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Research article

Medical robotic engineering selection based on square root neutrosophic normal interval-valued sets and their aggregated operators

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Abstract: We introduce the concepts of multiple attribute decision-making (MADM) using square root neutrosophic normal interval-valued sets (SRNSNIVS). The square root neutrosophic (SRNS), interval-valued NS, and neutrosophic normal interval-valued (NSNIV) sets are extensions of SRNSNIVS. A historical analysis of several aggregating operations is presented in this article. In this article, we discuss a novel idea for the square root NSNIV weighted averaging (SRNSNIVWA), NSNIV weighted geometric (SRNSNIVWG), generalized SRNSNIV weighted averaging (GSRNSNIVWA), and generalized SRNSNIV weighted geometric (GSRNSNIVWG). Examples are provided for the use of Euclidean distances and Hamming distances. Various algebraic operations will be applied to these sets in this communication. This results in more accurate models and is closed to an integer Δ. A medical robotics system is described as combining computer science and machine tool technology. There are five types of robotics such as Pharma robotics, Robotic-assisted biopsy, Antibacterial nano-materials, AI diagnostics, and AI epidemiology. A robotics system should be selected based on four criteria, including robot controller features, affordable off-line programming software, safety codes, and the manufacturer's experience and reputation. Using expert judgments and criteria, we will be able to decide which options are the most appropriate. Several of the proposed and current models are also compared in order to demonstrate the reliability and usefulness of the models under study. Additionally, the findings of the study are fascinating and intriguing.

Keywords: aggregating operators; decision making; generalized square root neutrosophic normal interval-valued weighted averaging; generalized square root neutrosophic normal interval-valued weighted geometric
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1. Introduction

Decision-makers are increasingly having difficulty identifying the optimal solution to real-world problems as they become increasingly complex. It is possible to choose the best option despite the difficulty of deciding between the alternatives. A challenge many firms face is creating opportunities, objectives, and viewpoint constraints. As a result, individuals and groups must consider multiple objectives at the same time while engaging in decision-making (DM). The result is that we need to improve our ability to deal with difficult situations. A variety of methods have been used by researchers to contribute to this field of study. The uncertainties have been addressed by several theories, such as fuzzy set (FS) proposed by Zadeh [1], intuitionistic FS (IFS) proposed by Atanassov [2], Pythagorean FS (PFS) proposed by Yager [3] and NSS proposed by Smarandache [4]. An FS in which each element of the universe and the grades are called the membership degree (MD). As a result, Atanassov [2] introduces the IFS that requires the total value of the MD and non-membership degree (NMD) to be less than or equal to 1. If the MD and NMD sum to greater than 1, we might have difficulty with DM. Yazbek et al. [5] introduced the Novel Approach to Model the Economic Characteristics of an Organization by interval-valued complex Pythagorean fuzzy information. PFS logic was developed by Yager [3] as a generalization of IFS characterized by an MD and NMD with a square sum that must be less than or equal to 1.

There are many applications based on PFS, as discussed by Akram et al. [6–8]. This paper extends Rahman et al. [9] discussion of a geometric aggregation operator (AO) based on interval-valued PFS (IVPFS) to the framework of group DM. Peng et al. [10] proposed a Pythagorean fuzzy AO with interval values. As an alternative to group with MADM, Rahman et al. [11] have proposed a Pythagorean IV fuzzy Einstein AO. The idea of IVPFS for MADM with normal AOs were discussed by Yang et al. [12]. Shami et al. [13] studied the square root FS (SRFS) and its weighted AOs in the context of DM. The NSS is developed by Smarandache [4]. The neutral mind is known as "neutrosophy" and it is this neutrality that distinguishes FS from IFS. The NSS has a truth degree (TD), indeterminacy degree (ID) and falsehood degree (FD). There are three components of TD, ID and FD which lie between 0 and 1. The Pythagorean NSIV set (PNSIVS) were first discussed by Smarandache et al. [14]. An NSS with a single value is applied to medical diagnostics and context analysis [15]. We evaluated MCDM and MADM problems using Ejegwa [16] extended distance measures for IFSs, including Hamming distance (HD), Euclidean distance (ED) and normalized ED (NED). A MADM for Pythagorean NSNIV with AOs were introduced by Palanikumar et al. [17]. The majority of distance functions for PNSNIVSs as a generalization of the PNSIVS. Xu et al. [18] discussed the concept of regression prediction for fuzzy time series. Yang [19] introduced the notion of a class of fuzzy c-numbers clustering procedures for fuzzy data.

Peng et al. [20] proposed neutrosophic based on Under MABAC and TOPSIS. Zhang et al. [21] discussed generalizing PFS based on TOPSIS to MCDM. A number of practical MADM applications were discussed by Hwang et al. [22]. Jana et al. discussed a generalization of fuzzy soft sets [23]. A fuzzy bipolar MABAC-based MAGDM approach was presented by Jana [24]. An approach for robust linear MCDM with bipolar fuzzy soft AOs has been developed by Jana et al. [25]. The Pythagorean fuzzy dombi AOs have been introduced by Jana et al. [26]. Ullah et al. [27] discussed the concept of complex PFS with practical applications. The proposed AOs are based on the trapezoidal neutrosophic MADM by Jana et al. [28]. Jana et al. [29] was developed the neutrosophic dombi power AO. The MCDM approach was presented by Jana et al. [30] using single-valued trigonometric numbers (SVTrNs).

Recently, Hesitant FSs with real-life applications were discussed by [31, 32]. Lu et al. [33, 34] discussed the consensus progress for DM in social networks with incomplete probabilistic hesitant fuzzy and hesitant multiplicative preference relations. Yazdi et al. [35] introduced the application of an artificial intelligence in DM for the selection of maintenance strategy. Rojek et al. [36] discussed an artificial intelligence to supervise machine failures and support their repair. Huang et al. [37] discussed the concept of design alternative assessment and selection, a Z-cloud rough number-based BWM-MABAC model. Xiao [38] introduced q-rung orthopair fuzzy DM with new score function and best-worst method for manufacturer selection. Huang et al. [39] discussed the failure mode and effect analysis using T-spherical fuzzy maximizing deviation and combined comparison solution methods. Mahmood et al. [40] discussed the concept of Prioritized muirhead mean AOs under the complex single-valued neutrosophic settings and their application in MADM. The purpose of this work is to expand the definition of SRNSNIV sets. We obtain SRNSNIVS based on AOs. We will apply these operators to DM problems and develop a ranking based on them.

(1) There are new ED and HD measures introduced for SRNSNIVS.

(2) MADM and SRNSNIVN aggregation operators are shown in an example using the new definition.

(3) Determine positive and negative ideal values based on SRNSNIVWA, SRNSNIVWG, GSRNSNIVWA, and GSRNSNIVWG.

(4) In order to arrive at a result, Δ is used to make a decision.

The following seven sections make up the paper. A brief explanation of the related ideas can be found in Section 2. MADM using square root NSNIV number (SRNSNIVN) is discussed in Section 3. In Section 4 uses SRNSNIVN based on various distances. A few AOs are discussed in Section 5. The SRNSNIV set is discussed using MADM in Section 6, which includes a numerical example, analysis, and algorithm. Section 7 contains the conclusion.

2. Preliminaries

For our future studies, we will quickly review some fundamental terms in this section.

Definition 2.1. *[3] Let Ξ be the universe. The PFS $\mho$ in Ξ is $\mho = \left\{\mu, \langle \Theta_{\mho}^{\mathscr{T}}(\mu), \Theta_{\mho}^{\mathscr{F}}(\mu)\rangle \middle| \mu \in \Xi\right\}$, where $\Theta_{\mho}^{\mathscr{T}} : \Xi \to [0,1]$ and $\Theta_{\mho}^{\mathscr{F}} : \Xi \to [0,1]$ are denotes the MD and NMD of $\mu \in \Xi$ to $\mho$, respectively and $0 \leq (\Theta_{\mho}^{\mathscr{T}}(\mu))^2 + (\Theta_{\mho}^{\mathscr{F}}(\mu))^2 \leq 1$. For, $\mho = \langle \Theta_{\mho}^{\mathscr{T}}, \Theta_{\mho}^{\mathscr{F}} \rangle$ is represent a Pythagorean fuzzy number (PFN).*

Definition 2.2. *[13] The SRFS $\mho$ in Ξ is $\mho = \left\{\mu, \langle \Theta_{\mho}^{\mathscr{T}}(\mu), \Theta_{\mho}^{\mathscr{F}}(\mu)\rangle \middle| \mu \in \Xi\right\}$, where $\Theta_{\mho}^{\mathscr{T}} : \Xi \to [0,1]$ and $\Theta_{\mho}^{\mathscr{F}} : \Xi \to [0,1]$ are denotes the MD and NMD of $\mu \in \Xi$ to $\mho$, respectively and $0 \leq (\Theta_{\mho}^{\mathscr{T}}(\mu))^2 + \sqrt{\Theta_{\mho}^{\mathscr{F}}(\mu)} \leq 1$. For, $\mho = \langle \Theta_{\mho}^{\mathscr{T}}, \Theta_{\mho}^{\mathscr{F}} \rangle$ is represent a square root fuzzy number (SRFN).*

Definition 2.3. *[10] The PIVFS $\mho$ in Ξ is $\widetilde{\mho} = \left\{\mu, \left\langle \widetilde{\Theta_{\mho}^{\mathscr{T}}}(\mu), \widetilde{\Theta_{\mho}^{\mathscr{F}}}(\mu)\right\rangle \middle| \mu \in \Xi\right\}$, where $\widetilde{\Theta_{\mho}^{T}} : \Xi \to Int([0,1])$ and $\widetilde{\Theta_{\mho}^{F}} : \Xi \to Int([0,1])$ denotes the MD and NMD of $\mu \in \Xi$ to $\mho$, respectively, and $0 \leq (\Theta_{\mho}^{\mathscr{T}\mathscr{U}}(\mu))^2 + (\Theta_{\mho}^{\mathscr{F}\mathscr{U}}(\mu))^2 \leq 1$. For, $\widetilde{\mho} = \left\langle \left[\Theta_{\mho}^{\mathscr{T}\mathscr{L}}, \Theta_{\mho}^{\mathscr{T}\mathscr{U}}\right], \left[\Theta_{\mho}^{\mathscr{F}\mathscr{L}}, \Theta_{\mho}^{\mathscr{F}\mathscr{U}}\right]\right\rangle$ is represent a Pythagorean interval-valued fuzzy number (PIVFN).*

Definition 2.4. *[4] The NSS $\mho$ in Ξ is $\mho = \left\{\mu, \langle \Theta_{\mho}^{\mathscr{T}}(\mu), \Theta_{\mho}^{\mathscr{I}}(\mu), \Theta_{\mho}^{\mathscr{F}}(\mu)\rangle \middle| \mu \in \Xi\right\}$, $\Theta_{\mho}^{\mathscr{T}} : \Xi \to [0,1]$, $\Theta_{\mho}^{\mathscr{I}} : \Xi \to [0,1]$ and $\Theta_{\mho}^{\mathscr{F}} : \Xi \to [0,1]$ are denotes the TD, ID and FD of $\mu \in \Xi$ to $\mho$, respectively and $0 \leq \Theta_{\mho}^{\mathscr{T}}(\mu) + \Theta_{\mho}^{\mathscr{I}}(\mu) + \Theta_{\mho}^{\mathscr{F}}(\mu) \leq 3$. For, $\mho = \langle \Theta_{\mho}^{\mathscr{T}}, \Theta_{\mho}^{\mathscr{I}}, \Theta_{\mho}^{\mathscr{F}} \rangle$ is represent a neutrosophic number (NSN).*

Definition 2.5. *[14] The PNSS $\mho$ in Ξ is $\mho = \left\{\mu, \langle \Theta_{\mho}^{\mathscr{T}}(\mu), \Theta_{\mho}^{\mathscr{I}}(\mu), \Theta_{\mho}^{\mathscr{F}}(\mu)\rangle \middle| \mu \in \Xi\right\}$, $\Theta_{\mho}^{\mathscr{T}} : \Xi \to [0,1]$, $\Theta_{\mho}^{\mathscr{I}} : \Xi \to [0,1]$ and $\Theta_{\mho}^{\mathscr{F}} : \Xi \to [0,1]$ are denotes the TD, ID and FD of $\mu \in \Xi$ to $\mho$, respectively and $0 \leq (\Theta_{\mho}^{\mathscr{T}}(\mu))^2 + (\Theta_{\mho}^{\mathscr{I}}(\mu))^2 + (\Theta_{\mho}^{\mathscr{F}}(\mu))^2 \leq 2$. For, $\mho = \langle \Theta_{\mho}^{\mathscr{T}}, \Theta_{\mho}^{\mathscr{I}}, \Theta_{\mho}^{\mathscr{F}} \rangle$ is represent a Pythagorean neutrosophic number (PNSN).*

Definition 2.6. *[10] Let $\widetilde{\mho} = \left\langle [\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}], [\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}]\right\rangle$, $\widetilde{\mho}_1 = \left\langle [\Theta_1^{\mathscr{T}\mathscr{L}}, \Theta_1^{\mathscr{T}\mathscr{U}}], [\Theta_1^{\mathscr{F}\mathscr{L}}, \Theta_1^{\mathscr{F}\mathscr{U}}]\right\rangle$ and $\widetilde{\mho}_2 = \left\langle [\Theta_2^{\mathscr{T}\mathscr{L}}, \Theta_2^{\mathscr{T}\mathscr{U}}], [\Theta_2^{\mathscr{F}\mathscr{L}}, \Theta_2^{\mathscr{F}\mathscr{U}}]\right\rangle$ be the PIVFNs, and $\Delta > 0$. Then,*

(1) $$\widetilde{\mho}_1 \oplus \mho_2 = \begin{bmatrix} \left[\sqrt{(\Theta_1^{\mathscr{T}\mathscr{L}})^2 + (\Theta_2^{\mathscr{T}\mathscr{L}})^2 - (\Theta_1^{\mathscr{T}\mathscr{L}})^2 \cdot (\Theta_2^{\mathscr{T}\mathscr{L}})^2}, \sqrt{(\Theta_1^{\mathscr{T}\mathscr{U}})^2 + (\Theta_2^{\mathscr{T}\mathscr{U}})^2 - (\Theta_1^{\mathscr{T}\mathscr{U}})^2 \cdot (\Theta_2^{\mathscr{T}\mathscr{U}})^2}\right], \\ \left[\Theta_1^{\mathscr{F}\mathscr{L}} \cdot \Theta_2^{\mathscr{F}\mathscr{L}}, \Theta_1^{\mathscr{F}\mathscr{U}} \cdot \Theta_2^{\mathscr{F}\mathscr{U}}\right] \end{bmatrix},$$

(2) $$\widetilde{\mho}_1 \otimes \widetilde{\mho}_2 = \begin{bmatrix} \left[\Theta_1^{\mathscr{T}\mathscr{L}} \cdot \Theta_2^{\mathscr{T}\mathscr{L}}, \Theta_1^{\mathscr{T}\mathscr{U}} \cdot \Theta_2^{\mathscr{T}\mathscr{U}}\right], \\ \left[\sqrt{(\Theta_1^{\mathscr{F}\mathscr{L}})^2 + (\Theta_2^{\mathscr{F}\mathscr{L}})^2 - (\Theta_1^{\mathscr{F}\mathscr{L}})^2 \cdot (\Theta_2^{\mathscr{F}\mathscr{L}})^2}, \sqrt{(\Theta_1^{\mathscr{F}\mathscr{U}})^2 + (\Theta_2^{\mathscr{F}\mathscr{U}})^2 - (\Theta_1^{\mathscr{F}\mathscr{U}})^2 \cdot (\Theta_2^{\mathscr{F}\mathscr{U}})^2}\right] \end{bmatrix},$$

(3) $$\Delta \cdot \widetilde{\mho} = \left[\left[\sqrt{1 - (1 - (\Theta^{\mathscr{T}\mathscr{L}})^2)^{\Delta}}, \sqrt{1 - (1 - (\Theta^{\mathscr{T}\mathscr{U}})^2)^{\Delta}}\right], \left[(\Theta^{\mathscr{F}\mathscr{L}})^{\Delta}, (\Theta^{\mathscr{F}\mathscr{U}})^{\Delta}\right]\right],$$

(4) $$\widetilde{\mho}^{\Delta} = \left[\left[(\Theta^{\mathscr{T}\mathscr{L}})^{\Delta}, (\Theta^{\mathscr{T}\mathscr{U}})^{\Delta}\right], \left[\sqrt{1 - (1 - (\Theta^{\mathscr{F}\mathscr{L}})^2)^{\Delta}}, \sqrt{1 - (1 - (\Theta^{\mathscr{F}\mathscr{U}})^2)^{\Delta}}\right]\right].$$

Definition 2.7. *[10] For any PIVFN $\widetilde{\mho} = \left\langle [\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}], [\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}]\right\rangle$ and score function $S(\widetilde{\mho})$ is defined as $S(\widetilde{\mho}) = \frac{1}{2}\left((\Theta^{\mathscr{T}\mathscr{L}})^2 + (\Theta^{\mathscr{T}\mathscr{U}})^2 - (\Theta^{\mathscr{F}\mathscr{L}})^2 - (\Theta^{\mathscr{F}\mathscr{U}})^2\right)$, $S(\widetilde{\mho}) \in [-1,1]$, and the accuracy function $H(\widetilde{\mho})$ is defined as $H(\widetilde{\mho}) = \frac{1}{2}\left((\Theta^{\mathscr{T}\mathscr{L}})^2 + (\Theta^{\mathscr{T}\mathscr{U}})^2 + (\Theta^{\mathscr{F}\mathscr{L}})^2 + (\Theta^{\mathscr{F}\mathscr{U}})^2\right)$, $H(\widetilde{\mho}) \in [0,1]$.*

Definition 2.8. *For any SRIVFN* $\widetilde{\mho} = \left\langle [\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}], [\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}] \right\rangle$ *and score function* $S(\widetilde{\mho})$ *is defined as* $S(\widetilde{\mho}) = \frac{1}{2}\left((\Theta^{\mathscr{T}\mathscr{L}})^2 + (\Theta^{\mathscr{T}\mathscr{U}})^2 - \sqrt{\Theta^{\mathscr{F}\mathscr{L}}} - \sqrt{\Theta^{\mathscr{F}\mathscr{U}}}\right)$, $S(\widetilde{\mho}) \in [-1, 1]$, *and the accuracy function* $H(\widetilde{\mho})$ *is defined as* $H(\widetilde{\mho}) = \frac{1}{2}\left((\Theta^{\mathscr{T}\mathscr{L}})^2 + (\Theta^{\mathscr{T}\mathscr{U}})^2 + \sqrt{\Theta^{\mathscr{F}\mathscr{L}}} + \sqrt{\Theta^{\mathscr{F}\mathscr{U}}}\right)$, $H(\widetilde{\mho}) \in [0, 1]$.

