T\Theta^T, \tag{3.26}$$

where

$$\Upsilon = \begin{bmatrix} \Upsilon(1) \\ 0 \\ 0 \\ 0 \end{bmatrix},\ \mathcal{R} = \Theta^{-1}\begin{bmatrix} \tilde{\mathcal{K}} & -\tilde{\mathcal{K}} & 0 & 0 \end{bmatrix}, \tag{3.27}$$

in which $\Upsilon(1) = \mathcal{L}_1 B - B\Theta$.

Based on (3.25) and (3.26), **W** can be expressed as:

$$\mathbf{W} = \begin{bmatrix} \Lambda & \kappa\Upsilon + \mathcal{R}^T\Theta^T \\ \bullet & -\kappa(\{\Theta\}_*) \end{bmatrix}. \tag{3.28}$$

From the expression of **W** in (3.28), Lemma 2.1 applied to (3.22) leads to the following inequality:

$$\Lambda + \{\Upsilon\mathcal{R}\}_* \prec 0. \tag{3.29}$$

Using $\mathcal{K} = \Theta^{-1}\tilde{\mathcal{K}}$ form (3.24) and the forms of Υ and $\mathcal{R}$ in (3.27), we get:

$$\{\Upsilon\mathcal{R}\}_* = \begin{bmatrix} \{\mathcal{L}_1 B\mathcal{K} - B\tilde{\mathcal{K}}\}_* & -\mathcal{L}_1 B\mathcal{K} + B\tilde{\mathcal{K}} & 0 & 0 \\ \bullet & 0 & 0 & 0 \\ \bullet & \bullet & 0 & 0 \\ \bullet & \bullet & \bullet & 0 \end{bmatrix}, \tag{3.30}$$

which shows that (3.29) can be rewritten as (3.5), thereby ensuring the σ-E stability of the system. □

Remark 6. *Recently, the asymptotic stabilizing OB control of QOSL nonlinear systems with time delay has been investigated in [23–25]. However, exponential stability was not addressed for this class of models. Moreover, the asymptotic stability results can be derived as corollaries of Theorems 3.1 and 3.2 by setting* $\sigma = 0$.

Remark 7. *Authors in [27] focused on designing OB controllers for standard Lipschitz nonlinear systems. In contrast, our study addresses QOSL systems, which reduce conservatism, explicitly account for time-delay effects, and guarantees exponential rather than asymptotic stability.*

Discussion. *Although the proposed approach provides significant advantages by explicitly considering time-delay effects, ensuring exponential instead of asymptotic stability, and addressing quasi-one-sided rather than standard or one-sided Lipschitz conditions, its applicability could be further expanded by extending the framework to more complex problems, such as:*

- *Accounting for matched perturbations, as in [28].*
- *Considering the tracking control problem, as in [29].*
- *Addressing DoS attacks, as in [30, 31].*

4. Numerical example

Consider the QOSL nonlinear system expressed in the form (2.7), where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}, \; \bar{A} = \begin{bmatrix} 0.1 & 0 & 0.2 \\ 0 & 0.1 & 0.2 \\ 0 & 0 & 0.1 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$\phi(\nu, \nu(t-\varrho(t))) = \begin{bmatrix} 2\sin(\nu_1) + 0.5\sin(\nu_1(t-\varrho(t)))) \\ -2\nu_2 + 0.5\sin(\nu_2(t-\varrho(t)))) \\ -2\nu_3 + 0.5\sin(\nu_3(t-\varrho(t)))) \end{bmatrix}.$$

Figure 1 illustrates the dynamic evolution of v_1 for $\varrho(t) = 0.2 + 0.1\sin(t)$, ($\bar{\varrho} = 0.3$ and $\breve{\varrho} = 0.1$), $d = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$, and $\mu = 0$, revealing that the system exhibits divergent behavior, which indicates open-loop instability.

