Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(11): 25137–25153.
DOI: 10.3934/math.20251112
Received: 31 August 2025
Revised: 07 October 2025
Accepted: 20 October 2025
Published: 03 November 2025

Research article

Finite-time transient control of uncertain nonlinear systems based on prescribed performance and disturbance observer

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Abstract: For uncertain nonlinear systems with dead-zone input, this paper focuses on designing a disturbance observer-based adaptive fuzzy finite-time prescribed performance controller. To meet practical requirements, a fixed-time disturbance observer is constructed to estimate the system uncertainties. To confine the tracking error within a predefined range, a finite-time performance function is introduced. Subsequently, the constrained problem is transformed into a stability problem via a transformation function. Based on Lyapunov stability theory, the proposed control method can ensure that the closed-loop signals are bounded and the tracking error is restricted within the prescribed range. The simulation results confirm the validity of the proposed approach.

Keywords: nonlinear system; disturbance observer; performance function; finite time
Mathematics Subject Classification: 93C10, 93C42

1. Introduction

In recent decades, the stability control of nonlinear systems has continued to be an open area of research. In practical engineering, common nonlinear systems include horizontal platform systems, electromechanical systems, unmanned aerial vehicle systems, etc. Correspondingly, various control methods have been proposed by researchers, such as adaptive control [1–6], feedback control [7–10], fuzzy control [11–14], sliding mode control [15–17], impulsive control [18] and so on. In [3], the output state tracking problem of three kinds of dynamic nonlinear systems was investigated by using an adaptive control approach. To solve the stabilization problem of uncertain nonlinear strict-feedback systems, a dynamic event-triggered adaptive control method was proposed in [6]. To realize the finite-time stability of uncertain stochastic nonlinear systems, Fang et al. [7] presented a finite-time state-feedback control scheme. Liu et al. [11] focused on overcoming the effects of time-varying state constraints and backlash-like hysteresis, and thus proposed an adaptive fuzzy tracking control method applicable to nonstrict feedback nonlinear systems. Zhang et al. [17] proposed a fuzzy integral sliding mode control method to investigate the stabilization problem of nonlinear descriptor systems. Reference [18] investigated the problem of stochastic fixed-time stability in impulsive stochastic nonlinear time-varying systems via the multiple Lyapunov method and stochastic analysis theory. The control methods in the aforementioned studies can ensure that the tracking error converges to a small neighborhood of the origin; however, the range of this neighborhood cannot be preset.

To address this limitation, the prescribed performance control method [19–27] has been widely applied, and its key feature lies in the ability to pre-adjust the predefined range. The design steps of the prescribed performance control method can be summarized into three steps: 1) Selecting appropriate performance function and transformation function; 2) Transforming the restricted problem of the tracking error into the stability problem of the transformation variable; 3) Designing the control scheme to ensure that the transformation variable is bounded. For example, in order to overcome the impact of unknown control gain signs, Xiang and Liu [20] proposed two types of prescribed performance control strategies, which ensured that the tracking error was limited within a preset range. For nonlinear Markov jumping systems with dead-zone output, an adaptive global preassigned performance control strategy was put forward in [22], and this strategy guaranteed that all signals within the closed-loop system remained bounded in probability. A finite-time prescribed performance control method was developed by Gao et al. [23], which offered an effective solution to the spacecraft attitude tracking problem. Qiu et al. [25] investigated the problem of finite-time prescribed performance control for strict-feedback nonlinear systems, with the control design based on a disturbance observer. Compared with traditional prescribed performance control methods, finite-time performance control enables better presetting of the time required to reach the predefined range. In contrast to the finite/fixed-time control method, the time to reach the predefined range does not need to be derived theoretically and can be directly adjusted and set in the controller.

Regarding external disturbances, the disturbance observer constructed in [25, 26] can only achieve asymptotically bounded stability. Furthermore, it is necessary to consider whether accurate estimation of external disturbances can be achieved under nonlinear inputs and how the prescribed performance method can be combined with a disturbance observer.

Based on the above discussions, the presented disturbance observer-based adaptive fuzzy finite-time prescribed performance control strategy in this paper has the following highlights.

1) The uncertainty within the system, external disturbance, and unknown nonlinear dead-zone component are regarded as a mixed disturbance. The fixed-time disturbance observer designed in this paper can effectively estimate this mixed disturbance and can also reduce the observation error by adjusting its own parameters.

2) The proposed prescribed-time performance control strategy can ensure that the system's output signal gradually approaches the reference signal within the prescribed time, and the tracking error enters the preset range after the prescribed time. Among them, both the preset error range and the preset time are adjustable.

The remainder of this paper is structured as follows. Section 2 presents the system description, along with the corresponding assumptions and lemmas. Section 3 details the analysis process and outlines the main results. Section 4 provides a simulation example to verify the proposed method. Finally, Section 5 offers a concise conclusion.

2. Preliminaries

2.1. System models

Consider the following uncertain nonlinear systems:

$$\begin{cases} \dot{\xi}_i(t) = \xi_{i+1}(t),\ i = 1, 2, \cdots, n-1, \\ \dot{\xi}_n(t) = f(t, \boldsymbol{\xi}) + u_\mu(t) + d(t), \\ y(t) = \xi_1(t), \end{cases} \tag{2.1}$$

where $\boldsymbol{\xi} = [\xi_1(t), \xi_2(t), \cdots, \xi_n(t)]^T \in \mathbb{R}^n$ denotes the measurable system state vector, $f(t, \boldsymbol{\xi})$ represents an unknown nonlinear function. $y(t) \in \mathbb{R}$ stands for the system output. $d(t) \in \mathbb{R}$ is the unknown external disturbance. $u_\mu(t) \in \mathbb{R}$ represents the output of dead-zone in the actuator, which is expressed as

$$u_\mu(t) = \begin{cases} m(\mu(t) - \bar{\mu}),\ \mu(t) \geq \bar{\mu}, \\ 0,\ -\underline{\mu} < \mu(t) < \bar{\mu}, \\ m(\mu(t) + \underline{\mu}),\ \mu(t) > -\underline{\mu}, \end{cases} \tag{2.2}$$

where $\mu(t)$ is the dead-zone input, m is the known slope of the dead-zone, $\bar{\mu}$ and $\underline{\mu}$ are the unknown breakpoints. Obviously, $u_\mu(t)$ can be rewritten as

$$u_\mu(t) = m\mu(t) + \Delta\mu(t), \tag{2.3}$$

with

$$\Delta\mu(t) = \begin{cases} -m\bar{\mu},\ \mu(t) \geq \bar{\mu}, \\ -m\mu(t),\ -\underline{\mu} < \mu(t) < \bar{\mu}, \\ m\underline{\mu},\ \mu(t) > -\underline{\mu}. \end{cases} \tag{2.4}$$

Combining (2.1) and (2.3), one obtains

$$\begin{cases} \dot{\xi}_i(t) = \xi_{i+1}(t),\ i = 1, 2, \cdots, n-1, \\ \dot{\xi}_n(t) = \bar{f}(t, \boldsymbol{\xi}) + m\mu(t), \\ y(t) = \xi_1(t), \end{cases} \tag{2.5}$$

where $\bar{f}(t, \boldsymbol{\xi}) = f(t, \boldsymbol{\xi}) + \Delta\mu(t) + d(t)$ is regarded as a mixed disturbance.

To facilitate subsequent analysis, the following assumptions and Lemmas are introduced.

