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*Editorial*

# Special issue "Discrete optimization: Theory, algorithms and new applications"

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Discrete optimization is an important area of applied mathematics and computer science, where functions are optimized over a discrete set of feasible solutions. In the Call for Papers for this issue, I asked for submissions presenting the latest research for solving discrete optimization problems and also related ones. Both theoretical results and practical issues could be addressed. In particular, the aim was to present new theoretical results, structural investigations, or new models and algorithmic approaches. Among the possible subjects, nonlinear and linear integer programming, combinatorial optimization, problems on graphs and networks, robust optimization, scheduling, logistics, and optimization in production, logistics and transport were mentioned, to name a few.

After a careful refereeing with at least two referees per submission, 21 papers have been accepted for this special issue. They cover a broad spectrum, not only classical discrete optimization problems but also adjoining subjects. Subsequently, the published works are briefly discussed in the sequence of their publication dates.

In the first accepted paper, Ali, Noureen, Bhatti and Albalahi deal with optimal molecular trees with respect to Sombor indices. In particular, the best possible upper bounds on these indices for such molecular trees in terms of the order and the number of branching vertices or vertices of degree 2 are determined, and the molecular trees achieving these bounds are completely characterized.

The second published paper by Lv, Liu and Zhong presents a new approach to solve the optimization problem of an uncertain network system based on uncertainty theory. The authors redefine the concept of an uncertain network system and suggest a unified identification method. In addition, they present a network optimization model for the shortest path problem based on a conditional uncertain measure matrix. The developed hybrid algorithm is tested on an example of an underground logistics system.

In the next paper, Tangkhawiwetkul suggests a neural network to solve the generalized inverse

mixed variational inequality problem in Hilbert spaces. The stability and existence of a solution of the neural network under consideration is proven. Finally, some algorithms are presented and illustrated by a numerical example.

Then, Vakhania deals with preemptive scheduling on unrelated machines with the objective to minimize the maximum job completion time by applying linear programming. In particular, he presents a polynomial-time algorithm for simultaneously released jobs. Another linear program is derived for non-simultaneously released jobs, and it is shown that an optimal schedule can be found in $O(m)$ time from the optimal solution of the linear program, where $m$ denotes the number of unrelated parallel machines. However, if job splitting is not allowed, even a restricted version of the problem on three machines turns out to be *NP*-hard.

The next paper by Arunchai, Seangwattana, Sitthithakerngkiet and Sombut deals with image restoration by applying a modified proximal point algorithm. In particular, the proposed algorithm for solving the common problem between convex constrained minimization and modified variational inclusion problems has also been demonstrated to generate a robust convergence theorem.

The paper by Wang, Wang, Hou and Yang presents a motif-based spectral clustering method for directed weighted networks. This work also supplements the approach of obtaining matrix expressions of the motif adjacency matrix in directed unweighted networks. It is shown that the applied clustering method can correctly identify the partition structure of the benchmark network,

The next paper by Ofem, Isik, Ugwannadi, George and Narain deals with the approximation of the solution of a nonlinear delay integral equation by applying an efficient iterative algorithm in hyperbolic spaces. Several convergence results are obtained, and the efficiency of the algorithm is demonstrated on numerical examples. In addition, a nonlinear integral equation with two delays is solved using the main results of this paper.

Akhter, Ali and Bilal discuss special type convex functions on Riemann manifolds. Some fundamental properties of special type functions are presented, and the application to optimization problems is shown for these special type convex functions.

Xing, Ozalan and Liu deal with the degree sequence on tensor and Cartesian products of graphs and the omega index. From the results, they conclude that the set of graphs forms an Abelian semigroup in the case of a tensor product, and this set is an Abelian monoid in the case of a Cartesian product.

Pazhaniraja, Basheer, Thirugnanasambandam, Ramilingam, Rashid and Kalaivani present a multi-objective Boolean grey wolf optimization-based decomposition algorithm applied to high-frequency and high-utility itemset mining. The developed algorithm is tested on 12 different real-time datasets, and the results are compared with seven different existing multi-objective models, where also a statistical analysis was performed.

