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Research article

Hyers-Ulam stability of coupled systems for stochastic differential equations with random impulses driven by Poisson jumps

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Abstract: The problem of existence, uniqueness, and stability in the Hyers-Ulam sense of solutions for random impulsive stochastic functional differential equations driven by Poisson jumps with finite delays is considered. Based on techniques combining the generalized Banach fixed-point theorem and the expansion of the Perov-type fixed-point theorem, two significant quantitative and qualitative results are analyzed, and then an example is presented to illustrate our results.

Keywords: stochastic analysis; impulsive differential equations; matrix convergent to zero; iterative methods; Hyers-Ulam stability; Wiener process; Poisson process
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1. Introduction and position of problem

Although the first studies considering the impact of aftereffects/delays on the dynamics of systems of various classes began in the mid-20th century, such studies have only recently started to develop intensively, primarily due to practical needs. Many areas of application for such problems can be found in real life, including, in particular, management, mechanics, physics, chemistry, economics, biology, medicine, nuclear energy, and information theory; please see [1–3].

Many real phenomena and complex processes allow for their mathematical formulation in the form of abstract differential equations with impulse action in the case of an infinite phase space. Examples of such abstract equations include delay, hysteresis, and distributed parameter systems. Impulse ordinary differential equations and systems of such evolution equations are studied in depth in [4–6], where a classification of such systems is provided depending on the nature of the actions. There exist three different classes (types) of such systems: - systems subjected to impulse action at fixed moments in time; - systems subjected to impulse action at the moment when the representative point P hits the given surfaces of the extended phase space; - discontinuous dynamic systems; see [7–9]. For the abstract equations, one cannot proceed from this classification; therefore, it is necessary to introduce the concept of a solution in a different way. Hence, analysis of the qualitative behavior of the stability of such a system has attracted many researchers in recent years; see [10–12].

Differential and integral equations have become standard models for financial quantities, such as asset prices, interest rates, and their derivatives. Unlike deterministic models, such as ordinary differential equations, which have a unique solution for each corresponding initial condition, stochastic differential equations have solutions that are continuous-time stochastic processes. Methods for solving stochastic differential equations are based on similar techniques used for solving ordinary differential equations but are generalized to accommodate stochastic dynamics; see [13–15]. A stochastic system for drug distribution in a different real-life context can be found in [16, 17].

This paper deals with a coupled system of impulsive stochastic functional differential equations given for $t \geq t_0$ as

$$\begin{cases} dy(t) &= f_1(t, y_t, z_t) + g_1(t, y_t, z_t)dB(t) + \int_{\mathcal{A}} h_1(t, y_t, z_t, s)\tilde{\mathcal{K}}(dt, ds), \quad t \neq \lambda_k \\ dz(t) &= f_2(t, y_t, z_t) + g_2(t, y_t, z_t)dB(t) + \int_{\mathcal{A}} h_2(t, y_t, z_t, s)\tilde{\mathcal{K}}(dt, ds), \quad t \neq \lambda_k \\ y(\lambda_k) &= b_k^1(\zeta_k)y(\lambda_k^-), \quad k \in \mathbb{N}, \\ z(\lambda_k) &= b_k^2(\zeta_k)z(\lambda_k^-), \quad k \in \mathbb{N}, \\ y_{t_0} &= \varpi_1 = \{\varpi_1(\alpha) : \alpha \in [-t_1, 0]\} \\ z_{t_0} &= \varpi_2 = \{\varpi_2(\alpha) : \alpha \in [-t_1, 0]\}. \end{cases} \tag{1.1}$$

Let Ω be an open domain of $\mathbb{R}^d$, $d \geq 1$, $t_1 > 0$. Here ζ_k is a random variable defined from Ω to $M_k = (0, d_k), \forall k \in \mathbb{N}$, where $0 < d_k < +\infty$. Assume that ζ_i and ζ_j are independent for $i \neq j$, $i, j \in \{1, 2\}, \ldots$. Suppose that $T \in (t_0, +\infty)$; the functional

$$f_1, f_2 : [t_0, T] \times \mathcal{D}_0 \times \mathcal{D}_0 \to \mathbb{R}^d,$$

$$g_1, g_2 : [t_0, T] \times \mathcal{D}_0 \times \mathcal{D}_0 \to \mathbb{R}^{d\times m},$$

$$h_1, h_2 : [t_0, T] \times \mathcal{D}_0 \times \mathcal{A} \to \mathbb{R}^d,$$

and

$$b_k^1, b_k^2 : M_k \to \mathbb{R}^{d\times d},$$

are Borel functions that are measurable. Let $(\Omega, \mathcal{P}, \mathcal{P}_t, \mathbb{P})$ be a complete probability space, furnished with a family of continuous and increasing σ-algebras $\{\mathcal{P}_t, t \in J\}$ such that $\mathcal{P}_t \subset \mathcal{P}$. By $u_t = (y_t, z_t)$ we mean the segment solution; if

$$y(.,.), z(.,.) : [-t_1, T] \times \Omega \to \mathbb{R}^d,\ t_1 > T,$$

then

$$y_t(.,.), z_t(.,.) : [-t_1, 0] \times \Omega \to \mathbb{R}^d,$$

is defined as

$$y_t(\alpha,\beta) = y(t+\alpha,\beta), \qquad \alpha \in [-t_1, 0],\ \beta \in \Omega,$$

and

$$z_t(\alpha,\beta) = z(t+\alpha,\beta), \qquad \alpha \in [-t_1, 0],\ \beta \in \Omega.$$

Let us define $\mathcal{D}_0$ as the space of all piecewise continuous processes

$$\varpi_l : [-t_1, 0] \times \Omega \to \mathbb{R}^d,\ l = \{1, 2\},$$

such that $\varpi_l(\alpha, .)$ is $\mathcal{P}_0$-measurable for $\alpha \in [-t_1, 0]$ where

$$\int_{-t_1}^{0} \mathbb{E}|\varpi_l(t)|^2 dt < \infty.$$

In $\mathcal{D}_0$, the norm:

$$\|\varpi_l(t)\|^2_{\mathcal{D}_0} = \int_{-t_1}^{0} \mathbb{E}|\varpi_l(t)|^2 dt,$$

is considered. For $T > 0$, we introduce the space

$$\mathcal{D}_T = \{y : y \in C(-t_1, T; L^2(\Omega; \mathbb{R}^d)) : \sup_{[0,T]} \mathbb{E}(|y(t)|^2) < \infty, \quad \text{such that} \quad \int_{-t_1}^{0} \mathbb{E}|\varpi_1(t)|^2 dt < \infty\},$$

with the norm

$$\|u\|_{\mathcal{D}_T} = \sup_{[0,T]} \sqrt{(\mathbb{E}(|u(t)|^2)} + \|\varpi_1\|_{\mathcal{D}_0}.$$

The impulsive moments λ_k form a strictly increasing sequence

$$\lambda_0 < \lambda_1 < \lambda_2 < ... < \lambda_k < ... < \lim_{k\to\infty} \lambda_k = \infty,$$

and

$$y(\lambda_k^-) = \lim_{t\to\lambda_{k-0}} y(t).$$

Let

$$\lambda_0 = t_0,\ \lambda_k = \lambda_{k-1} + \zeta_k,\ k = 1, 2, \ldots.$$

Then $\{\lambda_k\}$ is a process with independent increments. Let $\{\mathcal{K}(t), \quad t \geq 0\}$ be the simple counting process generated by $\{\lambda_k\}$, and let $\{B(t), t \geq 0\}$ be a given m-dimensional Brownian motion. Denoting by $\mathcal{P}_t^{(1)}$ the σ-algebra generated by $\{\mathcal{K}(t) : t \geq 0\}$, and let $\mathcal{P}_t^{(2)}$ be the σ-algebra generated by $\{B(s) : s \geq t\}$; see [18–20]. We have

$$u \wedge v = \min(u, v) \quad \text{and} \quad u \vee v = \max(u, v).$$

The work [21] investigated the stability of multi-linked stochastic delayed complex networks that are subject to stochastic hybrid impulses. This study utilized Dupire's functional Itô calculus to analyze systems with path-dependent dynamics and impulsive effects, focusing primarily on the stability of such networks under stochastic hybrid impulses and time delays. In [22], semi-global synchronization of stochastic mixed time-delay systems influenced by Lévy noise is considered. The authors proposed a control strategy based on aperiodic intermittent delayed sampled-data control, aiming to achieve synchronization despite the challenges posed by jump noise and mixed delays. These two works used Dupire's functional Itô calculus and specialized control methodologies for synchronization. Our manuscript, on the other hand, develops maximal inequalities for stochastic integrals with respect to compensated Poisson random measures and investigates the filtration structures related to Lévy jumps, thereby providing fundamental probabilistic tools.