Assumption 2.1. *The reference signal $\xi_d(t)$ and its derivatives are known and bounded.*

Assumption 2.2. *The unknown function $f(t, \boldsymbol{\xi})$ and the external disturbance $d(t)$ are bounded.*

Remark 2.1. *It should be pointed out that the control inputs in References [28–31] are all linear; however, in practical applications, control inputs may be affected by uncertainties. Therefore, investigating nonlinear inputs such as saturation and dead-zone is meaningful. Furthermore, the external disturbance $d(t)$ in practice is bounded, which means that effective disturbance observers can be designed to estimate external disturbances.*

Lemma 2.1. *[32] Consider a nonlinear system: $\dot{\varsigma}(t) = f(\varsigma(t))$, $\varsigma(0) = 0$, where $\varsigma(t) \in \mathbb{R}^n$ and $f(\varsigma(t))$ is a continuous function. If there exists a positive definite function $V(\varsigma(t))$ such that $\dot{V}(\varsigma(t)) \leq -a_1 V(\varsigma(t))^p - a_2 V(\varsigma(t))^q + a_3$ where a_1, a_2, a_3 are positive constants, $p \in (0, 1)$ and $q > 1$, then the nonlinear system is practically fixed-time stable, and its convergence time is bounded.*

Lemma 2.2. *[33] For any two variables ς_1, ς_2, if three positive parameters l_1, l_2 and l_3 are selected, then the following inequality holds:*

$$|\varsigma_1|^{l_1}|\varsigma_2|^{l_2} \leq \frac{l_1}{l_1+l_2} l_3 |\varsigma_1|^{l_1+l_2} + \frac{l_2}{l_1+l_2} l_3^{-\frac{l_1}{l_2}} |\varsigma_2|^{l_1+l_2}. \tag{2.6}$$

Lemma 2.3. *[34] For the first-order filter*

$$\dot{\zeta}(t) = -\tau(\zeta(t) - \alpha(t)), \tag{2.7}$$

where $\alpha(t)$ is the input signal and $\alpha(0) = \zeta(0)$. τ is a suitable positive parameter, which can adjust estimation error $\zeta_c(t) = \zeta(t) - \alpha(t)$ such that $|\zeta_c(t)| \leq \varepsilon$, where ε is a given positive constant.

2.2. Finite-time performance function

Definition 2.1. *A smooth function $h(t)$ is referred to as a finite-time performance function when it meets the following criteria: 1) $h(t)$ is a decreasing function; 2) $\lim_{t\to t^s} h(t) = l_a$ and $h(t) = l_a$ for $t \geq t^s$, where t^s denotes the preset time and l_a represents the preset boundary value.*

This paper develops the following quadratic polynomial finite-time performance function $h(t, l_0, l_a, t^s)$:

$$h(t, l_0, l_a, t^s) = \begin{cases} l_2 t^2 + l_1 t + l_0, & t < t^s, \\ l_a, \ t \geq t^s, \end{cases} \tag{2.8}$$

where l_0, l_1, l_2, l_a, and t^s are preset parameters. To ensure that $h(t, l_0, l_a, t^s)$ and its derivative $\dot{h}(t, l_0, l_a, t^s)$ are continuous at $t = t^s$, l_1 and l_2 satisfy the following conditions:

$$\begin{cases} l_1 = \frac{2(l_a - l_0)}{t^s}, \\ l_2 = \frac{l_0 - l_a}{t^{s2}}. \end{cases} \tag{2.9}$$

Define the tracking error as $e(t) = \xi_1(t) - \xi_d(t)$ and assume $e(t)$ that satisfies the following prescribed performance boundary (PPB) condition:

$$h(t, l_0, l_a, t^s) < e(t) < h(t, k_0, k_a, t^s), \quad t \geq 0, \tag{2.10}$$

where parameters l_0, l_a, k_0, k_a satisfy $l_a < l_0 < k_0$ and $l_a < k_a < k_0$. The transformation function is introduced as

$$s_1(t) = \ \ln \frac{\nu(t)}{1 - \nu(t)}, \tag{2.11}$$

where $\nu(t) = \frac{e(t) - h(t, l_0, l_a, t^s)}{h(t, k_0, k_a, t^s) - h(t, l_0, l_a, t^s)}$. The derivative of $s_1(t)$ is obtained as

$$\dot{s}_1(t) = \ b(t)(\xi_2(t) - \dot{\xi}_d(t) + c(t)), \tag{2.12}$$

where $b(t) = \frac{1}{\nu(t)(1-\nu(t))(h(t,k_0,k_a,t^s) - h(t,l_0,l_a,t^s))}$, $c(t) = \frac{J(t)}{h(t,k_0,k_a,t^s) - h(t,l_0,l_a,t^s)}$, $J(t) = h(t, l_0, l_a, t^s)\dot{h}(t, k_0, k_a, t^s) - \dot{h}(t, l_0, l_a, t^s)h(t, k_0, k_a, t^s) - e(t)(\dot{h}(t, k_0, k_a, t^s) - \dot{h}(t, l_0, l_a, t^s))$.

To ensure the tracking error $e(t)$ satisfies the PPB condition as specified in (2.10), the subsequent lemma is necessary.

Lemma 2.4. *If the transformation function $s_1(t)$ is bounded, the tracking error $e(t)$ is capable of being restricted within the PPB condition* (2.10).

Remark 2.2. *If $s_1(t)$ is bounded, one has $v(t) = \frac{e^{s_1(t)}}{1+e^{s_1(t)}}$ according to* (2.11)*, which ensures that $0 < v(t) = \frac{e(t)-h(t,l_0,l_a,t^s)}{h(t,k_0,k_a,t^s)-h(t,l_0,l_a,t^s)} < 1$. Clearly, $e(t)$ satisfies PPB condition* (2.10)*. In other words, the boundedness of $s_1(t)$ guarantees that the tracking error $e(t)$ cannot cross prescribed performance functions $h(t, l_0, l_a, t^s)$ and $h(t, k_0, k_a, t^s)$. In addition, when $e(t)$ is limited within PPB* (2.10)*, $b(t)$ in* (2.12) *satisfies $b(t) > 0,\ t \geq 0$.*

Figures 1 and 2 show the trajectories of the tracking error $e_1(t)$, under different preset times, adjusted by the two proposed finite-time performance functions (2.8).

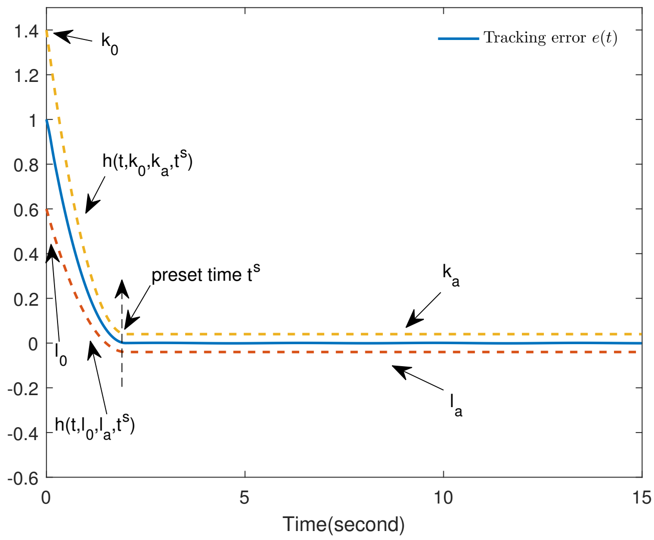

Figure 1. Schematic diagram of fast tracking under the preset time t^s.

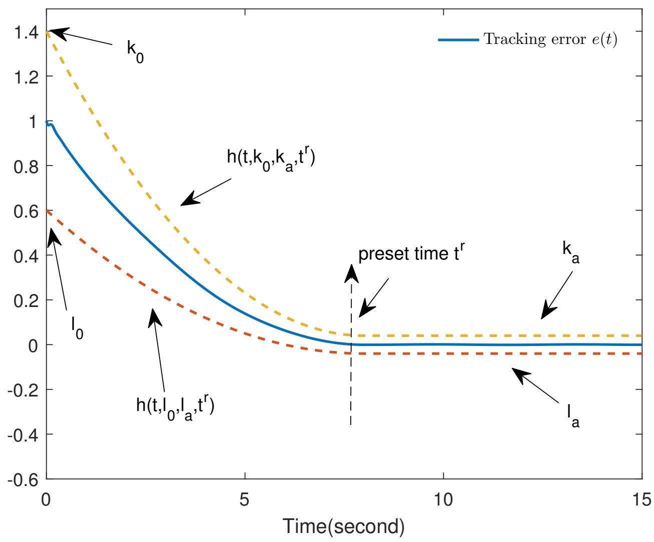

Figure 2. Schematic diagram of slow tracking under the preset time t^r.

