AIMS Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(10): 23821–23848.
DOI: 10.3934/math.20251059
Received: 28 March 2025
Revised: 25 June 2025
Accepted: 03 July 2025
Published: 21 October 2025

Research article

Existence of a solution to the satellite web coupling problem and generating neutrosophic fractals via a novel contraction

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Abstract: In this paper, we provide the notion of a generalized neutrosophic contraction, which extends the concepts of neutrosophic non-expansive mappings and neutrosophic Banach contractions. Using this approach, we proved a fixed point theorem and demonstrated the existence and uniqueness of a solution in the context of neutrosophic metric space and discussed its importance with some appealing applications such as the satellite web coupling problem. We also investigated a new avenue in fractal function production, where fractal structures are constructed using neutrosophic Hutchinson-Barnsley (NHB) operators. We have provided a variety of very interesting examples to illustrate the efficiency of our work in complicated dynamical systems, fractal geometry, and iterated function systems (IFS). We set the stage for future studies in applied mathematics, stability analysis, and fixed point theory by utilizing neutrosophic contraction principles.

Keywords: fixed point approximation; generalized neutrosophic iterated function system; satellite web coupling problem; generalized neutrosophic contraction; attractor; fractals
Mathematics Subject Classification: 54H25, 47H10

1. Introduction

The Banach fixed point theorem, also known as the contraction mapping principle, is a fundamental result in fixed point theory. It guarantees the existence and uniqueness of fixed points for contraction mappings defined on complete metric spaces. Due to its wide applicability in various fields such as differential equations, dynamic systems, and numerical analysis, many authors have worked on generalizing the classical Banach contraction principle.

Over the years, several researchers have introduced extensions and generalizations of Banach-type contractions to various abstract spaces. In particular, Berinde [1] introduced the concept of an enriched Banach contraction, which allowed for more general conditions under which fixed point results could still be obtained. These developments significantly broadened the scope of the classical theorem.

Later, the concept of a fuzzy metric space was introduced by Kramosil [2] to deal with uncertainty and imprecision in measurement. In such spaces, the notion of distance is generalized using fuzzy logic, which allows a better handling of real-world applications where exact values may not be available. For further details on the generalizations of fuzzy metric spaces, readers may refer to the following key references [3, 4] that provide significant insights and developments in this area.

In 2024, a new generalization of the fuzzy Banach contraction was proposed by the author [5], incorporating both fuzzy logic and a broader class of contractive conditions. A corresponding fixed point result was established under this generalized framework, contributing to the ongoing efforts to extend classical fixed point theorems to more complex and realistic settings.

The neutrosophic Banach fixed point (BFP) theorem [6] plays an important role in the framework of neutrosophic fixed point (NFP) theory. It has numerous applications across mathematical domains, particularly in solving linear and nonlinear ordinary differential and integral equations. This theorem establishes the foundation for demonstrating the existence and uniqueness of fixed points in neutrosophic metric spaces (NMSs), facilitating the resolution of complex mathematical problems. Additionally, its relevance extends beyond mathematics into applied sciences, such as engineering, where neutrophilic sets are instrumental in addressing uncertainty and imprecision [7–9]. The neutrosophic Banach contraction (NBC) theorem, by integrating three measures—the degree of nearness, non-nearness, and naturalness—has proven valuable in areas like modeling uncertainty, iterative algorithms, and system stability.

Theorem 1.1. *[6] Let* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *be a complete neutrosophic metric space. Let* $\mathfrak{g} : I \to I$ *be a mapping satisfying*

$$\mathfrak{A}(\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{z}) \leq \mathfrak{A}(\mathfrak{g}\mathfrak{u}_1, \mathfrak{g}\mathfrak{u}_2, \lambda^* \mathfrak{z}),$$

$$\mathfrak{B}(\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{z}) \geq \mathfrak{B}(\mathfrak{g}\mathfrak{u}_1, \mathfrak{g}\mathfrak{u}_2, \lambda^* \mathfrak{z}),$$

$$\mathfrak{C}(\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{z}) \geq \mathfrak{C}(\mathfrak{g}\mathfrak{u}_1, \mathfrak{g}\mathfrak{u}_2, \lambda^* \mathfrak{z}),$$

for every $\mathfrak{u}_1, \mathfrak{u}_2 \in I$, $0 < \lambda^* < 1$. *Then* $\mathfrak{g}$ *possesses a uniquely determined fixed point.*

Hutchinson's foundational study on self-similarity [10] marked a pivotal development in the evolution of fractal theory. Building on this, Barnsley [11] formulated the theory of iterated function systems (ifs), demonstrating that a finite set of contractive maps is sufficient to generate fractal structures within any metric space. The contributions of Hutchinson and Barnsley ignited a surge of interest in self-similar sets, extending applications to image processing, signal modeling, and pattern recognition [12–14]. In recent years, remote sensing and satellite imagery have leveraged these concepts to enhance feature detection, reconstruction, and classification of terrain and spatial data [15–17]. In the realm of remote sensing and electronic systems, notable contributions have been made toward echo signal detection, crop classification using deep learning, simultaneous wireless information, and power transfer systems [18–20].

This article broadens the scope of traditional contraction mappings by proposing a new class called generalized neutrosophic contractions, which unify and extend the known neutrosophic contractions and non-expansive mappings. The primary aim is to establish the existence and uniqueness of fixed points under these generalized mappings via the Krasnoselskij iterative scheme, and to demonstrate strong convergence. These developments have practical significance by tackling satellite web coupling challenges [21–23].

In a future direction, the proposed framework will be extended to neutrosophic Banach spaces (NBSs), along with the definition of a new IFS governed by a Hutchinson–Barnsley-type operator. We aim to utilize fixed point theorems to demonstrate the existence of unique attractors for these systems. As in classical IFS theory, this generalized approach is expected to play a pivotal role in modeling fractal structures and constructing self-similar objects in high-dimensional uncertainty environments [24–27]. The proposed methodology, supported by analytical and graphical examples, is particularly promising for real-world applications involving complex geometric and dynamic systems.

2. Preliminaries

Definition 2.1. *[28] A mapping* $o : [0,1] \times [0,1] \to [0,1]$ *is referred to as a continuous t-norm if it satisfies the following:*

(1) $\mathfrak{u}_1 o 1 = \mathfrak{u}_1$, *for all* $\mathfrak{u}_1 \in [0,1]$;
(2) $\mathfrak{u}_1 o \mathfrak{u}_2 = \mathfrak{u}_2 o \mathfrak{u}_1$ *for all* $\mathfrak{u}_1, \mathfrak{u}_2 \in [0,1]$;
(3) $\mathfrak{u}_1 o(\mathfrak{u}_2 o \mathfrak{u}_3) = (\mathfrak{u}_1 o \mathfrak{u}_2) o \mathfrak{u}_3$, *for all* $\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3 \in [0,1]$;
(4) If $\mathfrak{u}_1 \leq \mathfrak{u}_2$ *and* $\mathfrak{u}_3 \leq \mathfrak{u}_4$, *then* $\mathfrak{u}_1 o \mathfrak{u}_3 \leq \mathfrak{u}_2 o \mathfrak{u}_4$, *for all* $\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \mathfrak{u}_4 \in [0,1]$;
(5) o is continuous.

Definition 2.2. *[28] A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is a continuous t-conorm if* $*$ *satisfies the following conditions:*

(a) $*$ *is commutative and associative;*
(b) $*$ *is continuous;*
(c) $0 * \mathfrak{u}_1 = \mathfrak{u}_1$, $\forall\, \mathfrak{u}_1 \in [0,1]$;
(d) $\mathfrak{u}_3 * \mathfrak{u}_4 \geq \mathfrak{u}_1 * \mathfrak{u}_2$, *whenever* $\mathfrak{u}_1 \leq \mathfrak{u}_3$ *and* $\mathfrak{u}_4 \geq \mathfrak{u}_2$, *and* $\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \mathfrak{u}_4 \in [0,1]$.

Definition 2.3. *[6] We say a 6-tuple* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *is a neutrosophic normed space if a vector space* I, *with a continuous t-norm and t-conorm o and* $*$ *respectively, and* $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}$ *are neutrosophic sets on* $I \times (0, \infty)$, *fulfills the conditions given below for all* $\mathfrak{u}_1, \mathfrak{u}_2 \in I$, *with* $\mathfrak{z}, \mu > 0$:

(a) $\mathfrak{A}(\mathfrak{u}_1, \mathfrak{z}) + \mathfrak{B}(\mathfrak{u}_1, \mathfrak{z}) + \mathfrak{C}(\mathfrak{u}_1, \mathfrak{z}) \leq 3$,
(b) $\mathfrak{A}(\mathfrak{u}_1, \mathfrak{z}) > 0$,
(c) $\mathfrak{A}(\mathfrak{u}_1, \mathfrak{z}) = 1$ *iff* $\mathfrak{u}_1 = 0$,
(d) $\mathfrak{A}(\alpha \mathfrak{u}_1, \mathfrak{z}) = \mathfrak{A}\left(\mathfrak{u}_1, \frac{\mathfrak{z}}{|\alpha|}\right)$ *for all* $\alpha \neq 0$,
(e) $\mathfrak{A}(\mathfrak{u}_1, \mathfrak{z}) o \mathfrak{A}(\mathfrak{u}_2, \mu) \leq \mathfrak{A}(\mathfrak{u}_1 + \mathfrak{u}_2, \mathfrak{z} + \mu)$,
(f) $\mathfrak{A}(\mathfrak{u}_1, \cdot) : (0, \infty) \to [0,1]$ *is continuous,*
(g) $\lim_{\mathfrak{z} \to \infty} \mathfrak{A}(\mathfrak{u}_1, \mathfrak{z}) = 1$ *and* $\lim_{\mathfrak{z} \to 0} \mathfrak{A}(\mathfrak{u}_1, \mathfrak{z}) = 0$,
(h) $\mathfrak{B}(\alpha \mathfrak{u}_1, \mathfrak{z}) = \mathfrak{B}\left(\mathfrak{u}_1, \frac{\mathfrak{z}}{|\alpha|}\right)$ *for each* $\alpha \neq 0$,
(i) $\mathfrak{B}(\mathfrak{u}_1, \mathfrak{z}) * \mathfrak{B}(\mathfrak{u}_2, \mu) \geq \mathfrak{B}(\mathfrak{u}_1 + \mathfrak{u}_2, \mathfrak{z} + \mu)$,
(j) $\mathfrak{B}(\mathfrak{u}_1, \cdot) : (0, \infty) \to [0,1]$ *is continuous,*
(k) $\lim_{\mathfrak{z} \to 0} \mathfrak{B}(\mathfrak{u}_1, \mathfrak{z}) = 1$,

(l) $\mathfrak{C}(\alpha\mathfrak{u}_1, \mathfrak{z}) = \mathfrak{C}\left(\mathfrak{u}_1, \frac{\mathfrak{z}}{|\alpha|}\right)$ *for each* $\alpha \neq 0$,

(m) $\mathfrak{C}(\mathfrak{u}_1, \mathfrak{z}) * \mathfrak{C}(\mathfrak{u}_2, \mu) \geq \mathfrak{C}(\mathfrak{u}_1 + \mathfrak{u}_2, \mathfrak{z} + \mu)$,

(n) $\mathfrak{C}(\mathfrak{u}_1, \cdot) : (0, \infty) \to [0, 1]$ *is continuous,*

(o) $\lim_{\mathfrak{z}\to 0} \mathfrak{C}(\mathfrak{u}_1, \mathfrak{z}) = 1$.

In this case $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C})$ *is called a neutrosophic norm with respect to o and* $*$.

Definition 2.4. *[6] Let* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *be a neutrosophic normed space. For* $\mathfrak{z} > 0$*, we define the open ball* $B(\mathfrak{u}_1, \mathfrak{r}, \mathfrak{z})$ *with center* $\mathfrak{u}_1 \in I$ *and radius* $0 < \mathfrak{r} < 1$*, as*

$$B(\mathfrak{u}_1, \mathfrak{r}, \mathfrak{z}) = \{\mathfrak{u}_2 \in I : \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) > 1 - \mathfrak{r},\ \mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) < \mathfrak{r},\ \mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) < \mathfrak{r}\}.$$

A subset $A \subseteq I$ *is called open if for each* $\mathfrak{u} \in A$*, there exist* $\mathfrak{z} > 0$ *and* $0 < \mathfrak{r} < 1$ *such that* $B(\mathfrak{u}, \mathfrak{r}, \mathfrak{z}) \subseteq A$. *Let* $\mathfrak{g}_{(\mathfrak{A},\mathfrak{B},\mathfrak{C})}$ *denote the family of all open subsets of I.* $\mathfrak{g}_{(\mathfrak{A},\mathfrak{B},\mathfrak{C})}$ *is called the topology induced by the neutrosophic norm.*

Note that this topology is the same as the topology induced by the neutrosophic metric.

Definition 2.5. *[6] The sequence* $\mathfrak{u}_n$ *is said to be convergent to* $\mathfrak{u} \in I$ *in the NNS* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *and denoted by* $\mathfrak{u}_n \xrightarrow{(\mathfrak{A},\mathfrak{B},\mathfrak{C})} \mathfrak{u}$ *if*

$$\mathfrak{A}(\mathfrak{u}_n - \mathfrak{u}, \mathfrak{z}) \to 1, \quad \mathfrak{B}(\mathfrak{u}_n - \mathfrak{u}, \mathfrak{z}) \to 0, \quad \text{and} \quad \mathfrak{C}(\mathfrak{u}_n - \mathfrak{u}, \mathfrak{z}) \to 0$$

whenever $n \to \infty$ *for every* $\mathfrak{z} > 0$.

Definition 2.6. *[6] A sequence* $\mathfrak{u}_n$ *in an NNS* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *is called a Cauchy sequence if for each* $0 < \epsilon < 1$ *and* $\mathfrak{z} > 0$*, there exists* $n_0 \in \mathbb{N}$ *such that*

$$\mathfrak{A}(\mathfrak{u}_n - \mathfrak{u}_m, \mathfrak{z}) > 1 - \epsilon, \quad \mathfrak{B}(\mathfrak{u}_n - \mathfrak{u}_m, \mathfrak{z}) < \epsilon, \quad \text{and} \quad \mathfrak{C}(\mathfrak{u}_n - \mathfrak{u}_m, \mathfrak{z}) < \epsilon$$

for each $n, m \geq n_0$.

Definition 2.7. *[6] A Neutrosophic Banach space is an NNS in which every Cauchy sequence is convergent.*

Now, we are going to define the generalized neutrosophic Contraction (GNC).

