Mathematics

AIMS Mathematics, 9(3): 5330–5348.
DOI: 10.3934/math.2024257
Received: 05 December 2023
Revised: 11 January 2024
Accepted: 16 January 2024
Published: 26 January 2024

http://www.aimspress.com/journal/Math

Research article

On the boundedness of solutions of some fuzzy dynamical control systems

Omar Kahouli[1,*], Amina Turki[2], Mohamed Ksantini[2], Mohamed Ali Hammami[3] and Ali Aloui[1]

[1] Department of Electronics Engineering, Applied College, University of Hail, Hail 2440, Saudi Arabia
[2] Control and Energies Management Laboratory (CEM-Lab), National Engineering School of Sfax, University of Sfax, Tunisia
[3] Tunisia Faculty of Sciences of Sfax, Department of Mathematics, University of Sfax, Tunisia

* **Correspondence:** Email: a.kahouli@uoh.edu.sa.

Abstract: The asymptotic behavior of solutions of fuzzy control systems is a component of the study of fuzzy control theory. The study of stability for T-S (Takagi-Sugeno) fuzzy systems, which process qualitative data through linguistic expressions, is the subject of this paper. Asymptotic stability is conservative in many real-world applications due to measurement noise and other disruptions. The ultimate limit, which indicates that the mistakes stay in a specific area close to the origin after a long enough amount of time, is a crucial characteristic that is frequently defined for such systems. We are interested with the problem of the state feedback controller for T-S fuzzy models with uncertainties where the global exponential ultimate boundedness of solutions is studied for certain fuzzy control systems. We use common quadratic Lyapunov function and parallel distributed compensation controller techniques to study the asymptotic behavior of the solutions of fuzzy control system in presence of perturbations. An example demonstrating the validity of the main result is discussed.

Keywords: fuzzy systems; uncertainties; boundedness of solutions; convergence and stability
Mathematics Subject Classification: 93C10, 93C42, 93D15

1. Introduction

Fuzzy systems have become more and more essential in control engineering and industry since Zadeh [40] originally presented them in 1965. In the last twenty years, extensive research has been conducted on nonlinear systems using Takagi-Sugeno (T-S) fuzzy systems [4–6,8,16,18,21–24,26–29,31].

Fuzzy controllers or models utilize fuzzy rules, which consist of linguistic if-then statements integrating fuzzy sets, fuzzy logic, and fuzzy inference. These rules are crucial for establishing a connection between the input and output variables in fuzzy controllers and models. They serve the purpose of encapsulating knowledge and expertise derived from expert control and modeling [13–15,17,19,20,31,32]. The stability and performance criteria of T-S-fuzzy dynamical systems have been the subject of numerous techniques in the literature [29–32]. Essentially, the configuration that is most often taken into consideration is known as parallel-distributed compensation (PDC), wherein the plant that needs to be regulated shares membership functions with the controller. The principle of approaches has been to create less conservative LMI conditions, like those found in [33,34]. T-S fuzzy models [35,36] are nonlinear systems that offer local linear representations of an underlying system by employing an if-then rule set. Numerous nonlinear systems may be accurately simulated by this family of models. As such, it is imperative to study their stability or design stabilizing controllers. Nonlinear system stability analysis has received a lot of attention [1,2,7,9–12]. The authors in [41] have constructed a unified control rule in order to improve the accuracy of both the disturbance estimates and stabilization of nonlinear T-S fuzzy semi-Markovian jump systems, which are modelled by using an equivalent-input-disturbance technique based on proportional-integral observers.

In the presence of perturbations such as unmodeled dynamics, measurement noises, and disturbances [3,12,37], asymptotic stability tends to be conservative. For such systems, ultimate boundedness implies the solution's capacity to stay within a specific vicinity around the origin for a prolonged duration, which is commonly the most achievable property of the solutions. Certain constraints are typically imposed on the perturbation terms to establish this behavior [12]. It may be challenging to find a Lyapunov function for nonlinear systems, but it is simple for linear systems. Actually, by linearizing the equations and calculating a local linear Lyapunov function that should be valid in the region of a fixed point, this technique is frequently used to propose candidate Lyapunov functions for nonlinear systems. This paper presents sufficient conditions to ensure the ultimate exponential convergence of solutions for a certain category of uncertain fuzzy control systems. Furthermore, a numerical example is included to illustrate the validity of the primary outcome.

The remainder of this paper is organized as follows. In section two, we present the T-S fuzzy system where we present the conventional T-S fuzzy model and issues about stability. In section three, we study the boundedness and convergence analysis for uncertain fuzzy systems where some feedback controllers are constructed. The section four treats an example as an application of the obtained results.

2. T-S fuzzy dynamical system

The majority of physical systems have complex and ambiguous mathematical models that are challenging to obtain. For small range motion, these systems dynamics could exhibit linear or nonlinear characteristics. In order to study the asymptotic behaviors of the solutions via local approximation, Lyapunov's linearization method is frequently used. Takagi and Sugeno have suggested utilizing fuzzy inferences to provide an efficient method of aggregating these models. To design a T-S fuzzy controller, one needs a T-S fuzzy model for a nonlinear system. As a result, creating a fuzzy model is a crucial and fundamental step in this process. Let us examine a particular kind of continuous-time T-S fuzzy control system that is characterized by the following fuzzy rules, Rule i: If $z_1(t)$ is F_{i1} and $z_2(t)$ is F_{i2} ... and $z_p(t)$ is F_{ip}, then

$$\dot{x}(t) = A_i x(t) + B_i u(t), i = 1,2,\ldots,r, \tag{2.1}$$

where $x(t)$ is the state of the system, $u(t) \in \mathbb{R}^m$ represents the inputs and r is the number of fuzzy rules. $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times n}$ represent the input matrices, where $i = 1,2,\ldots,r$ serves as the index of the fuzzy rules. F_{ij} represents the inputs fuzzy sets, while $z(t) = [z_1(t),\ldots,z_p(t)]^T$ constitutes measurable variables, termed as premise variables. Utilizing weighted average defuzzifiers, an aggregated fuzzy model can be derived:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z)\big(A_i x(t) + B_i u(t)\big)}{\sum_{i=1}^{r} w_i(z)}, \ where \ w_i(z) = \prod_{i=1}^{r} F_{ij}(z_j). \tag{2.2}$$

The membership functions, denoted as $\mu_i(z)$, belong to the class C^l, implying that they are continuously differentiable and defined as:

$$\mu_i(z) = \frac{w_i(z)}{\sum_{i=1}^{r} w_i(z)}, \tag{2.3}$$

where $\mu_i = \mu_i(z) > 0$ for all $i = 1,2,\ldots,r$ and satisfy $\sum_{i=1}^{r} \mu_i(z) = 1$. Then the fuzzy system has the state-space form:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z)(A_i x(t) + B_i u(t)). \tag{2.4}$$

The PDC concept is used in many published results related to fuzzy system control. The design of the fuzzy controller incorporates a linear state feedback control in its consequent part, sharing the same antecedent as the fuzzy system. The controller for each local dynamic is defined as:

Rule i: If $z_1(t)$ is F_{i1} and $z_2(t)$ is F_{i2} ... and $z_p(t)$ is F_{ip}, then

$$u(t) = K_i x(t), i = 1,2,\ldots,r, \tag{2.5}$$

The local state feedback gain is denoted as K_i. Consequently, the defuzzified outcome is:

$$u(t) = \sum_{j=1}^{r} \mu_j(z) K_j x(t). \tag{2.6}$$

The system (2.2) in closed-loop with the fuzzy controller (2.4) yields the following fuzzy system,

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z)\mu_j(z)(A_i + B_i K_j)x(t). \tag{2.7}$$

For the asymptotic behaviors of the solutions, where some adequate requirements for the stability are inferred, we shall here employ the Lyapunov technique. Assume that the following criteria are met in order for there to be a common positive definite matrix P:

$$(A_i + B_iK_i)^T P + P(A_i + B_iK_i) < 0, \qquad i = 1,2,\ldots,r, \tag{2.8}$$

and

$$\frac{1}{2}(A_i + B_iK_j + A_j + B_jK_i)^T P + \frac{1}{2}P(A_i + B_iK_j + A_j + B_jK_i) < 0, \;\; 1 \le i \le j \le r. \tag{2.9}$$

Or, without loss of generality,

$$(A_i + B_iK_i)^T P + P(A_i + B_iK_i) < -Q_i, \qquad i = 1,2,\ldots,r, \tag{2.10}$$

and

$$\frac{1}{2}(A_i + B_iK_j + A_j + B_jK_i)^T P + \frac{1}{2}P(A_i + B_iK_j + A_j + B_jK_i) < -Q_{ij}, \;\; 1 \le i \le j \le r. \tag{2.11}$$

The matrices $Q_i, i = 1,2,\ldots,r,$ and Q_{ij}, $i < j \le r$, are symmetric positive definite. In such case, the fuzzy system (2.5) is asymptotically stable when these requirements are met. Thus, this may be converted into a convex problem [29–31], which can then be resolved using the optimization of linear matrix inequalities. Feedback controllers can be built if the solution is viable, which means that the stabilization restrictions are satisfied. The fuzzy system stability precondition theorems are consistent with Lyapunov's definition of stability. Numerous studies have demonstrated (see, for example, [30–32]) that identifying a common positive definite matrix P ensures the stability of the T-S system as a whole.

Choosing $V(x) = x^T P x$ as the Lyapunov function of the T-S system, the following stability conditions for ensuring stability is derived by using the Lyapunov approach.

3. Convergence analysis of uncertain fuzzy systems

Inspired by the findings from the previous section regarding the fuzzy-model concept of control, our objective is to expand the T-S fuzzy system to include external disturbances. Let's look at the perturbed model below:

$$\dot{x} = A_i x + B_i u + \Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon), i = 1,\ldots,r \tag{3.1}$$

$\Lambda_i: \mathbb{R}^+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ are smooth functions, $\Gamma_i: \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^+ \longrightarrow \mathbb{R}^n$ represents the perturbations terms for $i = 1,\ldots,r$.

In order to improve the stability analysis condition by placing limitations on the term of uncertainties, the first section of this work focuses on utilizing the property of the system's shape, which consists of using the controller connected to the linear part. The first portion of this work focuses on employing the form property of the system, i.e., using the controller associated with the linear part, to improve the stability analysis condition by limiting the term of uncertainty, $u(t) = \sum_{i=1}^{r} \mu_i(z) K_i x(t)$.

The T-S fuzzy model is given by:

$$\begin{aligned} &\text{Rule } i\text{: If } z_1(t) \text{ is } F_{i1} \text{ and } z_2(t) \text{ is } F_{i2} \ldots \text{ and } z_p(t) \text{ is } F_{ip}, \text{ then} \\ &\dot{x} = A_i x + B_i u + \Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon), i = 1,\ldots,r \end{aligned} \tag{3.2}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $A_i(n,n)$ constant matrix, $B_i(n,n)$ matrix control input and the functions Λ_i represent the uncertainties of each fuzzy subsystem and are time-varying are of appropriate dimension. The perturbed terms are the functions Γ_i with $\varepsilon > 0$, for $i = 1,\ldots,r$. F_{ik} is the fuzzy set *(k = 1, 2, ..., p), z(t)=T (z_1 (t), ..., z_p (t))* is the premise variable vector associated with the system states and inputs and *r* is the number of fuzzy rules. This yields the output of fuzzy system:

$$\dot{x} = \frac{\sum_{i=1}^{r} w_i(z)(A_i x(t) + B_i u(t) + \Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon))}{\sum_{i=1}^{r} w_i(z)} \tag{3.3}$$

where $w_i(z) = \prod_{j=1}^{p} F_{ij}(z_i)$ and $F_{ij}(z_i)$ denotes the grade of the membership function F_{ij}, corresponding to $z_i(t)$.

Let $\mu_i(z)$ defines as:

$$\mu_i(z) = \frac{w_i(z)}{\sum_{j=1}^{r} w_j(z)}. \tag{3.4}$$

Then the fuzzy system has the state-space form:

$$\dot{x} = \sum_{i=1}^{r} \mu_i(z)\big(A_i x(t) + B_i u(t) + \Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon)\big). \tag{3.5}$$

Clearly, $\sum_{i=1}^{r} \mu_i(z) = 1$ and $\mu_i(z) \geq 0$ for $i = 1,\ldots,r$.

Regarding the T-S fuzzy system (3.2), we assume that the pairs $(A_i, B_i), i = 1,\ldots,r$ are controllable. Based on this assumption, a stabilizing controller gain K_i can be obtained by using the pole placement design or Ackerman's formula, such that each local dynamic is stably controlled. The global control input matrix, represented by *B*, is represented as follows:

$$B = \sum_{i=1}^{r} \mu_i B_i. \tag{3.6}$$

This indicates that the control performance is dominated by the global control input matrix. The fuzzy controller's design can be seen as a linear state feedback control for the system (3.2), which is defined as follows:

$$\begin{gathered} \text{Rule } i\text{: If } z_1(t) \text{ is } F_{i1} \text{ and } z_2(t) \text{ is } F_{i2} \ldots \text{ and } z_p(t) \text{ is } F_{ip}, \text{ then} \\ u(t) = K_i x(t),\ i = 1,2,\ldots,r, \end{gathered} \tag{3.7}$$

where K_i is the local state feedback gain, where the defuzzified result is $u(t) = \sum_{i=1}^{r} \mu_i(z) K_i x(t)$.

For dynamic systems, we must first remember what uniformly eventually (ultimate) boundedness and exponential convergence in the sense of [1,12,38]. Let consider the following system described by:

$$\dot{x} = F^{\varepsilon}(t,x) \tag{3.8}$$

with $F^{\varepsilon}$are smooth functions, $t \in \mathbb{R}_+$ is the time and $x \in \mathbb{R}^n$ is the state, $\varepsilon > 0$.

The system (2.3) is said uniformly ultimately bounded if their exists $R(\varepsilon) > 0$, such that for all $R_1 > 0$, there exists a $T = T\ x\ (R_1, \varepsilon) > 0$ such that:

$$\|x(t_0)\| \leq R_1 \Rightarrow \|x(t)\| \leq R(\varepsilon) \text{ for all } t \geq t_0 + T \text{ and } t_0 \geq 0. \tag{3.9}$$

The system (2.3) is said to be globally uniformly ultimately bounded if the above property holds for arbitrarily large.

