AIMS Mathematics

AIMS Mathematics, 8(9): 21700–21731.
DOI: 10.3934/math.20231107
Received: 23 March 2023
Revised: 18 May 2023
Accepted: 22 May 2023
Published: 07 July 2023

http://www.aimspress.com/journal/Math

Research article

Solving bi-objective bi-item solid transportation problem with fuzzy stochastic constraints involving normal distribution

T. K. Buvaneshwari and D. Anuradha*

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore (Tamil Nadu), India

* **Correspondence:** Email: anuradhadhanapal1981@gmail.com.

Abstract: In today's competitive world, entrepreneurs cannot argue for transporting a single product. It does not provide much profit to the entrepreneur. Due to this reason, multiple products need to be transported from various origins to destinations through various types of conveyances. Real-world decision-making problems are typically phrased as multi-objective optimization problems because they may be effectively described with numerous competing objectives. Many real-life problems have uncertain objective functions and constraints due to incomplete or uncertain information. Such uncertainties are dealt with in fuzzy/interval/stochastic programming. This study explored a novel integrated model bi-objective bi-item solid transportation problem with fuzzy stochastic inequality constraints following a normal distribution. The entrepreneur's objectives are minimizing the transportation cost and duration of transit while maximizing the profit subject to constraints. The chance-constrained technique is applied to transform the uncertainty problem into its equivalent deterministic problem. The deterministic problem is then solved with the proposed method, namely, the global weighted sum method (GWSM), to find the optimal compromise solution. A numerical example is provided to test the efficacy of the method and then is solved using the Lingo 18.0 software. To highlight the proposed method, comparisons of the solution with the existing solution methods are performed. Finally, to understand the sensitivity of parameters in the proposed model, sensitivity analysis (SA) is conducted.

Keywords: solid transportation problem; stochastic programming; fuzzy random variables; normal distribution; global weighted sum method
Mathematics Subject Classification: 90B06, 90C15

1. Introduction

The current business environment is becoming more competitive on a daily basis, and every organization strives to improve how to deliver products to customers with minimum duration of transit and minimum cost, so that each customer's demand is met by the entrepreneur. A conventional mathematical programming problem that considers source and destination constraints is known as the transportation problem (TP), and it was developed by Hitchcock in 1941. In a transportation problem, there are different modes of transportation available for the transportation of products which we might transport from various sources to various destinations by different types of conveyances (such as goods trains, ships, load flights, trucks, etc.) to save the duration of transit and cost. In such a situation, the solid transportation problem (STP) or three-dimensional transportation problem, which considers three constraints, is appropriate. It was proposed by Shell [1]. Haley [2] expanded the modified distribution approach by providing a solution procedure for the STP. Several researchers have carried out extensive research on the single-objective STP [3,4]. In most real-world decision-making problems (DMPs), there are several competing criteria to be considered and optimized at the same time. Such problems are referred to as multi-objective optimization problems (MOOPs). Moreover, in today's rapidly changing market, the trading of a single product does not yield a high profit to entrepreneurs. As a result, all entrepreneurs in the field of transportation trade many products. The problem of transporting multiple products with multiple objectives involving three types of constraints, namely, source, destination and conveyance, is termed as the multi-objective multi-item solid transportation problem (MOMISTP). This type of problem yields a high profit to the entrepreneur with a minimum duration of transit and minimum cost.

In most DMPs like transportation, the values of the parameters cannot always be deterministic and fluctuate from time to time due to bad weather conditions, road conditions, etc. This introduces some uncertainty in the problem which can be dealt with in any of four ways: (i) fuzzy, (ii) interval, (iii) stochastic and (iv) both fuzzy and stochastic together. Dealing with different types of uncertainties in many DMPs is still an emerging problem. Various types of multi-objective STP (MOSTP) and MOMISTP under uncertainty have been investigated by many researchers so far. Khalifa et al. [5] employed a fuzzy geometric programming approach to solve the fractional two-stage MOSTP. Chhibber et al. [6] examined the Pareto-optimal solution for fixed-charge STP under an intuitionistic fuzzy environment. Anithakumari et al. [7] solved a fully interval integer MOSTP. Baidya and Bera [8] proposed an STP model under a fully fuzzy environment. Ghosh et al. [9] solved a type-2 zigzag uncertain multi-objective fixed-charge STP using time window versus preservation technology. Pramanik et al. [10] introduced MOSTP in imprecise environments. Kar et al. [11] investigated the MOMISTP under the fuzzy environment with volume, vehicle cost and weight capacity. Rani and Gulati [12] developed the uncertain multi-objective multi-product STP and obtained the fuzzy optimal compromise solution using the fuzzy programming approach (FPA). Rani et al. [13] introduced fuzzy the MOMISTP using the FPA. Kundu et al. [14] solved the MOMISTP in the fuzzy environment with the help of two different existing methods.

In reality, sometimes the data are insufficient because they cannot be measured or collected precisely. This uncertainty occurs in fuzzy, stochastic or both fuzzy and stochastic environment. The data or parameters are stochastic in nature and defined by random variables with known probability distributions called stochastic TPs. Several studies on stochastic TPs have been published by researchers like Kataoka [15], Szwarc [16], Singh et al.[17], Williams [18], Holmberg [19] and

Charnes [20]. Mahapatra [21–24] presented a stochastic TP where demand and supply parameters are random variables (RVs) with different distributions (normal, Weibull, lognormal, etc.). The stochastic programming problems were solved by Agrawal et al. [25] using a gaining-sharing knowledge-based algorithm. The data or parameters uncertain in both fuzzy and stochastic environments are called fuzzy random/stochastic parameters. Kwakernaak [26] introduced the concept of fuzzy stochastic optimization using fuzzy random parameters. The occurrence of these parameters increases the persuasiveness of the randomness and fuzziness together. For instance, the yield of crops depends greatly on the availability of productive resources, which are frequently fuzzy in nature. Rainfall, a critical indicator of the supply of water during the planning period, is a crucial agricultural resource, and the parameters involved in the problem are intrinsically stochastic. The unpredictable crop production leads to high uncertainty as to the location and scale of indemnities, and fuzzy random parameters are employed in these environments. After recognizing the previously stated circumstance, active researchers have considered both fuzzy programming (FP) and SP in the planning and programming environment. A fuzzy random chance-constrained programming model has been proposed by Zhao and Cao [27] for the vehicle routing problem of hazardous materials transportation. Maity et al. [28] have created a new approach for addressing the TP by adding multi-modal transportation systems in a fuzzy stochastic environment. Based on fuzzy mathematical programming, Nasseri and Bavandi [29] solved a fuzzy stochastic linear fractional programming problem. Most of the research has used various distributions to define the variable when modeling a fuzzy stochastic TP [30–32]. Sensitivity analysis is used as a post optimality tool to analyze the effect of the changes in the objective function coefficients and the effect of changes of the right hand side constraints on the optimal value of the objective function as well as the validity ranges of these effects. Studies on the sensitivity analysis in linear programming and multi-objective programming have been performed [33–35]. The implementation of sensitivity analysis is based primarily on an optimal compromise solution. Table 1 shows a comparative analysis of the methodology with the existing works on stochastic and fuzzy stochastic parameters.

In Table 1, it is noted that the majority of research has considered the parameter as a stochastic parameter or fuzzy stochastic parameter following Normal, Log-normal, Exponential, Extreme value and Weibull distributions for TP, fuzzy TP, fractional TP, fuzzy fractional TP, bi-objective TP and Multi-item interval valued STP.

In the above literature survey, there are some gaps in the analysis of the MOMISTP which are summarized below.

- It is noted that the work that has been done till now is mostly with the stochastic parameter or fuzzy parameter.
- To the best of our knowledge, few articles with both fuzzy and stochastic parameters together follow a Normal Distribution(ND).
- In STP multiple objectives with multiple items with fuzzy stochastic parameters are also very rare.
- In DMPs multi-item multiple objectives with fuzzy stochastic parameter are of great importance, though there are very few such models in the literature. Transporting multi-items from one source to another destination using different types of conveyances is one of the important problems in improving the economic growth rate of a country. The shipment of multiple items using various modes of transportation per day establishes the role of the multi-objective transportation problem as a science, and it is economically important. This motivates

us to study the solid transportation problem with multiple objectives with multiple items under a fuzzy stochastic environment.

- In this study, considering all these lacunas, the bi-objective bi-item solid transportation problem with fuzzy stochastic parameters is formulated and discussed in a fuzzy stochastic environment.

Table 1. A comparative analysis of the methodology with the existing works.

References	Objective nature		Problem objective		Constraint parameter		Distribution
	Cr	Im	SO	MO	S	FS	
[36] Gessesse et al. (2019)	√			√	√		Normal
[37] Roy (2014)	√		√		√		Weibull
[22] Mahapatra et al. (2013)	√			√	√		Extreme value
[21] Roy & Mahapatra (2011)		√		√	√		Lognormal
[38] Mahapatra (2013)	√		√		√		Weibull
[39] Das & Lee (2021)	√			√	√		Weibull
[40] Osman et al. (2017)		√		√		√	Using alpha-cut approach with normal, Weibull and expoential distriution for Fractional Programming Problem
[32] Agrawal & Ganesh (2020)		√	√			√	Using fuzzy membership function with Exponential distribution for fuzzy fractional TP
[41] Giri et al. (2013)		√		√		√	Using average method with Normal Distribution for Multi-item interval valued STP
[30] Acharya et al. (2014)	√			√		√	Using alpha cut with Normal Distribution for bi-objective TP
Proposed article	√			√		√	Using alpha cut with Normal Distribution for bi-objective bi-item STP

Note: Cr: Crisp; Im: Imprecise; SO: Single objective; MO: Multi-objective; S: Stochastic; FS: Fuzzy Stochastic.

Hence, the main facets of the paper are briefed as follows:

i. In this study, a bi-objective bi-item solid transportation problem is formulated, in which the first objective represents the transportation cost, the other is the duration of transit in a deterministic environment, and the constraints are considered in a fuzzy stochastic environment.
ii. We extended the transformation technique [30] for the formulated three-dimensional multi-item problem with two objectives. Then, the extended chance-constrained (transformation)

technique is applied to transform the fuzzy stochastic bi-objective bi-item solid transportation problem into its equivalent deterministic problem.

iii. The reduced bi-objective bi-item solid transportation problem cannot be solved explicitly. The entrepreneur is always keen on minimizing the cost of transportation and duration of transit while shipping multiple items to obtain a maximum profit. In some unavoidable situations, minimizing both objectives is impossible. We have to give minimum priority to any one of the objectives. According to the priority of the objective in the problem, a weight proportion has to be provided to it. Therefore, the weights and priority ranking are used for goal functions in our proposed method, namely, the global weighted sum method. Using these factors with the objective via the global weighted sum method can help the entrepreneur to balance the situation.

iv. The proposed method is used to transform the deterministic bi-objective bi-item solid transportation problem into the single-objective problem. Our method can also be used to transform any multi-objective optimization problem into a single objective problem. The transformed single objective problem is solved using Lingo 18.0 software to obtain the optimal compromise solution of the BOBISTP(bi-objective bi-item solid transportation problem) under a fuzzy stochastic environment.

The rest of the paper is arranged as follows: Essential definitions and theorems are provided in Section 2. The bi-objective bi-item solid transportation problem with fuzzy stochastic constraint is mathematically formulated in Section 3, and its equivalence deterministic model is explained in Section 4. The proposed global weighted sum method and working methodology are presented in Section 5. The numerical example for Model 1 is discussed, and it is compared with other existing approaches along with some managerial implications in Section 6. The sensitivity analysis and discussion of the suggested research are covered in Section 7. Finally, the conclusions and directions for future study are given.

2. Preliminaries

Basic definitions of fuzzy sets, fuzzy numbers, alpha cut of a fuzzy number and triangular fuzzy numbers can be referred to in [42] (Zadeh, 1965), and we have presented some necessary definitions and notations related to the theory of uncertainty.

Definition 2.1. (Triangular fuzzy number, Zadeh, [42])

A fuzzy number $\tilde{R}$ is denoted as a triangular fuzzy number by (r_1, r_2, r_3) where r_1, r_2 and r_3 are real numbers, and its membership function $\mu_{\tilde{R}}(x)$ is given below.

$$\boldsymbol{\mu_{\tilde{R}}(x)} = \begin{cases} \dfrac{\boldsymbol{x - r_1}}{\boldsymbol{r_2 - r_1}} & \boldsymbol{r_1 \le x \le r_2}, \\ \dfrac{\boldsymbol{x - r_3}}{\boldsymbol{r_2 - r_3}} & \boldsymbol{r_2 \le x \le r_3}, \\ \boldsymbol{0} & \textbf{otherwise}. \end{cases}$$

Definition 2.2. ([42] Zadeh, 1965) The deterministic interval by α-cut operation, interval R_α can be attained as follows $\forall\, \alpha \in [0,1]$. Thus, $A_\alpha = [r_1 + (r_2 - r_1)\alpha, r_3 - (r_3 - r_2)\alpha]$.

Example: Let A=0. $\tilde{3}$ = (0.2,0.3,0.4) be a triangular fuzzy number, and then using the α-cut operation, we get $(0.2,0.3,0.4) = (0.2 + 0.1\alpha, 0.4 - 0.1\alpha)$.