This article is structured as follows. Some useful notation and necessary preliminaries are stated in Section 2, and the main results related to quantitative studies are presented and proved in Section 3. We provide the type of stability of the problem in Section 4.

2. Preliminaries and fixed-point results

Let $(\Omega, \mathcal{P}, \mathbb{P})$ be a probability space with filtration $\mathcal{P}_t\}$, $t \geq 0$ such that $\mathcal{P}_t = \mathcal{P}_t^{(1)} \vee \mathcal{P}_t^{(2)}$ and denoting by $\mathcal{B}_\sigma(\mathbb{R}^d)$ the Borel σ-algebra of $\mathbb{R}^d$; see [23–25]. Let $(p(t))_{t\geq 0}$ be an $\mathbb{R}^d$-valued, σ-finite stationary $\mathfrak{F}_t$-adapted Poisson point process on $(\Omega, \mathfrak{F}, \{\mathfrak{F}\}_{t\geq 0}, \mathbb{P})$. The counting random measure $\mathcal{K}$ is given by

$$\mathcal{K}((t_1, t_2], \times \mathcal{A})(\beta) = \sum_{t_1 < s \leq t_2} 1_{\mathcal{A}}(p(s)(\beta)), \text{ for any } \mathcal{A} \in \mathcal{B}_\sigma(\mathbb{R}^d - \{0\}).$$

where $0 \notin \bar{\mathcal{A}}$. We will denote this by $\mathcal{K}(t, \mathcal{A}) = \mathcal{K}((0, t] \times \mathcal{A})$. A σ-finite measure $\mathcal{A}$ is given by

$$\begin{cases} E(\mathcal{K}(t, \mathcal{A})) = \tau(\mathcal{A})t, \\ \mathbb{P}(\mathcal{K}(t, \mathcal{A}) = k) = \frac{e^{-t\tau(\mathcal{A}))(t\tau(\mathcal{A}))^k}}{k!}, \end{cases}$$

where the measure τ is the Lévy measure. The compensated Poisson random measure $\tilde{\mathcal{K}}$ is given as

$$\tilde{\mathcal{K}}((0, t] \times \mathcal{A}) = \mathcal{K}((0, t] \times \mathcal{A})t\tau(\mathcal{U}).$$

Here, $dt\tau(\mathcal{A})$ is the compensator; for more details, see [26].

Theorem 2.1. *[27, Theorem 4.4.23] Let $p > 0$. Then, for all processes y, we have*

$$\int_0^t |Y|^2(s)ds < \infty, \ t \in [0, \infty),$$

we have

$$c_p \mathbb{E}\left(\int_0^t |Y|^2(s)ds\right)^{\frac{p}{2}} \leq \mathbb{E}\left(\sup_{s\in[0,t]} \int_0^t y(s)dB(s)\right)^p \leq C_p \mathbb{E}\left(\int_0^t |Y|^2(s)ds\right)^{\frac{p}{2}}, \tag{2.1}$$

where $c_p, C_p > 0$.

Lemma 2.1. *[28, Theorem 7.3.2] Let $p \in \{1,2\}$ and $\sigma : \mathbb{R}^+ \times \mathbb{R}^d \to \mathbb{R}^d$ be measurable functions (progressively) such that*

$$\int_0^t E\left(\int_{\mathcal{A}} |\sigma(s,y)|^p \tau(dy)ds\right) < \infty.$$

Then, for $\bar{c}_p > 0$, we have

$$\mathbb{E}\left[\sup_{0\le s\le t}\left(\int_0^s \int_Z \sigma(s,y)\tilde{\mathcal{K}}(ds,dy)\right)^p\right] \le \bar{c}_p \mathbb{E}\int_0^t \int_Z |\sigma(s,y)|^p \tau(dy)ds. \tag{2.2}$$

The classical contraction principles were improved for contractive maps in spaces endowed with vector-valued metrics, and the fixed point theorem in a complete generalized metric space can be found in [29–31].

Definition 2.1. *[32] Let $T \in (t_0, +\infty)$, an $\mathbb{R}^d \times \mathbb{R}^d-$ valued stochastic process $u = (y,z) \in \mathcal{D}_T \times \mathcal{D}_T$, be a solution of (1.1) in $(\Omega, \mathcal{P}, \mathbb{P})$, if*

1) *$u(t)$ is $\mathcal{P}_t$-adapted for $t \ge t_0$,*
2) *$u(t)$ is right continuous and has a limit on the left almost surely;*
3) *$u(t_0 + s) = (\varpi_1(s), \varpi_2(s))$ for $s \in [-t_1, 0]$*
4) *u(t) satisfies, for $t \in [-t_1, T], \beta \in \Omega$*

$$\mathbb{P}\left(\int_0^t \int_{\mathcal{A}} |h_l(r, y_r, z_r, s)|_y^2 \tau(ds)dr < \infty\right) = 1,$$

$$\begin{cases} y(t) = & \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i^1(\zeta_i)\varpi_1(0) + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} f_1(s,y_s,z_s)ds + \int_{\lambda_k}^{t} f_1(s,y_s,z_s)ds \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} g_1(s,y_s,z_s)d\beta(s) + \int_{\lambda_k}^{t} g_1(s,y_s,z_s)dB(s) \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}} h_1(r,y_r,z_r,s)\tilde{\mathcal{K}}(dr,ds) + \int_{\lambda_k}^{t}\int_{\mathcal{A}} h_1(r,y_r,z_r,s)\tilde{\mathcal{K}}(dr,ds)\Bigg] \\ & \times I_{(\lambda_k,\lambda_{k-1}]}(t), \quad \mathbb{P}-a.s, \\ z(t) = & \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i^2(\zeta_i)\varpi_2(0) + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} f_2(s,y_s,z_s)ds + \int_{\lambda_k}^{t} f_2(s,y_s,z_s)ds \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} g_2(s,y_s,z_s)d\beta(s) + \int_{\lambda_k}^{t} g_2(s,y_s,z_s)dB(s) \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}} h_2(r,y_r,z_r,s)\tilde{\mathcal{K}}(dr,ds) + \int_{\lambda_k}^{t}\int_{\mathcal{A}} h_2(r,y_r,z_r,s)\tilde{\mathcal{K}}(dr,ds)\Bigg] \\ & \times I_{(\lambda_k,\lambda_{k-1}]}(t), \quad \mathbb{P}-a.s, \end{cases} \tag{2.3}$$

here

$$\prod_{j=i}^{k} b_j(\zeta_j) = b_k(\zeta_k)b_{k-1}(\zeta_k - 1),$$

and $I_G(.)$ is given by

$$I_E(t) = \begin{cases} 1, & \text{if } t \in E, \\ 0, & \text{if } t \notin E. \end{cases}$$

Definition 2.2. *[33] Let $(\bar{y}(t), \bar{z}(t))$ be an $\mathbb{R}^d \times \mathbb{R}^d$–valued stochastic process. We say that the coupled impulsive stochastic functional differential system* (1.1) *is* Hyers-Ulam stable *if there exists a constant $\eta > 0$ such that for every $\varepsilon_1, \varepsilon_2 > 0$, whenever a pair of adapted processes $(\bar{y}, \bar{z})$ satisfies, for all $t \in [t_0, T]$,*

$$\mathbb{E}\left\|\bar{y}(t) - \Phi_1(\bar{y}, \bar{z})(t)\right\|^2 \le \varepsilon_1, \qquad \mathbb{E}\left\|\bar{z}(t) - \Phi_2(\bar{y}, \bar{z})(t)\right\|^2 \le \varepsilon_2,$$

where Φ_1 and Φ_2 denote the stochastic integral operators associated with system (1.1). *Then there exists an exact solution $(y(t), z(t))$ of* (1.1) *such that*

$$\mathbb{E}\|y(t) - \bar{y}(t)\|^2 \le \eta \max\{\varepsilon_1, \varepsilon_2\}, \qquad \mathbb{E}\|z(t) - \bar{z}(t)\|^2 \le \eta \max\{\varepsilon_1, \varepsilon_2\}, \quad \forall\, t \in [t_0, T].$$

3. Quantitative analysis of solution

Here, we consider the existence and uniqueness of a mild solution for (1.1) based on the Perov fixed point theorem. The following hypotheses are assumed.