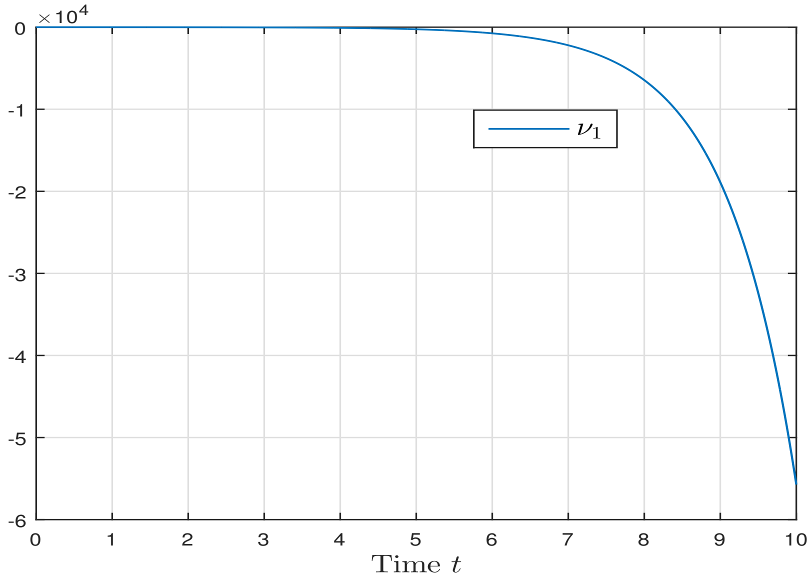

Figure 1. Dynamic evolution of v_1 for $\mu = 0$.

$\phi(v, v(t - \varrho(t)))$ is defined as [24]. Therefore, in accordance with Remark 1, it satisfies the QOSL condition involving the matrices $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{N}$, as described in (2.5) and (2.6).

To solve condition in Theorem 3.1, it is necessary to restrict $\mathcal{L}_1$, $\mathcal{M}_1$ and $\mathcal{N}_1$ to constant matrices. For example, we can fix $\ell_{11} = \ell_{22} = \ell_{33} = 1$ and $\ell_{23} = 0$. However, $\mathcal{L}_2$, $\mathcal{M}_2$ and $\mathcal{N}_2$ are allowed to be decision-variable matrices with a fixed structure, which will be computed in the subsequent design process.

For Theorem 3.2, $\mathcal{L}_1$, $\mathcal{M}_1$ and $\mathcal{N}_1$ are treated as decision-variable matrices with a fixed structure, rather than being restricted to constant values.

We set $\kappa = 100$ when solving the conditions in Theorem 3.2. Table 1 lists the comparison results on the maximum allowed decay rate σ via Theorems 3.1 and 3.2. In this table, "Inf" means that the corresponding LMIs are not feasible, while "N/A" indicates that the LMIs cannot be applied. The theorems in [23, 24] are applicable only when the delay is constant, that is, when $\breve{\varrho} = 0$. Furthermore, as they guarantee asymptotic stability rather than exponential stability, the maximum allowable decay rate is $\sigma = 0$. It is evident that Theorem 3.2 yields less conservative results than Theorem 3.1. This improvement is due to the greater freedom in selecting the matrices $\mathcal{L}_1$, $\mathcal{M}_1$ and $\mathcal{N}_1$.

Table 1. Comparison of maximum allowable decay rates σ.

	$\bar{\varrho} = 0.3$ $\check{\varrho} = 0.15$	$\bar{\varrho} = 0.3$ $\check{\varrho} = 0.1$	$\bar{\varrho} = 0.3$ $\check{\varrho} = 0$
Max σ via Theorem 3.1	Inf	0.009	0.04
Max σ via Theorem 3.2	0.05	0.07	0.1
Max σ via Theorems in [23, 24]	N/A	N/A	0

The feasible solution of Theorem 3.1 for $\varrho(t) = 0.2 + 0.1\sin(t)$ ($\bar{\varrho} = 0.3$ and $\check{\varrho} = 0.1$) and $\sigma = 0.07$ is:

$$\mathcal{L}_1 = \begin{bmatrix} 0.2076 & 0 & 0 \\ 0 & 17.0555 & -6.0637 \\ 0 & -6.0637 & 6.1028 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} 10.7356 & 0 & 0 \\ 0 & 10.2840 & 1.0018 \\ 0 & 1.0018 & 10.5079 \end{bmatrix},$$

$$\mathcal{K} = \begin{bmatrix} -77.7114 & -0.2658 & -0.0215 \end{bmatrix}, \mathcal{H} = \begin{bmatrix} 4.4603 & -0.0042 \\ 0.0683 & 0.1410 \\ 0.0306 & -0.0960 \end{bmatrix}, \mathcal{G} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0 & 0 \end{bmatrix}.$$

Figures 2–4 illustrate the system states and their estimates for initial conditions $d = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ and $\bar{d} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$. As shown in the figures, each system state converges to zero while the corresponding observer estimates closely track the true states, causing the estimation errors to converge to zero. This illustrates that the proposed OB controller simultaneously achieves accurate state estimation and system stabilization.

