Research article

Near neutrosophic soft set

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Abstract: In this article, the notion of near neutrosophic soft sets (Nss) is obtained by combining the notion of Nss and the notion of near approximation space. Accordingly, a new set was obtained by restricting the set of features with the help of the indifferentiable relation defined on the set. The features and definitions that the set will provide are given, and, based on these features, the benefits that will be provided when they are implemented are investigated in the example.

Keywords: near soft set; neutrosophic soft set; approximation space

Mathematics Subject Classification: 54H25, 97E60, 54A05

1. Introduction

The concept of near sets is a concept given by Peters [1], and is related to the nearness of objects. According to this notion, new equivalence classes are obtained by a determined selection of the properties of objects. The nearness of the objects of the sets to each other is determined according to the defined relation. Another notion, the soft set, was stated by Molodtsov [2], and soft sets and soft topological spaces have been studied by many scientists [3–6]. Some concepts related to these notions were examined by Hussain and Ahmad in [7]. In [8], Wardowski has studied on soft mapping. The notion of near soft sets emerged after the soft set approach and near set theory were jointly discussed by Tasbozan et al. [9]. By introducing proximity-based sets with this concept, the topology has also been defined.

To eliminate ambiguities, the notion of neutrosophic sets (Ns) is used. Smarandache defined this, and different features were described for this concept [10, 11]. Smarandache considered a situation with truth, falsehood, and uncertainty, and called this the membership function. The notion of the Ns is a special notion given in fuzzy logic. Nss are expressed with triple membership functions of objects given by their features [11]. In 2013, Maji defined the notion of Nss [12]. Later, Deli and Broumi changed this notion [13]. Additionally, this notion was studied by many different authors [14–17]. In 2013, in Broumi’s study, a new concept called the generalized Nss was defined. Moreover, after
giving some features of this notion, the verdict-making question with the aid of generalized \( N_{ss} \) was discussed as an application [18]. In a study conducted in 2014, both \( N_{s} \) theory and rough set theory emerged as strong candidates for directing uncertain, true, and false knowledge in order to resolve uncertainty. In addition, a unified structure called rough neutrosophic sets was developed, and its features were examined [15]. In 2020, this notion was introduced into pre-open (formerly closed) sets, and preliminary separation axioms were defined along with its topology [14]. In a study conducted in 2019, the notion of a \( N_{s} \) point was defined after redefining some basic notions of \( N_{ss} \). Additionally, \( N_{s} \) separation axioms and the relationship between them was given in [12]. Then, Al-Quran et al. described \( N_{s} \) rough sets [19]. Das et al. [20] defined roughness on the neutrosophic soft set in a new way, different from the definitions of Al-Quran et al. [19]. Al-Quran et al. used fully soft sets to achieve a neutrosophic soft rough set [19]. This definition was determined to be a more effective approach for writing ambiguous and imprecise data because it does not use software content [20]. A series of neutrosophic soft sets were identified. Additionally, in this series, an algorithm for the decision-making problem is proposed and applied to a real-life experiment [21]. Neutrosophic vague \( N \)-soft sets consisting of neutrosophic vague sets and \( N \)-soft sets were identified. Additionally, a new method based on neutrosophic vague \( N \)-soft sets was given. The method was applied to decision-making problems. [22]. Definitions of \( N_{s} \) topological spaces and some properties were given with examples [11]. Recently, Deli [23] studied this notion at interval values. This notion has also been studied by many authors [24, 25].

In this study, obtaining near \( N_{ss} \) is considered when lower and upper approximations are obtained with equivalence classes which are taken from objects for a subset of \( N_{ss} \) features. Additionally, our aim in the study is to ensure that the concept of \( N_{ss} \) in near approximation spaces can be used in practical solutions related to \( N_{ss} \). With the help of this set, set properties near to \( N_{ss} \) were transferred and defined with lower and upper approaches. Moreover, the advantage provided by the set is explained with an example that can be effective in practice. The concepts defined in this study allow finding close objects with restrictive features encountered in daily life.