(H_1) There exist constants $a_{f_l},\ b_{f_l}, c_{f_l} \in \mathbb{R}^+$ for each $l = 1, 2, \forall t \in [t_0, T]$, such that

$$\begin{cases} \mathbb{E}\|f_l(t, y, z) - f_l(t, \bar{y}, \bar{z})\|^2 \le a_{f_l}\|y - \bar{y}\|^2_{\mathcal{D}_0} + b_{f_l}\|z - \bar{z}\|^2_{\mathcal{D}_0}, \\ \mathbb{E}\|f_l(t, y, z)\|^2 \le c_{f_l}\left(1 + \|y\|^2_{\mathcal{D}_0} + \|z\|^2_{\mathcal{D}_0}\right), \end{cases}$$

where $y, z, \bar{y}, \bar{z} \in \mathcal{D}_0$.

(H_2) Let $g \in L^p\left([t_0, T] \times \mathcal{D}_0 \times \mathcal{D}_0 : \mathbb{R}^{d\times m}\right)$ be continuous function. Then, there exist constants $a_{g_l},\ b_{g_l}, c_{g_l} \in \mathbb{R}^+$ for each $l \in \{1, 2\}$, for all $t \in [t_0, T]$ such that

$$\begin{cases} \mathbb{E}\|g_l(t, y, z) - g_l(t, \bar{y}, \bar{z})\|^2 \le a_{g_l}\|y - \bar{y}\|^2_{\mathcal{D}_0} + b_{g_l}\|z - \bar{z}\|^2_{\mathcal{D}_0}, \\ \mathbb{E}\|g_l(t, y, z)\|^2 \le c_{g_l}\left(1 + \|y\|^2_{\mathcal{D}_0} + \|z\|^2_{\mathcal{D}_0}\right), \end{cases}$$

where $y, z, \bar{y}, \bar{z} \in \mathcal{D}_0$.

(H_3) Let $h_l : [t_0, T] \times \mathcal{D}_0 \times \mathcal{D}_0 \times \mathcal{A} \to \mathbb{R}^d$ be a function. Then there exist constants $a_{h_l},\ b_{h_l}, c_{h_l} \in \mathbb{R}^+$ for each $l \in \{1, 2\}$, and $t \in [t_0, T]$ where

$$\begin{cases} \int_{\mathcal{A}} \mathbb{E}\|h_l(t, y, z, s) - h_l(t, \bar{y}, \bar{z}, z)\|^2 \tau(ds) \le a_{h_l}\|y - \bar{y}\|^2_{\mathcal{D}_0} + b_{h_l}\|z - \bar{z}\|^2_{\mathcal{D}_0}, \\ \int_{\mathcal{A}} \mathbb{E}\|h_l(t, y, z, s)\|^2 \tau(ds) \le c_{h_l}\left(1 + \|y\|^2_{\mathcal{D}_0} + \|z\|^2_{\mathcal{D}_0}\right). \end{cases}$$

(H_4) The condition

$$\mathbb{E}\left(\max_{i,k}\left\{\prod_{j=i}^{k}\|b_j(\zeta_j)\|\right\}\right)^2,$$

is uniformly bounded. Then, there exist constants $\bar{B} > 0$ where

$$\mathbb{E}\left(\max_{i,k}\left\{\prod_{j=i}^{k}\|b_j(\zeta_j)\|\right\}\right)^2 \le \bar{B},$$

and $\zeta_j \in D_j,\ j \in \{1, 2, \ldots\}$.

Theorem 3.1. *Let* (H_1)–(H_4) *hold and introduce the matrix*

$$M_{trice} = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_3 \end{pmatrix}, \ m_j \geq 0, \quad j = 1, 2, 3, 4,$$

where

$$m_1 = \left(3\max\{1, \bar{B}\}\left((t-t_0)^2 a_{f_1} + C_2(t-t_0)a_{g_1} + \bar{c}_2(t-t_0)^2 a_{h_1}\right)\right)^{\frac{1}{2}},$$

$$m_2 = \left(3\max\{1, \bar{B}\}\left((t-t_0)^2 b_{f_1} + C_2(t-t_0)b_{g_1} + \bar{c}_2(t-t_0)^2 b_{h_1}\right)\right)^{\frac{1}{2}},$$

and

$$m_3 = \left(3\max\{1, \bar{B}\}\left((t-t_0)^2 a_{f_2} + C_2(t-t_0)a_{g_2} + \bar{c}_2(t-t_0)^2 a_{h_2}\right)\right)^{\frac{1}{2}},$$

$$m_4 = \left(3\max\{1, \bar{B}\}\left((t-t_0)^2 b_{f_2} + C_2(t-t_0)b_{g_2} + \bar{c}_2(t-t_0)^2 b_{h_2}\right)\right)^{\frac{1}{2}}.$$

If M converges to 0*, then* (1.1) *admits a unique solution.*

Proof. Consider the operator

$$\mathcal{K} : \mathcal{D}_T \times \mathcal{D}_T \to \mathcal{D}_T \times \mathcal{D}_T,$$

defined by

$$\mathcal{K}(y,z) = (\mathcal{K}_1(y,z), \mathcal{K}_2(y,z)), \ (y,z) \in \mathcal{D}_T \times \mathcal{D}_T,$$

where

$$\mathcal{K}_1(y,z) = \begin{cases} \varpi_1(t), \quad t \in [-t_1, 0] \\ \sum\limits_{k=0}^{+\infty} \Bigg[\prod\limits_{i=1}^{k} b_i^1(\zeta_i)\varpi_1(0) + \sum\limits_{i=1}^{k}\prod\limits_{j=i}^{k} b_i^1(\zeta_j) \int_{\lambda_{i-1}}^{\lambda_i} f_1(s, y_s, z_s)ds + \int_{\lambda_k}^{t} f_1(s, y_s, z_s)ds \\ + \sum\limits_{i=1}^{k}\prod\limits_{j=i}^{k} b_i^1(\zeta_j) \int_{\lambda_{i-1}}^{\lambda_i} g_1(s, y_s, z_s)dB(s) + \int_{\lambda_k}^{t} g_1(s, y_s, z_s)dB(s) \\ + \sum\limits_{i=1}^{k}\prod\limits_{j=i}^{k} b_i^1(\zeta_j) \int_{\lambda_{i-1}}^{\lambda_i} \int_{\mathcal{A}} h_1(r, y_r, z_r, s)\tilde{\mathcal{K}}(dr, ds) + \int_{\lambda_k}^{t} \int_{\mathcal{A}} h_1(r, y_r, z_r, s)\tilde{\mathcal{K}}(dr, ds) \Bigg] \\ \times I_{(\lambda_k, \lambda_{k-1}]}(t), \quad \mathbb{P} - a.s, \end{cases}$$

and

$$\mathcal{K}_2(y,z) = \begin{cases} \varpi_2(t), \quad t \in [-t_1, 0] \\ \sum\limits_{k=0}^{+\infty} \Bigg[\prod\limits_{i=1}^{k} b_i^2(\zeta_i)\varpi_2(0) + \sum\limits_{i=1}^{k}\prod\limits_{j=i}^{k} b_i^2(\zeta_j) \int_{\lambda_{i-1}}^{\lambda_i} f_2(s, y_s, z_s)ds + \int_{\lambda_k}^{t} f_2(s, y_s, z_s)ds \\ + \sum\limits_{i=1}^{k}\prod\limits_{j=i}^{k} b_i^2(\zeta_j) \int_{\lambda_{i-1}}^{\lambda_i} g_2(s, y_s, z_s)dB(s) + \int_{\lambda_k}^{t} g_2(s, y_s, z_s)dB(s) \\ + \sum\limits_{i=1}^{k}\prod\limits_{j=i}^{k} b_i^2(\zeta_j) \int_{\lambda_{i-1}}^{\lambda_i} \int_{\mathcal{A}} h_2(r, y_r, z_r, s)\tilde{\mathcal{K}}(dr, ds) + \int_{\lambda_k}^{t} \int_{\mathcal{A}} h_2(r, y_r, z_r, s)\tilde{\mathcal{K}}(dr, ds) \Bigg] \\ \times I_{(\lambda_k, \lambda_{k-1}]}(t), \quad \mathbb{P} - a.s. \end{cases}$$