We consider the case where $t_0 = 0$, the solution denotes $x(t)$ of (2.3) with respect the initial condition $(0, x(t))$. System (2.3) is said to be globally exponentially ultimately bounded if:

$$\|x(t)\| \leq \theta(\varepsilon)\|x(0)\|e^{-\upsilon(\varepsilon)t} + r(\varepsilon), \text{ for all } t \geq 0, \tag{3.10}$$

with $\theta(\varepsilon)>0, \upsilon(\varepsilon)>0$.

Noting that, one can use Lyapunov analysis to prove the ultimate boundedness in the case of quadratic Lyapunov function of the form $V(x) = x^T Px$, if the derivative of Valong the trajectories satisfies $\dot{V} \leq -a\|x\|^2 + b\|x\|$, with a>0 and a>0 which implies that $\dot{V} \leq -a\|x\|^2 + a\theta\|x\|^2 - a\theta\|x\|^2 + b\|x\|$, for $0 < \theta < 1$.

Then one gets the following estimation: $\dot{V} \leq -a(1-\theta)\|x\|^2 - (a\theta\|x\| - b)\|x\|$, thus $\dot{V} \leq -a(1-\theta)\|x\|^2 < 0 \; for \; \|x\| > \frac{b}{a\theta}$, which gives the ultimate boundedness of solutions. Moreover, one can reach an estimation as (3.10) via a differential inequality of the form: $\dot{Z} \leq -\alpha Z + \beta\sqrt{Z}$, where $\alpha > 0, \beta > 0$ and $Z \geq 0$. In this case, one can obtain the following estimation:

$$\sqrt{Z(t)} \leq (\sqrt{Z(0)} - \frac{\beta}{\alpha})e^{-\frac{\alpha}{2}t} + \frac{\beta}{\alpha}. \tag{3.11}$$

Therefore, for a quadratic Lyapunov function of the form $V(x) = x^T Px$, one can take $Z(t) = V(x(t)) = x^T(t)Px(t)$ to get:

$$\sqrt{V(t)} \leq (\sqrt{V(0)} - \frac{\beta}{\alpha})e^{-\frac{\alpha}{2}t} + \frac{\beta}{\alpha}. \tag{3.12}$$

Thus, using the fact that $\lambda_{min}(P)\|x\|^2 \leq V(t,x) = x^T Px \leq \lambda_{max}(P)\|x\|^2$, one obtains $\sqrt{\lambda_{min}(P)}\|x(t)\| \leq \sqrt{V(x(t))} \leq (\sqrt{\lambda_{max}(P)}\|x(0)\| - \frac{\beta}{\alpha})e^{-\frac{\alpha}{2}t} + \frac{\beta}{\alpha}$.

Note that, we must have $\sqrt{\lambda_{max}(P)}\|x(0)\| - \frac{\beta}{\alpha} > 0$, this is because the initial conditions are taken outside the set $\{x \in \mathbb{R}^n / \|x\| \leq \frac{\beta}{\sqrt{\lambda_{max}(P)}\alpha}\}$. Thus, an estimation as in (3.10) can be obtained:

$$\|x(t)\| \leq \frac{1}{\sqrt{\lambda_{min}(P)}}((\sqrt{\lambda_{max}(P)}\|x(0)\| - \frac{\beta}{\alpha})e^{-\frac{\alpha}{2}t} + \frac{\beta}{\alpha}). \tag{3.13}$$

Remark: It is well known that most plants in industry show significant nonlinearities, which usually make the analysis and controller design difficult. However, exact mathematical models of most physical systems are difficult to obtain because of the existence of complexities and uncertainties. In order to overcome such difficulties, various schemes have been developed in the past two decades, among which a successful approach is the fuzzy control. The dynamics of these systems may include linear or nonlinear behaviors for small range motion. Lyapunov's linearization method is often implemented to deal with the local dynamics of nonlinear systems and to formulate local linearized approximation. So, the complex system can be divided into a set of local mathematical models. Takagi and Sugeno have proposed an effective means of aggregating these models by using the fuzzy inferences to construct the system. Therefore, an interesting line of research which gives promising prospects in several applications is the study of the stability of solutions. The asymptotic stability is conservative or difficult to obtain in many applications. So, the asymptotic behavior of the solutions can be studied throughout the boundedness property. The advantage of the exponential ultimate boundedness of solutions is that the solutions convergence toward a small neighborhood of the origin, which can be supposed not necessarily equilibrium point of the system.

Next, we will consider the following fuzzy systems based fuzzy controller:

$$\begin{cases} \dot{x} = \dfrac{\sum_{i=1}^{r} w_i(z)(A_i x(t) + B_i u(t) + \Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon))}{\sum_{i=1}^{r} w_i(z)} \\ u(t) = \displaystyle\sum_{i=1}^{r} \frac{w_i(z)}{\sum_{j=1}^{r} w_j(z)} K_i x(t). \end{cases} \tag{3.14}$$

Since the inception of fuzzy controllers, considerable interest has been shown in this field. The highly positive experimental results show that the fuzzy controller performed consistently better than the traditional controllers and was less susceptible to noise and parameters' changes. Our objective is to find some conditions on the bound of the term of perturbation such that the fuzzy system (3.2) is globally exponentially ultimately bounded in closed-loop with a fuzzy controller. In that case the solutions converge toward a neighborhood of the origin which is characterized by a small ball centered at the origin. This stability criterion enable the selection of the common positive-definite matrix P where the quadratic function $x^T Px$ can be used as a Lyapunov function candidate for the fuzzy system.

3.1. Control analysis of fuzzy systems under perturbations

Let consider the T-S fuzzy model (3.2):

$$\begin{aligned} &\text{Rule } i: \text{If } z_1(t) \text{ is } F_{i1} \text{ and } z_2(t) \text{ is } F_{i2} \dots \text{ and } z_p(t) \text{ is } F_{ip}, \text{then} \\ &\dot{x} = A_i x + B_i u + \Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon), i = 1,\dots,r. \end{aligned} \tag{3.15}$$

The function f_l represent the terms of uncertainties of each fuzzy subsystem that satisfy the following assumption:

$(\mathcal{H}_1)$ for all $i = 1,\dots,r,$

$$\|\Lambda_i(t,x)\| \le \varrho_i(x)\|x\|, \forall t \ge 0, \forall x \in \mathbb{R}^n \tag{3.16}$$

where ϱ_i are some nonnegative continuous functions, such that $\varrho_i(0) = 0, i = 1,\dots,r.$ The representation of the nonlinearities associated with the following bound which is a positive continuous function, named $\varrho: \mathbb{R}^n \to \mathbb{R}_+,$ such that $\varrho(0) = 0,$ which has the form:

$$\varrho(x) = \left[\sum_{i=1}^{r} \varrho_i^2(x)\right]^{\frac{1}{2}} \tag{3.17}$$

$(\mathcal{H}_2)$ assume that,

$$\|\Gamma_i(t,x,\varepsilon)\| \le v_i^{\varepsilon}(t), i = 1,2,\dots,r, \tag{3.18}$$

for all $t \ge 0$ and $x \in \mathbb{R}^n$ where v_i^{ε} are known nonnegative continuous functions for $i = 1,2,\dots,r,$ with

$$\left(\int_0^{+\infty} v^{\varepsilon}(t)^2 dt\right)^{\frac{1}{2}} \le \tilde{v}(\varepsilon) < +\infty, \tag{3.19}$$

where

$$v^{\varepsilon}(t):=\left(\sum_{i=1}^{r} v_i^{\varepsilon}(t)^2\right)^{\frac{1}{2}} \tag{3.20}$$

$\tilde{v}(\varepsilon)$is a nonnegative constant depending on the parameter ε.