Remark 2.3. *Similarly, a mth performance function can be constructed as*

$$h(t, l_0, l_a, t^s) = \begin{cases} l_m t^m + l_{m-1} t^{m-1} + \cdots + l_1 t + l_0, & t < t^s, \\ l_a, \ t \geq t^s, \end{cases} \tag{2.13}$$

where m is a positive integer greater than 2, parameters $l_1, l_2, \cdots, l_m$ need to satisfy the continuity condition that $h(t, l_0, l_a, t^s)$, $\dot{h}(t, l_0, l_a, t^s), \cdots, h^{(m-1)}(t, l_0, l_a, t^s)$ are continuous at $t = t^s$.

2.3. Fuzzy logic system (FLS)

Lemma 2.5. *[25, 29, 35] Suppose $\omega(\boldsymbol{\xi})$ is a continuous function defined on a compact set Ω_ω. Then, given a constant $\varepsilon_\omega > 0$, there exists an FLS $\theta_\omega^T \varphi_\omega(\boldsymbol{\xi})$ that satisfies*

$$\sup_{\boldsymbol{\xi} \in \Omega_\omega} |\omega(\boldsymbol{\xi}) - \theta_\omega^T \varphi_\omega(\boldsymbol{\xi})| \leq \varepsilon_\omega. \tag{2.14}$$

Thus, one gets

$$\omega(\boldsymbol{\xi}) = \theta_\omega^{*T} \varphi_\omega(\boldsymbol{\xi}) + \varepsilon_\omega(\boldsymbol{\xi}), \quad |\varepsilon_\omega(\boldsymbol{\xi})| \leq \bar{\varepsilon}_\omega, \tag{2.15}$$

where θ_ω^* is the ideal parameter vector, $\varepsilon_\omega(\boldsymbol{\xi})$ is the fuzzy estimation error, $\bar{\varepsilon}_\omega$ is the upper bound of $\varepsilon_\omega(\boldsymbol{\xi})$, $\varphi_\omega(\boldsymbol{\xi})$ is the membership function vector.

When Lemma 2.5 is utilized, the unknown function $\bar{f}(t, \boldsymbol{\xi})$ in (2.5) can be expressed as

$$\bar{f}(t, \boldsymbol{\xi}) = \theta_{\bar{f}}^{*T} \varphi_{\bar{f}}(\boldsymbol{\xi}) + \varepsilon_{\bar{f}}(\boldsymbol{\xi}), \tag{2.16}$$

where where $\theta_{\bar{f}}^*$ is the ideal parameter vector, $\varphi_{\bar{f}}(\boldsymbol{\xi})$ refers to the membership function vector and $\varepsilon_{\bar{f}}(\boldsymbol{\xi})$ represents the fuzzy estimation error. According to Lemma 2.5, $\varepsilon_{\bar{f}}(\boldsymbol{\xi})$ is bounded; namely, there exists an unknown positive constant β such that $|\varepsilon_{\bar{f}}(\boldsymbol{\xi})| \leq \beta$.

2.4. Disturbance observer

To construct a disturbance observer for $\bar{f}(t, \boldsymbol{\xi})$, two variables, $\sigma(t)$ and $w(t)$, are introduced and designed as follows:

$$\begin{cases} \hat{\bar{f}}(t, \boldsymbol{\xi}) = -c_1 \sigma(t)^{p_1} - c_2 \sigma(t)^{q_1} - \frac{1}{2}\sigma(t) + \hat{\theta}_{\bar{f}}^T \varphi_{\bar{f}}(\boldsymbol{\xi}), \\ \dot{w}(t) = -c_1 \sigma(t)^{p_1} - c_2 \sigma(t)^{q_1} - \frac{1}{2}\sigma(t) + \hat{\theta}_{\bar{f}}^T \varphi_{\bar{f}}(\boldsymbol{\xi}) + m\mu(t), \\ \sigma(t) = w(t) - \xi_n(t), \end{cases} \tag{2.17}$$

where c_1, c_2 are positive constant, $p_1 = \frac{w_1}{w_2} \in (0, 1)$, $q_1 = \frac{w_3}{w_4} \in (1, +\infty)$, w_1, w_2, w_3, w_4 are positive odd integers. $\hat{\theta}_{\bar{f}}$ is the estimation of $\hat{\theta}_{\bar{f}}^*$.

Define the estimation error $\tilde{\bar{f}}(t, \boldsymbol{\xi}) = \hat{\bar{f}}(t, \boldsymbol{\xi}) - \bar{f}(t, \boldsymbol{\xi})$. From (2.5) and (2.17), one has

$$\begin{aligned} \dot{\sigma}(t) &= \dot{w}(t) - \dot{\xi}_n(t) \\ &= -c_1 \sigma(t)^{p_1} - c_2 \sigma(t)^{q_1} - \tfrac{1}{2}\sigma(t) + \hat{\theta}_{\bar{f}}^T \varphi_{\bar{f}}(\boldsymbol{\xi}) + m\mu(t) - \bar{f}(t, \boldsymbol{\xi}) - m\mu(t) \\ &= -c_1 \sigma(t)^{p_1} - c_2 \sigma(t)^{q_1} - \tfrac{1}{2}\sigma(t) + \hat{\theta}_{\bar{f}}^T \varphi_{\bar{f}}(\boldsymbol{\xi}) - \bar{f}(t, \boldsymbol{\xi}) \\ &= \hat{\bar{f}}(t, \boldsymbol{\xi}) - \bar{f}(t, \boldsymbol{\xi}) = \tilde{\bar{f}}(t, \boldsymbol{\xi}). \end{aligned} \tag{2.18}$$

For $\hat{\theta}_{\bar{f}}$, the following adaptive law is designed as

$$\dot{\hat{\theta}}_{\bar{f}} = -\sigma(t)\varphi_{\bar{f}}(\boldsymbol{\xi}) - \gamma\hat{\theta}_{\bar{f}}, \tag{2.19}$$

where γ is a positive parameter.

Theorem 2.1. *If the disturbance observer* (2.17) *and the adaptive law* (2.19) *are designed for the system* (2.1)*, then $\sigma(t)$ can converge to a sufficiently small neighborhood of the origin within a fixed time. Furthermore, the disturbance observer $\hat{\bar{f}}(t,\boldsymbol{\xi})$ can effectively estimate $\bar{f}(t,\boldsymbol{\xi})$.*