As in [32], we shall use the fixed-point theorem in a complete generalized metric space to prove that $\mathcal{K}$ has a fixed point. In fact, let $(y,z), (\overline{y}, \overline{z}) \in \mathcal{D}_T \times \mathcal{D}_T$. Then, for $t \in [t_0 - t_1, T]$, we get

$$\mathbb{E}\|\mathcal{K}_1(y,z)(t) - \mathcal{K}_1(\bar{y}, \bar{z})(t)\|^2$$

$$
\begin{aligned}
\leq\ & 3\mathbb{E}\left[\max_{i,k}\left\{1,\prod_{j=i}^{k}\|b_j^1(\zeta_j)\|\right\}\right]^2\left(\int_{t_0}^{t}\mathbb{E}\|f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)\|ds\right)^2\\
&+3\mathbb{E}\left[\max_{i,k}\left\{1,\prod_{j=i}^{k}\|b_j^1(\zeta_j)\|\right\}\right]^2\left(\int_{t_0}^{t}\mathbb{E}\|g_1(s,y_s,z_s)-g_1(s,\bar{y}_s,\bar{z}_s)\|dB(s)\right)^2\\
&+3\mathbb{E}\left[\max_{i,k}\left\{1,\prod_{j=i}^{k}\|b_j^1(\zeta_j)\|\right\}\right]^2\left(\int_{t_0}^{t}\mathbb{E}\|h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)\|\tilde{\mathcal{K}}(dr,ds)\right)^2.
\end{aligned}
$$

By (2.2) and Burkholder-type inequality, we obtain

$$
\begin{aligned}
&\mathbb{E}\|\mathcal{K}_1(y,z)(t)-\mathcal{K}_1(\bar{y},\bar{z})(t)\|^2\\
\leq\ & 3\max\{1,\bar{B}\}(t-t_0)^2\left(a_{f_1}\|y_r-\bar{y}_r\|^2_{\mathcal{D}_0}+b_{f_1}\|z_r-\bar{z}_r\|^2_{\mathcal{D}_0}\right)\\
&+3\max\{1,\bar{B}\}C_2(t-t_0)\left(a_{g_1}\|y_r-\bar{y}_r\|^2_{\mathcal{D}_0}+b_{g_1}\|z_r-\bar{z}_r\|^2_{\mathcal{D}_0}\right)\\
&+3\max\{1,\bar{B}\}\bar{c}_2(t-t_0)^2\left(a_{h_1}\|y_r-\bar{y}_r\|^2_{\mathcal{D}_0}+b_{h_1}\|z_r-\bar{z}_r\|^2_{\mathcal{D}_0}\right).
\end{aligned}
$$

Since $(y,z)=(\bar{y},\bar{z})$ in $[-t_1,0]$, then

$$
\|\mathcal{K}_1(y,z)-\mathcal{K}_1(\bar{y},\bar{z})\|^2_{\mathcal{D}_T}\leq m_1^2\|y-\bar{y}\|^2_{\mathcal{D}_T}+m_2^2\|z-\bar{z}\|^2_{\mathcal{D}_T},
$$

and

$$
\|\mathcal{K}_2(y,z)-\mathcal{K}_2(\bar{y},\bar{z})\|^2_{\mathcal{D}_T}\leq m_3^2\|y-\bar{y}\|^2_{\mathcal{D}_T}+m_4^2\|z-\bar{z}\|^2_{\mathcal{D}_T}.
$$

Then

$$
\begin{aligned}
\|\mathcal{K}(y,z)-\mathcal{K}(\bar{y},\bar{z})\|_{\mathcal{D}_T} &= \begin{pmatrix}\|\mathcal{K}_1((y,z)-\mathcal{K}_1(\bar{y},\bar{z})\|_{\mathcal{D}_T}\\ \|\mathcal{K}_2(y,z)-\mathcal{K}_2(\bar{y},\bar{z})\|_{\mathcal{D}_T}\end{pmatrix}\\
&\leq \begin{pmatrix}m_1 & m_2\\ m_3 & m_4\end{pmatrix}\begin{pmatrix}\|y-\bar{y}\|_{\mathcal{D}_T}\\ \|z-\bar{z}\|_{\mathcal{D}_T}\end{pmatrix}.
\end{aligned}
$$

Therefore, we arrive at

$$
\|\mathcal{K}(y,z)-\mathcal{K}(\bar{y},\bar{z})\|_{\mathcal{D}_T}\leq M_{trice}\begin{pmatrix}\|y-\bar{y}\|_{\mathcal{D}_T}\\ \|z-\bar{z}\|_{\mathcal{D}_T}\end{pmatrix},\ \text{for all } (y,z),(\bar{y},\bar{z})\in\mathcal{D}_T\times\mathcal{D}_T.
$$

Due to the Preov fixed point theorem, the mapping $\mathcal{K}$ has a unique fixed $(y,z)\in\mathcal{D}_T\times\mathcal{D}_T$ that is exactly the unique solution of (1.1). □

4. Hyers-Ulam stability

The stability through the continuous dependence of solutions on the initial conditions is investigated.

Theorem 4.1. *Let* $(y(t),z(t))$, $(\bar{y}(t),\bar{z}(t))$ *be the solutions of* (1.1) *with initial values* (ϖ_1,ϖ_2) *and* $(\bar{\varpi}_1,\bar{\varpi}_2)$*, respectively. According to the assumptions of Theorem 3.1, the solution of* (1.1) *is stable in mean square.*

Proof. Let $(y(t), z(t))$ and $(\bar{y}(t), \bar{z}(t))$ be the two solutions of the system (1.1). Then, we have

$$\begin{aligned}
\mathbb{E}\|y(t)-\bar{y}(t)\|^2 \;\leq\;& 4\mathbb{E}\max_{k}\left\{1, \prod_{i=1}^{k}\|b_i^1(\zeta_i)\|^2\right\}\mathbb{E}\|\varpi_1-\bar{\varpi}_1\|^2 \\
&+4\mathbb{E}\left[\max_{i,k}\left\{1, \prod_{j=i}^{k}\|b_j^1(\zeta_j)\|\right\}\right]^2\left(\int_{t_0}^{t}\mathbb{E}\|f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)\|ds\right)^2 \\
&+4\mathbb{E}\left[\max_{i,k}\left\{1, \prod_{j=i}^{k}\|b_j^1(\zeta_j)\|\right\}\right]^2\left(\int_{t_0}^{t}\mathbb{E}\|g_1(s,y_s,z_s)-g_1(s,\bar{y}_s,\bar{z}_s)\|dB(s)\right)^2 \\
&+4\mathbb{E}\left[\max_{i,k}\left\{1, \prod_{j=i}^{k}\|b_j^1(\zeta_j)\|\right\}\right]^2\left(\int_{t_0}^{t}\mathbb{E}\|h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)\|\tilde{\mathcal{K}}(dr,ds)\right)^2.
\end{aligned}$$

Using inequality (2.2) and the hypotheses (H_1)–(H_4), we get

$$\begin{aligned}
&\sup_{t\in[t_0-t_1,T]}\mathbb{E}\|y(t)-\bar{y}(t)\|^2 \leq 4\max\left\{1,\bar{B}\right\}\mathbb{E}\|\varpi_1-\bar{\varpi}_1\|^2 \\
&+(4\max\{1,\bar{B}\}\Big((t-t_0)^2a_{f_1}+C_2(t-t_0)a_{g_1}+\bar{c}_2(t-t_0)^2a_{h_1}\Big)\Big)\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}\mathbb{E}\|y(s)-\bar{y}(s)\|^2ds \\
&+\Big(4\max\{1,\bar{B}\}\Big((t-t_0)^2b_{f_1}+C_2(t-t_0)b_{g_1}+\bar{c}_2(t-t_0)^2b_{h_1}\Big)\Big)\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}\mathbb{E}\|z(s)-\bar{z}(s)\|^2ds \\
&\leq 4\max\left\{1,\bar{B}\right\}\mathbb{E}\|\varpi_1-\bar{\varpi}_1\|^2+\widehat{m}_1\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}E\|y(s)-\bar{y}(s)\|^2ds+\widehat{m}_2\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}\mathbb{E}\|z(s)-\bar{z}(s)\|^2ds.
\end{aligned}$$

