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Research article

Bipolar complex fuzzy credibility aggregation operators and their application in decision making problem

Muhammad Qiyas[1], **Muhammad Naeem**[2], **Neelam Khan**[3] **and Lazim Abdullah**[4,*]

[1] Department of Mathematics, Riphah International University Faisalabad Campus, Pakistan
[2] Department of Mathematics Deanship of Applied Sciences Umm Al-Qura University, Makkah, Saudi Arabia
[3] Department of Mathematics, Abdul Wali khan University Mardan, KP, Pakistan
[4] Programme of Mathematics, Faculty of Ocean Engineering Technology and Informatics Universiti Malaysia Terengganu, 21030 Kuala Nerus Terengganu, Malaysia

* **Correspondence:** Email: lazim_m@umt.edu.my.

Abstract: A bipolar complex fuzzy credibility set (BCFCS) is a new approach in computational intelligence and decision-making under uncertainty. Bipolar complex fuzzy credibility (BCFC) information has been employed as a strategy for dealing with confusing and unreliable situations that arise in everyday life. In this paper, we used the concept of aggregation operators to diagnose the well-known averaging and geometric aggregation operators, as well as evaluate some properties and related results. Using described operators, an algorithm for multiple criteria group decision making is proposed. Then, a numerical example of a case study of Hospital selection is discussed. Lastly, the comparative analysis of suggested operators with existing operators are also given to discuss the rationality, efficiency and applicability of these operators.

Keywords: complex fuzzy credibility set; bipolar fuzzy set; bipolar complex fuzzy credibility sets; aggregation operators
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1. Introduction

In today's culture, scientific and technological advances have resulted in scientific and technological discoveries that have decreased the complications in our daily lives. Yet, despite scientific progress that has made life easier, some concerns, such as decision making (DM), remain complicated. DM, particularly multi-criteria group decision making (MCGDM), has been widely adopted in a variety of sectors where traditional methods have failed in recent years. As in real life, information is usually uncertain, and as information becomes more complex, additional solutions are required. In 1965, Zadeh [35] invented the concept of a fuzzy set (FS). Fuzzy set is a remarkable performance with numerous uses. An FS is defined by its membership degree (MD) for any value between 0 and 1. Atanassov expanded fuzzy set and defined intuitionistic fuzzy set (IFS) in [1]. An IFS is defined by two functions and expressed as and for each element of fixed set on the closed-interval 0 to 1, as well as their sum in the range. Xiao et al. [34] developed a q-rung orthopair fuzzy decision-making model with new score function and best-worst method for manufacturer selection. Xiao et al. [33] suggested an integrated risk assessment method using Z-fuzzy clouds and generalized TODIM.

In the complex fuzzy set (CFS), an upgraded variety of the classical fuzzy set can be utilized to handle fuzziness information). A complex fuzzy value may deal with information in two ways, since it has both a phase and an amplitude term. Ramot et al. [29] describe the basic operations for CFS. Merigó et al. [23], for example, offer a complicated fuzzy generalized aggregate operator (AO) and describe their applicability in DM. Hu et al. [14, 15] shows that CFSs now has approximate paralleling and orthogonality relations. Zhang et al. [37] present the concept of δ-equalities amongst CFSs. Dai et al. [8, 16], Alkouri and Salleh [3] provide CFS distance measurements have been defined. Bi et al. [7] proposed new entropy measure classes in CFSs. Liu et al. [21] defined distance between CFSs, their cross-entropy, and their application in DM are all measured. Dai [9] extended rotational in variance to CF operations. Ma et al. [22] provide CFS was defined as a concept for handling problems with many periodic factors. Several CFS applications have been investigated for various intellectuals, such as neighborhood operators Mahmood [24]. Huang et al. [17] developed an assessment and prioritization method of key engineering characteristics for complex products based on cloud rough numbers. Further information on CFSs can be found in [4, 5, 31].

The CFSs may only define complex-valued grades for positive membership functions and cannot express complex-valued grades for negative membership grades (NMG), as it is limited in their application. Alkouri et al. [2] proposed the CIFSs structure, which includes mainly two complex membership functions that express an element's positive and NMD. Greenfield et al. [10] introduced the definition of complex interval-valued fuzzy set (CIVFS), which clearly enhanced the paradigm of interval-valued fuzzy sets and helped to conceptualize CFSs. Kumar and Bajaj defined complex intuitionistic fuzzy soft sets with distance measurements and entropy in [20]. Garg and Rani [12] presented some generalized CIF aggregation operations and their application to the MCDM process. Garg and Rani created a strong correlation coefficient measure of CIFSs and its applications in decision-making. Rani and Garg defined CIF power aggregation operators and their applications in MCDM in [30]. Garg and Rani are the first to suggest a power aggregation operator and a ranking method for CIFSs, as well as their use in decision making. Garg and Rani [11] proposed generalized AOs for CIFS based on t-norm methods and discussed some of their DM applications. Huang et al. [18] proposed a design alternative assessment and selection: a novel Z-cloud rough number-based BWM-MABAC model.

We know that the CFS theory discussed just a supporting grade while leaving out the grade of negative supporting grade, and as a result, various issues have emerged in many cases. For this, Mahmood and Rehman [25] modified the theory of CFS and diagnosed the mathematical form of bipolar CFS (BCFS) with a terminology represented by positive and negative supporting grades in the shape of complex numbers, with real and imaginary parts belonging to the unit intervals [0,1] and [-1,0]. Gao et al. [13] developed dual hesitant bipolar fuzzy Hamacher aggregation operators and their applications to MADM. Limited number of researchers produced numerous applications, such as the Hamacher aggregation information examined by Mahmood et al. [26]. Also, Wei et al. [32] defined a bipolar fuzzy Hamacher aggregation operators in multiple attribute decision making. Additionally, Mahmood and Rehman [27] proposed the core theory of Dombi operators based on BCFS. Jana et al. [19] proposed bipolar fuzzy Dombi prioritized aggregation operators in MADM.

The main motivation for this analysis is explained below:

(1) To define new operations for bipolar complex fuzzy credibility numbers (BCFCNs), and to investigate properties these numbers. A BCFCN is superior that a bipolar complex fuzzy number as it carries more comprehensive and reasonable information. Bipolar complex fuzzy credibility numbers is the extension of bipolar complex fuzzy numbers to deal with two-sided contrasting features, which can describe the information with a bipolar complex fuzzy number and an credibility number simultaneously.

(2) A secondary objective of this paper is to introduce some fundamental operations on BCFCNs, their key properties, and related significant results. Suggested operations are very helpful to strengthen BCFCS theory.

(3) Since aggregation operators for bipolar complex fuzzy credibility numbers (BCFCNs) have not been established so far, motivated by the above discussion, this paper presents novel averaging and geometric aggregation operators under bipolar complex fuzzy credibility information are proposed.

(4) An algorithm for new MCDM technique is developed based on proposed aggregation operators using bipolar complex fuzzy credibility information. Proposed technique is also demonstrated by a numerical illustration.

(5) To demonstrate the validity and capability of the proposed technique, conduct a comparative examination of the developed operators with various current theories.

The framework of this study is as follows: Section 2 includes certain prevalent ideas such as BCFCS, aggregation operator, and their operational laws. In Section 3, we employed the theory of averaging/geometric aggregation operators to diagnose the well-known operators, such as bipolar complex fuzzy credibility weighted average (BCFCWA), bipolar complex fuzzy credibility ordered weighted average (BCFCOWA), bipolar complex fuzzy credibility hybrid average (BCFCHA), bipolar complex fuzzy credibility weighted geometric (BCFCWG), bipolar complex fuzzy credibility ordered weighted geometric (BCFCOWG), and bipolar complex fuzzy credibility hybrid geometric (BCFCHG) operators, as well as analyses their strategic features and related outcomes. Section 4 proposes an algorithm for multiple criteria group decision making utilizing stated operators. Then, a numerical example of a case study of Hospital selection is discussed. In Section 5, we compared the described operators to existing methodologies to demonstrate the validity and capabilities of the proposed approach. Finally, write the study's conclusion.

2. Preliminaries

The aim of this part is to present in a concise manner the preexisting basic definitions for CFS, CIFS, BFC, BCFS and BCFCS.

Definition 2.1. *[28] A CFS C on Q (fixed set) is defined,*

$$C = \{\langle \check{n}, \mu_C(\check{n})\rangle \, | \check{n} \in Q\}, \tag{2.1}$$

where $\mu_C : U \to \{z : z \in C, |z| \le 1\}$ *and* $\mu_C(\check{n}) = a + ib = \chi_C(\check{n}).e^{2\pi i \Theta_{C(\check{n})}}$. *Here,* $\chi_C(\check{n}) = \sqrt{a^2 + \mathcal{K}^2} \in R$ *and* $\chi_C(\check{n}), \Theta_{C(\check{n})} \in [0,1]$, *where* $i = \sqrt{-1}$.

Definition 2.2. *[2] A CIFS I on Q (fixed set) is define,*

$$I = \{\langle \check{n}, \mu_I(\check{n}), \upsilon_I(\check{n})\rangle \, | \check{n} \in Q\}, \tag{2.2}$$

where $\mu_I : U \to \{z_1 : z_1 \in I, |z_1| \le 1\}$, $\upsilon_I : U \to \{z_2 : z_2 \in I, |z_2| \le 1\}$, *such as* $\mu_I(\check{n}) = z_1 = a_1 + ib_1$ *and* $\upsilon_I(\check{n}) = z_2 = a_2 + ib_2$, *and* $0 \le |z_1| + |z_2| \le 1$ *or* $\mu_I(\check{n}) = \chi_I(\check{n}).e^{2\pi i \Theta_{\chi_I(\check{n})}}$ *and* $\upsilon_i(\check{n}) = \xi_i(\check{n}).e^{2\pi i \Theta_{\xi_I(\check{n})}}$ *satisfy the conditions;* $0 \le \chi_I(\check{n}) + \xi_I(\check{n}) \le 1$ *and* $0 \le \Theta_{\chi_I(\check{n})} + \Theta_{\xi_I(\check{n})} \le 1$. *The term* $H_I(\check{n}) = R.e^{2\pi i \Theta_R}$, *such that* $R = 1 - (|z_1| + |z_2|)$ *and* $\Theta_R(\check{n}) = 1 - \left(\Theta_{\chi_I(\check{n})} + \Theta_{\xi_I(\check{n})}\right)$ *be the hesitancy grade of Q. Furthermore,* $I = \left(\chi.e^{2\pi i \Theta_\chi}, \xi.e^{2\pi i \Theta_\xi}\right)$ *indicate the complex intuitionistic fuzzy number (CIFN).*

Definition 2.3. *[36] A BFS* $\mathcal{K}$ *on Q (fixed set) is of the form,*

$$\mathcal{K} = \left\{\left\langle \check{n}, \mu^+_{\mathcal{K}}(\check{n}), \mu^-_{\mathcal{K}}(\check{n})\right\rangle | \check{n} \in Q\right\}, \tag{2.3}$$

where, $\mu^+_{\mathcal{K}} : Q \to [0,1]$ *and* $\mu^-_{\mathcal{K}} : Q \to [-1,0]$.