Proof. Let $V_\sigma = \frac{1}{2}\sigma^2(t) + \frac{1}{2}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}}$, where $\tilde{\theta}_{\bar{f}} = \hat{\theta}_{\bar{f}} - \theta_{\bar{f}}^*$. The time derivative of $\dot{V}_\sigma$ is obtained as

$$\begin{aligned}
\dot{V}_\sigma &= \sigma(t)\dot{\sigma}(t) + \tilde{\theta}_{\bar{f}}^T\dot{\hat{\theta}}_{\bar{f}} \\
&= \sigma(t)(-c_1\sigma(t)^{p_1} - c_2\sigma(t)^{q_1} - \tfrac{1}{2}\sigma(t) + \hat{\theta}_{\bar{f}}^T\varphi_{\bar{f}}(\boldsymbol{\xi}) + m\mu(t) - \bar{f}(t,\boldsymbol{\xi}) - m\mu(t)) + \tilde{\theta}_{\bar{f}}^T\dot{\hat{\theta}}_{\bar{f}} \\
&= -c_1(\sigma(t)^2)^{\frac{p_1+1}{2}} - c_2(\sigma(t)^2)^{\frac{q_1+1}{2}} - \tfrac{1}{2}\sigma(t)^2 + \sigma(t)\tilde{\theta}_{\bar{f}}^T\varphi_{\bar{f}}(\boldsymbol{\xi}) - \sigma(t)\varepsilon_{\bar{f}}(\boldsymbol{\xi}) - \sigma(t)\tilde{\theta}_{\bar{f}}^T\varphi_{\bar{f}}(\boldsymbol{\xi}) - \gamma\tilde{\theta}_{\bar{f}}^T\hat{\theta}_{\bar{f}} \\
&\le -c_1(\sigma(t)^2)^{\frac{p_1+1}{2}} - c_2(\sigma(t)^2)^{\frac{q_1+1}{2}} - \tfrac{1}{2}\sigma(t)^2 + \beta|\sigma(t)| - \gamma\tilde{\theta}_{\bar{f}}^T(\tilde{\theta}_{\bar{f}} + \theta_{\bar{f}}^*) \\
&\le -c_1(\sigma(t)^2)^{\frac{p_1+1}{2}} - c_2(\sigma(t)^2)^{\frac{q_1+1}{2}} - \tfrac{1}{2}\sigma(t)^2 + \tfrac{\beta^2}{2} + \tfrac{\sigma(t)^2}{2} - \tfrac{\gamma}{2}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}} + \tfrac{\gamma}{2}\theta_{\bar{f}}^{*T}\theta_{\bar{f}}^* \\
& -c_1(\sigma(t)^2)^{\frac{p_1+1}{2}} - c_2(\sigma(t)^2)^{\frac{q_1+1}{2}} - \tfrac{\gamma}{2}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}} + \tfrac{\gamma}{2}\theta_{\bar{f}}^{*T}\theta_{\bar{f}}^* + \tfrac{\beta^2}{2} \\
&= c_1(\sigma(t)^2)^{\frac{p_1+1}{2}} - c_2(\sigma(t)^2)^{\frac{q_1+1}{2}} - (\tfrac{\gamma}{4}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}})^{\frac{p_1+1}{2}} - (\tfrac{\gamma}{4}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}})^{\frac{q_1+1}{2}} + r_1(t),
\end{aligned} \tag{2.20}$$

where $r_1(t) = (\frac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{p_1+1}{2}} + (\frac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{q_1+1}{2}} - \frac{\gamma}{2}||\tilde{\theta}_{\bar{f}}||^2 + \frac{\gamma}{2}\theta_{\bar{f}}^{*T}\theta_{\bar{f}}^* + \frac{\beta^2}{2}$. Suppose there exists a compact set $\Omega = \{\tilde{\theta}_{\bar{f}}\big| ||\tilde{\theta}_{\bar{f}}|| \le \nu\}$. If $\nu \ge \frac{2}{\sqrt{\gamma}}$, one has $\frac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2 \ge 1$, which implies

$$(\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{p_1+1}{2}} + (\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{q_1+1}{2}} - \tfrac{\gamma}{2}||\tilde{\theta}_{\bar{f}}||^2 \le (\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{q_1+1}{2}} - \tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2 \le (\tfrac{\gamma}{4}\nu^2)^{\frac{q_1+1}{2}} - \tfrac{\gamma}{4}\nu^2. \tag{2.21}$$

If $\nu < \frac{2}{\sqrt{\gamma}}$, one has $\frac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2 < 1$, which implies

$$(\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{p_1+1}{2}} + (\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{q_1+1}{2}} - \tfrac{\gamma}{2}||\tilde{\theta}_{\bar{f}}||^2 < (\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{p_1+1}{2}} - \tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2. \tag{2.22}$$

Select $\varsigma_1 = \frac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2$, $\varsigma_2 = 1$, $l_1 = \frac{p_1+1}{2}$, $l_2 = \frac{1-p_1}{2}$ and $l_3 = \frac{2}{p_1+1}$. It can be known from Lemma 2.2 that

$$(\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{p_1+1}{2}} - \tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2 \le \tfrac{1-p_1}{2}(\tfrac{p_1+1}{2})^{\frac{p_1+1}{1-p_1}}. \tag{2.23}$$

Substituting (2.23) into (2.22), one gets

$$(\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{p_1+1}{2}} + (\tfrac{\gamma}{4}||\tilde{\theta}_{\bar{f}}||^2)^{\frac{q_1+1}{2}} - \tfrac{\gamma}{2}||\tilde{\theta}_{\bar{f}}||^2 < \tfrac{1-p_1}{2}(\tfrac{p_1+1}{2})^{\frac{p_1+1}{1-p_1}}. \tag{2.24}$$

Obviously, $r(t) \le r_1^*$, where r_1^* is defined as

$$r_1^* = \begin{cases} \frac{1-p_1}{2}(\frac{p_1+1}{2})^{\frac{p_1+1}{1-p_1}} + \frac{\gamma}{2}\theta_{\bar{f}}^{*T}\theta_{\bar{f}}^* + \frac{\beta^2}{2}, & \nu < \frac{2}{\sqrt{\gamma}}, \\ (\frac{\gamma}{4}\nu^2)^{\frac{q_1+1}{2}} - \frac{\gamma}{4}\nu^2 + \frac{\gamma}{2}\theta_{\bar{f}}^{*T}\theta_{\bar{f}}^* + \frac{\beta^2}{2}, & \nu \ge \frac{2}{\sqrt{\gamma}}. \end{cases} \tag{2.25}$$

From (2.20) and (2.25), one obtains

$$\begin{aligned}\dot{V}_\sigma &\leq -c_1(\sigma(t)^2)^{\frac{p_1+1}{2}} - c_2(\sigma(t)^2)^{\frac{q_1+1}{2}} - (\tfrac{\gamma}{4}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}})^{\frac{p_1+1}{2}} - (\tfrac{\gamma}{4}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}})^{\frac{q_1+1}{2}} + r_1^* \\ &\leq -c_{\min}((\tfrac{\sigma(t)^2}{2})^{\frac{p_1+1}{2}} + (\tfrac{\sigma(t)^2}{2})^{\frac{q_1+1}{2}} + (\tfrac{1}{2}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}})^{\frac{p_1+1}{2}} + (\tfrac{1}{2}\tilde{\theta}_{\bar{f}}^T\tilde{\theta}_{\bar{f}})^{\frac{q_1+1}{2}}) + r_1^* \\ &\leq -c_{\min}(V_\sigma)^{\frac{p_1+1}{2}} - 2^{\frac{1-q_1}{2}}c_{\min}(V_\sigma)^{\frac{q_1+1}{2}} + r_1^*,\end{aligned} \tag{2.26}$$

where $c_{\min} = \min\{2^{\frac{p_1+1}{2}}, c_2 2^{\frac{q_1+1}{2}}, (\frac{\gamma}{2})^{\frac{p_1+1}{2}}, (\frac{\gamma}{2})^{\frac{q_1+1}{2}}\}$. It can be known from Lemma 2.1 that V_σ is practically fixed-time stable. $\sigma(t)$ and $\tilde{\theta}_{\bar{f}}$ will reach a small neighborhood of the origin within a fixed time. By applying (2.18), $\tilde{\bar{f}}(t,\boldsymbol{\xi})$ will also enter a certain neighborhood of the origin within a fixed time. This completes the proof.