In a similar way, we deduce that

$$\begin{aligned}
\sup_{t\in[t_0-t_1,T]}\mathbb{E}\|z(t)-\bar{z}(t)\|^2 \;\leq\;& 4\max\left\{1,\bar{B}\right\}\mathbb{E}\|\varpi_2-\bar{\varpi}_2\|^2 \\
&++\Big(4\max\{1,\bar{B}\}\Big((t-t_0)^2a_{f_2}+C_2(t-t_0)a_{g_2} \\
&+c_2(t-t_0)^2a_{h_2}\Big)\Big)\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}\mathbb{E}\|y(s)-\bar{y}(s)\|^2ds \\
&++\Big(4\max\{1,\bar{B}\}\Big((t-t_0)^2b_{f_2}+C_2(t-t_0)b_{g_2} \\
&+c_2(t-t_0)^2b_{h_2}\Big)\Big)\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}\mathbb{E}\|z(s)-\bar{z}(s)\|^2ds \\
\leq\;& 4\max\left\{1,\bar{B}\right\}\mathbb{E}\|\varpi_2-\bar{\varpi}_2\|^2+\widehat{m}_3\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}\mathbb{E}\|y(s)-\bar{y}(s)\|^2ds \\
&+\widehat{m}_4\int_{t_0}^{t}\sup_{s\in[t-t_1,t]}\mathbb{E}\|z(s)-\bar{z}(s)\|^2ds.
\end{aligned}$$

So, we get

$$\begin{aligned}
&\sup_{t\in[t_0-t_1,T]}\Big(\mathbb{E}\|y(t)-\bar{y}(t)\|^2+\mathbb{E}\|z(t)-\bar{z}(t)\|^2\Big) \\
\leq\;& K_1\left(\mathbb{E}\|\varpi_1-\bar{\varpi}_1\|^2+\mathbb{E}\|\varpi_2-\bar{\varpi}_2\|^2\right) \\
&+K_2\int_{t_0}^{T}\sup_{s\in[t-t_1,t]}\Big(\mathbb{E}\|y(s)-\bar{y}(s)\|^2+\mathbb{E}\|z(s)-\bar{z}(s)\|^2\Big)ds.
\end{aligned}$$

If we denote

$$K_1 = 4\max\left\{1, \bar{B}\right\},$$

and

$$K_2 = \max\{\widehat{m}_1 + \widehat{m}_3, \widehat{m}_2 + \widehat{m}_4\}.$$

Owing to Grownwall's inequality, we have the following.

$$\begin{aligned}
&\sup_{t\in[t_0-t_1,T]} \left(\mathbb{E}\|y(t) - \bar{y}(t)\|^2 + \mathbb{E}\|z(t) - \bar{z}(t)\|^2\right) \\
\leq\ & K_1 \left(\mathbb{E}\|\varpi_1 - \bar{\varpi}_1\|^2 + \mathbb{E}\|\varpi_2 - \bar{\varpi}_2\|^2\right) \exp(K_2) \\
\leq\ & \widetilde{M} \left(\mathbb{E}\|\varpi_1 - \bar{\varpi}_1\|^2 + \mathbb{E}\|\varpi_2 - \bar{\varpi}_2\|^2\right),
\end{aligned}$$

where

$$\widetilde{M} = K_1 \exp(K_2).$$

Let $\varepsilon > 0$ and $\delta = \varepsilon/\widetilde{M}$ so that

$$\mathbb{E}\|\varpi_1 - \bar{\varpi}_1\|^2 < \delta,$$

and

$$E\|\varpi_2 - \bar{\varpi}_2\|^2 < \delta.$$

Then

$$\sup_{t\in[t_0-t_1,T]} \mathbb{E}\|y(t) - \bar{y}(t)\|^2 < \varepsilon,$$

and

$$\sup_{t\in[t_0-t_1,T]} \mathbb{E}\|z(t) - \bar{z}(t)\|^2 < \varepsilon.$$

In summarizing the above, (1.1) is stable in the mean square. This completes the proof. □

Our next main result regarding the Hyers-Ulam stability of the system (1.1) with assumptions (H_1)–(H_4) is presented.

Theorem 4.2. *Assumption that* (H_1)–(H_4) *holds. Then, (1.1) is stable in the Ulam-Hyers sense.*

Proof. Let us define $(\bar{y}(t), \bar{z}(t))$ as the solution of the system (1.1).

$$\begin{cases} \bar{y}(t) = & \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i^1(\zeta_i)\varpi_1(0) + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} f_1(s,\bar{y}_s,\bar{z}_s)ds + \int_{\lambda_k}^{t} f_1(s,\bar{y}_s,\bar{z}_s)ds \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} g_1(s,\bar{y}_s,\bar{z}_s)dB(s) + \int_{\lambda_k}^{t} g_1(s,\bar{y}_s,\bar{z}_s)dB(s) \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}} h_1(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds) + \int_{\lambda_k}^{t}\int_{\mathcal{A}} h_1(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds)\Bigg] \\ & \times I_{(\lambda_k,\lambda_{k-1}]}(t), \quad \mathbb{P}-a.s, \\ \bar{z}(t) = & \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i^2(\zeta_i)\varpi_2(0) + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} f_2(s,\bar{y}_s,\bar{z}_s)ds + \int_{\lambda_k}^{t} f_2(s,\bar{y}_s,\bar{z}_s)ds \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} g_2(s,\bar{y}_s,\bar{z}_s)d\beta(s) + \int_{\lambda_k}^{t} g_2(s,\bar{y}_s,\bar{z}_s)dB(s) \\ & + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}} h_2(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds) + \int_{\lambda_k}^{t}\int_{\mathcal{A}} h_2(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds)\Bigg] \\ & \times I_{(\lambda_k,\lambda_{k-1}]}(t), \quad \mathbb{P}-a.s. \end{cases}$$

Then

$$\mathbb{E}\Bigg\|\bar{y}(t) - \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i^1(\zeta_i)\varpi_1(0) + \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} f_1(s,\bar{y}_s,\bar{z}_s)ds + \int_{\lambda_k}^{t} f_1(s,\bar{y}_s,\bar{z}_s)ds$$

$$+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} g_1(s,\bar{y}_s,\bar{z}_s)dB(s) + \int_{\lambda_k}^{t} g_1(s,\bar{y}_s,\bar{z}_s)dB(s)$$

$$+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}} h_1(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds) + \int_{\lambda_k}^{t}\int_{\mathcal{A}} h_1(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds)\Bigg] I_{(\lambda_k,\lambda_{k+1}]}(t)\Bigg\|^2 \le \varepsilon.$$

Similarly,

$$\mathbb{E}\Bigg\|\bar{z}(t) - \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i^2(\zeta_i)\varpi_2(0) + \textstyle\sum_{i=1}^{k}\displaystyle\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} f_2(s,\bar{y}_s,\bar{z}_s)ds + \int_{\lambda_k}^{t} f_2(s,\bar{y}_s,\bar{z}_s)ds$$

$$+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i} g_2(s,\bar{y}_s,\bar{z}_s)dB(s) + \int_{\lambda_k}^{t} g_2(s,\bar{y}_s,\bar{z}_s)dB(s)$$

$$+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}} h_2(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds) + \int_{\lambda_k}^{t}\int_{\mathcal{A}} h_2(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds)\Bigg] I_{(\lambda_k,\lambda_{k+1}]}(t)\Bigg\|^2 \le \varepsilon,$$

when $t \in [t_0 - t_1, t_0]$, we have

$$\mathbb{E}\|\bar{y}(t) - y(t)\|^2 = 0,$$

and

$$\mathbb{E}\|\bar{z}(t) - z(t)\|^2 = 0.$$

Hence for each $t \in [t_0, T]$, we have

$$\mathbb{E}\|\bar{y}(t) - y(t)\|^2$$

$$
\begin{aligned}
\leq\ & 2E\left\|\bar{y}(t)-\sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k}b_i^1(\zeta_i)\varpi_1(0)+\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}f_1(s,\bar{y}_s,\bar{z}_s)ds+\int_{\lambda_k}^{t}f_1(s,\bar{y}_s,\bar{z}_s)ds\right.\right.\\
&+\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}g_1(s,\bar{y}_s,\bar{z}_s)dB(s)+\int_{\lambda_k}^{t}g_1(s,\bar{y}_s,\bar{z}_s)dB(s)\\
&\left.\left.+\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}}h_1(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds)\ +\int_{\lambda_k}^{t}\int_{\mathcal{A}}h_1(r,\bar{y}_r,\bar{z}_r,s)\tilde{\mathcal{K}}(dr,ds)\right]I_{(\lambda_k,\lambda_{k+1}]}(t)\right\|^2\\
&+2\mathbb{E}\left\|\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=i}^{k}b_i(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)]ds\right.\right.\\
&+\int_{\lambda_k}^{t}[f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)]ds+\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[g_1(s,y_s,z_s)-g_1(s,\bar{y}_s,\bar{z}_s)]dB(s)\\
&+\int_{\lambda_k}^{t}[g_1(s,y_s,z_s)-g_1(s,\bar{y}_s,\bar{z}_s)]dB(s)\\
&+\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}}[h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\\
&\left.\left.+\int_{\lambda_k}^{t}\int_{\mathcal{A}}[h_1(r,y_r,z_r,s)-h(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\right]I_{(\lambda_k,\lambda_{k+1}]}(t)\right\|^2\\
\leq\ & 2\varepsilon+2\Delta_1,
\end{aligned}
$$