### 2. Preliminaries

**Definition 1.** Let \( O \) be an object set, \( P(O) \) be the power set of \( O \), \( E \) be a set of all parameters, and \( B \subseteq E \). Then, a soft set \( V_{B} \) over \( O \) is a set defined by an approximate function \( v_{B} : E \rightarrow P(O) \) such that \( v_{E}(\phi) = \emptyset \) if \( \phi \notin B \). The value \( v_{B}(\phi) \) is a set called the \( \phi \)-element of the soft set for all \( \phi \in E \). Hence, a soft set can be denoted by the set of ordered pairs [13]

\[
V_{B} = \{(\phi, v_{B}(\phi)) : \phi \in B, v_{B}(\phi) \in P(O)\}.
\]

**Definition 2.** Let \( (\mathcal{U}, \mathcal{E}, \sim_{Br}, N_{r}, v_{N_{r}}) \) be a nearness approximation space (NAS), \( \sigma = V_{B} \) a soft set(\( SS \)) over \( \mathcal{U} \), \( E \) a set of all features, and \( B \subseteq E \).

\[
N_{s}^{*}(V_{B}) = (N_{s}^{*}(V_{B}) = \cup\{u \in \mathcal{U} : [u]_{Br} \subseteq v_{B}(\phi), \phi \in B\}
\]

and

\[
N_{r}^{*}(V_{B}) = (N_{r}^{*}(V_{B}) = \cup\{u \in \mathcal{U} : [u]_{Br} \cap v_{B}(\phi) \neq \emptyset, \phi \in B\}
\]

are lower and upper near approximation operators. The soft set \( N_{s}((V_{B})) \) with

\[
Bnd_{N_{s}(B)}((V_{B})) = (N_{r}^{*}(V_{B}) - N_{s}^{*}(V_{B})) \geq 0
\]
is called a near soft set (NSS) [17].

**Definition 3.** Let $O$ be an object set, $E$ be the object set of features, and $A, B \subseteq E$

1. $V_A$ is called a relative NSS if $v_A(\emptyset) = \emptyset, \forall \phi \in A$,
2. $V_B$ is called a relative whole NSS if $v_B(\phi) = O, \forall \phi \in B$ [17].

**Definition 4.** The NSS $(V_A)^c = (V_A^c)$ is a complement of $(V_A)$ if $V_A^c(e) = O - V(e), \forall e \in A$ [17].

**Definition 5.** Let $(F, B)$ be an NSS over $O$, and $\tau$ be the collection of near soft subsets of $O$. If the following are provided:

i) $(\emptyset, B), (O, B) \in \tau$,

ii) $(F_1, B), (F_2, B) \in \tau$ then $(F_1, B) \cap (F_2, B) \in \tau$,

iii) $(F_i, B), \forall \phi \in B$ then $\cup_i(F_i, B) \in \tau$,

then $(O, \tau, B)$ is a near soft topological space (NSTS) [17].

**Definition 6.** Let $O$ be an object set, with $u \in O$. A neutrosophic set $K$ in $O$ is described by the truth-membership, indeterminacy, and falsity functions $D_K, I_K,$ and $Y_K,$ respectively, defined as

$$K = \{u, (D_K(u), I_K(u), Y_K(u)) : u \in O\}.$$ $D_K(u), I_K(u),$ and $Y_K(u)$ are real standard or nonstandard subsets of $[0, 1]$ [23].

**Definition 7.** Let $O$ be an object set, with $u \in O$, $E$ a set of features that are described as the elements of $O$, $B \subseteq E$, $V$ a neutrosophic set in $O$, and $\hat{v} : E \to P(O)$. A neutrosophic soft set $\hat{V}_E$ in $O$ is described by the truth-membership, indeterminacy, and falsity are $D_{\hat{v}(\phi)}(u), I_{\hat{v}(\phi)}(u),$ and $Y_{\hat{v}(\phi)}(u),$ respectively. Thus, the specialized family of some elements of the NSS $P(O)$ are defined as

$$\hat{V}_B = \{(\phi, < u, (D_{\hat{v}(\phi)}(u), I_{\hat{v}(\phi)}(u), Y_{\hat{v}(\phi)}(u)) : u \in O, \phi \in B\}.$$ $D_{\hat{v}(\phi)}(u), I_{\hat{v}(\phi)}(u),$ and $Y_{\hat{v}(\phi)}(u)$ are real standard or nonstandard subsets of $[0, 1]$ [16].