Remark 2.4. *Unlike the disturbance observers in [25, 26], the disturbance observer proposed in this paper is combined with the fuzzy logic system, and the closed-loop system is guaranteed to be bounded. The unknown mixed disturbance $\bar{f}(t,\boldsymbol{\xi})$ can be effectively estimated by adjusting the observer parameter.*

3. Main results

According to Lemma 2.4, it is necessary to design the dead-zone input $\mu(t)$ to ensure that $s_1(t)$ is bounded. To avoid repeated differentiation of $c(t)$, the first-order filter (2.7) is employed. From (2.12), the virtual controller $\bar{\xi}_2(t)$ is selected as

$$\bar{\xi}_2(t) = -k_1 s_1(t) - c(t) + \dot{\xi}_d(t), \tag{3.1}$$

where k_1 is a positive constant. Let $s_2(t) = \xi_2(t) - \zeta_1(t)$, where $\zeta_1(t)$ is the estimate of $\bar{\xi}_2(t)$ and $\zeta_1(t)$ satisfies

$$\dot{\zeta}_1(t) = -\tau_1(\zeta_1(t) - \bar{\xi}_2(t)), \tag{3.2}$$

where τ_1 is the design parameter and $\zeta_1(0) = \bar{\xi}_2(0)$. Let $\zeta_1^c(t) \triangleq \zeta_1(t) - \bar{\xi}_2(t)$. With the application of Lemma 2.3, there exists a positive constant ε_1 such that $|\zeta_1^c(t)| \leq \varepsilon_1$.

Similarly, let $s_i(t) = \xi_i(t) - \zeta_{i-1}(t)$, $i = 2, 3, \cdots, n-1$; one has

$$\dot{s}_i(t) = \xi_{i+1}(t) - \dot{\zeta}_{i-1}(t). \tag{3.3}$$

The virtual controller $\bar{\xi}_{i+1}(t)$ is designed as

$$\bar{\xi}_{i+1}(t) = -k_i s_i(t) + \dot{\zeta}_{i-1}(t) - s_{i-1}(t), \tag{3.4}$$

where k_i is a positive constant. $\zeta_i(t)$ is generated by the following filter:

$$\dot{\zeta}_i(t) = -\tau_i(\zeta_i(t) - \bar{\xi}_{i+1}(t)), \tag{3.5}$$

where τ_i is the design parameter and $\zeta_i(0) = \bar{\xi}_{i+1}(0)$. Let $\zeta_i^c(t) \triangleq \zeta_i(t) - \bar{\xi}_{i+1}(t)$; there exists a positive constant ε_i such that $|\zeta_i^c(t)| \leq \varepsilon_i$.

Let $s_n(t) = \xi_n(t) - \zeta_{n-1}(t) + \sigma(t)$; one gets

$$\begin{aligned}\dot{s}_n(t) &= \dot{\xi}_n(t) - \dot{\zeta}_{n-1}(t) + \dot{\sigma}(t) \\ &= \bar{f}(t,\boldsymbol{\xi}) + m\mu(t) - \dot{\zeta}_{n-1}(t) + \dot{\sigma}(t).\end{aligned} \tag{3.6}$$

Furthermore, we design the following dead-zone input as

$$\mu(t) = \frac{-k_n s_n(t) - \hat{\bar{f}}(t,\boldsymbol{\xi}) + \dot{\zeta}_{n-1}(t) - s_{n-1}(t)}{m}. \tag{3.7}$$

Theorem 3.1. *If the dead-zone input* (3.7) *and the disturbance observer* (2.17) *are designed for the system* (2.1)*, then the tracking error $e(t)$ can be limited to the interval (l_a, k_a) after the preset time t^s.*

Proof. Consider the following Lyapunov function

$$V(t) = \frac{1}{2}\frac{s_1^2(t)}{b(t)} + \sum_{j=2}^{n}\frac{1}{2}s_j^2(t). \tag{3.8}$$

Substituting (2.12), (3.3), and (3.6) into $\dot{V}(t)$, one has

$$\begin{aligned}
\dot{V}(t) &= \frac{s_1(t)}{b(t)}\dot{s}_1(t) - \frac{\dot{b}(t)s_1(t)^2}{2b^2(t)} + \sum_{j=2}^{n-1} s_j(t)\dot{s}_j(t) + s_n(t)\dot{s}_n(t)\\
&= s_1(t)(\xi_2(t) - \dot{\xi}_d(t) + c(t)) - \sum_{j=2}^{n-1} s_j(t)(\xi_{j+1}(t) - \dot{\zeta}_{j-1}(t))\\
&\quad -s_n(t)(\dot{\xi}_n(t) - \dot{\zeta}_{n-1}(t) + \dot{\sigma}(t)) - \tfrac{\dot{b}(t)s_1(t)^2}{2b^2(t)}\\
&= s_1(t)(s_2(t) + \bar{\xi}_2(t) + \zeta_1^c(t) - \dot{\xi}_d(t) + c(t)) - \sum_{j=2}^{n-1} s_j(t)(s_{j+1}(t) + \bar{\xi}_{j+1}(t) + \zeta_j^c(t) - \dot{\zeta}_{j-1}(t))\\
&\quad -s_n(t)(\bar{f}(t,\boldsymbol{\xi}) + m\mu(t) - \dot{\zeta}_{n-1}(t) + \dot{\sigma}(t)) - \tfrac{\dot{b}(t)s_1(t)^2}{2b^2(t)}\\
&= -\sum_{i=1}^{n} k_i s_i(t)^2 - \sum_{j=1}^{n-1} s_j(t)\zeta_j(t)^c - \frac{\dot{b}(t)s_1(t)^2}{2b^2(t)} + \bar{f}(t,\boldsymbol{\xi}) - \hat{\bar{f}}(t,\boldsymbol{\xi}) + \dot{\sigma}(t)\\
&\le -\sum_{i=1}^{n} k_i s_i(t)^2 + \frac{1}{2}\sum_{j=1}^{n-1} s_j(t)^2 + \frac{1}{2}\sum_{j=1}^{n-1}\varepsilon_j^2 - \tilde{\bar{f}}(t,\boldsymbol{\xi}) + \underbrace{\dot{\sigma}(t)}_{=\tilde{\bar{f}}(t,\boldsymbol{\xi})} - \frac{\dot{b}(t)s_1(t)^2}{2b^2(t)}\\
&= -(k_1 - \frac{1}{2} + \frac{\dot{b}(t)}{2b(t)^2})s_1^2(t) - \sum_{j=2}^{n-1}(k_j - \frac{1}{2})s_j(t)^2 - k_n s_n(t)^2 + r_2,
\end{aligned} \tag{3.9}$$

where $r_2 = \sum_{i=1}^{n-1}\frac{\varepsilon_i^2}{2}$. Select $k_1, k_2, \cdots, k_n$ such that $k_1 - \frac{1}{2} + \frac{\dot{b}(t)}{2b^2(t)} > 0,\ k_j - \frac{1}{2} > 0,\ k_n > 0,\ j = 2, 3, \cdots, n-1$. Define the following compact sets:

$$\begin{aligned}
\Omega_{s_1} &= \{s_1(t) \| s_1(t)| \le \sqrt{\tfrac{r_2}{k_1 - \frac{1}{2} + \frac{\dot{b}(t)}{2b^2(t)}}}\},\\
\Omega_{s_j} &= \{s_j(t) \| s_j(t)| \le \sqrt{\tfrac{r_2}{k_j - \frac{1}{2}}}\},\\
\Omega_{s_n} &= \{s_n(t) \| s_j(t)| \le \sqrt{\tfrac{r_2}{k_n}}\}.
\end{aligned} \tag{3.10}$$

Obviously, if $s_1(t) \notin \Omega_{s_1}$ or $s_j(t) \notin \Omega_{s_j}$ or $s_n(t) \notin \Omega_{s_n}$, one has $\dot{V}(t) < 0$. Thus, $s_1(t), s_2(t), \cdots, s_n(t)$ are bounded. According to Lemma 2.4, $e(t)$ can be limited within the interval (l_a, k_a) after the preset time t^s. This completes the proof.

The finite-time preset performance control strategy proposed in this paper and the corresponding overall control framework are shown in Figure 3.