Definition 2.8. *Suppose* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *is an NNS. A map* $\mathfrak{g} : I \to I$ *is known to be a generalized neutrosophic contraction if there exist* $\mathfrak{d} \in [0, +\infty)$ *and* $\mathfrak{x} \in [0, \mathfrak{d} + 1)$ *such that*

$$\left.\begin{aligned} \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) &\leq \mathfrak{A}(\mathfrak{g}(\mathfrak{u}_1) - \mathfrak{g}(\mathfrak{u}_2) + \mathfrak{d}(\mathfrak{u}_1 - \mathfrak{u}_2), \mathfrak{x}\mathfrak{z}) \\ \mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) &\geq \mathfrak{B}(\mathfrak{g}(\mathfrak{u}_1) - \mathfrak{g}(\mathfrak{u}_2) + \mathfrak{d}(\mathfrak{u}_1 - \mathfrak{u}_2), \mathfrak{x}\mathfrak{z}) \\ \mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) &\geq \mathfrak{C}(\mathfrak{g}(\mathfrak{u}_1) - \mathfrak{g}(\mathfrak{u}_2) + \mathfrak{d}(\mathfrak{u}_1 - \mathfrak{u}_2), \mathfrak{x}\mathfrak{z}) \end{aligned}\right\} \tag{2.1}$$

for all $\mathfrak{u}_1, \mathfrak{u}_2 \in I$ *and* $\mathfrak{z} \geq 0$.

Keep in mind that all GNC mappings are continuous.

Example 2.1. *1. Any neutrosophic contraction* $\mathfrak{g}$ *is a generalized neutrosophic contraction with* $\mathfrak{d} = 0$ *and* $\mathfrak{x} = \lambda^*$*, with contraction constant* λ^**, i.e.,* $\mathfrak{g}$ *satisfies the system of inequalities* (2.1) *with* $\mathfrak{d} = 0$ *and* $\mathfrak{x} = \lambda^* \in [0, 1)$.

2. *Assume that* $I = [0, 1]$ *has the neutrosophic norm, which is defined by*

$$\mathfrak{A}(\mathfrak{u}, \mathfrak{z}) = \begin{cases} 0 & if \quad \mathfrak{z} = 0, \quad \mathfrak{u} \in [0, 1] \\ \frac{\mathfrak{z}}{\mathfrak{z}+|\mathfrak{u}|} & if \quad \mathfrak{z} > 0, \quad \mathfrak{u} \in [0, 1] \end{cases}$$

$$\mathfrak{B}(\mathfrak{u}, \mathfrak{z}) = \begin{cases} 1 & if \quad \mathfrak{z} = 0, \quad \mathfrak{u} \in [0, 1] \\ \frac{|\mathfrak{u}|}{\mathfrak{z}+|\mathfrak{u}|} & if \quad \mathfrak{z} > 0, \quad \mathfrak{u} \in [0, 1] \end{cases}$$

$$\mathfrak{C}(\mathfrak{u}, \mathfrak{z}) = \begin{cases} 1 & if \quad \mathfrak{z} = 0, \quad \mathfrak{u} \in [0, 1] \\ \frac{|\mathfrak{u}|}{\mathfrak{z}+2|\mathfrak{u}|} & if \quad \mathfrak{z} > 0, \quad \mathfrak{u} \in [0, 1] \end{cases}$$

The function $\mathfrak{g}$ *preserves distances when defined on* I *as* $\mathfrak{g}(\mathfrak{u}) = 1 - \mathfrak{u}$, *making it neutrosophically non-expansive.*

Nevertheless, $\mathfrak{g}$ *does not fit the description of an NC. It fulfills the requirements for the GNC, though. If* $\mathfrak{g}$ *was an NC, a hypothetical scenario would support this claim, implying the existence of* $\lambda^* \in [0, 1)$ *such that*

$$\mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \leq \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_2, \lambda^* \mathfrak{z}),$$

$$\mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \geq \mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_2, \lambda^* \mathfrak{z}),$$

and also,

$$\mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \geq \mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_2, \lambda^* \mathfrak{z})$$

for any $\mathfrak{u}_1, \mathfrak{u}_2 \in [0, 1]$. *But this presumption results in a contradiction for any* $\mathfrak{u}_1 \neq \mathfrak{u}_2$. *Alternatively, the GNC condition* (2.1) *may be stated as*

$$\mathfrak{A}((\mathfrak{d} - 1)(\mathfrak{u}_1 - \mathfrak{u}_2), \mathfrak{x}\mathfrak{z}) \geq \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}),$$

$$\mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \geq \mathfrak{B}((\mathfrak{d} - 1)(\mathfrak{u}_1 - \mathfrak{u}_2), \mathfrak{x}\mathfrak{z}),$$

and also,

$$\mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \geq \mathfrak{C}((\mathfrak{d} - 1)(\mathfrak{u}_1 - \mathfrak{u}_2), \mathfrak{x}\mathfrak{z})$$

$\forall\ \mathfrak{u}_1, \mathfrak{u}_2 \in [0, 1]$, *where* $\mathfrak{x} \in [0, \mathfrak{d} + 1)$. *This inequality applies for* $\mathfrak{u}_1, \mathfrak{u}_2 \in [0, 1]$ *when* $\mathfrak{d} \in (0, 1)$ *and* $\mathfrak{x} = 1 - \mathfrak{d}$. *Therefore, for any* $\mathfrak{d} \in (0, 1)$, $\mathfrak{g}$ *constitutes a generalized neutrosophic contraction. It should be noted that* $\mathfrak{g}(\frac{1}{2}) = \frac{1}{2}$.

Remark 2.1. *For any initial value* x_0, *the sequence described by* $x_{n+1} = 1 - x_n$ *does not converge, as shown in Example 2.1 (2), unless* x_0 *is already a fixed point of* $\mathfrak{g}$. *This suggests that in this instance, the Picard iterative method is ineffective. Consequently, another iterative technique—like the Krasnoselskij iterative scheme—is needed to approximate the fixed point of a generalized neutrosophic contraction (GNC). We prove the efficiency of the Krasnoselskij iterative scheme in the context of generalized neutrosophic contractions by establishing an effective convergence result for it.*

Remark 2.2. *It is important to note that for* $\mathfrak{g}$, *a self-mapping defined on C, a convex subset of a linear space I, the mapping* $\mathfrak{g}_\mathfrak{e}$ *is defined as follows:*

$$\mathfrak{g}_\mathfrak{e}(\mathfrak{u}) = (1 - \mathfrak{e})\mathfrak{u} + \mathfrak{e}\mathfrak{g}(\mathfrak{u}) \tag{2.2}$$

for all $\mathfrak{u} \in C$. *Specifically,* $Fix(\mathfrak{g}_\mathfrak{e}) = Fix(\mathfrak{g})$ *is a feature of this mapping.*

3. Results

In this section we prove a fixed point result for this newly defined contraction.

Theorem 3.1. *Let* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *be an NBS and* $\mathfrak{g} : I \to I$ *is a generalized neutrosophic contraction. Then*

1. *Fix*$(\mathfrak{g}) = \{p\}$.
2. *There exists* $0 < \mathfrak{e} \leq 1$ *such that p is the limit of the iterative scheme* $\{\mathfrak{u}_n\}_{n=0}^{\infty}$, *given as:*

$$\mathfrak{u}_{n+1} = (1 - \mathfrak{e})\mathfrak{u}_n + \mathfrak{e}\mathfrak{g}\mathfrak{u}_n, \tag{3.1}$$

for any $\mathfrak{u}_0 \in I$ *and* $n \geq 0$.

Proof. ***Case (i):*** *Let* $\mathfrak{e} = \frac{1}{\mathfrak{d}+1}$ *and* $\mathfrak{d} > 0$. *Then we have*

$$\mathfrak{A}(\mathfrak{g}_\mathfrak{e}\mathfrak{u}_1 - \mathfrak{g}_\mathfrak{e}\mathfrak{u}_2, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \geq \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}), \tag{3.2}$$

$$\mathfrak{B}(\mathfrak{g}_\mathfrak{e}\mathfrak{u}_1 - \mathfrak{g}_\mathfrak{e}\mathfrak{u}_2, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \leq \mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}), \tag{3.3}$$

and

$$\mathfrak{C}(\mathfrak{g}_\mathfrak{e}\mathfrak{u}_1 - \mathfrak{g}_\mathfrak{e}\mathfrak{u}_2, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \leq \mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \tag{3.4}$$

for every $\mathfrak{u}_1, \mathfrak{u}_2 \in I$*, as* $\mathfrak{x} \in (0, \mathfrak{d} + 1)$ *implies that* $\mathfrak{e}\mathfrak{x} \in (0, 1)$. *Hence* $\mathfrak{g}_\mathfrak{e}$ *is a neutrosophic contraction.*

The Krasnoselskij iterative process $\{\mathfrak{u}_n\}_{n=0}^{\infty}$ *is precisely the Picard iteration associated with* $\mathfrak{g}_\mathfrak{e}$ *in relation to Eq* (2.2).

$$\mathfrak{u}_{n+1} = \mathfrak{g}_\mathfrak{e}(\mathfrak{u}_n), \quad n \geq 0. \tag{3.5}$$

Take $\mathfrak{u}_1 = \mathfrak{u}_n$ *and* $\mathfrak{u}_2 = \mathfrak{u}_{n-1}$ *in* (3.2) *to get*

$$\mathfrak{A}(\mathfrak{u}_{n+1} - \mathfrak{u}_n, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \geq \mathfrak{A}(\mathfrak{u}_n - \mathfrak{u}_{n-1}, \mathfrak{z}), \ \textit{for } n \geq 1. \tag{3.6}$$

From inequality (3.6)*, one obtains routinely the estimate:*

$$\mathfrak{A}(\mathfrak{u}_{n+1} - \mathfrak{u}_n, \mathfrak{z}) \geq \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_0, \frac{\mathfrak{z}}{(\mathfrak{e}\mathfrak{x})^n}), \ \textit{for } n \geq 1. \tag{3.7}$$

Let $m \in N$ *and* $\mathfrak{e}\mathfrak{x} = \mathfrak{a}$. *Observe that*

$$\mathfrak{z}(1 - \mathfrak{a})(1 + \mathfrak{a} + \mathfrak{a}^2 + ... + \mathfrak{a}^{m-1}) = \mathfrak{z}(1 - \mathfrak{a}^m) < \mathfrak{z}. \tag{3.8}$$

Thus using (3.8)*, we have*

$$\mathfrak{A}(\mathfrak{u}_n - \mathfrak{u}_{n+m}, \mathfrak{z}) \geq \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_0, \frac{\mathfrak{z}(1 - \mathfrak{a})}{\mathfrak{a}^n}).$$

Take $\mathfrak{u}_1 = \mathfrak{u}_n$ *and* $\mathfrak{u}_2 = \mathfrak{u}_{n-1}$ *in* (3.3) *to get*

$$\mathfrak{B}(\mathfrak{u}_{n+1} - \mathfrak{u}_n, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \leq \mathfrak{B}(\mathfrak{u}_n - \mathfrak{u}_{n-1}, \mathfrak{z}), \ \textit{for } n \geq 1. \tag{3.9}$$

Similarly, using (3.8)*, we obtain*

$$\mathfrak{B}(\mathfrak{u}_n - \mathfrak{u}_{n+m}, \mathfrak{z}) \leq \mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n})$$

and using (3.4)*, we get*

$$\mathfrak{C}(\mathfrak{u}_n - \mathfrak{u}_{n+m}, \mathfrak{z}) \leq \mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}).$$

This implies that $\{\mathfrak{u}_n\}_{n=0}^{\infty}$ *becomes a Cauchy sequence so it is convergent in neutrosophic Banach space* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$*. Let us indicate*

$$\lim_{n\to\infty} \mathfrak{u}_n = \mathfrak{u}. \tag{3.10}$$

By letting $n \to \infty$ *in* (3.5) *and using the continuity of* $\mathfrak{g}_\mathfrak{e}$*, we can get*

$$\mathfrak{u} = \mathfrak{g}_\mathfrak{e}(\mathfrak{u}),$$

and hence $\mathfrak{u} \in Fix(\mathfrak{g}_\mathfrak{e})$*. Now suppose another FP of* $\mathfrak{g}_\mathfrak{e}$ *is* $\mathfrak{u}^*$*. Then, using* (3.2)–(3.4)*, we obtain a contradiction, proving uniqueness, as we know that Fix(*$\mathfrak{g}$*) = Fix(*$\mathfrak{g}_\mathfrak{e}$*) by Remark 2.2.*

Case (ii): *Let* $\mathfrak{e} = 1$ *and* $\mathfrak{d} = 0$*, and then by using similar steps as in case (i), but substituting* $\mathfrak{g} = \mathfrak{g}_1$ *for* $\mathfrak{g}_\mathfrak{e}$*, then we can prove that the Krasnoselskij iteration contracts, and it reduces to the Picard iteration associated with* $\mathfrak{g}$*;*

$$\mathfrak{u}_{n+1} = \mathfrak{g}(\mathfrak{u}_n), \quad n \geq 0.$$

Remark 3.1. *In the particular case, we derive the classical NBC fixed point theorem [6] by virtue of Theorem 3.1:* $\mathfrak{d} = 0$.

4. Application to a satellite web coupling problem

The specified non-linear boundary value problem that describes radiation from the web coupling between two satellites at a temperature $w(t)$ is:

$$-\frac{d^2w}{dt^2} = mw^4, \quad 0 < t < 1, \quad w(0) = w(1) = 0, \tag{4.1}$$

where the radiation temperature at any position $t \in [0, 1]$ is represented by $w(t)$, and

$$m = \frac{2al^2K^3}{\psi h} > 0$$

is a non-dimensional positive constant. The constant absolute temperature of both satellites is denoted by K, meaning that radiation from the web's surface is emitted into space at 0 degrees Celsius. The distance between the two satellites is represented by l, and the radiation properties of the web's surface are described by a—a positive constant that takes into account radiation from both the top and bottom surfaces. The thermal conductivity is represented by ψ, and the thickness is denoted by h.