### 3. Near neutrosophic soft set

**Definition 8.** Let

$$\hat{V}_E = \{(\phi, < u, (D_{\hat{v}(\phi)}(u), I_{\hat{v}(\phi)}(u), Y_{\hat{v}(\phi)}(u)) : u \in O, \phi \in E\}$$

be an NSS on a universe $O$ with $\sim_B$ being the equivalence relation. Let $(O, E, \sim_B, N_r, \nu_N)$ be a nearness approximation space (NAS), $\hat{V}_B = (V, B)$ an NSS over $O, E$ a set of all features, and $B \subseteq E$. The neutrosophic soft lower and upper approximations of any subset $V$ based on $\sim_B$ respectively,

$$N_r(\hat{V}_B) = (N_r(\hat{V}_B) = \cup\{u \in O : [u]_{Br} \subseteq v_B(\phi), \phi \in B\}$$
and

\[ N_{r}^{\ast}(\hat{V}_{B}) = (N_{r}^{\ast}(V_{B}) = \cup\{u \in O : [u]_{Br} \cap \nu_{B}(\phi) \neq \emptyset, \phi \in B\}. \]

\( N_{r}^{\ast} \) and \( N_{r}^{\ast} \) can be defined as near neutrosophic soft approximations of \( V \) with regard to \( B \). The neutrosophic soft set \( N_{r}((\hat{V}_{B})) \) with \( \text{Bnd}_{N_{r}(B)}(V_{B}) = (N_{r}(V_{B}) - N_{r}(V_{B})) \geq 0 \) is called a near neutrosophic soft set (NNSS), denoted by \( \hat{V}_{B} \).

**Definition 9.** Let \( \hat{V}_{B} \) be an NSS on an object \( O \) and \( V \subseteq O \). Then, the accuracy measure of \( V \) is defined as

\[
C_{NSS}^{\hat{V}_{B}} = \frac{|N_{r}^{\ast}(\hat{V}_{B})|}{|N_{r}^{\ast}(\hat{V}_{B})|}
\]

where \( V \neq \emptyset \) and \( |\cdot| \) denotes the cardinality of sets.

**Definition 10.** Let \( \hat{V}_{B} \) be an NNSS on an object \( O \) and \( V \subseteq O \). Then, the affiliation function of an element \( v \) to a set \( V \) is described as

\[
N_{V}(v) = \frac{|v_{B} \cap V|}{|v_{B}|}
\]

where \( \hat{v} : E \rightarrow P(U) \).

**Proposition 11.** Let \( \hat{V}_{B} \) be an NNSS on an object set \( O \) and \( M, Z \subseteq O \). Thus, the following properties apply:

1. \( N_{r}^{\ast}(M_{B}) \subseteq M \subseteq N_{r}^{\ast}(\hat{M}_{B}) \).
2. \( N_{r}^{\ast}(\emptyset) = \emptyset = N_{r}^{\ast}(\emptyset) \).
3. \( N_{r}^{\ast}(O) = O = N_{r}^{\ast}(O) \).
4. \( M \subseteq D \Rightarrow N_{r}^{\ast}(\hat{M}_{B}) \subseteq N_{r}^{\ast}(\hat{D}_{B}) \).
5. \( M \subseteq D \Rightarrow N_{r}^{\ast}(\hat{M}_{B}) \subseteq N_{r}^{\ast}(\hat{D}_{B}) \).
6. \( N_{r}^{\ast}(M \cap D)_{B} \subseteq N_{r}^{\ast}(\hat{M}) \cap N_{r}^{\ast}(\hat{D}) \).
7. \( N_{r}^{\ast}(M \cup D)_{B} \subseteq N_{r}^{\ast}(\hat{M}) \cap N_{r}^{\ast}(\hat{D}) \).
8. \( N_{r}^{\ast}(M \cap D)_{B} \subseteq N_{r}^{\ast}(\hat{M}) \cap N_{r}^{\ast}(\hat{D}) \).
9. \( N_{r}^{\ast}(M \cup D)_{B} = N_{r}^{\ast}(\hat{M}) \cup N_{r}^{\ast}(\hat{D}) \).

**Proof.** 1. As in the definition of NNSS, we can induce that \( N_{r}^{\ast}(\hat{M}_{B}) \subseteq M \). Moreover, let \( u \in M \) and \( R \) be an equivalence relation. Hence, \( u \in N_{r}^{\ast}(\hat{M}_{B}) \). Thus, \( N_{r}^{\ast}(\hat{M}_{B}) \subseteq M \subseteq N_{r}^{\ast}(\hat{M}_{B}) \).
2. This is obvious from the definition.
3. As in property (i), we get \( O \subseteq N_{r}^{\ast}(\hat{O}_{B}) \). Since \( O \) is the object set \( O = N_{r}^{\ast}(\hat{O}_{B}) \), from equivalence relation, we get \( N_{r}^{\ast}(\hat{O}_{B}) = O \). Hence, \( N_{r}^{\ast}(O) = O = N_{r}^{\ast}(O) \).
4. Let \( M \subseteq D \) and \( u \in N_{r}^{\ast}(\hat{M}_{B}) \). There exists \([u]_{Br}\) such that \( u \in [u]_{Br} \). Thus, \( u \in [u]_{Br} \subseteq D \). Hence, \( u \in N_{r}^{\ast}(\hat{D}_{B}) \). As a result, \( N_{r}^{\ast}(\hat{M}_{B}) \subseteq N_{r}^{\ast}(\hat{D}_{B}) \).
5. Let $M \subseteq D$ and $u \in N^*_r(M_B^\wedge)$. There exists $[u]_{B^r}$ such that $[u]_{B^r} \cap v_B(\phi) \neq \emptyset, \phi \in B$. Thus, $u \in N^*_r(M_B^\wedge)$. As a result, $N^*_r(M_B^\wedge) \subseteq N^*_r(D_B^\wedge)$.