In the next paper, Babadag gives a new approach to Jacobsthal, Jacobsthal-Lucas numbers as well as dual vectors. In particular, some formulas and properties for these numbers are presented. Some fundamental identities such as the generating function, the Binet formulas, the Cassini's, Catalan's and d'Ocagne identities for the numbers under consideration have been obtained.

Liu and Yuan deal with the prediction of the air quality index of Hefei/China based on an improved model. They also investigate the changes in the monthly air quality index of Hefei from 2014 to 2020 and in particular the influence of the annual Spring Festival. Finally, the authors also present some recommendations for the air quality management policy in Hefei based on the obtained results.

Then, Hezam presents a bi-level humanitarian plan design model with considering both equity and efficiency on the example of Yemen. The upper-level model aims to minimize the unmet demand,

and the lower-level model wants to maximize the funds sent to affected areas fulfilling the required needs. The model has been applied to real data collected from Yemen in 2021 and turned out to ensure the continuous flow of aid from donors to beneficiaries.

The paper by Okeke, Adamu, Promkam and Sunthrayuth describes a two-step inertial method for solving the split common null point problem with multiple output sets in Hilbert spaces. A weak convergence analysis has been made, and numerical tests confirm the theoretical analysis of the given algorithm.

The next paper by Fiqhi Roslan, Zamri, Mansor and Mohd Kasihmuddin deals with a major 3 satisfiability logic in a discrete Hopfield neural network integrated with a multi-objective election algorithm. The developed model is compared for different order logical combinations. The performance of the logical combinations under consideration is evaluated by the mean absolute error, global minimum energy, total neuron variation, the Jaccard similarity index and the Gower and Legendre similarity index.

Buvaneshwari and Anuradha present an approach to solve the bi-objective bi-item solid transportation problem with fuzzy stochastic constraints. The objectives are the minimization of the transportation cost and the duration of transit as well as the profit maximization. By means of the chance-constrained technique, the uncertainty problem is transformed into a deterministic one, which is then solved by the global weighted sum method. A numerical example as well as a sensitivity analysis are presented.

In the next paper, Alotaibi and Vuyyuru present a deep learning model for multi-modal emotion identification based on the fusion of electroencephalogram signals and facial expressions. Convolutional neural networks are used to acquire spatial features from the original electroencephalogram signals. The results of the experiments confirm that the suggested method can recognize emotions accurately.

Then, Wang, Huang, Jiang, Xu and Wang present a hybrid relaxed iterative algorithm with two half-spaces to solve the fixed-point problem and the split-feasibility problem. Under certain weak conditions, the strong convergence of the generated iterative sequence has been proven. Some numerical experiments are also presented which demonstrate the efficiency of the suggested iterative method.

Garg, Deep, Alnowibet, Wagdy Mohamed, Shokouhifar and Werner present a Laplacian biogeography-based sine cosine algorithm for structural engineering design optimization problems. This algorithm is tested on a set of 23 benchmark functions as well as five real-world structural engineering design problems. Statistical tests have been applied to validate the superiority of the new approach over various state-of-the-art algorithms.

Liang, Lu, Sun, Yu and Zuo deal with the online single machine scheduling problem to minimize the weighted makespan when only one restart is allowed. In the general case of arbitrary processing times, it is proven that no online algorithm with a competitive ratio of less than 2 can exist. For the special case of equal processing times, they present a best possible online algorithm with a competitive ratio of 1.4656.

In the last accepted paper, Moradi, Khalaj, Herat, Darigh and Yamcholo suggest a two-level swarm intelligence-based ensemble learning model that uses the whale optimization algorithm for feature selection and hyper-parameter optimization. They also present a method to combine $K$-member clustering with whale optimization. Detailed computational results show the superiority of this new approach.

Finally, as the guest editor, it is my pleasure to thank the Editorial staff of the journal AIMS Mathematics for their support and all reviewers for their insightful and timely reports. I hope the readers of this issue will find stimulating ideas for their future research.

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**Conflict of interest**

The author declares no conflict of interest.