The Green's function $g(\mathfrak{u}, c)$ is given by:

$$g(\mathfrak{u}, c) = \begin{cases} \mathfrak{u}(1-c), & 0 < \mathfrak{u} < c, \\ c(1-\mathfrak{u}), & c < \mathfrak{u} < 1. \end{cases}$$

The integral form of Eq (4.1) is:

$$w(\mathfrak{u}) = 1 - m\int_0^1 g(\mathfrak{u}, c)w^4(c)\,dc - \left(\frac{1-\mathfrak{e}}{\mathfrak{e}}\right)w(\mathfrak{u}).$$

Consider a collection of Riemann integrable functions defined on [0, 1] as $I = R[0,1]$. We define

$$\mathfrak{A}(\mathfrak{u}-\mathfrak{u}_1, \mathfrak{z}) = \sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{u}-\mathfrak{u}_1|^p}{\mathfrak{z}}},$$

$$\mathfrak{B}(\mathfrak{u}-\mathfrak{u}_1, \mathfrak{z}) = 1 - 2\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{u}-\mathfrak{u}_1|^p}{\mathfrak{z}}},$$

$$\mathfrak{C}(\mathfrak{u}-\mathfrak{u}_1, \mathfrak{z}) = 1 - \sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{u}-\mathfrak{u}_1|^p}{\mathfrak{z}}},$$

for all $\mathfrak{u}, \mathfrak{u}_1 \in I$, with the operation o such that $\mathfrak{u}_1 o \mathfrak{u}_2 = \mathfrak{u}_1\mathfrak{u}_2$, and $*$ such that $\mathfrak{u}_1 * \mathfrak{u}_2 = \max\{\mathfrak{u}_1, \mathfrak{u}_2\}$. It is easy to prove that $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ is an NBS. A mapping $\mathfrak{U} : I \to I$ is defined by

$$\mathfrak{U}(\Delta(\mathfrak{u})) = 1 - m\int_0^1 g(\mathfrak{u}, \mathfrak{u}_1)\Delta^4(\mathfrak{u}_1)d\mathfrak{u}_1 - \left(\frac{1-\mathfrak{e}}{\mathfrak{e}}\right)\Delta(\mathfrak{u}). \tag{4.2}$$

Then,

$$\mathfrak{U}_{\mathfrak{e}}(\Delta(\mathfrak{u})) = \mathfrak{e}\cdot\left(1 - m\cdot\int_0^1 g(\mathfrak{u}, \mathfrak{u}_1)\Delta^4(\mathfrak{u}_1)d\mathfrak{u}_1\right). \tag{4.3}$$

Theorem 4.1. *Let* $\mathfrak{U} : I \to I$ *be a mapping that is defined as in Eq* (4.2) *and for* $\mathfrak{d} = \frac{1-\mathfrak{e}}{\mathfrak{e}}$, *where* $\mathfrak{e} \in (0,1)$, *the following conditions are true:*

(i)

$$|(\Delta(\mathfrak{u}_1) + \Omega(\mathfrak{u}_1))(\Delta^2(\mathfrak{u}_1) + \Omega^2(\mathfrak{u}_1))|^p \le \frac{\mathfrak{e}\mathfrak{x}}{|\mathfrak{e}m|^p}.$$

(ii) There exists a continuous function $g : [0,1]\times[0,1] \to \mathbb{R}^+$ *such that*

$$\sup_{\mathfrak{u}\in[0,1]} \int_0^g g(\mathfrak{u}, \mathfrak{u}_1)d\mathfrak{u}_1 \le 1.$$

Then, the satellite web coupling boundary value problem (4.1) *has only one solution.*
Proof. Consider the following for all $\Delta, \Omega \in I$*:*

$$\begin{aligned}
\mathfrak{A}(\mathfrak{d}(\Delta(\mathfrak{u}) - \Omega(\mathfrak{u})) + \mathfrak{U}(\Delta(\mathfrak{u})) - \mathfrak{U}(\Omega(\mathfrak{u})), \mathfrak{x}\mathfrak{z}) &= \mathfrak{A}(\frac{1-\mathfrak{e}}{\mathfrak{e}}(\Delta(\mathfrak{u}) - \Omega(\mathfrak{u})) + \mathfrak{U}(\Delta(\mathfrak{u})) - \mathfrak{U}(\Omega(\mathfrak{u})), \mathfrak{x}\mathfrak{z}) \\
&= \mathfrak{A}(\frac{(1-\mathfrak{e})\Delta(\mathfrak{u}) - (1-\mathfrak{e})\Omega(\mathfrak{u}) + \mathfrak{e}\mathfrak{U}(\Delta(\mathfrak{u})) - \mathfrak{e}\mathfrak{U}(\Omega(\mathfrak{u}))}{\mathfrak{e}}, \mathfrak{x}\mathfrak{z}) \\
&= \mathfrak{A}(\mathfrak{U}_{\mathfrak{e}}(\Delta(\mathfrak{u})) - \mathfrak{U}_{\mathfrak{e}}(\Omega(\mathfrak{u})), \mathfrak{e}\mathfrak{x}\mathfrak{z}) \\
&= \sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{U}_{\mathfrak{e}}(\Delta(\mathfrak{u})) - \mathfrak{U}_{\mathfrak{e}}(\Omega(\mathfrak{u}))|^p}{\mathfrak{e}\mathfrak{x}\mathfrak{z}}}
\end{aligned}$$

$$= \sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Delta^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)-\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Omega^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}}$$

$$\begin{aligned}
\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Delta^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)-\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Omega^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} &= \sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{e}m|^p|(\Delta^4(\mathfrak{u}_1)-\Omega^4(\mathfrak{u}_1))\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)d\mathfrak{u}_1|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} \\
&\geq \sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1))\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)d\mathfrak{u}_1|^p}{\mathfrak{z}}} \\
&\geq \sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1))|^p}{\mathfrak{z}}} \\
&= \mathfrak{A}(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1),\mathfrak{z}).
\end{aligned}$$

$$\begin{aligned}
\mathfrak{B}(\mathfrak{d}(\Delta(\mathfrak{u})-\Omega(\mathfrak{u}))+\mathfrak{U}(\Delta(\mathfrak{u}))-\mathfrak{U}(\Omega(\mathfrak{u})),\mathfrak{x}\mathfrak{z}) &= \mathfrak{B}(\frac{1-\mathfrak{e}}{\mathfrak{e}}(\Delta(\mathfrak{u})-\Omega(\mathfrak{u}))+\mathfrak{U}(\Delta(\mathfrak{u}))-\mathfrak{U}(\Omega(\mathfrak{u})),\mathfrak{x}\mathfrak{z}) \\
&= \mathfrak{B}(\mathfrak{U}_{\mathfrak{e}}(\Delta(\mathfrak{u}))-\mathfrak{U}_{\mathfrak{e}}(\Omega(\mathfrak{u})),\mathfrak{e}\mathfrak{x}\mathfrak{z}) \\
&= 1-2\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{U}_{\mathfrak{e}}(\Delta(\mathfrak{u}))-\mathfrak{U}_{\mathfrak{e}}(\Omega(\mathfrak{u}))|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} \\
&= 1-2\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Delta^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)-\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Omega^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} \\
&= 1-2\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{e}m|^p|(\Delta^4(\mathfrak{u}_1)-\Omega^4(\mathfrak{u}_1))\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)d\mathfrak{u}_1|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} \\
&\leq 1-2\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1))\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)d\mathfrak{u}_1|^p}{\mathfrak{z}}} \\
&\leq 1-2\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1))|^p}{\mathfrak{z}}} \\
&= \mathfrak{B}(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1),\mathfrak{z}).
\end{aligned}$$

$$\begin{aligned}
\mathfrak{C}(\mathfrak{d}(\Delta(\mathfrak{u})-\Omega(\mathfrak{u}))+\mathfrak{U}(\Delta(\mathfrak{u}))-\mathfrak{U}(\Omega(\mathfrak{u})),\mathfrak{x}\mathfrak{z}) &= \mathfrak{C}(\frac{1-\mathfrak{e}}{\mathfrak{e}}(\Delta(\mathfrak{u})-\Omega(\mathfrak{u}))+\mathfrak{U}(\Delta(\mathfrak{u}))-\mathfrak{U}(\Omega(\mathfrak{u})),\mathfrak{x}\mathfrak{z}) \\
&= \mathfrak{C}(\mathfrak{U}_{\mathfrak{e}}(\Delta(\mathfrak{u}))-\mathfrak{U}_{\mathfrak{e}}(\Omega(\mathfrak{u})),\mathfrak{e}\mathfrak{x}\mathfrak{z}) \\
&= 1-\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{U}_{\mathfrak{e}}(\Delta(\mathfrak{u}))-\mathfrak{U}_{\mathfrak{e}}(\Omega(\mathfrak{u}))|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} \\
&= 1-\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Delta^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)-\mathfrak{e}\cdot\left(1-m\cdot\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)\Omega^4(\mathfrak{u}_1)d\mathfrak{u}_1\right)|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} \\
&= 1-\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|\mathfrak{e}m|^p|(\Delta^4(\mathfrak{u}_1)-\Omega^4(\mathfrak{u}_1))\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)d\mathfrak{u}_1|^p}{\mathfrak{e}\mathfrak{X}\mathfrak{z}}} \\
&\leq 1-\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1))\int_0^1 g(\mathfrak{u},\mathfrak{u}_1)d\mathfrak{u}_1|^p}{\mathfrak{z}}} \\
&\leq 1-\sup_{\mathfrak{z}\in[0,1]} e^{\frac{-|(\Delta(\mathfrak{u}_1)-\Omega(\mathfrak{u}_1))|^p}{\mathfrak{z}}}
\end{aligned}$$

$$= \mathfrak{C}(\Delta(\mathfrak{u}_1) - \Omega(\mathfrak{u}_1), \mathfrak{z}).$$

Therefore, $\mathfrak{U}$ *has a unique FP since all of the assumptions of Theorem 3.1 are met. So, the diffrential equation* (4.1) *has a unique solution.*

5. Application to fractals

In this section, we generate fractals using the generalized neutrosophic contraction and illustrate how this concept applies to the creation of intricate and captivating fractal patterns.

Definition 5.1. *[6] Consider the NNS* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *and the topology* $\Upsilon_{(\mathfrak{A},\mathfrak{B},\mathfrak{C})}$ *that is produced by the neutrosophic norm* $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C})$*. The set of all non-empty compact subsets of* $(I, \Upsilon_{(\mathfrak{A},\mathfrak{B},\mathfrak{C})})$ *will be denoted by the symbol* $\Theta(I)$*.*

Define, $\mathfrak{A}(\mathfrak{u}-\Gamma, \mathfrak{z}) = \sup_{\lambda\in\Gamma} \mathfrak{A}(\mathfrak{u}-\lambda, \mathfrak{z})$*,* $\mathfrak{B}(\mathfrak{u}-\Gamma, \mathfrak{z}) = \inf_{\lambda\in\Gamma} \mathfrak{C}(\mathfrak{u}-\lambda, \mathfrak{z})$*, and* $\mathfrak{C}(\mathfrak{u}-\Gamma, \mathfrak{z}) = \inf_{\lambda\in\Gamma} \mathfrak{C}(\mathfrak{u}-\lambda, \mathfrak{z})$*. Similarly* $\mathfrak{A}(\Delta-\Gamma, \mathfrak{z}) = \inf_{\mathfrak{u}\in\Delta} \mathfrak{A}(\mathfrak{u}-\Gamma, \mathfrak{z})$*,* $\mathfrak{B}(\Delta-\Gamma, \mathfrak{z}) = \sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u}-\Gamma, \mathfrak{z})$*, and* $\mathfrak{C}(\Delta-\Gamma, \mathfrak{z}) = \sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u}-\Gamma, \mathfrak{z})$ *for all* $\mathfrak{u} \in I$ *and* $\Delta, \Gamma \in \Theta(I)$*. The Hausdorff neutrosophic norm is denoted by* $H_{(\mathfrak{A},\mathfrak{B},\mathfrak{C})}$*, where* $H_{\mathfrak{A}}, H_{\mathfrak{B}}, H_{\mathfrak{C}} : \Theta(I) \times (0, \infty) \to [0, 1]$ *are neutrosophic sets defined by*

$$H_{\mathfrak{A}}(\Delta - \Gamma, \mathfrak{z}) = \min\{\mathfrak{A}(\Delta - \Gamma, \mathfrak{z}), \mathfrak{A}(\Gamma - \Delta, \mathfrak{z})\},$$

$$H_{\mathfrak{B}}(\Delta - \Gamma, \mathfrak{z}) = \max\{\mathfrak{B}(\Delta - \Gamma, \mathfrak{z}), \mathfrak{B}(\Gamma - \Delta, \mathfrak{z})\},$$

and

$$H_{\mathfrak{C}}(\Delta - \Gamma, \mathfrak{z}) = \max\{\mathfrak{C}(\Delta - \Gamma, \mathfrak{z}), \mathfrak{C}(\Gamma - \Delta, \mathfrak{z})\}.$$

Hence, $(\Theta(I), H_{\mathfrak{A}}, H_{\mathfrak{B}}, H_{\mathfrak{C}}, o, *)$ *is a Hausdorff NNS.*
If $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *is a complete NNS then* $(\Theta(I), H_{\mathfrak{A}}, H_{\mathfrak{B}}, H_{\mathfrak{C}}, o, *)$ *is a complete Hausdorff NNS.*

Definition 5.2. *Let* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *be an NNS and* $\{\mathfrak{g}_N : I \to I : n = 1, 2, 3..., N\}$ *is a finite collection of generalized neutrosophic contractions. The operator* $T : \Theta(I) \to \Theta(I)$ *defined by*

$$T(\Delta) = \mathfrak{g}_1(\Delta) \cup \mathfrak{g}_2(\Delta) \cup ... \cup \mathfrak{g}_N(\Delta) = \bigcup_{i=1}^{N} \mathfrak{g}_i(\Delta),$$

for all $\Delta \in \Theta(I)$*, is a generalized neutrosophic H-B operator.*

Definition 5.3. *Let* $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ *be an NNS. If* $\{\mathfrak{g}_N : I \to I : n = 1, 2, 3..., N\}$ *is a finite collection of generalized neutrosophic contractions, then* $(I : \mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, ..., \mathfrak{g}_N)$ *is called the generalized neutrosophic IFS* $(GNIFS)$*.*

Definition 5.4. *A compact set* Δ *that is not empty serves as an attractor for the GNIFS if*

1. $T(\Delta) = \Delta$.
2. *An element* Ω *exists in the set* $\Upsilon_{(\mathfrak{A},\mathfrak{B},\mathfrak{C})}$ *such that* $\Delta \subset \Omega$ *and* $\lim_{k\to\infty}(T^k(\Gamma), \Delta, \mathfrak{z}) = 1$*, considering any compact set* $\Gamma \subset \Omega$ *and* $\mathfrak{z} > 0$*.*

To bolster our next conclusion, we established the following lemma.