and then

$$\mathbb{E}\|\bar{z}(t)-z(t)\|^2\leq 2\varepsilon+2\Delta_2.$$

We first estimate the third part

$$
\begin{aligned}
\Delta_1\ =\ & \mathbb{E}\left\|\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)]ds\right.\right.\\
&+\int_{\lambda_k}^{t}[f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)]ds+\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[g_1(s,y_s,z_s)-g_1(s,\bar{y}_s,\bar{z}_s)]dB(s)\\
&+\int_{\lambda_k}^{t}[g_1(s,y_s,z_s)-g_1(s,\bar{y}_s,\bar{z}_s)]dB(s)\\
&+\sum_{i=1}^{k}\prod_{j=i}^{k}b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}}[h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\\
&\left.\left.+\int_{\lambda_k}^{t}\int_{\mathcal{A}}[h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\right]I_{(\lambda_k,\lambda_{k+1}]}(t)\right\|^2\\
\leq\ & 3(\bar{I}_1+\bar{I}_2+\bar{I}_3),
\end{aligned}
$$

and

$$\begin{aligned}\Delta_2 &= \mathbb{E}\left\|\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[f_2(s,y_s,z_s)-f_2(s,\bar{y}_s,\bar{z}_s)]ds\right.\right.\\
&\quad+\int_{\lambda_k}^{t}[f_2(s,y_s,z_s)-f_2(s,\bar{y}_s,\bar{z}_s)]ds+\sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[g_2(s,y_s,z_s)-g_2(s,\bar{y}_s,\bar{z}_s)]dB(s)\\
&\quad+\int_{\lambda_k}^{t}[g_2(s,y_s,z_s)-g_2(s,\bar{y}_s,\bar{z}_s)]dB(s)\\
&\quad+\sum_{i=1}^{k}\prod_{j=i}^{k} b_i^2(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}\int_{\mathcal{A}}[h_2(r,y_r,z_r,s)-h_2(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\\
&\quad\left.\left.+\int_{\lambda_k}^{t}\int_{\mathcal{A}}[h_2(r,y_r,z_r,s)-h_2(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\right]I_{(\lambda_k,\lambda_{k+1}]}(t)\right\|^2\\
&\le 3(\overline{I}_1^1+\overline{I}_2^1+\overline{I}_3^1).\end{aligned}$$

First

$$\begin{aligned}\overline{I}_1 &= \mathbb{E}\left\|\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)]ds\right.\right.\\
&\quad\left.\left.+\int_{\lambda_k}^{t}[f_1(s,y_s,z_s)-f_1(s,\bar{y}_s,\bar{z}_s)]ds\right]I_{(\lambda_k,\lambda_{k+1}]}(t)\right\|^2\\
&\le 2(\bar{B}^2+1)(T-t_0)\left(a_{f_1}\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+b_{f_1}\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds\right).\end{aligned}$$

Similarly, we have

$$\overline{I}_1^1\le 2(\bar{B}^2+1)(T-t_0)\left(a_{f_2}\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+b_{f_2}\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds\right).$$

By condition (H_2) and using inequality (2.1), we obtain

$$\begin{aligned}\overline{I}_2 &= \mathbb{E}\left\|\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=i}^{k} b_i(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[g_1(s,\bar{y}_s,z_s)-g_1(s,\bar{y}_s,\bar{z}_s)]dB(s)\right.\right.\\
&\quad\left.\left.+\int_{\lambda_k}^{t}[g_1(s,y_s,z_s)-g(s,\bar{y}_s,\bar{z}_s)]dB(s)\right]I_{(\lambda_k,\lambda_{k+1}]}(t)\right\|^2\\
&\le 2C_2(\bar{B}^2+1)(T-t_0)\left(a_{g_1}\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+b_{g_1}\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds\right).\end{aligned}$$

Similarly,

$$\overline{I}_2^2\le 2C_2(\bar{B}^2+1)(T-t_0)\left(a_{g_2}\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+b_{g_2}\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds\right).$$

Finally, by the inequality (2.2) and (H_3) and (H_4),

$$\begin{aligned}\bar{I}_3 &= \mathbb{E}\left\|\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=i}^{k} b_i^1(\zeta_j)\int_{\lambda_{i-1}}^{\lambda_i}[h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\right.\right.\\&\quad\left.\left.+\int_{\lambda_k}^{t}[h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)]\tilde{\mathcal{K}}(dr,ds)\right]I_{(\lambda_k,\lambda_{k+1}]}(t)\right\|^2\\&\le (\bar{B}^2+1)(T-t_0)\bar{c}_2\left[\int_{t_0}^{t}\int_{\mathcal{A}}\mathbb{E}\|h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)\|^2\nu(ds)dr\right.\\&\quad\left.+\left(\int_{t_0}^{t}\int_{\mathcal{A}}\mathbb{E}\|h_1(r,y_r,z_r,s)-h_1(r,\bar{y}_r,\bar{z}_r,s)\|^2\nu(ds)dr\right)\right]\\&< 2(\bar{B}^2+1)(T-t_0)\bar{c}_2\left(a_{h_1}\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+b_{h_1}\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds\right),\end{aligned}$$

and

$$\bar{I}_2^3 \le 2(\bar{B}^2+1)(T-t_0)\bar{\bar{c}}_2\left(a_{h_2}\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+b_{h_2}\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds\right).$$

Then,

$$\mathbb{E}\|\bar{y}(t)-y(t)\|^2 \le 2\varepsilon_1+2K_1\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+2K_2\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds,$$

where

$$K_1 = 3(2(\bar{B}^2+1)(T-t_0)\left(a_{f_1}+C_2a_{g_1}+\bar{\bar{c}}_2a_{h_1}\right),$$

and

$$K_2 = 3(2(\bar{B}^2+1)(T-t_0)\left(b_{f_1}+C_2b_{g_1}+\bar{c}_2b_{h_1}\right).$$

Similarly,

$$\mathbb{E}\|\bar{z}(t)-z(t)\|^2 \le 2\varepsilon_2+2\bar{K}_1\int_{t_0}^{t}\|y_s-\bar{y}_s\|^2ds+2\bar{K}_2\int_{t_0}^{t}\|z_s-\bar{z}_s\|^2ds,$$

where

$$\bar{K}_1 = 3(2(\bar{B}^2+1)(T-t_0)\left(a_{f_2}+C_2a_{g_2}+\bar{c}_2a_{h_2}\right),$$

and

$$\bar{K}_2 = 3(2(\bar{B}^2+1)(T-t_0)\left(b_{f_2}+C_2b_{g_2}+\bar{\bar{c}}_2b_{h_2}\right).$$

Considering

$$\begin{aligned}\int_{t_0}^{t}\|\bar{y}(s)-y(s)\|^2_{\mathcal{D}_0}ds &= \int_{t_0}^{t}\sup_{\alpha\in[-s,0]}\mathbb{E}\|\bar{y}(s+\alpha)-y(s+\alpha)\|^2ds\\&= \sup_{\alpha\in[-s,0]}\int_{t_0}^{t}\mathbb{E}\|\bar{y}(s+\alpha)-y(s+\alpha)\|^2ds\\&= \sup_{\alpha\in[-s,0]}\int_{t_0+\alpha}^{t+\alpha}\mathbb{E}\|\bar{y}(s)-y(s)\|^2ds.\end{aligned}$$

When $t\in[t_0-t_1,t_0]$, we have

$$\mathbb{E}\|\bar{y}(t)-y(t)\|^2 = 0.$$

Then,

$$\int_{t_0}^{t} \|\bar{y}_s - y_s\|^2_{\mathcal{D}_0} ds = \sup_{\alpha\in[-s,0]} \int_{t_0}^{t+\alpha} \mathbb{E}\|\bar{y}(s) - y(s)\|^2 ds$$
$$= \int_{t_0}^{t} \mathbb{E}\|\bar{y}(s) - y(s)\|^2 ds.$$

