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Research article

Pythagorean fuzzy N-Soft PROMETHEE approach: A new framework for group decision making

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Abstract: The use of Pythagorean fuzzy N-soft sets (PFNSs) enables the examination of belongingness and non-belongingness of membership degrees, as well as their combinations with N-grading, in the unpredictable nature of individuals. This research aims to enhance our understanding of a popular multi-criteria group decision making (MCGDM) technique, Preference Ranking Organization Method for Enrichment of Evaluations, under the PFNS environment, aiding in making effective decisions for real-life problems, as fuzzy set theory is directly relevant to real-life applications. The PROMETHEE technique's main principle is to calculate the inflow and outflow streams of alternatives based on the deviation of their score degrees, ultimately providing partial and complete rankings of the given options. To capture the uncertainty of human nature, which demands both the association and disassociation of the considered criteria and provision of N-grading, the PFNS PROMETHEE technique is introduced in this research article. First, an Analytic Hierarchy Process AHP is used to check the feasibility of the standard weights of the criteria. The article then explains the detailed method of the fuzzy N-soft PROMETHEE technique to rank alternatives, with all the steps presented in an extensive flowchart for better understanding of the methodology. Furthermore, the practicality and viability of the proposed technique are demonstrated through an example of selecting the best chemical element in cloud seeding, where the most suitable choice is identified using an outranking directed graph. The credibility of the PFNS PROMETHEE technique is assessed by comparison with an existing method. Finally, the proposed technique's strengths and weaknesses are discussed to demonstrate its efficiency and drawbacks.

Keywords: N-soft sets; Pythagorean fuzzy sets; PROMETHEE method; outranking directed graph
Mathematics Subject Classification: 03E72, 03E75, 90B50

1. Introduction

As the modern world is more complex and sophisticated than in previous decades, practitioners are often faced with complex, ambiguous, and uncertain problems that cannot be solved using the existing crisp set theory, which offers no intermediate assessment between true and false. Most hurdles arise when the considered options or choices have more than one feature endowed with ambiguity. To address uncertainty and refine it into mathematical certainty, Zadeh [1] proposed the notion of fuzzy sets, which capture the partial belongingness of each alternative to its respective attributes. However, the question arises as to whether only the belongingness or membership degree should be considered, or if the non-belongingness or disassociation of the membership degree is also relevant. Atanassov [2, 3] clarified this confusion by introducing the notion of intuitionistic fuzzy sets which deal with both membership degree (μ) and non-membership degree (ν), whose sum must not exceed 1, i.e., $\mu + \nu \leq 1$. However this relaxed restriction still hinders the acquisition of the best results for many real-world problems. To overcome this, Yager and Abbasov [4] further relaxed this restriction by imposing an extended constraint that the sum of the squares of the membership and non-membership degrees must not exceed 1, i.e., $(\mu)^2 + (\nu)^2 \leq 1$. These sets are known as Pythagorean fuzzy sets (PFSs). Recent studies have demonstrated that PFSs can operate well within popular multi-criteria decision-making methodologies. Let us cite a few inspirational examples. Zeng et al. [5] and Zhang [6] proposed hybrid Pythagorean multi-criteria decision making techniques. Akram et al. [7] presented an outranking method for multicriteria decision making with complex Pythagorean fuzzy information. Recently, Kirisci and Simsek [8] presented a decision-making method, and Deveci et al. [9] published a survey on recent applications of Pythagorean fuzzy sets. Their state-of-the-art presentation ranges between 2013 and 2020.

With the passage of time, fuzzy set theory was challenged by the crisp soft set theory ,which explores parameterizations by a family of attributes. This task was accomplished with the help of soft sets [10]. However, this was not the end of the story, because soft set theory embraces crisp behavior only when the parameters are either satisfied or not. In order to make this concept applicable to the real-world problems, Fatimah et al. [11] introduced N-soft set theory including N-grading within a crisp soft set theory, and Alcantud [12] has discussed the semantics of N-soft sets. Other works found applications of this notion [13] ; for example, [14] made an analysis of tourism facilities using N-soft set decision making procedures. The introduction of Ordered Weighted Averaging (aggregation) operators allowed Alcantud et al. [15] to produce the first multi-agent decision methods with N-soft sets. From a different position, and in order to introduce uncertainty in the originally crisp soft set theory, Maji et al. [16, 17] suggested fuzzy soft set theory with the innovation of fuzziness. Later on, valuation fuzzy soft sets (Alcantud et al. [18]) extended their model and produced an application to the case of real estate information. Both [19] and Adeel et al. [20] incorporated fuzziness with N-soft grading and came up with fuzzy N-soft set (FNSS) theory, also producing the corresponding decision making analysis. The development of new hybrid models serves as evidence of the influential role of these ideas in the field of decision making. To name but a few, we can cite N-soft rough sets [21] and N-soft sets approach to rough sets [13], multi-FNSS [22], complex FNSSs (CFNSS) [23] and picture FNSSs (PFNSS) [24]. For further study, the readers are referred to [25–29].

When a problem has been examined thoroughly by considering all of its aspects, features, restrictions, etc., the next step is to formulate the appropriate decision making techniques or strategies that should help us to provide reasonable solutions. Many developments have been made in this regard. As hinted above, several decision making strategies have been developed in recent years that provide more suitable solutions in various situations, inclusive of the novel models introduced after the formulation of the crisp techniques. The origin of this branch can be traced back to Bellman and Zadeh [30], who initiated fuzzy decision-making by considering the fuzziness that had been introduced a few years earlier. With time, abundant techniques and methods have been introduced to deal with other forms of uncertainty of human nature, acceptance and non-acceptance of criteria and N-grading. Among these techniques, we have AHP [31] for the consistency of the weights of the criteria, TOPSIS [32], which works on the basis of positive and negative ideal solutions, ELECTRE [33, 34], VIKOR [35] and MULTIMOORA [36]. Each technique has its own unique characteristics, which guarantee merits and impose demerits. Other methodologies are available. For example, Ali and Akram [37] proposed a decision-making method based on fuzzy N-soft expert sets. Zhang et al. [38] developed multi-attribute decision making methods based on Pythagorean fuzzy N-soft sets. Jana [39] proposed a multiple attribute group decision-making method based on extended bipolar fuzzy MABAC approach. A robust aggregation operator was used for multi-criteria decision-making method in a bipolar fuzzy soft environment [40]. But our proposed PFNS PROMETHEE technique excels in making the more appropriate decision in our daily life problems due to its ability to capture the majority of aspects of association or disassociation of the attributes having fuzzy nature, along with ranking them from top to bottom.

1.1. Literature review of the PROMETHEE technique

As we mentioned above, various MCDM techniques have been developed to make use of fuzzy set theory in our real-life challenges. Their goal is to identify the most suitable decision, yielding the right choice for us according to the terms, conditions and raking of the alternatives. Among these scientific strategies, the family of PROMETHEE techniques stands out because of its notable feature of providing the complete ranking of the alternatives from most to least preferable. PROMETHEE [41] (Preference Ranking Organization Method for Enrichment of Evaluation) works by comparing the given options with others as per the related attributes. Incomplete and complete rankings of the alternatives are deduced by the PROMETHEE I and PROMETHEE II strategies, respectively. The study of the PROMETHEE method was expanded by Goumas and Lygerou [42] for DM in a fuzzy scenario. Gul et al. [43] presented a PROMETHEE method based on fuzzy logic and used fuzzy numbers for an MCDM problem. Feng et al. [44] developed an extension of the PROMETHEE method with fuzzy soft sets, and Krishankumar et al. [45] considered the PROMETHEE method with intuitionistic fuzzy sets. Zhang et al. [46] put forward the PROMETHEE method in interval Pythagorean terms. Feng et al. [47, 48] revisited generalized intuitionistic fuzzy soft sets and related multi-attribute decision making methods. Liao and Xu [49] proposed the PROMETHEE method in intuitionistic fuzzy environment by using intuitionistic fuzzy sets as the criteria value of the alternatives. Considering Pythagorean information, many developments have been made since Yager [50] introduced the Pythagorean membership grades and utilized them in multi-criteria decision making techniques. Chen [51] discussed a well-known multi-criteria decision making technique PROMETHEE-based outranking approach with Pythagorean fuzzy sets. Mardani et al. [52] surveyed both its theory and applications with the recent fuzzy developments. For further study on various methodologies, one may refer to [53–64]. Table 1 summarizes contributions related to the MCDM techniques under various fuzzy models.