Lemma 5.1. *Let $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ be an NNS. Then, for every $\Delta, \Gamma, \Omega, \Lambda \in \Theta(I)$ belonging to $\Theta(I)$, the following conditions are satisfied:*

i) If $\Gamma \subset \Omega$, then

$$\inf_{\mathfrak{u}\in\Delta} \mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z}) \geq \inf_{\mathfrak{u}\in\Delta} \mathfrak{A}(\mathfrak{u} - \Gamma, \mathfrak{z})$$
$$\sup_{\mathfrak{u}\in\Delta} \mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z}) \leq \sup_{\mathfrak{u}\in\Delta} \mathfrak{B}(\mathfrak{u} - \Gamma, \mathfrak{z})$$
$$\sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z}) \leq \sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u} - \Gamma, \mathfrak{z}).$$

ii)

$$\inf_{\mathfrak{u}\in\Delta\cup\Gamma} \mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z}) = \min\{\inf_{\mathfrak{u}\in\Delta} \mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z}), \inf_{\lambda\in\Gamma} \mathfrak{A}(\lambda - \Omega, \mathfrak{z})\}$$
$$\sup_{\mathfrak{u}\in\Delta\cup\Gamma} \mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z}) = \max\{\sup_{\mathfrak{u}\in\Delta} \mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z}), \sup_{\lambda\in\Gamma} \mathfrak{B}(\lambda - \Omega, \mathfrak{z})\}$$
$$\sup_{\mathfrak{u}\in\Delta\cup\Gamma} \mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z}) = \max\{\sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z}), \sup_{\lambda\in\Gamma} \mathfrak{C}(\lambda - \Omega, \mathfrak{z})\}.$$

iii)

$$H_{\mathfrak{A}}(\Delta \cup \Gamma - \Omega \cup \Lambda, \mathfrak{z}) \geq \min\{H_{\mathfrak{A}}(\Delta - \Omega, \mathfrak{z}), H_{\mathfrak{A}}(\Gamma - \Lambda, \mathfrak{z})\}$$
$$H_{\mathfrak{B}}(\Delta \cup \Gamma - \Omega \cup \Lambda, \mathfrak{z}) \leq \max\{H_{\mathfrak{B}}(\Delta - \Omega, \mathfrak{z}), H_{\mathfrak{B}}(\Gamma - \Lambda, \mathfrak{z})\}$$
$$H_{\mathfrak{C}}(\Delta \cup \Gamma - \Omega \cup \Lambda, \mathfrak{z}) \leq \max\{H_{\mathfrak{C}}(\Delta - \Omega, \mathfrak{z}), H_{\mathfrak{C}}(\Gamma - \Lambda, \mathfrak{z})\}.$$

Proof. (i): Given that $\Gamma \subset \Omega$, then for all $\mathfrak{u} \in \Delta$, it follows that

$$\mathfrak{A}(\mathfrak{u} - \Gamma, \mathfrak{z}) = \sup_{\lambda\in\Gamma} \mathfrak{A}(\mathfrak{u} - \lambda, \mathfrak{z}) \leq \sup_{\zeta\in\Omega} \mathfrak{A}(\mathfrak{u} - \zeta, \mathfrak{z}) = \mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z})$$

which implies that

$$\inf_{\mathfrak{u}\in\Delta} \mathfrak{A}(\mathfrak{u} - \Gamma, \mathfrak{z}) \leq \inf_{\mathfrak{u}\in\Delta} \mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z})$$
$$\mathfrak{B}(\mathfrak{u} - \Gamma, \mathfrak{z}) = \inf_{\lambda\in\Gamma} \mathfrak{B}(\mathfrak{u} - \lambda, \mathfrak{z}) \geq \inf_{\zeta\in\Omega} \mathfrak{B}(\mathfrak{u} - \zeta, \mathfrak{z}) = \mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z})$$

which implies that

$$\sup_{\mathfrak{u}\in\Delta} \mathfrak{B}(\mathfrak{u} - \Gamma, \mathfrak{z}) \geq \sup_{\mathfrak{u}\in\Delta} \mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z})$$
$$\mathfrak{C}(\mathfrak{u} - \Gamma, \mathfrak{z}) = \inf_{\lambda\in\Gamma} \mathfrak{C}(\mathfrak{u} - \lambda, \mathfrak{z}) \geq \inf_{\zeta\in\Omega} \mathfrak{C}(\mathfrak{u} - \zeta, \mathfrak{z}) = \mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z})$$

which implies that

$$\sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u} - \Gamma, \mathfrak{z}) \geq \sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z}).$$

(ii):

$$\inf_{\mathfrak{u}\in\Delta\cup\Gamma} \mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z}) = \inf\{\mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z}) : \mathfrak{u} \in \Delta \cup \Gamma\} = \min\{\inf_{\mathfrak{u}\in\Delta} \mathfrak{A}(\mathfrak{u} - \Omega, \mathfrak{z}), \inf_{\lambda\in\Gamma} \mathfrak{A}(\lambda - \Omega, \mathfrak{z})\}$$
$$\sup_{\mathfrak{u}\in\Delta\cup\Gamma} \mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z}) = \sup\{\mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z}) : \mathfrak{u} \in \Delta \cup \Gamma\} = \max\{\sup_{\mathfrak{u}\in\Delta} \mathfrak{B}(\mathfrak{u} - \Omega, \mathfrak{z}), \sup_{\lambda\in\Gamma} \mathfrak{B}(\lambda - \Omega, \mathfrak{z})\}$$
$$\sup_{\mathfrak{u}\in\Delta\cup\Gamma} \mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z}) = \sup\{\mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z}) : \mathfrak{u} \in \Delta \cup \Gamma\} = \max\{\sup_{\mathfrak{u}\in\Delta} \mathfrak{C}(\mathfrak{u} - \Omega, \mathfrak{z}), \sup_{\lambda\in\Gamma} \mathfrak{C}(\lambda - \Omega, \mathfrak{z})\}.$$

(iii): It follows from (ii) that

$$\begin{aligned}\inf_{\mathfrak{u}\in\Delta\cup\Gamma}\mathfrak{A}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}) &= \min\{\inf_{\mathfrak{u}\in\Delta}\mathfrak{A}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}),\inf_{\lambda\in\Gamma}\mathfrak{A}(\lambda-\Omega\cup\Lambda,\mathfrak{z})\}\\ &\geq \min\{\inf_{\mathfrak{u}\in\Delta}\mathfrak{A}(\mathfrak{u}-\Omega,\mathfrak{z}),\inf_{\lambda\in\Gamma}\mathfrak{A}(\lambda-\Lambda,\mathfrak{z})\}\\ &\geq \min\{\min\{\inf_{\mathfrak{u}\in\Delta}\mathfrak{A}(\mathfrak{u}-\Omega,\mathfrak{z}),\inf_{\zeta\in\Omega}\mathfrak{A}(\Delta-\zeta,\mathfrak{z})\},\\ &\qquad \min\{\inf_{\lambda\in\Gamma}\mathfrak{A}(\lambda-\Lambda,\mathfrak{z}),\inf_{\beta\in\Lambda}\mathfrak{A}(\Gamma-\beta,\mathfrak{z})\}\}\\ &= \min\{H_{\mathfrak{A}}(\Delta-\Omega,\mathfrak{z}),H_{\mathfrak{A}}(\Gamma-\Lambda,\mathfrak{z})\}.\end{aligned}$$

In a similar way, we obtain that

$$\inf_{\mathfrak{u}\in\Omega\cup\Lambda}\mathfrak{A}(\mathfrak{u}-\Delta\cup\Gamma,\mathfrak{z}) \geq \min\{H_{\mathfrak{A}}(\Delta-\Omega,\mathfrak{z}),H_{\mathfrak{A}}(\Gamma-\Lambda,\mathfrak{z})\}.$$

Hence it follows that

$$\begin{aligned}H_{\mathfrak{A}}(\Delta\cup\Gamma-\Omega\cup\Lambda,\mathfrak{z}) &= \min\{\inf_{\mathfrak{u}\in\Delta\cup\Gamma}\mathfrak{A}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}),\inf_{\mathfrak{u}\in\Omega\cup\Lambda}\mathfrak{A}(\mathfrak{u}-\Delta\cup\Gamma,\mathfrak{z})\}\\ &\geq \min\{H_{\mathfrak{A}}(\Delta-\Omega,\mathfrak{z}),H_{\mathfrak{A}}(\Gamma-\Lambda,\mathfrak{z})\}.\end{aligned}$$

$$\begin{aligned}\sup_{\mathfrak{u}\in\Delta\cup\Gamma}\mathfrak{B}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}) &= \max\{\sup_{\mathfrak{u}\in\Delta}\mathfrak{B}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}),\sup_{\lambda\in\Gamma}\mathfrak{B}(\lambda-\Omega\cup\Lambda,\mathfrak{z})\}\\ &\leq \max\{\sup_{\mathfrak{u}\in\Delta}\mathfrak{B}(\mathfrak{u}-\Omega,\mathfrak{z}),\sup_{\lambda\in\Gamma}\mathfrak{B}(\lambda-\Lambda,\mathfrak{z})\}\\ &\leq \max\{\max\{\sup_{\mathfrak{u}\in\Delta}\mathfrak{B}(\mathfrak{u}-\Omega,\mathfrak{z}),\sup_{\zeta\in\Omega}\mathfrak{B}(\Delta-\zeta,\mathfrak{z})\},\\ &\qquad \max\{\sup_{\lambda\in\Gamma}\mathfrak{B}(\lambda-\Lambda,\mathfrak{z}),\sup_{\beta\in\Lambda}\mathfrak{B}(\Gamma-\beta,\mathfrak{z})\}\}\\ &= \max\{H_{\mathfrak{B}}(\Delta-\Omega,\mathfrak{z}),H_{\mathfrak{B}}(\Gamma-\Lambda,\mathfrak{z})\}.\end{aligned}$$

In a similar way, we obtain that

$$\sup_{\mathfrak{u}\in\Omega\cup\Lambda}\mathfrak{B}(\mathfrak{u}-\Delta\cup\Gamma,\mathfrak{z}) \leq \max\{H_{\mathfrak{B}}(\Delta-\Omega,\mathfrak{z}),H_{\mathfrak{B}}(\Gamma-\Lambda,\mathfrak{z})\}.$$

Hence it follows that

$$\begin{aligned}H_{\mathfrak{B}}(\Delta\cup\Gamma-\Omega\cup\Lambda,\mathfrak{z}) &= \max\{\sup_{\mathfrak{u}\in\Delta\cup\Gamma}\mathfrak{B}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}),\sup_{\mathfrak{u}\in\Omega\cup\Lambda}\mathfrak{B}(\mathfrak{u}-\Delta\cup\Gamma,\mathfrak{z})\}\\ &\leq \max\{H_{\mathfrak{B}}(\Delta-\Omega,\mathfrak{z}),H_{\mathfrak{B}}(\Gamma-\Lambda,\mathfrak{z})\}.\end{aligned}$$

$$\begin{aligned}\sup_{\mathfrak{u}\in\Delta\cup\Gamma}\mathfrak{C}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}) &= \max\{\sup_{\mathfrak{u}\in\Delta}\mathfrak{C}(\mathfrak{u}-\Omega\cup\Lambda,\mathfrak{z}),\sup_{\lambda\in\Gamma}\mathfrak{C}(\lambda-\Omega\cup\Lambda,\mathfrak{z})\}\\ &\leq \max\{\sup_{\mathfrak{u}\in\Delta}\mathfrak{C}(\mathfrak{u}-\Omega,\mathfrak{z}),\sup_{\lambda\in\Gamma}\mathfrak{C}(\lambda-\Lambda,\mathfrak{z})\}\\ &\leq \max\{\max\{\sup_{\mathfrak{u}\in\Delta}\mathfrak{C}(\mathfrak{u}-\Omega,\mathfrak{z}),\sup_{\zeta\in\Omega}\mathfrak{C}(\Delta-\zeta,\mathfrak{z})\},\\ &\qquad \max\{\sup_{\lambda\in\Gamma}\mathfrak{C}(\lambda-\Lambda,\mathfrak{z}),\sup_{\beta\in\Lambda}\mathfrak{C}(\Gamma-\beta,\mathfrak{z})\}\}\\ &= \max\{H_{\mathfrak{C}}(\Delta-\Omega,\mathfrak{z}),H_{\mathfrak{C}}(\Gamma-\Lambda,\mathfrak{z})\}.\end{aligned}$$

In a similar way, we obtain that

$$\sup_{\mathfrak{u}\in\Omega\cup\Lambda} \mathfrak{C}(\mathfrak{u} - \Delta \cup \Gamma, \mathfrak{z}) \leq \max\{H_{\mathfrak{C}}(\Delta - \Omega, \mathfrak{z}), H_{\mathfrak{C}}(\Gamma - \Lambda, \mathfrak{z})\}.$$

Hence it follows that

$$\begin{aligned} H_{\mathfrak{C}}(\Delta \cup \Gamma - \Omega \cup \Lambda, \mathfrak{z}) &= \max\{\sup_{\mathfrak{u}\in\Delta\cup\Gamma} \mathfrak{C}(\mathfrak{u} - \Omega \cup \Lambda, \mathfrak{z}), \sup_{\mathfrak{u}\in\Omega\cup\Lambda} \mathfrak{C}(\mathfrak{u} - \Delta \cup \Gamma, \mathfrak{z})\} \\ &\leq \max\{H_{\mathfrak{C}}(\Delta - \Omega, \mathfrak{z}), H_{\mathfrak{C}}(\Gamma - \Lambda, \mathfrak{z})\}. \end{aligned}$$

Theorem 5.2. *Let $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ be an NNS and $\{\mathfrak{g}_n : n = 1, 2, 3, ..., N\}$ a finite family of generalized neutrosophic contraction mappings on I. Define $T\colon \Theta(I) \to \Theta(I)$ by*

$$T(\Delta) = \mathfrak{g}_1(\Delta) \cup \mathfrak{g}_2(\Delta) \cup \mathfrak{g}_3(\Delta) \cup ... \cup \mathfrak{g}_N(\Delta),$$

for each $\Delta \in \Theta(I)$. Then T is a generalized neutrosophic contraction on $\Theta(I)$.
Proof. We will show for $N = 2$.