Definition 2.3. ([43] Nanda and Kar, 1992) Let $\tilde{r} = (r_1, r_2, r_3)$ and $\tilde{s} = (s_1, s_2, s_3)$ be two fuzzy numbers with α-cut $\tilde{r}[\alpha] = (r_*, r^*)$ and $\tilde{s}[\alpha] = (s_*, s^*)$, respectively. Then, $\tilde{t} \le \tilde{s}$ iff $r^* \le s_*$.

Definition 2.4. ([44] Buckley and Eslami, 2004) Let X be a continuous random variable with probability density function f (x, θ), where θ is a parameter describing the density function. Suppose θ is uncertain and estimated from confidence interval, and then θ can be generated as a fuzzy number. If we denote by $\tilde{X}$ a continuous random variable with fuzzy parameter θ and δ as fuzzy probability, then $\tilde{X}$ is said to be a continuous fuzzy random variable with density f $(x, \tilde{\theta})$, P($\tilde{X} \leq$ x)= $\tilde{\theta}$.

Let $\tilde{X}$= $[\tilde{p};\ \tilde{q}]$ be an event. Then, the probability of the event δ of continuous FRV (fuzzy random variable) on X is a fuzzy number whose α-cut is defined as follows:

$$\tilde{P}(\tilde{p} \leq \tilde{X} \leq \tilde{q})[\alpha]=\begin{pmatrix} min:\{\int_c^d f(x,\theta)\,dx/\tilde{\theta}[\alpha], \int_{-\infty}^{\infty} f(x,\theta)\,dx = 1\}, \\ max: \int_c^d f(x,\theta)\,dx/\tilde{\theta}[\alpha], \int_{-\infty}^{\infty} f(x,\theta)\,dx = 1\} \end{pmatrix} = (\delta_*[\alpha], \delta^*[\alpha])$$

Theorem 2.1. ([45] Nanda et al., 2008) Let $\tilde{Z} = pX + qY$ be the linear combination of the FRVs X and Y, which is also a FRV, whose mean and variance are fuzzy numbers.

Theorem 2.2. ([30] Acharyaet al.,2014) If $\tilde{a}_i$, $i=1,2,....m,$ are independent fuzzy random variable (RV) distributed normally, then $\tilde{P}(\sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i) \geq \tilde{\beta}_i$ s equivalent to $\sum_{j=1}^{n} x_{ij} \leq \phi^{-1}(1-\beta_i^*)\sigma_{*a_i} + \mu_{*a_i}$, and $\tilde{\mu}_{a_i}$and $\tilde{\sigma}_{a_i}$ are mean and variances of $\tilde{a}_i$ which follow normal distributions.

Theorem 2.3. ([30] Acharyaet al., 2014) If $\tilde{b}_j$, j=1,2,….n, are independent fuzzy RV distributed normally, then $\tilde{P}(\sum_{i=1}^{m} x_{ij} \geq \tilde{b}_j) \geq \tilde{\gamma}_j$ is equivalent to $\sum_{i=1}^{m} x_{ij} \geq \phi^{-1}(\gamma_j^*)\sigma_{b_j}^* + \mu_{b_j}^*$, and $\tilde{\mu}_{b_j}$ and $\tilde{\sigma}_{b_j}$ are the mean and standard deviation (SD) of $\tilde{b}_j$ which follow normal distributions.

3. Mathematical formulation

This section deals with notations, formulation of the BOBISTP and formulation of the BOBISTP with fuzzy stochastic constraints. The notations used in this model are as follows:

3.1. Notations

i, the number of sources (1, 2, . . . , m),
j, the number of destinations (1, 2, . . . , n),
k, the number of conveyances (1, 2, . . ., s),
p, the number of items (1, 2).

c_{ijk}^p is the cost for transporting one unit of item p from source *i* to destination *j* by conveyance k,

t_{ijk}^p is the duration of transit for transporting one unit of item p from source *i* to destination j by conveyance k,

$\tilde{a}^p{}_i$ is the fuzzy availability of products with item p at the source *i*.

$\tilde{b}^p{}_j$ is the fuzzy demand of products with item p at the destination j,

$\tilde{e}_k$ is the fuzzy transportation capacity at the conveyance k,

x_{ijk}^{p} is the number of item p to be transported from source i to destination j with the aid of conveyance k.

$\tilde{\beta}^p$ is the fuzzy probability for supply constraints for item p.

$\tilde{\gamma}^p{}_j$ is the fuzzy probability for demand constraints for item p.

$\tilde{\eta}_k$ is the fuzzy probability for conveyance constraints for item p.

$\tilde{\mu}_{a_i^p}$ is the mean of supply constraints for item p.

$\tilde{\sigma}_{a_i^p}$ is the standard deviation of supply constraints for item p**.**

$\tilde{\mu}_{b_j^p}$ is the mean of demand constraints for item p.

$\tilde{\sigma}_{b_j^p}$ is the standard deviation of demand constraints for item p**.**

$\tilde{\mu}_{e_k}$ is the mean of conveyance constraints.

$\tilde{\sigma}_{e_k}$ is the standard deviation of conveyance constraints**.**

3.2. Formulation for bi-objective bi-item solid transportation problem (BOBISTP)

In real-world DMPs, there are several competing objectives to be considered and optimized at the same time. The objectives may be considered as minimizing the total transportation cost, minimizing the total transportation duration, minimizing the deterioration of goods, minimizing the total loss during transportation, etc. In this study, we have formulated a STP with two objectives, where one of the objectives is considered as z_1, minimizing the total transportation cost, and the other is z_2, minimizing the duration of transit in which two distinct items are shipped from the i^{th} source to the j^{th} destination by the k^{th} conveyance. Now, the mathematical model for the bi-objective bi-item solid transportation problem is as follows.

$$\text{(Q)}\quad \text{Minimize } z_1 = \sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} c_{ijk}^{p}\, x_{ijk}^{p}, \tag{1}$$

$$\text{Minimize } z_2 = \sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} t_{ijk}^{p}\, x_{jk}^{p}, \tag{2}$$

subject to the constraints.

If the total quantity carried from source i is no more than a_i^p, we obtain

$$\textstyle\sum_{j=1}^{n}\sum_{k=1}^{s} x_{ijk}^{p} \le a_i^p \,, i = 1,2,\ldots,m\,, \quad p = 1,2. \tag{3}$$

If the total quantity carried from source i should meet the b_j^p demand of destination j, we obtain

$$\sum_{i=1}^{m}\sum_{k=1}^{s} x_{ijk}^{p} \geq b_j^p \;, j = 1,2,\ldots,n, \quad p = 1,2. \tag{4}$$

If the total quantity carried by conveyance k is not more than its transportation capacity e_k, we obtain

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} x_{ijk}^{p} \leq e_k \;, k = 1,2,\ldots,s; \quad p = 1,2. \tag{5}$$

$$x_{ijk}^{p} \geq 0, \text{ for all } i \text{ , } j \text{ and } k. \tag{6}$$

3.3. Formulation for BOBISTP with fuzzy stochastic constraints

Modeling real-world problems requires data as input parameters which include information represented in the state of uncertainty. This introduces some uncertainty in the problem which can be dealt with in any of the four ways: (i) fuzzy, (ii) interval, (iii) stochastic and (iv) both fuzzy and stochastic together. The parameters uncertain in both fuzzy and stochastic environment are called fuzzy stochastic parameters. In this study, parameters in the constraints are all considered as fuzzy stochastic parameters. These parameters are assumed to follow any of the probability distributions, where in our formulation we follow a Normal distribution.

The mathematical model for BOBISTP with fuzzy stochastic inequality constraints is given as follows:

$$\text{(R)Minimize } z_1 = \sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} c_{ijk}^{p}\, x_{ijk}^{p},$$

$$\text{Minimize } z_2 = \sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} t_{ijk}^{p}\, x_{ijk}^{p},$$

subject to the constraints.

$$P(\textstyle\sum_{j=1}^{n}\sum_{k=1}^{s} x_{ijk}^{p} \leq \tilde{a}^{p}{}_{i}) \geq \tilde{\beta}^{p}{}_{i}, i = 1,2,\ldots,m \;, p = 1,2. \tag{7}$$

$$P(\textstyle\sum_{i=1}^{m}\sum_{k=1}^{s} x_{ijk}^{p} \geq \tilde{b}_j^p) \geq \tilde{\gamma}^{p}{}_{j}, j = 1,2,\ldots,n, \quad p = 1,2. \tag{8}$$

$$P(\textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} x_{ijk}^{p} \leq \tilde{e}_k) \geq \tilde{\eta}_k, k = 1,2,\ldots,s. \tag{9}$$

$$x_{ijk}^{p} \geq 0, \text{ for all } i \text{ , } j \text{ and } k. \tag{10}$$

The above DMPs with fuzzy stochastic constraints will consume more time to compute the objectives subject to the constraints directly. To reduce the time and simplify the calculation process, transformation of uncertain data or parameters into deterministic data or parameters is required, and it is supported by uncertainty theory. The extended transformation technique for bi-item bi-objective fuzzy stochastic constraints to the equivalent deterministic bi-item bi-objective constraints is discussed in the next section.

4. Equivalent deterministic formulation

An equivalent deterministic model for the fuzzy stochastic problem is discussed in this section. The parameters of the constraints with bi-item in the mathematical model (7–10) are fuzzy stochastic. These constraints cannot be solved directly, as it consumes more time. Therefore, the uncertainty is to be removed using the extended chance-constraint programming technique following a normal distribution. This method permits violations of the constraints up to a predefined probability level. Sometimes, all the parameters in the three-dimensional transportation problem are uncertain, in situations such as lack of information, fluctuation in market value and so on. In rare cases, supply at the origin may be uncertain due to unavoidable delays in production and so on. Uncertainty in demand can occur due to bad weather conditions and so on. Similarly, uncertainness can also occur in conveyances. Due to the aforementioned reasons, the problem (R) is categorized into four models, which are stated as follows:

Model 1 (M1): Problem (R) is considered as problem (M1) where supplies $\tilde{a}^{p}{}_{i}$, $(i = 1, 2 \ldots m)$ $(p = 1,2)$, demands $\tilde{b}^{p}{}_{j}$,$(j = 1, 2 \ldots n)$ $(p = 1,2)$, and conveyances $\tilde{e}_{k}$ *(k=1, 2. . .s)* follow ND.

Model 2 (M2): In problem (R) only supply availabilities $\tilde{a}^{p}{}_{i}, (i = 1, 2 \ldots m)$ $(p = 1,2)$, follow ND.

Model 3 (M3): In problem (R) only demand requirements $\tilde{b}^{p}{}_{j}$,$(j = 1, 2 \ldots n)$ $(p = 1,2)$, follow ND.

Model 4 (M4): In problem (R) only conveyance capacities $\tilde{e}_{k}$, $(k = 1, 2 \ldots s)$, follow ND.

In the above four models, all or any one of the constraints are/is fuzzy stochastic. These models consume more time to compute the objectives subject to the constraints directly. To overcome this, the following theorems are required. The theorem is for transforming the fuzzy stochastic constraint(s) to deterministic constraint(s) of a bi-objective bi-item solid transportation problem, which is stated as follows.

Theorem 4.1. If $\tilde{a}^{p}{}_{i}$, *i=1,2,….,m, p=1,2*, are independent fuzzy RV distributed normally, then $\tilde{P}(\sum_{j=1}^{n}\sum_{k=1}^{s} x_{ijk}^{p} \leq \tilde{a}^{p}{}_{i}) \geq \tilde{\beta}^{p}{}_{i}$ is equivalent to $\sum_{j=1}^{n}\sum_{k=1}^{s} x_{ijk}^{p} \leq \phi^{-1}(1-\beta_{i}^{*p})\sigma_{*a_{i}^{p}} + \mu_{*a_{i}^{p}}$, $\tilde{\mu}_{a_{i}^{p}}$and $\tilde{\sigma}_{a_{i}^{p}}$ are the mean and standard deviation (SD) of $\tilde{a}^{p}{}_{i}$ which follow ND. $\tilde{\beta}^{p}$ is the fuzzy probability for supply constraints for item p.

Proof.

It is assumed that $\tilde{a}^{p}{}_{i} : i = 1,2,\ldots,m\,, p = 1,2$, are independent RVs which follow ND with mean (location parameter) and standard deviation (scale parameter), respectively.