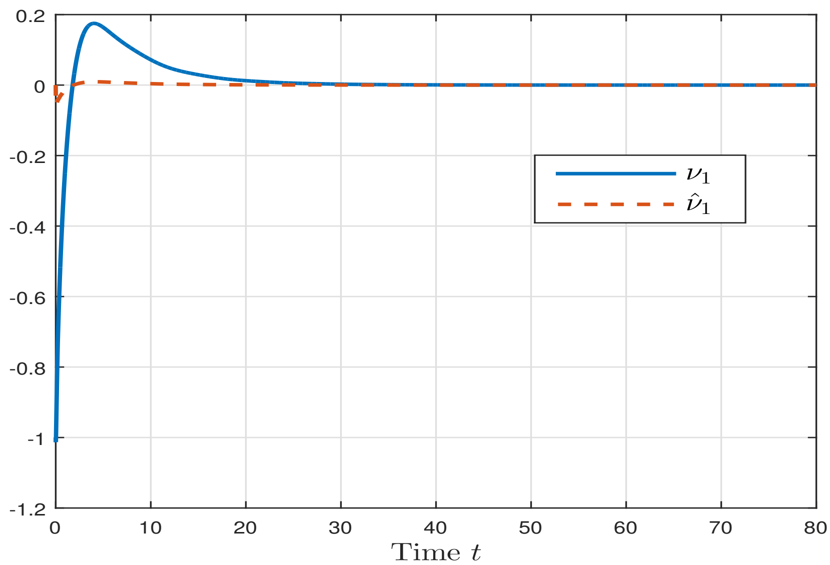

Figure 2. Dynamic evolution of ν_1 and $\hat{\nu}_1$.

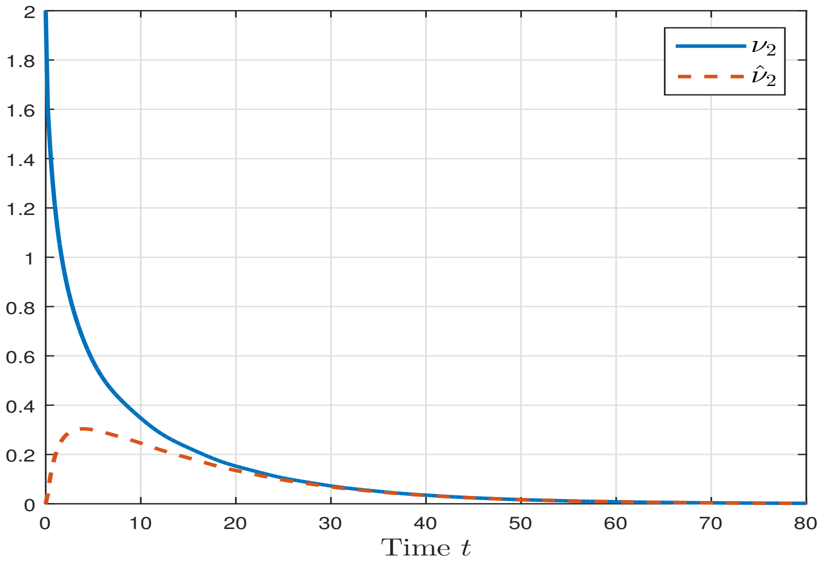

Figure 3. Dynamic evolution of ν_2 and $\hat{\nu}_2$.