Definition 2.4. *[25] A BCFS* $\mathcal{K}$ *on Q (fixed set) is of the form,*

$$\mathcal{K} = \left\{\left\langle \check{n}, \mu^+_{\mathcal{K}}(\check{n}), \mu^-_{\mathcal{K}}(\check{n})\right\rangle | \check{n} \in Q\right\}, \tag{2.4}$$

where, $\mu^+_{\mathcal{K}} : Q \to [0,1] + i[0,1]$ *and* $\mu^-_{\mathcal{K}} : Q \to [-1,0] + i[-1,0]$ *is called membership degree.* $\mu^+_{\mathcal{K}}(\check{n}) = a^+_{\mathcal{K}}(\check{n}) + ib^+_{\mathcal{K}}(\check{n})$ *and* $\mu^-_{\mathcal{K}}(\check{n}) = a^-_{\mathcal{K}}(\check{n}) + ib^-_{\mathcal{K}}(\check{n})$ *with* $a^+_{\mathcal{K}}(\check{n}), ib^+_{\mathcal{K}}(\check{n}) \in [0,1]$ *and* $a^-_{\mathcal{K}}(\check{n}), ib^-_{\mathcal{K}}(\check{n}) \in [-1,0]$. *The bipolar complex fuzzy number is represented by* $\mathcal{K} = \left(a^+_{\mathcal{K}}(\check{n}) + ib^+_{\mathcal{K}}(\check{n}), a^-_{\mathcal{K}}(\check{n}) + ib^-_{\mathcal{K}}(\check{n})\right)$.

Definition 2.5. *[6] A BCFCS* $\mathcal{K}$ *on Q (fixed set) is of the form,*

$$\mathcal{K} = \left\{\left\langle \check{n}, \left(\mu^+_{\mathcal{K}}(\check{n}), \mu^-_{\mathcal{K}}(\check{n})\right), \left(\upsilon^+_{\mathcal{K}}(\check{n}), \upsilon^-_{\mathcal{K}}(\check{n})\right)\right\rangle | \check{n} \in Q\right\}, \tag{2.5}$$

where, $\mu^+_{\mathcal{K}} : Q \to [0,1] + i[0,1], \upsilon^+_{\mathcal{K}} : Q \to [0,1] + i[0,1]$, $\mu^-_{\mathcal{K}} : Q \to [-1,0] + i[-1,0]$ *and* $\upsilon^-_{\mathcal{K}} : Q \to [-1,0] + i[-1,0]$ *as known as the MG and credibility degree.* $\mu^+_{\mathcal{K}}(\check{n}) = a^+_{\mathcal{K}}(\check{n}) + ib^+_{\mathcal{K}}(\check{n}), \upsilon^+_{\mathcal{K}}(\check{n}) = c^+_{\mathcal{K}}(\check{n}) + id^+_{\mathcal{K}}(\check{n})$, $\mu^-_{\mathcal{K}}(\check{n}) = a^-_{\mathcal{K}}(\check{n}) + ib^-_{\mathcal{K}}(\check{n})$ *and* $\upsilon^-_{\mathcal{K}} = c^-_{\mathcal{K}} + id^-_{\mathcal{K}}$ *with* $a^+_{\mathcal{K}}(\check{n}), ib^+_{\mathcal{K}}(\check{n}), c^+_{\mathcal{K}}(\check{n}), id^+_{\mathcal{K}}(\check{n}) \in [0,1]$ *and* $a^-_{\mathcal{K}}(\check{n}), ib^-_{\mathcal{K}}(\check{n}), c^-_{\mathcal{K}}(\check{n}), id^-_{\mathcal{K}}(\check{n}) \in [-1,0]$. *The bipolar complex fuzzy credibility number is represented as,* $\mathcal{K} = \left(\left\langle\left(a^+_{\mathcal{K}} + ib^+_{\mathcal{K}}\right), \left(a^-_{\mathcal{K}} + ib^-_{\mathcal{K}}\right)\right\rangle, \left\langle\left(c^+_{\mathcal{K}} + id^+_{\mathcal{K}}\right), \left(c^-_{\mathcal{K}} + id^-_{\mathcal{K}}\right)\right\rangle\right)$.

Definition 2.6. *For any two BCFCNs* $\mathcal{K}_1 = \left(\left\langle a^+_{\mathcal{K}_1} + ib^+_{\mathcal{K}_1}, a^-_{\mathcal{K}_1} + ib^-_{\mathcal{K}_1}\right\rangle, \left\langle c^+_{\mathcal{K}_1} + id^+_{\mathcal{K}_1}, c^-_{\mathcal{K}_1} + id^-_{\mathcal{K}_1}\right\rangle\right)$ *and* $\mathcal{K}_2 = \left(\left\langle a^+_{\mathcal{K}_2} + ib^+_{\mathcal{K}_2}, a^-_{\mathcal{K}_2} + ib^-_{\mathcal{K}_2}\right\rangle, \left\langle c^+_{\mathcal{K}_2} + id^+_{\mathcal{K}_2}, c^-_{\mathcal{K}_2} + id^-_{\mathcal{K}_2}\right\rangle\right)$, *and for any* $\lambda > 0$. *The following operation are defined as*

$$(1)\ \mathcal{K}_1 \oplus \mathcal{K}_2 = \left\{\begin{array}{c} \left\langle a^+_{\mathcal{K}_1} + a^+_{\mathcal{K}_2} - a^+_{\mathcal{K}_1}a^+_{\mathcal{K}_2} + i\left(b^+_{\mathcal{K}_1} + b^+_{\mathcal{K}_2} - b^+_{\mathcal{K}_1}b^+_{\mathcal{K}_2}\right), -\left(a^-_{\mathcal{K}_1}a^-_{\mathcal{K}_2}\right) + i\left(-\left(b^-_{\mathcal{K}_1}b^-_{\mathcal{K}_2}\right)\right)\right\rangle, \\ \left\langle c^+_{\mathcal{K}_1} + c^+_{\mathcal{K}_2} - c^+_{\mathcal{K}_1}c^+_{\mathcal{K}_2} + i\left(c^+_{\mathcal{K}_1} + c^+_{\mathcal{K}_2} - c^+_{\mathcal{K}_1}c^+_{\mathcal{K}_2}\right), -\left(c^-_{\mathcal{K}_1}c^-_{\mathcal{K}_2}\right) + i\left(-\left(d^-_{\mathcal{K}_1}d^-_{\mathcal{K}_2}\right)\right)\right\rangle \end{array}\right\};$$

(2) $\mathcal{K}_1 \otimes \mathcal{K}_2 = \left\{ \begin{array}{c} \left\langle \left(a^+_{\mathcal{K}_1} a^+_{\mathcal{K}_2}\right) + i\left(b^+_{\mathcal{K}_1} b^+_{\mathcal{K}_2}\right), a^-_{\mathcal{K}_1} + a^-_{\mathcal{K}_2} - a^-_{\mathcal{K}_1} a^-_{\mathcal{K}_2} + i\left(b^-_{\mathcal{K}_1} + b^-_{\mathcal{K}_2} - b^-_{\mathcal{K}_1} b^-_{\mathcal{K}_2}\right)\right\rangle \\ \left\langle \left(c^+_{\mathcal{K}_1} c^+_{\mathcal{K}_2}\right) + i\left(d^+_{\mathcal{K}_1} d^+_{\mathcal{K}_2}\right), c^-_{\mathcal{K}_1} + c^-_{\mathcal{K}_2} - c^-_{\mathcal{K}_1} c^-_{\mathcal{K}_2} + i\left(c^-_{\mathcal{K}_1} + c^-_{\mathcal{K}_2} - c^-_{\mathcal{K}_1} c^-_{\mathcal{K}_2}\right)\right\rangle \end{array} \right\}$;

(3) $\lambda\mathcal{K}_1 = \left\{ \begin{array}{c} \left\langle 1 - \left(1 - a^+_{\mathcal{K}_1}\right)^\lambda + i\left(1 - \left(1 - b^+_{\mathcal{K}_1}\right)^\lambda\right), -\left(a^-_{\mathcal{K}_1}\right)^\lambda + i\left(-\left(b^-_{\mathcal{K}_1}\right)^\lambda\right)\right\rangle \\ \left\langle 1 - \left(1 - c^+_{\mathcal{K}_1}\right)^\lambda + i\left(1 - \left(1 - d^+_{\mathcal{K}_1}\right)^\lambda\right), -\left(c^-_{\mathcal{K}_1}\right)^\lambda + i\left(-\left(d^-_{\mathcal{K}_1}\right)^\lambda\right)\right\rangle \end{array} \right\}$;

(4) $\mathcal{K}_1^\lambda = \left\{ \begin{array}{c} \left\langle \left(a^+_{\mathcal{K}_1}\right)^\lambda + i\left(b^+_{\mathcal{K}_1}\right)^\lambda, -1 + \left(1 + a^+_{\mathcal{K}_1}\right)^\lambda + i\left(-1 + \left(1 + b^+_{\mathcal{K}_1}\right)^\lambda\right)\right\rangle, \\ \left\langle \left(c^+_{\mathcal{K}_1}\right)^\lambda + i\left(d^+_{\mathcal{K}_1}\right)^\lambda, -1 + \left(1 + c^+_{\mathcal{K}_1}\right)^\lambda + i\left(-1 + \left(1 + d^+_{\mathcal{K}_1}\right)^\lambda\right)\right\rangle \end{array} \right\}$.

Definition 2.7. *Let* $\mathcal{K} = \left(\left\langle a^+_{\mathcal{K}} + ib^+_{\mathcal{K}}, a^-_{\mathcal{K}} + ib^-_{\mathcal{K}}\right\rangle, \left\langle c^+_{\mathcal{K}} + id^+_{\mathcal{K}}, c^-_{\mathcal{K}} + id^-_{\mathcal{K}}\right\rangle\right)$ *be the BCFCN. The score function is defined as*

$$So(\mathcal{K}) = \frac{1}{8}\left(4 + a^+_{\mathcal{K}} + ib^+_{\mathcal{K}} + a^-_{\mathcal{K}} + ib^-_{\mathcal{K}} + c^+_{\mathcal{K}} + id^+_{\mathcal{K}} + c^-_{\mathcal{K}} + id^-_{\mathcal{K}}\right). \tag{2.6}$$

3. Bipolar complex fuzzy credibility average operator

Here, we are going to define some aggregation operators like, BCFCWA, BCFCOWA, and BCFCHA operators.

3.1. Bipolar complex fuzzy credibility weighted average operator

Definition 3.1. *Let* $\mathcal{K}_i = \left(\left\langle a^+_{\mathcal{K}_i} + ib^+_{\mathcal{K}_i}, a^-_{\mathcal{K}_i} + ib^-_{\mathcal{K}_i}\right\rangle, \left\langle c^+_{\mathcal{K}_i} + id^+_{\mathcal{K}_i}, c^-_{\mathcal{K}_i} + id^-_{\mathcal{K}_i}\right\rangle\right)(i = 1, ..., n)$ *be a set of BCFCNs with the weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{i=1}^n \Phi_i = 1$ *and* $0 \leq \Phi_i \leq 1$. *Then, the BCFCWA operator is obtain as*

$$BCFCWA\left(\mathcal{K}_1, ..., \mathcal{K}_n\right) = \bigoplus_{\hat{j}=1}^{n} \Phi_{\hat{j}}\mathcal{K}_{\hat{j}}, \tag{3.1}$$

utilizing Definition 3.1, aggregated value for BCFCWA operator is shown in Theorem 3.2.