Table 1. Related work on MCDM in various environments.

Reference	Year	Main contribution
Bellman and Zadeh [30]	1970	Proposed decision making in fuzzy environment
Hwang and Yoon [32]	1981	Proposed methods for multiple attribute decision making
Brans and Vincke [41]	1985	Put forward the PROMETHEE technique
Brans et al. [58]	1986	Used PROMETHEE method for selecting and ranking the projects
Saaty [31]	1986	Led the foundation of AHP technique
Roy [33]	1990	Proposed ELECTRE method
Goumas and Lygerou [42]	2000	Extended PROMETHEE in fuzzy environment
Opricovic and Tzeng [35]	2004	Comparative analysis between VIKOR and TOPSIS
Yager [50]	2013	Introduced Pythagorean membership grades in MCDM
Mardani et al. [52]	2015	Presented two decades review (1994-2014) on fuzzy MCDM techniques and applications
Zeng et al. [5]	2016	Proposed a hybrid method for Pythagorean fuzzy MCDM
Zhang [6]	2016	Proposed MC Pythagorean fuzzy decision analysis
Krishankumar [45]	2017	Introduced PROMETHEE under intuitionistic fuzzy set
Chen [51]	2018	Proposed PROMETHEE with Pythagorean fuzzy MCDM
Jana and Pal [63]	2019	Proposed a robust single-valued neutrosophic soft aggregation operators in MCDM
Feng et al. [44]	2020	Combined PROMETHEE with intuitionistic fuzzy soft set
Kirsci and Simesk [8]	2021	Used Pythagorean fuzzy soft set in infectious disease application
Molla et al. [10]	2021	Extended PROMETHE with Pythagorean fuzzy set for medical diagnosis
Alipour et al. [64]	2021	Developed a new Pythagorean fuzzy based DM method for fuel cell and hydrogen components supplier selection
Ye and Chen [59]	2022	Used Pythagorean fuzzy set under PROMETHEE for selection of cotton fabric
Kirsci et al. [60]	2022	Used VIKOR with Pythagorean fuzzy soft set in COVID-19 disease
Hua and Jing [61]	2023	Proposed generalized shapley index based interval with Pythagorean fuzzy PROMETHEE for GDM

1.2. Motivation of this article

Our research is motivated by the exceptional ability of PFNSSs to address the association or disassociation of a parameterized family of attributes and to rank them as a function of the given information. The incorporation of PFNSSs to the PROMETHEE technique enables us to make the most appropriate decision in our daily life, as it is based on faithful information. Therefore we are motivated to suggest a groundbreaking MCGDM method within this exceptional family, which we call PFNS PROMETHEE. A summary of important elements for motivation follows:

- Fuzzy sets [1] have opened the gates of the treatment of the ambiguity and uncertainty that we observe in human nature, but still these sets are too restrictive to take into account the approval and disapproval of the attributes, that is, membership and non-membership degrees.

- By dispensing with the possibility of N-ordered grades of the alternatives with respect to the relevant criteria, the performance of Zadeh's fuzzy sets was hindered.
- Fuzzy soft sets [16] combine them with a parameterized family of qualities or criteria of the alternatives, but they are unable to capture N-grading, despite the fact that this property has numerous real-world uses.
- The PFNS PROMETHEE technique overcomes two limitations: It enables us to handle fuzzy information along with N-grading of the alternatives, and with its help we can tackle the association and disassociation of the membership degrees with respect to their attributes. Both features contribute to its applicability to many real-life applications.
- Consequently, PFNS sets become a very powerful tool in terms of their ability to encapsulate the vagueness of attributes in the inputs, and the technique is also powerful as it allows us to rank the alternatives from top to bottom in the output.
- The main task in our daily life chores is to choose the best selection among various choices, rank the selections from finest to worst and assign the N-ordered grading according to the extent of acceptance and non-acceptance of the criteria. All these hurdles can easily be passed by our proposed MCGDM technique, PFNS PROMETHEE.
- A familiar MCDM strategy such as the AHP method [31] is employed to normalize the weights of the criteria, check their consistency, and limit the decision makers' personal interest or influence on their choice.

1.3. Contribution of this article

The comparison between any two alternatives is the foundation of the suggested methodology, called PFNS PROMETHEE, and it is done by evaluating the inconsistency of the given choices by formulating score degrees, preference function, multiple attribute preference index and their positive and negative movements. This article contributes with the following points:

- The PFNS PROMETHEE approach is formulated. A thorough presentation produces all the necessary formulas and operations.
- A comprehensive flowchart is drawn for better understanding of the technique. It shows a complete course of action.
- A numerical example about the selection of the best chemical compound in cloud seeding is solved with the help of our proposed PFNS PROMETHEE technique. It allows us to pick the most suitable and harmless chemical in cloud seeding.
- An AHP technique is used to normalize the weights of the criteria when solving the numerical example in order to reduce the risk of involvement of personal interests of the decision makers in their final selection or decision.
- The PFNS PROMETHEE I technique, in the application of selection of the best chemical in cloud seeding, gives us the incomplete outranking of the alternatives, while PFNS PROMETHEE II gives us proper outranking of the alternatives.
- To prove the accuracy and reliability of the proposed PFNS PROMETHEE technique, a comparison is made with the Pythagorean fuzzy PROMETHEE approach [62].
- At last, merits and demerits of the PFNS PROMETHEE technique are discussed to clarify its benefits and potential issues.

1.4. Structure of the paper

The following structure organizes this article's content. The PFNS PROMETHEE technique's methodology is detailed in Section 2. Through the use of the PFNS PROMETHEE, Section 3 incorporates the application of choosing the appropriate chemical for cloud seeding. The comparison between PFNS PROMETHEE and Pythagorean fuzzy PROMETHEE is covered in Section 4 [62]. The pros and demerits of our suggested method are discussed in Section 5.

2. Structure of Pythagorean fuzzy N-soft PROMETHEE method

Prior to the formal definition of PFNSS, we review the basic definitions of N-soft sets and fuzzy N-soft sets that are extensively used in the article. We recommend [12] for the semantic interpretation of N-soft sets.

Definition 2.1 ([11]). Let U be a set of a universe, E be a set of attributes and $B \subseteq E$. Let $G_N = \{0, 1, 2, \ldots, N-1\}$ be a set of N ordered grades, where $N > 1$ is a natural number. An N-soft set on U is a triplet (δ, B, N), where $\delta : B \to 2^{\delta(U)\times G_N}$, with the property that for each $b \in B$ there exists a unique $(u, g_b) \in U \times G$ such that $(u, g_b) \in \delta(b)$, $u \in U$ and $g_b \in G$.

Definition 2.2 ([19]). Consider $G_N = \{0, 1, 2, \ldots, N-1\}$ to be a set of N-ordered grades, where $N > 1$ is a natural number. A triplet (Γ, A, N) is said to be a fuzzy N-soft set on $F(U)$, where $F(U)$ is a set of all fuzzy sets on U and $A \subseteq U$, if $\Gamma : A \to 2^{F(U)\times G_N}$ follows the condition for each $a \in A$ and $u \in F(U)$ such that there exists a unique $r \in G_N$ such that $(u, r) \in F(a)$.