We will use the following fuzzy controller:

$$u(t)=\sum_{i=1}^{r} \mu_i(z) K_i x(t). \tag{3.21}$$

The closed-loop system is given by

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z)\mu_j(z)[A_i+B_iK_j]x(t)+\sum_{i=1}^{r}\mu_i(\Lambda_i(t,x)+\Gamma_i(t,x,\varepsilon)) \tag{3.22}$$

$$=\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i{}^2 G_{ii}x(t)+\sum_{i<1}^{r} 2\mu_i\mu_j G_{ii}x(t)+\sum_{i=1}^{r}\mu_i(\Lambda_i(t,x)+\Gamma_i(t,x,\varepsilon)) \tag{3.23}$$

where

$$G_{ii}=A_i+B_iK_i \tag{3.24}$$

$$G_{ij}=\frac{1}{2}(A_i+B_iK_j+A_j+B_jK_i). \tag{3.25}$$

The stability of the local fuzzy dynamics is taken into account first in the controller synthesis. That is, each subsystem's steady feedback gains are identified. Assume that certain matrices K_i, $i=1,2,\dots,r,$ and a symmetric positive definite matrix P that met the following stability criteria:

$$(A_i+B_iK_i)^T P+P(A_i+B_iK_i)<-Q_i, i=1,\dots,r, \tag{3.26}$$

where Q_i is a positive define matrix.

Considering these assumptions, each subsystem is locally controllable and therefore a stabilizing feedback gain is obtained. Let consider the function $V(x)=x^T P x$ as a Lyapunov candidate. The derivative of $V(x)$ with respect to time is given by,

$$\begin{aligned}\dot{V}(x)=&\sum_{i=1}^{r}\mu_i^2 x^T\left(G_{ii}^T P+PG_{ii}\right)x+2\sum_{i<j}^{r}\mu_i\mu_j x^T(G_{ij}^T P+PG_{ij})x\\&+2x^T P\sum_{i=1}^{r}\mu_i(\Lambda_i(t,x)+\Gamma_i(t,x,\varepsilon)).\end{aligned} \tag{3.27}$$

Regarding each matrix $(G_{ii}^T P+PG_{ii})$, one has

$$\lambda_{min}(G_{ii}^T P + PG_{ii})\|x\|^2 \leq x^T(G_{ii}^T P + PG_{ii})x \leq \lambda_{max}(G_{ii}^T P + PG_{ii})\|x\|^2, \tag{3.28}$$

where $\lambda_{min}(.)(resp. \lambda_{max}(.))$ denotes the smallest (resp. the largest) eigenvalue of the matrix.

Define

$$\alpha = \max_{i,j} \lambda_{max}\left(G_{ii}^T P + PG_{ii}\right). \tag{3.29}$$

For $1 \leq i < j \leq r$. A relaxed condition concerning the coupling effect is expressed as:

$$\sum_{i<j}^{r} \mu_i \mu_j x^T(G_{ij}^T P + PG_{ij})x \leq k\|x\|^2 \tag{3.30}$$

where $k = \frac{r(r-1)}{2}\alpha$. Indeed, one has

$$\sum_{i<j}^{r} \mu_i \mu_j x^T(G_{ij}^T P + PG_{ij})x \leq \sum_{i<j}^{r} \mu_i \mu_j \lambda_{max}(G_{ij}^T P + PG_{ij})\|x\|^2. \tag{3.31}$$

It follows that,

$$\sum_{i<j}^{r} \mu_i \mu_j x^T(G_{ij}^T P + PG_{ij})x \leq \sum_{i<j}^{r} \mu_i \mu_j max_{i,j} \lambda_{max}(G_{ij}^T P + PG_{ij})\|x\|^2. \tag{3.32}$$

Hence,

$$\sum_{i<j}^{r} \mu_i \mu_j x^T(G_{ij}^T P + PG_{ij})x \leq \alpha \sum_{i<j}^{r} \mu_i \mu_j \|x\|^2 = \alpha \frac{r(r-1)}{2}\|x\|^2. \tag{3.33}$$

Then one can state the following theorem.

Theorem 3.1. If the conditions $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$ are verified for the fuzzy system (3.2) and there exist a common define positive matrix P and feedback gain matrices K_i, $i = 1,2,\dots,r$, such that the conditions givin in (3.26) hold, then the closed-loop system is globally exponentially ultimately bounded with the control law (3.21) where ϱ verifies

$$\varrho(x) < \frac{1}{2\|P\|} \frac{(1-\theta)}{(\sum_{i=1}^{r} \mu_i^2)^{\frac{1}{2}}} (\inf_{i=1,\dots,r} \lambda_{min}(Q_i) \sum_{i=1}^{r} \mu_i^2 - 2k), \tag{3.34}$$

with $\theta \in [0,1]$ and $\lambda_0 =: \inf_{i=1,\dots,r} \lambda_{min}(Q_i) \sum_{i=1}^{r} \mu_i^2 - 2k > 0$.

Proof. Using the Lyapunov function $V(x) = x^T Px$. The derivative of $V(x)$ along the trajectories of (3.2) in closed-loop with (2.6) with respect to time is given by,

$$\dot{V}(x) = \sum_{i=1}^{r} \mu_i^2 x^T (G_{ii}^T P + PG_{ii})x + 2\sum_{i<j}^{r} \mu_i \mu_j x^T (G_{ij}^T P + PG_{ij})x + 2x^T P \sum_{i=1}^{r} \mu_i (\Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon)). \tag{3.35}$$

Since, $\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j = 1$, then, $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$, from we have:

$$\| \sum_{i=1}^{r} \mu_i \Lambda_i(t,x) \| \leq \sum_{i=1}^{r} \mu_i \varrho_i(x) \|x\|^2 \tag{3.36}$$

and

$$\| \sum_{i=1}^{r} \mu_i \Gamma_i(t,x,\varepsilon) \| \leq \sum_{i=1}^{r} \mu_i \upsilon_i^{\varepsilon}(t) \|x\|. \tag{3.37}$$

Taking into account the last expressions, it follows that
Thus,

$$\dot{V}(x) \leq -\sum_{i=1}^{r} \mu_i^2 \lambda_{min}(Q_i) \|x\|^2 + 2K\|x\|^2 + 2\|P\| \sum_{i=1}^{r} \mu_i \varrho_i(x) \|x\|^2 + 2\|P\| \sum_{i=1}^{r} \mu_i \upsilon_i^{\varepsilon}(t) \|x\|. \tag{3.38}$$