Let us consider the fuzzy stochastic supply constraint (7) as $\tilde{P}\left(\sum_{j=1}^{n}\sum_{k=1}^{s} x_{ijk}^{p} \leq \tilde{a}^{p}{}_{i}\right) \geq \tilde{\beta}^{p}{}_{i}, i =$

$1,2,\ldots,m\,,\quad p=1,2.$

Using Definition 2.2, the alpha cut representation for (7) is

$$\tilde{P}\left(\sum_{j=1}^{n}\sum_{k=1}^{s}x_{ijk}^{p}\leq \tilde{a}^{p}{}_{i}\,[\alpha]\right)\geq \tilde{\beta}_{i}{}^{p}[\alpha] \tag{11}$$

$$\Rightarrow \tilde{P}(A_i\leq \tilde{a}^{p}{}_{i}\,[\alpha])\geq \tilde{\beta}_{i}{}^{p}[\alpha],\, where\; A_i=\sum_{j=1}^{n}\sum_{k=1}^{s}x_{ijk}^{p}$$

$$\Rightarrow \{P(A_i\leq a_i^p\,)\geq \beta_i{}^{p}/\,a_i^p\in \tilde{a}_i^p[\alpha],\beta_i{}^{p}\in \tilde{\beta}_{i}{}^{p}[\alpha]\}$$

$$\Rightarrow \{(1-P(a_i{}^{p}\leq A_i))\geq \beta_i{}^{p}/\,a_i{}^{p}\in \tilde{a}_i{}^{p}[\alpha],\beta_i{}^{p}\in \tilde{\beta}_{i}{}^{p}[\alpha]\} \tag{12}$$

$$\Rightarrow \{1-P\left(\frac{a_i^p-\mu_{a_i^p}}{\sigma_{a_i^p}}\leq \frac{A_i-\mu_{a_i^p}}{\sigma_{a_i^p}}\right)\geq \beta_i{}^{p}/a_i^p\in \tilde{a}_i^p[\alpha],\mu_{a^{p}{}_{i}}\in \tilde{\mu}_{a^{p}{}_{i}}[\alpha],\sigma_{a^{p}{}_{i}}\in \tilde{\sigma}_{a^{p}{}_{i}}[\alpha],\beta_i{}^{p}\in \tilde{\beta}_{i}{}^{p}[\alpha]\}.$$

We know that $P(\,z\leq x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{\left(\frac{-(z)^2}{2}\right)}dz=\phi(x)$ is the cumulative distribution function of a one-dimensional standard normal variable. Hence, the probabilistic constraint (12) can be presented as

$$\Rightarrow \{1-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{A_i-\mu_{a_i^p}}{\sigma_{a_i^p}}}e^{\left(\frac{-\left(\frac{a_i^p-\mu_{a_i^p}}{\sigma_{a_i^p}}\right)^2}{2}\right)}dz\geq \beta_i{}^{p}/a_i^p\in \tilde{a}_i^p[\alpha],\mu_{a_i^p}\in \tilde{\mu}_{a_i^p}[\alpha],\sigma_{a_i^p}\in \tilde{\sigma}_{a_i^p}[\alpha],\beta_i{}^{p}\in \tilde{\beta}_{i}{}^{p}[\alpha]\} \tag{13}$$

$$\Rightarrow \{1-\phi\left(\frac{A_i-\mu_{a_i^p}}{\sigma_{a_i^p}}\right)\geq \beta_i{}^{p}/a_i^p\in \tilde{a}_i^p[\alpha],\mu_{a_i^p}\in \tilde{\mu}_{a_i^p}[\alpha],\sigma_{a_i^p}\in \tilde{\sigma}_{a_i^p}[\alpha],\beta_i{}^{p}\in \tilde{\beta}_{i}{}^{p}[\alpha]\}.$$

By Definition 2.4, the fuzzy RV on the alpha cut is represented as lower and upper bounds. Then, $\tilde{a}_i{}^{p}[\alpha]=[a_{i\;*}^{p},a_i^{p^*}],\tilde{\mu}_{a_i^p}[\alpha]=[\mu_{*\,a_i^p},\mu^*{}_{a_i^p}],\tilde{\sigma}_{a_i^p}[\alpha]=[\sigma_{*\,a_i^p},\sigma^*{}_{a_i^p}],\tilde{\beta}_{i}{}^{p}[\alpha]=[\beta_{i\;*}^{p},\beta_i^{p^*}].$

For the minimization problem, the random fuzzy number should reach its minimum value. Then, the supply parameter of (11), the supplies ($\tilde{a}^{p}{}_{i}\,[\alpha]$) are replaced by its lower bound and its probability value ($\tilde{\beta}_{i}{}^{p}[\alpha]$) in upper bound. Then, the inequality constraint is transformed as

$$min\colon\left\{1-\phi\left(\frac{A_i-\mu_{a_i^p}}{\sigma_{a_i^p}}\right)\geq \beta_i^p\right\}=\left\{1-\phi\left(\frac{A_i-\mu_{a_{i\,*}^p}}{\sigma_{a_{i\,*}^p}}\right)\geq \beta_i^{*p}\right\}. \tag{14}$$

Similarly, for the maximization problem, the fuzzy number should reach its maximum value. Then, the supply parameter $\tilde{a}^p{}_i\,[\alpha]$ and its probability value $\tilde{\beta}_i{}^p[\alpha]$ are replaced by its upper bound. Then, the inequality constraint is transformed as

$$max:\left\{1-\phi\left(\frac{A_i-\mu_{a_i^p}}{\sigma_{a_i^p}}\right)\geq\beta_i^p\right\}=\left\{1-\phi\left(\frac{A_i-\mu^*{}_{a_i^p}}{\sigma^*{}_{a_i^p}}\right)\geq\beta_i^{*p}\right\}. \tag{15}$$

Now, the supply constraint for the minimization problem (14) is

$$1-\phi\left(\frac{A_i-\mu_{*a_i^p}}{\sigma_{*a_i^p}}\right)\geq\beta_i^{*p},$$

On rearranging, we obtain

$$\Rightarrow\phi\left(\frac{A_i-\mu_{*a_i^p}}{\sigma_{*a_i^p}}\right)\leq 1-\beta_i^{*p}$$

$$\Rightarrow\left(\frac{A_i-\mu_{*a_i^p}}{\sigma_{*a_i^p}}\right)\leq\phi^{-1}(1-\beta_i^{*p})$$

$$\Rightarrow A_i\leq\phi^{-1}(1-\beta_i^{*p})\sigma_{*a_i^p}+\mu_{*a_i^p}.$$

Thus, the probabilistic constraints (7) can be transformed into a deterministic supply constraint as

$$\sum_{j=1}^n\sum_{k=1}^s x_{ijk}^p\leq\phi^{-1}(1-\beta_i^{*p})\sigma_{*a_i^p}+\mu_{*a_i^p}. \tag{16}$$

Here, we use Z-Score Percentile ND Table to find the $\phi^{-1}(1-\beta_i^{*p})$.

Similarly, we can prove for the maximization problem.

Theorem 4.2. If $\tilde{b}^p{}_j$, j=1,2,…,n, p=1,2, are independent fuzzy RV distributed normally, then $\tilde{P}(\sum_{i=1}^m\sum_{k=1}^s x_{ijk}^p\geq\tilde{b}^p{}_j)\geq\tilde{\gamma}^p{}_j$ is equivalent to $\sum_{i=1}^m\sum_{k=1}^s x_{ijk}^p\geq\phi^{-1}(\gamma_j^{*p})\sigma_{b_j^p}^*+\mu_{b_j^p}^*$, and $\tilde{\mu}_{b_j^p}$ and $\tilde{\sigma}_{b_j^p}$ are the mean and SD of $\tilde{b}^p{}_j$ which follow ND. $\tilde{\gamma}^p{}_j$ is the fuzzy probability for demand constraints for item p.

Proof.

It is assumed that $\tilde{b}^p{}_j: j=1,2,\ldots,n, p=1,2,$ are independent RVs which follow ND with mean (location parameter) and standard deviation (scale parameter), respectively.

Let us consider the fuzzy stochastic demand constraint (8) as $\tilde{P}(\sum_{i=1}^m\sum_{k=1}^s x_{ijk}^p\geq\tilde{b}^p{}_j)\geq$

$\tilde{\gamma}^{p}{}_{j}, j = 1,2,\dots,n\,, p = 1,2.$

Using Definition 2.2, the alpha cut representation of (8) is

$$\tilde{P}\left(\sum_{i=1}^{m}\sum_{k=1}^{s} x_{ijk}^{p} \geq \tilde{b}^{p}{}_{j}\,[\alpha]\right) \geq \tilde{\gamma}_{j}{}^{p}[\alpha] \tag{17}$$

$$\Rightarrow \tilde{P}(B_j \geq \tilde{b}^{p}{}_{j}\,[\alpha]) \geq \tilde{\gamma}_{j}{}^{p}[\alpha], where\ B_j = \sum_{i=1}^{m}\sum_{k=1}^{s} x_{ijk}^{p}$$

$$\Rightarrow \{P(B_j \geq b_j^p) \geq \gamma_j^p / b_j^p\ \in \tilde{b}^{p}{}_{j}\,[\alpha], \gamma_j{}^{p} \in \tilde{\gamma}_j{}^{p}[\alpha]\}$$

$$\Rightarrow \{P(b_j^p \leq B_j) \geq \gamma_j{}^{p} / b^{p}{}_{j}\ \in \tilde{b}^{p}{}_{j}\,[\alpha], \gamma_j^p \in \tilde{\gamma}_j{}^{p}[\alpha]\}$$

$$\Rightarrow \{P\left(\frac{b_j^p - \mu_{b_j^p}}{\sigma_{b_j^p}} \leq \frac{B_j - \mu_{b_j^p}}{\sigma_{b_j^p}}\right) \geq \gamma_j^p / b_j^p \in \tilde{b}^{p}{}_{j}\,[\alpha], \mu_{b_j}^p \in \tilde{\mu}_{b_j}^p[\alpha], \sigma_{b_j}^p \in \tilde{\sigma}_{b_j}^p[\alpha], \gamma_j^p \in \tilde{\gamma}_j^p[\alpha]\}.$$

Hence, the probabilistic constraint can be presented as

$$\Rightarrow \{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{B_j-\mu_{b_j^p}}{\sigma_{b_j^p}}} e^{\left(\frac{-\left(\frac{b_j^p-\mu_{b_j^p}}{\sigma_{b_j^p}}\right)^2}{2}\right)} dz \geq \gamma_j{}^{p} / b_j^p \in \tilde{b}^{p}{}_{j}[\alpha], \mu_{b_j^p} \in \tilde{\mu}_{b_j}{}^{p}[\alpha], \sigma_{b_j^p} \in \tilde{\sigma}_{b_j^p}[\alpha], \gamma_j^p \in \tilde{\gamma}_j^p[\alpha]\}.$$

The above integral can be expressed as

$$\Rightarrow \left\{\phi\left(\frac{B_j - \mu_{b^{p}{}_{j}}}{\sigma_{b^{p}{}_{j}}}\right) \geq \gamma_j{}^{p}, b_j^p \in \tilde{b}^{p}{}_{j}[\alpha], \mu_{b_j^p} \in \tilde{\mu}_{b_j}{}^{p}[\alpha], \sigma_{b_j^p} \in \tilde{\sigma}_{b_j^p}[\alpha], \gamma_j^p \in \tilde{\gamma}_j^p[\alpha]\right\}. \tag{18}$$

By Definition 2.4, the alpha cut of $\tilde{b}_j{}^{p}[\alpha] = [b_{j*}{}^{p}, b_j{}^{p*}]$, $\tilde{\mu}_{b_j{}^p}[\alpha] = \left[\mu_{*b_j{}^p}, \mu_{b_j^p}^*\right]$, $\tilde{\sigma}_{b_j{}^p}[\alpha] = \left[\sigma_{*b_j{}^p}, \sigma_{b_j^p}^*\right], \tilde{\gamma}_j{}^{p}[\alpha] = [\gamma_{*j}^{p}, \gamma_j^{*p}]$.

For the minimization problem, the random fuzzy number should reach its minimum value. Therefore, both the parameters of demand constraint (17) that is requirement ($\tilde{b}_j{}^{p}[\alpha]$) and its probability value$\left(\tilde{\gamma}_j{}^{p}[\alpha]\right)$ are replaced by their upper bound. Then, the inequality constraint is transformed as

$$min:\left\{\phi(\frac{B_j-\mu_{b_j{}^p}}{\sigma_{b_j{}^p}})\geq \gamma_j{}^p\right\}=\left\{\phi(\frac{B_j-\mu^*_{b_j^p}}{\sigma^*_{b_j^p}})\geq \gamma_j^{*p}\right\}. \tag{19}$$

Similarly, for the maximization problem, the fuzzy number should reach its maximum value. Then, the demand parameter of (17), the requirement ($\tilde{b}_j{}^p[\alpha]$) is replaced by its lower bound, and its probability value $(\tilde{\gamma}_j{}^p[\alpha])$ is replaced by its upper bound. Then, the inequality constraint is transformed as

$$max:\left\{\phi\left(\frac{B_j-\mu_{b_j{}^p}}{\sigma_{b_j{}^p}}\right)\geq \gamma_j^p\right\}=\left\{\phi\left(\frac{B_j-\mu_{*b_j{}^p}}{\sigma_{*b_j{}^p}}\right)\right\}\geq \gamma_j^{*p}. \tag{20}$$

Now, the demand constraint for the minimization problem (19) is

$$\phi\left(\frac{B_j-\mu^*_{b_j{}^p}}{\sigma^*_{b_j^p}}\right)\geq \gamma_j^{*p}$$

$$\Rightarrow \phi\left(\frac{B_j-\mu^*_{b_j{}^p}}{\sigma^*_{b_j^p}}\right)\geq \phi^{-1}(\gamma_j^{*p})$$

$$\Rightarrow B_j\geq \phi^{-1}(\gamma_j^{*p})\sigma^*_{b_j^p}+\mu^*_{b_j^p}$$

$$\textstyle\sum_{i=1}^m\sum_{k=1}^s x_{ijk}^p\geq \phi^{-1}(\gamma_j^{*p})\sigma^*_{b_j^p}+\mu^*_{b_j^p}. \tag{21}$$

Theorem 4.3. If $\tilde{e}_k$, k=1,2,….,s, are independent fuzzy RV distributed normally, then $P(\sum_{i=1}^m\sum_{j=1}^n\sum_{p=1}^2 x_{ijk}^p\leq \tilde{e}_k)\geq \eta_k, k=1,2,\dots,s$ is equivalent to $\sum_{i=1}^m\sum_{j=1}^n\sum_{p=1}^2 x_{ijk}^p\leq \phi^{-1}(1-\eta_k^*)\sigma_{*e_k}+\mu_{*e_k}$, and $\tilde{\mu}_{e_k}$and $\tilde{\sigma}_{e_k}$ are the mean and SD of $\tilde{e}_k$which follow ND. $\tilde{\eta}_k$ is the fuzzy probability for conveyance constraints for item p.