Let $\mathfrak{g}_1, \mathfrak{g}_2 : I \to I$ be two generalized neutrosophic contractions. Take $\Delta, \Lambda \in \Theta(I)$ with $H_{\mathfrak{A}}(T(\Delta), T(\Lambda), \mathfrak{z}) \neq 0$, $H_{\mathfrak{B}}(T(\Delta), T(\Lambda), \mathfrak{z}) \neq 0$, and $H_{\mathfrak{C}}(T(\Delta), T(\Lambda), \mathfrak{z}) \neq 0$. Lemma 5.1 (iii) clearly implies that

$$\begin{aligned} H_{\mathfrak{A}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) &= H_{\mathfrak{A}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_1(\Delta) \cup \mathfrak{g}_2(\Delta) - \mathfrak{g}_1(\Lambda) \cup \mathfrak{g}_2(\Lambda), \mathfrak{x}\mathfrak{z}) \\ &\geq \min\{H_{\mathfrak{A}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_1(\Delta) - \mathfrak{g}_1(\Lambda), \mathfrak{x}\mathfrak{z}), \\ &\qquad H_{\mathfrak{A}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_2(\Delta) - \mathfrak{g}_2(\Lambda), \mathfrak{x}\mathfrak{z})\} \\ &\geq H_{\mathfrak{A}}(\Delta - \Lambda, \mathfrak{z})) \end{aligned}$$

$$\begin{aligned} H_{\mathfrak{B}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) &= H_{\mathfrak{B}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_1(\Delta) \cup \mathfrak{g}_2(\Delta) - \mathfrak{g}_1(\Lambda) \cup \mathfrak{g}_2(\Lambda), \mathfrak{x}\mathfrak{z}) \\ &\leq \max\{H_{\mathfrak{B}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_1(\Delta) - \mathfrak{g}_1(\Lambda), \mathfrak{x}\mathfrak{z}), \\ &\qquad H_{\mathfrak{B}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_2(\Delta) - \mathfrak{g}_2(\Lambda), \mathfrak{x}\mathfrak{z})\} \\ &\leq H_{\mathfrak{B}}(\Delta - \Lambda, \mathfrak{z})) \\ H_{\mathfrak{C}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) &= H_{\mathfrak{C}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_1(\Delta) \cup \mathfrak{g}_2(\Delta) - \mathfrak{g}_1(\Lambda) \cup \mathfrak{g}_2(\Lambda), \mathfrak{x}\mathfrak{z}) \\ &\leq \max\{H_{\mathfrak{C}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_1(\Delta) - \mathfrak{g}_1(\Lambda), \mathfrak{x}\mathfrak{z}), \\ &\qquad H_{\mathfrak{C}}(\mathfrak{d}(\Delta - \Lambda) + \mathfrak{g}_2(\Delta) - \mathfrak{g}_2(\Lambda), \mathfrak{x}\mathfrak{z})\} \\ &\leq H_{\mathfrak{C}}(\Delta - \Lambda, \mathfrak{z})). \end{aligned}$$

Hence, T is a GNC on $\Theta(I)$.

Theorem 5.3. *Consider $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ is an NBS and $\{\mathfrak{g}_n : n = 1, 2, 3, ..., N\}$ a finite collection of GNCs on I. On $\Theta(I)$, define a mapping T as*

$$T(\Delta) = \mathfrak{g}_1(\Delta) \cup \mathfrak{g}_2(\Delta) \cup \mathfrak{g}_3(\Delta) \cup ... \cup \mathfrak{g}_N(\Delta),$$

for each $\Delta \in \Theta(I)$.

Then i) $T\colon \Theta(I) \to \Theta(I)$.

ii) T has a unique FP $\Delta \in \Theta(I)$, that is, $\Delta = T(\Delta)$.

Proof. i) Since every $\mathfrak{g}_i$ is a generalized neutrosophic contraction, by Theorem 5.2 and the definition of T, the result follows immediately.

ii) From Theorem 5.2, $T\colon \Theta(I) \to \Theta(I)$ is a generalized neutrosophic contraction. Furthermore, $(\Theta(I), H_{\mathfrak{A}}, H_{\mathfrak{B}}, H_{\mathfrak{C}}, o, *)$ is a Hausdorff NBS because $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ is an NBS. Thus, Theorem 3.1 implies (ii).

Definition 5.5. *Let $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ be an NNS. A mapping $T\colon \Theta(I) \to \Theta(I)$ is a generalized neutrosophic $(\mathfrak{A}_T, \mathfrak{B}_T, \mathfrak{C}_T)$ contraction if there exist $\mathfrak{d} \in [0, +\infty)$ and $\mathfrak{x} \in [0, \mathfrak{d}+1)$ such that the following holds for each $\Delta, \Lambda \in \Theta(I)$ with $H_{\mathfrak{A}}(T(\Delta), T(\Lambda), \mathfrak{z}) \neq 0$, $H_{\mathfrak{B}}(T(\Delta), T(\Lambda), \mathfrak{z}) \neq 0$, $H_{\mathfrak{C}}(T(\Delta), T(\Lambda), \mathfrak{z}) \neq 0$:*

$$H_{\mathfrak{A}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) \geq \mathfrak{A}_T(\Delta - \Lambda, \mathfrak{z}) \tag{5.1}$$

where

$$\mathfrak{A}_T(\Delta - \Lambda, \mathfrak{z}) = \min\{H_{\mathfrak{A}}(\Delta - \Lambda, \mathfrak{z}), H_{\mathfrak{A}}(\Delta - T(\Delta), \mathfrak{z}), H_{\mathfrak{A}}(\Lambda - T(\Lambda), \mathfrak{z})\}$$

$$H_{\mathfrak{B}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) \leq \mathfrak{B}_T(\Delta - \Lambda, \mathfrak{z}) \tag{5.2}$$

where

$$\mathfrak{B}_T(\Delta - \Lambda, \mathfrak{z}) = \max\{H_{\mathfrak{B}}(\Delta - \Lambda, \mathfrak{z}), H_{\mathfrak{B}}(\Delta - T(\Delta), \mathfrak{z}), H_{\mathfrak{B}}(\Lambda - T(\Lambda), \mathfrak{z})\}$$

$$H_{\mathfrak{C}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) \leq \mathfrak{C}_T(\Delta - \Lambda, \mathfrak{z}) \tag{5.3}$$

where

$$\mathfrak{C}_T(\Delta - \Lambda, \mathfrak{z}) = \max\{H_{\mathfrak{C}}(\Delta - \Lambda, \mathfrak{z}), H_{\mathfrak{C}}(\Delta - T(\Delta), \mathfrak{z}), H_{\mathfrak{C}}(\Lambda - T(\Lambda), \mathfrak{z})\}.$$

It should be noted that T is trivially a generalized neutrosophic $(\mathfrak{A}_T, \mathfrak{B}_T, \mathfrak{C}_T)$ contraction if T, as outlined in Theorem 5.2, is a GNC.

Theorem 5.4. *Let $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{a}, o, *)$ be an NBS and $\{I; \mathfrak{g}_n, n = 1, 2, 3, ...N\}$ is a GNIFS. Let $T\colon \Theta(I) \to \Theta(I)$ be a generalized neutrosophic $(\mathfrak{A}_T, \mathfrak{B}_T, \mathfrak{a}_T)$ contraction operator defined by*

$$T(\Delta) = \mathfrak{g}_1(\Delta) \cup \mathfrak{g}_2(\Delta) \cup ... \cup \mathfrak{g}_N(\Delta), \quad \textit{for each } \Delta \in \Theta(I).$$

Then

1. *T has only one FP.*
2. *for some $\mathfrak{e} \in (0, 1]$, the iterative scheme $\{\Delta_n\}_{n=0}^{\infty}$, given by*

$$\Delta_{n+1} = (1 - \mathfrak{e})\Delta_n + \mathfrak{e}T(\Delta_n), \tag{5.4}$$

for $n \geq 0$, converges to the fixed point of T, for any $\Delta_0 \in I$.

Proof. (i): Let $\mathfrak{e} = \frac{1}{\mathfrak{d}+1}$ and $\mathfrak{d} > 0$. Then for $\mathfrak{e} \in (0, 1)$, T is a generalized neutrosophic $(\mathfrak{A}_T, \mathfrak{B}_T, \mathfrak{C}_T)$ contraction, and therefore for each $\Delta, \Gamma \in \Theta(I)$ with $H_{\mathfrak{A}}(T(\Delta), T(\Gamma), \mathfrak{z}) \neq 0$, $H_{\mathfrak{B}}(T(\Delta), T(\Gamma), \mathfrak{z}) \neq 0$, $H_{\mathfrak{C}}(T(\Delta), T(\Gamma), \mathfrak{z}) \neq 0$,

$$H_{\mathfrak{A}}(\mathfrak{d}(\Delta-\Gamma)+T(\Delta)-T(\Gamma),\mathfrak{x}\mathfrak{z}) \geq \mathfrak{A}_T(\Delta-\Gamma,\mathfrak{z}) \tag{5.5}$$

where

$$\mathfrak{A}_T(\Delta-\Gamma,\mathfrak{z}) = \min\{H_{\mathfrak{A}}(\Delta-\Gamma,\mathfrak{z}), H_{\mathfrak{A}}(\Delta-T(\Delta),\mathfrak{z}), H_{\mathfrak{A}}(\Gamma-T(\Gamma),\mathfrak{z})\}$$

$$H_{\mathfrak{B}}(\mathfrak{d}(\Delta-\Gamma)+T(\Delta)-T(\Gamma),\mathfrak{x}\mathfrak{z}) \leq \mathfrak{B}_T(\Delta-\Gamma,\mathfrak{z}) \tag{5.6}$$

where

$$\mathfrak{B}_T(\Delta-\Gamma,\mathfrak{z}) = \max\{H_{\mathfrak{B}}(\Delta-\Gamma,\mathfrak{z}), H_{\mathfrak{B}}(\Delta-T(\Delta),\mathfrak{z}), H_{\mathfrak{B}}(\Gamma-T(\Gamma),\mathfrak{z})\}.$$

And then

$$H_{\mathfrak{C}}(\mathfrak{d}(\Delta-\Gamma)+T(\Delta)-T(\Gamma),\mathfrak{x}\mathfrak{z}) \leq \mathfrak{C}_T(\Delta-\Gamma,\mathfrak{z}) \tag{5.7}$$

where

$$\mathfrak{C}_T(\Delta-\Gamma,\mathfrak{z}) = \max\{H_{\mathfrak{C}}(\Delta-\Gamma,\mathfrak{z}), H_{\mathfrak{C}}(\Delta-T(\Delta),\mathfrak{z}), H_{\mathfrak{C}}(\Gamma-T(\Gamma),\mathfrak{z})\}$$

$$\begin{aligned} H_{\mathfrak{A}}(\mathfrak{d}(\Delta-\Gamma)+T(\Delta)-T(\Gamma),\mathfrak{x}\mathfrak{z}) &= H_{\mathfrak{A}}((\frac{1}{\mathfrak{e}}-1)(\Delta-\Gamma)+T(\Delta)-T(\Gamma),\mathfrak{x}\mathfrak{z}) \\ &= H_{\mathfrak{A}}((\frac{1-\mathfrak{e}}{\mathfrak{e}})(\Delta-\Gamma)+T(\Delta)-T(\Gamma),\mathfrak{x}\mathfrak{z}) \\ &= H_{\mathfrak{A}}((1-\mathfrak{e})(\Delta-\Gamma)+\mathfrak{e}T(\Delta)-\mathfrak{e}T(\Gamma),\mathfrak{e}\mathfrak{x}\mathfrak{z}) \\ &= H_{\mathfrak{A}}(T_{\mathfrak{e}}(\Delta)-T_{\mathfrak{e}}\Gamma,\mathfrak{e}\mathfrak{x}\mathfrak{z}). \end{aligned}$$

By using inequality (5.5)*, we have*

$$H_{\mathfrak{A}}(T_{\mathfrak{e}}(\Delta)-T_{\mathfrak{e}}(\Gamma),\mathfrak{e}\mathfrak{x}\mathfrak{z}) \geq \mathfrak{A}_T(\Delta-\Gamma,\mathfrak{z}). \tag{5.8}$$

Similarly,

$$H_{\mathfrak{B}}(T_{\mathfrak{e}}(\Delta)-T_{\mathfrak{e}}(\Gamma),\mathfrak{e}\mathfrak{x}\mathfrak{z}) \leq \mathfrak{B}_T(\Delta-\Gamma,\mathfrak{z}) \tag{5.9}$$

$$H_{\mathfrak{C}}(T_{\mathfrak{e}}(\Delta)-T_{\mathfrak{e}}(\Gamma),\mathfrak{e}\mathfrak{x}\mathfrak{z}) \leq \mathfrak{C}_T(\Delta-\Gamma,\mathfrak{z}). \tag{5.10}$$

Let Δ_0 *be any element of* $\Theta(I)$*. If* $\Delta_0 = T_{\mathfrak{e}}(\Delta_0)$*, the evidence is complete. Therefore, we assume that* $\Delta_0 \neq T_{\mathfrak{e}}(\Delta_0)$*. Define*

$$\Delta_1 = T_{\mathfrak{e}}(\Delta_0), \quad \Delta_2 = T_{\mathfrak{e}}(\Delta_1), \quad \ldots, \quad \Delta_{m+1} = T_{\mathfrak{e}}(\Delta_m)$$

for $m \in \mathbb{N}$*.*

Suppose that $\Delta_m = \Delta_{m+1}$ *for all* $m \in \mathbb{N}$*.*

$$\begin{aligned} H_{\mathfrak{A}}(\Delta_{m+1}-\Delta_{m+2},\mathfrak{e}\mathfrak{x}\mathfrak{z}) &= H_{\mathfrak{A}}(T_{\mathfrak{e}}(\Delta_m)-T_{\mathfrak{e}}(\Delta_{m+1}),\mathfrak{e}\mathfrak{x}\mathfrak{z}) \\ &\geq \mathfrak{A}_T(\Delta_m-\Delta_{m+1},\mathfrak{z}) \end{aligned}$$

where

$$\mathfrak{A}_T(\Delta_m - \Delta_{m+1}, \mathfrak{z}) = \min\{H_{\mathfrak{A}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{A}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{A}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{z})\}.$$

This implies that

$$\begin{aligned} H_{\mathfrak{A}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{e}\mathfrak{x}\mathfrak{z}) &\geq \min\{H_{\mathfrak{A}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{A}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{z})\} \\ &= H_{\mathfrak{A}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}). \end{aligned}$$

The estimate is typically obtained from the above inequality:

$$H_{\mathfrak{A}}(\Delta_{n+1} - \Delta_n, \mathfrak{z}) \geq H_{\mathfrak{A}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}}{(\mathfrak{e}\mathfrak{x})^n}), \ \ for\ n \geq 1. \tag{5.11}$$

Suppose $m \in N$ and $\mathfrak{e}\mathfrak{x} = \mathfrak{a}$. Note that

$$\mathfrak{z}(1 - \mathfrak{a})(1 + \mathfrak{a} + \mathfrak{a}^2 + ... + \mathfrak{a}^{m-1}) = \mathfrak{z}(1 - \mathfrak{a}^m) < \mathfrak{z}. \tag{5.12}$$