Theorem 3.2. *Let* $\mathcal{K}_{\hat{j}} = \left(a^+_{\mathcal{K}_{\hat{j}}} + ib^+_{\mathcal{K}_{\hat{j}}}, c^+_{\mathcal{K}_{\hat{j}}} + id^+_{\mathcal{K}_{\hat{j}}}, a^-_{\mathcal{K}_{\hat{j}}} + ib^-_{\mathcal{K}_{\hat{j}}}, c^-_{\mathcal{K}_{\hat{j}}} + id^-_{\mathcal{K}_{\hat{j}}}\right)(\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^n \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$. *Then, BCFCWA operator is obtained as*

$$BCFCWA\left(\mathcal{K}_1, ..., \mathcal{K}_n\right) = \bigoplus_{\hat{j}=1}^{n} \Phi_{\hat{j}}\mathcal{K}_{\hat{j}}$$

$$= \left\{ \begin{array}{c} \left(1 - \prod_{\hat{j}=1}^{n}\left(1 - a^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(1 - \prod_{\hat{j}=1}^{n}\left(1 - b^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right), -\prod_{\hat{j}=1}^{n}\left(a^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(-\prod_{\hat{j}=1}^{n}\left(b^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(1 - \prod_{\hat{j}=1}^{n}\left(1 - c^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_i} + i\left(1 - \prod_{\hat{j}=1}^{n}\left(1 - d^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right), -\prod_{\hat{j}=1}^{n}\left(c^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(-\prod_{\hat{j}=1}^{n}\left(d^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}. \tag{3.2}$$

Proof. To prove this theorem, we used mathematical induction principle. As we know that

$$\mathcal{K}_1 \oplus \mathcal{K}_2 = \bigoplus_{\hat{j}=2}^{n} \Phi_{\hat{j}}\mathcal{K}_{\hat{j}}$$

and

$$\Phi_1\mathcal{K}_1=\left\{\begin{array}{c}\left(1-\left(1-a^+_{\mathcal{K}_1}\right)^{\Phi_1}+i\left(1-\left(1-b^+_{\mathcal{K}_1}\right)^{\Phi_1}\right),-\left(a^-_{\mathcal{K}_1}\right)^{\Phi_1}+i\left(-\left(b^-_{\mathcal{K}_1}\right)^{\Phi_1}\right)\right),\\ \left(1-\left(1-c^+_{\mathcal{K}_1}\right)^{\Phi_1}+i\left(1-\left(1-d^+_{\mathcal{K}_1}\right)^{\Phi_1}\right),-\left(c^-_{\mathcal{K}_1}\right)^{\Phi_1}+i\left(-\left(d^-_{\mathcal{K}_1}\right)^{\Phi_1}\right)\right)\end{array}\right\}.$$

Let Eq (3.2) is true for $n=2$. Then,

$$BCFCWA\left(\mathcal{K}_1,\mathcal{K}_2\right)=\bigoplus_{\hat{j}=1}^{2}\Phi_{\hat{j}}\mathcal{K}_{\hat{j}}$$

$$=\left\{\begin{array}{c}\left(1-\prod_{\hat{j}=1}^{2}\left(1-a^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{2}\left(1-b^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{2}\left(a^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{2}\left(b^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right),\\ \left(1-\prod_{\hat{j}=1}^{2}\left(1-c^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{2}\left(1-d^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{2}\left(c^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{2}\left(d^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right)\end{array}\right\}.$$

The result hold for $n=2$.

Now, let Eq (3.2) is true for $n=\tau$. Then, we get

$$BCFCWA\left(\mathcal{K}_1,...,\mathcal{K}_n\right)=\bigoplus_{\hat{j}=1}^{n}\Phi_{\hat{j}}\mathcal{K}_{\hat{j}}$$

$$=\left\{\begin{array}{c}\left(1-\prod_{\hat{j}=1}^{\tau}\left(1-a^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{\tau}\left(1-b^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{\tau}\left(a^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{\tau}\left(b^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right),\\ \left(1-\prod_{\hat{j}=1}^{\tau}\left(1-c^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{\tau}\left(1-d^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{\tau}\left(c^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{\tau}\left(d^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right)\end{array}\right\}.$$

Next, let Eq (3.2) is true for $n=\tau+1$,

$$BCFCWA\left(\mathcal{K}_1,...,\mathcal{K}_\tau\oplus\mathcal{K}_{\tau+1}\right)=\left(\bigoplus_{\hat{j}=1}^{\tau}\Phi_{\hat{j}}\mathcal{K}_{\hat{j}}\right)\oplus\left(\Phi_{\tau+1}\mathcal{K}_{\tau+1}\right)$$

$$=\left\{\begin{array}{c}\left(1-\prod_{\hat{j}=1}^{\tau}\left(1-a^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{\tau}\left(1-b^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{\tau}\left(a^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{\tau}\left(b^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right),\\ \left(1-\prod_{\hat{j}=1}^{\tau}\left(1-c^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{\tau}\left(1-d^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{\tau}\left(c^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{\tau}\left(d^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right)\end{array}\right\}$$

$$\oplus\left\{\begin{array}{c}\left(1-\left(1-a^+_{\mathcal{K}_{\tau+1}}\right)^{\Phi_{\tau+1}}+i\left(1-\left(1-b^+_{\mathcal{K}_{\tau+1}}\right)^{\Phi_{\tau+1}}\right),-\left(a^-_{\mathcal{K}_{\tau+1}}\right)^{\Phi_{\tau+1}}+i\left(-\left(b^-_{\mathcal{K}_{\tau+1}}\right)^{\Phi_{\tau+1}}\right)\right),\\ \left(1-\left(1-c^+_{\mathcal{K}_{\tau+1}}\right)^{\Phi_{\tau+1}}+i\left(1-\left(1-d^+_{\mathcal{K}_{\tau+1}}\right)^{\Phi_{\tau+1}}\right),-\left(c^-_{\mathcal{K}_i}\right)^{\Phi_{\tau+1}}+i\left(-\left(d^-_{\mathcal{K}_{\tau+1}}\right)^{\Phi_{\tau+1}}\right)\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left(1-\prod_{\hat{j}=1}^{\tau+1}\left(1-a^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{\tau+1}\left(1-b^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{\tau+1}\left(a^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{\tau+1}\left(b^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right),\\ \left(1-\prod_{\hat{j}=1}^{\tau+1}\left(1-c^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{\tau+1}\left(1-d^+_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{\tau+1}\left(c^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{\tau+1}\left(d^-_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right)\end{array}\right\},$$

which show that Eq (3.2) true for $n=\tau+1$. Hence, the given result is hold for $n\geq 1$.

The BCFCWA operator satisfied the following properties.

Theorem 3.3 (**Idempotency**). *Let* $\mathcal{K}_{\hat{j}} = \left(a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weight vector* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then*

$$BCFCWA(\mathcal{K}_1, ..., \mathcal{K}_n) = \mathcal{K}. \tag{3.3}$$

Proof. As we know that;

$$\begin{aligned}
BCFCWA\left(\mathcal{K}_1, ..., \mathcal{K}_n\right) &= \bigoplus_{\hat{j}=1}^{n} \Phi_{\hat{j}} \mathcal{K}_{\hat{j}} \\
&= \left\{ \begin{array}{l} \left(1 - \prod_{\hat{j}=1}^{n}\left(1 - a^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(1 - \prod_{\hat{j}=1}^{n}\left(1 - b^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right), - \prod_{\hat{j}=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(- \prod_{\hat{j}=1}^{n}\left(b^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(1 - \prod_{\hat{j}=1}^{n}\left(1 - c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(1 - \prod_{\hat{j}=1}^{n}\left(1 - d^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right), - \prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(- \prod_{\hat{j}=1}^{n}\left(d^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\} \\
&\quad \left\{ \begin{array}{l} \left(1 - \left(1 - a^{+}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}} + i\left(1 - \left(1 - b^{+}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}}\right), - \left(a^{-}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}} + i\left(- \left(b^{-}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}}\right)\right), \\ \left(1 - \left(1 - c^{+}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}} + i\left(1 - \left(1 - d^{+}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}}\right), - \left(c^{-}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}} + i\left(- \left(d^{-}_{\mathcal{K}}\right)^{\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}}\right)\right) \end{array} \right\} \\
&= \left(\left\langle a^{+}_{\mathcal{K}} + ib^{+}_{\mathcal{K}}, a^{-}_{\mathcal{K}} + ib^{-}_{\mathcal{K}}\right\rangle, \left\langle c^{+}_{\mathcal{K}} + id^{+}_{\mathcal{K}},, c^{-}_{\mathcal{K}} + id^{-}_{\mathcal{K}}\right\rangle\right) \\
&= \mathcal{K}.
\end{aligned}$$

Theorem 3.4 (**Monotonicity**). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right)$ *and* $\mathcal{K}^{/}_{i} = \left(\left\langle a^{/+}_{\mathcal{K}_{\hat{j}}} + ib^{/+}_{\mathcal{K}_{\hat{j}}}, a^{/-}_{\mathcal{K}_{\hat{j}}} + ib^{/-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{/+}_{\mathcal{K}_{\hat{j}}} + id^{/+}_{\mathcal{K}_{\hat{j}}}, c^{/-}_{\mathcal{K}_{\hat{j}}} + id^{/-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weight vector* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*, if* $a^{+}_{\mathcal{K}_{\hat{j}}} \geq a^{/+}_{\mathcal{K}_{\hat{j}}}, ib^{+}_{\mathcal{K}_{\hat{j}}} \geq ib^{/+}_{\mathcal{K}_{\hat{j}}}, ib^{-}_{\mathcal{K}_{\hat{j}}} \leq ib^{/-}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} \leq a^{/-}_{\mathcal{K}_{\hat{j}}}$*,* $c^{+}_{\mathcal{K}_{\hat{j}}} \geq c^{/+}_{\mathcal{K}_{\hat{j}}}, id^{+}_{\mathcal{K}_{\hat{j}}} \geq id^{/+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} \leq c^{/-}_{\mathcal{K}_{\hat{j}}}$*, and* $id^{-}_{\mathcal{K}_{\hat{j}}} \leq id^{/-}_{\mathcal{K}_{\hat{j}}}$*. Then,*

$$BCFCWA(\mathcal{K}_1, ..., \mathcal{K}_n) \geq BCFCWA(\mathcal{K}^{/}_1, ..., \mathcal{K}^{/}_n). \tag{3.4}$$

Proof. We know that $a^{+}_{\mathcal{K}_{\hat{j}}} \geq a^{/+}_{\mathcal{K}_{\hat{j}}}, ib^{+}_{\mathcal{K}_{\hat{j}}} \geq ib^{/+}_{\mathcal{K}_{\hat{j}}}, ib^{-}_{\mathcal{K}_{\hat{j}}} \leq ib^{/-}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} \leq a^{/-}_{\mathcal{K}_{\hat{j}}}$*,* $c^{+}_{\mathcal{K}_{\hat{j}}} \geq c^{/+}_{\mathcal{K}_{\hat{j}}}, id^{+}_{\mathcal{K}_{\hat{j}}} \geq id^{/+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} \leq c^{/-}_{\mathcal{K}_{\hat{j}}}$*, and* $id^{-}_{\mathcal{K}_{\hat{j}}} \leq id^{/-}_{\mathcal{K}_{\hat{j}}}$*. Then,*

$$\begin{aligned}
1 - a^{+}_{\mathcal{K}_{\hat{j}}} &\leq 1 - a^{/+}_{\mathcal{K}_{\hat{j}}} \\
&\implies 1 - \prod_{\hat{j}=1}^{n}\left(1 - a^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \geq 1 - \prod_{\hat{j}=1}^{n}\left(1 - a^{/+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}.
\end{aligned}$$