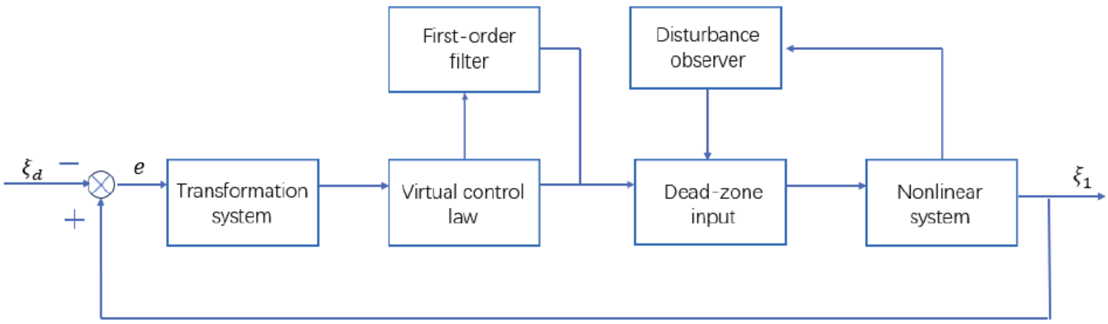

Figure 3. The control framework of this paper.

4. Simulation results

To demonstrate the effectiveness of our result, a one-link manipulator [25] is considered and described as follows:

$$J\ddot{\xi}(t) + B\dot{\xi}(t) + MGq\sin(\xi(t)) = u_\mu(t) + d(t), \tag{4.1}$$

where $\xi(t)$ and $\dot{\xi}(t)$ represent the angle and angular velocity of the link, respectively. $u_d(t)$ is the output of dead-zone in the actuator and $d(t)$ is the external disturbance. The specific meanings of other symbols J, B, M, G, and q in system (4.1) can be found in [25]. Let $\xi_1(t) = \xi(t)$, $\dot{\xi}(t) = \xi_2(t)$; we can rewrite (4.1) as

$$\begin{cases} \dot{\xi}_1(t) = \xi_2(t), \\ \dot{\xi}_2(t) = f(\boldsymbol{\xi}) + u_\mu(t) + d(t), \\ y(t) = \xi_1(t), \end{cases} \tag{4.2}$$

where $\boldsymbol{\xi} = [\xi_1, \xi_2]^T$, $f(\boldsymbol{\xi}) = -\frac{B}{J}\xi_2(t) - \frac{MGq}{J}\sin(\xi_1(t))$. Firstly, we introduce the finite-time performance function and its corresponding control strategy from [25] as follows:

$$\begin{cases} q(t) = \begin{cases} (q_0^l - l\varpi t)^{\frac{1}{l}} + q_\infty, \ 0 \le t < t^s, \\ q_\infty, \ t \ge t^s, \end{cases} \\ e(t) = \xi_1(t) - \xi_d(t), \\ s_1(t) = \tan(\frac{\pi e(t)}{2q(t)}), \ e(0) < q(0), \\ \bar{\xi}_2(t) = -\frac{2q(t)}{\pi(1+s_1^2(t))}c_1 s_1^{2\eta-1}(t) - \frac{\pi(1+s_1^2(t))}{2q(t)}s_1(t) + \dot{\xi}_d(t) + \frac{2}{\pi}\arctan(s_1(t)), \\ s_2(t) = \xi_2(t) - \zeta_1(t), \\ \dot{\zeta}_1(t) = -\tau_1(\zeta_1(t) - \bar{\xi}_2(t)), \\ \zeta_1(0) = \bar{\xi}_2(0), \\ \mu(t) = \frac{-c_2 s_2^{2\eta-1}(t) - \frac{3}{2}s_2(t) - \hat{\theta}_{\bar{f}}^T\varphi_{\bar{f}}(\boldsymbol{\xi}) - \hat{\bar{d}}(t) + \dot{\zeta}_1(t)}{m}, \\ \hat{\bar{d}}(t) = \hat{\kappa}(t) - \delta\xi_2(t), \\ \dot{\hat{\kappa}}(t) = -\delta(\hat{\theta}_{\bar{f}}^T\varphi_{\bar{f}}(\boldsymbol{\xi}) + \hat{\sigma}(t) + \delta\xi_2(t) + m\mu(t)), \\ \dot{\hat{\theta}}_{\bar{f}} = s_2(t)\varphi_{\bar{f}}(\boldsymbol{\xi}) - \gamma\hat{\theta}_{\bar{f}}, \end{cases} \tag{4.3}$$

where $l = \frac{n_0}{m_0} < 1$, n_0 and m_0 are positive even and odd integers, respectively. The performance function $q(t)$ is continuous at $t = t^s$. The relationship between parameters l, q_0, ν and t^s satisfies $t^s = \frac{q_0^l}{l\varpi}$. Let $\bar{d}(t) = d(t) + \Delta\mu(t)$ and $\kappa(t) = \bar{d}(t) - \delta\xi_2(t)$, $\hat{\bar{d}}(t)$ and $\hat{\kappa}(t)$ are the estimations of $\bar{d}(t)$ and $\kappa(t)$, respectively. Here we specify that $\hat{\bar{d}}(t) = \hat{\kappa}(t) + \delta\xi_2(t)$. Define $\tilde{\bar{d}}(t) = \bar{d}(t) - \hat{\bar{d}}(t)$ and $\tilde{\kappa}(t) = \kappa(t) - \hat{\kappa}(t)$; one has $\dot{\tilde{\bar{d}}}(t) = \dot{\tilde{\kappa}}(t)$. Theorem 1 in [25] proves that the closed-loop system composed of $s_1(t), s_2(t), \zeta_1(t), \tilde{\bar{d}}(t)$, and $\tilde{\theta}_{\bar{f}}$ is practically finite-time stable. The following fuzzy membership functions are selected to estimate $f(\boldsymbol{\xi})$:

$$\varphi(\varrho) = \exp\left(-\frac{(\varrho+6-l)^2}{2}\right), \tag{4.4}$$

where $\varrho = \xi_1, \xi_2$, $l = 1, 2, \cdots, 11$. The reference signal $\xi_d = \sin(t)$ and parameters in one-link manipulator (4.1) are selected as $J = 1, M = 1, B = 2, G = 10, q = 1$. In addition, $d(t) = 3\sin(2\pi t)$, $m = 5, \bar{\mu} = \underline{\mu} = 6$, $[\xi_1(0), \xi_2(0)]^T = [5.0, 0.2]^T, \hat{\kappa}(0) = 0, \hat{\theta}_{\bar{f}}(0) = \mathbf{0}$. The design parameters for the control strategy (4.3) are given as $l = \frac{10}{13}, t^s = 10, q_0 = 5.9, \varpi = \frac{q_0^l}{lt^s}, \eta = \frac{1}{2}, c_1 = c_2 = 5, \tau_1 = 20, \delta = 5, \gamma = 0.5$. Correspondingly, the control strategy of this article is as follows:

$$\begin{cases} e(t) = \xi_1(t) - \xi_d(t), \\ s_1(t) = \ln\frac{\nu(t)}{1-\nu(t)}, \\ \nu(t) = \frac{e(t)-h(t,l_0,l_a,t^s)}{h(t,k_0,k_a,t^s)-h(t,l_0,l_a,t^s)}, \\ \bar{\xi}_2(t) = -c_1 s_1(t) + \dot{\xi}_d(t) - c(t), \\ s_2(t) = \xi_2(t) - \zeta_1(t) + \sigma(t), \\ \dot{\zeta}_1(t) = -\tau_1(\zeta_1(t) - \bar{\xi}_2(t)), \\ \zeta_1(0) = \bar{\xi}_2(0), \\ \mu(t) = \frac{-c_2 s_2(t) - \hat{\bar{f}}(t,\boldsymbol{\xi}) + \dot{\zeta}_1(t) - s_1(t)}{m}, \\ \hat{\bar{f}}(t,\boldsymbol{\xi}) = -c_1\sigma(t)^{p_1} - c_2\sigma(t)^{q_1} - \frac{1}{2}\sigma(t) + \hat{\theta}_{\bar{f}}^T\varphi_{\bar{f}}(\boldsymbol{\xi}), \\ \sigma(t) = w(t) - \xi_n(t), \\ \dot{w}(t) = -c_1\sigma(t)^{p_1} - c_2\sigma(t)^{q_1} - \frac{1}{2}\sigma(t) + \hat{\theta}_{\bar{f}}^T\varphi_{\bar{f}}(\boldsymbol{\xi}) + m\mu(t), \\ \dot{\hat{\theta}}_{\bar{f}} = -\sigma(t)\varphi_{\bar{f}}(\boldsymbol{\xi}) - \gamma\hat{\theta}_{\bar{f}}. \end{cases} \tag{4.5}$$

The same parameters and initial values are kept consistent. In addition, performance functions in (4.5) are selected as $h(t, 6.4, 0.05, 15)$ and $h(t, 3.6, -0.05, 15)$. The values of other parameters and initial conditions are selected as $p_1 = \frac{47}{51}, q_1 = \frac{55}{51}$ and $w(0) = 0$.