The definition of a Pythagorean fuzzy N-soft set is as follows:

Definition 2.3 ([38]). Let $G_N = \{0, 1, 2, \ldots, N-1\}$ be a set of N ordered grades, where $N > 1$ is a natural number, and $A \subseteq Z$ be a subset of the set of attributes. A triplet (F_P, A, N) is said to be a Pythagorean fuzzy N-soft set on a universal set U if $F_P : A \to 2^{U\times G_N} \times PFN$, in which $F : A \to 2^{U\times G_N}$ and $P : A \to PFN$, PFN denote the Pythagorean fuzzy numbers as $\mu : A \to [0, 1]$ and $\nu : A \to [0, 1]$ satisfying $(\mu)^2 + (\nu)^2 \leq 1$.

Both N-soft sets and fuzzy N-soft sets deal with a parameterized family of attributes, but the latter model succeeds in considering the fuzziness or uncertainty of the attributes. Nevertheless, both techniques fail to encompass a sufficiently large family or membership and non-membership degrees of the attributes, and at this point PFNSSs make their appearance. The uncertainty of the fuzzy set theory incorporated with the association and disassociation of the criteria and the N-grading of the alternatives with respect to the criteria are integrated with an MCGDM technique, PROMETHEE, which enables us to utilize these aspects in our daily life problems to make the accurate decisions or the best choices under the required circumstances. The methodology of the PFNS PROMETHEE technique depends on positive and negative movement of selected choices with the construction of the preference function as per characteristics of attributes. The detailed procedure of the PFNS PROMETHEE technique is summed up in the succeeding steps:

(1) Each individual professional put forward their assessments by considering all the features of the selected options under consideration and summed up their results in a separate set of matrices. A collection of q professionals, $E = \{e_t\}$, $t = 1, 2, \ldots, q$, inspect all u alternatives, $A = \{a_l\}$, $l =$

$1, 2, \ldots, u$, as per the p criteria as $C = \{c_s\},\ s = 1, 2, \ldots, p$ and do the calculations in the structure of Pythagorean fuzzy N-soft values ($PFNSV$) as $(d_{ls}, \mu_{ls}, \nu_{ls})$:

$$\Gamma^t = \begin{array}{c} \\ a_1 \\ \\ a_2 \\ \vdots \\ a_r \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_p \end{array} \\ \begin{pmatrix} \langle d_{11}, (b^t_{11}, k^t_{11})\rangle & \langle d_{12}, (b^t_{12}, k^t_{12})\rangle & \ldots & \langle d_{1p}, (b^t_{1p}, k^t_{1p})\rangle \\ \langle d_{21}, (b^t_{21}, k^t_{21})\rangle & \langle d_{22}, (b^t_{22}, k^t_{22})\rangle & \ldots & \langle d_{2p}, (b^t_{2p}, k^t_{2p}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle d_{r1}, (b^t_{r1}, k^t_{r1})\rangle & \langle d_{r2}, (b^t_{r2}, k^t_{r2})\rangle & \ldots & \langle d_{rp}, (b^t_{rp}, k^t_{up})\rangle \end{pmatrix} \end{array}.$$

(2) As every single professional has his own worth in the relevant field, having the combined effect of their abilities a Pythagorean fuzzy N-soft averaging ($PFNSA$) operator helps us to benefit from the single decision of each expert in a combined single persuasive decision as:

$$\Upsilon_{ls} = PFNSA\ ((d^{(1)}_{ls}, \ldots, d^{(q)}_{ls}), (b^{(1)}_{ls}, \ldots, b^{(q)}_{ls})) \tag{2.1}$$

$$= (max(d^{(1)}_{ls}, \ldots, d^{(q)}_{ls}), \frac{b^{(1)}_{ls} + \ldots + b^{(q)}_{ls}}{q}, \frac{k^{(1)}_{ls} + \ldots + k^{(q)}_{ls}}{q}) \tag{2.2}$$

$$= (d^*_{ls}, \mu^*_{ls}, \nu^*_{ls}). \tag{2.3}$$

(3) To position the alternatives, it is essential to pick out the pre-dominant option from the rest of the others, and for this sole purpose we calculate score degree and matrix by using Eq 2.4:

$$\widetilde{s} = \frac{d^*_{ls}}{N-1} + (\mu^*_{ls})^2 - (\nu^*_{ls})^2. \tag{2.4}$$

(4) The pairwise inconsistency of the selected choices according to the difference between their score degrees is the basic feature of the PROMETHE technique. This inconsistency is evaluated by Eq 2.5:

$$S_k(a_l, a_m) = \widetilde{t}_k(a_l) - \widetilde{t}_k(a_m), \quad l,\ m =\ 1, 2, \ldots, v. \tag{2.5}$$

(5) To consider the relative worth of each attribute, a preference function $\xi_k(h_l, h_m)$ is utilized whose value lies in between 0 and 1, where 1 indicates the strong preference, and 0 makes the options equivalent as per the relative attribute.

(6) For the positive and negative movements of the selected options, a multiple attribute preference index based on the preference function is defined by utilizing the AHP technique [31]. This index is estimated by Eq 2.6 as:

$$Z(a_l, a_m) = \frac{\sum_{k=1}^{p} w(k)\psi_k(a_l, a_m)}{\sum_{k=1}^{p} w(k)}, \quad l \neq\ m,\ \ l,\ m =\ 1, 2, \ldots, u. \tag{2.6}$$

Because the criterion weights calculated by the AHP technique are normalized, $\sum_{k=1}^{p} v(k) = 1$. As a result, Eq 2.6 reduces to Eq 2.7 as follows:

$$Z(a_l, a_m) = \sum_{k=1}^{p} w(k)\psi_k(a_l, a_m), \quad l \neq m, \; l, \; m = 1, 2, \ldots, u. \tag{2.7}$$

This preference index exhibits the supremacy of an alternative over others according to all the considered attributes. Its values lie between 0 and 1 as follows:

- $Z(a_l, a_m) \approx 1$ indicates superiority of alternative a_l over a_m as per the attributes.
- $Z(a_l, a_m) \approx 0$ indicates the inferiority of alternative a_l over a_m as per the attributes.

(7) The PROMETHEE I technique is used to obtain an incomplete outranking, and the PROMETHEE II technique is used to obtain a complete outranking. The outgoing flow of the alternatives is calculated as follows using Eq 2.8:

$$\lambda^{+}(a_l) = \frac{1}{r-1} \sum_{a_l \in H} Z(a_l, a_m), \quad l \neq m, \; l, m = 1, 2, \ldots, u. \tag{2.8}$$

This flow assesses an alternative's dominance over others. As shown in Figure 1, it is the average of all outgoing arcs drawn from *hl*.

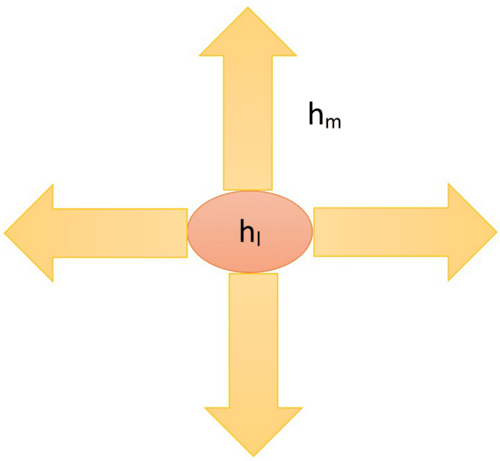

Figure 1. Outgoing arcs from h_l.

Meanwhile, the incoming movement of the considered choices is calculated by using the Eq 2.9 as follows:

$$\lambda^{-}(a_l) = \frac{1}{r-1} \sum_{a_m \in H} Z(h_m, h_l), \quad l \neq m, \; l, m = 1, 2, \ldots, u. \tag{2.9}$$

This flow estimates the inferiority of an option over all other choices. It is actually the average of all incoming arcs towards h_l as shown in Figure 2.