And

$$\prod_{\hat{j}=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \leq \prod_{\hat{j}=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}.$$

For imaginary part

$$\left(1 - \prod_{\hat{j}=1}^{n}\left(1 - ib^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right) \geq \left(1 - \prod_{\hat{j}=1}^{n}\left(1 - ib^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right).$$

And

$$\prod_{\hat{j}=1}^{n}\left(ib^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \leq \prod_{\hat{j}=1}^{n}\left(ib^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}.$$

Similarly, we find for the credibility degree,

$$1-\prod_{\hat{j}=1}^{n}\left(1-c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \geq 1-\prod_{\hat{j}=1}^{n}\left(1-c^{/+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}$$

and

$$\prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \leq \prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}.$$

For imaginary part

$$\left(1-\prod_{\hat{j}=1}^{n}\left(1-id^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right) \geq \left(1-\prod_{\hat{j}=1}^{n}\left(1-id^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right).$$

And

$$\prod_{\hat{j}=1}^{n}\left(id^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \leq \prod_{\hat{j}=1}^{n}\left(id^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}.$$

By the combination of real and imaginary parts, we get

$$\left\{\begin{array}{c}\left(1-\prod_{\hat{j}=1}^{n}\left(1-a^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-b^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(b^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right),\\ \left(1-\prod_{\hat{j}=1}^{n}\left(1-c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-d^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(d^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right)\end{array}\right\}$$

$$\geq \left\{\begin{array}{c}\left(1-\prod_{\hat{j}=1}^{n}\left(1-a^{/+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-b^{/+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(a^{/-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(b^{/-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right),\\ \left(1-\prod_{\hat{j}=1}^{n}\left(1-c^{/+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-d^{/+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(c^{/-}_{\mathcal{K}_{i}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(d^{/-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right)\end{array}\right\}.$$

We assume that, $BCFCWA(\mathcal{K}_1, ..., \mathcal{K}_n) = \mathcal{K}_1$ and $BCFCWA(\mathcal{K}^{/}_1, ..., \mathcal{K}^{/}_n) = \mathcal{K}^{/}_1$. So, utilized Eq (2.6), we get

$$Sc(\mathcal{K}_1) \geq Sc(\mathcal{K}^{/}_1).$$

Then, we have two possibility,

1) When, $Sc(\mathcal{K}_1) \geq Sc(\mathcal{K}^{/}_1)$, we get

$$BCFCWA(\mathcal{K}_1, ..., \mathcal{K}_n) \geq BCFCRWA(\mathcal{K}^{/}_1, ..., \mathcal{K}^{/}_n).$$

2) When, $Sc(\mathcal{K}_1) = Sc(\mathcal{K}^{/}_1)$, we get

$$\left\{\begin{array}{c}\left(1-\prod_{\hat{j}=1}^{n}\left(1-a^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-b^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(b^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right),\\ \left(1-\prod_{\hat{j}=1}^{n}\left(1-c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{i=1}^{n}\left(1-d^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(d^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right)\end{array}\right\}$$

$$= \left\{ \begin{array}{c} \left(1-\prod_{\hat{j}=1}^{n}\left(1-a_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{i=1}^{n}\left(1-b_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(a_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(b_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(1-\prod_{\hat{j}=1}^{n}\left(1-c_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-d_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(c_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(d_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}.$$

We utilized the accuracy function because the score functions are equal.

$$\left\{ \begin{array}{c} \left(1-\prod_{\hat{j}=1}^{n}\left(1-a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-b_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(a_{\mathcal{K}_{\hat{j}}}^{-}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(b_{\mathcal{K}_{\hat{j}}}^{-}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(1-\prod_{\hat{j}=1}^{n}\left(1-c_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-d_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(c_{\mathcal{K}_{\hat{j}}}^{-}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(d_{\mathcal{K}_{\hat{j}}}^{-}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}$$

$$\geq \left\{ \begin{array}{c} \left(1-\prod_{i=1}^{n}\left(1-a_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-b_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}\right),-\prod_{i=1}^{n}\left(a_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{\hat{j}}}+i\left(-\prod_{\hat{j}=1}^{n}\left(b_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(1-\prod_{\hat{j}=1}^{n}\left(1-c_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}+i\left(1-\prod_{\hat{j}=1}^{n}\left(1-d_{\mathcal{K}_{\hat{j}}}^{/+}\right)^{\Phi_{\hat{j}}}\right),-\prod_{\hat{j}=1}^{n}\left(c_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{i}}+i\left(-\prod_{\hat{j}=1}^{n}\left(d_{\mathcal{K}_{\hat{j}}}^{/-}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}.$$

From cases (1) and (2), we have the required proof.

Theorem 3.5 (Boundedness). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a_{\mathcal{K}_{\hat{j}}}^{+} + ib_{\mathcal{K}_{\hat{j}}}^{+}, a_{\mathcal{K}_{\hat{j}}}^{-} + ib_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle, \left\langle c_{\mathcal{K}_{\hat{j}}}^{+} + id_{\mathcal{K}_{\hat{j}}}^{+}, c_{\mathcal{K}_{\hat{j}}}^{-} + id_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be a set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_i = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*, if* $\mathcal{K}_{\hat{j}}^{+}, \mathcal{K}_{\hat{j}}^{-}$ *are the maximum and minimum BCFCNs. Then,*

$$\mathcal{K}_{\hat{j}}^{+} \leq BCFCWA(\mathcal{K}_1, ..., \mathcal{K}_n) \leq \mathcal{K}_{\hat{j}}^{-}. \tag{3.5}$$

Proof. We studied two cases (for real and imagined components) separately for MG and credibility degree.

(1) For membership degree, we have

$$\begin{aligned} & \left(1-\prod_{\hat{j}=1}^{n}\left(1-\min_{1\leq\hat{j}\leq n}a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}\right) \leq \left(1-\prod_{\hat{j}=1}^{n}\left(1-a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}\right) \\ \leq\ & \left(1-\prod_{\hat{j}=1}^{n}\left(1-\max_{1\leq\hat{j}\leq n}a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}\right) \\ \Longrightarrow\ & \left(1-\left(1-\min_{1\leq\hat{j}\leq n}a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}}\right) \leq \left(1-\prod_{\hat{j}=1}^{n}\left(1-a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}\right) \\ \leq\ & \left(1-\left(1-\max_{1\leq\hat{j}\leq n}a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}}\right). \end{aligned}$$

As $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$, so

$$\Longrightarrow \min_{1\leq\hat{j}\leq n} a_{\mathcal{K}_{\hat{j}}}^{+} \leq \left(1-\prod_{\hat{j}=1}^{n}\left(1-a_{\mathcal{K}_{\hat{j}}}^{+}\right)^{\Phi_{\hat{j}}}\right) \leq \max_{1\leq\hat{j}\leq n} a_{\mathcal{K}_{\hat{j}}}^{+}.$$

Similarly, we can prove for $ib_{\mathcal{K}_{\hat{j}}}^{+}$, and

$$
\begin{aligned}
&\prod_{\hat{j}=1}^{n} \min_{1\le \hat{j}\le n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \le \prod_{i=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \le \prod_{\hat{j}=1}^{n} \max_{1\le \hat{j}\le n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \\
\Longrightarrow \quad & \min_{1\le \hat{j}\le n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}} \le \prod_{\hat{j}=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \\
\le \quad & \max_{1\le \hat{j}\le n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}}.
\end{aligned}
$$

As $\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}} = 1$. Then

$$
\min_{1\le \hat{j}\le n} a^{-}_{\mathcal{K}_{\hat{j}}} \le \prod_{\hat{j}=1}^{n}\left(a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \le \max_{1\le \hat{j}\le n} a^{-}_{\mathcal{K}_{\hat{j}}}.
$$

Similarly, we can prove for $ib^{-}_{\mathcal{K}_i}$.
(2) For credibility degree, we have

$$
\begin{aligned}
&\left(1-\prod_{\hat{j}=1}^{n}\left(1-\min_{1\le \hat{j}\le n} c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right) \le \left(1-\prod_{\hat{j}=1}^{n}\left(1-c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right) \\
\le \quad & \left(1-\prod_{\hat{j}=1}^{n}\left(1-\max_{1\le \hat{j}\le n} c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right) \\
\Longrightarrow \quad & \left(1-\left(1-\min_{1\le \hat{j}\le n} c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}}\right) \le \left(1-\prod_{\hat{j}=1}^{n}\left(1-c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right) \\
\le \quad & \left(1-\left(1-\max_{1\le \hat{j}\le n} c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}}\right).
\end{aligned}
$$

As $\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}} = 1$, so

$$
\Longrightarrow \min_{1\le \hat{j}\le n} c^{+}_{\mathcal{K}_{\hat{j}}} \le \left(1-\prod_{\hat{j}=1}^{n}\left(1-c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right) \le \max_{1\le \hat{j}\le n} c^{+}_{\mathcal{K}_{\hat{j}}}.
$$

Similarly, we can prove for $id^{+}_{\mathcal{K}_{\hat{j}}}$. Additionally,

$$
\begin{aligned}
&\prod_{\hat{j}=1}^{n} \min_{1\le \hat{j}\le n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \le \prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \le \prod_{\hat{j}=1}^{n} \max_{1\le \hat{j}\le n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \\
\Longrightarrow \quad & \min_{1\le \hat{j}\le n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}} \le \prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \le \max_{1\le \hat{j}\le n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}}}.
\end{aligned}
$$

As $\sum_{\hat{j}=1}^{n}\Phi_{\hat{j}} = 1$. Then

$$
\min_{1\le \hat{j}\le n} c^{-}_{\mathcal{K}_{\hat{j}}} \le \prod_{\hat{j}=1}^{n}\left(c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} \le \max_{1\le \hat{j}\le n} c^{-}_{\mathcal{K}_{\hat{j}}}.
$$

Similarly, we have for $id^{-}_{\mathcal{K}_{\hat{j}}}$.