Definition 2.9. *[19] The fuzzy number* $M(\mu) = e^{-\left(\frac{\mu-\varrho}{\tau}\right)^2}, (\tau > 0)$ *is called a normal fuzzy number (NFN) if* $M = (\varrho, \tau)$ *and* $\widetilde{N}$ *is the NFN set (NFNS), where R is a real number.*

Definition 2.10. *[18] Let* $\widetilde{N}$ *be the NFNS,* $M_1 = (\varrho_1, \tau_1) \in \widetilde{N}$*and* $M_2 = (\varrho_2, \tau_2) \in \widetilde{N}$, $(\tau_1, \tau_2 > 0)$. *Then* $\mathbb{D}(M_1, M_2) = \sqrt{(\varrho_1 - \varrho_2)^2 + \frac{1}{2}(\tau_1 - \tau_2)^2}$.

3. Basic operations for SRNSNIVN approach

In this article, we define the SRNSNIVN and its operations in connection with square root NSIVNs (SRNSIVNs) and NFNs.

Definition 3.1. *The SRNSIV set* $\widetilde{\mho}$ *in* Ξ *is* $\widetilde{\mho} = \left\{\mu, \left\langle \widetilde{\Theta^{\mathscr{T}}_{\mho}}(\mu), \widetilde{\Theta^{\mathscr{I}}_{\mho}}(\mu), \widetilde{\Theta^{\mathscr{F}}_{\mho}}(\mu) \right\rangle \middle| \mu \in \Xi\right\}$, *where* $\widetilde{\Theta^{T}_{\mho}} : \Xi \to Int([0, 1])$, $\widetilde{\Theta^{I}_{\mho}} : \Xi \to Int([0, 1])$ *and* $\widetilde{\Theta^{F}_{\mho}} : \Xi \to INT([0, 1])$ *are denotes the TD, ID and FD of* $\mu \in \Xi$ *to* $\widetilde{\mho}$, *respectively and* $0 \le (\widetilde{\Theta^{\mathscr{T}}_{\mho}}(\mu))^2 + \sqrt{\widetilde{\Theta^{\mathscr{I}}_{\mho}}(\mu)} + \sqrt{\widetilde{\Theta^{\mathscr{F}}_{\mho}}(\mu)} \le 2$, *implies* $0 \le (\Theta^{\mathscr{T}\mathscr{U}}_{\mho}(\mu))^2 + \sqrt{\Theta^{\mathscr{I}\mathscr{U}}_{\mho}(\mu)} + \sqrt{\Theta^{\mathscr{F}\mathscr{U}}_{\mho}(\mu)} \le 2$. *For,* $\widetilde{\mho} = \left\langle \left[\Theta^{\mathscr{T}\mathscr{L}}_{\mho}, \Theta^{\mathscr{T}\mathscr{U}}_{\mho}\right], \left[\Theta^{\mathscr{I}\mathscr{L}}_{\mho}, \Theta^{\mathscr{I}\mathscr{U}}_{\mho}\right], \left[\Theta^{\mathscr{F}\mathscr{L}}_{\mho}, \Theta^{\mathscr{F}\mathscr{U}}_{\mho}\right] \right\rangle$ *is called a square root neutrosophic interval-valued number(SRNSIVN).*

Definition 3.2. *For any SRNSIVN* $\widetilde{\mho} = \left\langle \left[\Theta^{\mathscr{T}\mathscr{L}}_{\mho}, \Theta^{\mathscr{T}\mathscr{U}}_{\mho}\right], \left[\Theta^{\mathscr{I}\mathscr{L}}_{\mho}, \Theta^{\mathscr{I}\mathscr{U}}_{\mho}\right], \left[\Theta^{\mathscr{F}\mathscr{L}}_{\mho}, \Theta^{\mathscr{F}\mathscr{U}}_{\mho}\right] \right\rangle$, *the score function* $S(\widetilde{\mho}) = \frac{\varrho}{2}\left(\frac{(\Theta^{\mathscr{T}\mathscr{L}})^2 + (\Theta^{\mathscr{T}\mathscr{U}})^2}{2} - \frac{\sqrt{\Theta^{\mathscr{I}\mathscr{L}}} + \sqrt{\Theta^{\mathscr{I}\mathscr{U}}}}{2} + 1 - \frac{\sqrt{\Theta^{\mathscr{F}\mathscr{L}}} + \sqrt{\Theta^{\mathscr{F}\mathscr{U}}}}{2}\right)$, *where* $S(\widetilde{\mho}) \in [-1, 1]$.

Definition 3.3. *Let* $\widetilde{\mho} = \left\langle (\varrho, \tau); [\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}], [\Theta^{\mathscr{I}\mathscr{L}}, \Theta^{\mathscr{I}\mathscr{U}}], [\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}] \right\rangle$ *is a SRNSNIVN. The FD, ID and FD are defined as* $[\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}] = \left[\Theta^{\mathscr{T}\mathscr{L}} e^{-\left(\frac{\mu-\varrho}{\tau}\right)^2}, \Theta^{\mathscr{T}\mathscr{U}} e^{-\left(\frac{\mu-\varrho}{\tau}\right)^2}\right]$, $[\Theta^{\mathscr{I}\mathscr{L}}, \Theta^{\mathscr{I}\mathscr{U}}] = \left[\Theta^{\mathscr{I}\mathscr{L}} e^{-\left(\frac{\mu-\varrho}{\tau}\right)^2}, \Theta^{\mathscr{I}\mathscr{U}} e^{-\left(\frac{\mu-\varrho}{\tau}\right)^2}\right]$ *and* $[\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}] = \left[1 - (1 - \Theta^{\mathscr{F}\mathscr{L}}) e^{-\left(\frac{\mu-\varrho}{\tau}\right)^2}, 1 - (1 - \Theta^{\mathscr{F}\mathscr{U}}) e^{-\left(\frac{\mu-\varrho}{\tau}\right)^2}\right]$, $x \in X$ *respectively, where X is a non-empty set and* $[\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}], [\Theta^{\mathscr{I}\mathscr{L}}, \Theta^{\mathscr{I}\mathscr{U}}], [\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}] \in ([0, 1])$ *and* $0 \le (\Theta^{\mathscr{T}\mathscr{U}}(\mu))^2 + \sqrt{\Theta^{\mathscr{I}\mathscr{U}}(\mu)} + \sqrt{\Theta^{\mathscr{F}\mathscr{U}}(\mu)} \le 2$, *where* $(\varrho, \tau) \in \widetilde{N}$.

Definition 3.4. *Let* $\widetilde{\mho} = \left\langle (\varrho, \tau); [\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}], [\Theta^{\mathscr{I}\mathscr{L}}, \Theta^{\mathscr{I}\mathscr{U}}], [\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}] \right\rangle$, $\widetilde{\mho}_1 = \left\langle (\varrho_1, \tau_1); [\Theta_1^{\mathscr{T}\mathscr{L}}, \Theta_1^{\mathscr{T}\mathscr{U}}], [\Theta_1^{\mathscr{I}\mathscr{L}}, \Theta_1^{\mathscr{I}\mathscr{U}}], [\Theta_1^{\mathscr{F}\mathscr{L}}, \Theta_1^{\mathscr{F}\mathscr{U}}] \right\rangle$ *and* $\widetilde{\mho}_2 = \left\langle (\varrho_2, \tau_2); [\Theta_2^{\mathscr{T}\mathscr{L}}, \Theta_2^{\mathscr{T}\mathscr{U}}], [\Theta_2^{\mathscr{I}\mathscr{L}}, \Theta_2^{\mathscr{I}\mathscr{U}}], [\Theta_2^{\mathscr{F}\mathscr{L}}, \Theta_2^{\mathscr{F}\mathscr{U}}] \right\rangle$ *be any three SRNSNIVNs, and* $\Delta > 0$. *Then,*

$$\text{(1)}\ \widetilde{\mho}_1 \oplus \widetilde{\mho}_2 = \left[\begin{array}{c} (\varrho_1 + \varrho_2, \tau_1 + \tau_2; \\ \left[\begin{array}{c} \left(\sqrt[2\Delta]{\Theta_1^{\mathscr{T}\mathscr{L}}} + \sqrt[2\Delta]{\Theta_2^{\mathscr{T}\mathscr{L}}} - \sqrt[2\Delta]{\Theta_1^{\mathscr{T}\mathscr{L}}} \cdot \sqrt[2\Delta]{\Theta_2^{\mathscr{T}\mathscr{L}}} \right)^{2\Delta}, \\ \left(\sqrt[2\Delta]{\Theta_1^{\mathscr{T}\mathscr{U}}} + \sqrt[2\Delta]{\Theta_2^{\mathscr{T}\mathscr{U}}} - \sqrt[2\Delta]{\Theta_1^{\mathscr{T}\mathscr{U}}} \cdot \sqrt[2\Delta]{\Theta_2^{\mathscr{T}\mathscr{U}}} \right)^{2\Delta} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{L}}} + \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{L}}} - \sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{L}}} \cdot \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{L}}} \right)^{\Delta}, \\ \left(\sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{U}}} + \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{U}}} - \sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{U}}} \cdot \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{U}}} \right)^{\Delta} \end{array}\right], \\ \left[\Theta_1^{\mathscr{F}\mathscr{L}} \cdot \Theta_2^{\mathscr{F}\mathscr{L}}, \Theta_1^{\mathscr{F}\mathscr{U}} \cdot \Theta_2^{\mathscr{F}\mathscr{U}}\right] \end{array}\right],$$

$$\text{(2)}\ \widetilde{\mho}_1 \otimes \widetilde{\mho}_2 = \left[\begin{array}{c} (\varrho_1 \cdot \varrho_2, \tau_1 \cdot \tau_2; \left[\Theta_1^{\mathscr{T}\mathscr{L}} \cdot \Theta_2^{\mathscr{T}\mathscr{L}}, \Theta_1^{\mathscr{T}\mathscr{U}} \cdot \Theta_2^{\mathscr{T}\mathscr{U}}\right], \\ \left[\begin{array}{c} \left(\sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{L}}} + \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{L}}} - \sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{L}}} \cdot \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{L}}} \right)^{\Delta}, \\ \left(\sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{U}}} + \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{U}}} - \sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{U}}} \cdot \sqrt[\Delta]{\Theta_2^{\mathscr{I}\mathscr{U}}} \right)^{\Delta} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[2\Delta]{\Theta_1^{\mathscr{F}\mathscr{L}}} + \sqrt[2\Delta]{\Theta_2^{\mathscr{F}\mathscr{L}}} - \sqrt[2\Delta]{\Theta_1^{\mathscr{F}\mathscr{L}}} \cdot \sqrt[2\Delta]{\Theta_2^{\mathscr{F}\mathscr{L}}} \right)^{2\Delta}, \\ \left(\sqrt[2\Delta]{\Theta_1^{\mathscr{F}\mathscr{U}}} + \sqrt[2\Delta]{\Theta_2^{\mathscr{F}\mathscr{U}}} - \sqrt[2\Delta]{\Theta_1^{\mathscr{F}\mathscr{U}}} \cdot \sqrt[2\Delta]{\Theta_2^{\mathscr{F}\mathscr{U}}} \right)^{2\Delta} \end{array}\right] \end{array}\right],$$

$$\text{(3)}\ \Delta \cdot \widetilde{\mho} = \left[\begin{array}{c} (\Delta \cdot \varrho, \Delta \cdot \tau); \\ \left[\left(1 - (1 - \sqrt[2\Delta]{\Theta^{\mathscr{T}\mathscr{L}}})^{\Delta}\right)^{2\Delta}, \left(1 - (1 - \sqrt[2\Delta]{\Theta^{\mathscr{T}\mathscr{U}}})^{\Delta}\right)^{2\Delta}\right], \\ \left[\sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{L}}}, \sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{U}}}\right], \left[(\Theta^{\mathscr{F}\mathscr{L}})^{\Delta}, (\Theta^{\mathscr{F}\mathscr{U}})^{\Delta}\right] \end{array}\right],$$

$$\text{(4)}\ \widetilde{\mho}^{\Delta} = \left[\begin{array}{c} (\varrho^{\Delta}, \tau^{\Delta}); \left[(\Theta^{\mathscr{T}\mathscr{L}})^{\Delta}, (\Theta^{\mathscr{T}\mathscr{U}})^{\Delta}\right], \left[\sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{L}}}, \sqrt[\Delta]{\Theta_1^{\mathscr{I}\mathscr{U}}}\right], \\ \left[\left(1 - (1 - \sqrt[2\Delta]{\Theta^{\mathscr{F}\mathscr{L}}})^{\Delta}\right)^{2\Delta}, \left(1 - (1 - \sqrt[2\Delta]{\Theta^{\mathscr{F}\mathscr{U}}})^{\Delta}\right)^{2\Delta}\right] \end{array}\right].$$

4. Various distance measure for SRNSNIVN approach

The ED and HD measures for SRNSNIVNs are introduced along with some mathematical features of the model.

Definition 4.1. *For SRNSNIVNs* $\widetilde{\mho}_1 = \left\langle(\varrho_1, \tau_1; [\Theta_1^{\mathscr{T}\mathscr{L}}, \Theta_1^{\mathscr{T}\mathscr{U}}], [\Theta_1^{\mathscr{I}\mathscr{L}}, \Theta_1^{\mathscr{I}\mathscr{U}}], [\Theta_1^{\mathscr{F}\mathscr{L}}, \Theta_1^{\mathscr{F}\mathscr{U}}]\right\rangle$ *and* $\widetilde{\mho}_2 = \left\langle(\varrho_2, \tau_2; [\Theta_2^{\mathscr{T}\mathscr{L}}, \Theta_2^{\mathscr{T}\mathscr{U}}], [\Theta_2^{\mathscr{I}\mathscr{L}}, \Theta_2^{\mathscr{I}\mathscr{U}}], [\Theta_2^{\mathscr{F}\mathscr{L}}, \Theta_2^{\mathscr{F}\mathscr{U}}]\right\rangle$. *Then*

$$\mathbb{D}_E\left(\widetilde{\mho}_1, \widetilde{\mho}_2\right) = \frac{1}{2}\sqrt{\begin{array}{l} \left[\begin{array}{l} \frac{1+(\Theta_1^{\mathscr{T}\mathscr{L}})^2 - \sqrt{\Theta_1^{\mathscr{I}\mathscr{L}}} - \sqrt{\Theta_1^{\mathscr{F}\mathscr{L}}} + 1 + (\Theta_1^{\mathscr{T}\mathscr{U}})^2 - \sqrt{\Theta_1^{\mathscr{I}\mathscr{U}}} - \sqrt{\Theta_1^{\mathscr{F}\mathscr{U}}}}{4}\varrho_1 \\ -\frac{1+(\Theta_2^{\mathscr{T}\mathscr{L}})^2 - \sqrt{\Theta_2^{\mathscr{I}\mathscr{L}}} - \sqrt{\Theta_2^{\mathscr{F}\mathscr{L}}} + 1 + (\Theta_2^{\mathscr{T}\mathscr{U}})^2 - \sqrt{\Theta_2^{\mathscr{I}\mathscr{U}}} - \sqrt{\Theta_2^{\mathscr{F}\mathscr{U}}}}{4}\varrho_2 \end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{l} \frac{1+(\Theta_1^{\mathscr{T}\mathscr{L}})^2 - \sqrt{\Theta_1^{\mathscr{I}\mathscr{L}}} - \sqrt{\Theta_1^{\mathscr{F}\mathscr{L}}} + 1 + (\Theta_1^{\mathscr{T}\mathscr{U}})^2 - \sqrt{\Theta_1^{\mathscr{I}\mathscr{U}}} - \sqrt{\Theta_1^{\mathscr{F}\mathscr{U}}}}{4}\tau_1 \\ -\frac{1+(\Theta_2^{\mathscr{T}\mathscr{L}})^2 - \sqrt{\Theta_2^{\mathscr{I}\mathscr{L}}} - \sqrt{\Theta_2^{\mathscr{F}\mathscr{L}}} + 1 + (\Theta_2^{\mathscr{T}\mathscr{U}})^2 - \sqrt{\Theta_2^{\mathscr{I}\mathscr{U}}} - \sqrt{\Theta_2^{\mathscr{F}\mathscr{U}}}}{4}\tau_2 \end{array}\right]^2 \end{array}}$$

where $\mathbb{D}_E\left(\widetilde{\mho}_1, \widetilde{\mho}_2\right)$ *is denote the ED between* $\widetilde{\mho}_1$ *and* $\widetilde{\mho}_2$.