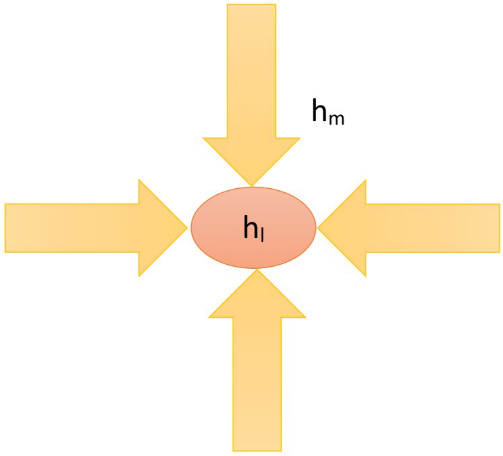

Figure 2. Incoming arcs towards h_l.

The option with maximum outgoing movement and minimum incoming movement is preferable. The preferences based on the outgoing and incoming flows can be computed by using the following Eqs 2.10 and 2.11:

$$\begin{aligned} &a_l J^+ a_m, \Leftrightarrow M^+(a_l) > M^+(a_m) \forall a_l, a_m \in H \ ; \\ &a_l L^+ a_m, \Leftrightarrow M^+(a_l) = M^+(a_m) \forall a_l, a_m \in H. \end{aligned} \tag{2.10}$$

$$\begin{aligned} &a_l J^- a_m, \Leftrightarrow M^-(a_l) < M^-(a_m) \forall a_l, a_m \in H \ ; \\ &a_l L^- a_m, \Leftrightarrow M^-(a_l) = M^-(a_m) \forall a_l, a_m \in H. \end{aligned} \tag{2.11}$$

The intersection of these rankings allows us to perform a partial outranking of $(\widetilde{F}, \widetilde{G}, \widetilde{K})$ of PROMETHEE I using the following Eq 2.12:

$$\begin{aligned} &a_l \widetilde{J} a_m, if \ a_l J^+ a_m \text{ and } a_l J^- a_m, \\ &\qquad or \ a_i P^+ a_j \text{ and } a_l K^- a_m, \\ &\qquad or \ a_l L^+ a_m \text{ and } a_l J^- a_m \ ; \\ &a_l \widetilde{L} a_m, iff \ a_l L^+ a_m \text{ and } a_l L^- a_m; \\ &a_l \widetilde{R} a_m, \text{otherwise.} \end{aligned} \tag{2.12}$$

In Eq 2.12, $h_l \widetilde{F} h_m$ indicates that h_l outranks h_m, $h_l \widetilde{G} h_m$ indicates that h_l is equivalent to h_m, and $h_l \widetilde{K} h_m$ indicates that h_l cannot be compared with h_m. As a result, PROMETHEE I provides an incomplete outranking because some of the alternatives remain incomparable. We have a thorough outranking of the options thanks to PROMETHEE II. Eq 2.13 is used to calculate the alternatives' net outranking:

$$M(a_l) = M^+(a_l) - M^-(a_l). \tag{2.13}$$

Equation 2.13 illustrates the distinction between the outgoing and incoming flows of the alternatives, which gives us the whole outranking (P^*, I^*) in PROMETHEE II of the alternatives with the help of Eq 2.14:

$$a_l J^* a_m, iff\ M(a_l) > M(a_m)\ ;$$
$$a_l L^* a_m, iff\ M(a_l) = M(a_m). \tag{2.14}$$

In Eq 2.14, $h_l F^* h_m$ states that h_l outranks h_m, and $h_l G^* h_m$ denotes that both h_l and h_m are at the same position and equivalent to each other. The option with the highest outranking movement is therefore regarded as the best option. As a result, PROMETHEE II completely outranks the alternatives and is necessary for the multi-criteria decision-making process.

This F$\mathcal{N}$S PROMETHEE step-by-step process is presented in a flowchart, as seen in Figure 3.

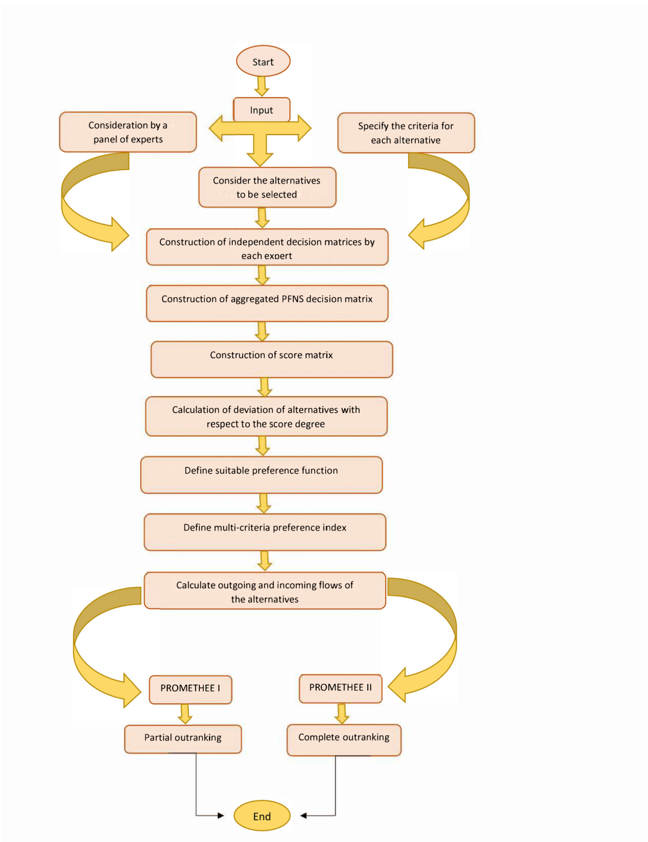

Figure 3. Pictorial representation of proposed method.

3. Applications

This segment is summarized with the practical implementation of an impressive MCGDM technique, PF$\mathcal{N}$S PROMETHEE, for making accurate decisions or choosing the best option in a multi-criteria environment where dealing with the belongingness and non-belongingness of each criterion to its specific set is our top priority. This section discuss the implementation of our presented technique in the selection of the most suited chemical in a cloud seeding process, a process of artificial rain.

3.1. Selection of the best chemical element in cloud seeding

Cloud seeding

Water travels in clouds. When there is a drought as in Figure 4, with less rainfall, hail, or thick fog as in Figure 5 in a region, water droplets may not get sufficiently dense to fall to the ground. In this situation, artificial precipitation is required to boost precipitation in the affected areas. By inserting microscopic ice nuclei into specific kinds of subfreezing clouds, cloud seeding is a method of weather modification that increases a cloud's capacity to generate rain or snow. These nuclei act as a foundation for the growth of snowflakes. Following cloud seeding, the freshly created snowflakes quickly develop and descend from the clouds back to the Earth's surface, boosting the snowpack and stream flow. Figure 6 represents two techniques to introduce particles to clouds : using massive guns to fire a cloud of dust into the air and using aircraft to drop the granules from the air.

Figure 4. Drought due to less rainfall.

Figure 5. Smog due to less rainfall.

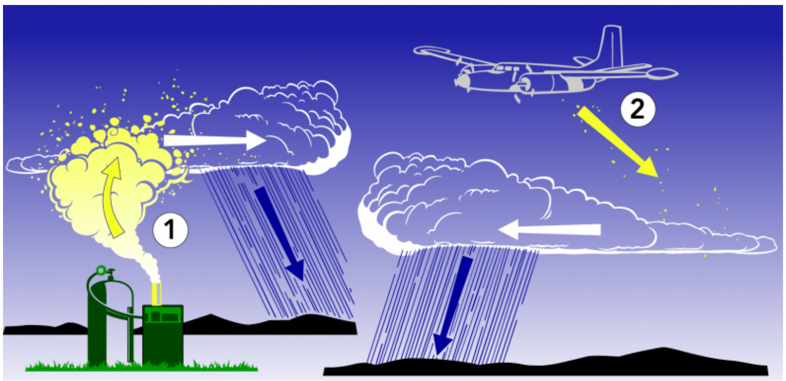

Figure 6. Two ways of cloud seeding.