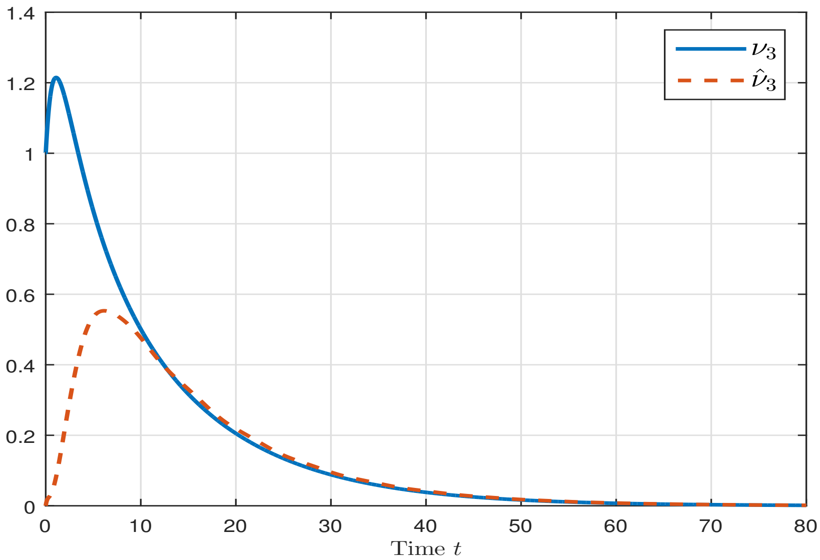

Figure 4. Dynamic evolution of ν_3 and $\hat{\nu}_3$.

The dynamic evolution of μ over time is depicted in Figure 5.

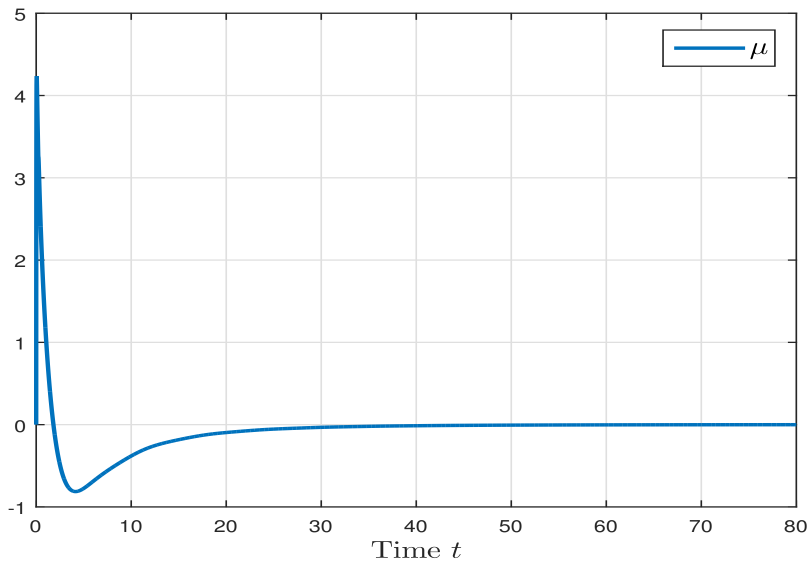

Figure 5. Dynamic evolution of μ.

5. Conclusions

This paper addresses the problem of OB control design for QOSL systems with time varying delay. The primary objective is to derive sufficient conditions in the form of LMI to guarantee exponential stability of the resulting closed-loop system. We first establish an LMI condition by constructing an appropriate Lyapunov-Krasovskii functional. This initial derivation applies to the case where the matrices ensuing the QOSL condition for the controller are constant. In this first design, the main advantage is that the controller and observer gains are computed simultaneously in a single step, which significantly reduces the computational burden. However, a key drawback is that the matrices satisfying the QOSL condition for the controller are constrained to be constant. This restriction offers no design freedom and leads to conservative results. This limitation is addressed through the application of a specific decoupling methodology, which alleviates the aforementioned conservatism. The efficacy of the proposed method is validated through a numerical example, which also illustrates the comparative advantage of the second design over the initial one. As a future perspective, the framework could be extended to explicitly account for the effects of input saturation in the stability analysis, for example by incorporating saturation nonlinearities into the model or by integrating anti-windup strategies into the controller design.

Author contributions

O. Kahouli: Methodology, writing original draft preparation; L. El Amraoui: Conceptualization, software; M. Ayari: visualization, validation; H. Gassara: methodology, writing original draft preparation; A. El Hajjaji: formal analysis, supervision; All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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