Using Cauchy-Schwartz inequality

$$\dot{V}(x) \leq (-inf_{i=1,\dots,r} \lambda_{min}(Q_i) \sum_{i=1}^{r} \mu_i^2 + 2k + 2\|P\| \varrho(x) (\sum_{i=1}^{r} \mu_i^2)^{\frac{1}{2}})) \|x\|^2 + 2\|P\| \sum_{i=1}^{r} \mu_i \upsilon_i^{\varepsilon}(t) \|x\|. \tag{3.39}$$

Since

$$\lambda_{min}(P) \|x\|^2 \leq V(t,x) = x^T P x \leq \lambda_{max}(P) \|x\|^2, \tag{3.40}$$

then by taking $\|P\| = \lambda_{max}(P)$, yields:

$$\dot{V}(t,x) \leq -\frac{\lambda_0 \theta}{\lambda_{max}(P)} V(t,x) + 2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)} \upsilon^{\varepsilon}(t) V(t,x)^{\frac{1}{2}}. \tag{3.41}$$

Let

$$\zeta = \frac{\lambda_0 \theta}{\lambda_{max}(P)} > 0, \tag{3.42}$$

and

$$\overline{\upsilon}^{\varepsilon}(t)=2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}\upsilon^{\varepsilon}(t). \tag{3.43}$$

Given the earlier notations, it can be inferred that:

$$\dot{V}(t,x)\leq-\zeta V(t,x)+\overline{\upsilon}^{\varepsilon}(t)V(t,x)^{\frac{1}{2}}. \tag{3.44}$$

The variable in the last expression is changed as follows: $\Upsilon(t)=V(t,x)^{\frac{1}{2}}$. The derivative with respect to time is given by:

$$\dot{\Upsilon}(t)=\frac{\dot{V}(t,x)}{2V(t,x)^{\frac{1}{2}}}. \tag{3.45}$$

This implies that,

$$\dot{\Upsilon}(t)\leq-\frac{1}{2}\zeta\Upsilon(t)+\frac{1}{2}\overline{\upsilon}^{\varepsilon}(t). \tag{3.46}$$

Thus,

$$\Upsilon(t)\leq\Upsilon(0)e^{-\frac{1}{2}\zeta t}+\frac{1}{2}e^{-\frac{1}{2}\zeta t}.\int_{0}^{t}\overline{\upsilon}^{\varepsilon}(s)e^{\frac{1}{2}\zeta s}ds. \tag{3.47}$$

It follows that,

$$\lambda_{min}^{\frac{1}{2}}(P)\|x(t)\|\leq\lambda_{max}^{\frac{1}{2}}(P)\|x(0)\|e^{-\frac{1}{2}\zeta t}+\frac{1}{2}e^{-\frac{1}{2}\zeta t}.((\int_{0}^{t}\overline{\upsilon}^{\varepsilon}(s)^{2}ds)^{\frac{1}{2}}.(\int_{0}^{t}(e^{\frac{1}{2}\zeta s})^{2}ds)^{\frac{1}{2}}). \tag{3.48}$$

So,

$$\lambda_{min}^{\frac{1}{2}}(P)\|x(t)\|\leq\lambda_{max}^{\frac{1}{2}}(P)\|x(0)\|e^{-\frac{1}{2}\zeta t}+\frac{1}{2}e^{-\frac{1}{2}\zeta t}.((\int_{0}^{+\infty}\overline{\upsilon}^{\varepsilon}(s)^{2}ds)^{\frac{1}{2}}.(\int_{0}^{t}e^{\zeta s}ds)^{\frac{1}{2}}). \tag{3.49}$$

One gets,

$$\lambda_{min}^{\frac{1}{2}}(P)\|x(t)\|\leq\lambda_{max}^{\frac{1}{2}}(P)\|x(0)\|e^{-\frac{1}{2}\zeta t}+2\tilde{\upsilon}(\varepsilon)(\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)})^{2}e^{-\frac{1}{2}\zeta t}.(\frac{1}{\zeta}(e^{\frac{1}{\zeta t}}-1))^{\frac{1}{2}}. \tag{3.50}$$

Hence,

$$\|x(t)\|\leq\frac{\lambda_{max}^{\frac{1}{2}}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}\|x(0)\|e^{-\frac{1}{2}\zeta t}+2\tilde{\upsilon}(\varepsilon)\frac{\lambda_{max}^{2}(P)}{\lambda_{min}^{\frac{3}{2}}(P)}e^{-\frac{1}{2}\zeta t}.(\frac{1}{\zeta}e^{\frac{1}{\zeta t}})^{\frac{1}{2}}. \tag{3.51}$$

Then,

$$\|x(t)\| \leq \frac{\lambda_{max}^{\frac{1}{2}}(P)}{\lambda_{min}^{\frac{1}{2}}(P)} \|x(0)\| e^{-\frac{1}{2}\zeta t} + 2\frac{\lambda_{max}^{2}(P)}{\lambda_{min}^{\frac{3}{2}}(P)} \frac{\tilde{\upsilon}(\varepsilon)}{\zeta^{\frac{1}{2}}}. \tag{3.52}$$

We obtain an estimation on the trajectories as the one given in (3.3), with

$$\theta(\varepsilon) = \frac{\lambda_{max}^{\frac{1}{2}}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}, \tag{3.53}$$

$$\upsilon(\varepsilon) = \frac{1}{2}\zeta \tag{3.54}$$

and

$$r(\varepsilon) = 2\frac{\lambda_{max}^{2}(P)}{\lambda_{min}^{\frac{3}{2}}(P)} \frac{\tilde{\upsilon}(\varepsilon)}{\zeta^{\frac{1}{2}}}. \tag{3.55}$$

Hence, the system is globally exponentially ultimately bounded. It follows that $\mathcal{B}_{r(\varepsilon)}$, with $r(\varepsilon) = 2\frac{\lambda_{max}^{2}(P)}{\lambda_{min}^{\frac{3}{2}}(P)} \frac{\tilde{\upsilon}(\varepsilon)}{\zeta^{\frac{1}{2}}}$, is globally uniformly exponentially stable.

Remark. Note that the last inequality implies that system (3.2) in closed loop is uniformly ultimately bounded in the sense that the trajectories satisfy the inequality (3.10). Moreover, in the case when $r = r(\varepsilon)$ goes to zero as ε tends to zero, then the error approaches the origin exponentially as *t* tends to infinity.

The design process for T-S fuzzy systems can be summed up as follows using the last method.

Step 1: Verify the controllability of (A_i, B_i), for $i = 1,2,\dots,r$.

Step 2: The conditions $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$ are satisfied.

Step 3: Solve the equations (3.26) to obtain P, K_i, Q_i for $i = 1,2,\dots,r$.

Note that, for simplicity one can choose $Q_i = I$.

Step 4: By using Matlab (control-toolbox), consider the nonlinear program based on equations:

$$G_{ii}^T P + PG_{ii}), 1 \leq i < j \leq r.$$

To ascertain the value of *k*, the nonlinear programming is formulated in the following manner: Determine $\lambda_{max}\left(G_{ii}^T P + PG_{ii}\right)$ and then for $1 \leq i < j \leq r$.

$$\alpha = \max_{i,j} \lambda_{max}(G_{ii}^T P + PG_{ii}) \tag{3.56}$$

Step 5: Use the fuzzy controller (3.21) for (3.2).