Then, combined the above two cases, by the score function, we obtain

$$Sc(\mathcal{K}_{\hat{j}}^{+}) \leq Sc(\mathcal{K}_{\hat{j}}) \leq Sc(\mathcal{K}_{\hat{j}}^{-}).$$

So, based on cases (1) and (2) and the definition of the score function, we get

$$\mathcal{K}^{+} \leq BCFCWA(\mathcal{K}_1, ..., \mathcal{K}_n) \leq \mathcal{K}^{-}.$$

3.2. Bipolar complex fuzzy credibility ordered weighted average operator

Definition 3.6. *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a_{\mathcal{K}_{\hat{j}}}^{+} + ib_{\mathcal{K}_{\hat{j}}}^{+}, a_{\mathcal{K}_{\hat{j}}}^{-} + ib_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle, \left\langle c_{\mathcal{K}_{\hat{j}}}^{+} + id_{\mathcal{K}_{\hat{j}}}^{+}, c_{\mathcal{K}_{\hat{j}}}^{-} + id_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle\right)(\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then, the BCFCOWA operator is determined as*

$$BCFCOWA\,(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigoplus_{\hat{j}=1}^{n} \Phi_{\hat{j}} \mathcal{K}_{\sigma(\hat{j})}, \tag{3.6}$$

and for $\mathcal{K}_{\sigma(\hat{j}-1)} \geq \mathcal{K}_{\sigma(\hat{j})}$ *the permutation is* $\sigma(1), ..., \sigma(n)$ *for all* $\hat{j} = 1, ..., n$*. Utilizing Definition 3.6, aggregated value for BCFCOWA operator is shown in Theorem 3.7.*

Theorem 3.7. *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a_{\mathcal{K}_{\hat{j}}}^{+} + ib_{\mathcal{K}_{\hat{j}}}^{+}, a_{\mathcal{K}_{\hat{j}}}^{-} + ib_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle, \left\langle c_{\mathcal{K}_{\hat{j}}}^{+} + id_{\mathcal{K}_{\hat{j}}}^{+}, c_{\mathcal{K}_{\hat{j}}}^{-} + id_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle\right)(\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then, BCFCOWA operator is obtained as*

$$BCFCOWA\,(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigoplus_{\hat{j}=1}^{n} \Phi_{\hat{j}} \mathcal{K}_{\sigma(\hat{j})} \tag{3.7}$$

$$= \left\{ \begin{array}{c} \left(1 - \prod_{\hat{j}=1}^{n} \left(1 - a_{\mathcal{K}_{\sigma(\hat{j})}}^{+}\right)^{\Phi_{\hat{j}}} + i\left(1 - \prod_{\hat{j}=1}^{n} \left(1 - b_{\mathcal{K}_{\sigma(\hat{j})}}^{+}\right)^{\Phi_{\hat{j}}}\right), - \prod_{\hat{j}=1}^{n} \left(a_{\mathcal{K}_{\sigma(\hat{j})}}^{-}\right)^{\Phi_{\hat{j}}} + i\left(- \prod_{\hat{j}=1}^{n} \left(b_{\mathcal{K}_{\sigma(\hat{j})}}^{-}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(1 - \prod_{\hat{j}=1}^{n} \left(1 - c_{\mathcal{K}_{\sigma(\hat{j})}}^{+}\right)^{\Phi_{\hat{j}}} + i\left(1 - \prod_{\hat{j}=1}^{n} \left(1 - d_{\mathcal{K}_{\sigma(\hat{j})}}^{+}\right)^{\Phi_{\hat{j}}}\right), - \prod_{\hat{j}=1}^{n} \left(c_{\mathcal{K}_{\sigma(\hat{j})}}^{-}\right)^{\Phi_{\hat{j}}} + i\left(- \prod_{\hat{j}=1}^{n} \left(d_{\mathcal{K}_{\sigma(\hat{j})}}^{-}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}$$

where $\sigma(1), ..., \sigma(n)$ *be the permutation of the* $(\hat{j} = 1, ..., n)$*, for each* $\mathcal{K}_{\sigma(\hat{j}-1)} \geq \mathcal{K}_{\sigma(\hat{j})}$ *for all* $(\hat{j} = 1, ..., n)$ *.*

Proof. Proof is follow from Theorem 3.2.

The BCFCOWA operator satisfied the following properties.

Theorem 3.8 (Idempotency). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a_{\mathcal{K}_{\hat{j}}}^{+} + ib_{\mathcal{K}_{\hat{j}}}^{+}, a_{\mathcal{K}_{\hat{j}}}^{-} + ib_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle, \left\langle c_{\mathcal{K}_{\hat{j}}}^{+} + id_{\mathcal{K}_{\hat{j}}}^{+}, c_{\mathcal{K}_{\hat{j}}}^{-} + id_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle\right)(\hat{j} = 1, ..., n)$ *be the set of BCFCNs with the weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then*

$$BCFCOWA(\mathcal{K}_1, ..., \mathcal{K}_n) = \mathcal{K}. \tag{3.8}$$

Proof. Similar to Theorem 4.12.

Theorem 3.9 (Monotonicity). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a_{\mathcal{K}_{\hat{j}}}^{+} + ib_{\mathcal{K}_{\hat{j}}}^{+}, a_{\mathcal{K}_{\hat{j}}}^{-} + ib_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle, \left\langle c_{\mathcal{K}_{\hat{j}}}^{+} + id_{\mathcal{K}_{\hat{j}}}^{+}, c_{\mathcal{K}_{\hat{j}}}^{-} + id_{\mathcal{K}_{\hat{j}}}^{-}\right\rangle\right)$ *and* $\mathcal{K}_{\hat{j}}^{/} = \left(\left\langle a_{\mathcal{K}_{\hat{j}}}^{/+} + ib_{\mathcal{K}_{\hat{j}}}^{/+}, a_{\mathcal{K}_{\hat{j}}}^{/-} + ib_{\mathcal{K}_{\hat{j}}}^{/-}\right\rangle, \left\langle c_{\mathcal{K}_{\hat{j}}}^{/+} + id_{\mathcal{K}_{\hat{j}}}^{/+}, c_{\mathcal{K}_{\hat{j}}}^{/-} + id_{\mathcal{K}_{\hat{j}}}^{/-}\right\rangle\right)$ $(\hat{j} = 1, ..., n)$ *be the set of BCFCNs with the weight vector* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*, if* $a_{\mathcal{K}_{\hat{j}}}^{+} \geq a_{\mathcal{K}_{\hat{j}}}^{/+}, ib_{\mathcal{K}_{\hat{j}}}^{+} \geq ib_{\mathcal{K}_{\hat{j}}}^{/+}, ib_{\mathcal{K}_{\hat{j}}}^{-} \leq ib_{\mathcal{K}_{\hat{j}}}^{/-}, a_{\mathcal{K}_{\hat{j}}}^{-} \leq a_{\mathcal{K}_{\hat{j}}}^{/-}, c_{\mathcal{K}_{\hat{j}}}^{+} \geq c_{\mathcal{K}_{\hat{j}}}^{/+}, id_{\mathcal{K}_{\hat{j}}}^{+} \geq id_{\mathcal{K}_{\hat{j}}}^{/+}, c_{\mathcal{K}_{\hat{j}}}^{-} \leq c_{\mathcal{K}_{\hat{j}}}^{/-}$*, and* $id_{\mathcal{K}_{\hat{j}}}^{-} \leq id_{\mathcal{K}_{\hat{j}}}^{/-}$*. Then*

$$BCFCOWA(\mathcal{K}_1, ..., \mathcal{K}_n) \geq BCFCOWA(\mathcal{K}_1^{/}, ..., \mathcal{K}_n^{/}). \tag{3.9}$$

Proof. Similar to Theorem 3.4.

Theorem 3.10 (Boundedness). *Let*

$$\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right)$$

$(\hat{j} = 1, ..., n)$ be the set of BCFCNs with the weight vector $\Phi = (\Phi_1, ..., \Phi_n)^T$, such as $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ and $0 \leq \Phi_{\hat{j}} \leq 1$, if $\mathcal{K}^{+}_{\hat{j}}, \mathcal{K}^{-}_{\hat{j}}$ are the maximum and minimum BCFCNs. Then

$$\mathcal{K}^{+}_{\hat{j}} \leq BCFCOWA(\mathcal{K}_1, ..., \mathcal{K}_n) \leq \mathcal{K}^{-}_{\hat{j}}. \tag{3.10}$$

Proof. Similar to Theorem 3.5.

3.3. Bipolar complex fuzzy credibility hybrid average operator

Definition 3.11. *Let $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ be the set of BCFCN with weights $\Phi = (\Phi_1, ..., \Phi_n)^T$, such as $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ and $0 \leq \Phi_{\hat{j}} \leq 1$, and $\vartheta = (\vartheta_1, ..., \vartheta_n)^T$, such that $\sum_{\hat{j}=1}^{n} \vartheta_{\hat{j}} = 1$ and $0 \leq \vartheta_{\hat{j}} \leq 1$ be the associated weights of BCFCNs. Then, the BCFCHA operator is obtained as*

$$BCFCHA(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigoplus_{\hat{j}=1}^{n} \Phi_{\hat{j}} \mathcal{K}^{*}_{\sigma(\hat{j})}, \tag{3.11}$$

and for $\mathcal{K}_{\sigma(\hat{j}-1)} \geq \mathcal{K}_{\sigma(\hat{j})}$ the permutation is $\sigma(1), ..., \sigma(n)$ for all $(\hat{j} = 1, ..., n)$. Utilizing Definition 4.11, aggregated value for BCFCHA operator is shown in Theorem 4.12.

Theorem 3.12. *Let $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ be the set of BCFCNs with weights $\Phi = (\Phi_1, ..., \Phi_n)^T$, such as $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ and $0 \leq \Phi_{\hat{j}} \leq 1$, and $\vartheta = (\vartheta_1, ..., \vartheta_n)^T$, such as $\sum_{\hat{j}=1}^{n} \vartheta_{\hat{j}} = 1$ and $0 \leq \vartheta_{\hat{j}} \leq 1$ be the associated weight vector of the given set of BCFCNs. Then, BCFCHA operator is obtained as*

$$BCFCHA(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigoplus_{\hat{j}=1}^{n} \Phi_{\hat{j}} \mathcal{K}^{*}_{\sigma(\hat{j})} \tag{3.12}$$

$$= \left\{ \begin{array}{l} \left(1 - \prod_{\hat{j}=1}^{n} \left(1 - a^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(1 - \prod_{\hat{j}=1}^{n} \left(1 - b^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right), - \prod_{\hat{j}=1}^{n} \left(a^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(- \prod_{\hat{j}=1}^{n} \left(b^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(1 - \prod_{\hat{j}=1}^{n} \left(1 - c^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(1 - \prod_{\hat{j}=1}^{n} \left(1 - d^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right), - \prod_{\hat{j}=1}^{n} \left(c^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(- \prod_{\hat{j}=1}^{n} \left(d^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}$$

where $\sigma(1), ..., \sigma(n)$ be the permutation of the $(\hat{j} = 1, ..., n)$, for each $\mathcal{K}_{\sigma(\hat{j}-1)} \geq \mathcal{K}_{\sigma(\hat{j})}$ for all $(\hat{j} = 1, ..., n)$. The largest permutation value from the family of BCFCNs is represented by $\mathcal{K}^{*}_{\sigma(\hat{j})} = n\vartheta_{\hat{j}}\mathcal{K}_{\hat{j}}$, and n stands for the balancing coefficient.

Proof. Proof is follow from Theorem 3.2.

4. Bipolar complex fuzzy credibility geometric operator

Here, we are going to defined some aggregation operators like as, BCFCWG, BCFCOWG, and BCFCHG operators.

4.1. Bipolar complex credibility fuzzy weighted geometric operator

Definition 4.1. *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_1} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be a set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then, the BCFCWG operator is determined as*

$$BCFCWG(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigotimes_{\hat{j}=1}^{n} \left(\mathcal{K}_{\hat{j}}\right)^{\Phi_{\hat{j}}}, \tag{4.1}$$

utilizing Definition 4.1, aggregated value for BCFCWG operator is shown in Theorem refthE1.