Also,

$$\mathbb{D}_H\left(\widetilde{\mho}_1, \widetilde{\mho}_2\right) = \frac{1}{2}\left[\begin{array}{l} \left| \begin{array}{l} \dfrac{1 + (\Theta_1^{\mathscr{TL}})^2 - \sqrt{\Theta_1^{\mathscr{IL}}} - \sqrt{\Theta_1^{\mathscr{FL}}} + 1 + (\Theta_1^{\mathscr{TU}})^2 - \sqrt{\Theta_1^{\mathscr{IU}}} - \sqrt{\Theta_1^{\mathscr{FU}}}}{4}\varrho_1 \\ -\dfrac{1 + (\Theta_2^{\mathscr{TL}})^2 - \sqrt{\Theta_2^{\mathscr{IL}}} - \sqrt{\Theta_2^{\mathscr{FL}}} + 1 + (\Theta_2^{\mathscr{TU}})^2 - \sqrt{\Theta_2^{\mathscr{IU}}} - \sqrt{\Theta_2^{\mathscr{FU}}}}{4}\varrho_2 \end{array} \right| \\ +\frac{1}{2}\left| \begin{array}{l} \dfrac{1 + (\Theta_1^{\mathscr{TL}})^2 - \sqrt{\Theta_1^{\mathscr{IL}}} - \sqrt{\Theta_1^{\mathscr{FL}}} + 1 + (\Theta_1^{\mathscr{TU}})^2 - \sqrt{\Theta_1^{\mathscr{IU}}} - \sqrt{\Theta_1^{\mathscr{FU}}}}{4}\tau_1 \\ -\dfrac{1 + (\Theta_2^{\mathscr{TL}})^2 - \sqrt{\Theta_2^{\mathscr{IL}}} - \sqrt{\Theta_2^{\mathscr{FL}}} + 1 + (\Theta_2^{\mathscr{TU}})^2 - \sqrt{\Theta_2^{\mathscr{IU}}} - \sqrt{\Theta_2^{\mathscr{FU}}}}{4}\tau_2 \end{array} \right| \end{array}\right]$$

where $\mathbb{D}_H\left(\widetilde{\mho}_1, \widetilde{\mho}_2\right)$ *denote the HD between* $\widetilde{\mho}_1$ *and* $\widetilde{\mho}_2$.

Theorem 4.1. *If any three SRNSNIVNs* $\widetilde{\mho}_1 = \left\langle(\varrho_1, \tau_1; [\Theta_1^{\mathscr{TL}}, \Theta_1^{\mathscr{TU}}], [\Theta_1^{\mathscr{IL}}, \Theta_1^{\mathscr{IU}}], [\Theta_1^{\mathscr{FL}}, \Theta_1^{\mathscr{FU}}]\right\rangle$, $\widetilde{\mho}_2 = \left\langle(\varrho_2, \tau_2; [\Theta_2^{\mathscr{TL}}, \Theta_2^{\mathscr{TU}}], [\Theta_2^{\mathscr{IL}}, \Theta_2^{\mathscr{IU}}], [\Theta_2^{\mathscr{FL}}, \Theta_2^{\mathscr{FU}}]\right\rangle$, $\widetilde{\mho_3} = \left\langle(\varrho_3, \tau_3; [\Theta_3^{\mathscr{TL}}, \Theta_3^{\mathscr{TU}}], [\Theta_3^{\mathscr{IL}}, \Theta_3^{\mathscr{IU}}], [\Theta_3^{\mathscr{FL}}, \Theta_3^{\mathscr{FU}}]\right\rangle$*, then* $\mathbb{D}_E(\mho_1, \mho_2)$ *satisfies the following properties are holds.*

(1) $\mathbb{D}_E(\widetilde{\mho}_1, \widetilde{\mho}_2) = 0$ *iff* $\widetilde{\mho}_1 = \widetilde{\mho}_2$.

(2) $\mathbb{D}_E(\widetilde{\mho}_1, \widetilde{\mho}_2) = \mathbb{D}_E(\widetilde{\mho}_2, \widetilde{\mho}_1)$.

(3) $\mathbb{D}_E(\widetilde{\mho}_1, \widetilde{\mho_3}) \leq \mathbb{D}_E(\widetilde{\mho}_1, \widetilde{\mho}_2) + \mathbb{D}_E(\widetilde{\mho}_2, \widetilde{\mho_3})$.

Proof. Now,

$$\left(\mathbb{D}_E(\mho_1, \mho_2 + \mathbb{D}_E(\mho_2, \mho_3)\right)^2 = \left[\begin{array}{l} \frac{1}{2}\sqrt{\begin{array}{l} \left[\begin{array}{l} \frac{1+(\Theta_1^{\mathscr{TL}})^2 - \sqrt{\Theta_1^{\mathscr{IL}}} - \sqrt{\Theta_1^{\mathscr{FL}}} + 1 + (\Theta_1^{\mathscr{TU}})^2 - \sqrt{\Theta_1^{\mathscr{IU}}} - \sqrt{\Theta_1^{\mathscr{FU}}}}{4}\varrho_1 \\ -\frac{1+(\Theta_2^{\mathscr{TL}})^2 - \sqrt{\Theta_2^{\mathscr{IL}}} - \sqrt{\Theta_2^{\mathscr{FL}}} + 1 + (\Theta_2^{\mathscr{TU}})^2 - \sqrt{\Theta_2^{\mathscr{IU}}} - \sqrt{\Theta_2^{\mathscr{FU}}}}{4}\varrho_2 \end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{l} \frac{1+(\Theta_1^{\mathscr{TL}})^2 - \sqrt{\Theta_1^{\mathscr{IL}}} - \sqrt{\Theta_1^{\mathscr{FL}}} + 1 + (\Theta_1^{\mathscr{TU}})^2 - \sqrt{\Theta_1^{\mathscr{IU}}} - \sqrt{\Theta_1^{\mathscr{FU}}}}{4}\tau_1 \\ -\frac{1+(\Theta_2^{\mathscr{TL}})^2 - \sqrt{\Theta_2^{\mathscr{IL}}} - \sqrt{\Theta_2^{\mathscr{FL}}} + 1 + (\Theta_2^{\mathscr{TU}})^2 - \sqrt{\Theta_2^{\mathscr{IU}}} - \sqrt{\Theta_2^{\mathscr{FU}}}}{4}\tau_2 \end{array}\right]^2 \end{array}} \\ +\frac{1}{2}\sqrt{\begin{array}{l} \left[\begin{array}{l} \frac{1+(\Theta_2^{\mathscr{TL}})^2 - \sqrt{\Theta_2^{\mathscr{IL}}} - \sqrt{\Theta_2^{\mathscr{FL}}} + 1 + (\Theta_2^{\mathscr{TU}})^2 - \sqrt{\Theta_2^{\mathscr{IU}}} - \sqrt{\Theta_2^{\mathscr{FU}}}}{4}\varrho_2 \\ -\frac{1+(\Theta_3^{\mathscr{TL}})^2 - \sqrt{\Theta_3^{\mathscr{IL}}} - \sqrt{\Theta_3^{\mathscr{FL}}} + 1 + (\Theta_3^{\mathscr{TU}})^2 - \sqrt{\Theta_3^{\mathscr{IU}}} - \sqrt{\Theta_3^{\mathscr{FU}}}}{4}\varrho_3 \end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{l} \frac{1+(\Theta_2^{\mathscr{TL}})^2 - \sqrt{\Theta_2^{\mathscr{IL}}} - \sqrt{\Theta_2^{\mathscr{FL}}} + 1 + (\Theta_2^{\mathscr{TU}})^2 - \sqrt{\Theta_2^{\mathscr{IU}}} - \sqrt{\Theta_2^{\mathscr{FU}}}}{4}\tau_2 \\ -\frac{1+(\Theta_3^{\mathscr{TL}})^2 - \sqrt{\Theta_3^{\mathscr{IL}}} - \sqrt{\Theta_3^{\mathscr{FL}}} + 1 + (\Theta_3^{\mathscr{TU}})^2 - \sqrt{\Theta_3^{\mathscr{IU}}} - \sqrt{\Theta_3^{\mathscr{FU}}}}{4}\tau_3 \end{array}\right]^2 \end{array}} \end{array}\right]^2$$

implies

$$\begin{array}{l} \frac{1}{4}\left((\Upsilon_1\varrho_1 - \Upsilon_2\varrho_2^2 + \frac{1}{2}(\Upsilon_1\tau_1 - \Upsilon_2\tau_2^2\right) + \frac{1}{4}\left((\Upsilon_2\varrho_2 - \Upsilon_3\varrho_3^2 + \frac{1}{2}(\Upsilon_2\tau_2 - \Upsilon_3\tau_3^2\right) \\ +\frac{1}{2}\left(\sqrt{(\Upsilon_1\varrho_1 - \Upsilon_2\varrho_2^2 + \frac{1}{2}(\Upsilon_1\tau_1 - \Upsilon_2\tau_2^2} \times \sqrt{(\Upsilon_2\varrho_2 - \Upsilon_3\varrho_3^2 + \frac{1}{2}(\Upsilon_2\tau_2 - \Upsilon_3\tau_3^2}\right). \end{array}$$

Since,

$$\Upsilon_1 = \frac{1 + (\Theta_1^{\mathscr{TL}})^2 - \sqrt{\Theta_1^{\mathscr{IL}}} - \sqrt{\Theta_1^{\mathscr{FL}}} + 1 + (\Theta_1^{\mathscr{TU}})^2 - \sqrt{\Theta_1^{\mathscr{IU}}} - \sqrt{\Theta_1^{\mathscr{FU}}}}{4},$$

$$\Upsilon_2 = \frac{1 + (\Theta_2^{\mathscr{T}\mathscr{L}})^2 - \sqrt{\Theta_2^{\mathscr{I}\mathscr{L}}} - \sqrt{\Theta_2^{\mathscr{F}\mathscr{L}}} + 1 + (\Theta_2^{\mathscr{T}\mathscr{U}})^2 - \sqrt{\Theta_2^{\mathscr{I}\mathscr{U}}} - \sqrt{\Theta_2^{\mathscr{F}\mathscr{U}}}}{4},$$

$$\Upsilon_3 = \frac{1 + (\Theta_3^{\mathscr{T}\mathscr{L}})^2 - \sqrt{\Theta_3^{\mathscr{I}\mathscr{L}}} - \sqrt{\Theta_3^{\mathscr{F}\mathscr{L}}} + 1 + (\Theta_3^{\mathscr{T}\mathscr{U}})^2 - \sqrt{\Theta_3^{\mathscr{I}\mathscr{U}}} - \sqrt{\Theta_3^{\mathscr{F}\mathscr{U}}}}{4}.$$

Hence, $\Big(\mathbb{D}_E(\mho_1, \mho_2 + \mathbb{D}_E(\mho_2, \mho_3)\Big)^2$

$$\begin{aligned}
&\geq \frac{1}{4}\Big((\Upsilon_1\varrho_1 - \Upsilon_2\varrho_2^2 + \frac{1}{2}(\Upsilon_1\tau_1 - \Upsilon_2\tau_2^2\Big) + \frac{1}{4}\Big((\Upsilon_2\varrho_2 - \Upsilon_3\varrho_3^2 + \frac{1}{2}(\Upsilon_2\tau_2 - \Upsilon_3\tau_3^2\Big) \\
&\quad + \frac{1}{2}\Big((\Upsilon_1\varrho_1 - \Upsilon_2\varrho_2 \times (\Upsilon_2\varrho_2 - \Upsilon_3\varrho_3 + \frac{1}{2}(\Upsilon_1\tau_1 - \Upsilon_2\tau_2 \times (\Upsilon_2\tau_2 - \Upsilon_3\tau_3\Big) \\
&= \frac{1}{4}\Big((\Upsilon_1\varrho_1 - \Upsilon_2\varrho_2^2 + (\Upsilon_2\varrho_2 - \Upsilon_3\varrho_3^2 + 2(\Upsilon_1\varrho_1 - \Upsilon_2\varrho_2 \times (\Upsilon_2\varrho_2 - \Upsilon_3\varrho_3\Big) \\
&\quad + \frac{1}{4}\Big(\frac{1}{2}(\Upsilon_1\tau_1 - \Upsilon_2\tau_2^2 + \frac{1}{2}(\Upsilon_2\tau_2 - \Upsilon_3\tau_3^2 + (\Upsilon_1\tau_1 - \Upsilon_2\tau_2 \times (\Upsilon_2\tau_2 - \Upsilon_3\tau_3\Big) \\
&= \frac{1}{4}(\Upsilon_1\varrho_1 - \Upsilon_2\varrho_2 + \Upsilon_2\varrho_2 - \Upsilon_3\varrho_3^2 + \frac{1}{8}(\Upsilon_1\tau_1 - \Upsilon_2\tau_2 + \Upsilon_2\tau_2 - \Upsilon_3\tau_3^2 \\
&= \frac{1}{4}(\Upsilon_1\varrho_1 - \Upsilon_3\varrho_3^2 + \frac{1}{8}(\Upsilon_1\tau_1 - \Upsilon_3\tau_3^2 \\
&= \frac{1}{4}\Big[(\Upsilon_1\varrho_1 - \Upsilon_3\varrho_3^2 + \frac{1}{2}(\Upsilon_1\tau_1 - \Upsilon_3\tau_3^2\Big] \\
&= \mathbb{D}_E(\mho_1, \mho_3^2.
\end{aligned}$$

□

Corollary 4.1. *If any three SRNSNIVNs* $\widetilde{\mho}_1 = \Big\langle(\varrho_1, \tau_1; [\Theta_1^{\mathscr{T}\mathscr{L}}, \Theta_1^{\mathscr{T}\mathscr{U}}], [\Theta_1^{\mathscr{I}\mathscr{L}}, \Theta_1^{\mathscr{I}\mathscr{U}}], [\Theta_1^{\mathscr{F}\mathscr{L}}, \Theta_1^{\mathscr{F}\mathscr{U}}]\Big\rangle, \widetilde{\mho}_2 = \Big\langle(\varrho_2, \tau_2; [\Theta_2^{\mathscr{T}\mathscr{L}}, \Theta_2^{\mathscr{T}\mathscr{U}}], [\Theta_2^{\mathscr{I}\mathscr{L}}, \Theta_2^{\mathscr{I}\mathscr{U}}], [\Theta_2^{\mathscr{F}\mathscr{L}}, \Theta_2^{\mathscr{F}\mathscr{U}}]\Big\rangle,$ $\widetilde{\mho_3} = \Big\langle(\varrho_3, \tau_3; [\Theta_3^{\mathscr{T}\mathscr{L}}, \Theta_3^{\mathscr{T}\mathscr{U}}], [\Theta_3^{\mathscr{I}\mathscr{L}}, \Theta_3^{\mathscr{I}\mathscr{U}}], [\Theta_3^{\mathscr{F}\mathscr{L}}, \Theta_3^{\mathscr{F}\mathscr{U}}]\Big\rangle$. *Then*

(1) $\mathbb{D}_H(\widetilde{\mho}_1, \widetilde{\mho}_2) = 0$ *iff* $\widetilde{\mho}_1 = \widetilde{\mho}_2$.

(2) $\mathbb{D}_H(\widetilde{\mho}_1, \widetilde{\mho}_2)$ *and* $\mathbb{D}_H(\widetilde{\mho}_2, \widetilde{\mho}_1)$ *are co-occur.*

(3) $\mathbb{D}_H(\widetilde{\mho}_1, \widetilde{\mho_3}) \leq \mathbb{D}_H(\widetilde{\mho}_1, \widetilde{\mho}_2) + \mathbb{D}_H(\widetilde{\mho}_2, \widetilde{\mho}_3)$.

5. Different aggregation operators for SRNSNIVN

Here we introduced the new operators for SRNSNIVWA, SRNSNIVWG, GSRNSNIVWA, and GSRNSNIVWG.

5.1. SRNSNIV weighted averaging(SRNSNIVWA) operator

Definition 5.1. *Let* $\widetilde{\mho}_i = \Big\langle(\varrho_i, \tau_i); [\Theta_i^{\mathscr{T}\mathscr{L}}, \Theta_i^{\mathscr{T}\mathscr{U}}], [\Theta_i^{\mathscr{I}\mathscr{L}}, \Theta_i^{\mathscr{I}\mathscr{U}}], [\Theta_i^{\mathscr{F}\mathscr{L}}, \Theta_i^{\mathscr{F}\mathscr{U}}]\Big\rangle$ *be the family of SRNSNIVNs,* $W = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)$ *be the weight of* $\widetilde{\mho}_i$, $\varepsilon_i \geq 0$ *and* $\oplus_{i=1}^n \varepsilon_i = 1$. *Then SRNSNIVWA* $(\widetilde{\mho}_1, \widetilde{\mho}_2, ..., \widetilde{\mho}_n) = \oplus_{i=1}^n \varepsilon_i \widetilde{\mho}_i$.