Proof.

The proof is similar to that of Theorem (4.1).

$$\text{Thus, } \textstyle\sum_{i=1}^m\sum_{j=1}^n\sum_{p=1}^2 x_{ijk}^p\leq \phi^{-1}(1-\eta_k^*)\sigma_{*e_k}+\mu_{*e_k}. \tag{22}$$

The equivalent deterministic model of the problem is established using Theorems (4.1)–(4.3), in which all resource parameters follow NDs with known mean and SD. Construct the following 4 Models as deterministic models.

***Model 1**: Supply* $\tilde{a}^p{}_i$, $(i=1,2\dots,m)$ $(p=1,2)$, *demand* $\tilde{b}^p{}_j$, $(j=1,2\dots,n)$ $(p=1,2)$ *and conveyance* $\tilde{e}_k$, $(k=1,2\dots,s)$ *follow ND.*

In Model 1, $\tilde{a}^p{}_i,\tilde{b}^p{}_j$ and $\tilde{e}_k$are uncertain values. By using results of Theorems (4.1)–(4.3), convert the fuzzy stochastic supply, demand and conveyance constraint into an equivalent deterministic constraint. Now, the problem (P_1) is constructed with Eqs (1), (2), (16), (21), (22) and (6) and is formulated as follows.

$$\text{(P1)Minimize } z_1 = \sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} c_{ijk}^{p}\, x_{ijk}^{p},$$

$$\text{Minimize } z_2 = \sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} t_{ijk}^{p}\, x_{ijk}^{p},$$

subject to the constraints

$$\sum_{j=1}^{n}\sum_{k=1}^{s} x_{ijk}^{p} \le \phi^{-1}(1-\beta_i^{*p})\sigma_{*a_i^p} + \mu_{*a_i^p}, i = 1,2,\dots,m, p = 1,2$$

$$\sum_{i=1}^{m}\sum_{k=1}^{s} x_{ijk}^{p} \ge \phi^{-1}(\gamma_j^{*p})\sigma^{*}_{b_j^p} + \mu^{*}_{b_j^p}, j = 1,2,\dots,n, \quad p = 1,2$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{l} x_{ijk}^{p} \le \phi^{-1}(1-\eta_k^{*})\sigma_{*e_k} + \mu_{*e_k}, k = 1,2,\dots,s$$

$x_{ijk}^{p} \ge 0$, for all i, j and k.

Model 2: *Only supply availabilities* $\tilde{a}^{p}{}_{i}$, $(i = 1,2\dots m)$ $(p = 1,2)$, *follow ND;* $b^{p}{}_{j}$ *and* e_k *remain precise values.*

In the same manner, construct the problem (P_2) with two objective functions, (1) and (2), subject to the constraints (16), (4), (5) and (6).

Model 3: *Only demand requirements* $\tilde{b}^{p}{}_{j}$, $(j = 1,2\dots n)$ $(p = 1,2)$, *follow ND;* $a^{p}{}_{i}$ *and* e_k *remain precise values.*

Construct the problem (P_3) with two objective functions, (1) and (2), subject to the constraints (21), (3), (5) and (6).

Model 4: *Only conveyance capacities* $\tilde{e}_k$, $(k = 1,2\dots,s)$ $(p = 1,2)$, *follow ND;* $a^{p}{}_{i}$ *and* $b^{p}{}_{j}$ *remain precise values.*

Construct the problem (P_4) with two objective functions, (1) and (2), subject to the constraints (22), (3), (4) and (6).

Now, the reduced deterministic problems (P_1), (P_2), (P_3) and (P_4) that have been achieved involve two objectives. The reduced deterministic problems cannot be solved explicitly. Even though, many methods are available in the literature to convert a multi-objective optimization problem to a single objective optimization problem. In this article, another improved method is proposed, namely, the global weighted sum method, based on the global criteria method and weighted sum method, to reduce the MOOP into the single objective problem, which is discussed in the following section.

5. Global weighted sum method

Many approaches, such as the global criteria approach, goal programming approach, constraint approach, fuzzy programming approach, fuzzy goal programming approach, etc., are available in the literature to transform the MOOP into the single objective problem. In the aforementioned approaches, the range between the ideal solution and the feasible solution to the problem is minimized. It is also acknowledged that all objective functions are equally significant. In some unavoidable situations, minimizing all the objectives is impossible. We have to give minimum priority to any one of the objectives. According to the priority of the objective in the problem, a weight proportion has to be provided to it. Therefore, the weights and priority ranking are used for goal functions in our proposed method. Using these factors with the objective function via the global weighted sum method can help the entrepreneur to balance the situation. The procedure for converting the MOOP to the single objective optimization problem is stated as follows:

Step a: Choosing one objective together with all the constraints at a time, ignoring other objectives, solve the MOOP as a single objective optimization problem to find the optimal solutions, say $(\underline{z}_1, \underline{z}_2, \underline{z}_3, \ldots, \underline{z}_r)$.

Step b: Consider the optimal solutions obtained in Step a as the ideal solutions $(\underline{z}_1, \underline{z}_2, \underline{z}_3, \ldots, \underline{z}_r)$.

Step c: Using Step b, formulate the following auxiliary problem:

$$\text{(G) Minimize } \lambda \tag{23}$$

subject to the constraints (16), (21), (22), (6) and $\lambda \geq 0$.

$$\text{Here } \lambda = \text{Minimize}\left\{\sum_{r=1}^{q} w_r \left(\frac{z_r(x) - \underline{z}_r}{\underline{z}_r}\right)^d\right\}^{\frac{1}{d}}. \tag{24}$$

where $1 \leq d \leq \infty$, the usual value of d is 2, w_r is the weight of the r^th^ objective function satisfying the conditions $\sum_{r=1}^{q} w_r = 1$, and $w_r > 0$. This method is called the global weighted sum method.

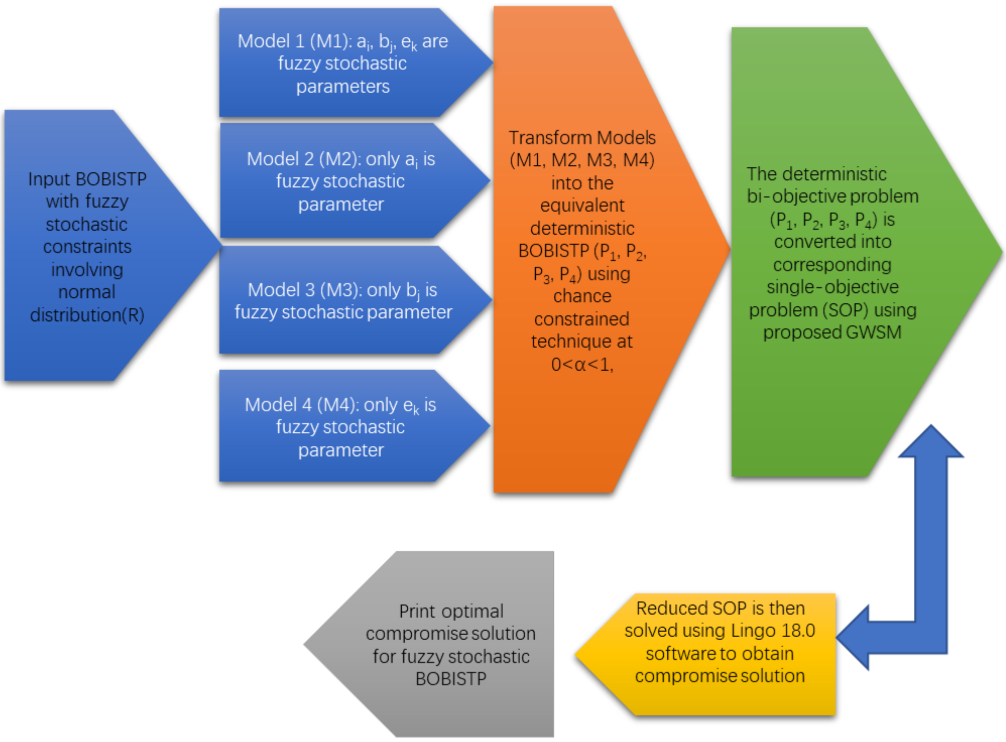

Figure 1. Flow chart for working methodology.

5.1. Working methodology

The working method for solving the BOBISTP with fuzzy stochastic constraints involving normal distribution is presented in the flow chart.

A real-life numerical example is presented for Model 1 to illustrate the model and methods. In the first step, we have to transform the bi-objective bi-item fuzzy stochastic constraint(s) into bi-objective bi-item deterministic constraint(s) using the extended chance constrained technique following normal distribution. Second, we have to transform the bi-objective bi-item solid transportation problem into a single objective BISTP using the proposed global weighted sum method (GWSM). Then, the reduced problem is solved using the LINGO 18.0 software.

6. Numerical example

A chemical company named Bharath Alkalies and Chemicals Ltd. is a fast-growing company. The company is involved in producing chemical products like ammonia (Item 1) and benzene (Item 2). Currently, the company has two production plants, which are in Chennai (A1) and Ranipet (A2). The company with two branches (A1 and A2) supplies two types of chemical products (Item 1 and Item 2) to three states in India, Kerala (D1), Karnataka (D2) and Telangana (D3), through two different conveyances, truck (E1) and train (E2). The administrator of the company plans to transport the goods from the upcoming month. To start his project, he has to collect the primary records related to delivery capacity, demand, total profit, cost of a unit product, duration of transit and so on. However, he is not able to get the necessary data due to uncertain human and natural phenomena as given below. The demands for different products vary from state to state. The manufacturing company has to take responsibility to provide the appropriate conveyances. The supply of products is hindered due to unexpected events at the factory and weather conditions. Rainfall during the planning period is a critical indicator of getting raw material, a crucial resource. The parameters involved in the problem are intrinsically stochastic and labor availability is frequently fuzzy in nature. Thus, for the supplier a_i^p, the probability that the required number of products with item p available is $\tilde{\beta}_i^{\,p}$. The demand for products is uncertain by nature, too. It arises from inaccurate demand forecasting, fluctuating demand or unforeseen delivery delays. Therefore, for cities b_j^p, the probability of the expected demand for item p is $\tilde{\gamma}_j^{\,p}$. Similar to the transportation capacities, traffic jams and roadblocks make them unclear. A probability $\tilde{\eta}_k$ is defined as the probability of two conveyances having available capacities e_k with item p. Relying on prior experience, the company expects the supply, demand and conveyance parameter to follow a fuzzy normal uncertain distribution with known mean and variance. The transportation cost (c_{ijk}) and the duration of transit (t_{ijk}) from source to destination for the conveyance of two items are given in Tables 2 and 3.

In Tables 2 and 3, the fuzzy stochastic constraint parameters are normally distributed as $N(\tilde{\mu}_{a_i^p};\tilde{\sigma}^2{}_{a_i^p}), N(\tilde{\mu}_{b_j^p};\tilde{\sigma}^2{}_{b_j^p}), N(\tilde{\mu}_{e_k};\tilde{\sigma}^2{}_{e_k})$. The entrepreneurs might select the probabilities based on insights. Now, the probabilities of supplies, demands and conveyances are denoted as $\tilde{\beta}_i^{\,p}, \tilde{\gamma}_j^{\,p}, \tilde{\eta}_k$, which are expressed in triangular fuzzy numbers provided in Table 4.

It is essential to think about how to minimize the duration of transit while keeping expenses down. As a result, the entrepreneur seeks to minimize the transportation cost and the duration of transit.

Table 2. c_{ijk} and t_{ijk} for Item 1.

							Capacity $N(\tilde{\mu}_{e_k};\tilde{\sigma}^2{}_{e_k})$
Conveyance	E1		E1		E1		N((49,50,51); (2,3,4))
		E2		E2		E2	N((53,54,55); (3,4,5))
	D1		D2		D3		**Supply** $N(\tilde{\boldsymbol{\mu}}_{a_i};\tilde{\boldsymbol{\sigma}}^2{}_{a_i})$
A1	8(c_{ijk})	12	8	9	13	11	N((24,25,26);
	6(t_{ijk})	8	6	7	10	8	(1,1.5,2))
A2	12	12	8	9	14	17	N((32,33,34);
	9	7	7	10	8	11	(5,6,7))
Demand $N(\tilde{\mu}_{b_j};\tilde{\sigma}^2{}_{b_j})$	N((16,17,18); (5,6,8))		N((20,21,22); (0,1,2))		N((14,15,16); (5,6,7))		

Table 3. c_{ijk} and t_{ijk} for Item 2.

							Capacity $N(\tilde{\mu}_{e_k};\tilde{\sigma}^2{}_{e_k})$
Conveyance	E1		E1		E1		N((49,50,51); (2,3,4))
		E2		E2		E2	N((53,54,55); (3,4,5))
	D1		D2		D3		Supply $N(\tilde{\mu}_{a_i};\tilde{\sigma}^2{}_{a_i})$
A1	11	13	9	8	12	11	N((34,35,36);
	8	9	7	6	12	11	(2,3,4))
A2	14	17	11	12	15	15	N((26,27,28);
	12	9	8	8	10	11	(5,6,7))
Demand $N(\tilde{\mu}_{b_j};\tilde{\sigma}^2{}_{b_j})$	N((23,24,25); (0,1,2))		N((16,17,18); (5,6,8))		N((16,17,18); (1,2,3))		

Table 4. Probabilities of supplies, demands and conveyances.