Theorem 4.2. *Let* $\mathcal{K}_{\hat{j}} = \left(a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then, BCFCWG operator is obtained as*

$$BCFCWG(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigotimes_{\hat{j}=1}^{n} \left(\mathcal{K}_{\hat{j}}\right)^{\Phi_{\hat{j}}} \tag{4.2}$$

$$= \left\{ \begin{array}{c} \left(\prod_{\hat{j}=1}^{n} \left(a^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(\prod_{\hat{j}=1}^{n} \left(b^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right), -1 + \prod_{\hat{j}=1}^{n} \left(1 + a^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(-1 + \prod_{\hat{j}=1}^{n} \left(1 + b^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(\prod_{\hat{j}=1}^{n} \left(c^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(\prod_{\hat{j}=1}^{n} \left(d^{+}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right), -1 + \prod_{\hat{j}=1}^{n} \left(1 + c^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}} + i\left(-1 + \prod_{\hat{j}=1}^{n} \left(1 + d^{-}_{\mathcal{K}_{\hat{j}}}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}.$$

Proof. Proof is same as Theorem 3.2.

The BCFCWG operator satisfied the following properties.

Theorem 4.3 (**Idempotency**). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with the weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then,*

$$BCFCWG(\mathcal{K}_1, ..., \mathcal{K}_n) = \mathcal{K}. \tag{4.3}$$

Proof. Proof is same as Theorem 3.3.

Theorem 4.4 (**Monotonicity**). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right)$ *and* $\mathcal{K}^{/}_{\hat{j}} = \left(\left\langle a^{/+}_{\mathcal{K}_{\hat{j}}} + ib^{/+}_{\mathcal{K}_{\hat{j}}}, a^{/-}_{\mathcal{K}_{\hat{j}}} + ib^{/-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{/+}_{\mathcal{K}_{\hat{j}}} + id^{/+}_{\mathcal{K}_{\hat{j}}}, c^{/-}_{\mathcal{K}_{\hat{j}}} + id^{/-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weight vector* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*, if* $a^{+}_{\mathcal{K}_{\hat{j}}} \geq a^{/+}_{\mathcal{K}_{\hat{j}}}$, $ib^{+}_{\mathcal{K}_{\hat{j}}} \geq ib^{/+}_{\mathcal{K}_{\hat{j}}}$, $ib^{-}_{\mathcal{K}_{\hat{j}}} \leq ib^{/-}_{\mathcal{K}_{\hat{j}}}$, $a^{-}_{\mathcal{K}_{\hat{j}}} \leq a^{/-}_{\mathcal{K}_{\hat{j}}}$, $c^{+}_{\mathcal{K}_{\hat{j}}} \geq c^{/+}_{\mathcal{K}_{\hat{j}}}$, $id^{+}_{\mathcal{K}_{\hat{j}}} \geq id^{/+}_{\mathcal{K}_{\hat{j}}}$, $c^{-}_{\mathcal{K}_{\hat{j}}} \leq c^{/-}_{\mathcal{K}_{\hat{j}}}$*, and* $id^{-}_{\mathcal{K}_{\hat{j}}} \leq id^{/-}_{\mathcal{K}_{\hat{j}}}$*. Then,*

$$BCFCWG(\mathcal{K}_1, ..., \mathcal{K}_n) \geq BCFCWG(\mathcal{K}_1, ..., \mathcal{K}_n). \tag{4.4}$$

Proof. Proof is same as (3.4).

Theorem 4.5 (**Boundedness**). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right)$ $(\hat{j} = 1, ..., n)$ *be a set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*, if* $\mathcal{K}^{+}_{\hat{j}}, \mathcal{K}^{-}_{\hat{j}}$ *are the maximum and minimum BCFCNs. Then*

$$\mathcal{K}^{+}_{\hat{j}} \leq BCFCWG(\mathcal{K}_1, ..., \mathcal{K}_n) \leq \mathcal{K}^{-}_{\hat{j}}. \tag{4.5}$$

Proof. Proof is same as Theorem 3.5.

4.2. Bipolar complex fuzzy credibility ordered weighted geometric operator

Definition 4.6. *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weight vector* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then, the BCFCOWG operator is determined as*

$$BCIFOWG(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigotimes_{\hat{j}=1}^{n} \left(\mathcal{K}_{\sigma(\hat{j})}\right)^{\Phi_{\hat{j}}}, \tag{4.6}$$

and for $\mathcal{K}_{\sigma(\hat{j}-1)} \geq \mathcal{K}_{\sigma(\hat{j})}$ *the permutation is* $\sigma(1), ..., \sigma(n)$ *for all* $(\hat{j} = 1, ..., n)$ *. Utilizing Definition 4.6, aggregated value for BCFCOWG operator is shown in Theorem 4.7.*

Theorem 4.7. *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then, BCFCOWG operator is obtained as*

$$BCFCOWG(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigotimes_{\hat{j}=1}^{n} \left(\mathcal{K}_{\sigma(\hat{j})}\right)^{\Phi_{\hat{j}}} \tag{4.7}$$

$$= \left\{ \begin{array}{l} \left(\prod_{\hat{j}=1}^{n} \left(a^{-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(\prod_{\hat{j}=1}^{n} \left(b^{-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right), -1 + \prod_{\hat{j}=1}^{n} \left(1 + a^{+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(-1 + \prod_{\hat{j}=1}^{n} \left(1 + b^{+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(\prod_{\hat{j}=1}^{n} \left(c^{-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(\prod_{\hat{j}=1}^{n} \left(d^{-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right), -1 + \prod_{\hat{j}=1}^{n} \left(1 + c^{+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(-1 + \prod_{\hat{j}=1}^{n} \left(1 + d^{+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}$$

where $\sigma(1), ..., \sigma(n)$ *be the permutation of the* $(\hat{j} = 1, ..., n)$*, for each* $\mathcal{K}_{\sigma(\hat{j}-1)} \geq \mathcal{K}_{\sigma(\hat{j})}$ *for all* $(\hat{j} = 1, ..., n)$ *.*

Proof. Proof follows from Theorem 3.2.

The BCFCOWG operator satisfied the following properties.

Theorem 4.8 (**Idempotency**)**.** *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with the weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*. Then*

$$BCFCOWG(\mathcal{K}_1, ..., \mathcal{K}_n) = \mathcal{K}. \tag{4.8}$$

Proof. Similar to Theorem 3.3.

Theorem 4.9 (**Monotonicity**)**.** *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right)$ *and* $\mathcal{K}^{/}_{\hat{j}} = \left(\left\langle a^{/+}_{\mathcal{K}_{\hat{j}}} + ib^{/+}_{\mathcal{K}_{\hat{j}}}, a^{/-}_{\mathcal{K}_{\hat{j}}} + ib^{/-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{/+}_{\mathcal{K}_{\hat{j}}} + id^{/+}_{\mathcal{K}_{\hat{j}}}, c^{/-}_{\mathcal{K}_{\hat{j}}} + id^{/-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \leq \Phi_{\hat{j}} \leq 1$*, if* $a^{+}_{\mathcal{K}_{\hat{j}}} \geq a^{/+}_{\mathcal{K}_{\hat{j}}}, ib^{+}_{\mathcal{K}_{\hat{j}}} \geq ib^{/+}_{\mathcal{K}_{\hat{j}}}, ib^{-}_{\mathcal{K}_{\hat{j}}} \leq ib^{/-}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} \leq a^{/-}_{\mathcal{K}_{\hat{j}}}$*,* $c^{+}_{\mathcal{K}_{\hat{j}}} \geq c^{/+}_{\mathcal{K}_{\hat{j}}}, id^{+}_{\mathcal{K}_{\hat{j}}} \geq id^{/+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} \leq c^{/-}_{\mathcal{K}_{\hat{j}}}$*, and* $\hat{j}d^{-}_{\mathcal{K}_{\hat{j}}} \leq \hat{j}d^{/-}_{\mathcal{K}_{\hat{j}}}$*. Then*

$$BCFCOWG(\mathcal{K}_1, ..., \mathcal{K}_n) \geq BCFCOWG(\mathcal{K}^{/}_1, ..., \mathcal{K}^{/}_n). \tag{4.9}$$

Proof. Similar to Theorem 3.4.

Theorem 4.10 (**Boundedness**). *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right)$ $(\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weight are* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \le \Phi_{\hat{j}} \le 1$*, if* $\mathcal{K}^{+}_{\hat{j}}, \mathcal{K}^{-}_{\hat{j}}$ *are the maximum and minimum BCFCNs. Then*

$$\mathcal{K}^{+}_{\hat{j}} \le BCFCOWG(\mathcal{K}_1, ..., \mathcal{K}_n) \le \mathcal{K}^{-}_{\hat{j}}. \tag{4.10}$$

Proof. Similar to Theorem 3.5.

4.3. Bipolar complex fuzzy credibility hybrid geometric operator

Definition 4.11. *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \le \Phi_{\hat{j}} \le 1$*, and* $\vartheta = (\vartheta_1, ..., \vartheta_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \vartheta_{\hat{j}} = 1$ *and* $0 \le \vartheta_{\hat{j}} \le 1$ *be the associated weights of BCFCNs. Then, the BCFCHG operator is obtained as;*

$$BCFCHG\,(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigotimes_{\hat{j}=1}^{n} \left(\mathcal{K}^{*}_{\sigma(\hat{j})}\right)^{\Phi_{\hat{j}}}, \tag{4.11}$$

and $\mathcal{K}_{\sigma(\hat{j}-1)} \ge \mathcal{K}_{\sigma(\hat{j})}$ *the permutation is* $\sigma(1), ..., \sigma(n)$ *for all* $(\hat{j} = 1, ..., n)$*. Utilizing Definition 4.11, aggregated value of BCFCHG operator is given in Theorem 4.12.*

Theorem 4.12. *Let* $\mathcal{K}_{\hat{j}} = \left(\left\langle a^{+}_{\mathcal{K}_{\hat{j}}} + ib^{+}_{\mathcal{K}_{\hat{j}}}, a^{-}_{\mathcal{K}_{\hat{j}}} + ib^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle, \left\langle c^{+}_{\mathcal{K}_{\hat{j}}} + id^{+}_{\mathcal{K}_{\hat{j}}}, c^{-}_{\mathcal{K}_{\hat{j}}} + id^{-}_{\mathcal{K}_{\hat{j}}}\right\rangle\right) (\hat{j} = 1, ..., n)$ *be the set of BCFCNs with weights* $\Phi = (\Phi_1, ..., \Phi_n)^T$ *, such as* $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}} = 1$ *and* $0 \le \Phi_{\hat{j}} \le 1$*, and* $\vartheta = (\vartheta_1, ..., \vartheta_n)^T$ *, such that* $\sum_{\hat{j}=1}^{n} \vartheta_{\hat{j}} = 1$ *and* $0 \le \vartheta_{\hat{j}} \le 1$ *be the associated weights of BCFCNs. Then, BCFCHG operator is obtained as*

$$BCFCHG\,(\mathcal{K}_1, ..., \mathcal{K}_n) = \bigotimes_{\hat{j}=1}^{n} \left(\mathcal{K}^{*}_{\sigma(\hat{j})}\right)^{\Phi_{\hat{j}}} \tag{4.12}$$

$$= \left\{ \begin{array}{c} \left(\prod_{\hat{j}=1}^{n} \left(a^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(\prod_{\hat{j}=1}^{n} \left(b^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right), -1 + \prod_{\hat{j}=1}^{n} \left(1 + a^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(-1 + \prod_{\hat{j}=1}^{n} \left(1 + b^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right)\right), \\ \left(\prod_{\hat{j}=1}^{n} \left(c^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(\prod_{\hat{j}=1}^{n} \left(d^{*-}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right), -1 + \prod_{\hat{j}=1}^{n} \left(1 + c^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}} + i\left(-1 + \prod_{\hat{j}=1}^{n} \left(1 + d^{*+}_{\mathcal{K}_{\sigma(\hat{j})}}\right)^{\Phi_{\hat{j}}}\right)\right) \end{array} \right\}$$

where $\sigma(1), ..., \sigma(n)$ *be the permutation of the* $(\hat{j} = 1, ..., n)$*, for each* $\mathcal{K}_{\sigma(\hat{j}-1)} \ge \mathcal{K}_{\sigma(\hat{j})}$ *for all* $(\hat{j} = 1, ..., n)$*. The largest permutation value from the set of BCFCNs is represented by* $\mathcal{K}^{*}_{\sigma(\hat{j})} = \left(\mathcal{K}_{\hat{j}}\right)^{n\vartheta_{\hat{j}}}$*, and n stands for the balancing coefficient.*

Proof. Proof is follow from Theorem 3.2.