Thus using inequality (5.12)*, we have*

$$\begin{aligned} H_{\mathfrak{A}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}) &\geq H_{\mathfrak{A}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}(1 - \mathfrak{a}^m)) \\ &= H_{\mathfrak{A}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}(1 - \mathfrak{a})(1 + \mathfrak{a} + \mathfrak{a}^2 + ... + \mathfrak{a}^{m-1})) \\ &\geq H_{\mathfrak{A}}(\Delta_n - \Delta_{n+1}, \mathfrak{z}(1 - \mathfrak{a})) o H_{\mathfrak{A}}(\Delta_{n+1} - \Delta_{n+2}, \mathfrak{z}(1 - \mathfrak{a})\mathfrak{a}) o \\ &\quad ... o H_{\mathfrak{A}}(\Delta_{n+m-1} - \Delta_{n+m}, \mathfrak{z}(1 - \mathfrak{a})\mathfrak{a}^{m-1}) \\ &\geq H_{\mathfrak{A}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1 - \mathfrak{a})}{\mathfrak{a}^n}) o H_{\mathfrak{A}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1 - \mathfrak{a})}{\mathfrak{a}^n}) o ... o H_{\mathfrak{A}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1 - \mathfrak{a})}{\mathfrak{a}^n}) \\ &= H_{\mathfrak{A}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1 - \mathfrak{a})}{\mathfrak{a}^n}). \end{aligned}$$

And

$$\begin{aligned} H_{\mathfrak{B}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{e}\mathfrak{x}\mathfrak{z}) &= H_{\mathfrak{B}}(T_{\mathfrak{e}}(\Delta_m) - T_{\mathfrak{e}}(\Delta_{m+1}), \mathfrak{e}\mathfrak{x}\mathfrak{z}) \\ &\leq \mathfrak{B}_T(\Delta_m - \Delta_{m+1}, \mathfrak{z}) \end{aligned}$$

where

$$\mathfrak{B}_T(\Delta_m - \Delta_{m+1}, \mathfrak{z}) = \max\{H_{\mathfrak{B}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{B}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{B}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{z})\}.$$

This implies that

$$\begin{aligned} H_{\mathfrak{B}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{e}\mathfrak{x}\mathfrak{z}) &\leq \max\{H_{\mathfrak{B}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{B}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{z})\} \\ &= H_{\mathfrak{B}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}). \end{aligned}$$

From the above inequality,

$$H_{\mathfrak{B}}(\Delta_{n+1} - \Delta_n, \mathfrak{z}) \leq H_{\mathfrak{B}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}}{(\mathfrak{e}\mathfrak{x})^n}), \ \ for\ n \geq 1. \tag{5.13}$$

Thus using inequality (5.12)*, we have*

$$\begin{aligned} H_{\mathfrak{B}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}) &\leq H_{\mathfrak{B}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}(1-\mathfrak{a}^m)) \\ &= H_{\mathfrak{B}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}(1-\mathfrak{a})(1+\mathfrak{a}+\mathfrak{a}^2+...+\mathfrak{a}^{m-1})) \\ &\leq H_{\mathfrak{B}}(\Delta_n - \Delta_{n+1}, \mathfrak{z}(1-\mathfrak{a})) * H_{\mathfrak{B}}(\Delta_{n+1} - \Delta_{n+2}, \mathfrak{z}(1-\mathfrak{a})\mathfrak{a}) * \\ &\quad ... * H_{\mathfrak{B}}(\Delta_{n+m-1} - \Delta_{n+m}, \mathfrak{z}(1-\mathfrak{a})\mathfrak{a}^{m-1}) \\ &\leq H_{\mathfrak{B}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}) * H_{\mathfrak{B}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}) * ... * H_{\mathfrak{B}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}) \\ &= H_{\mathfrak{B}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}). \end{aligned}$$

And

$$\begin{aligned} H_{\mathfrak{C}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{e}\mathfrak{x}\mathfrak{z}) &= H_{\mathfrak{C}}(T_{\mathfrak{e}}(\Delta_m) - T_{\mathfrak{e}}(\Delta_{m+1}), \mathfrak{e}\mathfrak{x}\mathfrak{z}) \\ &\leq \mathfrak{C}_T(\Delta_m - \Delta_{m+1}, \mathfrak{z}) \end{aligned}$$

where

$$\mathfrak{C}_T(\Delta_m - \Delta_{m+1}, \mathfrak{z}) = \max\{H_{\mathfrak{C}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{C}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{C}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{z})\}.$$

This implies that

$$\begin{aligned} H_{\mathfrak{C}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{e}\mathfrak{x}\mathfrak{z}) &\leq \max\{H_{\mathfrak{C}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}), H_{\mathfrak{C}}(\Delta_{m+1} - \Delta_{m+2}, \mathfrak{z})\} \\ &= H_{\mathfrak{C}}(\Delta_m - \Delta_{m+1}, \mathfrak{z}). \end{aligned}$$

From the above inequality,

$$H_{\mathfrak{C}}(\Delta_{n+1} - \Delta_n, \mathfrak{z}) \leq H_{\mathfrak{C}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}}{(\mathfrak{e}\mathfrak{x})^n}), \ \textit{for } n \geq 1. \tag{5.14}$$

Thus using inequality (5.12)*, we have*

$$\begin{aligned} H_{\mathfrak{C}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}) &\leq H_{\mathfrak{C}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}(1-\mathfrak{a}^m)) \\ &= H_{\mathfrak{C}}(\Delta_n - \Delta_{n+m}, \mathfrak{z}(1-\mathfrak{a})(1+\mathfrak{a}+\mathfrak{a}^2+...+\mathfrak{a}^{m-1})) \\ &\leq H_{\mathfrak{C}}(\Delta_n - \Delta_{n+1}, \mathfrak{z}(1-\mathfrak{a})) * H_{\mathfrak{C}}(\Delta_{n+1} - \Delta_{n+2}, \mathfrak{z}(1-\mathfrak{a})\mathfrak{a}) * \\ &\quad ... * H_{\mathfrak{C}}(\Delta_{n+m-1} - \Delta_{n+m}, \mathfrak{z}(1-\mathfrak{a})\mathfrak{a}^{m-1}) \\ &\leq H_{\mathfrak{C}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}) * H_{\mathfrak{C}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}) * ... * H_{\mathfrak{C}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}) \\ &= H_{\mathfrak{C}}(\Delta_1 - \Delta_0, \frac{\mathfrak{z}(1-\mathfrak{a})}{\mathfrak{a}^n}). \end{aligned}$$

So, $\{\Delta_n\}_{n=0}^{\infty}$ *is a Cauchy sequence and hence is convergent. Let us denote*

$$\lim_{n\to\infty} \Delta_n = \Delta. \tag{5.15}$$

As n approaches infinity in Eq (5.4)*, we readily obtain*

$$\Delta = T_{\mathfrak{e}}(\Delta),$$

so Δ is the FP of $T_{\mathfrak{e}}$. Now we are going to show that $T_{\mathfrak{e}}$ has only one FP. Let Γ be another fixed point of $T_{\mathfrak{e}}$. Afterward, by inequalities (5.8)–(5.10),

$$H_{\mathfrak{A}}(\Delta - \Gamma, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \geq H_{\mathfrak{A}}(\Delta - \Gamma, \mathfrak{z})$$

$$H_{\mathfrak{B}}(\Delta - \Gamma, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \leq H_{\mathfrak{B}}(\Delta - \Gamma, \mathfrak{z})$$

$$H_{\mathfrak{C}}(\Delta - \Gamma, \mathfrak{e}\mathfrak{x}\mathfrak{z}) \leq H_{\mathfrak{C}}(\Delta - \Gamma, \mathfrak{z})$$

which is contradictory. Hence, $Fix(T_{\mathfrak{e}}) = \Delta$ and by Remark 2.2, $Fix(T) = Fix(T_{\mathfrak{e}})$.

Case (ii): *Let $\mathfrak{e} = 1$ and $\mathfrak{d} = 0$, and then we can prove, by using similar steps as in case (i), but substituting $T = T_1$ for $\mathfrak{g}_{\mathfrak{e}}$, that the Krasnoselskij iteration contracts, it reduces to the Picard iteration associated with T:*

$$\Delta_{n+1} = T(\Delta_n), \quad n \geq 0.$$

Corollary 5.5. *Let $(I, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, o, *)$ be an NBS and $\{I; \mathfrak{g}_i, i = 1, 2, ...N\}$ a GNIFS. Let $\mathfrak{g} : I \to I$ be a self-mapping as defined in Remark 2.2. If there exist $\mathfrak{d} \in [0, +\infty)$ and $\mathfrak{x} \in [0, \mathfrak{d} + 1)$ such that the following is valid for each $\mathfrak{u}_1, \mathfrak{u}_2 \in I$:*

$$\mathfrak{A}(\mathfrak{d}(\mathfrak{u}_1 - \mathfrak{u}_2) + \mathfrak{g}(\mathfrak{u}_1) - \mathfrak{g}(\mathfrak{u}_2), \mathfrak{x}\mathfrak{z}) \geq \mathfrak{A}_{\mathfrak{g}}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \tag{5.16}$$

where $\mathfrak{A}_{\mathfrak{g}}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) = \min\{\mathfrak{A}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}), \mathfrak{A}(\mathfrak{u}_1 - \mathfrak{g}(\mathfrak{u}_1), \mathfrak{z}), \mathfrak{A}(\mathfrak{u}_2 - \mathfrak{g}(\mathfrak{u}_2), \mathfrak{z})\}$,

$$\mathfrak{B}(\mathfrak{d}(\mathfrak{u}_1 - \mathfrak{u}_2) + \mathfrak{g}(\mathfrak{u}_1) - \mathfrak{g}(\mathfrak{u}_2), \mathfrak{x}\mathfrak{z}) \leq \mathfrak{B}_{\mathfrak{g}}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \tag{5.17}$$

where $\mathfrak{B}_{\mathfrak{g}}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) = \max\{\mathfrak{B}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}), \mathfrak{B}(\mathfrak{u}_1 - \mathfrak{g}(\mathfrak{u}_1), \mathfrak{z}), \mathfrak{B}(\mathfrak{u}_2 - \mathfrak{g}(\mathfrak{u}_2), \mathfrak{z})\}$,

$$\mathfrak{C}(\mathfrak{d}(\mathfrak{u}_1 - \mathfrak{u}_2) + \mathfrak{g}(\mathfrak{u}_1) - \mathfrak{g}(\mathfrak{u}_2), \mathfrak{x}\mathfrak{z}) \leq \mathfrak{C}_{\mathfrak{g}}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) \tag{5.18}$$

where $\mathfrak{C}_{\mathfrak{g}}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}) = \max\{\mathfrak{C}(\mathfrak{u}_1 - \mathfrak{u}_2, \mathfrak{z}), \mathfrak{C}(\mathfrak{u}_1 - \mathfrak{g}(\mathfrak{u}_1), \mathfrak{z}), \mathfrak{C}(\mathfrak{u}_2 - \mathfrak{g}(\mathfrak{u}_2), \mathfrak{z})\}$,
then $\mathfrak{g}$ has a unique fixed point. Moreover, for any choice of $\mathfrak{u}_1 \in I$ and $n \in N$, the sequence $\{\mathfrak{u}_n\}$ defined by

$$\mathfrak{u}_{n+1} = (1 - \mathfrak{e})\mathfrak{u}_n + \mathfrak{e}\mathfrak{g}(\mathfrak{u}_n)$$

converges to the fixed point.

Example 5.1. *Let $I = R^2$ and $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C})$ be the neutrosophic norm defined by:*

$$\mathfrak{A}((\mathfrak{u}, \mathfrak{u}^*), \mathfrak{z}) = \exp^{\frac{-\sqrt{\mathfrak{u}^2 + (\mathfrak{u}^*)^2}}{\mathfrak{z}}},$$

$$\mathfrak{B}((\mathfrak{u}, \mathfrak{u}^*), \mathfrak{z}) = 1 - 2\exp^{\frac{-\sqrt{\mathfrak{u}^2 + (\mathfrak{u}^*)^2}}{\mathfrak{z}}},$$

and

$$\mathfrak{C}((\mathfrak{u}, \mathfrak{u}^*), \mathfrak{z}) = 1 - \exp^{\frac{-\sqrt{\mathfrak{u}^2 + (\mathfrak{u}^*)^2}}{\mathfrak{z}}}.$$

Let $\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3 : I \to I$ *be defined by:*

$$\mathfrak{g}_1(\mathfrak{u}, \mathfrak{u}^*) = (\frac{\mathfrak{u}}{2}, \frac{\mathfrak{u}^*}{2}), \quad \mathfrak{g}_2(\mathfrak{u}, \mathfrak{u}^*) = (\frac{\mathfrak{u}}{2} + \frac{1}{2}, \frac{\mathfrak{u}^*}{2}), \quad \text{and} \quad \mathfrak{g}_3(\mathfrak{u}, \mathfrak{u}^*) = (\frac{\mathfrak{u}}{2} + \frac{1}{4}, \frac{\mathfrak{u}^*}{2} + \frac{\sqrt{3}}{4}).$$

Let $\mathfrak{d} = 0$ *and* $\mathfrak{x} = \frac{1}{2}$*, and then*

$$\begin{aligned}
\mathfrak{A}(\mathfrak{d}((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)) + \mathfrak{g}_1(\mathfrak{u}_1, \mathfrak{u}_1^*) - \mathfrak{g}_1(\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{x}\mathfrak{z}) &= \mathfrak{A}((\frac{\mathfrak{u}_1}{2}, \frac{\mathfrak{u}_1^*}{2}) - (\frac{\mathfrak{u}_2}{2}, \frac{\mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{A}((\frac{\mathfrak{u}_1 - \mathfrak{u}_2}{2}, \frac{\mathfrak{u}_1^* - \mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{A}(\frac{1}{2}((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{A}(((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \mathfrak{z}) \\
&= \mathfrak{A}(((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)), \mathfrak{z})
\end{aligned}$$

$$\begin{aligned}
\mathfrak{B}(\mathfrak{d}((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)) + \mathfrak{g}_1(\mathfrak{u}_1, \mathfrak{u}_1^*) - \mathfrak{g}_1(\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{x}\mathfrak{z}) &= \mathfrak{B}((\frac{\mathfrak{u}_1}{2}, \frac{\mathfrak{u}_1^*}{2}) - (\frac{\mathfrak{u}_2}{2}, \frac{\mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{B}((\frac{\mathfrak{u}_1 - \mathfrak{u}_2}{2}, \frac{\mathfrak{u}_1^* - \mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{B}(\frac{1}{2}((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{B}(((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \mathfrak{z}) \\
&= \mathfrak{B}(((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)), \mathfrak{z})
\end{aligned}$$

and

$$\begin{aligned}
\mathfrak{C}(\mathfrak{d}((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)) + \mathfrak{g}_1(\mathfrak{u}_1, \mathfrak{u}_1^*) - \mathfrak{g}_1(\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{x}\mathfrak{z}) &= \mathfrak{C}((\frac{\mathfrak{u}_1}{2}, \frac{\mathfrak{u}_1^*}{2}) - (\frac{\mathfrak{u}_2}{2}, \frac{\mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{C}((\frac{\mathfrak{u}_1 - \mathfrak{u}_2}{2}, \frac{\mathfrak{u}_1^* - \mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{C}(\frac{1}{2}((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{C}(((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \mathfrak{z}) \\
&= \mathfrak{C}(((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)), \mathfrak{z}).
\end{aligned}$$