Step 6: Verify the condition (3.34) imposed on $\varrho(x)$ that holds for a suitable choice of $\varepsilon \in]0, \varepsilon^*[, \varepsilon^* > 0$ small enough.

4. Simulation examples

4.1. Example 1

Let's consider a dynamical system representing a translational oscillator coupled with an eccentric rotational proof mass actuator, as described in [3,25]. Here, x_1 and x_2 denote the translational position and velocity of the cart, with $x_1$being the derivative of x_2. Additionally, x_3 and x_4 signify the angular position and velocity for the rotational proof mass model, where x_3 corresponds to the derivative of x_4. The system's dynamics can be described by the nonlinear equation:

$$\dot{x} = A(x) + B(x)u + d. \tag{4.1}$$

In this equation:

- u represents the torque applied to the eccentric mass,
- d represents disturbances chosen to encompass the system's perturbation terms (3.2),
- $A(x) = \begin{bmatrix} x_2 & \frac{-x_1+\varepsilon x_4^2 \sin x_3}{1-\varepsilon^2 \cos^2 x_3} & x_4 & \frac{\varepsilon \cos x_3 (x_1 - \varepsilon x_4^2 \sin x_3)}{1-\varepsilon^2 \cos^2 x_3} \end{bmatrix}^T$, and
- $B(x) = [0 \quad \frac{-\varepsilon co \quad x_3}{1-\varepsilon^2 cos^2 \ x_3} \quad 0 \quad \frac{1}{1-\varepsilon^2 cos^2 \ x_3}]^T, x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \varepsilon \in]0, \frac{1}{2}[$.

The corresponding T-S fuzzy model associated to this nonlinear system is given by:

$$\dot{x} = A_i x + B_i(\varepsilon)u + \Lambda_i(t,x) + \Gamma_i(t,x,\varepsilon), i = 1,\ldots,4. \tag{4.2}$$

where,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & \frac{\varepsilon \sin(\alpha\pi)}{(\alpha\pi)} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-\varepsilon}{(1-\varepsilon^2)} & 0 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{(1-\varepsilon^2)} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 2\frac{\varepsilon}{\pi} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\varepsilon}{(1-\varepsilon^2)} & 0 & \frac{-\varepsilon^2}{(1-\varepsilon^2)} & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{(1-\varepsilon^2)} \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\varepsilon}{(1-\varepsilon^2)} & 0 & \frac{-15\varepsilon^2}{(1-\varepsilon^2)} & 0 \end{bmatrix}, B_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{(1-\varepsilon^2)} \end{bmatrix},$$

$$\Lambda_1(t,x)=\begin{bmatrix}\varrho sin\ x_1\\0\\0\\0\end{bmatrix},\Lambda_2(t,x)=\begin{bmatrix}\varrho sin\ x_2\\0\\0\\0\end{bmatrix},$$

$$\Lambda_3(t,x)=\begin{bmatrix}\varrho sin\ x_3\\0\\0\\0\end{bmatrix},\Lambda_4(t,x)=\begin{bmatrix}\varrho sin\ x_4\\0\\0\\0\end{bmatrix}.$$

$$\Gamma_1(t,x,\varepsilon)=\begin{bmatrix}0\\0\\0\\\dfrac{e^{-t}}{(1-\varepsilon^2)}\end{bmatrix},\Gamma_2(t,x,\varepsilon)=\begin{bmatrix}0\\0\\0\\\dfrac{-e^{-t}}{(1-\varepsilon^2)}\end{bmatrix},$$

$$\Gamma_3(t,x,\varepsilon)=\begin{bmatrix}0\\0\\0\\\dfrac{e^{-t}}{(1-\varepsilon^2)}\end{bmatrix},\Gamma_4(t,x,\varepsilon)=\begin{bmatrix}0\\0\\0\\\dfrac{-e^{-t}}{(1-\varepsilon^2)}\end{bmatrix}.$$

We use the following fuzzy controller: $u(t)=\sum_{j=1}^{4}\mu_j(z)K_jx(t)$ with $\varepsilon=0.01$ to stabilize the system, in the sense that all trajectories of the system are ultimately bounded (see Figure 1).

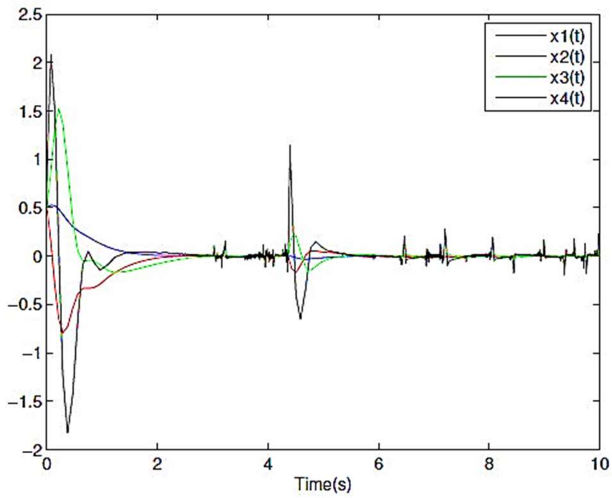

Figure 1. Time evolution of the states of the system.

The Figure 1 shows the time evolution of the states $(x_1(t);x_2(t);x_3(t);x_4(t))$ of the system with the initial states $(x_1(0);x_2(0);x_3(0);x_4(0))=(0.5;0.5;0.5;0.5)$. One can see that the trajectories are bounded and converge to a small neighborhood of the origin.

We need to select a positive definite matrix Q, such as Q=I, to address the algebraic Lyapunov equation (3.26) and determine the matrix P. For $\varrho(x) < 1$ we can take, $\varrho(x) = 0.1$ where $\varepsilon^* = \frac{1}{2}$ and by using MATLAB, we get the following solutions, for K_1, K_2, K_3, K_4, and the matrix P:

$$K_1 = \begin{bmatrix} -1.085 \\ 1.518 \\ 3.051 \\ 1.337 \end{bmatrix}, K_2 = \begin{bmatrix} -0.984 \\ 1.499 \\ 3.041 \\ 1.336 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -1.085 \\ 1.518 \\ 3.051 \\ 1.337 \end{bmatrix}, K_4 = \begin{bmatrix} -0.985 \\ 1.499 \\ 3.041 \\ 1.336 \end{bmatrix},$$

$$P = \begin{bmatrix} 1.649 & -0.003 & 0.001 & 1.212 \\ -0.003 & 1.649 & -1.213 & -0.005 \\ 0.001 & -1.213 & 1.053 & -0.112 \\ 1.212 & -0.005 & -0.112 & 1.232 \end{bmatrix}.$$

It should be noted that when $t \to +\infty$, the trajectory tends to the origin exponentially if the bounds of the nonlinearities go to zero when t approaches to infinity.

Consequently, $\lim_{\varepsilon \to 0} r(\varepsilon) = 0$ indicates that the state can converge to the origin exponentially as t goes to infinity if the perturbation term's bound can be made as small as desired.

4.2. Example 2

Consider the following nonlinear fuzzy planar system.

$$\begin{aligned} \dot{x} &= -2x_1 + \sin(x_1)u \\ \dot{x} &= x_1 \sin(x_1) + u, \end{aligned} \tag{4.3}$$

where $x(t) = [x_1(t) \;\; x_2(t)]^T \in \mathbb{R}^2$, the state vector and $u(t)$ is the input vector.