Chemical compounds used in cloud seeding

Scientists have used various chemical compounds in cloud seeding, such as silver iodide, potassium iodide, propane, dry ice (solid CO_2), calcium carbide, salt and urea compounds. All these chemical elements have their own benefits and hazardous effects on human life, aquatic life and the environment.

Beneficial and hazardous effects of cloud seeding

The practice of "cloud seeding" is used all over the world to augment local communities' natural water supply by increasing winter snowfall and mountain snowpack. Older research from a cloud seeding program in the Bridger Range of western Montana showed snowfall increases of up to 15 percent from cloud seeding using high altitude remote-controlled generators [56]. Now, let us take a look at hazardous effects of cloud seeding. The chemical compounds used in the cloud seeding have very toxic impacts on human and aquatic life. Silver iodide, the material used in cloud seeding, is toxic to aquatic life. Potassium iodide can cause severe allergy, skin rashes and swelling of the salivary glands. Dry ice causes headache, dizziness, difficulty breathing, tremors, confusion and ringing in the ears. Here arises a question of which type of chemical compounds should be considered for cloud seeding .

Consider a numerical example where among the various chemical compounds used in cloud seeding, a company wants to select the best chemical element for cloud seeding that has more beneficial effects and fewer toxic effects on human life and the environment. To get the benefits of the cloud seeding, suppose the company short listed the five chemical compounds used in cloud seeding as $\Phi = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}$ to be selected and a group of professionals as $\zeta = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\}$ equipped with the following professions:

- Health Expert (ς_1):
  A health expert will consider all the positive and negative effects of the cloud seeding process upon human and aquatic life. He will examine how the chemicals used in cloud seeding are beneficial for human health, how they have poisonous effects on human health conditions and for how much longer mankind can survive on this artificial rain.
- Chemist (ς_2):
  A chemist will analyze various features of the different chemical compounds used in cloud seeding. He will thoroughly examine the favorable and malignant aspects of each chemical compound and how these compounds can be used in producing a healthy and fruitful artificial rain.
- Finance Expert (ς_3):
  A finance expert will manage the budget of any type of artificial raining project, makes sure of the availability of the required chemicals, equipment , etc. His major work is to complete the project of cloud seeding in any area in the pre-defined budget assigned by the company or the government of a certain country.
- Environmentalist (ς_4):
  An environmentalist will investigate the environment conditions suitable or not for cloud seeding, the benefit and hazardous effects of chemicals on the environment, and how the environment changes as a result of the cloud seeding.

Figure 7. Criteria of chemicals used in cloud seeding.

The judgement is made by the aforementioned specialists following a careful examination of the relevant factors as $\xi = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ that are portrayed in Figure 7 and illustrated as follows:

- Easy to access (η_1):
  It is very important to have easy access to all the machinery, chemicals, and working tools required in cloud seeding so that within the specified budget or with the team a healthy and fruitful artificial rain can be produced. The machines and tools must not be very expensive or difficult to operate . Rather they must be easy to handle and easy to use.
- Eco-friendly (η_2):
  Chemicals used in the cloud seeding must be eco-friendly, as they must not have any toxic or poisonous effect on theenvironment. The natural impression of the environment must not be disturbed. However, such chemicals must be used that enhance the air quality index or make the environment less polluted.
- Safer to health (η_3):
  The chemicals used in cloud seeding must be safer to health. They must not have any noxious effects on human and aquatic life. Also, the machines used in the cloud seeding must not emit harmful smoke or waste that ruin aquatic life. Rather, the chemicals and machines used must fulfill the health conditions of the living organisms in that area.
- Chemically stable (η_4):
  Chemically stable elements must be used in cloud seeding. Chemically stable means that those chemicals must be used that are easy to operate, do not have harmful effects while using, and able to work in a healthy and stable environment.

3.2. Criteria weights by AHP technique

With the AHP approach [31], the weights of the criteria are determined. The table of contrast is being shown in Table 2 depending on a Saaty preference scale [31].

Table 2. Table of contrast.

B	η_1	η_2	η_3	η_4
η_1	1	0.125	0.20	0.167
η_2	8	1	4	1
η_3	5	0.333	1	0.25
η_4	6	1	3	1

The normalized weights are displayed in Table 3.

Table 3. Normalized weights.

B_{norm}	η_1	η_2	η_3	η_4
η_1	0.05	0.051	0.024	0.069
η_2	0.4	0.407	0.488	0.414
η_3	0.25	0.136	0.122	0.103
η_4	0.3	0.407	0.366	0.414

A weight vector is constructed in Table 4.

Table 4. Criteria weights (ϱ).

	ϱ
w_1	0.049
w_2	0.427
w_3	0.153
w_4	0.372

After taking the product of the table of contrast and criteria weights, the BV matrix is shown in Eq 3.1:

$$BV = \begin{pmatrix} 0.195 \\ 1.803 \\ 0.633 \\ 1.552 \end{pmatrix}. \tag{3.1}$$

The largest eigenvalue is calculated as follows:

$$\lambda_{max} = \frac{1}{4} \times [\frac{0.195}{0.049} + \frac{1.803}{0.427} + \frac{0.633}{0.153} + \frac{1.552}{0.372}] \tag{3.2}$$

$$= 4.128. \tag{3.3}$$

The consistency index is formulated as

$$S = 0.043.$$

The comparison values are consistent, as indicated by the consistency index being close to zero. The consistency ratio is determined by dividing the consistency index by the random index after the consistency index has been evaluated. Here, we have taken $n = 4$ and $T = 0.90$. The consistency ratio is estimated as $Y = 0.048$. As $0.048 < 0.10$, it is justifiable.

3.3. Steps for the selection of a best chemical element in cloud seeding by using PFNS PROMETHEE technique

Our presented *MCGDM* methodology, PFNS PROMETHEE can be very helpful in industrial phenomena as in the selection of the best chemical element used in cloud seeding. This technique compares any pair of given options depending on the positive and negative movement of the alternatives. This methodology comprises the divergence of the selected options as per the score degree, general and multi attribute priority function, and partial and proper positioning of the alternatives. The complete course of action of the PFNS PROMETHEE technique to choose the most suitable option is as follows:

1) The grades, membership degrees and non-membership degrees, in the form of Pythagorean fuzzy N-soft values (PFNSV), are arranged in Tables 5, 6, 7 and 8, respectively, given by each individual expert to each given option.

Table 5. $PF6SDM$ of the health expert (ς_1).

ς_1	η_1	η_2	η_3	η_4
φ_1	$(4, 0.79, 0.31)$	$(2, 0.35, 0.69)$	$(3, 0.48, 0.58)$	$(4, 0.73, 0.52)$
φ_2	$(2, 0.28, 0.67)$	$(3, 0.55, 0.63)$	$(2, 0.37, 0.77)$	$(1, 0.14, 0.89)$
φ_3	$(4, 0.68, 0.44)$	$(1, 0.16, 0.68)$	$(3, 0.49, 0.67)$	$(3, 0.57, 0.59)$
φ_4	$(3, 0.47, 0.72)$	$(4, 0.76, 0.36)$	$(4, 0.69, 0.42)$	$(3, 0.54, 0.61)$
φ_5	$(5, 0.95, 0.21)$	$(5, 0.84, 0.32)$	$(4, 0.77, 0.26)$	$(5, 0.89, 0.14)$

Table 6. $PF6SDM$ of chemist (ς_2).