Theorem 5.1. *Let* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}],[\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the family of SRNSNIVNs. Then*

$$SRNSNIVWA(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\left[\begin{array}{c}\left(\oplus_{i=1}^n\varepsilon_i\varrho_i,\oplus_{i=1}^n\varepsilon_i\tau_i\right);\\ \left[\left(1-\bigcirc_{i=1}^n\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta},\left(1-\bigcirc_{i=1}^n\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}\right],\\ \left[\left(1-\bigcirc_{i=1}^n\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta},\left(1-\bigcirc_{i=1}^n\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}\right],\\ \left[\bigcirc_{i=1}^n(\Theta_i^{\mathscr{F}\mathscr{L}})^{\varepsilon_i},\bigcirc_{i=1}^n(\Theta_i^{\mathscr{F}\mathscr{U}})^{\varepsilon_i}\right]\end{array}\right].$$

Proof. For the proof, we used the mathematical induction method.
Put $n=2$, SRNSNIVWA$(\widetilde{\mho}_1,\widetilde{\mho}_2)=\varepsilon_1\widetilde{\mho}_1\oplus\varepsilon_2\widetilde{\mho}_2$, where

$$\varepsilon_1\widetilde{\mho}_1=\left[\begin{array}{c}\left(\varepsilon_1\varrho_1,\varepsilon_1\tau_1\right);\\ \left[\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_1}\right)^{2\Delta},\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_1}\right)^{2\Delta}\right],\\ \left[\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_1}\right)^{\Delta},\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_1}\right)^{\Delta}\right],\\ \left[(\Theta_1^{\mathscr{F}\mathscr{L}})^{\varepsilon_1},(\Theta_1^{\mathscr{F}\mathscr{U}})^{\varepsilon_1}\right]\end{array}\right]$$

$$\varepsilon_2\widetilde{\mho}_2=\left[\begin{array}{c}\left(\varepsilon_2\varrho_2,\varepsilon_2\tau_2\right);\\ \left[\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_2}\right)^{2\Delta},\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_2}\right)^{2\Delta}\right],\\ \left[\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_2}\right)^{\Delta},\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_2}\right)^{\Delta}\right],\\ \left[(\Theta_2^{\mathscr{F}\mathscr{L}})^{\varepsilon_2},(\Theta_2^{\mathscr{F}\mathscr{U}})^{\varepsilon_2}\right]\end{array}\right].$$

Now,

$$\varepsilon_1\widetilde{\mho}_1\oplus\varepsilon_2\widetilde{\mho}_2=\left[\begin{array}{c}\left(\varepsilon_1\varrho_1+\varepsilon_2\varrho_2,\varepsilon_1\tau_1+\varepsilon_2\tau_2\right);\\ \left[\begin{array}{c}\left(\begin{array}{c}\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_1}\right)+\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_2}\right)\\ -\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_1}\right)\cdot\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_2}\right)\end{array}\right)^{2\Delta},\\ \left(\begin{array}{c}\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_1}\right)+\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_2}\right)\\ -\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_1}\right)\cdot\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_2}\right)\end{array}\right)^{2\Delta}\end{array}\right],\\ \left[\begin{array}{c}\left(\begin{array}{c}\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_1}\right)+\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_2}\right)\\ -\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_1}\right)\cdot\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_2}\right)\end{array}\right)^{\Delta},\\ \left(\begin{array}{c}\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_1}\right)+\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_2}\right)\\ -\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_1}\right)\cdot\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_2}\right)\end{array}\right)^{\Delta}\end{array}\right],\\ \left[(\Theta_1^{\mathscr{F}\mathscr{L}})^{\varepsilon_1}(\Theta_2^{\mathscr{F}\mathscr{L}})^{\varepsilon_2},(\Theta_1^{\mathscr{F}\mathscr{U}})^{\varepsilon_1}(\Theta_2^{\mathscr{F}\mathscr{U}})^{\varepsilon_2}\right]\end{array}\right]$$

$$= \begin{bmatrix} \left(\varepsilon_1\varrho_1 + \varepsilon_2\varrho_2, \varepsilon_1\tau_1 + \varepsilon_2\tau_2\right); \\ \begin{bmatrix} \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_1} \cdot \left(1 - \sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_2}\right)^{2\Delta}, \\ \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_1} \cdot \left(1 - \sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_2}\right)^{2\Delta} \end{bmatrix}, \\ \begin{bmatrix} \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_1} \cdot \left(1 - \sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_2}\right)^{\Delta}, \\ \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_1} \cdot \left(1 - \sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_2}\right)^{\Delta} \end{bmatrix}, \\ \left[(\Theta_1^{\mathscr{F}\mathscr{L}})^{\varepsilon_1} \cdot (\Theta_2^{\mathscr{F}\mathscr{L}})^{\varepsilon_2}, (\Theta_1^{\mathscr{F}\mathscr{U}})^{\varepsilon_1} \cdot (\Theta_2^{\mathscr{F}\mathscr{U}})^{\varepsilon_2}\right] \end{bmatrix}$$

$$SRNSNIVWA(\widetilde{\mho}_1, \widetilde{\mho}_2) = \begin{bmatrix} \left(\oplus_{i=1}^{2} \varepsilon_i\varrho_i, \oplus_{i=1}^{2}\varepsilon_i\tau_i\right); \\ \left[\left(1 - \bigcirc_{i=1}^{2}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}, \left(1 - \bigcirc_{i=1}^{2}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}\right], \\ \left[\left(1 - \bigcirc_{i=1}^{2}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta}, \left(1 - \bigcirc_{i=1}^{2}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}\right], \\ \left[\bigcirc_{i=1}^{2} (\Theta_i^{\mathscr{F}\mathscr{L}})^{\varepsilon_i}, \bigcirc_{i=1}^{2}(\Theta_i^{\mathscr{F}\mathscr{U}})^{\varepsilon_i}\right] \end{bmatrix}.$$

Also, valid for $n \geq 3$, hence $SRNSNIVWA(\widetilde{\mho}_1, \widetilde{\mho}_2, ..., \widetilde{\mho}_l) =$

$$\begin{bmatrix} \left(\oplus_{i=1}^{l} \varepsilon_i\varrho_i, \oplus_{i=1}^{l}\varepsilon_i\tau_i\right); \\ \left[\left(1 - \bigcirc_{i=1}^{l}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}, \left(1 - \bigcirc_{i=1}^{l}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}\right], \\ \left[\left(1 - \bigcirc_{i=1}^{l}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta}, \left(1 - \bigcirc_{i=1}^{l}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}\right], \\ \left[\bigcirc_{i=1}^{l} (\Theta_i^{\mathscr{F}\mathscr{L}})^{\varepsilon_i}, \bigcirc_{i=1}^{l}(\Theta_i^{\mathscr{F}\mathscr{U}})^{\varepsilon_i}\right] \end{bmatrix}.$$

If $n = l + 1$, then SRNSNIVWA $(\widetilde{\mho}_1, \widetilde{\mho}_2, ..., \widetilde{\mho}_l, \widetilde{\mho}_{l+1})$

$$= \begin{bmatrix} \left(\oplus_{i=1}^{l} \varepsilon_i\varrho_i + \varepsilon_{l+1}\varrho_{l+1}, \oplus_{i=1}^{l}\varepsilon_i\tau_i + \varepsilon_{l+1}\tau_{l+1}\right); \\ \begin{bmatrix} \begin{pmatrix} \oplus_{i=1}^{l}\left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right) + \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_{l+1}}\right) \\ - \bigcirc_{i=1}^{l} \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right) \cdot \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_{l+1}}\right) \end{pmatrix}^{2\Delta}, \\ \begin{pmatrix} \oplus_{i=1}^{l}\left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right) + \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_{l+1}}\right) \\ - \bigcirc_{i=1}^{l} \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right) \cdot \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_{l+1}}\right) \end{pmatrix}^{2\Delta} \end{bmatrix}, \\ \begin{bmatrix} \begin{pmatrix} \oplus_{i=1}^{l}\left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right) + \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_{l+1}}\right) \\ - \bigcirc_{i=1}^{l} \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right) \cdot \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_{l+1}}\right) \end{pmatrix}^{\Delta}, \\ \begin{pmatrix} \oplus_{i=1}^{l}\left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right) + \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_{l+1}}\right) \\ - \bigcirc_{i=1}^{l} \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right) \cdot \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_{l+1}}\right) \end{pmatrix}^{\Delta} \end{bmatrix}, \\ \left[\bigcirc_{i=1}^{l} (\Theta_i^{\mathscr{F}\mathscr{L}})^{\varepsilon_i} \cdot (\Theta_{l+1}^{\mathscr{F}\mathscr{L}})^{\varepsilon_{l+1}}, \bigcirc_{i=1}^{l}(\Theta_i^{\mathscr{F}\mathscr{U}})^{\varepsilon_i} \cdot (\Theta_{l+1}^{\mathscr{F}\mathscr{U}})^{\varepsilon_{l+1}}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left(\oplus_{i=1}^{l+1} \varepsilon_i \varrho_i, \oplus_{i=1}^{l+1} \varepsilon_i \tau_i\right); \\ \left[\left(1 - \bigcirc_{i=1}^{l+1}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}, \left(1 - \bigcirc_{i=1}^{l+1}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}\right], \\ \left[\left(1 - \bigcirc_{i=1}^{l+1}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta}, \left(1 - \bigcirc_{i=1}^{l+1}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}\right], \\ \left[\bigcirc_{i=1}^{l+1} (\Theta_i^{\mathscr{F}\mathscr{L}})^{\varepsilon_i}, \bigcirc_{i=1}^{l+1} (\Theta_i^{\mathscr{F}\mathscr{U}})^{\varepsilon_i}\right] \end{bmatrix}.$$

□

Theorem 5.2. *If all* $\widetilde{\mho}_i = \left\langle(\varrho_i, \tau_i); [\Theta_i^{\mathscr{T}\mathscr{L}}, \Theta_i^{\mathscr{T}\mathscr{U}}], [\Theta_i^{\mathscr{I}\mathscr{L}}, \Theta_i^{\mathscr{I}\mathscr{U}}][\Theta_i^{\mathscr{F}\mathscr{L}}, \Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *are equal, then* $SRNSNIVWA(\widetilde{\mho}_1, \widetilde{\mho}_2, ..., \widetilde{\mho}_n) = \widetilde{\mho}$*(idempotency property).*

Proof. Given that $(\varrho_i, \tau_i) = (\varrho, \tau)$, $[\Theta_i^{\mathscr{T}\mathscr{L}}, \Theta_i^{\mathscr{T}\mathscr{U}}] = [\Theta^{\mathscr{T}\mathscr{L}}, \Theta^{\mathscr{T}\mathscr{U}}]$, $[\Theta_i^{\mathscr{I}\mathscr{L}}, \Theta_i^{\mathscr{I}\mathscr{U}}] = [\Theta^{\mathscr{I}\mathscr{L}}, \Theta^{\mathscr{I}\mathscr{U}}]$ and $[\Theta_i^{\mathscr{F}\mathscr{L}}, \Theta_i^{\mathscr{F}\mathscr{U}}] = [\Theta^{\mathscr{F}\mathscr{L}}, \Theta^{\mathscr{F}\mathscr{U}}]$ and $\oplus_{i=1}^{n}\varepsilon_i = 1$. Now, $SRNSNIVWA(\widetilde{\mho}_1, \widetilde{\mho}_2, ..., \widetilde{\mho}_n)$

$$= \begin{bmatrix} \left(\oplus_{i=1}^{n} \varepsilon_i \varrho_i, \oplus_{i=1}^{n} \varepsilon_i \tau_i\right); \\ \left[\left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}, \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}\right], \\ \left[\left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta}, \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}\right], \\ \left[\bigcirc_{i=1}^{n} (\Theta_i^{\mathscr{F}\mathscr{L}})^{\varepsilon_i}, \bigcirc_{i=1}^{n} (\Theta_i^{\mathscr{F}\mathscr{U}})^{\varepsilon_i}\right] \end{bmatrix},$$

$$= \begin{bmatrix} \left(\varrho \oplus_{i=1}^{n} \varepsilon_i, \tau \oplus_{i=1}^{n} \varepsilon_i\right); \\ \left[\left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\oplus_{i=1}^{n}\varepsilon_i}\right)^{2\Delta}, \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\oplus_{i=1}^{n}\varepsilon_i}\right)^{2\Delta}\right], \\ \left[\left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\oplus_{i=1}^{n}\varepsilon_i}\right)^{\Delta}, \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\oplus_{i=1}^{n}\varepsilon_i}\right)^{\Delta}\right], \\ \left[(\Theta_i^{\mathscr{F}\mathscr{L}})^{\oplus_{i=1}^{n}\varepsilon_i}, (\Theta_i^{\mathscr{F}\mathscr{U}})^{\oplus_{i=1}^{n}\varepsilon_i}\right] \end{bmatrix},$$

$$= \begin{bmatrix} (\varrho, \tau); \\ \left[\left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)\right)^{2\Delta}, \left(1 - \left(1 - \sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)\right)^{2\Delta}\right], \\ \left[\left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)\right)^{\Delta}, \left(1 - \left(1 - \sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)\right)^{\Delta}\right], \\ \left[(\Theta_i^{\mathscr{F}\mathscr{L}}), (\Theta_i^{\mathscr{F}\mathscr{U}})\right] \end{bmatrix},$$

$$= \widetilde{\mho}.$$

□

Theorem 5.3. *Let* $\widetilde{\mho}_i = \left\langle(\varrho_{ij}, \tau_{ij}); [\Theta_{ij}^{\mathscr{T}\mathscr{L}}, \Theta_{ij}^{\mathscr{T}\mathscr{U}}], [\Theta_{ij}^{\mathscr{I}\mathscr{L}}, \Theta_{ij}^{\mathscr{I}\mathscr{U}}][\Theta_{ij}^{\mathscr{F}\mathscr{L}}, \Theta_{ij}^{\mathscr{F}\mathscr{U}}]\right\rangle(i = 1 \text{ to } n); (j = 1 \text{ to } i_j)$ *be the SRNSNIVWA, where*

$\widehat{\varrho} = \inf \varrho_{ij}$, $\underbrace{\varrho} = \sup \varrho_{ij}$, $\widehat{\tau} = \sup \tau_{ij}$, $\underbrace{\tau} = \inf \tau_{ij}$,

$\widehat{\Theta^{\mathscr{T}\mathscr{L}}} = \inf \Theta_{ij}^{\mathscr{T}\mathscr{L}}$, $\underbrace{\Theta^{\mathscr{T}\mathscr{L}}} = \sup \Theta_{ij}^{\mathscr{T}\mathscr{L}}$, $\widehat{\Theta^{\mathscr{T}\mathscr{U}}} = \inf \Theta_{ij}^{\mathscr{T}\mathscr{U}}$, $\underbrace{\Theta^{\mathscr{T}\mathscr{U}}} = \sup \Theta_{ij}^{\mathscr{T}\mathscr{U}}$,

$\widehat{\Theta^{\mathscr{I}\mathscr{L}}} = \inf \Theta_{ij}^{\mathscr{I}\mathscr{L}}$, $\underbrace{\Theta^{\mathscr{I}\mathscr{L}}} = \sup \Theta_{ij}^{\mathscr{I}\mathscr{L}}$, $\widehat{\Theta^{\mathscr{I}\mathscr{U}}} = \inf \Theta_{ij}^{\mathscr{I}\mathscr{U}}$, $\underbrace{\Theta^{\mathscr{I}\mathscr{U}}} = \sup \Theta_{ij}^{\mathscr{I}\mathscr{U}}$,
$\widehat{\Theta^{\mathscr{F}\mathscr{L}}} = \inf \Theta_{ij}^{\mathscr{F}\mathscr{L}}$, $\underbrace{\Theta^{\mathscr{F}\mathscr{L}}} = \sup \Theta_{ij}^{\mathscr{F}\mathscr{L}}$, $\widehat{\Theta^{\mathscr{F}\mathscr{U}}} = \inf \Theta_{ij}^{\mathscr{F}\mathscr{U}}$, $\underbrace{\Theta^{\mathscr{F}\mathscr{U}}} = \sup \Theta_{ij}^{\mathscr{F}\mathscr{U}}$.
Then, $\left\langle(\widehat{\varrho},\widehat{\tau});[\widehat{\Theta^{\mathscr{T}\mathscr{L}}},\widehat{\Theta^{\mathscr{T}\mathscr{U}}}],[\widehat{\Theta^{\mathscr{I}\mathscr{L}}},\widehat{\Theta^{\mathscr{I}\mathscr{U}}}],[\underbrace{\Theta^{\mathscr{F}\mathscr{L}}},\underbrace{\Theta^{\mathscr{F}\mathscr{U}}}]\right\rangle \leq SRNSNIVWA(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)$
$\leq \left\langle(\underbrace{\varrho},\underbrace{\tau});[\underbrace{\Theta^{\mathscr{T}\mathscr{L}}},\underbrace{\Theta^{\mathscr{T}\mathscr{U}}}],[\underbrace{\Theta^{\mathscr{I}\mathscr{L}}},\underbrace{\Theta^{\mathscr{I}\mathscr{U}}}],[\widehat{\Theta^{\mathscr{F}\mathscr{L}}},\widehat{\Theta^{\mathscr{F}\mathscr{U}}}]\right\rangle$, *where* $1 \leq i \leq n$, $j = 1,2,...,i_j$ *(boundedness property).*