One can represent exactly the system by the following two-rule fuzzy model:

Rule 1: If x_1 is M_{11} then $\dot{x}(t) = A_1 x(t) + B_1 u(t)$.

Rule 2: If x_1 is M_{21} then $\dot{x}(t) = A_2 x(t) + B_2 u(t)$,

where: $A_1 = \begin{bmatrix} -2 & 0 \\ -1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

We define the membership functions as:

$$\mu_1(x_1(t)) = \frac{1 - \sin(x_1(t))}{2} \;\; ; \;\; \mu_2(x_1(t)) = \frac{\sin(x_1(t)) + 1}{2}. \tag{4.4}$$

Using an LMI optimization algorithm, yields:

$$P = \begin{bmatrix} 0.0377 & 0.0000 \\ 0.0000 & 0.0183 \end{bmatrix},$$

The following feedback gains: $K_1 = [-0.0452 \quad 0.7962]$ and $K_2 = [0.0452 \quad 0.7962]$, and the matrices:

$$Q_1 = \begin{bmatrix} 0.0771 & -0.0063 \\ -0.0063 & 0.0145 \end{bmatrix}, Q_2 = \begin{bmatrix} 0.0771 & 0.0063 \\ 0.0063 & 0.0145 \end{bmatrix} \text{and } Q_{12} = \begin{bmatrix} 0.1024 & 0.0000 \\ 0.0000 & 0.0196 \end{bmatrix}.$$

Then, we have $\lambda_{min}(P) = 0.0183, \lambda_{max}(P) = \|P\| = 0.0377,$ and $\lambda_0 = 0.0001$ within $f\{(\lambda_{min}(Q_i); i = 1,2), (\lambda_{min}(Q_{12}))\} = 0.0139$. Now, we introduce parametric perturbations and external disturbances and we approximate the system by the following fuzzy models:

Rule 1: If x_1 is F_{11}then

$$\dot{x}(t) = A_1x(t) + B_1u(t) + \Lambda_1(t,x) + \Gamma_1(t,x,\varepsilon)$$

Rule 2: If x_1 is F_{21} then

$$\dot{x}(t) = A_2x(t) + B_2u(t) + \Lambda_2(t,x) + \Gamma_2(t,x,\varepsilon)$$

where, $\Lambda_1(t,x) = \Lambda_2(t,x) = \frac{1}{5}\frac{\sqrt{x_1^2+x_2^2}}{1+x_1^2}$, and $\Gamma_1(t,x,\varepsilon) = \Gamma_2(t,x,\varepsilon) = \frac{\varepsilon}{5}e^{-t}, t \geq 0$. One can take,

$$\rho(x) = \left(\sum_{i=1}^{2} \rho_i(x)^2\right)^{\frac{1}{2}} = \frac{1}{5}\frac{\sqrt{2}}{1+x_1^2}, \tag{4.5}$$

with $\varepsilon = 0.1, \upsilon(\varepsilon) < \frac{0.1}{5}0.95 = 0.019$. All the assumptions of Theorem 3.1 are satisfied, by using ρ as in (4.5), it follows that the trajectories of the system are bounded and converge toward a small neighborhood of the origin. Hence, the solutions of system (4.3) with a fuzzy controller of the form (2.6), under some restrictions on the perturbations as they are given in the system to be studied, are globally exponentially ultimately bounded.

5. Conclusions

In this paper, the feedback controller problem is treated for perturbed Takagi-Sugeno fuzzy models. In order to guarantee the exponential ultimate boundedness of solutions for fuzzy control systems with uncertainties related to a tiny parameter, certain new adequate criteria are provided. The applicability of the main result is demonstrated with an example and simulation results.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This research has been funded by Scientific Research Deanship at University of Ha'il-Saudi Arabia through project number (RG-23 070).

Conflict of interest

The authors declare no conflict of interest.

References

1. B. B. Hamed, I. Ellouze, M. A. Hammami, Practical uniform stability of nonlinear differential delay equations, *Mediterr. J. Math.*, **8** (2011), 603–616. https://doi.org/10.1007/s00009-010-0083-7
2. B. B. Hamed, M. A. Hammami, Practical stabilization of a class of uncertain time-varying non-linear delay systems, *J. Contr. Theo. Appl.*, **7** (2009), 175–180. https://doi.org/10.1007/s11768- 009-8017-2
3. R. T. Bupp, D. S. Bernstein, V. T. Coppola, A benchmark problem for nonlinear control design, *Int. J. Nonlin. Robust Contr.*, **8** (1998), 307–310. https://doi.org/10.1109/91.919253
4. P. Bergsten, R. Palm, D. Driankov, Observers for Takagi-Sugeno fuzzy systems, *IEEE T. Syst. Man Cy-S.*, *Part B (Cybernetics)*, **32** (2002), 114–121. https://doi.org/10.1109/3477.979966
5. R. Datta, R. Saravanakumar, R. Dey, B. Bhattacharya, Further results on stability analysis of Takagi-Sugeno fuzzy time-delay systems via improved Lyapunov-Krasovskii functional, *AIMS Math.*, **7** (2022), 16464–16481. https://doi.org/10.3934/math.2022901
6. F. Delmotte, T. M. Guerra, M. Ksontini, Continuous Takagi-Sugeno's models: Reduction of the number of LMI conditions in various fuzzy control design techniques, *IEEE Trans. Fuzzy Syst.*, **15** (2007), 426–438. https://doi.org/10.1109/TFUZZ.2006.889829
7. M. Dlala, M. A. Hammami, Uniform exponential practical stability of impulsive perturbed systems, *J. Dyn. Control Syst.*, **13** (2007), 373–386. https://doi.org/10.1007/s10883-007- 9020-x
8. C. Fantuzzi, R. Rovatti, *On the approximation capabilities of the homogeneous Takagi-Sugenomodel, in fuzzy systems*, Proceedings of the Fifth IEEE International Conference on Decision and Control, **2** (1996). https://doi.org/10.1109/FUZZY.1996.552326
9. M. A. Hammami, On the stability of nonlinear control systems with uncertaintly, *J. Dyn. Contr. Sys.*, **7** (2001), 171–179. https://doi.org/10.1023/A:1013099004015
10. Z. HajSalem, M. A. Hammami, M. Mohamed, On the global uniform asymptotic stability of time varying dynamical systems, *Stud. Univ. B. B. Math.*, **59** (2014), 57–67. Available from: https://www.cs.ubbcluj.ro/ studia-m/2014-1/2014-1/06-hajsalem-hammami-mabrouk-final.pdf.
11. M. A. Hammami, Global stabilization of a certain class of nonlinear dynamical systems using state detection, *Appl. Math. Lett.*, **14** (2001), 913–919. https://doi.org/10.1016/S0893- 9659(01)00065-9
12. H. Khalil, *Nonlinear systems*, 3 Eds., Englewood Cliffs, NJ: Prentice-Hall.
13. E. Kim, H. Lee, New approaches to relaxed quadratic stability condition of fuzzy control systems, *IEEE T. Fuzzy Syst.*, **8** (2000), 523–534. https://doi.org/10.1109/91.873576
14. H. K. Lam, F. H. F. Leung, Stability analysis of fuzzy control systems subject to uncertain grades of membership, *IEEE T. Syst. Man Cy-S.,* Part B, **35** (2005), 1322–1325. https://doi.org/10.1109/tsmcb.2005.850181
15. A. Larrache, M. Lhous, S. B. Rhila, M. Rachik, A. Tridane: An output sensitivity problem for a class of linear distributed systems with uncertain initial state, *Arch. Control Sci.,* **30** (2020), 139–155. https://doi.org/10.24425/acs.2020.132589
16. D. H. Lee, J. B. Park, Y. H. Joo, A fuzzy Lyapunov function approach to estimating the domain of attraction for continuous-time Takagi-Sugeno fuzzy systems, *Inform. Sciences,* **185** (2012), 230–248. https://doi.org/10.1016/j.ins.2011.06.008
17. J. Huang, *Nonlinear output regulation: Theory and application*, Philadelphia, USA: SIAM, 2004. Available from: https://epubs.siam.org/doi/pdf/10.1137/1.9780898718683.fm.
18. M. Ksantini, M. A. Hammami, F. Delmotte, On the global exponential stabilization of Takagi-Sugeno fuzzy uncertain systems, *Int. J. Innovative Comp. Inf. Contr.,* **11** (2015), 281–294. Available from: http://www.ijicic.org/ijicic-110120.pdf.