Now,

$$\begin{aligned}
\mathfrak{A}(\mathfrak{d}((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)) + \mathfrak{g}_2(\mathfrak{u}_1, \mathfrak{u}_1^*) - \mathfrak{g}_2(\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{x}\mathfrak{z}) &= \mathfrak{A}((\frac{\mathfrak{u}_1}{2} + \frac{1}{2}, \frac{\mathfrak{u}_1^*}{2}) - (\frac{\mathfrak{u}_2}{2} + \frac{1}{2}, \frac{\mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{A}((\frac{\mathfrak{u}_1 - \mathfrak{u}_2}{2}, \frac{\mathfrak{u}_1^* - \mathfrak{u}_2^*}{2}), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{A}(\frac{1}{2}((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \frac{1}{2}\mathfrak{z}) \\
&= \mathfrak{A}(((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \mathfrak{z}) \\
&= \mathfrak{A}(((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)), \mathfrak{z})
\end{aligned}$$

$$\begin{aligned}\mathfrak{B}(\mathfrak{d}((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*))+\mathfrak{g}_2(\mathfrak{u}_1,\mathfrak{u}_1^*)-\mathfrak{g}_2(\mathfrak{u}_2,\mathfrak{u}_2^*),\mathfrak{x}\mathfrak{z}) &= \mathfrak{B}((\frac{\mathfrak{u}_1}{2}+\frac{1}{2},\frac{\mathfrak{u}_1^*}{2})-(\frac{\mathfrak{u}_2}{2}+\frac{1}{2},\frac{\mathfrak{u}_2^*}{2}),\frac{1}{2}\mathfrak{z})\\ &= \mathfrak{B}((\frac{\mathfrak{u}_1-\mathfrak{u}_2}{2},\frac{\mathfrak{u}_1^*-\mathfrak{u}_2^*}{2}),\frac{1}{2}\mathfrak{z})\\ &= \mathfrak{B}(\frac{1}{2}((\mathfrak{u}_1-\mathfrak{u}_2),(\mathfrak{u}_1^*-\mathfrak{u}_2^*)),\frac{1}{2}\mathfrak{z})\\ &= \mathfrak{B}(((\mathfrak{u}_1-\mathfrak{u}_2),(\mathfrak{u}_1^*-\mathfrak{u}_2^*)),\mathfrak{z})\\ &= \mathfrak{B}(((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*)),\mathfrak{z})\end{aligned}$$

and

$$\begin{aligned}\mathfrak{C}(\mathfrak{d}((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*))+\mathfrak{g}_2(\mathfrak{u}_1,\mathfrak{u}_1^*)-\mathfrak{g}_2(\mathfrak{u}_2,\mathfrak{u}_2^*),\mathfrak{x}\mathfrak{z}) &= \mathfrak{C}((\frac{\mathfrak{u}_1}{2}+\frac{1}{2},\frac{\mathfrak{u}_1^*}{2})-(\frac{\mathfrak{u}_2}{2}+\frac{1}{2},\frac{\mathfrak{u}_2^*}{2}),\frac{1}{2}\mathfrak{z})\\ &= \mathfrak{C}((\frac{\mathfrak{u}_1-\mathfrak{u}_2}{2},\frac{\mathfrak{u}_1^*-\mathfrak{u}_2^*}{2}),\frac{1}{2}\mathfrak{z})\\ &= \mathfrak{C}(\frac{1}{2}((\mathfrak{u}_1-\mathfrak{u}_2),(\mathfrak{u}_1^*-\mathfrak{u}_2^*)),\frac{1}{2}\mathfrak{z})\\ &= \mathfrak{C}(((\mathfrak{u}_1-\mathfrak{u}_2),(\mathfrak{u}_1^*-\mathfrak{u}_2^*)),\mathfrak{z})\\ &= \mathfrak{C}(((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*)),\mathfrak{z}).\end{aligned}$$

Similarly,

$$\mathfrak{A}(\mathfrak{d}((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*))+\mathfrak{g}_3(\mathfrak{u}_1,\mathfrak{u}_1^*)-\mathfrak{g}_3(\mathfrak{u}_2,\mathfrak{u}_2^*),\mathfrak{x}\mathfrak{z}) = \mathfrak{A}(((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*)),\mathfrak{z}),$$

$$\mathfrak{B}(\mathfrak{d}((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*))+\mathfrak{g}_3(\mathfrak{u}_1,\mathfrak{u}_1^*)-\mathfrak{g}_3(\mathfrak{u}_2,\mathfrak{u}_2^*),\mathfrak{x}\mathfrak{z}) = \mathfrak{B}(((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*)),\mathfrak{z}),$$

and

$$\mathfrak{C}(\mathfrak{d}((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*))+\mathfrak{g}_3(\mathfrak{u}_1,\mathfrak{u}_1^*)-\mathfrak{g}_3(\mathfrak{u}_2,\mathfrak{u}_2^*),\mathfrak{x}\mathfrak{z}) = \mathfrak{C}(((\mathfrak{u}_1,\mathfrak{u}_1^*)-(\mathfrak{u}_2,\mathfrak{u}_2^*)),\mathfrak{z}),$$

which implies that $\mathfrak{g}_1$*,* $\mathfrak{g}_2$*, and* $\mathfrak{g}_3$ *are generalized neutrosophic contraction mappings.*

Consider the GNIFS $\{I;\mathfrak{g}_1,\mathfrak{g}_2,\mathfrak{g}_3\}$ *with the mapping* $T:\Theta(I)\to\Theta(I)$ *given as*

$$T(\Delta)=\mathfrak{g}_1(\Delta)\cup\mathfrak{g}_2(\Delta)\cup\mathfrak{g}_3(\Delta)$$

for all $\Delta\in\Theta(I)$*. We have, by Theorem 5.4,*

$$H_{\mathfrak{A}}(\mathfrak{d}(\Delta-\Lambda)+T(\Delta)-T(\Lambda),\mathfrak{x}\mathfrak{z})\geq H_{\mathfrak{A}}(\Delta-\Lambda,\mathfrak{z}),$$

$$H_{\mathfrak{B}}(\mathfrak{d}(\Delta-\Lambda)+T(\Delta)-T(\Lambda),\mathfrak{x}\mathfrak{z})\leq H_{\mathfrak{B}}(\Delta-\Lambda,\mathfrak{z}),$$

and

$$H_{\mathfrak{C}}(\mathfrak{d}(\Delta-\Lambda)+T(\Delta)-T(\Lambda),\mathfrak{x}\mathfrak{z})\leq H_{\mathfrak{C}}(\Delta-\Lambda,\mathfrak{z}).$$

As a result, the requirements listed in Theorem 5.4 are satisfied. Additionally, for any starting set $\Delta_0\in\Theta(I)$*, the sequence of compact sets* $\{\Delta_0,T(\Delta_0),T^2(\Delta_0),...\}$ *is convergent and has a limit that is the attractor of T.*

Example 5.2. *Let $I = R^2$ and $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C})$ be the neutrosophic norm defined by:*

$$\mathfrak{A}((\mathfrak{u}, \mathfrak{u}^*), \mathfrak{z}) = \exp^{\frac{-(|\mathfrak{u}|+|\mathfrak{u}^*|)}{\mathfrak{z}}},$$
$$\mathfrak{B}((\mathfrak{u}, \mathfrak{u}^*), \mathfrak{z}) = 1 - 2\exp^{\frac{-(|\mathfrak{u}|+|\mathfrak{u}^*|)}{\mathfrak{z}}},$$

and

$$\mathfrak{C}((\mathfrak{u}, \mathfrak{u}^*), \mathfrak{z}) = 1 - \exp^{\frac{-(|\mathfrak{u}|+|\mathfrak{u}^*|)}{\mathfrak{z}}}.$$

Let $\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \mathfrak{g}_4, \mathfrak{g}_5, \mathfrak{g}_6 : I \to I$ be defined by:

$$\mathfrak{g}_1(\mathfrak{u}, \mathfrak{u}^*) = \left(\frac{9}{20}\mathfrak{u} + \frac{3}{5}r\cos\frac{\pi}{3}, \frac{9}{20}\mathfrak{u}^* + \frac{3}{5}r\sin\frac{\pi}{3}\right), \mathfrak{g}_2(\mathfrak{u},) = \left(\frac{9}{20}\mathfrak{u} + \frac{3}{5}r\cos\frac{2\pi}{3}, \frac{9}{20}\mathfrak{u}^* + \frac{3}{5}r\sin\frac{2\pi}{3}\right),$$

$$\mathfrak{g}_3(\mathfrak{u}, \mathfrak{u}^*) = \left(\frac{9}{20}\mathfrak{u} + \frac{3}{5}r\cos\pi, \frac{9}{20}\mathfrak{u}^* + \frac{3}{5}r\sin\pi\right)\mathfrak{g}_4(\mathfrak{u}, \mathfrak{u}^*) = \left(\frac{9}{20}\mathfrak{u} + \frac{3}{5}r\cos\frac{4\pi}{3}, \frac{9}{20}\mathfrak{u}^* + \frac{3}{5}r\sin\frac{4\pi}{3}\right),$$

$$\mathfrak{g}_5(\mathfrak{u}, \mathfrak{u}^*) = \left(\frac{9}{20}\mathfrak{u} + \frac{3}{5}r\cos\frac{5\pi}{3}, \frac{9}{20}\mathfrak{u}^* + \frac{3}{5}r\sin\frac{5\pi}{3}\right), \mathfrak{g}_6(\mathfrak{u}, \mathfrak{u}^*) = \left(\frac{9}{20}\mathfrak{u} + \frac{3}{5}r\cos 2\pi, \frac{9}{20}\mathfrak{u}^* + \frac{3}{5}r\sin 2\pi\right).$$

Let $\mathfrak{d} = 0$ and $\mathfrak{x} = \frac{9}{20}$, and then

$$\begin{aligned}
\mathfrak{A}(\mathfrak{d}((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)) + \mathfrak{g}_1(\mathfrak{u}_1, \mathfrak{u}_1^*) - \mathfrak{g}_1(\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{x}\mathfrak{z}) &= \mathfrak{A}\left(\left(\begin{array}{c} \left(\frac{9}{20}\mathfrak{u}_1 + \frac{3}{5}r\cos\frac{\pi}{3}, \frac{9}{20}\mathfrak{u}_1^* + \frac{3}{5}r\sin\frac{\pi}{3}\right) \\ -\left(\frac{9}{20}\mathfrak{u}_2 + \frac{3}{5}r\cos\frac{\pi}{3}, \frac{9}{20}\mathfrak{u}_2^* + \frac{3}{5}r\sin\frac{\pi}{3}\right) \end{array}\right), \frac{9}{20}\mathfrak{z}\right) \\
&= \mathfrak{A}\left(\left(\frac{9}{20}(\mathfrak{u}_1 - \mathfrak{u}_2), \frac{9}{20}(\mathfrak{u}_1^* - \mathfrak{u}_2^*)\right), \frac{9}{20}\mathfrak{z}\right) \\
&= \mathfrak{A}\left(((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \mathfrak{z}\right) \\
&= \mathfrak{A}\left((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{z}\right)
\end{aligned}$$

$$\begin{aligned}
\mathfrak{B}(\mathfrak{d}((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)) + \mathfrak{g}_1(\mathfrak{u}_1, \mathfrak{u}_1^*) - \mathfrak{g}_1(\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{x}\mathfrak{z}) &= \mathfrak{B}\left(\left(\begin{array}{c} \left(\frac{9}{20}\mathfrak{u}_1 + \frac{3}{5}r\cos\frac{\pi}{3}, \frac{9}{20}\mathfrak{u}_1^* + \frac{3}{5}r\sin\frac{\pi}{3}\right) \\ -\left(\frac{9}{20}\mathfrak{u}_2 + \frac{3}{5}r\cos\frac{\pi}{3}, \frac{9}{20}\mathfrak{u}_2^* + \frac{3}{5}r\sin\frac{\pi}{3}\right) \end{array}\right), \frac{9}{20}\mathfrak{z}\right) \\
&= \mathfrak{B}\left(\left(\frac{9}{20}(\mathfrak{u}_1 - \mathfrak{u}_2), \frac{9}{20}(\mathfrak{u}_1^* - \mathfrak{u}_2^*)\right), \frac{9}{20}\mathfrak{z}\right) \\
&= \mathfrak{B}\left(((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \mathfrak{z}\right) \\
&= \mathfrak{B}\left((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{z}\right)
\end{aligned}$$

and

$$\begin{aligned}
\mathfrak{C}(\mathfrak{d}((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*)) + \mathfrak{g}_1(\mathfrak{u}_1, \mathfrak{u}_1^*) - \mathfrak{g}_1(\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{x}\mathfrak{z}) &= \mathfrak{C}\left(\left(\begin{array}{c} \left(\frac{9}{20}\mathfrak{u}_1 + \frac{3}{5}r\cos\frac{\pi}{3}, \frac{9}{20}\mathfrak{u}_1^* + \frac{3}{5}r\sin\frac{\pi}{3}\right) \\ -\left(\frac{9}{20}\mathfrak{u}_2 + \frac{3}{5}r\cos\frac{\pi}{3}, \frac{9}{20}\mathfrak{u}_2^* + \frac{3}{5}r\sin\frac{\pi}{3}\right) \end{array}\right), \frac{9}{20}\mathfrak{z}\right) \\
&= \mathfrak{C}\left(\left(\frac{9}{20}(\mathfrak{u}_1 - \mathfrak{u}_2), \frac{9}{20}(\mathfrak{u}_1^* - \mathfrak{u}_2^*)\right), \frac{9}{20}\mathfrak{z}\right) \\
&= \mathfrak{C}\left(((\mathfrak{u}_1 - \mathfrak{u}_2), (\mathfrak{u}_1^* - \mathfrak{u}_2^*)), \mathfrak{z}\right)
\end{aligned}$$

$$= \mathfrak{C}\left((\mathfrak{u}_1, \mathfrak{u}_1^*) - (\mathfrak{u}_2, \mathfrak{u}_2^*), \mathfrak{z}\right).$$

Hence, $\mathfrak{g}_1$ *is a GNC. Similarly we can prove that* $\mathfrak{g}_2, \mathfrak{g}_3, \mathfrak{g}_4, \mathfrak{g}_5$*, and* $\mathfrak{g}_6$ *are GNCs.*

Consider the GNIFS $\{I; \mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \mathfrak{g}_4, \mathfrak{g}_5, \mathfrak{g}_6, \mathfrak{g}_7\}$ *with the mapping* $T : \Theta(I) \to \Theta(I)$ *given as*

$$T(\Delta) = \bigcup_{i=1}^{7} \mathfrak{g}_i(\Delta)$$

for all $\Delta \in \Theta(I)$*. We have, by Theorem 5.4,*

$$H_{\mathfrak{A}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) \geq H_{\mathfrak{A}}(\Delta - \Lambda, \mathfrak{z}),$$

$$H_{\mathfrak{B}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) \leq H_{\mathfrak{B}}(\Delta - \Lambda, \mathfrak{z}),$$

and

$$H_{\mathfrak{C}}(\mathfrak{d}(\Delta - \Lambda) + T(\Delta) - T(\Lambda), \mathfrak{x}\mathfrak{z}) \leq H_{\mathfrak{C}}(\Delta - \Lambda, \mathfrak{z}).$$

As a result, the requirements listed in Theorem 5.4 are satisfied. Additionally, for any starting set $\Delta_0 \in \Theta(I)$*, the sequence of compact sets* $\{\Delta_0, T(\Delta_0), T^2(\Delta_0), ...\}$ *is convergent and has a limit that is the attractor of T.*

The convergence of T to the GNIFS attractor in Example 5.2 is depicted in Figures 1–6.