$\tilde{\beta}_1^1 = \widetilde{\mathbf{0.3}}$=(0.2,0.3,0.4)	$\tilde{\beta}_1^2 = \widetilde{\mathbf{0.2}}$= (0.1,0.2,0.3)	$\tilde{\beta}_2^1 = \widetilde{\mathbf{0.4}}$=(0.3,0.4,0.5)
$\tilde{\beta}_2^2 = \widetilde{\mathbf{0.1}}$= (0,0.1,0.2)	$\tilde{\gamma}_1{}^1$=$\widetilde{0.4}$= (0.4,0.5,0.6)	$\tilde{\gamma}_1{}^2$=$\widetilde{0.3}$=(0.3,0.4,0.5)
$\tilde{\gamma}_2{}^2 = \widetilde{\mathbf{0.5}}$= (0.4,0.5,0.6)	$\tilde{\gamma}_3{}^1 = \widetilde{0.6}$= (0.5,0.6,0.7)	$\tilde{\gamma}_3{}^2$=$\widetilde{0.7}$=(0.6,0.7,0.8)
$\tilde{\eta}_1$=$\widetilde{\mathbf{0.4}}$= (0.4,0.5,0.6)	$\tilde{\eta}_2$=$\widetilde{0.7}$= (0.7,0.8,0.9)	

Step 1a: Formulate the left-hand side alpha cut representation for bi-objective bi-item fuzzy stochastic constraints using Eqs (11) and (17) as shown in Tables 5 and 6.

Table 5. Alpha cut representation for Item 1.

							Capacity $\boldsymbol{N(\tilde{\mu}_{e_k};\tilde{\sigma}^2{}_{e_k})}$
Conveyance	E1		E1		E1		$N([49+\alpha,5-\alpha];$ $[2+\alpha,4-\alpha])$
		E2		E2		E2	$N([54+\alpha,5-\alpha];$ $[3+\alpha,5-\alpha])$
	D1		D2		D3		**Supply** $\boldsymbol{N(\tilde{\mu}_{a_i};\tilde{\sigma}^2{}_{a_i})}$
A1	8	12	8	9	13	11	$N([24+\alpha,2-\alpha];$
	6	8	6	7	10	8	$[1+0.5\alpha,-0.5\alpha])$
A2	12	12	8	9	14	17	$N([32+\alpha,3-\alpha];$
	9	7	7	10	8	11	$[5+\alpha,7-\alpha])$
Demand $N(\tilde{\mu}_{b_j};\tilde{\sigma}^2{}_{b_j})$	$N([16+\alpha,18-\alpha];$ $[5+\alpha,8-2\alpha])$		$N([20+\alpha,22-\alpha];$ $[0+\alpha,2-\alpha])$		$N([14+\alpha,16-\alpha];$ $[5+\alpha,7-\alpha])$		

Table 6. Alpha cut representation for Item 2.

							Capacity $N(\tilde{\mu}_{e_k};\tilde{\sigma}^2{}_{e_k})$
Conveyance	E1		E1		E1		$N([49+\alpha,51$ $-\alpha];$ $[2+\alpha,4-\alpha])$
		E2		E2		E2	$N([54+\alpha,56$ $-\alpha];$ $[3+\alpha,5-\alpha])$
	D1		D2		D3		**Supply** $\boldsymbol{N(\tilde{\mu}_{a_i};\tilde{\sigma}^2{}_{a_i})}$
A1	11	13	9	8	12	11	$N([34+\alpha,36$
	8	9	7	6	12	11	$-\alpha];$ $[2+\alpha,4-\alpha])$
A2	14	17	11	12	15	15	$N([26+\alpha,28$
	12	9	8	8	10	11	$-\alpha];$ $[5+\alpha,7-\alpha])$
Demand $N(\tilde{\mu}_{b_j};\tilde{\sigma}^2{}_{b_j})$	$N([23+\alpha,2-\alpha];$ $[0+\alpha,2-\alpha])$		$N([16+\alpha,1-\alpha];$ $[5+\alpha,8-2\alpha])$		$N([16+\alpha,18-\alpha];$ $[1+\alpha,3-\alpha])$		

Step 1b: Formulate the right-hand side alpha cut representation for Table 4 using Eqs (11) and (17) as shown below.

$\tilde{\beta}_1{}^1 = (0.2 + 0.1\alpha, 0.4 - 0.1\alpha)$ $\tilde{\beta}^2{}_1 = (0.1 + 0.1\alpha, 0.3 - 0.1\alpha)$ $\tilde{\beta}_2{}^1 = (0.3 + 0.1\alpha, 0.5 - 0.1\alpha)$

$\tilde{\beta}^2{}_2 = (0 + 0.1\alpha, 0.2 - 0.1\alpha)$ $\tilde{\gamma}_1{}^1 = (0.4 + 0.1\alpha, 0.6 - 0.1\alpha)$ $\tilde{\gamma}_1{}^2 = (0.3 + 0.1\alpha, 0.5 - 0.1\alpha)$

$\tilde{\gamma}_2{}^1 = (0.6 + 0.1\alpha, 0.8 - 0.1\alpha)$ $\tilde{\gamma}_2{}^2 = (0.4 + 0.1\alpha, 0.6 - 0.1\alpha)$ $\tilde{\gamma}_3{}^1 = (0.5 + 0.1\alpha, 0.7 - 0.1\alpha)$

$\tilde{\gamma}_3{}^2 = (0.6 + 0.1\alpha, 0.8 - 0.1\alpha)$ $\tilde{\eta}_1 = (0.4 + 0.1\alpha, 0.6 - 0.1\alpha)$ $\tilde{\eta}_2 = (0.1 + 0.1\alpha, 0.3 - 0.1\alpha)$

Step 2: Using Eqs (16), (21), (22), we have reduced the problem (R) into problem (P_1) as shown in Tables 7 and 8.

Table 7. (P1) for Item 1.

							Capacity $\phi^{-1}(1-\eta^*{}_k)\sigma_{*e_k} + \mu_{*e_k}$
Conveyance	E1		E1		E1		$= \varphi^{-1}(0.4 + 0.1\alpha) * \sqrt{2+\alpha} + (49+\alpha)$
		E2		E2		E2	$= \varphi^{-1}(0.7 + 0.1\alpha) * \sqrt{3+\alpha} + (54+\alpha)$
	D1		D2		D3		**Supply** $\phi^{-1}(1-\beta^{*p}{}_i)\sigma_{*a^p{}_i} + \mu_{*a^p{}_i}$
A1	8	12	8	9	13	11	$\varphi^{-1}(0.6 + 0.1\alpha) * \sqrt{1+0.5\alpha} + (24+\alpha)$
	6	8	6	7	10	8	
A2	12	12	8	9	14	17	$= \varphi^{-1}(0.5 + 0.1\alpha) * \sqrt{5+\alpha} + (32+\alpha)$
	9	7	7	10	8	11	
Demand $\phi^{-1}\left(\gamma^*{}_j{}^p\right)\sigma^*{}_{b_j^p} + \mu^*{}_{b_j^p}$	$= \varphi^{-1}(0.6 - 0.1\alpha) * \sqrt{8-2\alpha} + (18-\alpha)$		$= \varphi^{-1}(0.8 - 0.1\alpha) * \sqrt{2-\alpha} + (22-\alpha)$		$= \varphi^{-1}(0.7 - 0.1\alpha) * \sqrt{7-\alpha} + (16-\alpha)$		

Table 8. (P1) for Item 2.

							Capacity $\phi^{-1}(1-\eta^*{}_k)\sigma_{*e_k} + \mu_{*e_k}$
Conveyance	E1		E1		E1		$= \varphi^{-1}(0.4 + 0.1\alpha) * \sqrt{2+\alpha} + (49+\alpha)$
		E2		E2		E2	$= \varphi^{-1}(0.7 + 0.1\alpha) * \sqrt{3+\alpha} + (54+\alpha)$
	D1		D2		D3		**Supply** $\phi^{-1}(1-\beta^{*p}{}_i)\sigma_{*a^p{}_i} + \mu_{*a^p{}_i}$
A1	11	13	9	8	12	11	$= \varphi^{-1}(0.7 + 0.1\alpha) * \sqrt{2+\alpha} + (35+\alpha)$
	8	9	7	6	12	11	
A2	14	17	11	12	15	15	$= \varphi^{-1}(0.8 + 0.1\alpha) * \sqrt{5+\alpha} + (26+\alpha)$
	12	9	8	8	10	11	
Demand $\phi^{-1}(\gamma^*{}_j{}^p)\sigma^*{}_{b_j^p} + \mu^*{}_{b_j^p}$	$= \varphi^{-1}(0.5 - 0.1\alpha) * \sqrt{2-\alpha} + (25-\alpha)$		$= \varphi^{-1}(0.6 - 0.1\alpha) * \sqrt{8-2\alpha} + (18-\alpha)$		$= \varphi^{-1}(0.8 - 0.1\alpha) * \sqrt{3-\alpha} + (18-\alpha)$		

Step 3: Choose any alpha value between 0 and 1. Using Theorem 4.1, find ϕ^{-1} by choosing $\alpha = 0.5$. The equivalent deterministic constraints are shown in Tables 9 and 10.

Table 9. Deterministic constraints for Item 1.

							Capacity
Conveyance	E1		E1		E1		49.30
		E2		E2		E2	54.76
	D1		D2		D3		**Supply**
A1	8	12	8	9	13	11	
	6	8	6	7	10	8	24.93
A2	12	12	8	9	14	17	32.8
	9	7	7	10	8	11	
Demand	17.83		22.33		16.48		

Table 10. Deterministic constraints for Item 2.

							Capacity
Conveyance	E1		E1		E1		49.30
		E2		E2		E2	54.76
	D1		D2		D3		**Supply**
A1	11	13	9	8	12	11	35.57
	8	9	7	6	12	11	
A2	14	17	11	12	15	15	28.93
	12	9	8	8	10	11	
Demand	24.35		17.83		18.57		

Step 4: Tables 9 and 10 together are called a bi-objective bi-item solid transportation problem with deterministic constraints (BOBISTP-DC). Check if the BOBISTP-DC is balanced. If not, add a dummy availability or demand or conveyance. The costs of assigning dummy cells are always zero.

Step 5: Transform the balanced BOBISTP-DC into single objective BISTP-DC using the proposed Global weighted sum method (GWSM).

Step 5a: In the balanced BOBISTP-DC, take one objective function at a time subject to the constraints and exclude other objectives. Find the optimal solution to each objective using Lingo 18.0 software. The optimal solutions for first and second objectives are $\underset{\sim}{z}_1 = 970.245$ and $\underset{\sim}{z}_2 = 707.67$. The optimal number of quantities to be produced by the chemical company for z_1 is x_{111}^1=14.205, x_{132}^1=10.725, x_{212}^1=3.625, x_{221}^1=22.33, x_{231}^1=5.755, x_{242}^1=1.09, x_{122}^2=17, x_{132}^2=18.57, x_{211}^2=6.98, x_{213}^2=18.17, x_{221}^2=0.83, x_{242}^2=3.75. For z_2 it is x_{121}^1=22.33, x_{132}^1=2.6, x_{212}^1=17.83, x_{231}^1=13.88, x_{242}^1=1.09, x_{111}^2=13.09, x_{122}^2=17.83, x_{132}^2=0.4, x_{133}^2=4.25, x_{212}^2=11.26, x_{233}^2=13.75, x_{242}^2=3.75.

Step 5b: Consider the optimal solutions $\underset{\sim}{z}_1 = 970.245$ and $\underset{\sim}{z}_2 = 707.67$ obtained in Step 5a as the ideal solutions.

Step 5c: Using Step 5b, formulate the mathematical model (G) with weights 0.5 to each objective as follows:

(G) Minimize λ

subject to the constraints (16), (21), (22), (6) and $\lambda \geq 0$, where $\lambda = \text{Minimize}\left\{0.5\left(\frac{z_1-970.245}{970.245}\right)^2 + 0.5\left(\frac{z_2-707.67}{707.67}\right)^2\right\}^{\frac{1}{2}}$.

Solve the problem (G) using Lingo 18.0 software. The calculation of Lingo 18.0 is performed with an Intel(R) Core (TM) i3-7100U CPU @ 2.40GHz and 4 GB RAM. Then, the optimal number of quantities to be produced by the chemical company is x_{111}^1= 8.62, x_{132}^1= 16.31, x_{212}^1= 9.21, x_{221}^1=22.33, x_{231}^1=0.17, x_{242}^1=1.09, x_{111}^2=17.82, x_{122}^2= 17.75, x_{212}^2=5.09, x_{213}^2=1.44, x_{221}^2=0.029, x_{222}^2=0.053, x_{231}^2=0.33, x_{232}^2=1.5, x_{233}^2=16.72, x_{242}^2=3.75, and the optimal compromise solutions are z_1= 992.9835, z_2= 719.7665, $\lambda = 0.0205$.