Then, we obtain that

$$\mathbb{E}\|\bar{y}(t) - y(t)\|^2 + \mathbb{E}\|\bar{z}(t) - z(t)\|^2 \leq \varepsilon + \overline{M} \int_{t_0}^{t} \left(\|\bar{y}(s) - y(s)\|^2 + \|\bar{z}(s) - z(s)\|^2\right) ds,$$

where

$$\varepsilon = 2(\varepsilon_1 + \varepsilon_2), \qquad M = \max\{2K_1 + 2\bar{K}_1, 2K_2 + 2\bar{K}_2\}.$$

By applying Gronwall's inequality, we have

$$\mathbb{E}\|\bar{y}(t) - y(t)\|^2 + \mathbb{E}\|\bar{z}(t) - z(t)\|^2 \leq \varepsilon \exp(\overline{M}).$$

Therefore, there exists $\eta = \exp(\overline{M})$, such that

$$\mathbb{E}\|\bar{y}(t) - y(t)\|^2 \leq \eta\varepsilon,$$

and

$$\mathbb{E}\|\bar{z}(t) - z(t)\|^2 \leq \eta\varepsilon.$$

Then, (1.1) is Hyers-Ulam stable. □

4.1. Examples

Example 4.1. *Consider the stochastic differential equations with random impulses driven by Poisson jumps*

$$\begin{aligned}
d[u(t)] = & \left[\int_{-t_1}^{0} \vartheta_1(\alpha)(u(t+\alpha) + v(t+\alpha))d\alpha\right] dt + \left[\int_{-t_1}^{0} \vartheta_2(\alpha)(u(t+\alpha) + v(t+\alpha))\alpha\right] dB(t) \\
& + \left[\int_{-t_1}^{0} \int_{\mathcal{A}} \vartheta_3(\alpha)(u(t+\alpha) + v(t+\alpha))d\alpha\right] \tilde{\mathcal{K}}(dt, ds), \quad t \geq 0, t \neq \lambda_k, \\
d[v(t)] = & \left[\int_{-t_1}^{0} \vartheta_1(\alpha)(\tfrac{u(t+\alpha)+v(t+\alpha)}{3})d\alpha\right] dt + \left[\int_{-t_1}^{0} \vartheta_2(\tfrac{u(t+\alpha)+v(t+\alpha)}{3})\alpha\right] dB(t) \\
& + \left[\int_{-t_1}^{0} \int_{\mathcal{A}} \vartheta_3(\alpha)(u\tfrac{u(t+\alpha)+v(t+\alpha)}{3})d\alpha\right] \tilde{\mathcal{K}}(dt, ds), \quad t \geq 0, t \neq \lambda_k, \\
u(\lambda_k) = & \; b_1(k)\zeta_k u(\lambda_k^-), \quad k = 1, 2, ..., \\
v(\lambda_k) = & \; b_2(k)\zeta_k v(\lambda_k^-), \quad k = 1, 2, ..., \\
u_0 = & \; \lambda_1 = \{\lambda(\alpha) : -t_1 \leq \alpha \leq 0\}, \\
v_0 = & \; \lambda_2 = \{\lambda(\alpha) : -t_1 \leq \alpha \leq 0\}.
\end{aligned} \tag{4.1}$$

Let $r > 0$, u, v *in* $\mathbb{R}$*-valued stochastic process,*

$$\lambda \in \mathcal{D}_0 = C\left([-t_1, 0], L^2(\Omega, \mathbb{R})\right).$$

ζ_k is given from Ω to $D_k = (0, d_k)$, $\forall k = 1, 2, \ldots$. Let ζ_k follow an Erlang distribution, and ζ_i and ζ_j are independent as $i \neq j$, $i, j \in \mathbb{N}$, $t_0 = \lambda_0 < \lambda_1 < \lambda_2 < \ldots < \lambda_k < \ldots$,, and $\lambda_k = \lambda_k + \zeta_{k-1}$ for $k \in \mathbb{N}$. Let $\beta(t) \in \mathbb{R}$ be a one-dimensional Brownian motion, where b is a function of k. Let $\vartheta_1, \vartheta_2, \vartheta_3 : [-t_1, 0] \to \mathbb{R}$ be continuous functions. Define

$$f_1, f_2 : [t_0, T] \times \mathcal{D}_0 \times \mathcal{D}_0 \to \mathbb{R}^d,$$

$$g_1, g_2 : [t_0, T] \times \mathcal{D}_0 \times \mathcal{D}_0 \to \mathbb{R}^{d \times m},$$

$$h_1, h_2 : [t_0, T] \times \mathcal{D}_0 \times \mathcal{A} \to \mathbb{R}^d,$$

$$b_k^1, b_k^2 : D_k \to \mathbb{R}^{d \times d},$$

by

$$f_1(t, y(t), z(t))(.) = \int_{-t_1}^{0} \vartheta_1(u(t+\alpha) + v(t+\alpha))d\alpha(.),$$

$$g_1(t, y(t), z(t))(.) = \int_{-t_1}^{0} \vartheta_2(u(t+\alpha) + v(t+\alpha))d\alpha(.),$$

$$h_1(t, y(t), z(t))(.) = \int_{-t_1}^{0} \vartheta_3(u(t+\alpha) + v(t+\alpha))d\alpha(.),$$

and

$$f_2(t, y(t), z(t))(.) = \int_{-t_1}^{0} \vartheta_1(\frac{u(t+\alpha) + v(t+\alpha)}{3})d\alpha(.),$$

$$g_2(t, y(t), z(t))(.) = \int_{-t_1}^{0} \vartheta_2(\frac{u(t+\alpha) + v(t+\alpha)}{3})d\alpha(.),$$

$$h_2(t, y(t), z(t))(.) = \int_{-t_1}^{0} \vartheta_3(\frac{u(t+\alpha) + v(t+\alpha)}{3})d\alpha(.).$$

For

$$u(t+\alpha), v(t+\alpha) \in \mathcal{D}_0,$$

we assume that

i) $\max_{i,k}\left\{\prod_{j=i}^{k} E\|b^l(j)(\zeta_j)\|^2\right\} < \infty, \quad l \in \{1, 2\}.$

ii) $\int_{-t_1}^{0} \vartheta_1(\alpha)^2 d\alpha < \infty., \quad \int_{-t_1}^{0} \vartheta_2(\alpha)^2 d\alpha < \infty, \quad \int_{-t_1}^{0} \vartheta_3(\alpha)^2 d\alpha < \infty.$

Suppose that the states (i) *and* (ii) *are held, and then the assumptions* (H_1)–(H_4) *hold. Then, the system* (4.1) *has a unique solution* (y, z) *and is stable.*

Thus, the system (4.1) *has a unique mild solution.*

Example 4.2. *We consider the coupled stochastic system*

$$\begin{cases} dy(t) = [-a_1 y(t) + c z(t)]\, dt + \sigma_1 y(t)\, dB(t) + J_1(t)\, dN(t), \\ dz(t) = [-a_2 z(t) + c y(t)]\, dt + \sigma_2 z(t)\, dB(t) + J_2(t)\, dN(t), \\ y(\lambda_k) = (1+\rho_1) y(\lambda_k^-), \quad z(\lambda_k) = (1+\rho_2) z(\lambda_k^-), \end{cases}$$

where $B(t)$ is a Brownian motion and $N(t)$ a Poisson process with intensity λ. The parameters used are: $a_1 = 1$, $a_2 = 1.2$, $c = 0.5$, $\sigma_1 = 0.2$, $\sigma_2 = 0.25$, $\lambda = 0.8$, $\rho_1 = \rho_2 = 0.1$.

Using an Euler–Maruyama scheme implemented in R, we simulate two trajectories with initial conditions $y(0) = y_0$ and $y_\varepsilon(0) = y_0 + \varepsilon$, $z(0) = z_0$, $z_\varepsilon(0) = z_0 + \varepsilon$.

The numerical results show that

$$|y(t) - \bar{y}(t)| + |z(t) - \bar{z}(t)|$$

remains bounded and decays over time despite the impulsive and jump perturbations, confirming the Hyers-Ulam stability of the system; see Figures 1 and 2.