ς_1	η_1	η_2	η_3	η_4
φ_1	$(3, 0.42, 0.76)$	$(3, 0.58, 0.68)$	$(4, 0.73, 0.39)$	$(2, 0.31, 0.75)$
φ_2	$(1, 0.18, 0.93)$	$(2, 0.33, 0.76)$	$(2, 0.27, 0.88)$	$(3, 0.54, 0.63)$
φ_3	$(3, 0.57, 0.67)$	$(4, 0.68, 0.28)$	$(3, 0.47, 0.72)$	$(2, 0.37, 0.84)$
φ_4	$(5, 0.95, 0.16)$	$(3, 0.49, 0.77)$	$(3, 0.58, 0.69)$	$(4, 0.74, 0.37)$
φ_5	$(4, 0.72, 0.34)$	$(5, 0.92, 0.33)$	$(4, 0.69, 0.26)$	$(5, 0.87, 0.22)$

Table 7. $PF6SDM$ of finance expert (ς_3).

ς_1	η_1	η_2	η_3	η_4
φ_1	$(3, 0.54, 0.66)$	$(2, 0.42, 0.71)$	$(1, 0.11, 0.86)$	$(2, 0.28, 0.84)$
φ_2	$(2, 0.22, 0.86)$	$(1, 0.16, 0.94)$	$(2, 0.38, 0.82)$	$(1, 0.06, 0.98)$
φ_3	$(3, 0.57, 0.63)$	$(2, 0.39, 0.78)$	$(3, 0.58, 0.69)$	$(3, 0.46, 0.72)$
φ_4	$(4, 0.69, 0.44)$	$(3, 0.58, 0.68)$	$(3, 0.49, 0.73)$	$(4, 0.72, 0.34)$
φ_5	$(4, 0.78, 0.33)$	$(4, 0.75, 0.39)$	$(5, 0.98, 0.17)$	$(5, 0.88, 0.16)$

Table 8. $PF6SDM$ of environmentalist (ς_4).

ς_1	η_1	η_2	η_3	η_4
φ_1	$(1, 0.18, 0.83)$	$(2, 0.27, 0.88)$	$(3, 0.47, 0.73)$	$(2, 0.33, 0.74)$
φ_2	$(2, 0.36, 0.71)$	$(3, 0.52, 0.66)$	$(1, 0.11, 0.92)$	$(2, 0.29, 0.84)$
φ_3	$(2, 0.25, 0.84)$	$(3, 0.49, 0.71)$	$(3, 0.51, 0.64)$	$(2, 0.38, 0.79)$
φ_4	$(3, 0.56, 0.62)$	$(4, 0.68, 0.27)$	$(4, 0.78, 0.34)$	$(3, 0.47, 0.72)$
φ_5	$(5, 0.96, 0.13)$	$(5, 0.95, 0.17)$	$(4, 0.76, 0.27)$	$(4, 0.69, 0.34)$

2) These individual assessments are accumulated in a single matrix exhibiting the joint effect of the decisions of specialists by using the $PFNSA$ operator, given in Eqs 2.2 and 2.3, illustrated as $APF6SDM$ in Table 9.

Table 9. Aggregated P F6SDM.

ς_1	η_1	η_2	η_3	η_4
φ_1	$(4, 0.48, 0.64)$	$(3, 0.41, 0.74)$	$(4, 0.45, 0.64)$	$(4, 0.41, 0.71)$
φ_2	$(2, 0.26, 0.79)$	$(3, 0.39, 0.75)$	$(2, 0.28, 0.85)$	$(3, 0.26, 0.84)$
φ_3	$(4, 0.52, 0.65)$	$(4, 0.43, 0.61)$	$(3, 0.51, 0.68)$	$(3, 0.45, 0.74)$
φ_4	$(5, 0.67, 0.49)$	$(4, 0.63, 0.52)$	$(4, 0.64, 0.55)$	$(4, 0.62, 0.51)$
φ_5	$(5, 0.85, 0.25)$	$(5, 0.870.3)$	$(5, 0.8, 0.24)$	$(5, 0.83, 0.22)$

3) A score matrix is formed as in Table 10 by estimating score degrees by using Eq 2.4. It will exhibit the related deviation among the given choices.

Table 10. Score matrix.

	η_1	η_2	η_3	η_4
φ_1	0.62	0.22	0.59	0.46
φ_2	−0.16	0.19	−0.24	−0.03
φ_3	0.65	0.61	0.4	0.26
φ_4	1.21	0.92	0.91	0.92
φ_5	1.66	1.66	1.58	1.65

4) Compute the divergence of the selected options as per the score degree by using Eq 2.5. This deviation of the alternatives is arranged in Table 11.

Table 11. Deviation of the alternatives as per the attributes.

Alternatives	η_1	η_2	η_3	η_4	**Alternatives**	η_1	η_2	η_3	η_4
$\varphi_1\varphi_2$	0.78	0.03	0.83	0.49	$\varphi_1\varphi_3$	−0.03	−0.39	0.19	0.20
$\varphi_1\varphi_4$	−0.59	−0.7	−0.32	−0.46	$\varphi_1\varphi_5$	−1.04	−1.44	−0.99	−1.19
$\varphi_2\varphi_1$	−0.78	−0.03	−0.83	−0.49	$\varphi_2\varphi_3$	−0.81	−0.42	−0.64	−0.29
$\varphi_2\varphi_4$	−1.37	−0.73	−1.15	−0.95	$\varphi_2\varphi_5$	−1.82	−1.47	−1.82	−1.68
$\varphi_3\varphi_1$	0.03	0.39	−0.19	−0.2	$\varphi_3\varphi_2$	0.81	0.42	0.64	0.29
$\varphi_3\varphi_4$	−0.56	−0.31	−0.51	−0.66	$\varphi_3\varphi_5$	−1.01	−1.05	−1.18	−1.39
$\varphi_4\varphi_1$	0.59	0.7	0.32	0.46	$\varphi_4\varphi_2$	1.37	0.73	1.15	0.95
$\varphi_4\varphi_3$	0.56	0.31	0.51	0.66	$\varphi_4\varphi_5$	−0.45	−0.74	−0.67	−0.73
$\varphi_5\varphi_1$	1.04	1.44	0.99	1.19	$\varphi_5\varphi_2$	1.82	1.47	1.82	1.68
$\varphi_5\varphi_3$	1.01	1.05	1.18	1.39	$\varphi_5\varphi_4$	0.45	0.74	0.67	0.73

5) Define a suitable preference function from the six types of functions cited in [41] and [58] based on the indifference and preference thresholds in order to maintain the prominence of each option. The generalized preference functions listed in Table 12 define the different forms of criteria.

Table 12. Type of criteria and preference functions.

Criteria	Max or Min	Type of criteria	Parameters
φ_1	Max	I	$no\ value\ of\ k$
φ_2	Max	II	$k = 0.01$
φ_3	Max	II	$k = 0.01$
φ_4	Max	I	$no\ value\ of\ k$

6) The preference degree of every pair of alternatives with respect to every criterion is calculated by using a generalized criteria preference function by [41] and [58], as displayed in Table 13.

Table 13. Generalized criteria preference function.

Alternatives	η_1	η_2	η_3	η_4	Alternatives	η_1	η_2	η_3	η_4
$\varphi_1\varphi_2$	1	1	1	1	$\varphi_1\varphi_3$	0	0	1	1
$\varphi_1\varphi_4$	0	0	0	0	$\varphi_1\varphi_5$	0	0	0	0
$\varphi_2\varphi_1$	0	0	0	0	$\varphi_2\varphi_3$	0	0	0	0
$\varphi_2\varphi_4$	0	0	0	0	$\varphi_2\varphi_5$	0	0	0	0
$\varphi_3\varphi_1$	1	1	0	0	$\varphi_3\varphi_2$	1	1	1	1
$\varphi_3\varphi_4$	0	0	0	0	$\varphi_3\varphi_5$	0	0	0	0
$\varphi_4\varphi_1$	1	1	1	1	$\varphi_4\varphi_2$	1	1	1	1
$\varphi_4\varphi_3$	1	1	1	1	$\varphi_4\varphi_5$	0	0	0	0
$\varphi_5\varphi_1$	1	1	1	1	$\varphi_5\varphi_2$	1	1	1	1
$\varphi_5\varphi_3$	1	1	1	1	$\varphi_5\varphi_4$	1	1	1	1

7) Equation 2.7 can be used to calculate the multi-attribute preference index, which expresses the propensity of experts to favor one alternative over another. Table 14 shows this preference index.