The simulation results are shown in Figures 4–9. Figures 4 and 5 show that both the control method (4.3) in [25] and the proposed method (4.5) can overcome the influences of unknown functions, external disturbance, and unknown nonlinear dead-zone and ensure that the tracking error $e(t)$ is limited within the preset range at the preset time. Figure6 shows that the dead zone output $u_\mu(t)$ of the control method (4.3) in [25] has big chatter; however, the dead-zone output $u_\mu(t)$ of the proposed method (4.5) shows the chatter is weakened . Figure7 shows that the proposed method (4.5) can quickly and effectively estimate the mixed disturbance $\bar{f}(t,\boldsymbol{\xi})$. Due to the large chatter of the dead-zone output $u_\mu(t)$ by using the control method (4.3) in [25], the nonlinear dead-zone $\Delta\mu(t)$ also appears chatter and the estimation of the mixed disturbance $\bar{f}(t,\boldsymbol{\xi})$ is not as good as that of the proposed method (4.5). The comparison of simulation results shows that the proposed method (4.5) in this paper can overcome the mixed disturbance effectively and can estimate the mixed disturbance accurately and achieve that the tracking error is limited within the preset range in the preset time. To further demonstrate the controllability of the proposed method (4.5) in this paper, Figures 8 and 9 show the control effects at $t^s = 3s$ and $t^s = 15s$, respectively. Obviously, within the preset time, the output signal $\xi_1(t)$ will gradually approach the reference signal $\xi_d(t)$, and after the preset time, the tracking error will enter the preset range.

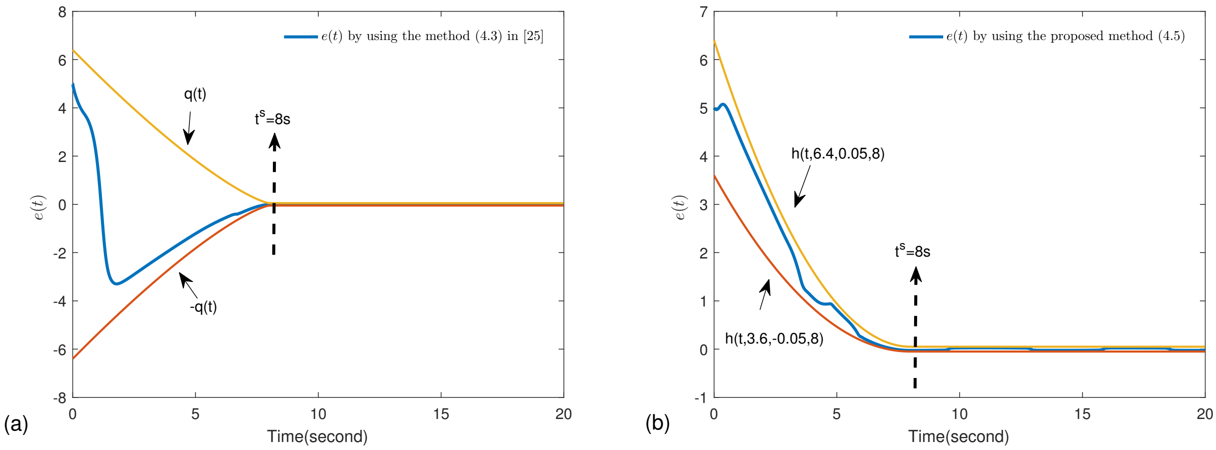

Figure 4. (a) The trajectory of tracking error $e(t)$ by using the control method (4.3) in [25]; (b) The trajectory of tracking error $e(t)$ by using the proposed method (4.5).

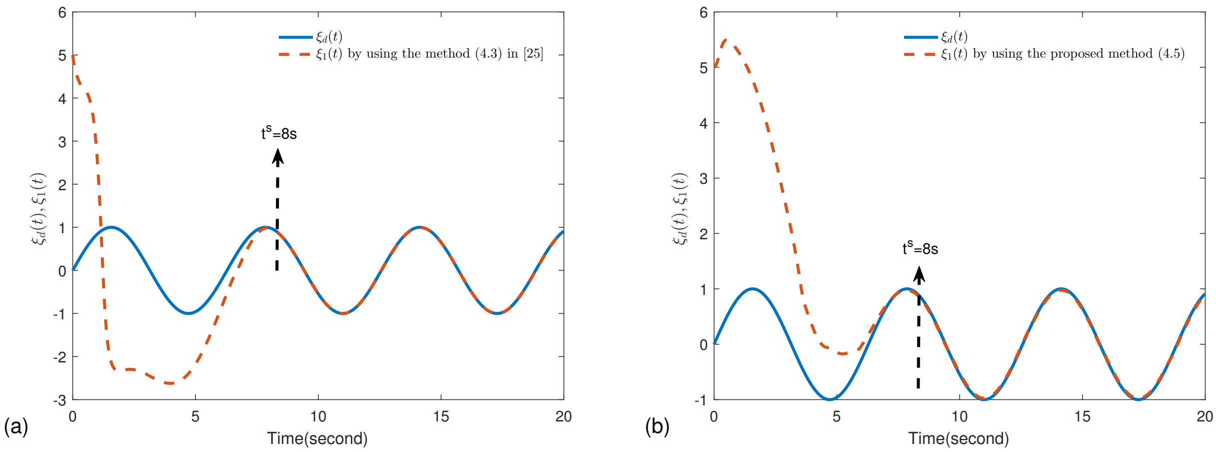

Figure 5. (a) The trajectory of reference signal $\xi_d(t)$ and output variable $\xi_1(t)$ by using the control method (4.3) in [25]; (b) The trajectory of tracking reference signal $\xi_d(t)$ and output variable $\xi_1(t)$ by using the proposed method (4.5).

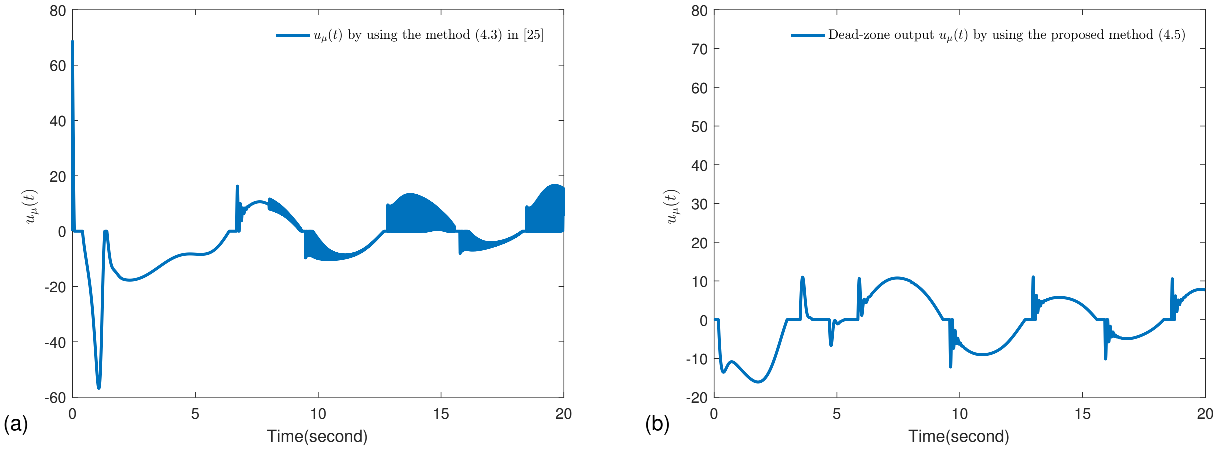

Figure 6. (a) Output trajectory of $u_\mu(t)$ by using the control method (4.3) in [25]; (b) Output trajectory of $u_\mu(t)$ by using the proposed method (4.5).