6.1. Results and discussion

In this paper, a BOBISTP with fuzzy stochastic constraints involving normal distributions has been constructed. In some situations, such as insufficient information in transportation, fluctuations in market value, etc., treat all the parameters in the BOBISTP as uncertain. In rare cases, uncertainty can occur in any one of the parameters in the problem. Due to this reason, BOBISTP with fuzzy stochastic constraints is categorized into four models. The fact that these models are constructed from different points of view is worth noting. The usage of the models is dependent on the entrepreneur's preference. A real-life example is presented for Model 1, and it is reduced to a single objective BISTP-DC by our proposed methods. The reduced problem is then solved using the Lingo 18.0 software to obtain the optimal compromise solution as z_1 = 992.9835, z_2 = 719.7665, $\lambda = 0.0205$.

If supplies at the origin alone are uncertain, due to unavoidable delays and troubles in the production and so on, then Model 2 is constructed. The same numerical example under section 6 is considered for Model 2 by choosing Item 1 and Item 2 demand values as $b_1^1 = 18$, $b_1^2 = 24$, $b_2^1 = 21$, $b_2^2 = 19$, $b_3^1 = 17$, $b_3^2 = 18$ and conveyance values as $e_1 = 50$, $e_2 = 55$, where all the objective values and supply constraint values remain the same as in Tables 2 and 3. The same procedure which is followed in Model 1 is adopted for solving Model 2 to obtain the optimal allocations and optimal compromise solution which is shown in Table 11. If demands at the destination alone are uncertain, due to fluctuating demands, unforeseen delivery delays and so on, then Model 3 is constructed. The same numerical example is considered for Model 3 by choosing Item 1 and Item 2 supply values as $a_1^1 = 25$, $a_1^2 = 36$, $a_2^1 = 33$, $a_2^2 = 29$ and conveyance values as $e_1 = 50$, $e_2 = 55$, where all the objective values and demand constraint values remain the same as in Tables 2 and 3. The same procedure which is followed in Model 1 is adopted for solving Model 3 to obtain the optimal allocations and optimal compromise solution which is shown in Table 11. If only conveyance capacities are uncertain, due to road blocks, insurgency, land slide, etc. in some routes, then, Model 4 is constructed. The same numerical example is considered for Model 3 by choosing Item 1 and Item 2 supply values as $a_1^1 = 25$, $a_1^2 = 36$, $a_2^1 = 33$, $a_2^2 = 29$ and conveyance values as $e_1 = 50$, $e_2 = 55$, where all the objective values and conveyance constraint values remain the same as in Tables 2 and 3. The same procedure which is followed in Model 1 is adopted for solving Model 4 to obtain the optimal allocations and optimal compromise solution which is shown in Table 11. To assess the performance of the proposed method, our solutions of all models are compared with the global criteria method (GCM) [46] and fuzzy programming approach (FPA) [47]. The solutions of these methods are shown in Table 11.

Table 11. The results of the four models and the compared methods.

Methods	**Model 1**			
	z_1	z_2	λ	Values of Decision variable
Proposed method	992.9835	719.7665	0.0205	x^1_{111}= 8.62, x^1_{132}= 16.31, x^1_{212}= 9.21, x^1_{221}=22.33, x^1_{231}=0.17, x^1_{242}=1.09, x^2_{111}=17.82, x^2_{122}= 17.75, x^2_{212}=5.09, x^2_{213}=1.44, x^2_{221}=0.029, x^2_{222}=0.053, x^2_{231}=0.33, x^2_{232}=1.5, x^2_{233}=16.72, x^2_{242}=3.75
GCM	992.9839	719.77	0.029	x^1_{111}= 8.53, x^1_{132}= 16.39, x^1_{212}= 9.21, x^1_{221}=22.33, x^1_{231}=0.083, x^1_{242}=1.09, x^2_{111}=18.08, x^2_{122}= 17.48, x^2_{212}=5.09, x^2_{213}=1.17, x^2_{221}=0.13, x^2_{222}=0.21, x^2_{231}=0.13, x^2_{232}=1.43, x^2_{233}=16.99, x^2_{242}=3.75
FPA	1044.23	927.68	0.0601	x^1_{111}= 13.29, x^1_{132}= 3.67, x^1_{133}= 6.87, x^1_{142}= 1.09 x^1_{213}=4.53, x^1_{222}=22.33, x^1_{233}=5.93, x^2_{121}=7.98, x^2_{122}= 9,02, x^2_{132}=18.57, x^2_{211}=24.35, x^2_{223}=0.83, x^2_{242}=5.75
	Model 2			
	z_1	z_2	λ	
Proposed method	1006.958	722.92	0.0222	x^1_{111}= 9.46, x^1_{132}= 15.47, x^1_{212}= 8.54, x^1_{221}=21, x^1_{231}=1.53, x^1_{242}=1.73, x^2_{111}=16.98, x^2_{122}= 18.58, x^2_{212}=6.05, x^2_{213}=0.95, x^2_{221}=0.182, x^2_{222}=0.23, x^2_{231}=0.847, x^2_{232}=0.881, x^2_{233}=16.27, x^2_{242}=3.5
GCM	1006.958	722.922	0.031	x^1_{111}= 8.76, x^1_{132}= 16.16, x^1_{212}= 9.23, x^1_{221}=21, x^1_{231}=0.83, x^1_{242}=1.73, x^2_{111}=17.83, x^2_{122}= 17.73, x^2_{212}=5.87, x^2_{213}=0.75, x^2_{221}=0.062, x^2_{222}=0.08, x^2_{231}=1.06, x^2_{232}=0.08, x^2_{233}=16.93, x^2_{242}=3.5
FPA	1103.01	968.67	0.0707	x^1_{111}= 3.388, x^1_{132}= 2.81, x^1_{131}= 15.811, x^1_{133}= 1.188, x^1_{142}= 1.73 x^1_{213}=11.8, x^1_{222}=21, x^1_{242}=2, x^2_{122}=17.57, x^2_{131}= 6.8, x^2_{132}=11.2, x^2_{211}=24, x^2_{223}=1.43, x^2_{242}=3.5
	Model 3			
	z_1	z_2	λ	
Proposed method	993.049	720.5601	0.0202	x^1_{111}= 8.69, x^1_{132}= 16.30, x^1_{212}= 9.13, x^1_{221}=21.33, x^1_{231}=1.78, x^1_{242}=1.36, x^2_{111}=18.27, x^2_{122}= 17.73, x^2_{212}=4.85, x^2_{213}=1.22, x^2_{221}=0.04, x^2_{222}=0.06, x^2_{231}=0.47, x^2_{232}=1.31, x^2_{233}=16.77, x^2_{242}=4.25
GCM	993.050	720.56	0.028	x^1_{111}= 8.61, x^1_{132}= 16.38, x^1_{212}= 9.21, x^1_{221}=22.33, x^1_{231}=0.09, x^1_{242}=1.36, x^2_{111}=18.58, x^2_{122}= 17.41, x^2_{212}=4.87, x^2_{213}=0.89, x^2_{221}=0.162, x^2_{222}=0.25, x^2_{231}=0.21, x^2_{232}=1.24, x^2_{233}=17.10, x^2_{242}=4.25
FPA	1096.84	969.89	0.0702	x^1_{111}= 2.98, x^1_{113}= 4.17, x^1_{131}= 13.72, x^1_{133}= 2.75, x^1_{142}= 1.36 x^1_{213}=10.67, x^1_{222}=22.33, x^2_{122}=17.43, x^2_{131}= 8.94, x^2_{132}=9.63, x^2_{211}=24.35, x^2_{223}=0.4, x^2_{242}=4.25

Continued on next page

Methods	Model 4			
	z_1	z_2	λ	
Proposed method	977.521	705.742	0.019	x_{111}^{1}= 9.37, x_{132}^{1}= 15.62, x_{212}^{1}= 8.62, x_{221}^{1}=21, x_{231}^{1}=1.37, x_{242}^{1}=2, x_{111}^{2}=17.26, x_{122}^{2}= 18.73, x_{212}^{2}=5.05, x_{213}^{2}=1.68, x_{221}^{2}=0.11, x_{222}^{2}=0.15, x_{231}^{2}=0.77, x_{232}^{2}=0.567, x_{233}^{2}=17.25, x^{2}_{242}=4
GCM	977.528	705.745	0.028	x_{111}^{1}= 8.94, x_{132}^{1}= 16.05, x_{212}^{1}= 9.05, x_{221}^{1}=21, x_{231}^{1}=0.94, x_{242}^{1}=2, x_{111}^{2}=17.76, x_{122}^{2}= 18.32, x_{212}^{2}=4.91, x_{213}^{2}=1.4, x_{221}^{2}=0.26, x_{222}^{2}=0.4, x_{231}^{2}=0.46, x_{233}^{2}=17.53, x_{242}^{2}=4
FPA	1086.12	956.12	0.0698	x_{111}^{1}= 2.179, x_{113}^{1}= 3.82, x_{131}^{1}= 14.88, x_{133}^{1}= 2.11, x_{142}^{1}= 2, x_{213}^{1}=12, x_{222}^{1}=21, x_{242}^{1}=2, x_{111}^{2}=17.26, x_{122}^{2}= 18, x_{131}^{2}=8.24, x_{132}^{2}=9.76, x_{211}^{2}=24, x_{223}^{2}=1, x_{242}^{2}=4

From Table 11, it is clear that the optimal compromise (OC) solution of the problem using our proposed method is very much closer to GCM and better than FPA. We can also note that the value of λ slightly deviates from all compared methods. The solutions obtained from all four models prove that the proposed method is a practical method to solve the bi-objective bi-item solid transportation problem with fuzzy stochastic constraints involving normal distribution. It is important to note that diverse different perspectives are used to build these models. Moreover, we cannot declare one Model to be superior to another in the decision-making process since the entrepreneur's preferences determine how the models are being used. The OC solutions for all four models are individually compared with the OC solutions of GCM and FPA, shown graphically in Figure 2. Figure 2 shows that the obtained transportation cost and duration of transit are similar to GCM and better than FPA.

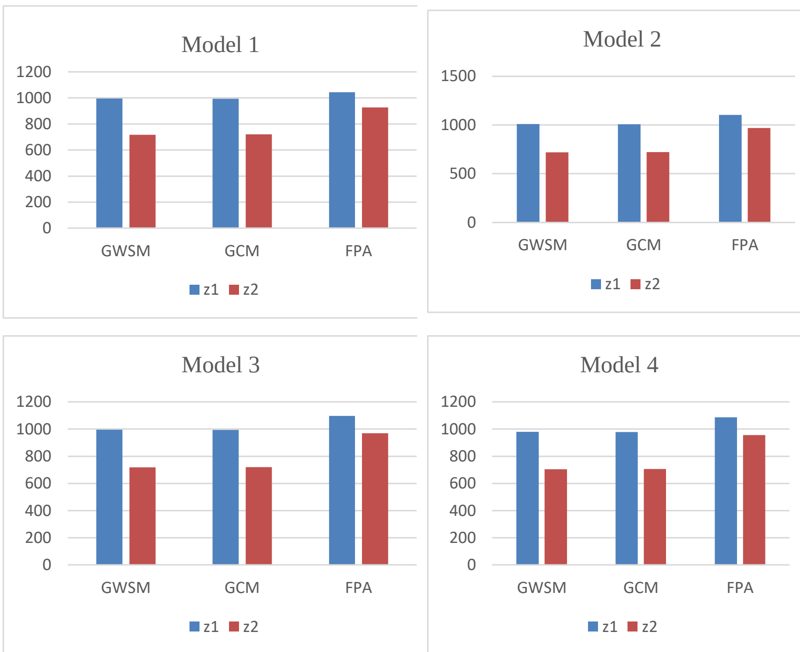

Figure 2. Comparison between existing methods.

6.2. Managerial insights

The managerial implications of this research are as follows:

The bi-objective bi-item item solid transportation problem with fuzzy stochastic constraints plays a vital role in many situations of managerial decision-making problems, such as planning of many complex three-dimensional resource allocation problems in the domain of industrial production, in which all the parameters are of fuzzy stochastic form. In such type of problem, the proposed method helps the entrepreneurs get to know more information about the objective functions and can be easily adjusted to suit the system constraints. Therefore, the entrepreneur is able to make more suitable decisions with the help of the present study.

7. Sensitivity analysis

Sensitivity analysis (SA) is carried out for the optimality in the BOBISTP with fuzzy stochastic constraint in respect of fluctuations in probabilities in uncertain parameters such as source, demand and conveyance. For the SA, we used the problem (M1) with an alpha value of 0.5. The problem (M1) is solved by changing the probability $(\tilde{0} \leq P \leq \tilde{1})$, where P is the probability on a_i^p or b_j^p or e_k. By choosing any two parameters' probabilities as constant at $\widetilde{0.5}$ and the remaining parameter's probability varying between $\tilde{0}$ and $\tilde{1}$, SA solves the problem (M1) with varying probabilities on constraint parameters to obtain an optimal compromise solution.