6. Let $u \in N_r^*(M \cap D_B^\wedge)$. There exists $[u]_{B^r}$ such that $[u]_{B^r} \subseteq M \cap D$. Then $u \in [u]_{B^r} \subseteq M$ and $u \in [u]_{B^r} \subseteq D$. As a result, $u \in N_r^*(M_B^\wedge)$ and $u \in N_r^*(D_B^\wedge)$, and we get $u \in N_r^*(M_B^\wedge) \cap N_r^*(D_B^\wedge)$. Thus, $N_r^*(M \cap D_B^\wedge) \subseteq N_r^*(M_B^\wedge) \cap N_r^*(D_B^\wedge)$.

7. Let $u \notin N_r^*(M \cup D_B^\wedge)$. There exists $[u]_{B^r}$ such that $[u]_{B^r} \subseteq M \cup D$. Then $\forall \phi \in B, u \notin [u]_{B^r} \subseteq M$ and $u \notin [u]_{B^r} \subseteq D$. As a result, $u \notin N_r^*(M_B^\wedge)$ and $u \notin N_r^*(D_B^\wedge)$, and we get $u \notin N_r^*(M_B^\wedge) \cup N_r^*(D_B^\wedge)$. Thus, $N_r^*(M \cup D_B^\wedge) \supseteq N_r^*(M_B^\wedge) \cup N_r^*(D_B^\wedge)$.

8. Let $u \in N^*_r(M \cap D_B^\wedge)$. There exists $[u]_{B^r}$ such that $[u]_{B^r} \cap v_B(\phi) \neq \emptyset, \phi \in B$. For this, $[u]_{B^r} \cap (M \cap D) \neq \emptyset, [u]_{B^r} \cap (M) \neq \emptyset$, and $[u]_{B^r} \cap (D) \neq \emptyset$. As a result, $u \in N^*_r(M_B^\wedge)$ and $u \in N^*_r(D_B^\wedge)$, and we get $u \in N^*_r(M_B^\wedge) \cap N^*_r(D_B^\wedge)$. Thus, $N^*_r(M \cap D_B^\wedge) \subseteq N^*_r(M_B^\wedge) \cap N^*_r(D_B^\wedge)$.

9. It is done similarly to item 7.

Example 12. Let $O = \{u_1, u_2, u_3, u_4, u_5\}$ be a set of patients, and $E = \{\phi_1, \phi_2, \phi_3\}$ be the set of each patient’s characteristics, such as fever, anemia, and weight, respectively. Then, an equivalence relation $R$ on $O$ is obtained from the Table 1, according to the properties of $O$ and $E$.

Table 1. $R$ equivalence relation on $O$.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
<th>$u_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Therefore, the equivalence classes of $R$ for $B = \{\phi_1, \phi_2\} \subseteq E$ are

$[u_1]_B = \{u_1, u_4\}$,  
$[u_2]_B = \{u_2, u_3\}$,  
$[u_3]_B = \{u_3\}$.

For $V = \{u_1, u_2, u_4\}$,

$V_B^\wedge = \{(\phi, < u, (D_{\phi_1}(u), L_{\phi_1}(u), Y_{\phi_1}(u)) > : u \in O, \phi \in B)\}$