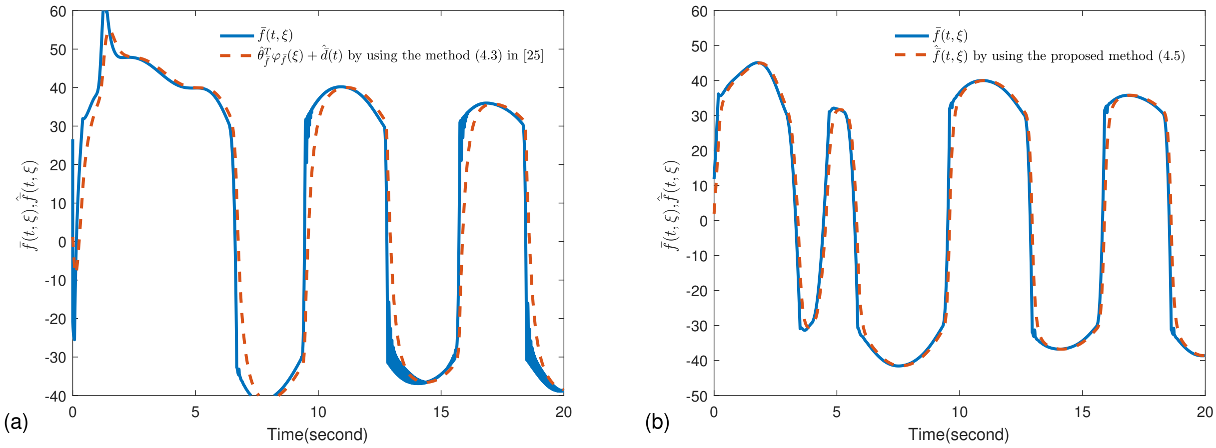

Figure 7. (a) The trajectory of estimate disturbance $\bar{f}(t,\boldsymbol{\xi})$ by using the control method (4.3) in [25]; (b) The trajectory of estimate disturbance $\bar{f}(t,\boldsymbol{\xi})$ by using the proposed method (4.5).

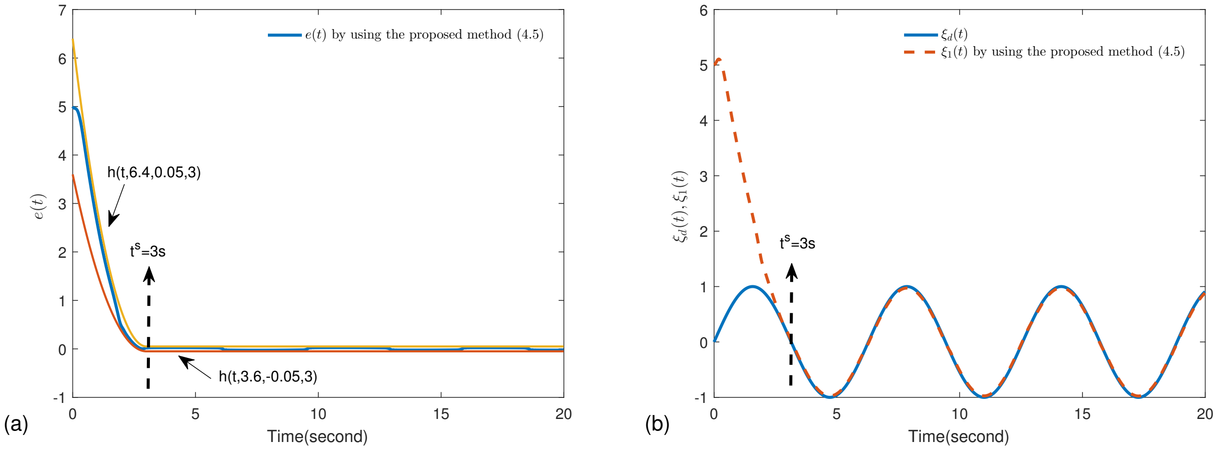

Figure 8. The trajectory of (a) $e(t)$; (b) $\xi_d(t)$ and $\xi_1(t)$ by using the proposed method (4.5) for $t^s = 3s$.

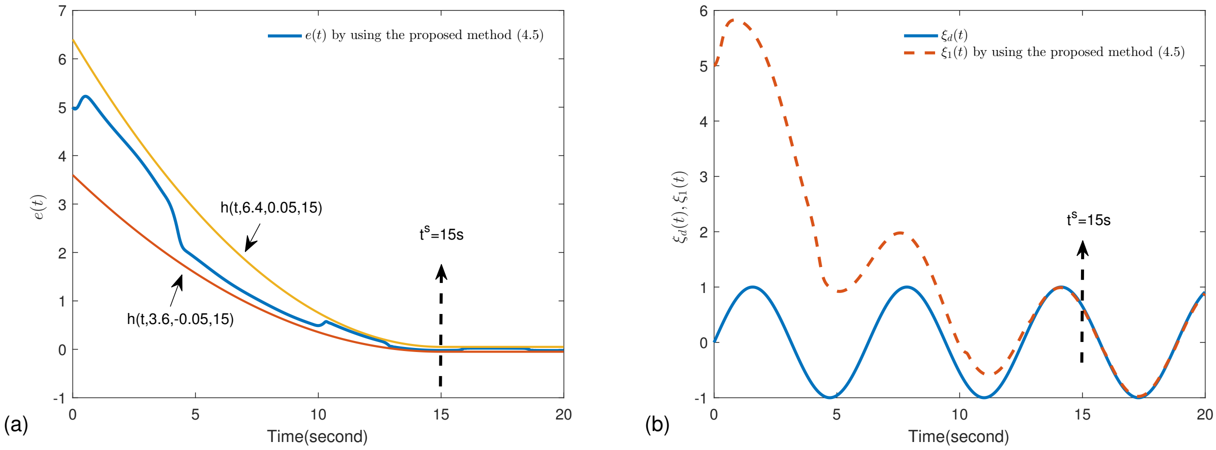

Figure 9. The trajectory of (a) $e(t)$; (b) $\xi_d(t)$ and $\xi_1(t)$ by using the proposed method (4.5) for $t^s = 15s$.

5. Conclusions

In this paper, a finite-time prescribed performance control strategy based on fixed-time disturbance is developed. Theoretical analysis illustrates that the tracking error $e(t)$ can be limited within a preset range and the system uncertainty, which is composed of the external disturbance, unknown system function and the unknown nonlinear dead-zone, can be estimated accurately based on the presented fixed-time observer disturbance. The simulation results validate the effectiveness of the proposed method. In the next step, we will combine interval excitation and the prescribed performance control technique to investigate the problem of accurate parameter estimation for strict-feedback nonlinear systems.

Author contributions

Yuhong Huo: Writing–original draft. Wei Xiang: Writing–review and editing.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was supported in part by the Anhui Province University Excellent Talents Funding Project (gxbjZD2021075) and in part by the Natural Science Foundation for the Higher Education Institutions of Anhui Province (2023AH051549).

Conflict of interest

The authors declare that they have no conflicts of interest.

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