Proof. Since, $\widehat{\Theta^{\mathscr{T}\mathscr{L}}} = \inf \Theta_{ij}^{\mathscr{T}\mathscr{L}}$, $\underbrace{\Theta^{\mathscr{T}\mathscr{L}}} = \sup \Theta_{ij}^{\mathscr{T}\mathscr{L}}$ $\widehat{\Theta^{\mathscr{T}\mathscr{U}}} = \inf \Theta_{ij}^{\mathscr{T}\mathscr{U}}$, $\underbrace{\Theta^{\mathscr{T}\mathscr{U}}} = \sup \Theta_{ij}^{\mathscr{T}\mathscr{U}}$ and $\widehat{\Theta^{\mathscr{T}\mathscr{L}}} \leq \Theta_{ij}^{\mathscr{T}\mathscr{L}} \leq \underbrace{\Theta^{\mathscr{T}\mathscr{L}}}$ and $\widehat{\Theta^{\mathscr{T}\mathscr{U}}} \leq \Theta_{ij}^{\mathscr{T}\mathscr{U}} \leq \underbrace{\Theta^{\mathscr{T}\mathscr{U}}}$. Now,

$$\begin{aligned}
\widehat{\Theta^{\mathscr{T}\mathscr{L}}} + \widehat{\Theta^{\mathscr{T}\mathscr{U}}} &= \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\widehat{\Theta^{\mathscr{T}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{2\Delta} + \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\widehat{\Theta^{\mathscr{T}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{2\Delta} \\
&\leq \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\Theta_{ij}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta} + \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\Theta_{ij}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta} \\
&\leq \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\underbrace{\Theta^{\mathscr{T}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{2\Delta} + \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[2\Delta]{(\underbrace{\Theta^{\mathscr{T}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{2\Delta} \\
&= \underbrace{\Theta^{\mathscr{T}\mathscr{L}}} + \underbrace{\Theta^{\mathscr{T}\mathscr{U}}}.
\end{aligned}$$

Since, $\widehat{\Theta^{\mathscr{I}\mathscr{L}}} = \inf \Theta_{ij}^{\mathscr{I}\mathscr{L}}$, $\underbrace{\Theta^{\mathscr{I}\mathscr{L}}} = \sup \Theta_{ij}^{\mathscr{I}\mathscr{L}}$ $\widehat{\Theta^{\mathscr{I}\mathscr{U}}} = \inf \Theta_{ij}^{\mathscr{I}\mathscr{U}}$, $\underbrace{\Theta^{\mathscr{I}\mathscr{U}}} = \sup \Theta_{ij}^{\mathscr{I}\mathscr{U}}$ and $\widehat{\Theta^{\mathscr{I}\mathscr{L}}} \leq \Theta_{ij}^{\mathscr{I}\mathscr{L}} \leq \underbrace{\Theta^{\mathscr{I}\mathscr{L}}}$ and $\widehat{\Theta^{\mathscr{I}\mathscr{U}}} \leq \Theta_{ij}^{\mathscr{I}\mathscr{U}} \leq \underbrace{\Theta^{\mathscr{I}\mathscr{U}}}$. Now,

$$\begin{aligned}
\widehat{\Theta^{\mathscr{I}\mathscr{L}}} + \widehat{\Theta^{\mathscr{I}\mathscr{U}}} &= \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\widehat{\Theta^{\mathscr{I}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{\Delta} + \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\widehat{\Theta^{\mathscr{I}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{\Delta} \\
&\leq \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\Theta_{ij}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta} + \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\Theta_{ij}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta} \\
&\leq \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\underbrace{\Theta^{\mathscr{I}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{\Delta} + \left(1 - \bigcirc_{i=1}^{n}\left(1 - \sqrt[\Delta]{(\underbrace{\Theta^{\mathscr{I}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{\Delta} \\
&= \underbrace{\Theta^{\mathscr{I}\mathscr{L}}} + \underbrace{\Theta^{\mathscr{I}\mathscr{U}}}.
\end{aligned}$$

Since, $\widehat{\Theta^{\mathscr{F}\mathscr{L}}} = \inf \Theta_{ij}^{\mathscr{F}\mathscr{L}}$, $\underbrace{\Theta^{\mathscr{F}\mathscr{L}}} = \sup \Theta_{ij}^{\mathscr{F}\mathscr{L}}$ $\widehat{\Theta^{\mathscr{F}\mathscr{U}}} = \inf \Theta_{ij}^{\mathscr{F}\mathscr{U}}$, $\underbrace{\Theta^{\mathscr{F}\mathscr{U}}} = \sup \Theta_{ij}^{\mathscr{F}\mathscr{U}}$ and $\widehat{\Theta^{\mathscr{F}\mathscr{L}}} \leq \Theta_{ij}^{\mathscr{F}\mathscr{L}} \leq \underbrace{\Theta^{\mathscr{F}\mathscr{L}}}$ and $\widehat{\Theta^{\mathscr{F}\mathscr{U}}} \leq \Theta_{ij}^{\mathscr{F}\mathscr{U}} \leq \underbrace{\Theta^{\mathscr{F}\mathscr{U}}}$. Now,

$$\begin{aligned}
\widehat{\Theta^{\mathscr{F}\mathscr{L}}} + \widehat{\Theta^{\mathscr{F}\mathscr{U}}} &= \bigcirc_{i=1}^{n}(\widehat{\Theta^{\mathscr{F}\mathscr{L}}})^{\varepsilon_i} + \bigcirc_{i=1}^{n}(\widehat{\Theta^{\mathscr{F}\mathscr{U}}})^{\varepsilon_i} \\
&\leq \bigcirc_{i=1}^{n}(\Theta_{ij}^{\mathscr{F}\mathscr{L}})^{\varepsilon_i} + \bigcirc_{i=1}^{n}(\Theta_{ij}^{\mathscr{F}\mathscr{U}})^{\varepsilon_i} \\
&\leq \bigcirc_{i=1}^{n}(\underbrace{\Theta^{\mathscr{F}\mathscr{L}}})^{\varepsilon_i} + \bigcirc_{i=1}^{n}(\underbrace{\Theta^{\mathscr{F}\mathscr{U}}})^{\varepsilon_i} \\
&= \underbrace{\Theta^{\mathscr{F}\mathscr{L}}} + \underbrace{\Theta^{\mathscr{F}\mathscr{U}}}.
\end{aligned}$$

Since, $\widehat{\varrho} = \inf \varrho_{ij}$, $\underbrace{\varrho} = \sup \varrho_{ij}$, $\widehat{\tau} = \sup \tau_{ij}$, $\underbrace{\tau} = \inf \tau_{ij}$ and $\widehat{\varrho} \leq \varrho_{ij} \leq \underbrace{\varrho}$ and $\underbrace{\tau} \leq \tau_{ij} \leq \widehat{\tau}$. Thus, $\oplus_{i=1}^{n}\varepsilon_i\widehat{\varrho} \leq \oplus_{i=1}^{n}\varepsilon_i\varrho_{ij} \leq \oplus_{i=1}^{n}\varepsilon_i\underbrace{\varrho}$ and $\oplus_{i=1}^{n}\varepsilon_i\underbrace{\tau} \leq \oplus_{i=1}^{n}\varepsilon_i\tau_{ij} \leq \oplus_{i=1}^{n}\varepsilon_i\widehat{\tau}$.

Hence,

$$\frac{\oplus_{i=1}^{n}\varepsilon_i\widehat{\varrho}}{2}\times\left[\begin{array}{c}\frac{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\widehat{\Theta^{\mathscr{T}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{2\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\widehat{\Theta^{\mathscr{T}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{2\Delta}}{2}\\ -\frac{\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\widehat{\Theta^{\mathscr{I}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{\Delta}}+\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\widehat{\Theta^{\mathscr{I}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{\Delta}}}{2}\\ +1-\frac{\sqrt{\left(\bigcirc_{i=1}^{n}(\underbrace{\Theta^{\mathscr{F}\mathscr{L}}})^{\varepsilon_i}\right)}+\sqrt{\left(\bigcirc_{i=1}^{n}(\underbrace{\Theta^{\mathscr{F}\mathscr{U}}})^{\varepsilon_i}\right)}}{2}\end{array}\right]$$

$$\leq\ \frac{\oplus_{i=1}^{n}\varepsilon_i\varrho_{ij}}{2}\times\left[\begin{array}{c}\frac{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{ij}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{ij}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}}{2}\\ -\frac{\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{ij}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta}}+\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{ij}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}}}{2}\\ +1-\frac{\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{ij}^{\mathscr{F}\mathscr{L}})^{\varepsilon_i}\right)}+\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{ij}^{\mathscr{F}\mathscr{U}})^{\varepsilon_i}\right)}}{2}\end{array}\right]$$

$$\leq\ \frac{\oplus_{i=1}^{n}\varepsilon_i\underbrace{\varrho}}{2}\times\left[\begin{array}{c}\frac{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\underbrace{\Theta_{ij}^{\mathscr{T}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{2\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\underbrace{\Theta_{ij}^{\mathscr{T}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{2\Delta}}{2}\\ -\frac{\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\underbrace{\Theta_{ij}^{\mathscr{I}\mathscr{L}}})}\right)^{\varepsilon_i}\right)^{\Delta}}+\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\underbrace{\Theta_{ij}^{\mathscr{I}\mathscr{U}}})}\right)^{\varepsilon_i}\right)^{\Delta}}}{2}\\ +1-\frac{\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{ij}^{\mathscr{F}\mathscr{L}})^{\varepsilon_i}\right)}+\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{ij}^{\mathscr{F}\mathscr{U}})^{\varepsilon_i}\right)}}{2}\end{array}\right].$$

Therefore, $\left\langle(\widehat{\varrho},\widehat{\tau});[\widehat{\Theta^{\mathscr{T}\mathscr{L}}},\widehat{\Theta^{\mathscr{T}\mathscr{U}}}],[\widehat{\Theta^{\mathscr{I}\mathscr{L}}},\widehat{\Theta^{\mathscr{I}\mathscr{U}}}],[\underbrace{\Theta^{\mathscr{F}\mathscr{L}}},\underbrace{\Theta^{\mathscr{F}\mathscr{U}}}]\right\rangle\leq SRNSNIVWA(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)$
$\leq\left\langle(\underbrace{\varrho},\underbrace{\tau});[\underbrace{\Theta^{\mathscr{T}\mathscr{L}}},\underbrace{\Theta^{\mathscr{T}\mathscr{U}}}],[\underbrace{\Theta^{\mathscr{I}\mathscr{L}}},\underbrace{\Theta^{\mathscr{I}\mathscr{U}}}],[\widehat{\Theta^{\mathscr{F}\mathscr{L}}},\widehat{\Theta^{\mathscr{F}\mathscr{U}}}]\right\rangle.$ □

Theorem 5.4. *Let* $\widetilde{\mho}_i=\left\langle(\varrho_{t_{ij}},\tau_{t_{ij}});[\Theta_{t_{ij}}^{\mathscr{T}\mathscr{L}},\Theta_{t_{ij}}^{\mathscr{T}\mathscr{U}}],[\Theta_{t_{ij}}^{\mathscr{I}\mathscr{L}},\Theta_{t_{ij}}^{\mathscr{I}\mathscr{U}}],[\Theta_{t_{ij}}^{\mathscr{F}\mathscr{L}},\Theta_{t_{ij}}^{\mathscr{F}\mathscr{U}}]\right\rangle$ *and* $\widetilde{W}_i=\left\langle(\varrho_{h_{ij}},\tau_{h_{ij}});[\Theta_{h_{ij}}^{\mathscr{T}\mathscr{L}},\Theta_{h_{ij}}^{\mathscr{T}\mathscr{U}}],[\Theta_{h_{ij}}^{\mathscr{I}\mathscr{L}},\Theta_{h_{ij}}^{\mathscr{I}\mathscr{U}}],[\Theta_{h_{ij}}^{\mathscr{F}\mathscr{L}},\Theta_{h_{ij}}^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the two families of SRNSNIVWAs. For any i, if there is* $\varrho_{t_{ij}}\leq\tau_{h_{ij}}$, $\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{T}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{T}\mathscr{U}}\right)}\leq\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{T}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{T}\mathscr{U}}\right)}$ *and* $\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{I}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{I}\mathscr{U}}\right)}\leq\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{I}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{I}\mathscr{U}}\right)}$ *and* $\left(\Theta_{t_{ij}}^{\mathscr{F}\mathscr{L}}\right)+\left(\Theta_{t_{ij}}^{\mathscr{F}\mathscr{U}}\right)\geq\left(\Theta_{h_{ij}}^{\mathscr{F}\mathscr{L}}\right)+\left(\Theta_{h_{ij}}^{\mathscr{F}\mathscr{U}}\right)$ *or* $\widetilde{\mho}_i\leq\widetilde{W}_i$*, then* $SRNSNIVWA\left(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n\right)\leq SRNSNIVWA\left(\widetilde{W}_1,\widetilde{W}_2,...,\widetilde{W}_n\right)$*, where* $(i=1\ to\ n);(j=1\ to\ i_j)$ *(monotonicity property).*

Proof. For any i, $\varrho_{t_{ij}}\leq\tau_{h_{ij}}$. Thus, $\oplus_{i=1}^{n}\varrho_{t_{ij}}\leq\oplus_{i=1}^{n}\tau_{h_{ij}}$.
For any i, $\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{T}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{T}\mathscr{U}}\right)}\leq\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{T}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{T}\mathscr{U}}\right)}$.
Therefore, $1-\sqrt[2\Delta]{\left(\Theta_{t_i}^{\mathscr{T}\mathscr{L}}\right)}+1-\sqrt[2\Delta]{\left(\Theta_{t_i}^{\mathscr{T}\mathscr{U}}\right)}\geq1-\sqrt[2\Delta]{\left(\Theta_{h_i}^{\mathscr{T}\mathscr{L}}\right)}+1-\sqrt[2\Delta]{\left(\Theta_{h_i}^{\mathscr{T}\mathscr{U}}\right)}$.
Hence,

$$\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{\left(\Theta_{t_i}^{\mathscr{T}\mathscr{L}}\right)}\right)^{\varepsilon_i}+\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{\left(\Theta_{t_i}^{\mathscr{T}\mathscr{U}}\right)}\right)^{\varepsilon_i}\geq\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{\left(\Theta_{h_i}^{\mathscr{T}\mathscr{L}}\right)}\right)^{\varepsilon_i}+\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{\left(\Theta_{h_i}^{\mathscr{T}\mathscr{U}}\right)}\right)^{\varepsilon_i}$$

and $\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{t_i}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{t_i}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}\leq$

$$\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{h_i}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{h_i}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}.$$

For any i, $\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{I}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{t_{ij}}^{\mathscr{I}\mathscr{U}}\right)}\leq\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{I}\mathscr{L}}\right)}+\sqrt{\left(\Theta_{h_{ij}}^{\mathscr{I}\mathscr{U}}\right)}$.
Therefore, $1-\sqrt[\Delta]{\left(\Theta_{t_i}^{\mathscr{I}\mathscr{L}}\right)}+1-\sqrt[\Delta]{\left(\Theta_{t_i}^{\mathscr{I}\mathscr{U}}\right)}\geq 1-\sqrt[\Delta]{\left(\Theta_{h_i}^{\mathscr{I}\mathscr{L}}\right)}+1-\sqrt[\Delta]{\left(\Theta_{h_i}^{\mathscr{I}\mathscr{U}}\right)}$.
Hence,
$\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{\left(\Theta_{t_i}^{\mathscr{I}\mathscr{L}}\right)}\right)^{\varepsilon_i}+\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{\left(\Theta_{t_i}^{\mathscr{I}\mathscr{U}}\right)}\right)^{\varepsilon_i}\geq$

$$\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{\left(\Theta_{h_i}^{\mathscr{I}\mathscr{L}}\right)}\right)^{\varepsilon_i}+\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{\left(\Theta_{h_i}^{\mathscr{I}\mathscr{U}}\right)2}\right)^{\varepsilon_i}$$

and $\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{t_i}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{t_i}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}\leq$

$$\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{h_i}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{h_i}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}.$$

For any i, $\left(\Theta_{t_{ij}}^{\mathscr{F}\mathscr{L}}\right)+\left(\Theta_{t_{ij}}^{\mathscr{F}\mathscr{U}}\right)\geq\left(\Theta_{h_{ij}}^{\mathscr{F}\mathscr{L}}\right)+\left(\Theta_{h_{ij}}^{\mathscr{F}\mathscr{U}}\right)$.
Therefore, $1-\frac{\left(\bigcirc_{i=1}^{n}\Theta_{t_{ij}}^{\mathscr{F}\mathscr{L}}\right)+\left(\bigcirc_{i=1}^{n}\Theta_{t_{ij}}^{\mathscr{F}\mathscr{U}}\right)}{2}\leq 1-\frac{\left(\bigcirc_{i=1}^{n}\Theta_{h_{ij}}^{\mathscr{F}\mathscr{L}}\right)+\left(\bigcirc_{i=1}^{n}\Theta_{h_{ij}}^{\mathscr{F}\mathscr{U}}\right)}{2}$. Hence,

$$\frac{\oplus_{i=1}^{n}\varrho_{tij}}{2}\times\left[\begin{array}{c}\frac{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{ti}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{ti}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}}{2}\\ -\frac{\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{ti}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}}+\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{ti}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}}}{2}\\ +1-\frac{\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{tij}^{\mathscr{F}\mathscr{L}})\right)}+\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{tij}^{\mathscr{F}\mathscr{U}})\right)}}{2}\end{array}\right]$$

$$\leq\frac{\oplus_{i=1}^{n}\varrho_{hij}}{2}\times\left[\begin{array}{c}\frac{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{hi}^{\mathscr{T}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}+\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_{hi}^{\mathscr{T}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}}{2}\\ -\frac{\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{hi}^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta}}+\sqrt{\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_{hi}^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}}}{2}\\ +1-\frac{\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{hij}^{\mathscr{F}\mathscr{L}})\right)}+\sqrt{\left(\bigcirc_{i=1}^{n}(\Theta_{hij}^{\mathscr{F}\mathscr{U}})\right)}}{2}\end{array}\right].$$

Hence, $SRNSNIVWA\left(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n\right)\leq SRNSNIVWA\left(\widetilde{W}_1,\widetilde{W}_2,...,\widetilde{W}_n\right)$. □

5.2. SRNSNIV weighted geometric(SRNSNIVWG) operator

Definition 5.2. *Let* $\widetilde{\mho}_i=\left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}],[\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the family of SRNSNIVNs. Then SRNSNIVWG* $(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\bigcirc_{i=1}^{n}\widetilde{\mho}_i^{\varepsilon_i}$.