5. An approach for MCGDM based on bipolar complex fuzzy credibility information

In this section, we construct an approach to tackle the MCGDM problem using the proposed bipolar complex intuitionistic fuzzy set. Assume that $\acute{E} = \left\{\acute{E}_1, ..., \acute{E}_n\right\}$ is the collection of n criteria and $\wp = \{\wp_1, ..., \wp_m\}$ is the set of m alternatives for a MCGDM problem. Let the weights for the criterion $\acute{E}_{\hat{j}}$ as $\Phi = (\Phi_1, ..., \Phi_n)^T$, such as $\sum_{\hat{j}=1}^{n} \Phi_{\hat{j}}$ and $0 \le \Phi_{\hat{j}}$. The following are the key steps:

Step 1: Create a decision matrix using the assessment data collected in accordance with the criteria $\acute{E}_{\hat{j}}$ for qualified experts for each alternative $\wp$;

$$\mathbf{M} = \begin{bmatrix} \Im_{11} & \Im_{12} & . & . & \Im_{1n} \\ \Im_{21} & \Im_{22} & . & . & \Im_{2n} \\ \Im_{31} & \Im_{32} & . & . & \Im_{3n} \\ . & . & . & . & . \\ \Im_{m1} & \Im_{m2} & . & . & \Im_{mn} \end{bmatrix}.$$

Step 2: Evaluate the aggregate information given by experts with the help of BCFCWA operators.
Step 3: Evaluate the aggregate information using BCFCOWA operators.
Step 4: Evaluate the score value of the information obtained with the help of operators.
Step 5: Give alternatives a ranking based on the score value.

6. Example

As per predictions, chronic diseases (CDs) can cause one-third of all deaths in the world and are one of the main causes of death and disability in the world. There are numerous diseases related with CDs, like diabetes, hypertension heart disease and cardiovascular diseases (CVDs), some of them present higher risks than others (in particular, CVD). It is one of the main causes of disability and presents threats to population vitality. Because it causes diseases like hypertension, arrhythmia, stroke, and heart attacks, as well as deaths, CVD is a global crisis. The diagnosis, monitoring, and treatment of CVD are necessary since it is a life-threatening disease. However, there are also other problems that can make diagnosis and treatment more difficult, like the lack of qualified cardiologists or patients who live in remote areas far from hospitals. Modern technologies, such as the Internet of Things (IoT), are utilized to monitor patients with CD in order to address these issues. IoT has set the stage for a variety of uses, and it has played a remarkable role in telemedicine in the health care field and in managing patients who are located elsewhere.

In this real life example we have to select the best hospital using our proposed work. There are four alternatives (Hospitals) and six criteria with the weights are $\Phi = (0.20, 0.30, 0.15, 0.25, 0.10)^T$, which is discussed as follows:

(1) Surgical Doctors (SD): In this type of criteria, we have discussed a special groups of doctors, which have the ability to treat all the surgical patient.

(2) Surgical Team (ST): In this criteria, there are one head doctor and having surgical doctors to handle the surgical problems.

(3) Surgical Room (SR): This criteria is about surgical room which is use for special purpose for any surgical patient.

(4) Oxygen Supplier (OS): In this criteria the surgical team can provide oxygen to any surgical patient.

(5) Send Ambulance (SA): This type of criteria is important to carry any surgical patient to the surgical team.

(6) Prove Medications (PM): In this types of criteria the surgical team provide specific medicine to any surgical patient.

Let, there are their experts with the weight vector $\kappa = (0.3, 0.4, 0.3)^T$, provided their individual assessment for each option and the corresponding assessments are presented in Tables 1–3.

Step 1: The total information given by experts for each alternative $\wp_i$ under the criteria $\acute{E}_{\jmath}$ in Tables 1–3.

Step 2: Using the BCFCWA operators and expert-provided data against their weights.

Table 1. BCF evaluation information given by expert one.

	$\acute{E}_1$	$\acute{E}_2$
$\wp_1$	$\begin{pmatrix} \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \\ \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.1 + i0.8, -0.1 - i0.8 \rangle, \\ \langle 0.8 + i0.1, -0.6 - i0.1 \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.2 + i0.3, -0.8 - i0.3 \rangle, \\ \langle 0.8 + i0.6, -0.2 - i0.6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \\ \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.5 + i0.7, -0.4 - i0.3 \rangle, \\ \langle 0.3 + i0.3, -0.6 - i0.6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.6 + i0.7, -0.6 - i0.2 \rangle, \\ \langle 0.4 + i0.2, -0.3 - i0.7 \rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.6 + i0.5, -0.3 - i0.2 \rangle, \\ \langle 0.4 + i0.3, -0.7 - i0.8 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.5, -0.3 - i0.5 \rangle, \\ \langle 0.5 + i0.4, -0.7 - i0.5 \rangle \end{pmatrix}$
	$\acute{E}_3$	$\acute{E}_4$
$\wp_1$	$\begin{pmatrix} \langle 0.3 + i0.3, -0.6 - i0.6 \rangle, \\ \langle 0.5 + i0.7, -0.4 - i0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.4, -0.3 - i0.2 \rangle, \\ \langle 0.7 + i0.4, -0.6 - i0.7 \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.4 + i0.2, -0.3 - i0.7 \rangle, \\ \langle 0.6 + i0.7, -0.6 - i0.2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.5 + i0.4, -0.7 - i0.5 \rangle, \\ \langle 0.2 + i0.5, -0.3 - i0.5 \rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \\ \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.4 + i0.2, -0.3 - i0.7 \rangle, \\ \langle 0.6 + i0.7, -0.6 - i0.2 \rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.8 + i0.6, -0.2 - i0.6 \rangle, \\ \langle 0.2 + i0.3, -0.8 - i0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.7 + i0.4, -0.6 - i0.7 \rangle, \\ \langle 0.2 + i0.4, -0.3 - i0.2 \rangle \end{pmatrix}$
	$\acute{E}_5$	$\acute{E}_6$
$\wp_1$	$\begin{pmatrix} \langle 0.4 + i0.3, -0.7 - i0.8 \rangle, \\ \langle 0.6 + i0.5, -0.3 - i0.2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.8 + i0.1, -0.6 - i0.1 \rangle, \\ \langle 0.1 + i0.8, -0.1 - i0.8 \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.3 + i0.3, -0.6 - i0.6 \rangle, \\ \langle 0.5 + i0.7, -0.4 - i0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.4, -0.3 - i0.2 \rangle, \\ \langle 0.7 + i0.4, -0.6 - i0.7 \rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.8 + i0.1, -0.6 - i0.1 \rangle, \\ \langle 0.1 + i0.8, -0.1 - i0.8 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.3, -0.8 - i0.3 \rangle, \\ \langle 0.8 + i0.6, -0.2 - i0.6 \rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.5 + i0.4, -0.7 - i0.5 \rangle, \\ \langle 0.2 + i0.5, -0.3 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.6 + i0.5, -0.3 - i0.2 \rangle, \\ \langle 0.4 + i0.3, -0.7 - i0.8 \rangle \end{pmatrix}$

Table 2. BCF evaluation information given by expert two.

	$\acute{E}_1$	$\acute{E}_2$
$\wp_1$	$\begin{pmatrix} \langle 0.2 + i0.4, -0.3 - i0.2 \rangle, \\ \langle 0.7 + i0.4, -0.6 - i0.7 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.1 + i0.8, -0.1 - i0.8 \rangle, \\ \langle 0.8 + i0.1, -0.6 - i0.1 \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.2 + i0.5, -0.3 - i0.5 \rangle, \\ \langle 0.5 + i0.4, -0.7 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \\ \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.5 + i0.7, -0.4 - i0.3 \rangle, \\ \langle 0.3 + i0.3, -0.6 - i0.6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.6 + i0.7, -0.6 - i0.2 \rangle, \\ \langle 0.4 + i0.2, -0.3 - i0.7 \rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.8 + i0.1, -0.6 - i0.1 \rangle, \\ \langle 0.1 + i0.8, -0.1 - i0.8 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.3, -0.8 - i0.3 \rangle, \\ \langle 0.8 + i0.6, -0.2 - i0.6 \rangle \end{pmatrix}$
	$\acute{E}_3$	$\acute{E}_4$
$\wp_1$	$\begin{pmatrix} \langle 0.3 + i0.3, -0.6 - i0.6 \rangle, \\ \langle 0.5 + i0.7, -0.4 - i0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.4 + i0.2, -0.3 - i0.7 \rangle, \\ \langle 0.6 + i0.7, -0.6 - i0.2 \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.2 + i0.3, -0.8 - i0.3 \rangle, \\ \langle 0.8 + i0.6, -0.2 - i0.6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.5, -0.3 - i0.5 \rangle \\ \langle 0.5 + i0.4, -0.7 - i0.5 \rangle, \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \\ \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.4 + i0.2, -0.3 - i0.7 \rangle, \\ \langle 0.6 + i0.7, -0.6 - i0.2 \rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.3 + i0.3, -0.6 - i0.6 \rangle, \\ \langle 0.5 + i0.7, -0.4 - i0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.7 + i0.4, -0.6 - i0.7 \rangle, \\ \langle 0.2 + i0.4, -0.3 - i0.2 \rangle \end{pmatrix}$
	$\acute{E}_5$	$\acute{E}_6$
$\wp_1$	$\begin{pmatrix} \langle 0.6 + i0.5, -0.3 - i0.2 \rangle \\ \langle 0.4 + i0.3, -0.7 - i0.8 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle 0.8 + i0.1, -0.6 - i0.1 \rangle, \\ \langle 0.1 + i0.8, -0.1 - i0.8 \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.5 + i0.4, -0.7 - i0.5 \rangle, \\ \langle 0.2 + i0.5, -0.3 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.7 + i0.4, -0.6 - i0.7 \rangle \\ \langle 0.2 + i0.4, -0.3 - i0.2 \rangle, \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.6 + i0.5, -0.3 - i0.2 \rangle, \\ \langle 0.4 + i0.3, -0.7 - i0.8 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \\ \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.8 + i0.6, -0.2 - i0.6 \rangle, \\ \langle 0.2 + i0.3, -0.8 - i0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.6 + i0.5, -0.3 - i0.2 \rangle, \\ \langle 0.4 + i0.3, -0.7 - i0.8 \rangle \end{pmatrix}$

Table 3. BCF evaluation information given by expert three.