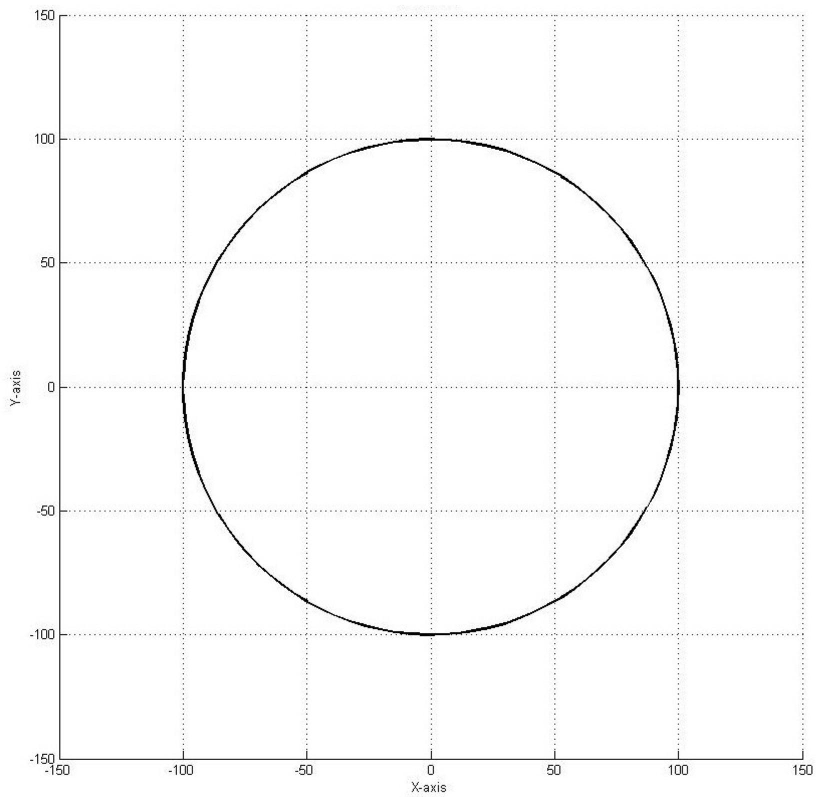

Figure 1. Δ_0.

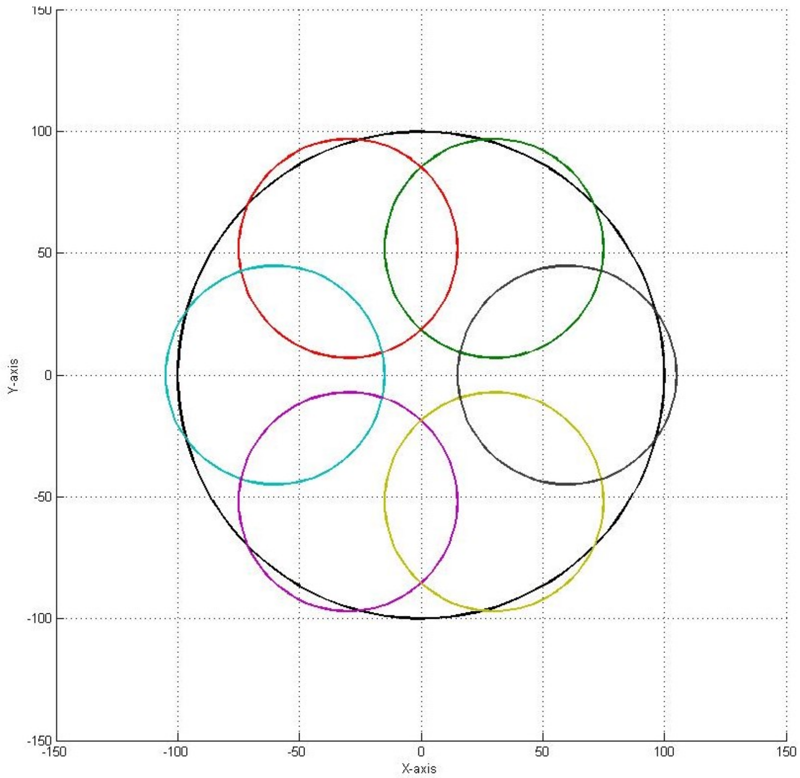

Figure 2. $\mathrm{T}(\Delta_0)$.

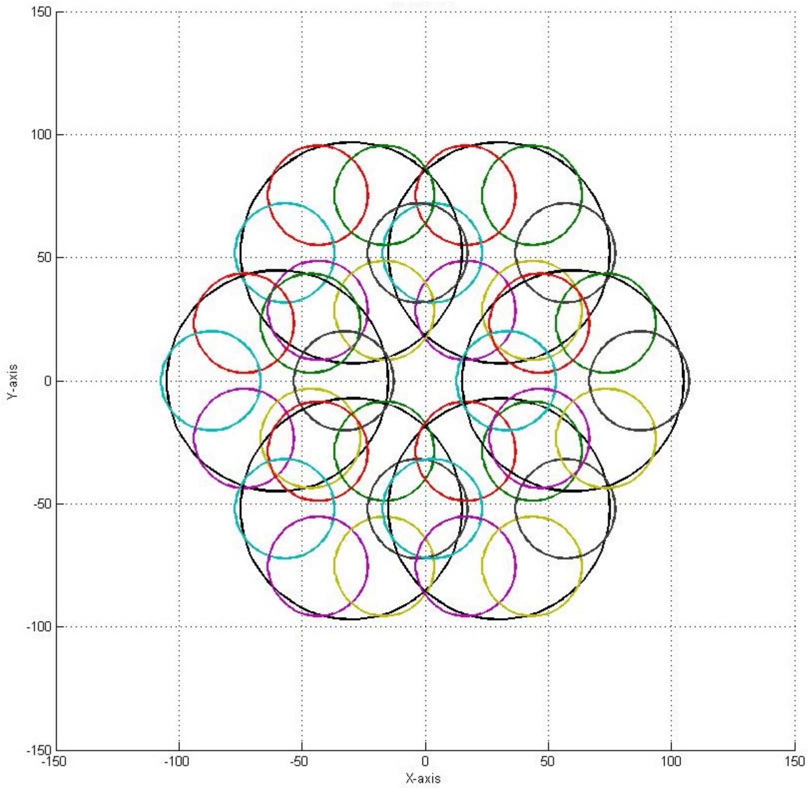

Figure 3. $\mathrm{T}^2(\Delta_0)$.

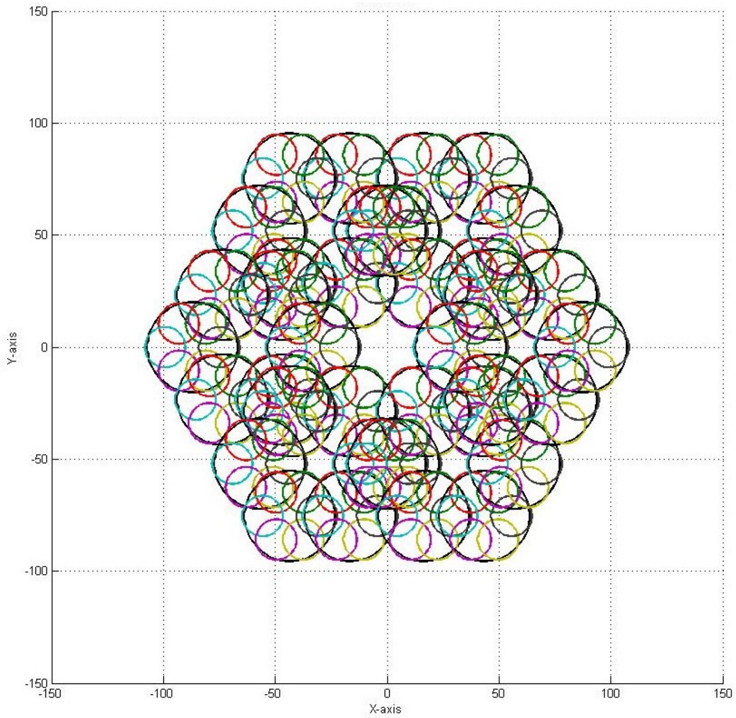

Figure 4. $\mathrm{T}^3(\Delta_0)$.

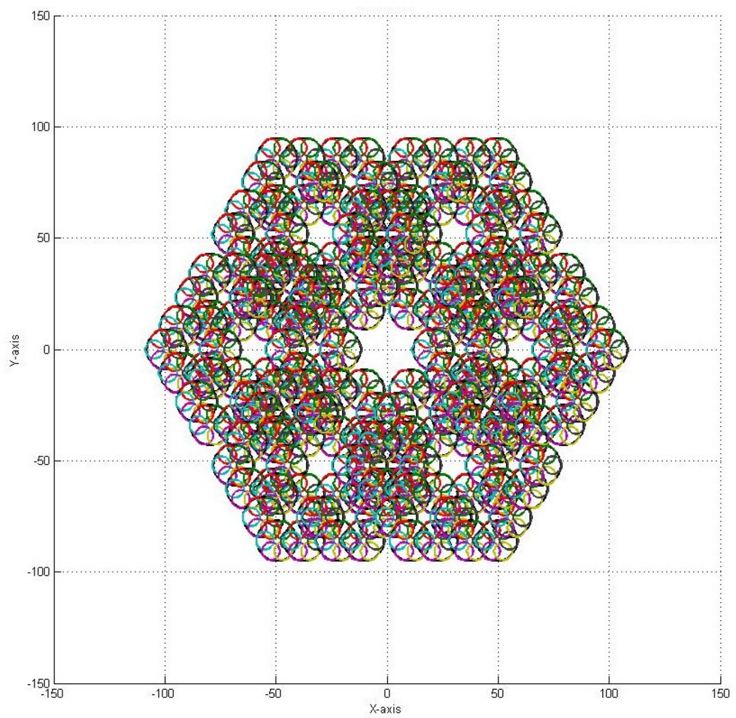

Figure 5. $\mathrm{T}^4(\Delta_0)$.

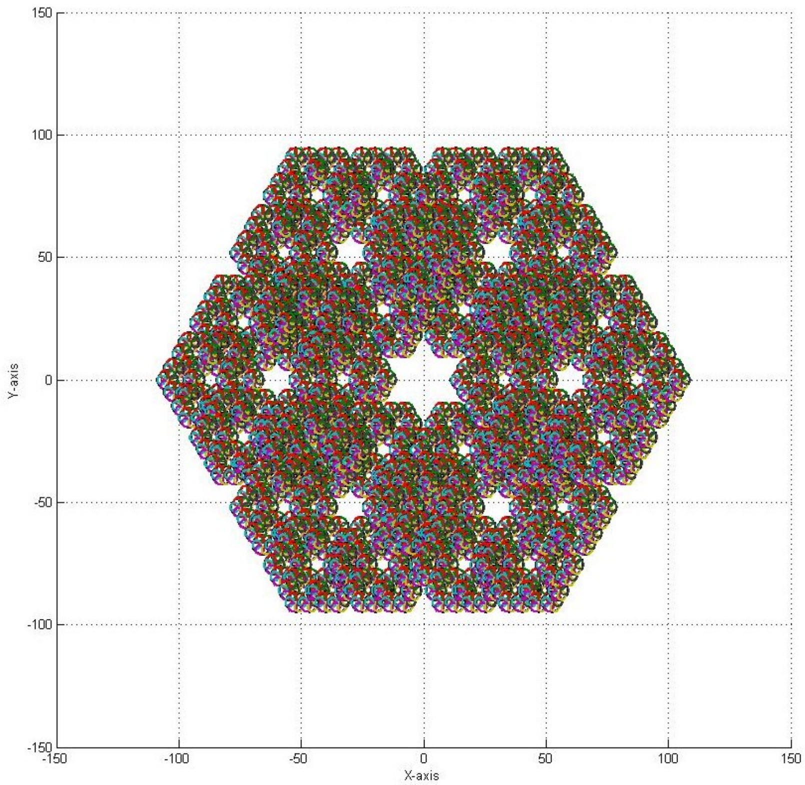

Figure 6. $T^5(\Delta_0)$.

6. Conclusions

We have presented a new class of mappings, called GNCs, which extends the notion of neutrosophic BCs and some neutrosophic non-expansive mappings. We have shown that the FP of any GNC may be efficiently found using the Krasnoselskij iterative approach. The classical neutrosophic BC principle emerges as a special case of our results. Moreover, we have shown that the class of neutrosophic BCs is contained in the class of GNCs. We have focused on mappings that are not neutrosophic contractions but are included in the category of GNCs. We have also identified a unique solution to the satellite web coupling problem. Moreover, we have demonstrated the adaptability of these GNCs in producing complex fractal structures by applying them to the construction of fractals utilizing Hutchinson–Barnsley operators. By using the FPT via a GNC, our result goes beyond conventional fractal creation techniques. We have set the stage for future studies in applied mathematics, stability analysis, and fixed point theory by utilizing neutrosophic contraction principles. Future investigations may explore the application of generalized neutrosophic contractions to other types of functional equations, such as Volterra or integro-differential equations. Further research could also focus on stochastic or random fixed point problems under the GNC framework. Additionally, the use of GNCs in higher-dimensional or fuzzy-neutrosophic metric spaces can lead to new insights in the modeling of uncertainty. Another interesting direction could be the design of algorithms for machine learning and optimization that are grounded in GNC-based fixed point principles. Finally, deeper exploration of the connection between GNCs and dynamical systems, especially in control theory or population models, may uncover rich mathematical structures and practical applications.

Author contributions

All authors equally conceptualized the study, contributed to its design and coordination, drafted the manuscript, participated in sequence alignment, and reviewed and approved the final manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no competing interests.

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