Theorem 5.5. *Let* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the family of SRNSNIVNs. Prove that*

$$\left[\begin{array}{c}\left(\bigcirc_{i=1}^{n}\varrho_i^{\varepsilon_i},\bigcirc_{i=1}^{n}\tau_i^{\varepsilon_i}\right);\left[\bigcirc_{i=1}^{n}(\Theta_i^{\mathscr{T}\mathscr{L}})^{\varepsilon_i},\bigcirc_{i=1}^{n}(\Theta_i^{\mathscr{T}\mathscr{U}})^{\varepsilon_i}\right],\\ \left[\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{\Delta},\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{\Delta}\right],\\ \left[\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\varepsilon_i}\right)^{2\Delta},\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\varepsilon_i}\right)^{2\Delta}\right]\end{array}\right].$$

Theorem 5.6. *If all* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}][\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle(i=1,2,...,n)$ *are equal, then SRNSNIVWG*$(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\widetilde{\mho}$.

Remark 5.1. *Using the SRNSNIVWG operator, the boundedness and monotonicity properties are met.*

5.3. Generalized SRNSNIVWA (GSRNSNIVWA) operator

Definition 5.3. *Let* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}],[\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the family of SRNSNIVN. Then GSRNSNIVWA* $(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\left(\oplus_{i=1}^{n}\varepsilon_i\widetilde{\mho}_i^{\Delta}\right)^{1/\Delta}$.

Theorem 5.7. *Let* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}],[\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the family of SRNSNIVNs. Then*

$$GSRNSNIVWA(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\left[\begin{array}{c}\left(\left(\oplus_{i=1}^{n}\varepsilon_i\varrho_i^{\Delta}\right)^{1/\Delta},\left(\oplus_{i=1}^{n}\varepsilon_i\tau_i^{\Delta}\right)^{1/\Delta}\right);\\ \left[\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{2\Delta}\right)^{\Delta},\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{2\Delta}\right)^{\Delta}\right],\\ \left[\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{\Delta}\right)^{\Delta},\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{\Delta}\right)^{\Delta}\right],\\ \left[\begin{array}{c}\left(1-\left(1-\sqrt[2\Delta]{\bigcirc_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}}\right)^{\Delta}\right)^{2\Delta},\\ \left(1-\left(1-\sqrt[2\Delta]{\bigcirc_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}}\right)^{\Delta}\right)^{2\Delta}\end{array}\right]\end{array}\right].$$

Proof. It must be demonstrated that,

$$\oplus_{i=1}^{n}\varepsilon_i\widetilde{\mho}_i^{\Delta}=\left[\begin{array}{c}\left(\left(\oplus_{i=1}^{n}\varepsilon_i\varrho_i^{\Delta}\right),\left(\oplus_{i=1}^{n}\varepsilon_i\tau_i^{\Delta}\right)\right);\\ \left[\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta},\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}\right],\\ \left[\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta},\left(1-\bigcirc_{i=1}^{n}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}\right],\\ \left[\bigcirc_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i},\bigcirc_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}\right]\end{array}\right].$$

Put $n = 2$, $\varepsilon_1\mho_1 \oplus \varepsilon_2\mho_2 =$

$$\left[\begin{array}{c}
\left(\varepsilon_1\varrho_1^{\Delta} + \varepsilon_2\varrho_2^{\Delta}, \varepsilon_1\tau_1^{\Delta} + \varepsilon_2\tau_2^{\Delta}\right); \\
\left[\begin{array}{c}
\left(\begin{array}{c}
\sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} + \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_2}\right)^{2\Delta}} \\
-\sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} \cdot \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_2}\right)^{2\Delta}}
\end{array}\right)^{2\Delta}, \\
\left(\begin{array}{c}
\sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} + \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_2}\right)^{2\Delta}} \\
-\sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} \cdot \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_2}\right)^{2\Delta}}
\end{array}\right)^{2\Delta}
\end{array}\right], \\
\left[\begin{array}{c}
\left(\begin{array}{c}
\sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} + \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_2}\right)^{\Delta}} \\
-\sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} \cdot \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_2}\right)^{\Delta}}
\end{array}\right)^{\Delta}, \\
\left(\begin{array}{c}
\sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} + \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_2}\right)^{\Delta}} \\
-\sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_1^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} \cdot \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_2^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_2}\right)^{\Delta}}
\end{array}\right)^{\Delta}
\end{array}\right], \\
\left[\begin{array}{c}
\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{F}\mathscr{L}})}\right)^{2\Delta}\right)\right)^{\varepsilon_1} \cdot \left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_1}, \\
\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_1^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_1} \cdot \left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_2^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_1}
\end{array}\right]
\end{array}\right]$$

$$= \left[\begin{array}{c}
\left(\left(\oplus_{i=1}^{2} \varepsilon_i\varrho_i^{\Delta}\right), \left(\oplus_{i=1}^{2} \varepsilon_i\tau_i^{\Delta}\right)\right); \\
\left[\left(1-\bigcirc_{i=1}^{2}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}, \left(1-\bigcirc_{i=1}^{2}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}\right], \\
\left[\left(1-\bigcirc_{i=1}^{2}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}, \left(1-\bigcirc_{i=1}^{2}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}\right], \\
\left[\bigcirc_{i=1}^{2}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}, \bigcirc_{i=1}^{2}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}\right]
\end{array}\right].$$

In general,

$$\left[\begin{array}{c}
\left(\left(\oplus_{i=1}^{l} \varepsilon_i\varrho_i^{\Delta}\right), \left(\oplus_{i=1}^{l} \varepsilon_i\tau_i^{\Delta}\right)\right); \\
\left[\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}, \left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}\right], \\
\left[\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}, \left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}\right], \\
\left[\bigcirc_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}, \bigcirc_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}\right]
\end{array}\right].$$

If $n = l + 1$, then $\oplus_{i=1}^{l}\varepsilon_i\widetilde{\mho}_i^{\Delta} + \varepsilon_{l+1}\widetilde{\mho}_{l+1}^{\Delta} = \oplus_{i=1}^{l+1}\varepsilon_i\widetilde{\mho}_i^{\Delta}$.
Now, $\oplus_{i=1}^{l}\varepsilon_i\widetilde{\mho}_i^{\Delta} + \varepsilon_{l+1}\widetilde{\mho}_{l+1}^{\Delta} = \oplus_{i=1}^{l+1}\varepsilon_i\widetilde{\mho}_i^{\Delta} = \varepsilon_1\widetilde{\mho}_1^{\Delta} \oplus \varepsilon_2\widetilde{\mho}_2^{\Delta} \oplus ... \oplus \varepsilon_l\widetilde{\mho}_l^{\Delta} \oplus \varepsilon_{l+1}\widetilde{\mho}_{l+1}^{\Delta}$

$$= \begin{bmatrix} \left(\varepsilon_i\varrho_i^{\Delta} + \varepsilon_{l+1}\varrho_{l+1}^{\Delta}, \varepsilon_i\tau_i^{\Delta} + \varepsilon_{l+1}\tau_{l+1}^{\Delta}\right); \\ \left[\begin{pmatrix} \sqrt[2\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}} + \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} \\ -\sqrt[2\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}} \cdot \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} \end{pmatrix}^{2\Delta}, \\ \begin{pmatrix} \sqrt[2\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}} + \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} \\ -\sqrt[2\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}} \cdot \sqrt[2\Delta]{\left(1-\left(1-\sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{2\Delta}} \end{pmatrix}^{2\Delta}\right], \\ \left[\begin{pmatrix} \sqrt[\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}} + \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} \\ -\sqrt[\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}} \cdot \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} \end{pmatrix}^{\Delta}, \\ \begin{pmatrix} \sqrt[\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}} + \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} \\ -\sqrt[\Delta]{\left(1-\bigcirc_{i=1}^{l}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}} \cdot \sqrt[\Delta]{\left(1-\left(1-\sqrt[\Delta]{(\Theta_{l+1}^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_1}\right)^{\Delta}} \end{pmatrix}^{\Delta}\right], \\ \left[\begin{matrix} \bigcirc_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i} \cdot \left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_1}, \\ \bigcirc_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i} \cdot \left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_{l+1}^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_1} \end{matrix}\right] \end{bmatrix}.$$

Hence,

$$\oplus_{i=1}^{l+1}\varepsilon_i\widetilde{\mho}_i^{\Delta} = \begin{bmatrix} \left(\left(\oplus_{i=1}^{l+1}\varepsilon_i\varrho_i^{\Delta}\right), \left(\oplus_{i=1}^{l+1}\varepsilon_i\tau_i^{\Delta}\right)\right); \\ \left[\left(1-\bigcirc_{i=1}^{l+1}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}, \left(1-\bigcirc_{i=1}^{l+1}\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{2\Delta}\right], \\ \left[\left(1-\bigcirc_{i=1}^{l+1}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}, \left(1-\bigcirc_{i=1}^{l+1}\left(1-\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)^{\varepsilon_i}\right)^{\Delta}\right], \\ \left[\bigcirc_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}, \bigcirc_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}\right] \end{bmatrix}.$$

Also, $\left(\bigcirc_{i=1}^{l+1}\varepsilon_i\widetilde{\mho}_i^{\Delta}\right)^{1/\Delta} =$

$$\left[\begin{array}{c}\left(\left(\bigcirc_{i=1}^{l+1}\varepsilon_i\varrho_i^{\Delta}\right)^{1/\Delta},\left(\bigcirc_{i=1}^{l+1}\varepsilon_i\tau_i^{\Delta}\right)^{1/\Delta}\right);\\ \left[\left(\left(1-\bigcirc_{i=1}^{l+1}\left(1-\left(\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{2\Delta}\right)^{\Delta},\left(\left(1-\bigcirc_{i=1}^{l+1}\left(1-\left(\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{2\Delta}\right)^{\Delta}\right],\\ \left[\left(\left(1-\bigcirc_{i=1}^{l+1}\left(1-\left(\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{\Delta}\right)^{\Delta},\left(\left(1-\bigcirc_{i=1}^{l+1}\left(1-\left(\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{\Delta}\right)^{\Delta}\right],\\ \left[\begin{array}{c}\left(1-\left(1-\sqrt[2\Delta]{\bigcirc_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}}\right)^{\Delta}\right)^{2\Delta},\\ \left(1-\left(1-\sqrt[2\Delta]{\bigcirc_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}}\right)^{\Delta}\right)^{2\Delta}\end{array}\right]\end{array}\right].$$

□

Remark 5.2. *The GSRNSNIVWA is switched to the SRNSNIVWA if* $\Delta = 1$.

Theorem 5.8. *If all* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}][\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle(i = 1$ *to* $n)$ *are equal, then* $GSRNSNIVWA(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\widetilde{\mho}$.

Remark 5.3. *Using the GSRNSNIVWA, the boundedness and monotonicity are met.*

5.4. Generalized SRNSNIVWG (GSRNSNIVWG) operator

Definition 5.4. *Let* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}],[\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the family of SRNSNIVNs. Then GSRNSNIVWG* $(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\frac{1}{\Delta}\left(\bigcirc_{i=1}^{n}(\Delta\widetilde{\mho}_i)^{\varepsilon_i}\right)$.

Theorem 5.9. *Let* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}],[\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle$ *be the family of SRNSNIVNs. Prove that GSRNSNIVWG*$(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)$=

$$\left[\begin{array}{c}\left(\frac{1}{\Delta}\bigcirc_{i=1}^{n}(\Delta\varrho_i)^{\varepsilon_i},\frac{1}{\Delta}\bigcirc_{i=1}^{n}(\Delta\tau_i)^{\varepsilon_i}\right);\\ \left[\begin{array}{c}\left(1-\left(1-\sqrt[2\Delta]{\bigcirc_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{L}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}}\right)^{\Delta}\right)^{2\Delta},\\ \left(1-\left(1-\sqrt[2\Delta]{\bigcirc_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Delta]{(\Theta_i^{\mathscr{T}\mathscr{U}})}\right)^{\Delta}\right)^{2\Delta}\right)^{\varepsilon_i}}\right)^{\Delta}\right)^{2\Delta}\end{array}\right],\\ \left[\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{L}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{\Delta}\right)^{\Delta},\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[\Delta]{(\Theta_i^{\mathscr{I}\mathscr{U}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{\Delta}\right)^{\Delta}\right],\\ \left[\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{L}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{2\Delta}\right)^{\Delta},\left(\left(1-\bigcirc_{i=1}^{n}\left(1-\left(\sqrt[2\Delta]{(\Theta_i^{\mathscr{F}\mathscr{U}})^{\Delta}}\right)\right)^{\varepsilon_i}\right)^{2\Delta}\right)^{\Delta}\right]\end{array}\right].$$

Remark 5.4. *The GSRNSNIVWG becomes the SRNSNIVWG operator if* $\Delta = 1$.

Remark 5.5. *Using the GSRNSNIVWG, the boundedness and monotonicity properties are met.*

Theorem 5.10. *If all* $\widetilde{\mho}_i = \left\langle(\varrho_i,\tau_i);[\Theta_i^{\mathscr{T}\mathscr{L}},\Theta_i^{\mathscr{T}\mathscr{U}}],[\Theta_i^{\mathscr{I}\mathscr{L}},\Theta_i^{\mathscr{I}\mathscr{U}}][\Theta_i^{\mathscr{F}\mathscr{L}},\Theta_i^{\mathscr{F}\mathscr{U}}]\right\rangle(i = 1$ *to* $n)$ *are equal, then* $GSRNSNIVWG(\widetilde{\mho}_1,\widetilde{\mho}_2,...,\widetilde{\mho}_n)=\widetilde{\mho}$.

6. MADM using SRNSNIV data

Let $\widetilde{\Xi} = \{\widetilde{\Xi}_1, \widetilde{\Xi}_2, ..., \widetilde{\Xi}_n\}$ be the n-alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the m-attributes, $w = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_m\}$ be the weights of attributes, $\widetilde{\widetilde{\Xi}}_{ij} = \Big\langle (\varrho_{ij}, \tau_{ij}); [\Theta_{ij}^{\mathscr{T}\mathscr{L}}, \Theta_{ij}^{\mathscr{T}\mathscr{U}}], [\Theta_{ij}^{\mathscr{I}\mathscr{L}}, \Theta_{ij}^{\mathscr{I}\mathscr{U}}]$ $[\Theta_{ij}^{\mathscr{F}\mathscr{L}}, \Theta_{ij}^{\mathscr{F}\mathscr{U}}]\Big\rangle$ is denote SRNSNIVN of $\widetilde{\Xi}_i$ in C_j. Here, $\Big[\Theta_{ij}^{\mathscr{T}\mathscr{L}}, \Theta_{ij}^{\mathscr{T}\mathscr{U}}\Big], \Big[\Theta_{ij}^{\mathscr{I}\mathscr{L}}, \Theta_{ij}^{\mathscr{I}\mathscr{U}}\Big],$ $\Big[\Theta_{ij}^{\mathscr{F}\mathscr{L}}, \Theta_{ij}^{\mathscr{F}\mathscr{U}}\Big] \in [0,1]$ and $0 \leq (\Theta_{ij}^{\mathscr{T}\mathscr{U}}(\mu))^2 + \sqrt{(\Theta_{ij}^{\mathscr{I}\mathscr{U}}(\mu))} + \sqrt{(\Theta_{ij}^{\mathscr{F}\mathscr{U}}(\mu))} \leq 2$.