Table 14. Multi-criteria preference index.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	1	0.53	0	0
φ_2	0	–	0	0	0
φ_3	0.57	1	–	0	0
φ_4	1	1	1	–	0
φ_5	1	1	1	1	–

8) F$\mathcal{N}$S PROMETHEE I evaluates the choices' incomplete outranking, and Eqs 2.8 and 2.9 are used to formulate the positive and negative movements of the chosen options. In Table 15, the outgoing and incoming flows of one option relative to another alternative are determined by Eqs 2.10 and 2.11, respectively.

Table 15. Positive and negative movements of the alternatives (PROMETHE I).

Alternatives	Outgoing flow (Υ^{+})	Incoming flow (Υ^{-})
φ_1	0.038	0.64
φ_2	0	1
φ_3	0.39	0.66
φ_4	0.75	0.25
φ_5	1	0

The intersection of these flows allows us to perform a partial outranking of the alternatives, which can be estimated using Eq 2.12 as follows:

$$\varphi_1\hat{P}\varphi_2, \varphi_3\hat{P}\varphi_1, \varphi_3\hat{P}\varphi_2, \varphi_4\hat{P}\varphi_1, \varphi_4\hat{P}\varphi_2, \varphi_4\hat{P}\varphi_3, \varphi_5\hat{P}\varphi_1, \varphi_5\hat{P}\varphi_2, \varphi_5\hat{P}\varphi_3, \varphi_5\hat{P}\varphi_4.$$

PROMETHEE I gives us partial outranking relations among the alternatives, but this is a special case when we have complete outranking relations with the help of PROMETHEE I. The outranking relationship among the alternatives by PROMETHEE I is shown in Figure 8.

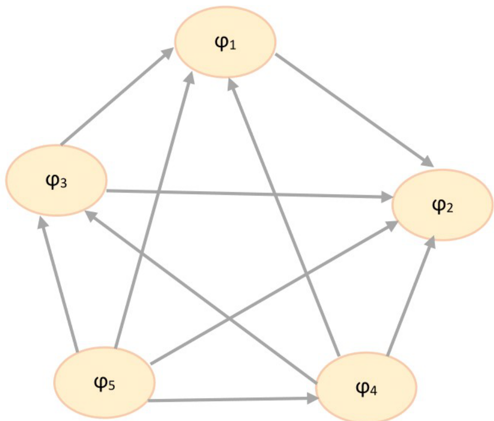

Figure 8. Outranking of the alternatives by PROMETHEE I.

9) Equation 2.13 calculates the net outranking of the options chosen. Using Eq 2.14, we can obtain the complete outranking of the alternatives, as shown in Table 16.

Table 16. Alternatives' net outranking flow (PROMETHEE II).

Alternatives	Net flow (Υ)
φ_1	-0.26
φ_2	-1
φ_3	-0.24
φ_4	0.5
φ_5	1

10) From all the above calculations, we come to know that alternative φ_5 is selected as the best robot butler to be launched in the market, and the ordering of the alternatives is as follows:
$\varphi_5 > \varphi_4 > \varphi_2 > \varphi_1 > \varphi_3$.

4. Comparative analysis

The proposed method is compared to an existing technique [62]. To compare the Pythagorean fuzzy technique with our suggested MCGDM technique, PF*N*S PROMETHEE, we apply it to the application of choosing the least hazardous chemical compound to utilize in cloud seeding [62]. We have taken into account a Pythagorean fuzzy environment for the sake of ease, and after conducting the computations, we will learn that the same optimal choice is chosen under both strategies. The phases of the Pythagorean fuzzy approach are as follows [62]:

(1) All the experts ($\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4$) assigned Pythagorean fuzzy membership and non-membership degrees to the considered alternatives after examining all the requirements for each option and the decision matrix constructed in Table 17.

Table 17. Decision Matrix.

ς_1	η_1	η_2	η_3	η_4
φ_1	$(0.48, 0.64)$	$(0.41, 0.74)$	$(0.45, 0.64)$	$(0.41, 0.71)$
φ_2	$(0.26, 0.79)$	$(0.39, 0.75)$	$(0.28, 0.85)$	$(0.26, 0.84)$
φ_3	$(0.52, 0.65)$	$(0.43, 0.61)$	$(0.51, 0.68)$	$(0.45, 0.74)$
φ_4	$(0.67, 0.49)$	$(0.63, 0.52)$	$(0.64, 0.55)$	$(0.62, 0.51)$
φ_5	$(0.85, 0.25)$	$(0.870.3)$	$(0.8, 0.24)$	$(0.83, 0.22)$

(2) Calculate the deviation between the two alternatives to evaluate the performance of each alternative and to compare their working progress with respect to the each criterion. The deviations or difference values between the two alternatives are shown in Table 18.

Table 18. Deviation of the alternatives.

	η_1	η_2	η_3	η_4		η_1	η_2	η_3	η_4
$D(\varphi_1, \varphi_2)$	0.38	0.02	0.43	0.29	$D(\varphi_2, \varphi_1)$	−0.38	−0.02	−0.43	−0.29
$D(\varphi_1, \varphi_3)$	−0.03	−0.19	−0.01	0	$D(\varphi_2, \varphi_3)$	−0.41	−0.22	−0.44	−0.29
$D(\varphi_1, \varphi_4)$	−0.39	−0.51	−0.32	−0.46	$D(\varphi_2, \varphi_4)$	−0.77	−0.53	−0.75	−0.75
$D(\varphi_1, \varphi_5)$	−0.84	−1.04	−0.79	−0.99	$D(\varphi_2, \varphi_5)$	−1.22	−1.06	−1.22	−1.28
$D(\varphi_3, \varphi_1)$	0.03	0.19	0.01	0	$D(\varphi_4, \varphi_1)$	0.39	0.51	0.32	0.46
$D(\varphi_3, \varphi_2)$	0.41	0.22	0.44	0.29	$D(\varphi_4, \varphi_2)$	0.77	0.53	0.75	0.75
$D(\varphi_2, \varphi_4)$	−0.36	−0.31	−0.31	−0.46	$D(\varphi_4, \varphi_3)$	0.36	0.31	0.31	0.46
$D(\varphi_3, \varphi_5)$	−0.81	−0.85	−0.78	−0.99	$D(\varphi_4, \varphi_5)$	−0.45	−0.53	−0.48	−0.53
$D(\varphi_5, \varphi_1)$	0.84	1.04	0.79	0.99					
$D(\varphi_5, \varphi_2)$	1.22	1.06	1.22	1.28					
$D(\varphi_5, \varphi_3)$	0.81	0.85	0.78	0.99					
$D(\varphi_5, \varphi_4)$	0.45	0.53	0.48	0.53					

(3) Construct the preference function according to the nature of the criteria. In the selection of the best chemical element in cloud seeding, we utilize V-type preference function as considered in [62]. We take the values of p and q as 0.8 and 0.1, respectively, where p indicates the strong preference threshold value, and q indicates the indifference value. The preference function value regarding the specified preference function is calculated in Table 19.

Table 19. Deviation of the alternatives.