The probability findings for a_i, b_j and e_k from SA are listed in Tables 12–14. In this SA, Figures 3 and 4 show the graphical representations of the transportation cost and duration of transit in relation to the probability for a_i. With the probability of a_i, the cost and duration of transit are gradually increasing. At $p(a_i) > \widetilde{0.5}$, transportation cost and time are gradually decreasing, as shown in Figures 3 and 4. It is worth mentioning that at a probability value of $\widetilde{0.6}$, transportation cost is sensitive to the variation of the probability of supply availability. A decision-maker can select appropriate probability for supply availability with the aid of this analysis. Likewise, we can determine the SA for b_j. In this analysis, Figures 5 and 6 illustrate how the transportation cost and time fluctuate in relation to the probability of b_j. As the probability of b_j increases, the cost of transportation gradually increases. At $\widetilde{0.5} < p(b_j) < \widetilde{0.6}$, transportation cost stagnates, and then at $p(b_j) > \widetilde{0.6}$ it gradually decreases, as shown in Figure 5. As the probability of b_j increases, the duration of transit gradually increases. At $p(b_j) > \widetilde{0.6}$ it gradually decreases, as shown in Figure 6. It is worth noting that at a probability value from $\widetilde{0.4}$ to $\widetilde{0.6}$, transportation cost and duration of transit are sensitive to the variation of the probability in demand requirements. Sensitivity analysis of the conveyance capacity reveals that there was substantial impact on the cost of transportation and impact on the duration of transit. This is because the capacity of the conveyance has bearing on how quickly the conveyance moves. According to Table 14, when the probability of conveyance capacity increases, the transportation cost and the duration of transit gradually decrease. Figure 7 and Figure 8 demonstrate the sensitivity to the probability for conveyance capacity. Some fascinating patterns can be seen in the SA of the probability. By comprehending the sensitivity patterns of probability for uncertain parameters, entrepreneurs can gain knowledge and the ability to construct the transportation system.

The results of the optimal transportation cost and duration of transit are displayed in Figures 3–8. The SA of BOBISTP with fuzzy stochastic constraint is accomplished by varying the probabilities of constraints in Tables 12–14.

Table 12. SA for varying probability of supplies.

S.no	Fuzzy stochastic supply $(\tilde{\beta}^{p}{}_{i})$	Fuzzy stochastic demand $(\tilde{\gamma}_j^p)$	Fuzzy stochastic conveyance $(\tilde{\eta}_k)$	Transportation Cost	Duration of transit
1	$\widetilde{0.1}$			882.1407	655.8256
2	$\widetilde{0.2}$			912.9049	721.0749
3	$\widetilde{0.3}$			946.6450	749.1950
4	$\widetilde{0.4}$			976.6450	774.1150
5	$\widetilde{0.5}$	$\widetilde{0.5}$	$\widetilde{0.5}$	1006.285	798.7550
6	$\widetilde{0.6}$			1016.275	806.4250
7	$\widetilde{0.7}$			998.7050	792.7550
8	$\widetilde{0.8}$			963.2750	765.9250
9	$\widetilde{0.9}$			903.7650	720.8550

Table 13. SA for varying probability of demands.

S.no	Fuzzy stochastic supply $(\tilde{\beta}^{p}{}_{i})$	Fuzzy stochastic demand $(\tilde{\gamma}_j^p)$	Fuzzy stochastic conveyance $(\tilde{\eta}_k)$	Transportation Cost	Duration of transit
1		$\widetilde{0.1}$		834.8800	665.6100
2		$\widetilde{0.2}$		887.2400	710.0100
3		$\widetilde{0.3}$		930.1700	742.8600
4		$\widetilde{0.4}$		968.7950	772.0850
5	$\widetilde{0.5}$	$\widetilde{0.5}$	$\widetilde{0.5}$	1006.285	798.755
6		$\widetilde{0.6}$		1001.585	790.4750
7		$\widetilde{0.7}$		945.3650	742.9350
8		$\widetilde{0.8}$		875.2400	683.5900
9		$\widetilde{0.9}$		757.4000	602.8700

Table 14. SA for varying probability of conveyances.

S.no	Fuzzy stochastic supply $(\tilde{\beta}^{p}{}_{i})$	Fuzzy stochastic demand $(\tilde{\gamma}_j^p)$	Fuzzy stochastic conveyance $(\tilde{\eta}_k)$	Transportation Cost	Duration of transit
1			$\widetilde{0.1}$	1094.331	785.5391
2			$\widetilde{0.2}$	1075.668	772.2717
3			$\widetilde{0.3}$	1060.741	761.6594
4			$\widetilde{0.4}$	1047.455	752.2150
5	$\widetilde{0.5}$	$\widetilde{0.5}$	$\widetilde{0.5}$	1034.316	742.8737
6			$\widetilde{0.6}$	1021.020	733.4396
7			$\widetilde{0.7}$	1005.595	723.3250
8			$\widetilde{0.8}$	986.3141	710.6759
9			$\widetilde{0.9}$	952.5489	690.8011

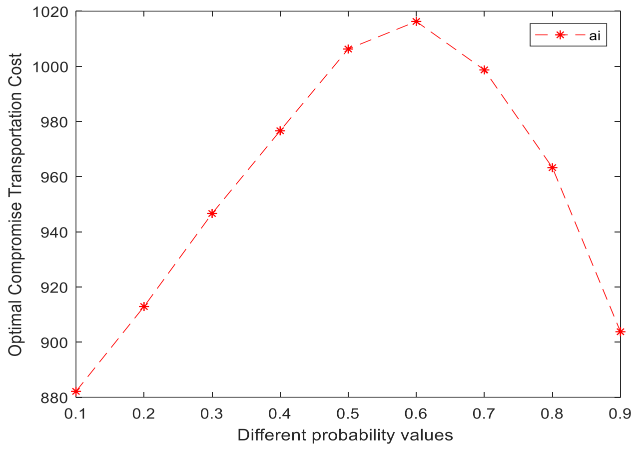

Figure 3. Optimal compromise transportation cost (a_i^p).

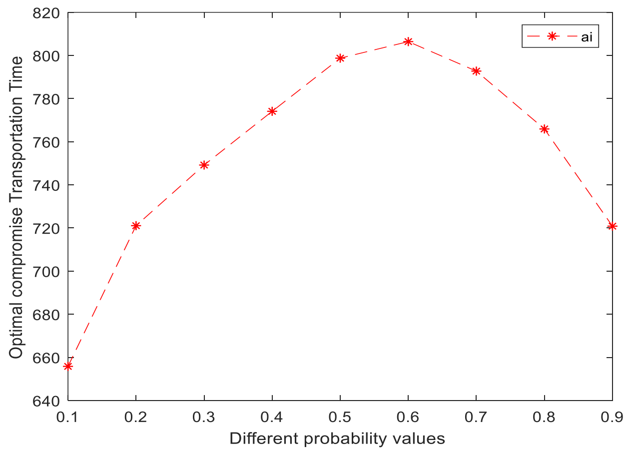

Figure 4. Optimal compromise transportation time (a_i^p).

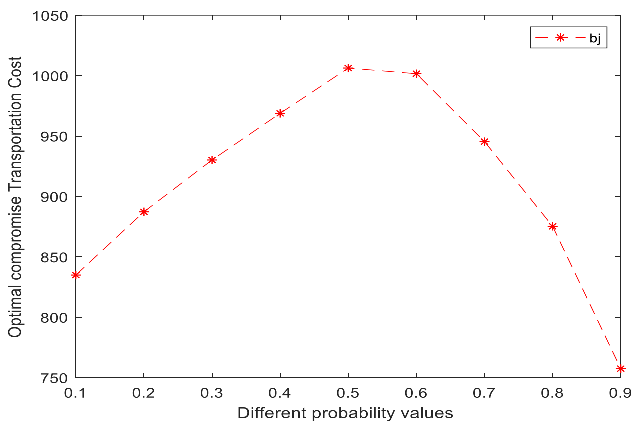

Figure 5. Optimal compromise transportation cost (b_j^p).

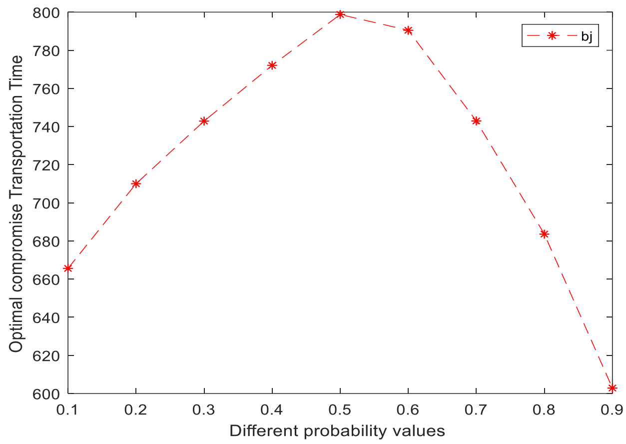

Figure 6. Optimal compromise transportation time (b_j^p).

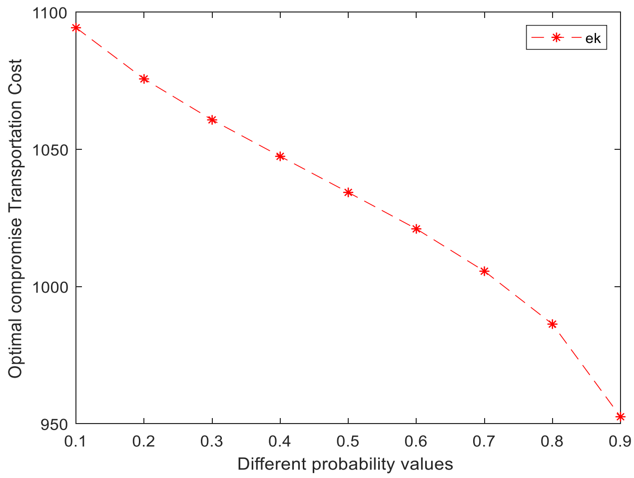

Figure 7. Optimal compromise transportation cost (e_k).

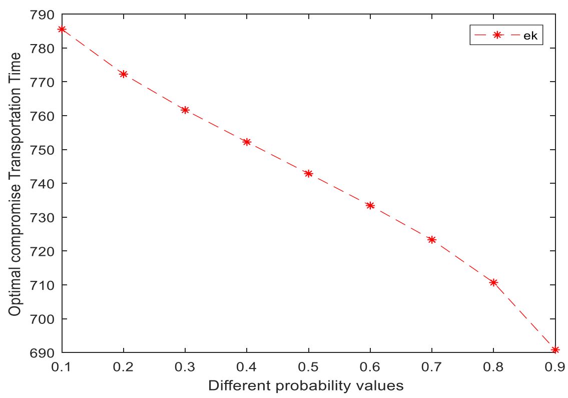

Figure 8. Optimal compromise transportation time (e_k).

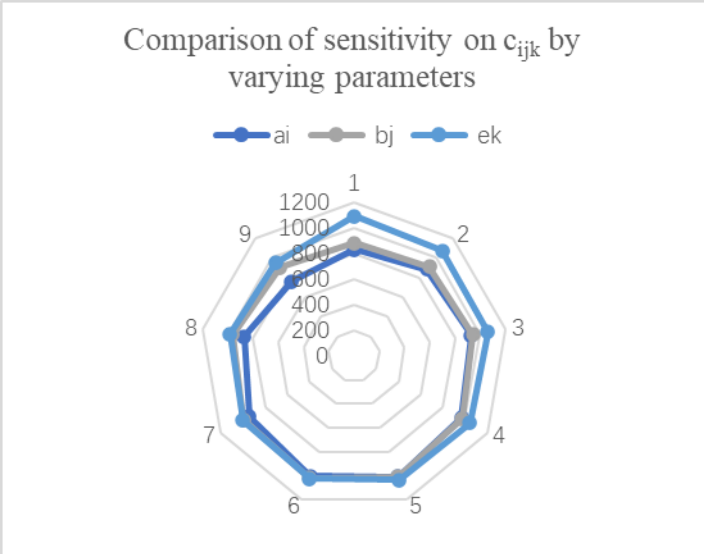

Figure 9. Comparison of SA on the cost by varying constraint parameters.

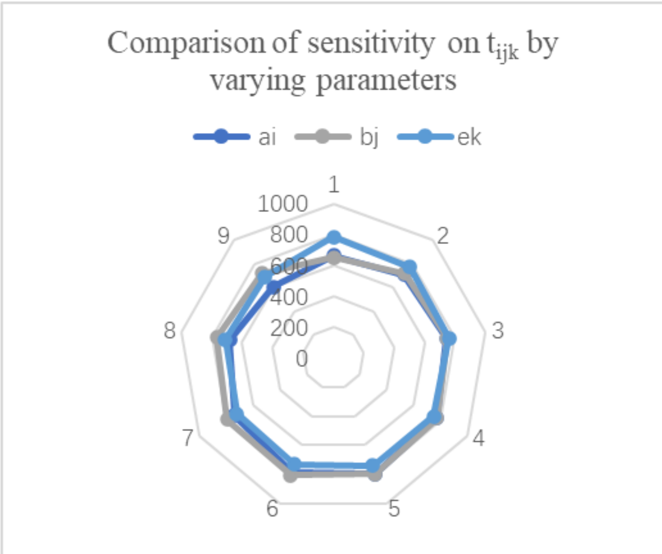

Figure 10. Comparison of SA on the time by varying constraint parameters.

In this study, in an imprecise environment, the best solution is obtained by varying the supply availabilities than by varying the other constraint probability parameters. To describe the difference graphically, the sensitivities to the probabilistic parameters varying, a_i, b_j and e_k, are picturized in Figures 9 and 10 with the help of radar charts. In this study, the problem (M1) attained the best solutions in an unpredictable situation. It is easy to contemplate more conservative decisions in extremely uncertain conditions. The results show that there is varying probability in constraints with a different value for the cost of transportation and duration of transit. When the optimization problem is more ambiguous, it is customary to shift toward more conservative solutions. It has been discovered that more conservative solutions are chosen as optimal when modeling BOBISTP with fuzzy stochastic constraints, the uncertainty introduced by the probabilities for a_i, b_j and e_k. On the other hand, this study shows the necessity of understanding the sensitivity of constraints in the face of increasing uncertainty. It aids entrepreneurs in selecting the appropriate degree of uncertainty for ambiguous parameters. Additionally, it offers entrepreneurs guidance in selecting acceptable restrictions for uncertain parameter level probability.