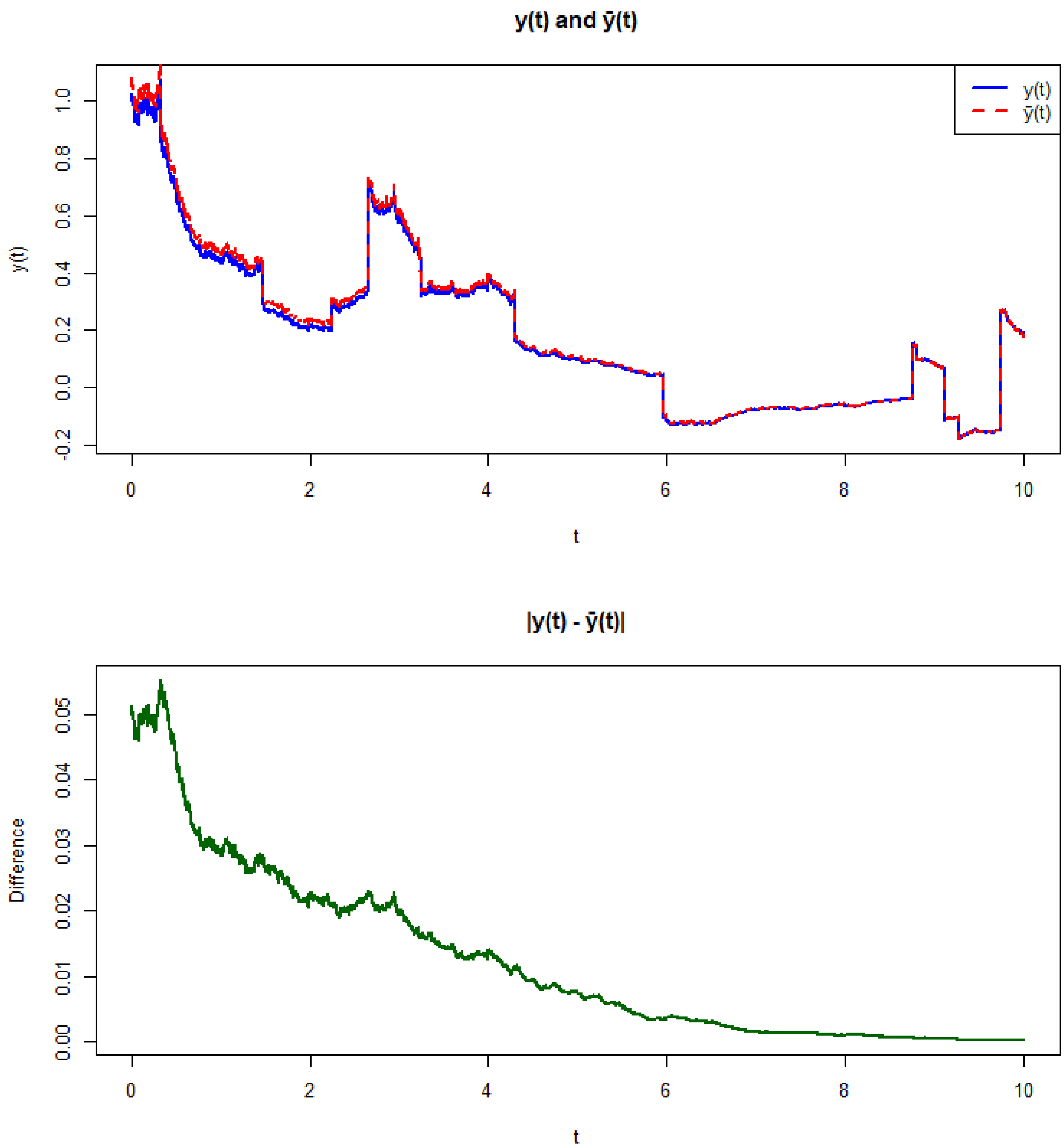

Figure 1. The Hyers-Ulam stability of the system in $y(t)$.

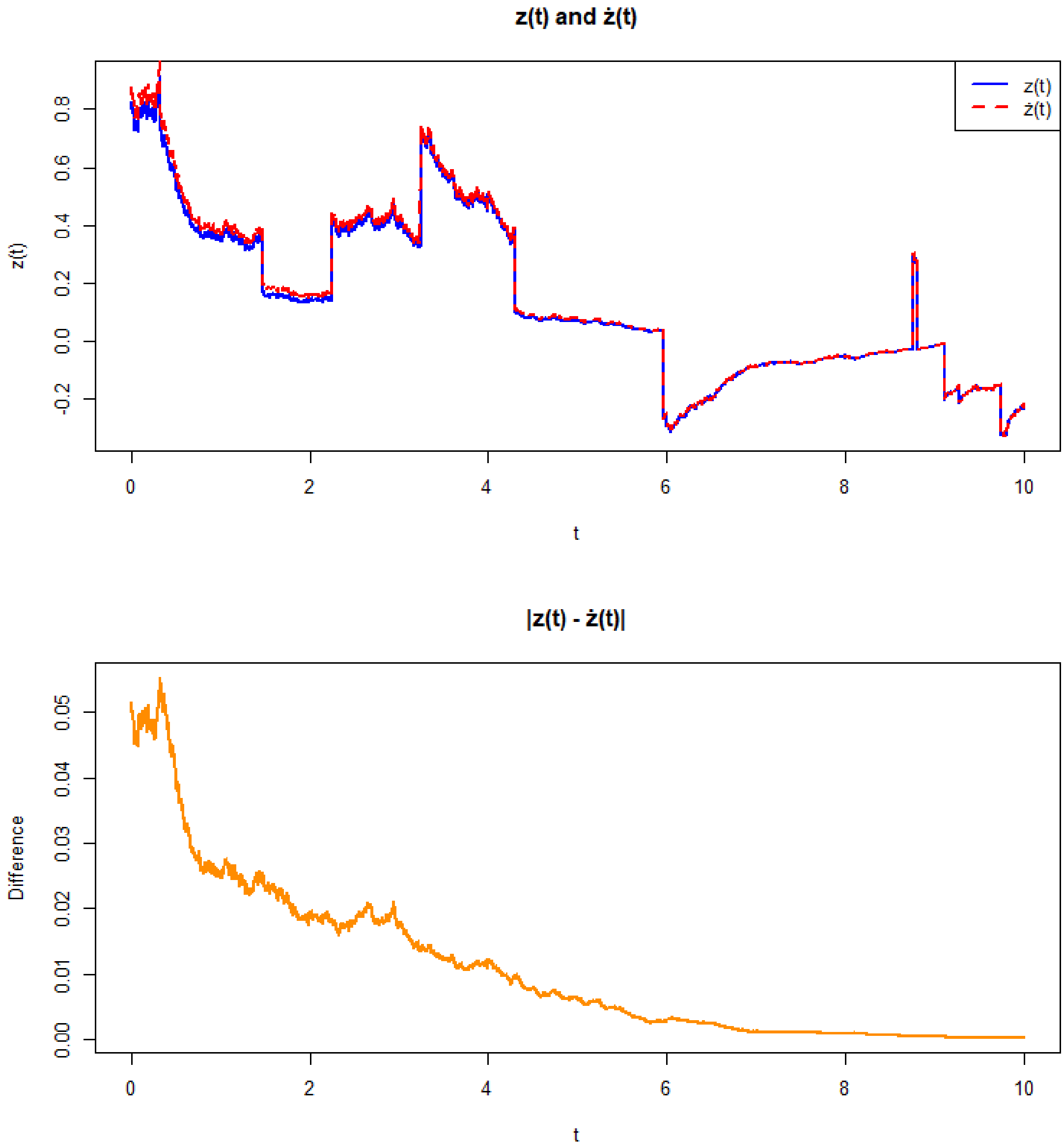

Figure 2. The Hyers-Ulam stability of the system in $z(t)$.

5. Conclusions

The quantitative and qualitative studies of the coupled system are investigated for stochastic differential equations with random impulses driven by Poisson jumps. The existence and uniqueness are established using the generalized Banach contraction mapping principle; together with some estimates, the result is given in theorem 3.1; see [34, 35]. Additionally, a Hyers-Ulam sense of solutions is given in theorem 4.2 by the expansion of the Perov-type fixed-point theorem under assumptions (H_1)–(H_4). The presence of the functions f_i, g_i and $h_i, i = 1, 2$, makes the system more interesting from an application point of view due to their great importance and extensive applications in real life. Potential topics include, but are not limited to: Stochastic differential games; Inverse stochastic differential equations; Random walks in random media; Stochastic analysis in biology and biomedicine; Markov processes; Population and evolutionary models; Random networks; and Stochastic analysis in finance. It can be applied in the field of ecological and biological modeling, particularly to stochastic predator-prey systems; please see [36–38].

Author contributions

Yasir A. Madani: Conceptualization, Formal analysis; Tayeb Blouhi: Writing–original draft preparation; Fatima Zohra Ladrani: Investigation, Methodology; Mohamed Bouye: Writing–review and editing; Khaled Zennir: Supervision; Keltoum Bouhali: Investigation, Methodology; Amin Benaissa Cherif: Writing–review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interest.

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