$= \{(\phi_1, < u_1, (0.7, 0.5, 0.3) >, < u_2, (0.8, 0.4, 0.2) >, < u_3, (0.8, 0.4, 0.2) >, < u_4, (0.7, 0.5, 0.4) >, < u_5, (0.9, 0.6, 0.1) >),$  
$< \phi_2, < u_1, (0.8, 0.4, 0.3) >, < u_2, (0.6, 0.5, 0.4) >, < u_3, (0.6, 0.5, 0.4) >, < u_4, (0.8, 0.4, 0.3) >, < u_5, (0.9, 0.6, 0.1) >)\}$

is an NSS. The tabular representation of the NSS $(V_B^\wedge)$ is described in Table 2.
Then, the approximations are defined as

\[
N_{r}(\hat{\nu}_{B}) = (N_{r}(\nu_{B})) = \{u_1, u_4 : [u_1]_B \in V, \phi_1, \phi_2 \in B\}
\]

and

\[
N_{r}^{<}(\hat{\nu}_{B}) = \bigcup\{u \in O : [u]_{Br} \cap \nu_{B}(e) \neq 0, e \in B\}
\]

The neutrosophic soft set \( N_{r}(\hat{\nu}_{B}) = (N_{r}(\nu_{B}), N_{r}(\nu_{B})) \) with \( Bnd_{N_{r}(\nu_{B})}(\nu_{B}) = (N_{r}(\nu_{B}) - N_{r}(\nu_{B})) \geq 0 \) is called a near neutrosophic soft set.

Then, accuracy measure of \( V = \{u_1, u_2, u_4\} \) is described by

\[
C_{NSS}^{\hat{\nu}_{B}} = \frac{\left|N_{r}(\hat{\nu}_{B})\right|}{\left|N_{r}^{<}(\hat{\nu}_{B})\right|} = \frac{\left|[u_1, u_4]\right|}{\left|[u_1, u_2, u_3, u_4]\right|} = \frac{1}{2}.
\]

Therefore, we get the accuracy measure for all alternatives from Table 3.

Table 3. The accuracy measure for all alternatives.

<table>
<thead>
<tr>
<th></th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( u_4 )</th>
<th>( u_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_1 )</td>
<td>(0.7, 0.5, 0.3)</td>
<td>(0.8, 0.4, 0.2)</td>
<td>(0.8, 0.4, 0.2)</td>
<td>(0.7, 0.5, 0.4)</td>
<td>(0.9, 0.6, 0.1)</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>(0.8, 0.4, 0.3)</td>
<td>(0.6, 0.5, 0.4)</td>
<td>(0.6, 0.5, 0.4)</td>
<td>(0.8, 0.4, 0.3)</td>
<td>(0.9, 0.6, 0.1)</td>
</tr>
<tr>
<td>( \phi_3 )</td>
<td>(0.9, 0.5, 0.2)</td>
<td>(0.6, 0.5, 0.4)</td>
<td>(0.8, 0.4, 0.2)</td>
<td>(0.8, 0.4, 0.2)</td>
<td>(0.6, 0.5, 0.4)</td>
</tr>
</tbody>
</table>

It is obvious that \( u_1 \) and \( u_4 \) are the two options with the maximum accuracy. Then, we get \( N_{r}^{\hat{\nu}_{B}} \) of the considered maximum options as in Table 3. Table 4 shows the tabular representation of \( \hat{\nu}_{B} \).

Table 4. The tabular representation of the NNS S.

<table>
<thead>
<tr>
<th></th>
<th>( u_1 )</th>
<th>( u_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_1 )</td>
<td>(0.7, 0.5, 0.3)</td>
<td>(0.7, 0.5, 0.4)</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>(0.8, 0.4, 0.3)</td>
<td>(0.8, 0.4, 0.3)</td>
</tr>
</tbody>
</table>
\[ S_{ij} = D_{\phi_j}(u_i) + I_{\phi_j}(u_i) - Y_{\phi_j}(u_i) \]

is calculated as the highest alternatives, as shown in Table 5.

**Table 5.** The highest alternatives.

<table>
<thead>
<tr>
<th></th>
<th>( u_1 )</th>
<th>( u_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_1 )</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

\[ C(u_i) = \sum_{j=1}^{m} S_{ij} \]

is called the score of the considered alternatives, as shown in Table 6.

**Table 6.** Score of the alternatives.

<table>
<thead>
<tr>
<th></th>
<th>( u_1 )</th>
<th>( u_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C(u_i) )</td>
<td>1.8</td>
<td>1.7</td>
</tr>
</tbody>
</table>

From Table 6, it is obvious that the \( u_1 \) has the maximum count of 1.8. Thus, the verdict is to select \( u_1 \) as the suitable answer.

### 4. Conclusions

A new approach to \( Nss \) called near \( Nss \) was proposed. While choosing the appropriate object by restricting the set of features as desired, near \( Nss \) allowed a closer section. In addition, near neutrosophic soft approximations were defined and their properties were verified. The contribution of the model to the selection was applied in order to select the objects.

### References