8. Conclusions and future scope

A bi-objective bi-item solid transportation problem with fuzzy stochastic constraints involving normal distribution has been formulated in this research. The formulated problem is with the coefficients of objective functions in the deterministic form, and the constraint parameters are in fuzzy stochastic form. In instances like a lack of information, a fluctuating market value, production delays, roadblocks, etc., all the parameters or any one of the parameters of the problem can occasionally be unclear, which might lead to uncertainty in supply, demand and conveyance. As a result, the problem is categorized into four models based on various circumstances. Initially, the extended chance-constrained technique with normal distribution is applied to transform the fuzzy stochastic constraint(s) of the BOBISTP to the equivalent deterministic constraint(s). Due to the conflicting nature of the objective functions, the proposed global weighted sum method is used to reduce the bi-objective bi-item solid transportation problem with deterministic constraint(s) into a single objective bi-item solid transportation problem with deterministic constraint(s). The reduced problem is then solved using Lingo 18.0 software to obtain the optimal compromise solution. We have considered one numerical example to test the effectiveness of our proposed method, and a comparison is made between the proposed method and other existing methods. The obtained solutions demonstrate that our proposed method provides an optimal compromise solution that is similar to GCM and better than FPA. The sensitivity analysis performed for the BOBISTP with all constraints considered fuzzy stochastic shows that it is crucial to understand the sensitivity of constraints against increasing uncertainty.

In the literature, there is a gap of study of STPs with multiple objectives with multiple items with fuzzy stochastic parameters. This study has removed this gap by introducing a bi-objective bi-item solid transportation problem with fuzzy stochastic constraints involving normal distribution. This study provides ideas about how fuzzy stochastic constraints in BOBISTP can be transformed into deterministic constraints and also about turning multi-objective optimization problems into single objective optimization problems. Still, in this study, we have some limitations where the cost and duration of transportation in the objectives are deterministic. To overcome this in our further study, we intend to examine BOBISTP situations while taking costs and duration of transit of transportation as uncertain because uncertainty can occur in the cost and time due to financial conditions, road congestion, etc. Based on the study of this paper, more decision making problems could be considered. In two stage bi-objective bi-item solid transportation problem with fuzzy stochastic constraints and two stage multi-objective multi-item fixed charge transportation problem, the same working methodology can be adopted to optimize the profit or loss. In spite of this, the problem can also be extended to a model using type-2 fuzzy parameters following distributions like the log-normal distribution, Weibull distribution, etc.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

The authors would like to thanks the reviewers for the deep comments to improve our work. We express our thanks to editorial office for their advice.

Also, we wish to thank the management of Vellore Institute of Technology, Vellore- 632014, India, for their continuous support and encouragement to carry out this research work.

Conflicts of interest

The authors declare that they have no conflicts of interest.

References

1. E. Shell, Distribution of a product by several properties, In: *Proceedings of the second symposium in linear programming*, **2** (1955), 615–642.
2. K. B. Haley, New methods in mathematical programming—The solid transportation problem, *Oper. Res.*, **10** (1962), 448–463. https://doi.org/10.1287/opre.10.4.448
3. P. Pandian, D. Anuradha, A new approach for solving solid transportation problems, *Appl. Math. Sci.*, **4** (2010), 3603–3610.
4. V. Vidhya, K. Ganesan, An alternate method for finding optimal solution to solid transportation problem under fuzzy environment, *IOP Conf. Ser. Mater. Sci. Eng.*, **912** (2020), 062047. https://doi.org/10.1088/1757-899X/912/6/062047
5. H. A. E. W. Khalifa, P. Kumar, M. G. Alharbi, On characterizing solution for multi-objective fractional two-stage solid transportation problem under fuzzy environment, *J. Intell. Syst.*, **30** (2021), 620–635. https://doi.org/10.1515/jisys-2020-0095
6. D. Chhibber, D. C. S. Bisht, P. K. Srivastava, Pareto-optimal solution for fixed-charge solid transportation problem under intuitionistic fuzzy environment, *Appl. Soft Comput.*, **107** (2021), 107368. https://doi.org/10.1016/j.asoc.2021.107368
7. T. Anithakumari, B. Venkateswarlu, A. Akilbasha, Optimizing a fully rough interval integer solid transportation problems, *J. Intell. Fuzzy Syst.*, **41** (2021), 2429–2439. https://doi.org/10.3233/JIFS-202373
8. A. Baidya, U. K. Bera, Solid transportation problem under fully fuzzy environment, *Int. J. Math. Oper. Res.*, **15** (2019), 498–539. https://doi.org/10.1504/IJMOR.2019.102997
9. S. Ghosh, K. H. Küfer, S. K. Roy, G. W. Weber, Type-2 zigzag uncertain multi-objective fixed-charge solid transportation problem: Time window vs. preservation technology, *Cent. Eur. J. Oper. Res.*, **31** (2023), 337–362. https://doi.org/10.1007/s10100-022-00811-7
10. S. Pramanik, D. Kumar, J. M. Maiti, Multi-objective solid transportation problem in imprecise environments, *J. Transp. Secur.*, **6** (2013), 131–150. https://doi.org/10.1007/s12198-013-0108-0
11. M. B. Kar, P. Kundu, S. Kar, T. Pal, A multi-objective multi-item solid transportation problem with vehicle cost, volume and weight capacity under fuzzy environment, *J. Intell. Fuzzy Syst.*, **35** (2018), 1991–1999. https://doi.org/10.3233/JIFS-171717
12. D. Rani, T. R. Gulati, Uncertain multi-objective multi-product solid transportation problems, *Sādhanā*, **41** (2016), 531–539. https://doi.org/10.1007/s12046-016-0491-x

13. D. Rani, T. R. Gulati, A. Kumar, On Fuzzy Multiobjective Multi-Item Solid Transportation Problem, *J. Optim.*, **2015** (2015), 787050. https://doi.org/10.1155/2015/787050
14. P. Kundu, S. Kar, M. Maiti, Multi-objective multi-item solid transportation problem in fuzzy environment, *Appl. Math. Model.*, **37** (2013), 2028–2038. https://doi.org/10.1016/j.apm.2012.04.026
15. S. Kataoka, A stochastic programing model, *Econometrica*, **31** (1963), 181–196.
16. W. Szwarc, The transportation problem with stochastic demand, *Manage. Sci.*, **11** (1964), 33–50.
17. S. Singh, A. Pradhan, M. P. Biswal, Multi-objective solid transportation problem under stochastic environment, *Sādhanā*, **44** (2019), 105. https://doi.org/10.1007/s12046-019-1094-0
18. A. C. Williams, A stochastic transportation problem, *Oper. Res.*, **11** (1963), 759–770.
19. K. Holmberg, H. Tuy, A production-transportation problem with stochastic demand and concave production costs, *Math. Program. Ser. B*, **85** (1999), 157–179. https://doi.org/10.1007/s101070050050
20. A. Charnes, W. W. Cooper, Chance-constrained programming, *Manage. Sci.*, **6** (1959), 73–79.
21. S. K. Roy, D. R. Mahapatra, Multi-objective interval-valued transportation probabilistic problem involving log-normal, *Int. J. Math. Sci. Comput.*, **1** (2011), 14–21.
22. D. R. Mahapatra, S. K. Roy, M. P. Biswal, Multi-choice stochastic transportation problem involving extreme value distribution, *Appl. Math. Model.*, **37** (2013), 2230–2240. https://doi.org/10.1016/j.apm.2012.04.024
23. S. K. Roy, D. R. Mahapatra, M. P. Biswal, Multi-choice stochastic transportation problem with exponential distribution, *J. Uncertain Syst.*, **6** (2012), 200–213.
24. D. R. Mahapatra, S. K. Roy, M. P. Biswal, Multi-objective stochastic transportation problem involving log-normal, *J. Phys. Sci.*, **14** (2010), 63–76.
25. P. Agrawal, K. Alnowibet, A. W. Mohamed, Gaining-sharing knowledge based algorithm for solving stochastic programming problems, *Comput. Mater. Contin.*, **71** (2022), 2847–2868. https://doi.org/10.32604/cmc.2022.023126
26. H. Kwakernaak, Fuzzy random variables-II. Algorithms and examples for the discrete case, *Inf. Sci.*, **17** (1979), 253–278. https://doi.org/10.1016/0020-0255(79)90020-3
27. L. Zhao, N. Cao, Fuzzy random chance-constrained programming model for the vehicle routing problem of hazardous materials transportation, *Symmetry*, **12** (2020), 1208. https://doi.org/10.3390/SYM12081208
28. G. Maity, V. F. Yu, S. K. Roy, Optimum intervention in transportation networks using multimodal system under fuzzy stochastic environment, *J. Adv. Transp.*, **2022** (2022), 3997396. https://doi.org/10.1155/2022/3997396
29. S. H. Nasseri, S. Bavandi, Fuzzy stochastic linear fractional programming based on fuzzy mathematical programming, *Fuzzy Inf. Eng.*, **10** (2018), 324–338. https://doi.org/10.1080/16168658.2019.1612605
30. S. Acharya, N. Ranarahu, J. K. Dash, M. M. Acharya, Computation of a multi-objective fuzzy stochastic transportation problem, *Int. J. Fuzzy Comput. Model.*, **1** (2014), 212–233. https://doi.org/10.1504/ijfcm.2014.067129
31. S. Dutta, S. Acharya, R. Mishra, Genetic algorithm based fuzzy stochastic transportation programming problem with continuous random variables, *Opsearch*, **53** (2016), 835–872. https://doi.org/10.1007/s12597-016-0264-7

32. P. Agrawal, T. Ganesh, Fuzzy fractional stochastic transportation problem involving exponential distribution, *Opsearch*, **57** (2020), 1093–1114. https://doi.org/10.1007/s12597-020-00458-5
33. T. Latunde, J. O. Richard, O. O. Esan, O. O. Dare, Sensitivity Analysis of Road Freight Transportation of a Mega Non-Alcoholic Beverage Industry, *J. Appl. Sci. Environ. Manage.*, **24** (2020), 449–454. https://doi.org/10.4314/jasem.v24i3.8
34. Y. Sun, M. Lang, Bi-objective optimization for multi-modal transportation routing planning problem based on pareto optimality, *J. Ind. Eng. Manage.*, **8** (2015), 1195–1217. https://doi.org/10.3926/jiem.1562
35. V. Kakran, J. Dhodiya, A belief-degree based multi-objective transportation problem with multi-choice demand and supply, *Int. J. Optim. Control Theor. Appl.*, **12** (2022), 99–112. https://doi.org/10.11121/ijocta.2022.1166
36. A. A. Gessesse, R. Mishra, M. M. Acharya, Solving multi-objective linear fractional stochastic transportation problems involving normal distribution using simulation-based genetic algorithm, *Int. J. Eng. Adv. Technol.*, **9** (2019), 9–17. https://doi.org/10.35940/ijeat.b3054.129219
37. S. K. Roy, Multi-choice stochastic transportation problem involving Weibull distribution, *Int. J. Oper. Res.*, **21** (2014), 38–58. https://doi.org/10.1504/IJOR.2014.064021
38. D. R. Mahapatra, Multi-choice stochastic transportation problem involving weibull distribution, *An Int. J. Optim. Control Theor. Appl.*, **4** (2013), 45–55. https://doi.org/10.11121/ijocta.01.2014.00154
39. A. Das, G. M. Lee, A multi-objective stochastic solid transportation problem with the supply, demand, and conveyance capacity following the weibull distribution, *Mathematics*, **9** (2021), 1757. https://doi.org/10.3390/math9151757
40. M. S. Osman, O. E. Emam, M. A. El Sayed, Stochastic Fuzzy Multi-level Multi-objective Fractional Programming Problem: A FGP Approach, *Opsearch*, **54** (2017), 816–840. https://doi.org/10.1007/s12597-017-0307-8
41. P. K. Giri, M. K. Maiti, M. Maiti, Fuzzy stochastic solid transportation problem using fuzzy goal programming approach, *Comput. Ind. Eng.*, **72** (2014), 160–168. https://doi.org/10.1016/j.cie.2014.03.001
42. L. A. Zadeh, Fuzzy sets, *Inf. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
43. S. Nanda, K. Kar, Convex fuzzy mappings, *Fuzzy Sets Syst.*, **48** (1992), 129–132. https://doi.org/10.1016/0165-0114(92)90256-4
44. J. J. Buckley, Uncertain probabilities III: The continuous case, *Soft Comput.*, **8** (2004), 200–206. https://doi.org/10.1504/ijfcm.2014.067129
45. S. Nanda, G. Panda, J. Dash, A new methodology for crisp equivalent of fuzzy chance constrained programming problem, *Fuzzy Optim. Decis. Mak.*, **7** (2008), 59–74.
46. M. Zelany, A concept of compromise solutions and the method of the displaced ideal, *Comput. Oper. Res.*, **1** (1974), 479–496. https://doi.org/10.1016/0305-0548(74)90064-1
47. H. J. Zimmermann, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets Syst.*, **1** (1978), 45–55. https://doi.org/10.1016/0165-0114(78)90031-3