19. Y. Menasria, H. Bouras, N. Debbache, An interval observer design for uncertain nonlinear systems based on the T-S fuzzy model, *Arch. Control Sci.,* **27** (2017), 397–407. https://doi.org/10.1515/acsc-2017-0025
20. H. Perez, B. Ogunnaike, S. Devasia, Output tracking between operating points for nonlinear process: Van de Vusse example, *IEEE Trans. Contr. Syst. Tech.,* **10** (2002) 611–617. https://doi.org/10.1109/TCST.2002.1014680
21. R. Sriraman, R. Samidurai, V. C. Amritha, G. Rachakit, P. Balaji, System decomposition-based stability criteria for Takagi-Sugeno fuzzy uncertain stochastic delayed neural networks in quaternion field, *AIMS Math.*, **8** (2023), 11589–11616. https://doi.org/10.3934/math.2023587
22. R. Sriraman, P. Vignesh, V. C. Amritha, G. Rachakit, P. Balaji, Direct quaternion method-based stability criteria for quaternion-valued Takagi-Sugeno fuzzy BAM delayed neural networks using quaternion-valued Wirtinger-based integral inequality, *AIMS Math.,* **8** (2023), 10486–10512. https://doi.org/10.3934/math.2023532
23. K. Tanaka, M. Sugeno, Stability analysis and design of fuzzy control systems, *Fuzzy Set. Syst.*, **45** (1992), 135–156. https://doi.org/10.1016/0165-0114(92)90113-I
24. T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modelling and control, *IEEE Trans. Syst. Man Cyber.*, **15** (1985), 116–132. http://dx.doi.org/10.1109/TSMC.1985.6313399
25. H. D. Tuan, P. Apkarian, T. Narikiyo, Y. Yamamoto, Parameterized linear matrix inequality techniques in Fuzzy control system design, *IEEE T. Fuzzy Syst.*, **9** (2001), 324–332. https://doi.org/10.1109/91.919253
26. N. Vafamand, M. H. Asemani, A. Khayatiyan, A robust L 1 controller design for continuous-time TS systems with persistent bounded disturbance and actuator saturation, *Eng. Appl. Artif. Intell.,* **56** (2016), 212–221. https://doi.org/10.1016/j.engappai.2016.09.002
27. W. B. Xie, H. Li, Z. H. Wang, J. Zhang, Observer-based controller design for a T-S fuzzy system with unknown premise variables, *Int. J. Control Autom. Syst.*, **17** (2019), 907–915. https://doi.org/10.1007/s12555-018-0245-0
28. J. Yang, S. Li, X. Yu, *Sliding-mode control for systems with mismatched uncertainties via a disturbance observer*, IECON 2011-37th Annual Conference of the IEEE Industrial Electronics Society, **60** (2013), 160–169. https://doi.org/10.1109/IECON.2011.6119961
29. M. Sugeno, G. T. Kang, Structure identification of fuzzy model, *Fuzzy Set. Syst.*, **28** (1988), 15–33. http://dx.doi.org/10.1016/0165-0114(88)90113-3
30. M. Sugeno, *Fuzzy control*, North-Holland, 1988.
31. T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. Syst. Man Cyber.,* **15** (1985), 116–132. http://dx.doi.org/10.1109/TSMC.1985.6313399
32. K. Tanaka, H. O. Wang, *Fuzzy control systems design and analysis*, John Wiley and Sons, New York, USA, 2001.
33. M. C. M. Teixeira, E. Assuncao, R. G. Avellar, On relaxed LMI-based designs for fuzzy regulators and fuzzy observers, *IEEE T. Fuzzy Syst.*, **11** (2003), 613–623. https://doi.org/10.1109/TFUZZ.2003.817840
34. H. O. Wang, K. Tanaka, M. Griffin, *Parallel distributed compensation of nonlinear systems by Takagi and Sugeno's mode*l, Proceedings of 1995 IEEE International Conference on Fuzzy Systems, **2** (1995). https://doi.org/10.1109/FUZZY.1995.409737
35. G. Yang, F. Hao, L. Zhang, L. X. Gao, Stabilization of discrete-time positive switched T-S fuzzy systems subject to actuator saturation, *AIMS Math.*, **8** (2023), 12708–12728. https://doi.org/10.3934/math.2023640

36. X. D. Liu, Q. L. Zhang, New approaches to H^{∞} controller designs based on fuzzy observers for T-S fuzzy systems via LMI, *Automatica*, **39** (2003), 1571–1582. https://doi.org/10.1016/S0005-1098(03)00172-9
37. L. X. Wang, Robust disturbance attenuation with stability for linear systems with norm-bounded nonlinear uncertainties, *IEEE Trans. Autom. Control,* **41** (1996), 886–888. https://doi.org/10.1109/WCICA.2004.1340782
38. L. Xua, S. S. Ge, The pth moment exponential ultimate boundedness of impulsive stochastic differential systems, *Appl. Math. Lett.*, **42** (2015), 22–29. http://dx.doi.org/10.1016/j.aml.2014.10.018
39. F. You, S. Cheng, K. Tian, X. Zhang, Robust fault estimation based on learning observer for Takagi-Sugeno fuzzy systems with interval time-varying delay, *Int. J. Adapt. Control,* **17** (2019). https://doi.org/10.1002/acs.3070
40. L. Zadeh, Fuzzy sets, *Fuzzy Sets, Fuzzy Logic and Fuzzy Systems*, Selected Papers, 1996. https://doi.org/10.1142/2895
41. A. Dharmarajan, P. Arumugam, S. Ramalingam, K. Ramasamy, Equivalent-Input-Disturbance based robust control design for fuzzy semi-markovian jump systems via the Proportional-Integral observer approach, *Mathematics*, 2023, 2543. https://doi.org/10.3390/math11112543

AIMS Press