6.1. *Algorithm for SRNSNIV*

Step-1: There should be a choice value for SRNSNIV.
Step-2: Choosing the values to be used for normalization. The matrix of choices $\mathbb{D} = (\widetilde{\Xi}_{ij})_{n\times m}$ is normalized into $\widehat{\mathbb{D}} = (\overline{\widetilde{\Xi}}_{ij})_{n\times m}$; put

$$\overline{\widetilde{\Xi}}_{ij} = \Big\langle (\widehat{\varrho_{ij}}, \widehat{\tau_{ij}}); [\widehat{\Theta_{ij}^{\mathscr{T}\mathscr{L}}}, \widehat{\Theta_{ij}^{\mathscr{T}\mathscr{U}}}], [\widehat{\Theta_{ij}^{\mathscr{I}\mathscr{L}}}, \widehat{\Theta_{ij}^{\mathscr{I}\mathscr{U}}}], [\widehat{\Theta_{ij}^{\mathscr{F}\mathscr{L}}}, \widehat{\Theta_{ij}^{\mathscr{F}\mathscr{U}}}]\Big\rangle$$

and

$$\widehat{\varrho_{ij}} = \frac{\varrho_{ij}}{\sup_i(\varrho_{ij})}, \widehat{\tau_{ij}} = \frac{\tau_{ij}}{\sup_i(\tau_{ij})} \cdot \frac{\tau_{ij}}{\varrho_{ij}}, \ \widehat{\Theta_{ij}^{\mathscr{T}\mathscr{L}}} = \Theta_{ij}^{\mathscr{T}\mathscr{L}}, \widehat{\Theta_{ij}^{\mathscr{T}\mathscr{U}}} = \Theta_{ij}^{\mathscr{T}\mathscr{U}}.$$

Step-3: Find the aggregate values, Using SRNSNIV operators, attribute C_j in $\widetilde{\Xi}_i$, $\overline{\widetilde{\Xi}}_{ij} = \Big\langle (\widehat{\varrho_{ij}}, \widehat{\tau_{ij}}); [\widehat{\Theta_{ij}^{\mathscr{T}\mathscr{L}}}, \widehat{\Theta_{ij}^{\mathscr{T}\mathscr{U}}}], [\widehat{\Theta_{ij}^{\mathscr{I}\mathscr{L}}}, \widehat{\Theta_{ij}^{\mathscr{I}\mathscr{U}}}], [\widehat{\Theta_{ij}^{\mathscr{F}\mathscr{L}}}, \widehat{\Theta_{ij}^{\mathscr{F}\mathscr{U}}}]\Big\rangle$ is aggregated into $\overline{\widetilde{\Xi}}_i = \Big\langle (\widehat{\varrho_i}, \widehat{\tau_i}); [\widehat{\Theta_i^{\mathscr{T}\mathscr{L}}}, \widehat{\Theta_i^{\mathscr{T}\mathscr{U}}}], [\widehat{\Theta_i^{\mathscr{I}\mathscr{L}}}, \widehat{\Theta_i^{\mathscr{I}\mathscr{U}}}], [\widehat{\Theta_i^{\mathscr{F}\mathscr{L}}}, \widehat{\Theta_i^{\mathscr{F}\mathscr{U}}}]\Big\rangle$.
Step-4: Calculate the positive and negative ideal values:

$$\overline{\widetilde{\Xi}}^P = \Big\langle \Big(\sup_{1\leq i\leq n} (\widehat{\varrho_{ij}}), \inf_{1\leq i\leq n} (\widehat{\tau_{ij}}) \Big); [1,1], [1,1], [0,0] \Big\rangle,$$

$$\overline{\widetilde{\Xi}}^N = \Big\langle \Big(\inf_{1\leq i\leq n} (\widehat{\varrho_{ij}}), \sup_{1\leq i\leq n} (\widehat{\tau_{ij}}) \Big); [0,0], [0,0], [1,1] \Big\rangle.$$

Step-5: Find the ED between each option with two ideal values:

$$\mathbb{D}_i^P = \mathbb{D}_E\Big(\overline{\widetilde{\Xi}}_i, \overline{\widetilde{\Xi}}^P\Big); \ \mathbb{D}_i^N = \mathbb{D}_E\Big(\overline{\widetilde{\Xi}}_i, \overline{\widetilde{\Xi}}^N\Big).$$

Step-6: Calculating relative closeness is as follows:

$$\mathbb{D}_i^\star = \frac{\mathbb{D}_i^N}{\mathbb{D}_i^P + \mathbb{D}_i^N}.$$

Step-7: In this case, the best output is $\sup \mathbb{D}_i^\star$ To solve a problem, a decision is to choose the best option.

6.2. Selection process based on medical robotic engineering

Educators, economists, managers, and politicians all face DM challenges every day. Before making your final robotic engineering choice, take into account the following five points. Based on professional evaluations against the criteria, we want to choose the best option from a large number of alternatives. Computer science and machine tool technology have been combined in robotics, a branch of applied engineering. Computer programming, microelectronics, machine design, and artificial intelligence are all covered. Currently, we have selected five types of robotic nurses: Pharma robotics, Robotic-assisted biopsy, Antibacterial nano-materials, AI diagnostics, and AI epidemiology. A robotics system should be chosen based on the following four criteria: robot controller features (C_1), affordable off line programming software (C_2), safety codes (C_3), experience and reputation of the robot manufacturer (C_4) and their weights are $w = \{0.4, 0.3, 0.2, 0.1\}$.

(1) ROBOTIC NURSES ($\widetilde{\Xi}_1$):
It is useful to have nursing robots in hospitals as well as in senior living facilities. Nurses may be able to focus on their work by having robots relieve them of their workloads. Automated machines have already been considered to support processes such as the distribution of food trays, medications, and laboratory specimens in hospitals, which are relevant to their primary duties. A robotic system designed to assist nurses in patient transfers, ambulation, and limiting their movements could significantly reduce the physical strain they face. Telemedicine might also be offered by nursing robots. Telepresence platforms equipped with robotic nurses enable doctors to communicate effectively with patients remotely. Robots usually travel to hospitals using their onboard screens to make visual contact with patients during routine vital visits. Additionally, the robot records the patient's vital signs at regular intervals in accordance with standard clinical protocols. Sales of health care robots are expected to reach 2.8 billion dollars by 2021. Health care seems to have a limitless number of applications. Robots for hospitals, housing, and surgery are all included in this market. Robots that can help with therapy, medication, logistics, telepresence, and cleaning can also be utilized. This includes surgical robots, training exoskeletons, intelligent prostheses and bionics, robotic nurses, and robots for assisting with therapy, medication, logistics, and telepresence. Further, touch sensors, speech recognition, gesture control, and machine vision are crucial technologies for health care robotics. Robert Din Sow is a patient in Thai and Japanese hospitals. While keeping an eye on elderly patients, Robert Din Sow conducts video talks with their families. Using a desktop or laptop computer, a robotic arm can be operated remotely.

(2) PHARMAROBOTICS ($\widetilde{\Xi}_2$):
Worldwide, dairy farmers administer millions of vaccines and reproductive items to cows. It takes a lot of training and work to administer these medications. In this sector, only about 70 % of cows achieve a 100 % compliance rate under current protocols, meaning they don't receive the full prescribed schedule of vaccines or reproductive products, which could negatively impact their health. We are pro-environment and pro-animal health farmers. As part of our efforts to build a healthier planet, we believe that using our patented technology to automate the process of giving essential drugs to animals will improve animal health standards. Sure Shot may administer up to three vaccinations and replica items. An automated gate system, RFID reader, camera ID reader, and dairy management software can be used to manufacture products based on each cow's RIFD tag. As a dairy farmer responsible for several agricultural activities, Alexander was mentored by

Marinus in 2017. As a first-year college student, Alexander had the idea of improving cow animal health standards and providing farmers with a sustainable economic future. The idea was patented and a new challenge was created. A robotic injection system for domestic herd animals. After that, the concept became a thought program. After the accelerator program, the team was able to develop a smart business model. February 2019 marked the founding of Pharm Robotics.

(3) ROBOTIC-ASSISTED BIOPSY ($\widetilde{\Xi}_3$):
The objective of our research was to develop a robot that could perform computer tomography (CT). The method's viability, accuracy, and efficacy were evaluated using ten peas placed inside a gel phantom (mean diameter 9.9t/0.4mm). Using CT imaging to identify the optimal access point, the position of the phantom was captured using an optical tracking device. The robot planning system (Linux-based industrial PC outfitted with a video capture card) received locational information about the phantom and its CT image after the correct angle, pitch, and location were determined. Using seven degrees of freedom, the robotic arm directed the needle's trajectory to the center of the target. A coaxial approach was used to perform the biopsy. After measuring the length of all harvested specimens, we pushed short bits of a guide into the target to determine the needle track's departure from the target. Resting biopsy specimens were available for all targets (mean length 5.6 t/1.4mm; only a needle pass was needed). As far as the needle tip was concerned, its mean deviation from the center of the target was 1.2 t/0.9 mm in both axis, and 0.6 t/0.4 mm in both axes. It was possible to test the perceptual and dexterity abilities of doctors using robotic assisted biopics in vitro utilizing CT guidance. These biopics early cancer diagnosis with enhanced detection and precise treatment. The procedures may be steered towards more accurate biopsies and focused medicines through the use of robotics in this situation. During a biopsy technique, which is stabilized by robotic manipulation, a tissue sample is obtained from the probable lesion.

(4) ANTIBACTERIAL NANOMATERIALS ($\widetilde{\Xi}_4$):
By fighting poisons and bacteria constantly, blood does not keep us alive. Due to the rise in antibiotic-resistant bacteria, robots are being called upon to save the day. It may one day be possible to save patients from bacterial illnesses that doctors are increasingly finding themselves unable to treat by using tiny robots. The tiny robots are made of gold nano wire coated with a membrane that kills bacteria and toxins and propelled by ultrasonic waves from outside the body. There is a major innovation in the device's covering. A red blood cell neutralizes bacteria's poisons, while a platelet attacks bacteria. Both work their magic through sensors on their exterior membranes. In order to create the toxin-fighting bot, gold nanowires are covered with the new membrane. Due to the effectiveness of their method, Ail claims that "we didn't have a substandard batch of robots". By making the nanowire segments concave at one end, Avila made the nanorobots asymmetrical, so that they would respond to ultrasound more effectively. Using ultrasound, the nanorobots move at a 35 micron per second speed. This setting does not allow for individual control of nabobs since they swarm in groups. A blood sample was found to contain an antibiotic-resistant bacterium called Staphylococcus Atreus. After just five minutes, there were three times fewer germs in the sample. The gold nanowire will be tested in vivo, among other things, to determine how hazardous it might be since gold reacts well to acoustic fields.

(5) AI DIAGNOSTICS AND AI EPIDEMIOLOGY ($\widetilde{\Xi}_5$):
In medicine, robots can do this work most effectively. Through machine learning, scientists can

train AIs to perform tasks more effectively than people by giving them thousands of instances. Diagnostic tools of this type have numerous applications, but a few stand out. In an effort to detect patients who are most likely to develop diabetes, heart failure, or stroke, New York University created an artificial intelligence that scans thousands of medical records. Over 8000 diseases and rare genetic disorders can be accurately diagnosed using facial recognition software in the Facial Dysmorphology Novel Analysis (FDNA) system. In the future, robotics, diagnostic image analysis, and precision medicine applications may become more and more dependent on robots due to future restrictions, trends. Computer vision powered by deep learning is one example of the use of medical machine learning (ML) in the clinical setting. Public health relies heavily on epidemiology. In the context of COVID-19, machine learning applications in epidemiology have certainly received increased attention. In addition to the development of highly linked databases of health records, larger amounts of data with a health impact are becoming available from a wider source, allowing machine learning to make almost magical predictions from large, high-dimensional data sets. In epidemiology, informal thinking plays a major role, which can be reconciled with the casual attitude of the ML. There can be no overstatement of how critical casual thinking is in epidemiology. Making flimsy claims is an issue worth considering.

A large number of options must be evaluated according to the criteria to determine which is the best. The following information is needed to make a decision:

Step-1: DM information are, (see Table 1).

Table 1. DM information.

	C_1	C_2	C_3	C_4
$\widetilde{\Xi}_1$	$\langle(0.8, 0.55); [0.5, 0.52], [0.6, 0.65], [0.52, 0.57]\rangle$	$\langle(0.6, 0.55); [0.5, 0.55], [0.6, 0.65], [0.65, 0.7]\rangle$	$\langle(0.7, 0.55); [0.6, 0.65], [0.5, 0.7], [0.4, 0.5]\rangle$	$\langle(0.85, 0.6); [0.45, 0.55], [0.7, 0.75], [0.55, 0.6]\rangle$
$\widetilde{\Xi}_2$	$\langle(0.8, 0.75); [0.3, 0.43], [0.19, 0.2], [0.4, 0.8]\rangle$	$\langle(0.8, 0.65); [0.55, 0.56], [0.8, 0.85], [0.3, 0.35]\rangle$	$\langle(0.55, 0.5); [0.4, 0.45], [0.7, 0.75], [0.55, 0.7]\rangle$	$\langle(0.8, 0.65); [0.5, 0.55], [0.35, 0.46], [0.45, 0.65]\rangle$
$\widetilde{\Xi}_3$	$\langle(0.85, 0.7); [0.5, 0.55], [0.4, 0.75], [0.3, 0.35]\rangle$	$\langle(0.75, 0.7); [0.2, 0.25], [0.9, 0.95], [0.4, 0.45]\rangle$	$\langle(0.7, 0.65); [0.6, 0.7], [0.65, 0.76], [0.3, 0.35]\rangle$	$\langle(0.7, 0.55); [0.6, 0.65], [0.65, 0.7], [0.35, 0.45]\rangle$
$\widetilde{\Xi}_4$	$\langle(0.65, 0.6); [0.36, 0.51], [0.37, 0.38], [0.6, 0.65]\rangle$	$\langle(0.7, 0.65); [0.49, 0.53], [0.65, 0.73], [0.55, 0.65]\rangle$	$\langle(0.8, 0.55); [0.5, 0.55], [0.55, 0.6], [0.65, 0.75]\rangle$	$\langle(0.8, 0.6); [0.5, 0.55], [0.4, 0.45], [0.5, 0.73]\rangle$
$\widetilde{\Xi}_5$	$\langle(0.7, 0.6); [0.45, 0.6], [0.5, 0.55], [0.25, 0.6]\rangle$	$\langle(0.6, 0.55); [0.25, 0.35], [0.45, 0.5], [0.75, 0.85]\rangle$	$\langle(0.65, 0.6); [0.45, 0.55], [0.55, 0.65], [0.5, 0.6]\rangle$	$\langle(0.6, 0.5); [0.5, 0.55], [0.65, 0.7], [0.45, 0.5]\rangle$

Table 2 shows the normalized decision matrix.

Step-2: Obtain the normalized decision matrix:

Table 2. Normalized decision values.

	C_1	C_2	C_3	C_4
$\widetilde{\Xi}_1$	$\langle(0.9412, 0.5042); [0.5, 0.52], [0.6, 0.65], [0.52, 0.57]\rangle$	$\langle(0.75, 0.7202); [0.5, 0.55], [0.6, 0.65], [0.65, 0.7]\rangle$	$\langle(0.875, 0.6648); [0.6, 0.65], [0.5, 0.7], [0.4, 0.5]\rangle$	$\langle(1, 0.6516); [0.45, 0.55], [0.7, 0.75], [0.55, 0.6]\rangle$
$\widetilde{\Xi}_2$	$\langle(0.9412, 0.9375); [0.3, 0.43], [0.19, 0.2], [0.4, 0.8]\rangle$	$\langle(1, 0.7545); [0.55, 0.56], [0.8, 0.85], [0.3, 0.35]\rangle$	$\langle(0.6875, 0.6993); [0.4, 0.45], [0.7, 0.75], [0.55, 0.7]\rangle$	$\langle(0.9412, 0.8125); [0.5, 0.55], [0.35, 0.46], [0.45, 0.65]\rangle$
$\widetilde{\Xi}_3$	$\langle(1, 0.7686); [0.5, 0.55], [0.4, 0.75], [0.3, 0.35]\rangle$	$\langle(0.9375, 0.9333); [0.2, 0.25], [0.9, 0.95], [0.4, 0.45]\rangle$	$\langle(0.875, 0.9286); [0.6, 0.7], [0.65, 0.76], [0.3, 0.35]\rangle$	$\langle(0.8235, 0.6648); [0.6, 0.65], [0.65, 0.7], [0.35, 0.45]\rangle$
$\widetilde{\Xi}_4$	$\langle(0.7647, 0.7385); [0.36, 0.51], [0.37, 0.38], [0.6, 0.65]\rangle$	$\langle(0.875, 0.8622); [0.49, 0.53], [0.65, 0.73], [0.55, 0.65]\rangle$	$\langle(1, 0.5817); [0.5, 0.55], [0.55, 0.6], [0.65, 0.75]\rangle$	$\langle(0.9412, 0.6923); [0.5, 0.55], [0.4, 0.45], [0.5, 0.73]\rangle$
$\widetilde{\Xi}_5$	$\langle(0.8235, 0.6857); [0.45, 0.6], [0.5, 0.55], [0.25, 0.6]\rangle$	$\langle(0.75, 0.7202); [0.25, 0.35], [0.45, 0.5], [0.75, 0.85]\rangle$	$\langle(0.8125, 0.8521); [0.45, 0.55], [0.55, 0.65], [0.5, 0.6]\rangle$	$\langle(0.7059, 0.641); [0.5, 0.55], [0.65, 0.7], [0.45, 0.5]\rangle$

Table 3 shows the SRNSNIVWG operator. The following aggregate data is available for each alternative.

Step-3: Aggregate information based on SRNSNIVWG operator for every alternative ($\Delta = 1$).

Table 3. SRNSNIVWG operator.

	$SRNSNIVWG$ $operator$ ($\Delta = 1$)
$\overset{\sqcap}{\Xi}_1$	$\langle(0.8717, 0.6084); [0.5131, 0.5561], [0.5936, 0.6719], [0.5443, 0.6038]\rangle$
$\overset{\sqcap}{\Xi}_2$	$\langle(0.9001, 0.8166); [0.4011, 0.4814], [0.5730, 0.6311], [0.4089, 0.6664]\rangle$
$\overset{\sqcap}{\Xi}_3$	$\langle(0.9366, 0.8339); [0.4012, 0.4633], [0.7018, 0.8442], [0.3356, 0.3911]\rangle$
$\overset{\sqcap}{\Xi}_4$	$\langle(0.8578, 0.7328); [0.4358, 0.5277], [0.5086, 0.5627], [0.5869, 0.6808]\rangle$
$\overset{\sqcap}{\Xi}_5$	$\langle(0.7864, 0.7219); [0.3812, 0.4973], [0.5139, 0.5759], [0.5037, 0.6917]\rangle$

Step-4: Consider the following alternatives and determine their optimum values, both positive and negative:

$$\overset{\sqcap}{\Xi}^{P} = \left[\langle(0.9366, 0.6084), 1, 1, 0\rangle\right],$$

$$\overset{\sqcap}{\Xi}^{P} = \left[\langle(0.7864, 0.8339), 0, 0, 1\rangle\right].$$

Step-5: ED between each alternative with both ideal values:

$$\mathbb{D}_1^P = 0.3224, \mathbb{D}_2^P = 0.3392, \mathbb{D}_3^P = 0.3382, \mathbb{D}_4^P = 0.3307, \mathbb{D}_5^P = 0.3303,$$

and

$$\mathbb{D}_1^N = 0.0647, \mathbb{D}_2^N = 0.0821, \mathbb{D}_3^N = 0.0810, \mathbb{D}_4^N = 0.0734, \mathbb{D}_5^N = 0.0732.$$

Step-6: Relative closeness is calculated as follows:

$$\mathbb{D}_1^\star = 0.1672, \mathbb{D}_2^\star = 0.1950, \mathbb{D}_3^\star = 0.1933, \mathbb{D}_4^\star = 0.1817, \mathbb{D}_5^\star = 0.1814.$$

Step-7: Ranking of alternatives are

$$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_5 \geq \widetilde{\Xi}_1.$$

As a result, Pharmarobotics is the best option.