	$\acute{E}_1$	$\acute{E}_2$
$\wp_1$	$\begin{pmatrix} \langle 0.7 + i0.4, -0.6 - i0.7 \rangle \\ \langle 0.2 + i0.4, -0.3 - i0.2 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle 0.8 + i0.1, -0.6 - i0.1 \rangle \\ \langle 0.1 + i0.8, -0.1 - i0.8 \rangle, \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.2 + i0.3, -0.8 - i0.3 \rangle, \\ \langle 0.8 + i0.6, -0.2 - i0.6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \\ \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.4 + i0.2, -0.3 - i0.7 \rangle, \\ \langle 0.6 + i0.7, -0.6 - i0.2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.8 + i0.1, -0.6 - i0.1 \rangle, \\ \langle 0.1 + i0.8, -0.1 - i0.8 \rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.6 + i0.7, -0.6 - i0.2 \rangle \\ \langle 0.4 + i0.2, -0.3 - i0.7 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.2, -0.3 - i0.5 \rangle, \\ \langle 0.5 + i0.4, -0.7 - i0.5 \rangle \end{pmatrix}$
	$\acute{E}_3$	$\acute{E}_4$
$\wp_1$	$\begin{pmatrix} \langle 0.5 + i0.7, -0.4 - i0.3 \rangle \\ \langle 0.3 + i0.3, -0.6 - i0.6 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle 0.5 + i0.7, -0.4 - i0.3 \rangle, \\ \langle 0.3 + i0.3, -0.6 - i0.6 \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.6 + i0.7, -0.6 - i0.2 \rangle, \\ \langle 0.4 + i0.2, -0.3 - i0.7 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.5, -0.3 - i0.5 \rangle \\ \langle 0.5 + i0.4, -0.7 - i0.5 \rangle, \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \\ \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.8 + i0.6, -0.2 - i0.6 \rangle \\ \langle 0.2 + i0.3, -0.8 - i0.3 \rangle, \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.7 + i0.4, -0.6 - i0.7 \rangle \\ \langle 0.2 + i0.4, -0.3 - i0.2 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.4, -0.3 - i0.2 \rangle \\ \langle 0.7 + i0.4, -0.6 - i0.7 \rangle, \end{pmatrix}$
	$\acute{E}_5$	$\acute{E}_6$
$\wp_1$	$\begin{pmatrix} \langle 0.6 + i0.5, -0.3 - i0.2 \rangle, \\ \langle 0.4 + i0.3, -0.7 - i0.8 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.1 + i0.8, -0.1 - i0.8 \rangle \\ \langle 0.8 + i0.1, -0.6 - i0.1 \rangle, \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.5 + i0.4, -0.7 - i0.5 \rangle, \\ \langle 0.2 + i0.5, -0.3 - i0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.4 + i0.3, -0.7 - i0.8 \rangle, \\ \langle 0.6 + i0.5, -0.3 - i0.2 \rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.6 + i0.5, -0.3 - i0.2 \rangle, \\ \langle 0.4 + i0.3, -0.7 - i0.8 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.7 + i0.5, -0.2 - i0.5 \rangle \\ \langle 0.3 + i0.4, -0.5 - i0.4 \rangle, \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.5 + i0.7, -0.4 - i0.3 \rangle \\ \langle 0.3 + i0.3, -0.6 - i0.6 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle 0.2 + i0.3, -0.8 - i0.3 \rangle \\ \langle 0.8 + i0.6, -0.2 - i0.6 \rangle, \end{pmatrix}$

Step 3: Using the BCFCOWA operator and the aggregated value of Table 4, with the weight vector $\Phi = (0.20, 0.30, 0.15, 0.25, 0.10)^T$. The total values of the alternatives $\wp_i (i = 1, ..., 4)$ as

$$\mathbb{R}_1 \quad (\langle 0.432 + i0.405, -0.228 - i0.321 \rangle, \langle 0.443 + i0.407, -0.327 - i0.332 \rangle).$$
$$\mathbb{R}_2 \quad (\langle 0.377 + i0.185, -0.328 - i0.326 \rangle, \langle 0.486 + i0.701, -0.325 - i0.341 \rangle).$$
$$\mathbb{R}_3 \quad (\langle 0.571 + i0.342, -0.690 - i0.325 \rangle, \langle 0.436 + i0.332, -0.170 - i0.438 \rangle).$$
$$\mathbb{R}_4 \quad (\langle 0.589 + i0.327, -0.489 - i0.211 \rangle, \langle 0.357 + i0.376, -0.329 - i0.462 \rangle).$$

Step 4: Analyze the score value of the data we collected using various operators.

Table 4. Aggregated values using BCFCWA operator.

	$\acute{E}_1$	$\acute{E}_2$
$\wp_1$	$\begin{pmatrix} \langle 0.361 + i0.491, -0.198 - i0.376\rangle, \\ \langle 0.475 + i0.403, -0.531 - i0.616\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.498 + i0.379, -0.291 - i0.436\rangle, \\ \langle 0.316 + i0.372, -0.187 - i0.439\rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.338 + i0.364, -0.229 - i0.375\rangle, \\ \langle 0.218 + i0.287, -0.251 - i0.336\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.382 + i0.385, -0.421 - i0.333\rangle, \\ \langle 0.498 + i0.405, -0.627 - i0.209\rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.223 + i0.590, -0.327 - i0.369\rangle, \\ \langle 0.319 + i0.497, -0.418 - i0.232\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.625 + i0.221, -0.339 - i0.195\rangle, \\ \langle 0.252 + i0.590, -0.393 - i0.284\rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.377 + i0.471, -0.370 - i0.305\rangle, \\ \langle 0.516 + i0.264, -0.409 - i0.260\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.442 + i0.408, -0.275 - i0.196\rangle, \\ \langle 0.610 + i0.384, -0.224 - i0.398\rangle \end{pmatrix}$
	$\acute{E}_3$	$\acute{E}_4$
$\wp_1$	$\begin{pmatrix} \langle 0.443 + i0.357, -0.509 - i0.287\rangle, \\ \langle 0.369 + i0.361, -0.432 - i0.339\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.542 + i0.215, -0.537 - i0.332\rangle, \\ \langle 0.437 + i0.118, -0.503 - i0.437\rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.400 + i0.325, -0.326 - i0.642\rangle, \\ \langle 0.455 + i0.452, -0.509 - i0.351\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.542 + i0.225, -0.417 - i0.499\rangle, \\ \langle 0.378 + i0.245, -0.352 - i0.350\rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.542 + i0.590, -0.431 - i0.376\rangle, \\ \langle 0.423 + i0.480, -0.548 - i0.327\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.246 + i0.324, -0.562 - i0.325\rangle, \\ \langle 0.411 + i0.325, -0.503 - i0.653\rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.480 + i0.531, -0.762 - i0.390\rangle, \\ \langle 0.437 + i0.531, -0.664 - i0.328\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.434 + i0.476, -0.438 - i0.332\rangle, \\ \langle 0.368 + i0.432, -0.666 - i0.598\rangle \end{pmatrix}$
	$\acute{E}_5$	$\acute{E}_6$
$\wp_1$	$\begin{pmatrix} \langle 0.243 + i0.437, -0.359 - i0.127\rangle, \\ \langle 0.392 + i0.315, -0.420 - i0.391\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.244 + i0.346, -0.248 - i0.432\rangle, \\ \langle 0.382 + i0.142, -0.266 - i0.258\rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.452 + i0.205, -0.474 - i0.349\rangle, \\ \langle 0.383 + i0.125, -0.532 - i0.302\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.240 + i0.351, -0.472 - i0.230\rangle, \\ \langle 0.147 + i0.501, -0.564 - i0.338\rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.252 + i0.325, -0.227 - i0.424\rangle, \\ \langle 0.419 + i0.189, -0.531 - i0.270\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.252 + i0.250, -0.241 - i0.360\rangle, \\ \langle 0.343 + i0.418, -0.358 - i0.137\rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.261 + i0.234, -0.252 - i0.235\rangle, \\ \langle 0.418 + i0.135, -0.363 - i0.363\rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.404 + i0.495, -0.365 - i0.462\rangle, \\ \langle 0.450 + i0.262, -0.259 - i0.316\rangle \end{pmatrix}$

Table 5. Score values of the alternatives.

Operators	$Sc(\acute{E}_1)$	$Sc(\acute{E}_2)$	$Sc(\acute{E}_3)$	$Sc(\acute{E}_4)$
BCFCWA	0.328	0.437	0.492	0.290

Step 5: Utilize the ranking values shown in Table 6 to determine which choice is the best.

Table 6. Alternative ranking.

BCFCWA	$\acute{E}_3 > \acute{E}_2 > \acute{E}_1 > \acute{E}_4$

Comparative study

The comparative study of determined techniques was discussed in this section, along with certain common operators based on accepted concept like as, BCFSs.

For this, we choose a few famous theories that are, Mahmood et al. [25], BCFSs and their applications in generalized similarity measures; Mahmood et al. [26], BCFS-based Hamacher aggregation information; Mahmood et al. [27], Dombi AOs under bipolar complex fuzzy information. Jana et al. [19], Bipolar fuzzy Dombi prioritized aggregation operators; Gao et al. [13], Dual hesitant bipolar fuzzy Hamacher aggregation operators.

The method described in [13, 19, 25–27] contains bipolar fuzzy set details, but the given model cannot be solved using this method. Reviewing Table 5 reveals that the methods now in use lack basic information and are unable to solve or rank the case that has been provided. Compared to other methods already in use, the strategy suggested in this study is more capable and dependable. The main analysis of the identified and proposed hypotheses is presented in Table 7.

Table 7. Ranking of the existing methods.

Methods	Score value				Ranking
	$Sc(\acute{E}_1)$	$Sc(\acute{E}_2)$	$Sc(\acute{E}_3)$	$Sc(\acute{E}_4)$	
Mahmood et al. [25]	0.760	0.792	0.826	0.731	$\acute{E}_3 > \acute{E}_2 > \acute{E}_1 > \acute{E}_4$
Mahmood et al. [26]	0.529	0.553	0.587	0.502	$\acute{E}_3 > \acute{E}_2 > \acute{E}_1 > \acute{E}_4$
Mahmood et al. [27]	0.831	0.842	0.889	0.782	$\acute{E}_3 > \acute{E}_1 > \acute{E}_4 > \acute{E}_2$
Jana et al. [19]	0.453	0.441	0.488	0.427	$\acute{E}_3 > \acute{E}_1 > \acute{E}_2 > \acute{E}_4$
Gao et al. [13]	0.663	0.684	0.699	0.650	$\acute{E}_3 > \acute{E}_2 > \acute{E}_1 > \acute{E}_4$

7. Conclusions

We define a number of operations, the scoring function, and the accuracy function for BCFCS in this paper. We also established several aggregation operators based on BCFC operational laws, such as BCFCWA, BCFCOWA, BCFCHA, BCFCWG, BCFCOWG, and BCFCHG operators. We explored the essential properties of the aforementioned operators' specific situations, such as idempotency, boundedness, and monotonous. Next, utilizing these operators, we solved the bipolar complex fuzzy MCGDM problem. To validate the interpreted techniques, we provided a numerical example of selecting fire extinguishers. Finally, we compared our findings to those of existing operators to establish the usefulness and applicability of our method.

In the future, we will use our proposed operators in different domains, like as, complex Pythagorean fuzzy set, complex picture fuzzy set, complex Spherical fuzzy set, and complex fractional orthotriple fuzzy set.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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