	η_1	η_2	η_3	η_4		η_1	η_2	η_3	η_4
$P(\varphi_1, \varphi_2)$	0.4	0	0.47	0.27	$P(\varphi_2, \varphi_1)$	0	0	0	0
$P(\varphi_1, \varphi_3)$	0	0	0	0	$P(\varphi_2, \varphi_3)$	0	0	0	0
$P(\varphi_1, \varphi_4)$	0	0	0	0	$P(\varphi_2, \varphi_4)$	0	0	0	0
$P(\varphi_1, \varphi_5)$	0	0	0	0	$P(\varphi_2, \varphi_5)$	0	0	0	0
$P(\varphi_3, \varphi_1)$	0	0.13	0	0	$P(\varphi_4, \varphi_1)$	0.41	0.59	0.31	0.51
$P(\varphi_3, \varphi_2)$	0.44	0.17	0.49	0.27	$P(\varphi_4, \varphi_2)$	0.96	0.61	0.93	0.93
$P(\varphi_2, \varphi_4)$	0	0	0	0	$P(\varphi_4, \varphi_3)$	0.37	0.3	0.3	0.51
$P(\varphi_3, \varphi_5)$	0	0	0	0	$P(\varphi_4, \varphi_5)$	0	0	0	0
$P(\varphi_5, \varphi_1)$	1	1	0.99	1					
$P(\varphi_5, \varphi_2)$	1	1	1	1					
$P(\varphi_5, \varphi_3)$	1	1	0.97	1					
$P(\varphi_5, \varphi_4)$	0.5	0.61	0.54	0.61					

(4) Calculate the aggregated preference index value by using the weights of the criteria calculated as: $[0.049, 0.427, 0.153, 0.372]^T$. Now, the preference matrix calculated is in Table 20.

Table 20. Multi-criteria preference index.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	−	0.35	0	0	0
φ_2	0	−	0	0	0
φ_3	0.02	0.4	−	0	0
φ_4	0.38	0.85	0.32	−	0
φ_5	1	1	1	0.51	−

(5) Calculate the positive or leaving outranking flow and negative or incoming flow of the alternatives by using PROMETHEE I, as calculated in Table 21.

Table 21. Positive and negative flows of the alternatives (PROMETHE I).

Alternatives	**Outgoing flow** (Υ^+)	**Incoming flow** (Υ^-)
φ_1	0.088	0.35
φ_2	0	0.65
φ_3	0.105	0.33
φ_4	0.38	0.13
φ_5	0.88	0

(6) Calculate the net outranking flow of the alternatives by using PROMETHEE II. The alternative having the largest net value is considered as the best alternative to be selected. The net flow of the alternatives is evaluated as follows in Table 22.

Table 22. Average outranking movement of the choices (PROMETHEE II).

Given options	Average movement (Υ)
φ_1	-0.26
φ_2	-0.65
φ_3	-0.23
φ_4	0.225
φ_5	0.88

(7) From Table 22, we conclude that φ_5 is considered as the most suitable chemical element for the cloud seeding, and the outranking flow of the alternatives is as follows: $\varphi_5 > \varphi_4 > \varphi_2 > \varphi_1 > \varphi_3$.

(8) Table 23 shows the comparative values of the alternatives by our proposed technique and the existing technique [62]. PROMETHEE technique under Pythagorean fuzzy environment and PF*N*S environment provides us with the same outranking relations among the alternatives, and the same best option is chosen under both of the circumstances.

Table 23. Comparison.

Techniques	Outranking relation	Best option
$PFNS\ PROMETHEE\ (suggested)$	$\varphi_5 > \varphi_4 > \varphi_2 > \varphi_1 > \varphi_3$	φ_5
$PF\ PROMETHEE$ [62]	$\varphi_5 > \varphi_4 > \varphi_2 > \varphi_1 > \varphi_3$	φ_5

Table 23 shows that the same out ranking relation is produced by applying both of the technique, our proposed technique and Pythagorean fuzzy PROMETHEE [62], and both of the techniques pick out the same best alternative which proves the viability and credibility of our proposed PF*N*S PROMETHEE approach.

5. Discussion

This section presents a detailed discussion of the contributions of this research paper , such as the working strategy, the numerical example exhibiting the application of our proposed technique, and the comparison of our proposed method with the existing technique. Getting the same results from both of the techniques proves the credibility of our proposed method.

- The most vital contribution of this research article is to extend the knowledge of a remarkable MCGDM technique, PROMETHEE under the Pythagorean fuzzy *N*-soft (PFNS) data containing the vagueness of the human decisions with the acceptance and non-acceptance of the criteria along with the *N*-grading of the parameterized family of attributes.
- PF*N*S PROMETHEE technique focuses mainly on the positive and negative movement of the given choices depending on the deviation of the options by constructing the score index and priority index.
- The PF*N*S PROMETHEE I gives the partial outranking of the alternatives while PF*N*S PROMETHEE II facilitates us with the proper outranking of the given choices, and this relation has been depicted by the outranking graphs.

- The practicality of our proposed technique is explained with the help of an example of selection of the best chemical compound in cloud seeding, where under Pythagorean fuzzy N-soft environment the PROMETHEE technique helps to choose the best alternative.
- The authenticity of our proposed technique is checked by applying the existing technique, Pythagorean fuzzy PROMETHEE [62], on the same example of selection of the best chemical element in cloud seeding, and the selection of the same best alternative from both of the techniques has proved the credibility of our proposed technique.

6. Insights of the PFNS PROMETHEE method

6.1. Merits of the proposed technique

- The PFNS PROMETHEE technique has been proved as an industrious technique to help us in our daily multi-criteria group decision courses where we face uncertain and vague information, association and disassociation of the criteria and the parameterized family of attributes of the alternatives.
- To associate an N-ordered grading or to position the given options as per the features they possess, PFNS PROMETHEE technique proves a helping hand to us and provides us with the N-ordered grading scenario.
- To examine the approval and disapproval of the criteria regarding each alternative, PFNS PROMETHEE helps us best to pass every hurdle in making the most suitable decision according to our circumstances.
- PFNS PROMETHEE allows us to choose the best alternative among various choices to be selected.
- The best option is suggested, as well as the outranking of the alternatives, in the outranking graph.

6.2. Limitations of the proposed technique

- PFNS PROMETHEE deals with the belongingness and non-belongingness of the criteria, but its performance decreases while dealing with negative aspects along with positive ones.
- The PFNS PROMETHEE technique fails when the sum of the squares of the membership and non-membership degrees exceeds 1 as $\mu^2 + \nu^2 > 1$, and there are many practical applications following this inequality.
- While dealing with positive and negative aspects or influence of the attributes of the alternatives, the PFNS PROMETHEE technique is of no use as it associates only with the membership and non-membership degrees but does not discuss the beneficial and harmful effects of the attributes over the alternatives.
- PFNS PROMETHEE provides us no help with multi-polar information. The efficiency of this technique is faded while dealing with the multi-polarity of the modern era.

7. Conclusions

Practical applications of fuzzy theory are found in many daily life problems. To extend the concept from theory to practical solutions, various MCGDM techniques have been developed and extended to increasingly general models in recent years. This research article focuses on a novel MCGDM technique that owes to the PROMETHEE approach. This methodology is generalized to the PFNS environment. The new technique enables accurate decision-making and ranking of alternatives while dealing with association and disassociation of attributes and assigning N-grading. A detailed step-by-step procedure is presented along with a comprehensive flowchart to facilitate understanding. To evaluate its effectiveness, a numerical example of selecting the best chemical in cloud seeding is provided and compared with an existing Pythagorean fuzzy PROMETHEE technique. Finally, the strengths and weaknesses of the proposed technique are discussed to highlight its advantages and limitations.

The PFNS PROMETHEE technique has very profound applications in future studies and development. It can be extensively used under multi-criteria study with a panel or group of decision making experts by considering the membership or non-membership degrees of fuzzy soft sets with N-grading. By analyzing the research on consensus process in multi-criteria group decision making [28, 29]. We can further extend our proposed technique in this consensus process to analyze its practicality. The proposed technique can be made extensively useful by applying it to an adaptive group decision making framework as in the work done by Dong et al. [27] and as an application of group decision making in the shipping industry as in the work done by Yang et al. [57].

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Conflict of interest

The authors declare no conflict of interest.

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