6.3. Comparison for proposed models and existing models

The suggested models are compared with a few existing models in this subsection. In this way, it demonstrates its value and advantages. Recently, Palanikumar et al. [17] introduced MADM approach for Pythagorean neutrosophic normal interval-valued aggregation operators. We proposed ED and HD is based on SRNSNIVWA, SRNSNIVWG, GSRNSNIVWA, and GSRNSNIVWG, respectively. In both cases, the Euclidean distance method and the Hamming distance method were used. In Tables 4 and 5, existing and proposed methods are compared. The following categories can be used to categorize distances:

Table 4. Comparison table.

$\Delta = 1$	$SRNSNIVWA$	$SRNSNIVWG$	$GSRNSNIVWA$	$GSRNSNIVWG$
$TOPSIS - Euclidean$ $distance\,(proposed)$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_3$ $\widetilde{\Xi}_1 \geq \widetilde{\Xi}_5$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_4$ $\widetilde{\Xi}_5 \geq \widetilde{\Xi}_1$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_3$ $\widetilde{\Xi}_1 \geq \widetilde{\Xi}_5$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_4$ $\widetilde{\Xi}_5 \geq \widetilde{\Xi}_1$
$TOPSIS - Hamming$ $distance\,(proposed)$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_3$ $\widetilde{\Xi}_1 \geq \widetilde{\Xi}_5$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_4$ $\widetilde{\Xi}_5 \geq \widetilde{\Xi}_1$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_3$ $\widetilde{\Xi}_1 \geq \widetilde{\Xi}_5$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_4$ $\widetilde{\Xi}_5 \geq \widetilde{\Xi}_1$
$Score\,(proposed)$	$\widetilde{\Xi}_5 \geq \widetilde{\Xi}_1 \geq \widetilde{\Xi}_3$ $\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4$	$\widetilde{\Xi}_1 \geq \widetilde{\Xi}_5 \geq \widetilde{\Xi}_4$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_2$	$\widetilde{\Xi}_5 \geq \widetilde{\Xi}_1 \geq \widetilde{\Xi}_3$ $\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4$	$\widetilde{\Xi}_1 \geq \widetilde{\Xi}_5 \geq \widetilde{\Xi}_4$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_2$

Table 5. Comparison table.

$\Delta = 1$	$SRNSNIVWA$	$SRNSNIVWG$	$GSRNSNIVWA$	$GSRNSNIVWG$
Euclidean distance [17]	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$ $\widetilde{\Xi}_1 \geq \widetilde{\Xi}_4$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_1$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$ $\widetilde{\Xi}_1 \geq \widetilde{\Xi}_4$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_1$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$
Hamming distance [17]	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$ $\widetilde{\Xi}_4 \geq \widetilde{\Xi}_1$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_1$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$ $\widetilde{\Xi}_4 \geq \widetilde{\Xi}_1$	$\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_1$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_5$
Score [17]	$\widetilde{\Xi}_5 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_1$ $\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4$	$\widetilde{\Xi}_1 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_5$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_2$	$\widetilde{\Xi}_5 \geq \widetilde{\Xi}_3 \geq \widetilde{\Xi}_1$ $\widetilde{\Xi}_2 \geq \widetilde{\Xi}_4$	$\widetilde{\Xi}_1 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_5$ $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_2$

EDs for proposed approaches and current methods based on these four aggregating operators are shown in Figure 1. Figure 1 shows the Graphical representation consisting of euclidean distances.

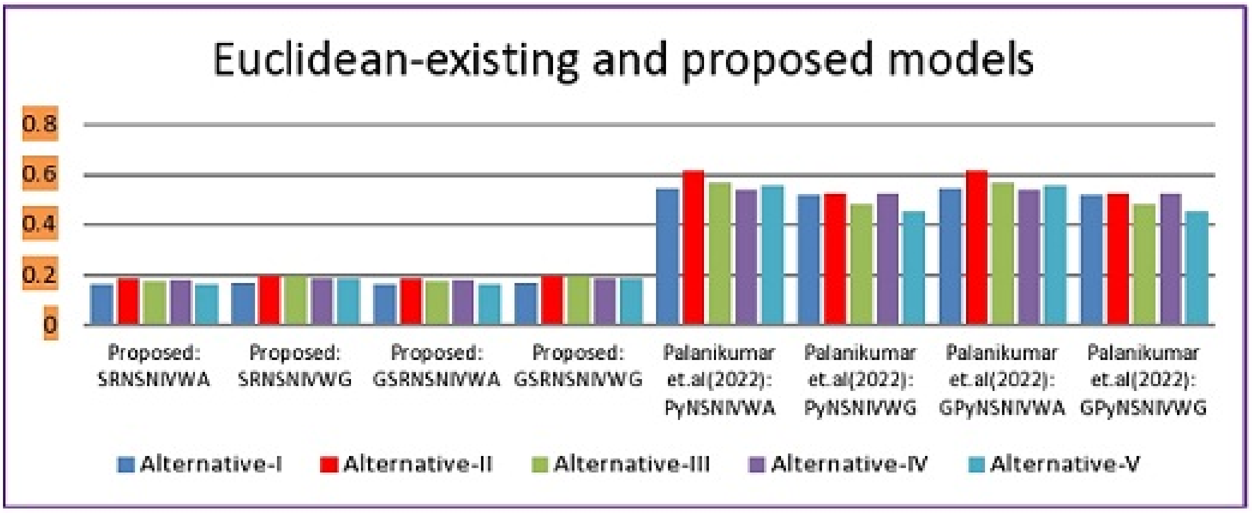

Figure 1. Euclidean distances.

HDs for proposed approaches and current methods based on these four aggregating operators are shown in Figure 2. Figure 2 shows the Graphical representation consisting of Hamming distances.

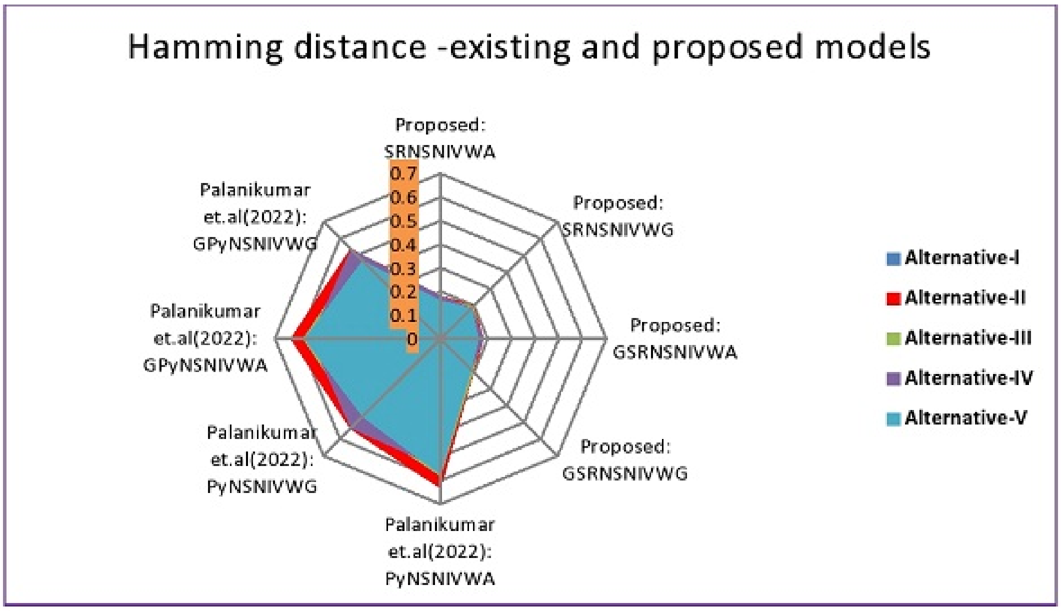

Figure 2. Hamming distances.

Score values for proposed approaches and current methods based on these four aggregating operators are shown in Figure 3. Figure 3 shows the Graphical representation consisting of score values.

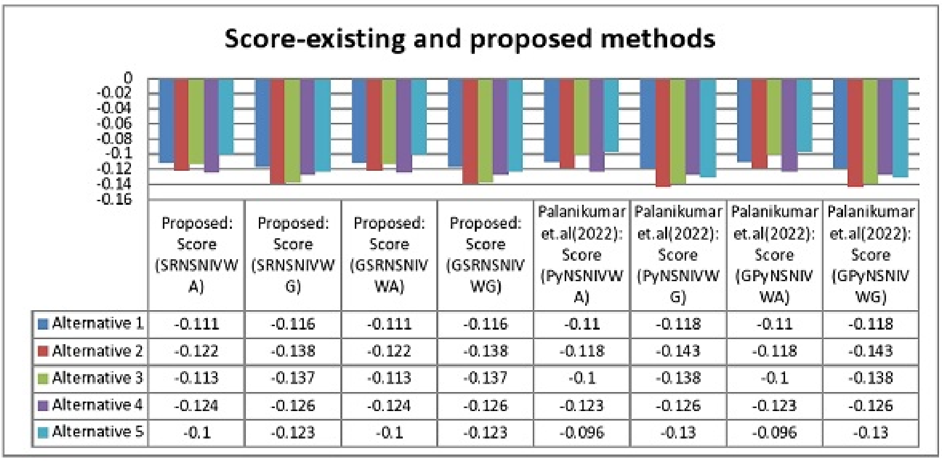

Figure 3. Score values.

6.4. *Data analysis*

MADM conditional dependability changes with the alternative. This test has prerequisites. Using the SRNSNIVWG approach, the proximity values and ranking are as follows. The $\Delta = 2$ values should be changed from the SRNSNIVWG approach. As shown below, the relative closeness values and orders are as follows: For each alternative, we provide the following aggregate data using the SRNSNIVWG operator (see Table 6):

Table 6. SRNSNIVWG operator.

$\overset{\sqcap}{\Xi}_1$	$\langle(0.8717, 0.6084); (0.5131, 0.5561), (0.5929, 0.6716), (0.5434, 0.6032)\rangle$
$\overset{\sqcap}{\Xi}_2$	$\langle(0.9001, 0.8166), [0.4011, 0.4814], [0.5508, 0.6096], [0.4076, 0.6628]\rangle$
$\overset{\sqcap}{\Xi}_3$	$\langle(0.9366, 0.8339); [0.4012, 0.4633], [0.6922, 0.8424], [0.3352, 0.3906]\rangle$
$\overset{\sqcap}{\Xi}_4$	$\langle(0.8578, 0.7328); [0.4358, 0.5277], [0.5041, 0.5565], [0.5867, 0.6806]\rangle$
$\overset{\sqcap}{\Xi}_5$	$\langle(0.7864, 0.7219); [0.3812, 0.4973], [0.5129, 0.5747], [0.4969, 0.6900]\rangle$

Determine the optimum values, both ideal values of the following alternatives:

$$\overset{\sqcap}{\Xi}^P = \langle(0.9366, 0.6084), [1, 1], [1, 1], [0, 0]\rangle$$

and

$$\overset{\sqcap}{\Xi}^N = \langle(0.7864, 0.8339), [0, 0], [0, 0], [1, 1]\rangle.$$

ED between each choice with both ideal values:

$$\mathbb{D}_1^P = 0.3222, \mathbb{D}_2^P = 0.3350, \mathbb{D}_3^P = 0.3371, \mathbb{D}_4^P = 0.3298, \mathbb{D}_5^P = 0.3294$$

$$\mathbb{D}_1^N = 0.0645, \mathbb{D}_2^N = 0.0779, \mathbb{D}_3^N = 0.0800, \mathbb{D}_4^N = 0.0725, \mathbb{D}_5^N = 0.0723.$$

Relative closeness are calculated as follows:

$$\mathbb{D}_1^\star = 0.1668, \mathbb{D}_2^\star = 0.1887, \mathbb{D}_3^\star = 0.1918, \mathbb{D}_4^\star 0.1802, \mathbb{D}_5^\star = 0.1800.$$

We can see from the data above that the SRNSNIVWG operator is used to determine the alternative ranking. If $\Delta = 2$, then new order is $\widetilde{\Xi}_3 \geq \widetilde{\Xi}_2 \geq \widetilde{\Xi}_4 \geq \widetilde{\Xi}_5 \geq \widetilde{\Xi}_1$. As a result, robotic-assisted biopsy becomes the preferred option to pharmarobotics. Similar to SRNSNIVWA, GSRNSNIVWA, and GSRNSNIVWG operators with Δ values are used to generate alternate ranks.

Figure 4 shows the Graphical representation consists of TOPSIS based on Hamming-SRNSNIVWG.

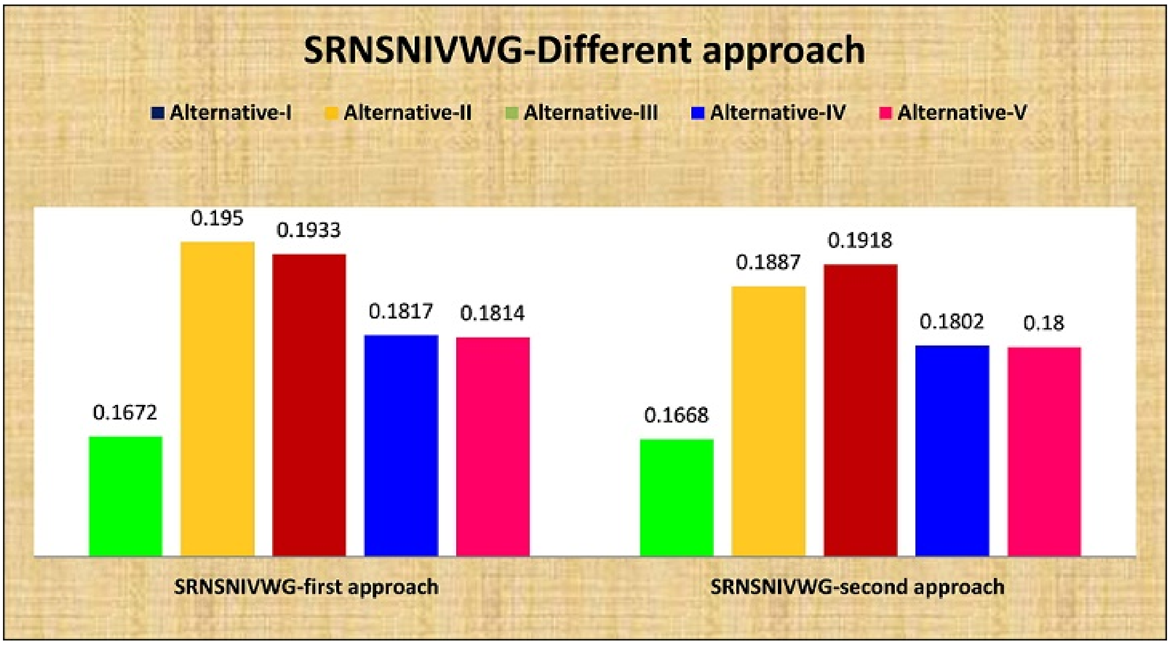

Figure 4. TOPSIS based on Hamming-SRNSNIVWG.

6.5. Advantages

According to the previous study, the applications have numerous advantages. SRNSNIVS is a combination of square root neutrosophic set and interval valued normal neutrosophic set. According to SRNSNIVS, human behavior and natural events follow a normal distribution in real life. By using the SRNSNIVS AOs, we find the most suitable alternative based on a set of options provided by the decision maker. Thus, the proposed MADM technique based on SRNSNIVS AOs provides another approach to finding the most effective alternative in DM. A decision maker can select the outcome according to Δ and their own preferences. With operators like SRNSNIVWA, SRNSNIVWG, GSRNSNIVWA, and GSRNSNIVWG, different ranking outcomes of each alternative could be produced dynamically.

7. Conclusions

We present the ED and HD measures for SRNSNIVSs. The mathematical simplicity of these distance measures makes them advantageous. It is demonstrated that ED and HD measures through the use of numerical examples. We have suggested AO rules for SRNSNIVWA, SRNSNIVWG, GSRNSNIVWA, and GSRNSNIVWG. As well as providing some examples, we discussed some of the features of these operators. It is possible for people to choose the best option under uncertain and inconsistent circumstances by implementing the SRNSNIV MADM. The SRNSNIVWA, SRNSNIVWG, GSRNSNIVWA, and GSRNSNIVWG operators have been used to solve MADM problems depending on Δ. Those generalized values of Δ significantly impact the ranking of alternatives as shown in the study. To make the decision, DM may select Δ values on the basis of the real life problems. The DM can select the method for obtaining results depending on Δ values. There are many practical applications of ED and HD measures in data analysis. This paper's discussion will be useful to future academics interested in this field. Further discussions will be held on the following topics are:

(1) Soft sets and expert sets are explored in terms of SRNSNIVS.

(2) Based on SRNSNIVS, we investigate generalized q-Rung interval valued normal neutrosophic set and cubic q-Rung interval valued normal fuzzy set.

(3) A generalized cubic interval valued normal neutrosophic set and complex interval valued normal neutrosophic set can be used to solve the problem of MADM.

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Conflict of interest

The authors declare no conflict of interest.

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